Dugong – HW 9

Garrett Carr

Due March 29, 2022

These data concern growth of dugongs (sometimes called sea cows, an aquatic mammal found primarily in the Indo-West Pacific). The data file is called 'dugong.dat' and contains two columns, the age of the animal in years, and the length of the animal in meters for 27 individuals. If you plot the data, you will see that growth is faster for young animals and then seems to stop in older animals. This type of growth is called nonlinear growth, and the simplest curve to describe such growth is:

$$y_i = a - bg^{x_i}, (1)$$

where y_i represents the length of the animal and x_i represents the age of the animal. As you can see, there are three parameters to estimate, a, b, and g. 'a' represents the asymptote or value at which growth stops, 'b' is constrained to be positive, and 'g' is constrained to be between 0 and 1.

You have two tasks:

- 1. Determine parameter estimates for the growth curve.
- 2. Compare the results from JAGS to those from Stan.

After making sure chains have converged appropriately, you will compare the results by examining both equal tail and HPD interval estimates of the parameters (and functions thereof) that matter.

You should hand in a no more than a one page summary of your results that should include at least one plot. Make sure to address all appropriate model diagnostics and convergence, but note that you might not be able to fit code output in your one page summary. All code should come in as an appendix to your one page summary.

Summary

I first ran the model in JAGS, and obtained the following results:

```
## Compiling model graph
##
      Resolving undeclared variables
##
      Allocating nodes
  Graph information:
##
##
      Observed stochastic nodes: 27
      Unobserved stochastic nodes: 4
##
##
      Total graph size: 128
##
## Initializing model
## Inference for Bugs model at "model1.txt", fit using jags,
  4 chains, each with 50000 iterations (first 2000 discarded)
```

```
n.sims = 192000 iterations saved
##
            mu.vect sd.vect
                                         25%
                                                                97.5% Rhat n.eff
                                2.5%
                                                  50%
                                                          75%
                                       2.606
## a
              2.658
                       0.077
                               2.529
                                                2.651
                                                        2.701
                                                                 2.831 1.001 36000
## b
              0.982
                       0.080
                               0.829
                                       0.929
                                                0.980
                                                        1.032
                                                                 1.146 1.001 31000
## g
              0.863
                       0.034
                               0.786
                                       0.845
                                                0.867
                                                        0.886
                                                                 0.919 1.001 11000
                      0.004
                               0.006
                                       0.008
                                                                 0.020 1.001 75000
              0.011
                                                0.010
                                                        0.013
## s2error
## deviance -48.609
                      3.474 -52.935 -51.161 -49.405 -46.897 -39.792 1.001 30000
## For each parameter, n.eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor (at convergence, Rhat=1).
## DIC info (using the rule, pD = var(deviance)/2)
## pD = 6.0 and DIC = -42.6
## DIC is an estimate of expected predictive error (lower deviance is better).
##
                    lower
                                  upper
## a
              2.518030089
                             2.81303670
## b
              0.823503674
                             1.13881610
## deviance -53.417501091 -41.79498584
              0.795065576
## g
                             0.92426316
              0.005212471
                             0.01835893
## s2error
## attr(,"Probability")
## [1] 0.95
       2.5%
##
               97.5%
## 2.529287 2.831464
##
        2.5%
                 97.5%
## 0.8294786 1.1459334
##
        2.5%
                 97.5%
## 0.7864701 0.9186779
##
          2.5%
                      97.5%
## 0.005984892 0.020167184
```

