

SpatialSlope

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Contents

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In [1]: using GaussianProcesses
        using GaussianProcesses: cov
        using Distributions
        import PyPlot; plt=PyPlot
        ;

In [2]: import PyPlot; plt=PyPlot
        using LaTeXStrings
        plt.rc("figure", dpi=300.0)
        plt.rc("figure", figsize=(12,8))
        plt.rc("savefig", dpi=300.0)
        plt.rc("text", usetex=true)
        plt.rc("font", family="serif")
        plt.rc("font", serif="Palatino")
        ;

In [3]: thresh = 0.0

        # data-generating parameters
        _σf_star = 1.0
        _Lstar  = 1.05
        _τstar  = 0.7
        _σy_star = 0.3

        _στ2 = 100.0 # diffuse normal prior on τ
        se = SEIso(log(_Lstar), log(√_σf_star))
        n_xx = 1000
        n_obs = 200
        xx = collect(linspace(-10.0,10.0,n_xx))
        _X = randn(n_obs)*2.0
        # create a gap
        _X[_X.>0.0] .+= 0.5
        _X[_X.<0.0] .-= 0.5
        gp_star = GP(;m=MeanZero(), k=se, logNoise=1e-8)

        _f = rand(gp_star, vcat(xx, _X)')
        _f_xx = _f[1:n_xx]
        _f_obs = _f[n_xx+1:end]

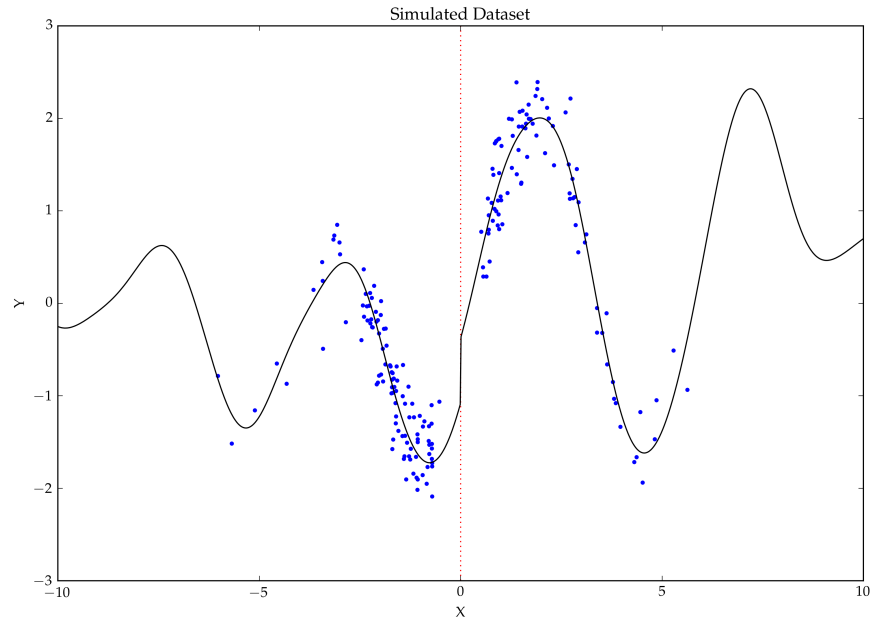
        noise = rand(Normal(0,_σy_star), n_obs)
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_Y = _f_obs .+ noise .+  $\tau$ star * (_X.>thresh)
;

In [4]: plt.svg(false)
plt.plot(_X, _Y, ".")
plt.axvline(x=0.0, color="red", linestyle=":")
plt.plot(xx, _f_xx+ $\tau$ star*(xx.>thresh), color="black", label="truth")
plt.title("Simulated Dataset")
plt.xlabel("X")
plt.ylabel("Y")
;

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In [5]: k_spatialslope = LinIso(0.0) * se + se

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Out[5]: Type: GaussianProcesses.SumKernel
        Type: GaussianProcesses.ProdKernel
        Type: GaussianProcesses.LinIso, Params: [0.0]
        Type: GaussianProcesses.SEIso, Params: [0.0487902,0.0]
        Type: GaussianProcesses.SEIso, Params: [0.0487902,0.0]

