# Two litte notes on interest rate derivatives gamma computation

Eurobanking 2010, Peter Caspers

# Disclaimer

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# Agenda

⇨ Gamma Computation with Smile

Cross Gammas

# Swaption Smile

**EUR Vol Skews (Vega)** float window

| | Receivers | | | | | | Payers | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | -200 | -150 | -100 | -50 | -25 | ATM | +25 | +50 | +100 | +150 | +200 |
| 1y2y | 32.67 | 16.62 | 7.16 | 2.22 | 0.84 | 34.48 | -0.45 | -0.62 | -0.46 | 0.03 | 0.64 |
| 1y5y | 19.96 | 11.71 | 6.1 | 2.3 | 0.97 | 24.93 | -0.65 | -1.04 | -1.2 | -0.9 | -0.38 |
| 1y10y | 13.82 | 8.77 | 4.87 | 1.95 | 0.85 | 19.41 | -0.61 | -1 | -1.23 | -0.99 | -0.53 |
| 1y20y | 15.94 | 10.51 | 6.09 | 2.55 | 1.13 | 18.15 | -0.81 | -1.3 | -1.44 | -0.93 | -0.16 |
| 1y30y | 18.43 | 12.23 | 7.2 | 3.11 | 1.42 | 19.89 | -1.11 | -1.89 | -2.52 | -2.3 | -1.69 |
| 2y2y | 20.98 | 11.46 | 5.58 | | | | | | | | |
| 2y5y | 15.4 | 9.52 | 5.12 | | | | | | | | |
| 2y10y | 12.52 | 8.05 | 4.54 | | | | | | | | |
| 2y20y | 13.65 | 9 | 5.23 | | | | | | | | |
| 2y30y | 16.08 | 10.66 | 6.27 | | | | | | | | |
| 5y2y | 10.31 | 6.51 | 3.56 | | | | | | | | |
| 5y5y | 9.56 | 6.09 | 3.35 | | | | | | | | |
| 5y10y | 9.89 | 6.47 | 3.68 | | | | | | | | |
| 5y20y | 11.1 | 7.35 | 4.28 | | | | | | | | |
| 5y30y | 13.41 | 8.88 | 5.21 | | | | | | | | |
| 10y2y | 6.58 | 4.23 | 2.34 | | | | | | | | |
| 10y5y | 6.74 | 4.33 | 2.4 | | | | | | | | |
| 10y10y | 8.32 | 5.45 | 3.12 | | | | | | | | |
| 10y20y | 10.5 | 6.91 | 4.02 | | | | | | | | |
| 10y30y | 12.4 | 8.07 | 4.65 | | | | | | | | |
| 20y2y | 8.02 | 5.01 | 2.76 | | | | | | | | |
| 20y5y | 10.41 | 6.54 | 3.63 | | | | | | | | |
| 20y10y | 14 | 8.84 | 4.97 | | | | | | | | |
| 20y20y | 15.95 | 9.77 | 5.43 | | | | | | | | |
| 20y30y | 16.44 | 10.03 | 5.58 | | | | | | | | |
| 30y2y | | 9.54 | 4.98 | | | | | | | | |
| 30y5y | | 11.55 | 6.11 | | | | | | | | |
| 30y10y | | 13.49 | 7.27 | | | | | | | | |
| 30y20y | | 11.13 | 6.05 | 2.49 | 1.13 | 16.64 | -0.91 | -1.61 | -2.51 | -2.92 | -3.03 |
| 30y30y | | 11.32 | 6.14 | 2.53 | 1.15 | 17.03 | -0.94 | -1.68 | -2.67 | -3.16 | -3.36 |