It seems like there is a high level of autocorrelation for the JAGS output, so using a different sampler would help solve the issue.

```
## Running MCMC with 4 parallel chains...
##
## Chain 4 Iteration:
                           1 / 14000 [ 0%]
                                               (Warmup)
## Chain 4 Iteration:
                         100 / 14000 [
                                         0%]
                                               (Warmup)
## Chain 3 Iteration:
                           1 / 14000 [
                                         0%]
                                               (Warmup)
## Chain 3 Iteration:
                         100 / 14000 [
                                         0%]
                                               (Warmup)
## Chain 3 Iteration:
                         200 / 14000 [
                                               (Warmup)
                                         1%]
## Chain 3 Iteration:
                         300 / 14000 [
                                         2%]
                                               (Warmup)
## Chain 3 Iteration:
                         400 / 14000 [
                                         2%]
                                               (Warmup)
## Chain 3 Iteration:
                         500 / 14000 [
                                         3%]
                                               (Warmup)
## Chain 3 Iteration:
                         600 / 14000 [
                                         4%]
                                               (Warmup)
## Chain 3 Iteration:
                         700 / 14000 [
                                         5%]
                                               (Warmup)
## Chain 3 Iteration:
                         800 / 14000 [
                                         5%]
                                              (Warmup)
```

```
## Chain 3 Iteration:
                          900 / 14000 [
                                          6%]
                                                (Warmup)
## Chain 3 Iteration:
                         1000 / 14000 [
                                          7%]
                                                (Warmup)
   Chain 4 Iteration:
                          200 / 14000
                                      Ε
                                          1%]
                                                (Warmup)
                                          2%]
## Chain 4 Iteration:
                          300 / 14000
                                                (Warmup)
                                      Ε
   Chain 4 Iteration:
                          400 / 14000
                                      2%]
                                                (Warmup)
   Chain 4 Iteration:
                                      Ε
                                          3%]
                                                (Warmup)
                          500 / 14000
   Chain 4 Iteration:
                                          4%1
                                                (Warmup)
                          600 / 14000
## Chain 4 Iteration:
                          700 / 14000
                                          5%]
                                                (Warmup)
                                       Γ
   Chain 4 Iteration:
                          800 / 14000
                                      Γ
                                          5%]
                                                (Warmup)
                                      [
                                          6%]
   Chain 4 Iteration:
                          900 / 14000
                                                (Warmup)
   Chain 4 Iteration:
                         1000 / 14000
                                       7%]
                                                (Warmup)
                                          7%]
   Chain 4 Iteration:
                                       1100 / 14000
                                                (Warmup)
   Chain 4 Iteration:
                         1200 / 14000
                                      8%]
                                                (Warmup)
                                      Chain 4 Iteration:
                         1300 / 14000
                                          9%]
                                                (Warmup)
   Chain 4 Iteration:
                         1400 / 14000 [ 10%]
                                                (Warmup)
   Chain 4 Iteration:
                         1500 / 14000
                                      [
                                        10%]
                                                (Warmup)
                                          0%]
   Chain 1 Iteration:
                            1 / 14000 [
                                                (Warmup)
   Chain 1 Iteration:
                          100 / 14000
                                          0%]
                                                (Warmup)
                          200 / 14000
   Chain 1 Iteration:
                                       1%]
                                                (Warmup)
  Chain 1 Iteration:
                          300 / 14000
                                       Γ
                                          2%]
                                                (Warmup)
   Chain 1 Iteration:
                          400 / 14000
                                      Γ
                                          2%]
                                                (Warmup)
   Chain 1 Iteration:
                          500 / 14000 [
                                          3%]
                                                (Warmup)
                          600 / 14000
                                          4%]
## Chain 1 Iteration:
                                                (Warmup)
                                      Ε
   Chain 1 Iteration:
                                      Γ
                                          5%]
                                                (Warmup)
                          700 / 14000
                          800 / 14000
                                      Ε
                                          5%]
                                                (Warmup)
   Chain 1 Iteration:
   Chain 1 Iteration:
                          900 / 14000
                                       Γ
                                          6%1
                                                (Warmup)
   Chain 1 Iteration:
                         1000 / 14000
                                       7%]
                                                (Warmup)
   Chain 1 Iteration:
                         1100 / 14000
                                          7%]
                                                (Warmup)
                                      [
   Chain 1 Iteration:
                         1200 / 14000
                                          8%]
                                                (Warmup)
   Chain 1 Iteration:
                         1300 / 14000
                                      9%]
                                                (Warmup)
                         1400 / 14000
   Chain 1 Iteration:
                                      [ 10%]
                                                (Warmup)
   Chain 1 Iteration:
                         1500 / 14000 [ 10%]
                                                (Warmup)
   Chain 1 Iteration:
                         1600 / 14000 [ 11%]
                                                (Warmup)
                        1700 / 14000 [ 12%]
   Chain 1 Iteration:
                                                (Warmup)
   Chain 1 Iteration:
                         1800 / 14000 [ 12%]
                                                (Warmup)
                         1900 / 14000 [ 13%]
   Chain 1 Iteration:
                                                (Warmup)
   Chain 1 Iteration:
                         2000 / 14000 [ 14%]
                                                (Warmup)
  Chain 1 Iteration:
                        2001 / 14000 [ 14%]
                                                (Sampling)
  Chain 2 Iteration:
                            1 / 14000 [
                                          0%]
                                                (Warmup)
                          100 / 14000 [
                                          0%]
                                                (Warmup)
   Chain 2 Iteration:
   Chain 2 Iteration:
                                          1%]
                          200 / 14000
                                      Ε
                                                (Warmup)
  Chain 2 Iteration:
                          300 / 14000
                                       Γ
                                          2%]
                                                (Warmup)
   Chain 2 Iteration:
                          400 / 14000
                                          2%]
                                                (Warmup)
                                      Γ
                                          3%]
   Chain 2 Iteration:
                                                (Warmup)
                          500 / 14000
                                          4%]
   Chain 2 Iteration:
                          600 / 14000
                                                (Warmup)
                          700 / 14000
                                       5%]
   Chain 2 Iteration:
                                                (Warmup)
   Chain 2 Iteration:
                          800 / 14000 [
                                          5%]
                                                (Warmup)
                                      6%]
                                                (Warmup)
   Chain 2 Iteration:
                          900 / 14000
                         1000 / 14000
   Chain 2 Iteration:
                                       Γ
                                          7%]
                                                (Warmup)
                                          7%]
   Chain 2 Iteration:
                         1100 / 14000
                                       Γ
                                                (Warmup)
                                          8%]
   Chain 2 Iteration:
                         1200 / 14000
                                       (Warmup)
   Chain 2 Iteration:
                         1300 / 14000
                                      [
                                          9%]
                                                (Warmup)
## Chain 2 Iteration:
                         1400 / 14000 [ 10%]
                                                (Warmup)
## Chain 2 Iteration:
                        1500 / 14000 [ 10%]
                                                (Warmup)
```

```
## Chain 2 Iteration:
                        1600 / 14000 [ 11%]
                                               (Warmup)
  Chain 2 Iteration:
                        1700 / 14000 [ 12%]
                                               (Warmup)
                        1800 / 14000 [ 12%]
                                               (Warmup)
   Chain 2 Iteration:
  Chain 2 Iteration:
                        1900 / 14000 [ 13%]
                                               (Warmup)
   Chain 2 Iteration:
                        2000 / 14000 [ 14%]
                                               (Warmup)
   Chain 2 Iteration:
                        2001 / 14000 [ 14%]
                                               (Sampling)
                        1100 / 14000 [
                                               (Warmup)
   Chain 3 Iteration:
                                         7%1
                        1200 / 14000
                                      Ε
  Chain 3 Iteration:
                                         8%]
                                               (Warmup)
                        1300 / 14000 [
   Chain 3 Iteration:
                                         9%]
                                               (Warmup)
                        1400 / 14000 [ 10%]
   Chain 3 Iteration:
                                               (Warmup)
   Chain 3 Iteration:
                        1500 / 14000 [ 10%]
                                               (Warmup)
   Chain 3 Iteration:
                        1600 / 14000 [ 11%]
                                               (Warmup)
   Chain 3 Iteration:
                        1700 / 14000 [ 12%]
                                               (Warmup)
   Chain 3 Iteration:
                        1800 / 14000 [ 12%]
                                               (Warmup)
   Chain 3 Iteration:
                        1900 / 14000 [ 13%]
                                               (Warmup)
