Problem: Accuracy

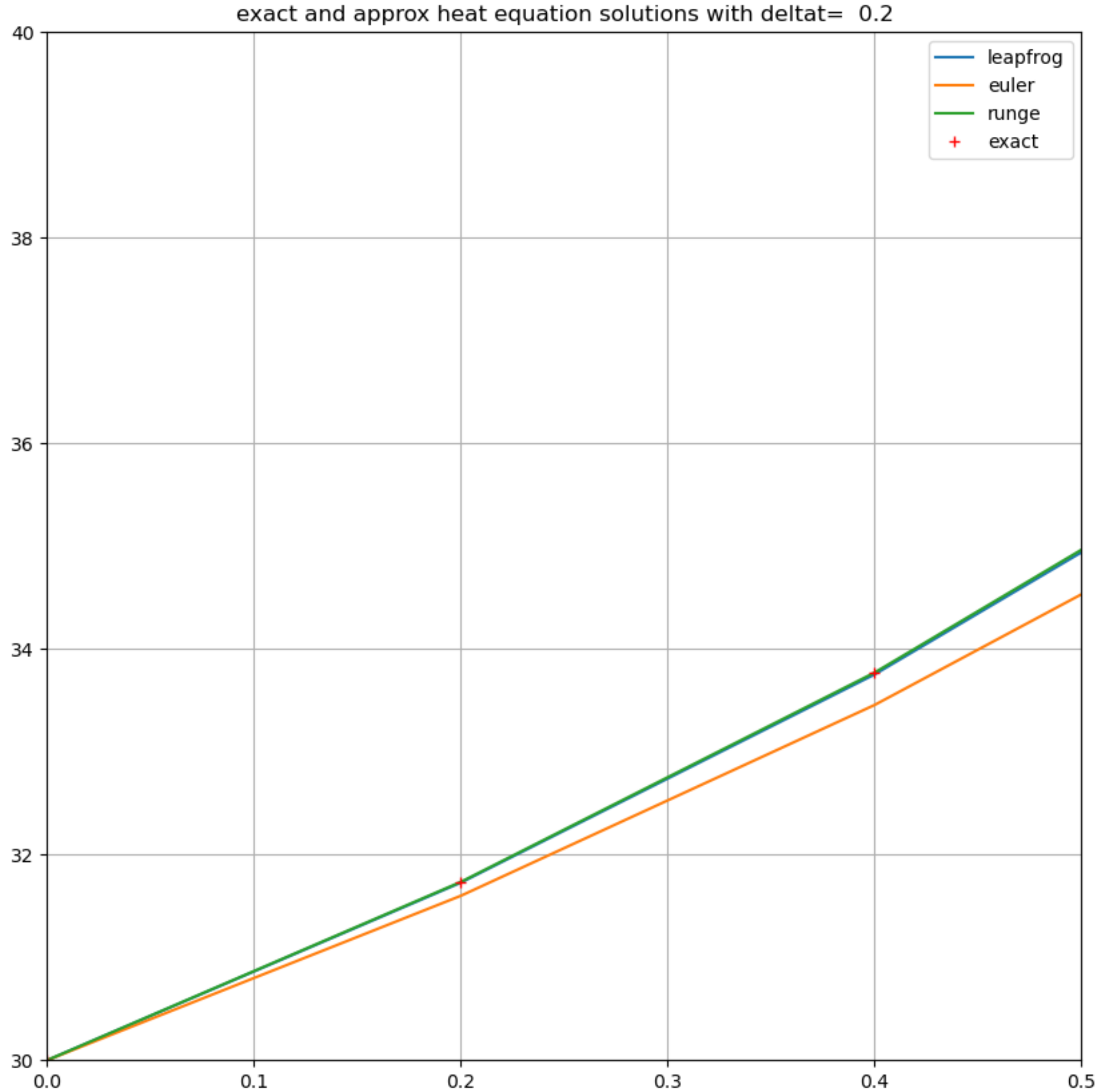
In [1]:
```python
# Import
import matplotlib.pyplot as plt
from lab2_functions import euler, leapfrog, runge, midpoint
import numpy as np
```