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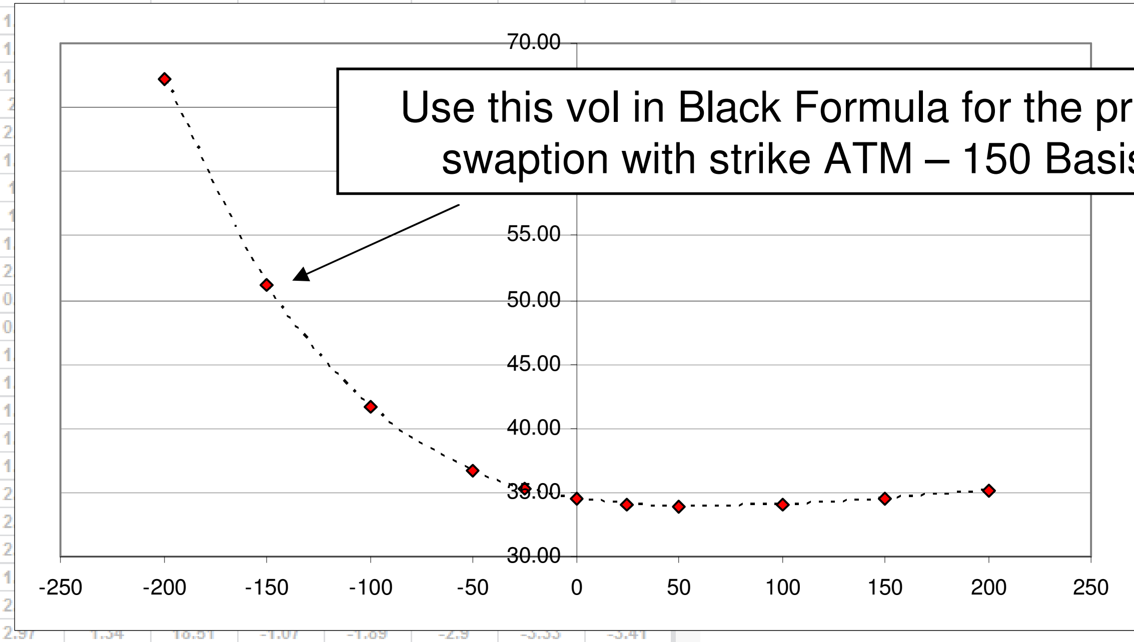
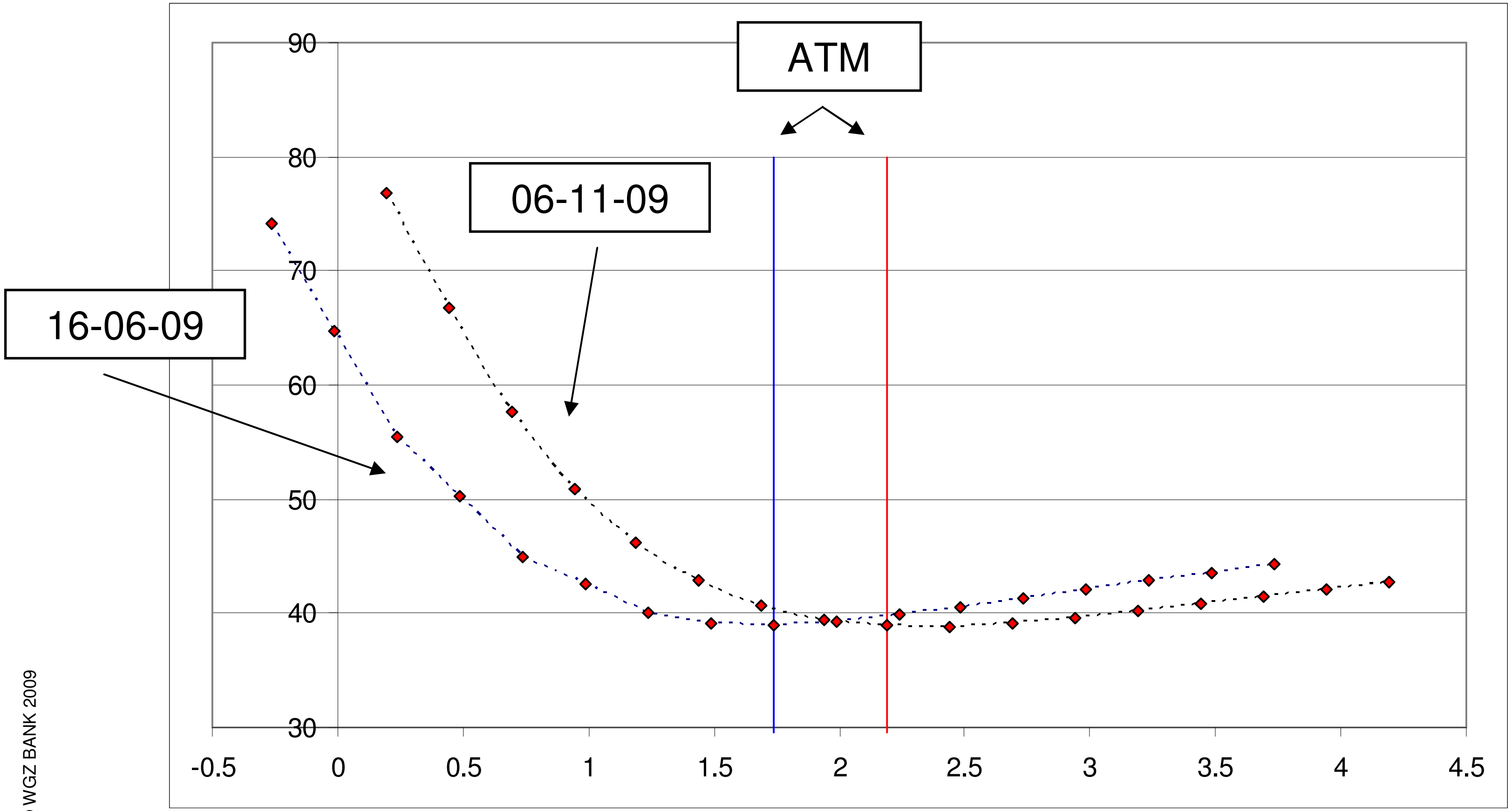