   Chain 3 Iteration:
                        2000 / 14000 [ 14%]
                                               (Warmup)
   Chain 3 Iteration:
                        2001 / 14000 [ 14%]
                                               (Sampling)
   Chain 4 Iteration:
                        1600 / 14000 [ 11%]
                                               (Warmup)
   Chain 4 Iteration:
                        1700 / 14000 [ 12%]
                                               (Warmup)
   Chain 4 Iteration:
                        1800 / 14000 [ 12%]
                                               (Warmup)
   Chain 4 Iteration:
                        1900 / 14000 [ 13%]
                                               (Warmup)
   Chain 4 Iteration:
                        2000 / 14000 [ 14%]
                                               (Warmup)
                        2001 / 14000 [ 14%]
  Chain 4 Iteration:
                                               (Sampling)
   Chain 4 Iteration:
                        2100 / 14000 [ 15%]
                                               (Sampling)
   Chain 4 Iteration:
                        2200 / 14000 [ 15%]
                                               (Sampling)
   Chain 1 Iteration:
                        2100 / 14000 [ 15%]
                                               (Sampling)
   Chain 1 Iteration:
                        2200 / 14000 [ 15%]
                                               (Sampling)
   Chain 1 Iteration:
                        2300 / 14000 [ 16%]
                                               (Sampling)
   Chain 1 Iteration:
                                               (Sampling)
                        2400 / 14000 [ 17%]
   Chain 1 Iteration:
                        2500 / 14000 [ 17%]
                                               (Sampling)
   Chain 1 Iteration:
                        2600 / 14000 [ 18%]
                                               (Sampling)
   Chain 1 Iteration:
                        2700 / 14000 [ 19%]
                                               (Sampling)
   Chain 1 Iteration:
                        2800 / 14000 [ 20%]
                                               (Sampling)
   Chain 2 Iteration:
                        2100 / 14000 [ 15%]
                                               (Sampling)
   Chain 2 Iteration:
                        2200 / 14000 [ 15%]
                                               (Sampling)
   Chain 2 Iteration:
                                               (Sampling)
                        2300 / 14000 [ 16%]
   Chain 2 Iteration:
                        2400 / 14000 [ 17%]
                                               (Sampling)
  Chain 2 Iteration:
                        2500 / 14000 [ 17%]
                                               (Sampling)
   Chain 2 Iteration:
                        2600 / 14000 [ 18%]
                                               (Sampling)
   Chain 2 Iteration:
                        2700 / 14000 [ 19%]
                                               (Sampling)
                                               (Sampling)
   Chain 2 Iteration:
                        2800 / 14000 [ 20%]
   Chain 2 Iteration:
                        2900 / 14000 [ 20%]
                                               (Sampling)
   Chain 3 Iteration:
                        2100 / 14000 [ 15%]
                                               (Sampling)
   Chain 3 Iteration:
                        2200 / 14000 [ 15%]
                                               (Sampling)
   Chain 3 Iteration:
                        2300 / 14000 [ 16%]
                                               (Sampling)
                        2400 / 14000 [ 17%]
   Chain 3 Iteration:
                                               (Sampling)
   Chain 3 Iteration:
                        2500 / 14000 [ 17%]
                                               (Sampling)
   Chain 3 Iteration:
                        2600 / 14000 [ 18%]
                                               (Sampling)
   Chain 4 Iteration:
                        2300 / 14000 [ 16%]
                                               (Sampling)
   Chain 4 Iteration:
                        2400 / 14000 [ 17%]
                                               (Sampling)
   Chain 4 Iteration:
                                               (Sampling)
                        2500 / 14000 [ 17%]
   Chain 4 Iteration:
                        2600 / 14000 [ 18%]
                                               (Sampling)
## Chain 4 Iteration:
                        2700 / 14000 [ 19%]
                                               (Sampling)
## Chain 4 Iteration:
                        2800 / 14000 [ 20%]
                                               (Sampling)
```

```
## Chain 4 Iteration:
                        2900 / 14000 [ 20%]
                                               (Sampling)
  Chain 4 Iteration:
                        3000 / 14000 [ 21%]
                                               (Sampling)
                                               (Sampling)
   Chain 4 Iteration:
                        3100 / 14000 [ 22%]
  Chain 4 Iteration:
                        3200 / 14000 [ 22%]
                                               (Sampling)
   Chain 4 Iteration:
                        3300 / 14000 [ 23%]
                                               (Sampling)
   Chain 1 Iteration:
                                               (Sampling)
                        2900 / 14000 [ 20%]
                                               (Sampling)
   Chain 1 Iteration:
                        3000 / 14000 [ 21%]
                        3100 / 14000 [ 22%]
  Chain 1 Iteration:
                                               (Sampling)
   Chain 1 Iteration:
                        3200 / 14000 [ 22%]
                                               (Sampling)
   Chain 1 Iteration:
                        3300 / 14000 [ 23%]
                                               (Sampling)
   Chain 2 Iteration:
                        3000 / 14000 [ 21%]
                                               (Sampling)
   Chain 2 Iteration:
                        3100 / 14000 [ 22%]
                                               (Sampling)
   Chain 2 Iteration:
                        3200 / 14000 [ 22%]
                                               (Sampling)
   Chain 3 Iteration:
                        2700 / 14000 [ 19%]
                                               (Sampling)
   Chain 3 Iteration:
                        2800 / 14000 [ 20%]
                                               (Sampling)
   Chain 3 Iteration:
                        2900 / 14000 [ 20%]
                                               (Sampling)
   Chain 3 Iteration:
                        3000 / 14000 [ 21%]
                                               (Sampling)
   Chain 4 Iteration:
                        3400 / 14000 [ 24%]
                                               (Sampling)
   Chain 4 Iteration:
                        3500 / 14000 [ 25%]
                                               (Sampling)
  Chain 4 Iteration:
                        3600 / 14000 [ 25%]
                                               (Sampling)
   Chain 4 Iteration:
                        3700 / 14000 [ 26%]
                                               (Sampling)
   Chain 1 Iteration:
                        3400 / 14000 [ 24%]
                                               (Sampling)
                        3500 / 14000 [ 25%]
  Chain 1 Iteration:
                                               (Sampling)
   Chain 2 Iteration:
                        3300 / 14000 [ 23%]
                                               (Sampling)
                        3400 / 14000 [ 24%]
   Chain 2 Iteration:
                                               (Sampling)
   Chain 2 Iteration:
                        3500 / 14000 [ 25%]
                                               (Sampling)
   Chain 3 Iteration:
                        3100 / 14000 [
                                        22%]
                                               (Sampling)
   Chain 3 Iteration:
                        3200 / 14000 [ 22%]
                                               (Sampling)
   Chain 3 Iteration:
                        3300 / 14000 [ 23%]
                                               (Sampling)
   Chain 4 Iteration:
                        3800 / 14000 [ 27%]
                                               (Sampling)
   Chain 4 Iteration:
                        3900 / 14000 [ 27%]
                                               (Sampling)
   Chain 4 Iteration:
                        4000 / 14000 [ 28%]
                                               (Sampling)
                        3600 / 14000 [ 25%]
   Chain 1 Iteration:
                                               (Sampling)
                        3700 / 14000 [ 26%]
                                               (Sampling)
   Chain 1 Iteration:
   Chain 1 Iteration:
                        3800 / 14000 [ 27%]
                                               (Sampling)
   Chain 2 Iteration:
                                               (Sampling)
                        3600 / 14000 [ 25%]
   Chain 3 Iteration:
                        3400 / 14000 [ 24%]
                                               (Sampling)
  Chain 4 Iteration:
                        4100 / 14000 [ 29%]
                                               (Sampling)
   Chain 4 Iteration:
                        4200 / 14000 [ 30%]
                                               (Sampling)
                        4300 / 14000 [ 30%]
                                               (Sampling)
   Chain 4 Iteration:
                                               (Sampling)
   Chain 1 Iteration:
                        3900 / 14000 [ 27%]
   Chain 1 Iteration:
                        4000 / 14000 [ 28%]
                                               (Sampling)
                                               (Sampling)
   Chain 1 Iteration:
                        4100 / 14000 [ 29%]
   Chain 1 Iteration:
                        4200 / 14000 [ 30%]
                                               (Sampling)
   Chain 2 Iteration:
                        3700 / 14000 [ 26%]
                                               (Sampling)
                        3800 / 14000 [ 27%]
   Chain 2 Iteration:
                                               (Sampling)
   Chain 2 Iteration:
                        3900 / 14000 [ 27%]
                                               (Sampling)
                        4000 / 14000 [ 28%]
   Chain 2 Iteration:
                                               (Sampling)
   Chain 3 Iteration:
                        3500 / 14000 [ 25%]
                                               (Sampling)
   Chain 3 Iteration:
                        3600 / 14000
                                      25%]
                                               (Sampling)
   Chain 3 Iteration:
                                               (Sampling)
                        3700 / 14000 [ 26%]
                                               (Sampling)
   Chain 4 Iteration:
                        4400 / 14000 [ 31%]
## Chain 4 Iteration:
                        4500 / 14000 [ 32%]
                                               (Sampling)
## Chain 4 Iteration:
                        4600 / 14000 [ 32%]
                                               (Sampling)
```

```
## Chain 4 Iteration:
                        4700 / 14000 [ 33%]
                                               (Sampling)
  Chain 1 Iteration:
                        4300 / 14000 [ 30%]
                                               (Sampling)
   Chain 1 Iteration:
                                               (Sampling)
                        4400 / 14000 [ 31%]
  Chain 1 Iteration:
                        4500 / 14000 [ 32%]
                                               (Sampling)
   Chain 2 Iteration:
                        4100 / 14000 [ 29%]
                                               (Sampling)
   Chain 2 Iteration:
                        4200 / 14000 [ 30%]
                                               (Sampling)
                                               (Sampling)
   Chain 2 Iteration:
                        4300 / 14000 [ 30%]
                        3800 / 14000 [ 27%]
  Chain 3 Iteration:
                                               (Sampling)
   Chain 3 Iteration:
                        3900 / 14000 [ 27%]
                                               (Sampling)
   Chain 3 Iteration:
                        4000 / 14000 [ 28%]
                                               (Sampling)
   Chain 3 Iteration:
                        4100 / 14000 [ 29%]
                                               (Sampling)
   Chain 4 Iteration:
                        4800 / 14000
                                      34%]
                                               (Sampling)