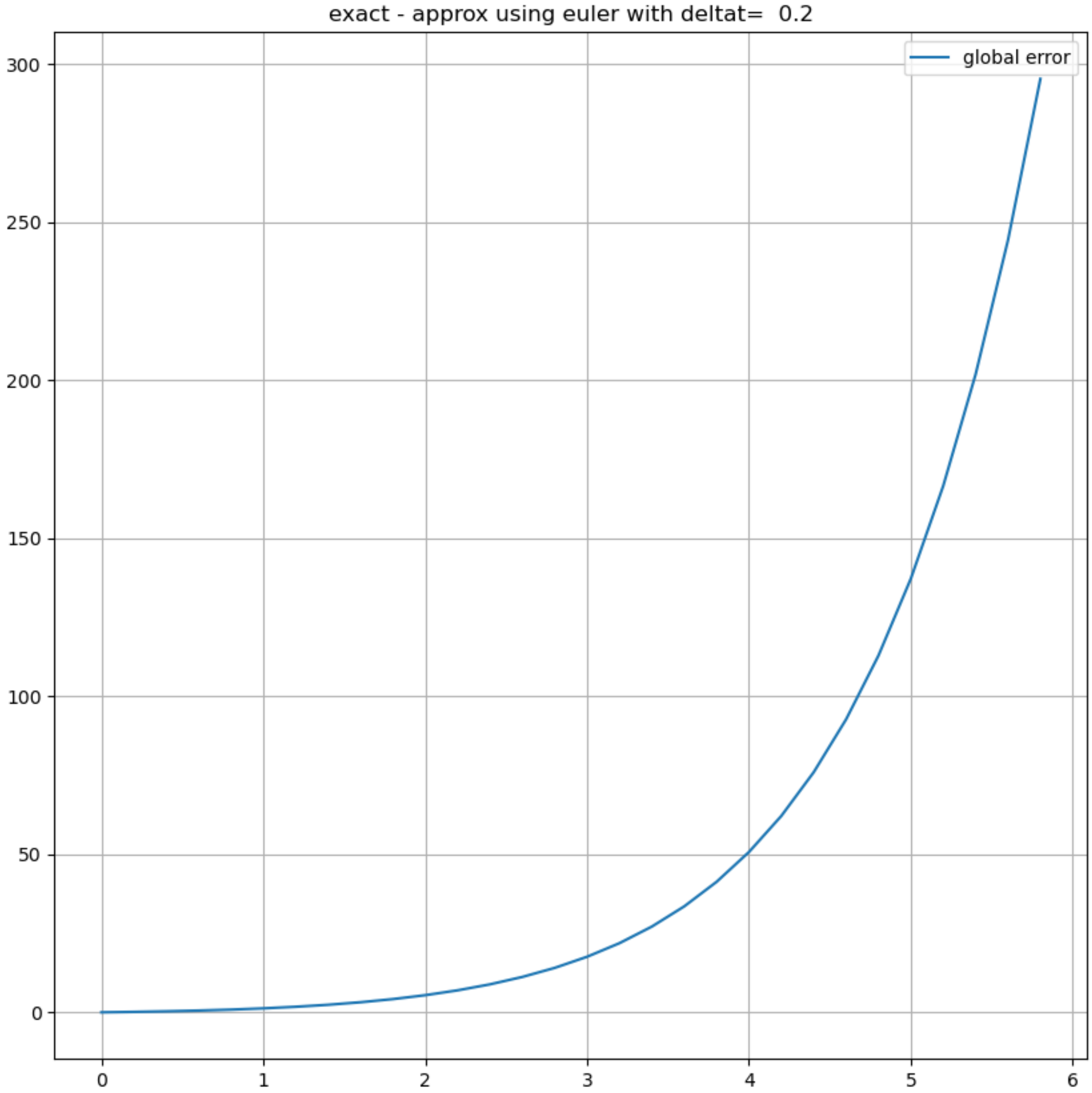
In [2]:
```python
# Definitions
theFuncs = {
    'euler': euler,
    'leapfrog': leapfrog,
    'runge': runge,
}


def eval_fun(fun_choice, arg_dict):
    """
    Parameters
    ----------
    fun_choice: str
       name of finite difference approx from lab2_functions
    arg_dict: dict
       dictionary of arguments for lab2_functions
    Returns
    -------
    approxTime, approxTemp: tuple
        tuple of ndarray float vectors with time and temperature
    """
    npts = arg_dict['npts']
    tend = arg_dict['tend']
    To = arg_dict['To']
    Ta = arg_dict['Ta']
    theLambda = arg_dict['theLambda']
    approxTime, approxTemp = theFuncs[fun_choice](npts, tend, To, Ta,
                                                   theLambda)
    return approxTime, approxTemp
```