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In [6]: left = _X .< 0.0

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lin_fit_left  = GP(_X[left]', _Y[left], MeanZero(), k_spatialslope, log( $\sigma$ y_star))
lin_fit_right = GP(_X[!left]', _Y[!left], MeanZero(), k_spatialslope, log( $\sigma$ y_star))

se_fit_left   = GP(_X[left]', _Y[left], MeanZero(), se, log( $\sigma$ y_star))
se_fit_right  = GP(_X[!left]', _Y[!left], MeanZero(), se, log( $\sigma$ y_star))
;

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In [17]: begin
          xx_left = xx .< 0.0

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xx_right = xx .> 0.0
fit_right = lin_fit_right
fit_left = lin_fit_left

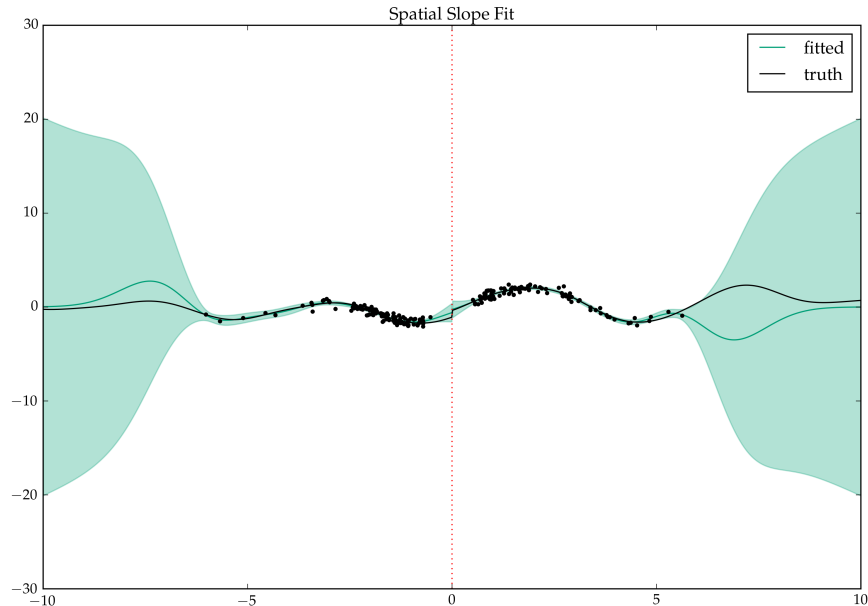
μ_left, Σ_left = predict(fit_left, xx[xx_left]')
fit_color = "#009F77"
plt.plot( xx[xx_left], μ_left, color=fit_color, label="fitted")
lower = μ_left-2.0*√Σ_left
upper = μ_left+2.0*√Σ_left
plt.fill_between(xx[xx_left], lower, upper, color=fit_color, alpha=0.3)

μ_right, Σ_right = predict(fit_right, xx[xx_right]')
fit_color = "#009F77"
plt.plot( xx[xx_right], μ_right, color=fit_color)
lower = μ_right-2.0*√Σ_right
upper = μ_right+2.0*√Σ_right
plt.fill_between(xx[xx_right], lower, upper, color=fit_color, alpha=0.3)

plt.plot(xx, _f_xx+_τstar*(xx.>thresh), color="black", label="truth")

