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Appendix A. The approach used to specify the observation error and system noise variance.

A : APPENDIX - Specifying the Observation Error and System Noise Variance

Determining the observation error variance, σ_o^2, and the system noise variance, σ_ε^2, is an important part of the state space model. Note that in principle σ_ε^2 and σ_o^2 could be estimated as part of the parameter estimation procedure using the state augmentation approach. However, it is often difficult to separately estimate these quantities using a state space model, as they may be confounded with one another or even with the movement parameters. It is therefore advisable to estimate any parameters off-line (where possible) to ensure optimal identifiability of the movement parameters. Such a procedure is outlined below for our application, and is used to specify the observation error variance in (2), as well as the magnitude of the system noise variance, which scales the normal mixture model (8).

The calculation of the sample auto-covariance (ACVF) function provides the basis for our estimation procedure. We assume the observation error is uncorrelated through time, and hence the observation error variance contributes to the ACVF only at zero lag, and has zero auto-covariance for all other lags. In contrast, the vertical velocity process, z_t, is correlated through time as dictated by the state evolution equation (5), and so has contributions to the ACVF at both zero and non-zero lags.

Based on the above assumptions, the theoretical ACVF, designated as $\gamma(k)$, for the measured vertical velocity, y_t, is given by

$$\gamma(k) = \sigma_o^2(0) + \sigma_z^2(k) \tag{A.1}$$

where k here denotes the lag. At zero lag the total variance, $\gamma(0)$, has contributions from observation error variance, $\sigma_o^2(0)$, and the process variance, $\sigma_z^2(0)$; at non-zero lags it has contributions only from $\sigma_z^2(k)$.

The sample ACVF, $\hat{\gamma}(k)$ of Figure 1, can therefore be used to separate σ_o^2 from σ_z^2 using (A.1). To do this, a quadratic was fit to $\hat{\gamma}(k)$ associated with small non-zero lags $k = 1, \ldots n$. We chose $n = 8$ to capture the quadratic trend near the origin. After fitting we then extrapolate to the origin, $k = 0$, to provide an estimate of the process variance at lag 0, or $\hat{\sigma}_z^2(0)$. Based on this, the estimated observation error variance is taken as $\hat{\sigma}_o^2 = \hat{\gamma}(0) - \hat{\sigma}_z^2(0)$. Note that this procedure is directly analogous to determining the nugget in kriging.

We next determine the system noise variance, σ_ε^2, using our estimate of the process noise variance, σ_z^2. If we assume that z_t follows an Markov process, or has an exponential correlation function, then the system noise variance can be estimated as (Priestley 2004, Section 3.5.3),

$$\hat{\sigma}_\varepsilon^2 = \left(1 - \left(\frac{\hat{\gamma}(1)}{\hat{\gamma}(0)}\right)^2\right) \hat{\sigma}_z^2(0). \tag{A.2}$$

This is an approximation and relies on the ratio of sample ACVF at lag 1 to lag 0 to describe the decay rate of the underlying autocovariance function, but acts as a variance adjustment due to the autocorrelation in our data, while making no assumptions about the movement process itself.

Both σ_o^2 and σ_ε^2 change over the dive record. The evolutionary sample ACVF (Figure 2a) was used with the above approach to compute the time evolution of the observation error variance and the system noise variance for each of our 110 time windows. These estimates are shown in Figure 3 i. The resultant values obtained for σ_ε^2 and σ_o^2 can be used directly as input to the state space model.