Use this vol in Black Formula for the pricing of a swaption with strike ATM – 150 Basispoints

# Smile Dynamics

Smile comoves with ATM



# Delta

$$\frac{\partial}{\partial S} c(S, \sigma(S)) = \frac{\partial c}{\partial S} + \frac{\partial c}{\partial \sigma} \frac{\partial \sigma}{\partial S}$$

Delta with constant σ

Correction Term when σ depends on S

# Hedging and risk calculation

Delta Hedges will be more stable, if the „usual“ market dynamics is captured appropriately.

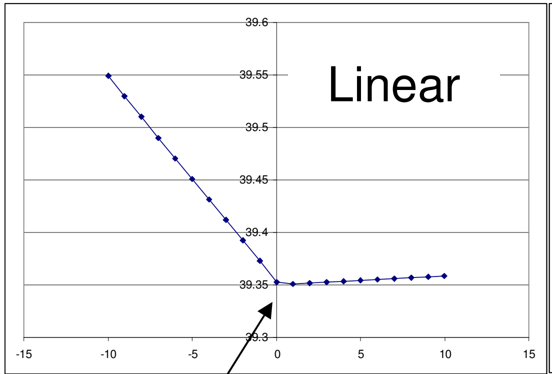
Also, risk calculation will be more suitable.

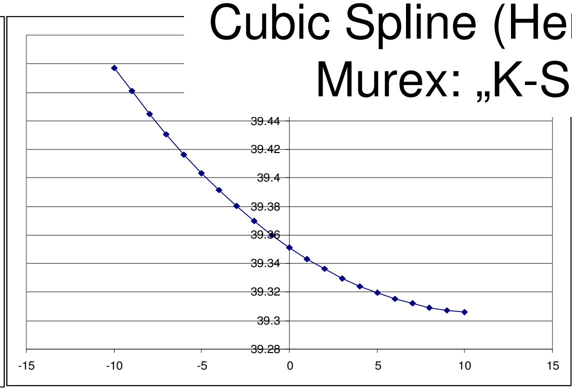
Therefore the correction term should be included both in the delta for hedging and in the delta for risk calculation.

Murex supports this for „money“ defined smiles.