plt.plot(_X, _Y, ".", color="black")
plt.axvline(x=0.0, color="red", linestyle=":")
plt.legend(loc="bottom left")
end
plt.title("Spatial Slope Fit")
;

```



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In [18]: begin
xx_left = xx .< 0.0
xx_right = xx .> 0.0
fit_right = se_fit_right

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fit_left = se_fit_left

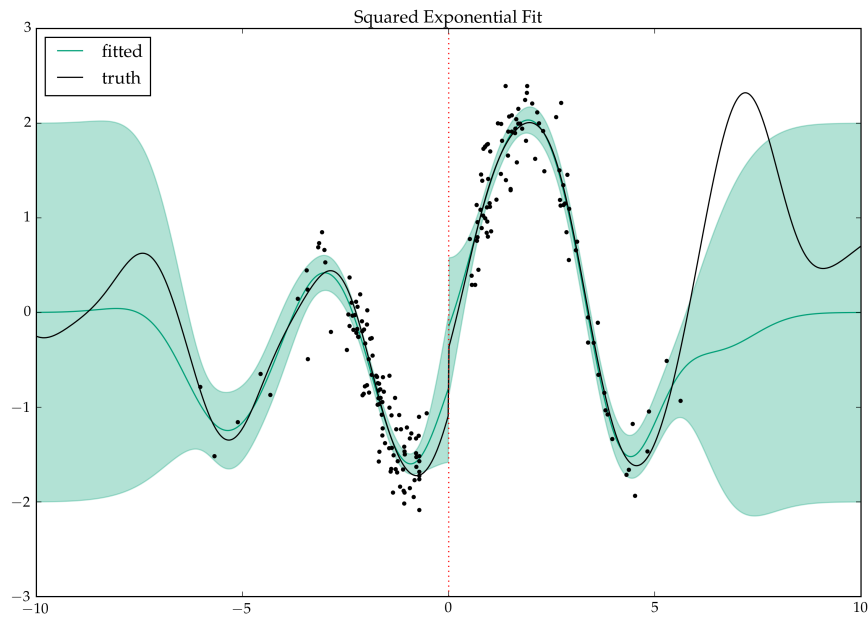
 $\mu_{\text{left}}$ ,  $\Sigma_{\text{left}}$  = predict(fit_left, xx[xx_left]')
fit_color = "#009F77"
plt.plot( xx[xx_left],  $\mu_{\text{left}}$ , color=fit_color, label="fitted")
lower =  $\mu_{\text{left}} - 2.0 * \sqrt{\Sigma_{\text{left}}}$ 
upper =  $\mu_{\text{left}} + 2.0 * \sqrt{\Sigma_{\text{left}}}$ 
plt.fill_between(xx[xx_left], lower, upper, color=fit_color, alpha=0.3)

 $\mu_{\text{right}}$ ,  $\Sigma_{\text{right}}$  = predict(fit_right, xx[xx_right]')
fit_color = "#009F77"
plt.plot( xx[xx_right],  $\mu_{\text{right}}$ , color=fit_color)
lower =  $\mu_{\text{right}} - 2.0 * \sqrt{\Sigma_{\text{right}}}$ 
upper =  $\mu_{\text{right}} + 2.0 * \sqrt{\Sigma_{\text{right}}}$ 
plt.fill_between(xx[xx_right], lower, upper, color=fit_color, alpha=0.3)

plt.plot(xx, _f_xx+_ $\tau$ star*(xx.>thresh), color="black", label="truth")

plt.plot(_X, _Y, ".", color="black")
plt.axvline(x=0.0, color="red", linestyle=":")
plt.legend(loc="bottom left")
end
plt.title("Squared Exponential Fit")
;

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In [19]: begin
xx_left = xx .< 0.0
xx_right = xx .> 0.0
fit_right = lin_fit_right
fit_left = lin_fit_left

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 $\mu_{\text{left}}, \Sigma_{\text{left}} = \text{predict}(\text{fit\_left}, \text{xx}[\text{xx\_left}])$ 
fit_color = "#009F77"
plt.plot( xx[xx_left],  $\mu_{\text{left}}$ , color=fit_color, label="fitted")
lower =  $\mu_{\text{left}} - 2.0 * \sqrt{\Sigma_{\text{left}}}$ 
upper =  $\mu_{\text{left}} + 2.0 * \sqrt{\Sigma_{\text{left}}}$ 
plt.fill_between(xx[xx_left], lower, upper, color=fit_color, alpha=0.3)

 $\mu_{\text{right}}, \Sigma_{\text{right}} = \text{predict}(\text{fit\_right}, \text{xx}[\text{xx\_right}])$ 
fit_color = "#009F77"
plt.plot( xx[xx_right],  $\mu_{\text{right}}$ , color=fit_color)
lower =  $\mu_{\text{right}} - 2.0 * \sqrt{\Sigma_{\text{right}}}$ 
upper =  $\mu_{\text{right}} + 2.0 * \sqrt{\Sigma_{\text{right}}}$ 
plt.fill_between(xx[xx_right], lower, upper, color=fit_color, alpha=0.3)

plt.plot(xx,  $_{\text{f\_xx}} + \tau_{\text{star}} * (\text{xx} > \text{thresh})$ , color="black", label="truth")

plt.plot(_X, _Y, ".", color="black")
plt.axvline(x=0.0, color="red", linestyle=":")
plt.legend(loc="bottom left")

end
plt.title("Spatial Slope Fit Zoomed in")
plt.ylim(-3.0, 3.0)
;

```