In [3]:
```python
# %%
if __name__ == "__main__":

    Ta = 20
    To = 30
    # run for tend = 3,6,10
    tend = 6.0
    theLambda = 0.8
    npts = 30


    default_args = dict(Ta=Ta,
                        To=To,
                        tend=tend,
                        theLambda=theLambda,
                        npts=npts)
```

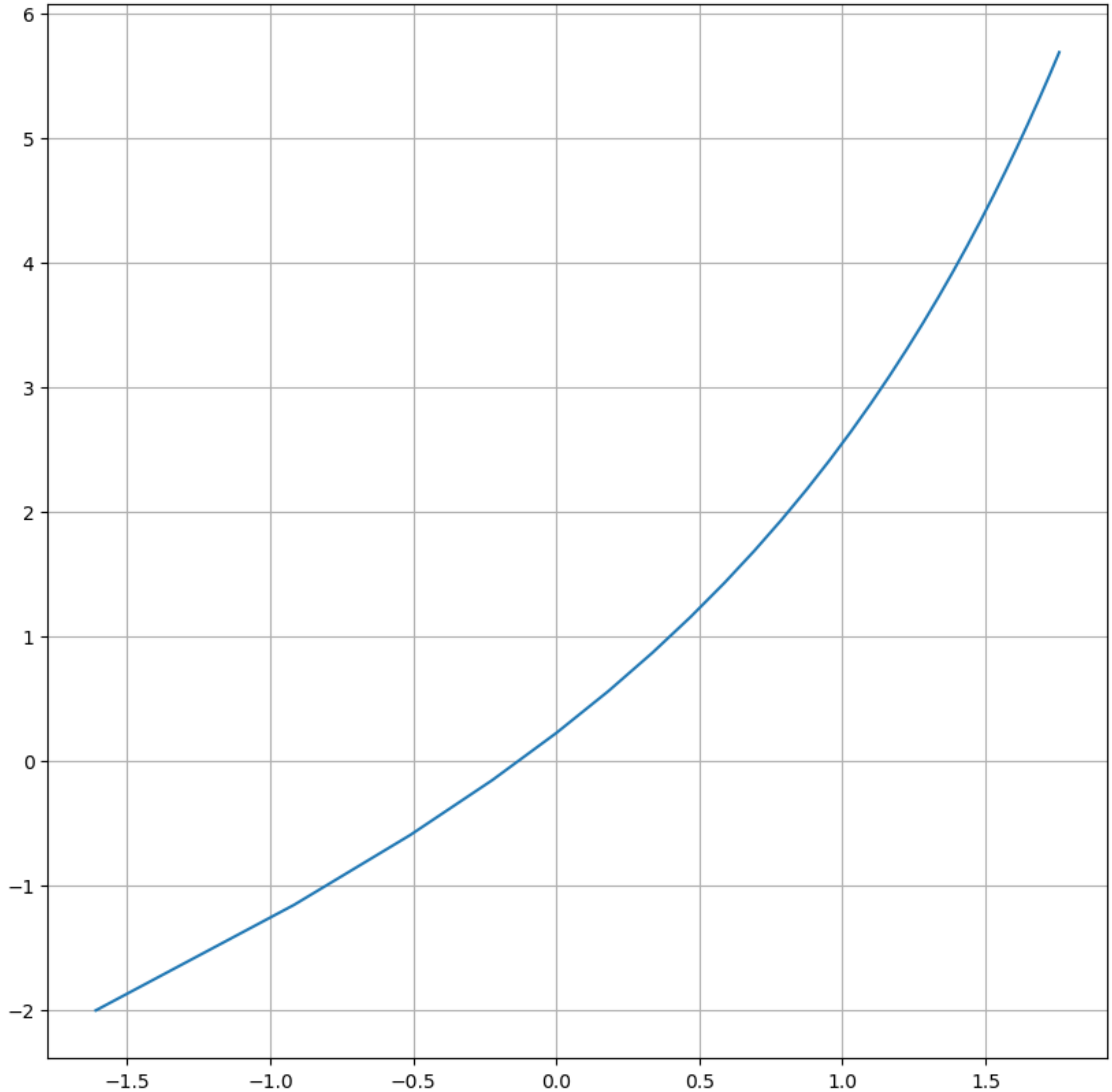
```
#
# start a plot to show all the functions
#
fig, ax1 = plt.subplots(1, 1, figsize=(10, 10))
plt.grid()
keep_curves = dict()
fun_list = ['leapfrog', 'euler', 'runge']
#
# add a curve for each function in fun_list
#
for fun_choice in fun_list:
    approxTime, approxTemp = eval_fun(fun_choice, default_args)
    ax1.plot(approxTime, approxTemp, label=fun_choice)
    keep_curves[fun_choice] = (approxTime,approxTemp)
#
# now add the exact solution for comparison
#
exactTime = np.empty_like(approxTime)
exactTemp = np.empty_like(exactTime)
for i in range(npts):
    exactTime[i] = tend * i / npts
    exactTemp[i] = Ta + (To - Ta) * np.exp(theLambda * exactTime[i])
ax1.plot(exactTime, exactTemp, 'r+', label='exact')
deltat=tend / npts
title = f"exact and approx heat equation solutions with deltat={deltat:5
ax1.set(title=title)
ax1.legend(loc='best')
ax1.set_xlim(0,0.5)
ax1.set_ylim(30, 40)
#
# g.w.added code here
#
locError = exactTemp[1] - approxTemp[1]
print('Local error at first time step = ', locError)
#
# Make a second plot that shows the difference between exact and euler
#
fig2, ax2 = plt.subplots(1, 1,figsize=(10,10))
plt.grid()
fun_choice = 'euler'
approxTemp = keep_curves[fun_choice][1]
difference = exactTemp - approxTemp
ax2.plot(exactTime, difference)
title = f"exact - approx using {fun_choice} with deltat={deltat:5.2g}"
ax2.set(title=title)
ax2.legend(['global error', 'local error'], loc='best')
plt.show()
#
# g.w.
#
fig3, ax3 = plt.subplots(1,1, figsize=(10,10))
plt.grid()
fun_choice = 'euler'
#fit = np.polyfit(np.log(exactTime), np.log(difference), 1)
ax3.plot(np.log(exactTime), np.log(difference))
#ax3.plot(np.log(exactTime), fit)
```

```
Local error at first time step =  8.9765847661738e-06
```






```
/tmp/ipykernel_1181031/3971561551.py:71: RuntimeWarning: divide by zero enco
untered in log
  ax3.plot(np.log(exactTime), np.log(difference))
```