# Gamma

$$\frac{\partial^2}{\partial S^2}c(S,\sigma(S)) = \frac{\partial^2 c}{\partial S^2} + f\left(\frac{\partial^2 c}{\partial S \partial \sigma}, \frac{\partial^2 c}{\partial \sigma^2}, \frac{\partial c}{\partial \sigma}, \frac{\partial \sigma}{\partial S}, \frac{\partial^2 \sigma}{\partial S^2}\right)$$




Cubic Spline (Hermite type)
Murex: „K-Spline“

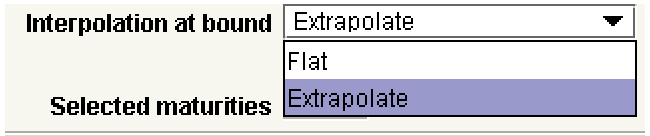
Here Smile Interpolation becomes crucial

Finite Difference Derivatives can be computed, but are not stable.
Second Derivative goes to +inf if stepsíze goes to 0.

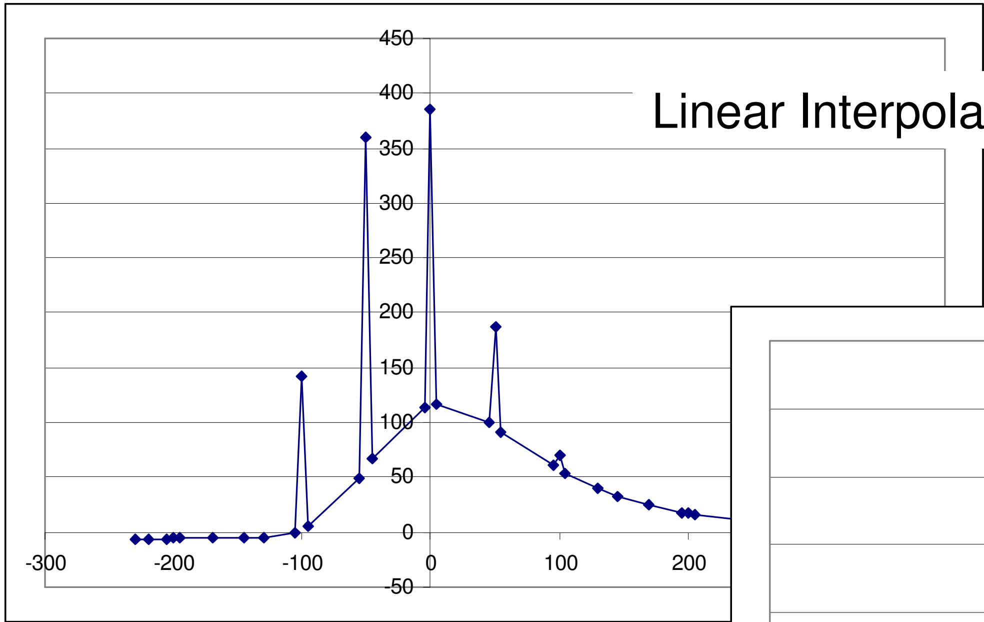
# Extrapolation

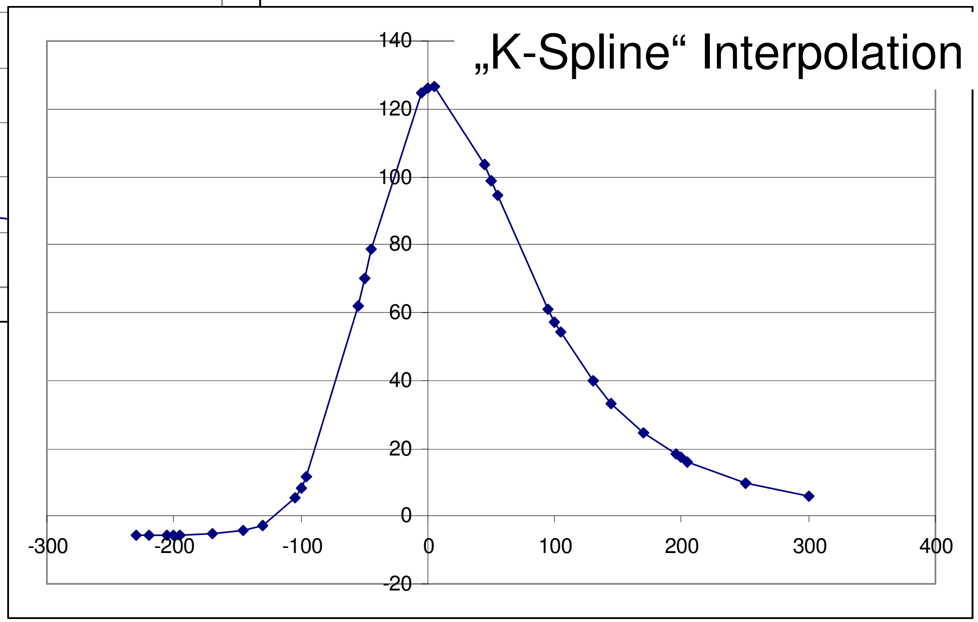

Outside the quoted range, one should use linear extrapolation (if not an own engineered „far strike“ - cms consistent smile anyway). Flat extrapolation produces „swings“ in the spline.

Murex Setting:

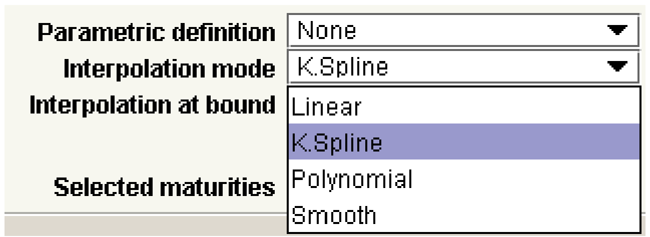


# Finite Difference Gammas




Murex Setting:



# Conclusion

Delta and Gamma computed for hedging and risk calculation should include the correction term expressing the dependency of the volatility on the underlying.

When using finite differences for Delta and Gamma computation, it is then vital to select a smooth interpolation of the volatility smile.

In Murex, best results are obtained with „K-Spline“ interpolation and „Extrapolate at bound“.