   Chain 4 Iteration:
                        4900 / 14000 [ 35%]
                                               (Sampling)
   Chain 4 Iteration:
                        5000 / 14000 [ 35%]
                                               (Sampling)
   Chain 1 Iteration:
                        4600 / 14000 [ 32%]
                                               (Sampling)
   Chain 1 Iteration:
                        4700 / 14000
                                      [ 33%]
                                               (Sampling)
                        4800 / 14000 [ 34%]
   Chain 1 Iteration:
                                               (Sampling)
   Chain 2 Iteration:
                        4400 / 14000
                                      [ 31%]
                                               (Sampling)
   Chain 2 Iteration:
                        4500 / 14000 [ 32%]
                                               (Sampling)
  Chain 2 Iteration:
                        4600 / 14000 [ 32%]
                                               (Sampling)
   Chain 3 Iteration:
                        4200 / 14000 [ 30%]
                                               (Sampling)
   Chain 3 Iteration:
                        4300 / 14000 [ 30%]
                                               (Sampling)
                        5100 / 14000 [ 36%]
  Chain 4 Iteration:
                                               (Sampling)
   Chain 4 Iteration:
                        5200 / 14000 [ 37%]
                                               (Sampling)
   Chain 4 Iteration:
                        5300 / 14000 [ 37%]
                                               (Sampling)
   Chain 1 Iteration:
                        4900 / 14000 [ 35%]
                                               (Sampling)
                        5000 / 14000
                                      35%]
                                               (Sampling)
   Chain 1 Iteration:
   Chain 1 Iteration:
                        5100 / 14000 [ 36%]
                                               (Sampling)
   Chain 2 Iteration:
                        4700 / 14000 [ 33%]
                                               (Sampling)
   Chain 2 Iteration:
                        4800 / 14000 [ 34%]
                                               (Sampling)
                        4900 / 14000
   Chain 2 Iteration:
                                      35%]
                                               (Sampling)
   Chain 3 Iteration:
                        4400 / 14000 [ 31%]
                                               (Sampling)
                                      [ 32%]
   Chain 3 Iteration:
                        4500 / 14000
                                               (Sampling)
                        5400 / 14000
                                        38%]
                                               (Sampling)
   Chain 4 Iteration:
                                      Chain 4 Iteration:
                        5500 / 14000
                                      39%]
                                               (Sampling)
   Chain 4 Iteration:
                                               (Sampling)
                        5600 / 14000 [ 40%]
   Chain 1 Iteration:
                        5200 / 14000 [ 37%]
                                               (Sampling)
  Chain 1 Iteration:
                        5300 / 14000 [ 37%]
                                               (Sampling)
   Chain 2 Iteration:
                        5000 / 14000 [ 35%]
                                               (Sampling)
                        4600 / 14000 [ 32%]
                                               (Sampling)
   Chain 3 Iteration:
                                               (Sampling)
   Chain 3 Iteration:
                        4700 / 14000 [ 33%]
   Chain 4 Iteration:
                        5700 / 14000 [ 40%]
                                               (Sampling)
                                               (Sampling)
   Chain 4 Iteration:
                        5800 / 14000 [ 41%]
   Chain 1 Iteration:
                        5400 / 14000 [ 38%]
                                               (Sampling)
   Chain 1 Iteration:
                        5500 / 14000 [ 39%]
                                               (Sampling)
                        5100 / 14000
   Chain 2 Iteration:
                                      36%]
                                               (Sampling)
   Chain 2 Iteration:
                        5200 / 14000 [ 37%]
                                               (Sampling)
                                      [ 37%]
   Chain 2 Iteration:
                        5300 / 14000
                                               (Sampling)
   Chain 3 Iteration:
                        4800 / 14000
                                      [ 34%]
                                               (Sampling)
   Chain 3 Iteration:
                        4900 / 14000
                                      [ 35%]
                                               (Sampling)
   Chain 4 Iteration:
                        5900 / 14000 [ 42%]
                                               (Sampling)
                                               (Sampling)
   Chain 4 Iteration:
                        6000 / 14000 [ 42%]
## Chain 4 Iteration:
                        6100 / 14000 [ 43%]
                                               (Sampling)
## Chain 1 Iteration:
                        5600 / 14000 [ 40%]
                                               (Sampling)
```

```
## Chain 1 Iteration:
                        5700 / 14000 [ 40%]
                                               (Sampling)
  Chain 1 Iteration:
                        5800 / 14000 [ 41%]
                                               (Sampling)
                                               (Sampling)
   Chain 2 Iteration:
                        5400 / 14000 [ 38%]
  Chain 2 Iteration:
                        5500 / 14000 [ 39%]
                                               (Sampling)
   Chain 3 Iteration:
                        5000 / 14000 [ 35%]
                                               (Sampling)
   Chain 3 Iteration:
                                               (Sampling)
                        5100 / 14000 [ 36%]
                                               (Sampling)
   Chain 3 Iteration:
                        5200 / 14000 [ 37%]
                        6200 / 14000 [ 44%]
  Chain 4 Iteration:
                                               (Sampling)
   Chain 4 Iteration:
                        6300 / 14000 [ 45%]
                                               (Sampling)
   Chain 4 Iteration:
                        6400 / 14000 [ 45%]
                                               (Sampling)
   Chain 4 Iteration:
                        6500 / 14000 [ 46%]
                                               (Sampling)
   Chain 1 Iteration:
                        5900 / 14000
                                      [ 42%]
                                               (Sampling)
   Chain 1 Iteration:
                        6000 / 14000 [ 42%]
                                               (Sampling)
   Chain 1 Iteration:
                        6100 / 14000 [ 43%]
                                               (Sampling)
   Chain 2 Iteration:
                        5600 / 14000 [ 40%]
                                               (Sampling)
   Chain 2 Iteration:
                        5700 / 14000 [ 40%]
                                               (Sampling)
   Chain 2 Iteration:
                        5800 / 14000 [ 41%]
                                               (Sampling)
   Chain 3 Iteration:
                        5300 / 14000 [ 37%]
                                               (Sampling)
   Chain 3 Iteration:
                        5400 / 14000 [ 38%]
                                               (Sampling)
   Chain 3 Iteration:
                        5500 / 14000 [ 39%]
                                               (Sampling)
   Chain 4 Iteration:
                        6600 / 14000 [ 47%]
                                               (Sampling)
   Chain 4 Iteration:
                        6700 / 14000 [ 47%]
                                               (Sampling)
                        6800 / 14000 [ 48%]
  Chain 4 Iteration:
                                               (Sampling)
   Chain 1 Iteration:
                        6200 / 14000 [ 44%]
                                               (Sampling)
   Chain 1 Iteration:
                        6300 / 14000 [ 45%]
                                               (Sampling)
   Chain 1 Iteration:
                        6400 / 14000 [ 45%]
                                               (Sampling)
   Chain 2 Iteration:
                        5900 / 14000
                                      [ 42%]
                                               (Sampling)
   Chain 2 Iteration:
                        6000 / 14000 [ 42%]
                                               (Sampling)
   Chain 2 Iteration:
                        6100 / 14000 [ 43%]
                                               (Sampling)
   Chain 3 Iteration:
                        5600 / 14000 [ 40%]
                                               (Sampling)
   Chain 3 Iteration:
                        5700 / 14000 [ 40%]
                                               (Sampling)
   Chain 4 Iteration:
                        6900 / 14000 [ 49%]
                                               (Sampling)
   Chain 4 Iteration:
                        7000 / 14000 [ 50%]
                                               (Sampling)
   Chain 4 Iteration:
                        7100 / 14000 [ 50%]
                                               (Sampling)
   Chain 1 Iteration:
                        6500 / 14000
                                      [ 46%]
                                               (Sampling)
   Chain 1 Iteration:
                                               (Sampling)
                        6600 / 14000 [ 47%]
   Chain 1 Iteration:
                        6700 / 14000 [ 47%]
                                               (Sampling)
  Chain 2 Iteration:
                        6200 / 14000 [ 44%]
                                               (Sampling)
   Chain 2 Iteration:
                        6300 / 14000 [ 45%]
                                               (Sampling)
   Chain 2 Iteration:
                        6400 / 14000 [ 45%]
                                               (Sampling)
                                               (Sampling)
   Chain 3 Iteration:
                        5800 / 14000 [ 41%]
   Chain 3 Iteration:
                        5900 / 14000 [ 42%]
                                               (Sampling)
                                               (Sampling)
   Chain 3 Iteration:
                        6000 / 14000 [ 42%]
   Chain 4 Iteration:
                        7200 / 14000 [ 51%]
                                               (Sampling)
   Chain 4 Iteration:
                        7300 / 14000 [ 52%]
                                               (Sampling)
                        7400 / 14000 [ 52%]
   Chain 4 Iteration:
                                               (Sampling)
   Chain 1 Iteration:
                        6800 / 14000 [ 48%]
                                               (Sampling)
                        6900 / 14000 [ 49%]
   Chain 1 Iteration:
                                               (Sampling)
   Chain 1 Iteration:
                        7000 / 14000 [ 50%]
                                               (Sampling)
   Chain 1 Iteration:
                        7100 / 14000
                                      [ 50%]
                                               (Sampling)
                                               (Sampling)
   Chain 2 Iteration:
                        6500 / 14000 [ 46%]
                                               (Sampling)
   Chain 2 Iteration:
                        6600 / 14000 [ 47%]
## Chain 2 Iteration:
                        6700 / 14000 [ 47%]
                                               (Sampling)
## Chain 3 Iteration:
                        6100 / 14000 [ 43%]
                                               (Sampling)
```

```
## Chain 3 Iteration:
                        6200 / 14000 [ 44%]
                                               (Sampling)
  Chain 4 Iteration:
                        7500 / 14000 [ 53%]
                                               (Sampling)
                                               (Sampling)
   Chain 4 Iteration:
                        7600 / 14000 [ 54%]
  Chain 4 Iteration:
                        7700 / 14000 [ 55%]
                                               (Sampling)
   Chain 4 Iteration:
                        7800 / 14000 [ 55%]
                                               (Sampling)