a) Increasing the order of the solution and decreasing the time step always improved the accuracy of the solution for the three methods and time steps I chose. b) The original difference plot is the global error of the euler method to the exact. To calculate the local error, you have to calculate the difference between the euler approx. at step t and the exact from the previous step t_1, not the exact from t_0. c) see below for how to do part c

In [ ]:

```

```

Problem: Backwards Euler

In [4]:

```
# Import
import matplotlib.pyplot as plt
from lab2_functions import euler,beuler,leapfrog
import numpy as np
```

In [5]:

```
# Define
theFuncs={'euler':euler,'beuler':beuler,'leapfrog':leapfrog}
```

In [6]:

```
# Plot
if __name__=="__main__":
```

```
tend=10.
Ta=20.
To=30.
theLambda=-8.
funChoice='beuler'
npts=40
approxTime,approxTemp=theFuncs[funChoice](npts,tend,To,Ta,theLambda)
exactTime=np.empty([npts,],float)
exactTemp=np.empty_like(exactTime)
for i in np.arange(0,npts):
   exactTime[i] = tend*i/npts
   exactTemp[i] = Ta + (To-Ta)*np.exp(theLambda*exactTime[i])
plt.close('all')
plt.figure(1)
plt.clf()
plt.plot(exactTime,exactTemp,'r+')
plt.plot(approxTime,approxTemp)
theAx=plt.gca()
theAx.set_xlim([0,10])
theAx.set_ylim([15,30])
theAx.set_title('stability')
plt.show()
```