   Chain 1 Iteration:
                                               (Sampling)
                        7200 / 14000 [ 51%]
                                               (Sampling)
   Chain 1 Iteration:
                        7300 / 14000 [ 52%]
                        7400 / 14000 [ 52%]
  Chain 1 Iteration:
                                               (Sampling)
   Chain 2 Iteration:
                        6800 / 14000 [ 48%]
                                               (Sampling)
   Chain 2 Iteration:
                        6900 / 14000 [ 49%]
                                               (Sampling)
   Chain 2 Iteration:
                        7000 / 14000 [ 50%]
                                               (Sampling)
   Chain 2 Iteration:
                        7100 / 14000
                                      [ 50%]
                                               (Sampling)
   Chain 3 Iteration:
                        6300 / 14000 [ 45%]
                                               (Sampling)
   Chain 3 Iteration:
                        6400 / 14000 [ 45%]
                                               (Sampling)
   Chain 3 Iteration:
                        6500 / 14000 [ 46%]
                                               (Sampling)
   Chain 3 Iteration:
                        6600 / 14000 [ 47%]
                                               (Sampling)
                        7900 / 14000 [ 56%]
   Chain 4 Iteration:
                                               (Sampling)
   Chain 4 Iteration:
                        8000 / 14000 [ 57%]
                                               (Sampling)
   Chain 4 Iteration:
                        8100 / 14000 [ 57%]
                                               (Sampling)
   Chain 1 Iteration:
                        7500 / 14000 [ 53%]
                                               (Sampling)
   Chain 1 Iteration:
                        7600 / 14000 [ 54%]
                                               (Sampling)
   Chain 1 Iteration:
                        7700 / 14000 [ 55%]
                                               (Sampling)
                        7200 / 14000 [ 51%]
  Chain 2 Iteration:
                                               (Sampling)
   Chain 2 Iteration:
                        7300 / 14000 [ 52%]
                                               (Sampling)
   Chain 2 Iteration:
                        7400 / 14000 [ 52%]
                                               (Sampling)
   Chain 3 Iteration:
                        6700 / 14000 [ 47%]
                                               (Sampling)
   Chain 3 Iteration:
                        6800 / 14000
                                      [ 48%]
                                               (Sampling)
   Chain 4 Iteration:
                        8200 / 14000 [ 58%]
                                               (Sampling)
   Chain 4 Iteration:
                        8300 / 14000 [ 59%]
                                               (Sampling)
   Chain 4 Iteration:
                        8400 / 14000 [ 60%]
                                               (Sampling)
                        8500 / 14000
   Chain 4 Iteration:
                                      60%]
                                               (Sampling)
   Chain 1 Iteration:
                        7800 / 14000 [ 55%]
                                               (Sampling)
   Chain 1 Iteration:
                        7900 / 14000 [ 56%]
                                               (Sampling)
                        8000 / 14000 [ 57%]
                                               (Sampling)
   Chain 1 Iteration:
   Chain 2 Iteration:
                        7500 / 14000
                                      [ 53%]
                                               (Sampling)
   Chain 2 Iteration:
                                               (Sampling)
                        7600 / 14000 [ 54%]
   Chain 3 Iteration:
                        6900 / 14000 [ 49%]
                                               (Sampling)
  Chain 3 Iteration:
                        7000 / 14000 [ 50%]
                                               (Sampling)
   Chain 4 Iteration:
                        8600 / 14000 [ 61%]
                                               (Sampling)
   Chain 4 Iteration:
                        8700 / 14000 [ 62%]
                                               (Sampling)
                                               (Sampling)
   Chain 4 Iteration:
                        8800 / 14000 [ 62%]
   Chain 1 Iteration:
                        8100 / 14000 [ 57%]
                                               (Sampling)
                                               (Sampling)
   Chain 1 Iteration:
                        8200 / 14000 [ 58%]
   Chain 1 Iteration:
                        8300 / 14000 [ 59%]
                                               (Sampling)
   Chain 2 Iteration:
                        7700 / 14000 [ 55%]
                                               (Sampling)
                        7800 / 14000 [ 55%]
   Chain 2 Iteration:
                                               (Sampling)
   Chain 3 Iteration:
                        7100 / 14000 [ 50%]
                                               (Sampling)
                        7200 / 14000 [ 51%]
   Chain 3 Iteration:
                                               (Sampling)
   Chain 4 Iteration:
                        8900 / 14000 [ 63%]
                                               (Sampling)
   Chain 4 Iteration:
                        9000 / 14000
                                      64%]
                                               (Sampling)
   Chain 4 Iteration:
                                               (Sampling)
                        9100 / 14000 [
                                        65%]
   Chain 1 Iteration:
                        8400 / 14000 [ 60%]
                                               (Sampling)
## Chain 1 Iteration:
                        8500 / 14000 [ 60%]
                                               (Sampling)
## Chain 2 Iteration:
                        7900 / 14000 [ 56%]
                                               (Sampling)
```

```
## Chain 2 Iteration:
                        8000 / 14000 [ 57%]
                                               (Sampling)
  Chain 3 Iteration:
                        7300 / 14000 [ 52%]
                                               (Sampling)
   Chain 3 Iteration:
                        7400 / 14000 [ 52%]
                                               (Sampling)
  Chain 4 Iteration:
                        9200 / 14000 [ 65%]
                                               (Sampling)
   Chain 4 Iteration:
                        9300 / 14000 [ 66%]
                                               (Sampling)
   Chain 1 Iteration:
                                               (Sampling)
                        8600 / 14000 [ 61%]
                                               (Sampling)
   Chain 1 Iteration:
                        8700 / 14000 [ 62%]
                        8800 / 14000 [ 62%]
  Chain 1 Iteration:
                                               (Sampling)
   Chain 2 Iteration:
                        8100 / 14000 [ 57%]
                                               (Sampling)
   Chain 2 Iteration:
                        8200 / 14000 [ 58%]
                                               (Sampling)
   Chain 2 Iteration:
                        8300 / 14000 [ 59%]
                                               (Sampling)
   Chain 3 Iteration:
                        7500 / 14000
                                      [ 53%]
                                               (Sampling)
   Chain 3 Iteration:
                        7600 / 14000 [ 54%]
                                               (Sampling)
   Chain 4 Iteration:
                        9400 / 14000 [ 67%]
                                               (Sampling)
   Chain 4 Iteration:
                        9500 / 14000 [ 67%]
                                               (Sampling)
   Chain 4 Iteration:
                        9600 / 14000 [ 68%]
                                               (Sampling)
                        8900 / 14000 [ 63%]
   Chain 1 Iteration:
                                               (Sampling)
   Chain 1 Iteration:
                        9000 / 14000
                                      [ 64%]
                                               (Sampling)
                        9100 / 14000 [ 65%]
                                               (Sampling)
   Chain 1 Iteration:
  Chain 2 Iteration:
                        8400 / 14000 [ 60%]
                                               (Sampling)
   Chain 2 Iteration:
                        8500 / 14000 [ 60%]
                                               (Sampling)
   Chain 2 Iteration:
                        8600 / 14000 [ 61%]
                                               (Sampling)
                        7700 / 14000 [ 55%]
  Chain 3 Iteration:
                                               (Sampling)
   Chain 3 Iteration:
                        7800 / 14000 [ 55%]
                                               (Sampling)
   Chain 4 Iteration:
                        9700 / 14000 [ 69%]
                                               (Sampling)
                                               (Sampling)
   Chain 4 Iteration:
                        9800 / 14000 [ 70%]
   Chain 4 Iteration:
                        9900 / 14000 [
                                        70%]
                                               (Sampling)
   Chain 1 Iteration:
                        9200 / 14000 [ 65%]
                                               (Sampling)
   Chain 1 Iteration:
                        9300 / 14000 [ 66%]
                                               (Sampling)
   Chain 1 Iteration:
                        9400 / 14000 [ 67%]
                                               (Sampling)
   Chain 2 Iteration:
                        8700 / 14000 [ 62%]
                                               (Sampling)
   Chain 2 Iteration:
                        8800 / 14000 [ 62%]
                                               (Sampling)
   Chain 3 Iteration:
                        7900 / 14000
                                      [ 56%]
                                               (Sampling)
                        8000 / 14000
                                               (Sampling)
  Chain 3 Iteration:
                                      [ 57%]
   Chain 3 Iteration:
                        8100 / 14000
                                      [ 57%]
                                               (Sampling)
   Chain 4 Iteration: 10000 / 14000 [ 71%]
                                               (Sampling)
   Chain 4 Iteration: 10100 / 14000 [ 72%]
                                               (Sampling)
  Chain 4 Iteration: 10200 / 14000 [ 72%]
                                               (Sampling)
   Chain 1 Iteration:
                        9500 / 14000 [ 67%]
                                               (Sampling)
                        9600 / 14000 [ 68%]
                                               (Sampling)
   Chain 1 Iteration:
                                               (Sampling)
   Chain 2 Iteration:
                        8900 / 14000 [ 63%]
   Chain 2 Iteration:
                        9000 / 14000 [ 64%]
                                               (Sampling)
                                               (Sampling)
   Chain 3 Iteration:
                        8200 / 14000 [ 58%]
   Chain 3 Iteration:
                        8300 / 14000 [ 59%]
                                               (Sampling)
   Chain 3 Iteration:
                        8400 / 14000 [ 60%]
                                               (Sampling)
## Chain 4 Iteration: 10300 / 14000 [ 73%]
                                               (Sampling)
   Chain 4 Iteration: 10400 / 14000 [ 74%]
                                               (Sampling)
   Chain 4 Iteration: 10500 / 14000 [ 75%]
                                               (Sampling)
   Chain 1 Iteration:
                        9700 / 14000 [ 69%]
                                               (Sampling)
   Chain 1 Iteration:
                        9800 / 14000
                                      [ 70%]
                                               (Sampling)
                        9900 / 14000 [ 70%]
                                               (Sampling)
   Chain 1 Iteration:
                                               (Sampling)
   Chain 2 Iteration:
                        9100 / 14000 [ 65%]
## Chain 2 Iteration:
                        9200 / 14000 [ 65%]
                                               (Sampling)
                                               (Sampling)
## Chain 2 Iteration:
                        9300 / 14000 [ 66%]
```

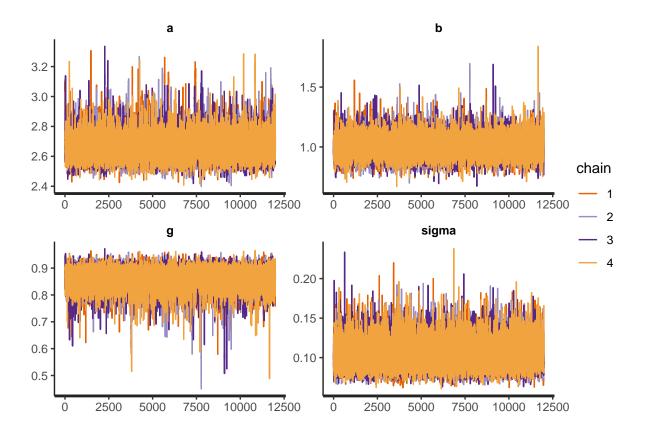
```
## Chain 3 Iteration:
                        8500 / 14000 [ 60%]
                                              (Sampling)
                                              (Sampling)
## Chain 3 Iteration:
                       8600 / 14000 [ 61%]
## Chain 4 Iteration: 10600 / 14000 [ 75%]
                                              (Sampling)
## Chain 4 Iteration: 10700 / 14000 [
                                       76%]
                                              (Sampling)
## Chain 4 Iteration: 10800 / 14000 [ 77%]
                                              (Sampling)
  Chain 1 Iteration: 10000 / 14000 [ 71%]
                                              (Sampling)
  Chain 1 Iteration: 10100 / 14000 [ 72%]
                                              (Sampling)
## Chain 1 Iteration: 10200 / 14000 [ 72%]
                                              (Sampling)
  Chain 2 Iteration:
                        9400 / 14000 [ 67%]
                                              (Sampling)
  Chain 2 Iteration:
                        9500 / 14000 [ 67%]
                                              (Sampling)
  Chain 2 Iteration:
                        9600 / 14000 [ 68%]
                                              (Sampling)
## Chain 3 Iteration:
                        8700 / 14000 [ 62%]
                                              (Sampling)
## Chain 3 Iteration:
                        8800 / 14000 [ 62%]
                                              (Sampling)
## Chain 4 Iteration: 10900 / 14000 [ 77%]
                                              (Sampling)
## Chain 4 Iteration: 11000 / 14000 [ 78%]
                                              (Sampling)
## Chain 4 Iteration: 11100 / 14000 [ 79%]
                                              (Sampling)
## Chain 4 Iteration: 11200 / 14000 [ 80%]
                                              (Sampling)
## Chain 1 Iteration: 10300 / 14000
                                              (Sampling)
## Chain 1 Iteration: 10400 / 14000 [ 74%]
                                              (Sampling)
## Chain 1 Iteration: 10500 / 14000 [ 75%]
                                              (Sampling)
  Chain 2 Iteration:
                       9700 / 14000 [ 69%]
                                              (Sampling)
## Chain 2 Iteration:
                        9800 / 14000 [ 70%]
                                              (Sampling)
## Chain 2 Iteration:
                        9900 / 14000 [ 70%]
                                              (Sampling)
## Chain 3 Iteration:
                        8900 / 14000 [ 63%]
                                              (Sampling)
                        9000 / 14000 [ 64%]
## Chain 3 Iteration:
                                              (Sampling)
  Chain 3 Iteration:
                       9100 / 14000 [ 65%]
                                              (Sampling)
## Chain 4 Iteration: 11300 / 14000
                                     [ 80%]
                                              (Sampling)
                                              (Sampling)
## Chain 4 Iteration: 11400 / 14000 [ 81%]
## Chain 4 Iteration: 11500 / 14000 [ 82%]
                                              (Sampling)
## Chain 1 Iteration: 10600 / 14000 [ 75%]
                                              (Sampling)
## Chain 1 Iteration: 10700 / 14000 [ 76%]
                                              (Sampling)
## Chain 1 Iteration: 10800 / 14000 [ 77%]
                                              (Sampling)
## Chain 1 Iteration: 10900 / 14000 [ 77%]
                                              (Sampling)
## Chain 2 Iteration: 10000 / 14000
                                              (Sampling)
                                     [ 71%]
## Chain 2 Iteration: 10100 / 14000 [ 72%]
                                              (Sampling)
## Chain 2 Iteration: 10200 / 14000 [ 72%]
                                              (Sampling)
## Chain 3 Iteration:
                       9200 / 14000 [ 65%]
                                              (Sampling)
## Chain 3 Iteration:
                        9300 / 14000 [ 66%]
                                              (Sampling)
## Chain 3 Iteration:
                        9400 / 14000 [ 67%]
                                              (Sampling)
  Chain 4 Iteration: 11600 / 14000 [ 82%]
                                              (Sampling)
  Chain 4 Iteration: 11700 / 14000 [ 83%]
                                              (Sampling)
## Chain 4 Iteration: 11800 / 14000 [ 84%]
                                              (Sampling)
## Chain 4 Iteration: 11900 / 14000 [ 85%]
                                              (Sampling)
## Chain 1 Iteration: 11000 / 14000 [ 78%]
                                              (Sampling)
## Chain 1 Iteration: 11100 / 14000 [ 79%]
                                              (Sampling)
## Chain 1 Iteration: 11200 / 14000 [ 80%]
                                              (Sampling)
## Chain 1 Iteration: 11300 / 14000 [ 80%]
                                              (Sampling)
                                     [ 73%]
## Chain 2 Iteration: 10300 / 14000
                                              (Sampling)
## Chain 2 Iteration: 10400 / 14000
                                     [ 74%]
                                              (Sampling)
## Chain 2 Iteration: 10500 / 14000
                                     [ 75%]
                                              (Sampling)
                                              (Sampling)
## Chain 3 Iteration:
                        9500 / 14000 [ 67%]
                                              (Sampling)
## Chain 3 Iteration:
                        9600 / 14000 [ 68%]
## Chain 4 Iteration: 12000 / 14000 [ 85%]
                                              (Sampling)
                                              (Sampling)
## Chain 4 Iteration: 12100 / 14000 [ 86%]
```

```
## Chain 4 Iteration: 12200 / 14000 [ 87%]
                                              (Sampling)
## Chain 1 Iteration: 11400 / 14000 [ 81%]
                                              (Sampling)
## Chain 1 Iteration: 11500 / 14000 [ 82%]
                                              (Sampling)
## Chain 1 Iteration: 11600 / 14000 [ 82%]
                                              (Sampling)
## Chain 2 Iteration: 10600 / 14000 [ 75%]
                                              (Sampling)
## Chain 2 Iteration: 10700 / 14000 [ 76%]
                                              (Sampling)
  Chain 2 Iteration: 10800 / 14000 [ 77%]
                                              (Sampling)
                        9700 / 14000 [ 69%]
## Chain 3 Iteration:
                                              (Sampling)
## Chain 3 Iteration:
                        9800 / 14000 [ 70%]
                                              (Sampling)
## Chain 3 Iteration:
                        9900 / 14000 [ 70%]
                                              (Sampling)
## Chain 4 Iteration: 12300 / 14000 [ 87%]
                                              (Sampling)
## Chain 4 Iteration: 12400 / 14000
                                     [ 88%]
                                              (Sampling)
## Chain 4 Iteration: 12500 / 14000 [ 89%]
                                              (Sampling)
## Chain 4 Iteration: 12600 / 14000 [ 90%]
                                              (Sampling)
## Chain 1 Iteration: 11700 / 14000 [ 83%]
                                              (Sampling)
## Chain 1 Iteration: 11800 / 14000 [ 84%]
                                              (Sampling)
## Chain 1 Iteration: 11900 / 14000 [ 85%]
                                              (Sampling)
## Chain 1 Iteration: 12000 / 14000
                                       85%]
                                              (Sampling)
## Chain 2 Iteration: 10900 / 14000 [ 77%]
                                              (Sampling)
## Chain 2 Iteration: 11000 / 14000 [ 78%]
                                              (Sampling)
## Chain 2 Iteration: 11100 / 14000 [ 79%]
                                              (Sampling)
## Chain 2 Iteration: 11200 / 14000 [ 80%]
                                              (Sampling)
## Chain 3 Iteration: 10000 / 14000 [ 71%]
                                              (Sampling)
## Chain 3 Iteration: 10100 / 14000 [ 72%]
                                              (Sampling)
## Chain 3 Iteration: 10200 / 14000 [ 72%]
                                              (Sampling)
## Chain 3 Iteration: 10300 / 14000 [ 73%]
                                              (Sampling)
## Chain 4 Iteration: 12700 / 14000
                                     90%]
                                              (Sampling)
## Chain 4 Iteration: 12800 / 14000 [ 91%]
                                              (Sampling)
## Chain 4 Iteration: 12900 / 14000 [ 92%]
                                              (Sampling)
## Chain 4 Iteration: 13000 / 14000 [ 92%]
                                              (Sampling)
## Chain 1 Iteration: 12100 / 14000
                                     [ 86%]
                                              (Sampling)
## Chain 1 Iteration: 12200 / 14000
                                     [ 87%]
                                              (Sampling)
                                     [ 87%]
## Chain 1 Iteration: 12300 / 14000
                                              (Sampling)
## Chain 1 Iteration: 12400 / 14000
                                       88%]
                                              (Sampling)
                                     Γ
## Chain 2 Iteration: 11300 / 14000
                                     (Sampling)
## Chain 2 Iteration: 11400 / 14000 [ 81%]
                                              (Sampling)
## Chain 2 Iteration: 11500 / 14000 [ 82%]
                                              (Sampling)
## Chain 2 Iteration: 11600 / 14000 [ 82%]
                                              (Sampling)
## Chain 3 Iteration: 10400 / 14000 [ 74%]
                                              (Sampling)
## Chain 3 Iteration: 10500 / 14000 [ 75%]
                                              (Sampling)
## Chain 3 Iteration: 10600 / 14000 [ 75%]
                                              (Sampling)
## Chain 3 Iteration: 10700 / 14000 [ 76%]
                                              (Sampling)
## Chain 4 Iteration: 13100 / 14000 [ 93%]
                                              (Sampling)
## Chain 4 Iteration: 13200 / 14000 [ 94%]
                                              (Sampling)
## Chain 4 Iteration: 13300 / 14000 [ 95%]
                                              (Sampling)
## Chain 4 Iteration: 13400 / 14000
                                     [ 95%]
                                              (Sampling)
## Chain 4 Iteration: 13500 / 14000 [ 96%]
                                              (Sampling)
                                     [ 89%]
## Chain 1 Iteration: 12500 / 14000
                                              (Sampling)
## Chain 1 Iteration: 12600 / 14000
                                     [ 90%]
                                              (Sampling)
## Chain 1 Iteration: 12700 / 14000
                                     [ 90%]
                                              (Sampling)
## Chain 1 Iteration: 12800 / 14000 [ 91%]
                                              (Sampling)
                                              (Sampling)
## Chain 2 Iteration: 11700 / 14000 [ 83%]
## Chain 2 Iteration: 11800 / 14000 [ 84%]
                                              (Sampling)
## Chain 2 Iteration: 11900 / 14000 [ 85%]
                                              (Sampling)
```

```
## Chain 3 Iteration: 10800 / 14000 [ 77%]
                                              (Sampling)
## Chain 3 Iteration: 10900 / 14000 [ 77%]
                                              (Sampling)
## Chain 3 Iteration: 11000 / 14000 [ 78%]
                                              (Sampling)
## Chain 4 Iteration: 13600 / 14000 [ 97%]
                                              (Sampling)
## Chain 4 Iteration: 13700 / 14000 [ 97%]
                                              (Sampling)
## Chain 4 Iteration: 13800 / 14000 [ 98%]
                                              (Sampling)
## Chain 1 Iteration: 12900 / 14000 [ 92%]
                                              (Sampling)
## Chain 1 Iteration: 13000 / 14000 [ 92%]
                                              (Sampling)
## Chain 1 Iteration: 13100 / 14000 [ 93%]
                                              (Sampling)
## Chain 1 Iteration: 13200 / 14000 [ 94%]
                                              (Sampling)
## Chain 2 Iteration: 12000 / 14000 [ 85%]
                                              (Sampling)
                                     [ 86%]
## Chain 2 Iteration: 12100 / 14000
                                              (Sampling)
## Chain 2 Iteration: 12200 / 14000 [ 87%]
                                              (Sampling)
## Chain 2 Iteration: 12300 / 14000 [ 87%]
                                              (Sampling)
## Chain 3 Iteration: 11100 / 14000 [ 79%]
                                              (Sampling)
## Chain 3 Iteration: 11200 / 14000 [ 80%]
                                              (Sampling)
## Chain 3 Iteration: 11300 / 14000 [ 80%]
                                              (Sampling)
## Chain 4 Iteration: 13900 / 14000 [ 99%]
                                              (Sampling)
## Chain 4 Iteration: 14000 / 14000
                                              (Sampling)
## Chain 4 finished in 5.7 seconds.
## Chain 1 Iteration: 13300 / 14000 [ 95%]
                                              (Sampling)
## Chain 1 Iteration: 13400 / 14000 [ 95%]
                                              (Sampling)
## Chain 2 Iteration: 12400 / 14000 [ 88%]
                                              (Sampling)
## Chain 2 Iteration: 12500 / 14000 [ 89%]
                                              (Sampling)
## Chain 3 Iteration: 11400 / 14000 [ 81%]
                                              (Sampling)
                                              (Sampling)
## Chain 3 Iteration: 11500 / 14000 [ 82%]
## Chain 1 Iteration: 13500 / 14000
                                     96%]
                                              (Sampling)
## Chain 1 Iteration: 13600 / 14000 [ 97%]
                                              (Sampling)
## Chain 1 Iteration: 13700 / 14000 [ 97%]
                                              (Sampling)
## Chain 1 Iteration: 13800 / 14000 [ 98%]
                                              (Sampling)
## Chain 2 Iteration: 12600 / 14000
                                     [ 90%]
                                              (Sampling)
## Chain 2 Iteration: 12700 / 14000 [ 90%]
                                              (Sampling)
## Chain 2 Iteration: 12800 / 14000 [ 91%]
                                              (Sampling)
## Chain 3 Iteration: 11600 / 14000
                                     [ 82%]
                                              (Sampling)
## Chain 3 Iteration: 11700 / 14000
                                     [ 83%]
                                              (Sampling)
## Chain 3 Iteration: 11800 / 14000 [ 84%]
                                              (Sampling)
## Chain 1 Iteration: 13900 / 14000 [ 99%]
                                              (Sampling)
## Chain 1 Iteration: 14000 / 14000 [100%]
                                              (Sampling)
## Chain 2 Iteration: 12900 / 14000 [ 92%]
                                              (Sampling)
## Chain 2 Iteration: 13000 / 14000 [ 92%]
                                              (Sampling)
## Chain 2 Iteration: 13100 / 14000 [ 93%]
                                              (Sampling)
## Chain 3 Iteration: 11900 / 14000 [ 85%]
                                              (Sampling)
## Chain 3 Iteration: 12000 / 14000 [ 85%]
                                              (Sampling)
## Chain 3 Iteration: 12100 / 14000
                                     [ 86%]
                                              (Sampling)
## Chain 1 finished in 5.9 seconds.
## Chain 2 Iteration: 13200 / 14000 [ 94%]
                                              (Sampling)
## Chain 2 Iteration: 13300 / 14000 [ 95%]
                                              (Sampling)
## Chain 2 Iteration: 13400 / 14000 [ 95%]
                                              (Sampling)
## Chain 3 Iteration: 12200 / 14000
                                     [ 87%]
                                              (Sampling)
## Chain 3 Iteration: 12300 / 14000
                                     [ 87%]
                                              (Sampling)
## Chain 3 Iteration: 12400 / 14000 [ 88%]
                                              (Sampling)
                                              (Sampling)
## Chain 2 Iteration: 13500 / 14000 [ 96%]
## Chain 2 Iteration: 13600 / 14000 [ 97%]
                                              (Sampling)
## Chain 2 Iteration: 13700 / 14000 [ 97%]
                                              (Sampling)
```

```
## Chain 3 Iteration: 12500 / 14000 [ 89%]
                                              (Sampling)
## Chain 3 Iteration: 12600 / 14000 [ 90%]
                                              (Sampling)
## Chain 3 Iteration: 12700 / 14000 [ 90%]
                                              (Sampling)
## Chain 2 Iteration: 13800 / 14000 [ 98%]
                                              (Sampling)
## Chain 2 Iteration: 13900 / 14000 [ 99%]
                                              (Sampling)
## Chain 2 Iteration: 14000 / 14000 [100%]
                                              (Sampling)
## Chain 3 Iteration: 12800 / 14000 [ 91%]
                                              (Sampling)
## Chain 3 Iteration: 12900 / 14000 [ 92%]
                                              (Sampling)
## Chain 3 Iteration: 13000 / 14000 [ 92%]
                                              (Sampling)
## Chain 3 Iteration: 13100 / 14000 [ 93%]
                                              (Sampling)
## Chain 2 finished in 6.4 seconds.
## Chain 3 Iteration: 13200 / 14000 [ 94%]
                                              (Sampling)
## Chain 3 Iteration: 13300 / 14000 [ 95%]
                                              (Sampling)
## Chain 3 Iteration: 13400 / 14000 [ 95%]
                                              (Sampling)
## Chain 3 Iteration: 13500 / 14000 [ 96%]
                                              (Sampling)
## Chain 3 Iteration: 13600 / 14000 [ 97%]
                                              (Sampling)
## Chain 3 Iteration: 13700 / 14000 [ 97%]
                                              (Sampling)
## Chain 3 Iteration: 13800 / 14000 [ 98%]
                                              (Sampling)
## Chain 3 Iteration: 13900 / 14000 [ 99%]
                                              (Sampling)
## Chain 3 Iteration: 14000 / 14000 [100%]
                                              (Sampling)
## Chain 3 finished in 6.6 seconds.
##
## All 4 chains finished successfully.
## Mean chain execution time: 6.1 seconds.
## Total execution time: 7.3 seconds.
## # A tibble: 5 x 10
##
     variable
                mean median
                                                            rhat ess_bulk ess_tail
                                 sd
                                        mad
                                                 q5
                                                       q95
                                                                     <dbl>
##
     <chr>>
               <dbl>
                      <dbl>
                              <dbl>
                                     <dbl>
                                              <dbl>
                                                     <dbl>
                                                           <dbl>
                                                                               <dbl>
              40.3
                      40.7
                             1.59
                                    1.36
                                            37.2
                                                    42.2
                                                             1.00
                                                                    13987.
                                                                             17560.
## 1 lp__
## 2 a
               2.67
                       2.66
                             0.0841 0.0741
                                             2.55
                                                     2.82
                                                             1.00
                                                                    13759.
                                                                             11269.
## 3 b
               1.01
                       1.00
                             0.0842 0.0781
                                             0.877
                                                     1.15
                                                             1.00
                                                                    18602.
                                                                             18475.
## 4 g
               0.863
                      0.867 0.0368 0.0319
                                             0.800
                                                     0.914
                                                             1.00
                                                                    14404.
                                                                             13412.
                      0.102 0.0168 0.0156
                                             0.0804
                                                     0.135
                                                             1.00
                                                                             22257.
## 5 sigma
               0.104
                                                                    21901.
```

Using the NUTS sampler provided with Stan, we have the following traceplots:



It appears to be doing pretty well. Lets compare the variables:

```
## # A tibble: 4 x 3
##
     variable '2.5%' '97.5%'
##
     <chr>
               <dbl>
                        <dbl>
## 1 a
              2.53
                        2.86
  2 b
              0.853
                        1.18
##
  3
              0.780
                        0.922
    g
              0.0772
## 4 sigma
                        0.142
##
           lower upper
## sigma 0.07501 0.1384
         2.51200 2.8320
## a
         0.84360 1.1700
## b
         0.78890 0.9267
## attr(,"Probability")
## [1] 0.95
```

Appendix

```
knitr::opts_chunk$set(echo = FALSE, fig.align = 'center', message = FALSE, eval = FALSE)
library(rstan)
library(R2jags)
```

```
library(cmdstanr)
library(loo)
dugong <- read.table("dugong.dat", header=TRUE)</pre>
dugong
plot(dugong, ylim = c(0,2.8))
model1 <- "
model {
for (i in 1:27) {
 y[i] ~ dnorm(mu[i], 1/s2error)
 mu[i] = a - b*g^(x[i])
 a ~ dnorm(3, 0.001)
 b ~ dgamma(6, 4)
g ~ dbeta(5.5, 1.5)
  s2error ~ dgamma(1.1, 0.05)
writeLines(model1, 'model1.txt')
y <- dugong$length
x <- dugong$age
data.jags \leftarrow c('y', 'x')
parms <- c('a', 'b', 'g', 's2error')
dugong.sim <- jags(model.file = 'model1.txt', data = data.jags, parameters.to.save = parms,</pre>
                  n.iter = 50000, n.burnin = 2000, n.chains = 4, n.thin = 1, inits = NULL)
dugong.sim
sims <- as.mcmc(dugong.sim)</pre>
gelman.diag(sims)
chains <- as.matrix(sims)</pre>
sims <- as.mcmc(chains)</pre>
raftery.diag(sims)
effectiveSize(sims)
autocorr.diag(sims)
geweke.diag(sims)
HPDinterval(sims, prob = 0.95)
library(dplyr)
quantile(sims[,1], c(0.025, 0.975))
quantile(sims[,2], c(0.025, 0.975))
quantile(sims[,4], c(0.025, 0.975))
quantile(sims[,5], c(0.025, 0.975))
mod <- cmdstan_model(stan_file='hw9.stan')</pre>
data_list \leftarrow list(x = x, y = y, N = length(dugong$age))
fit <- mod$sample(data_list,</pre>
                   chains = 4,
                   parallel_chains = 4,
                   sig_figs = 4,
```

```
iter_sampling = 12000,
    iter_warmup = 2000)

fit$summary(variables = c('lp__','a', 'b', 'g', 'sigma'))
    rstan::traceplot(read_stan_csv(fit$output_files()), pars = c('a', 'b', 'g', 'sigma'))
    fit$summary(variables = c('a', 'b', 'g', 'sigma'), ~quantile(., probs = c(0.025, 0.975)))

stanfit <- rstan::read_stan_csv(fit$output_files())
    samps <- extract(stanfit)
    chains <- cbind(samps[[1]], samps[[2]], samps[[3]], samps[[4]])
    colnames(chains) <- names(samps[1:4])
    sims <- as.mcmc(chains)

HPDinterval(sims, prob = 0.95)</pre>
```