# Agenda

Gamma Computation with Smile

⇨ Cross Gammas

# Bucket Deltas

5y / 7y European Payer-Swaption Deltas w.r.t. Zero Rate Buckets:

| Bucket | Delta |
|---|---|
| 1y | - |
| 2y | - |
| 3y | - |
| 4y | - |
| 5y | - 23,648.30 |
| 6y | 991.84 |
| 7y | 1,247.36 |
| 8y | 1,352.71 |
| 9y | 1,454.47 |
| 10y | 1,525.84 |
| 11y | 1,597.49 |
| 12y | 41,269.11 |
| 13y | 227.56 |
| 14y | - |
| 15y | - |
| Total | 26,018.07 |

5y Zero up
=> 5y / 1y Forward down
=> 5y / 7y Swap down

8y Zero up
=> 7y / 1y Forward up, 8y / 1y down
=> 5y / 7y Swap apprx. same

12y Zero up
=> 11y / 1y Forward up
=> 5y / 7y Swap up

# Bucket Gammas

5y / 7y European Payer-Swaption Gammas w.r.t. Zero Rate Buckets:

| Gamma | 5y | 6y | 7y | 8y | 9y | 10y | 11y | 12y | 13y |
|---|---|---|---|---|---|---|---|---|---|
| 5y | 100 | - 5 | - 6 | - 6 | - 7 | - 7 | - 7 | - 153 | - 1 |
| 6y | - 5 | - 0 | 0 | 0 | 0 | 0 | 0 | 8 | 0 |
| 7y | - 6 | 0 | - 1 | 0 | 0 | 0 | 0 | 10 | 0 |
| 8y | - 6 | 0 | 0 | - 1 | 0 | 1 | 1 | 11 | 0 |
| 9y | - 7 | 0 | 0 | 0 | - 1 | 1 | 1 | 12 | 0 |
| 10y | - 7 | 0 | 0 | 1 | 1 | - 1 | 1 | 12 | 0 |
| 11y | - 7 | 0 | 0 | 1 | 1 | 1 | - 1 | 13 | 0 |
| 12y | - 153 | 8 | 10 | 11 | 12 | 12 | 13 | 219 | 1 |
| 13y | - 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | - 0 |

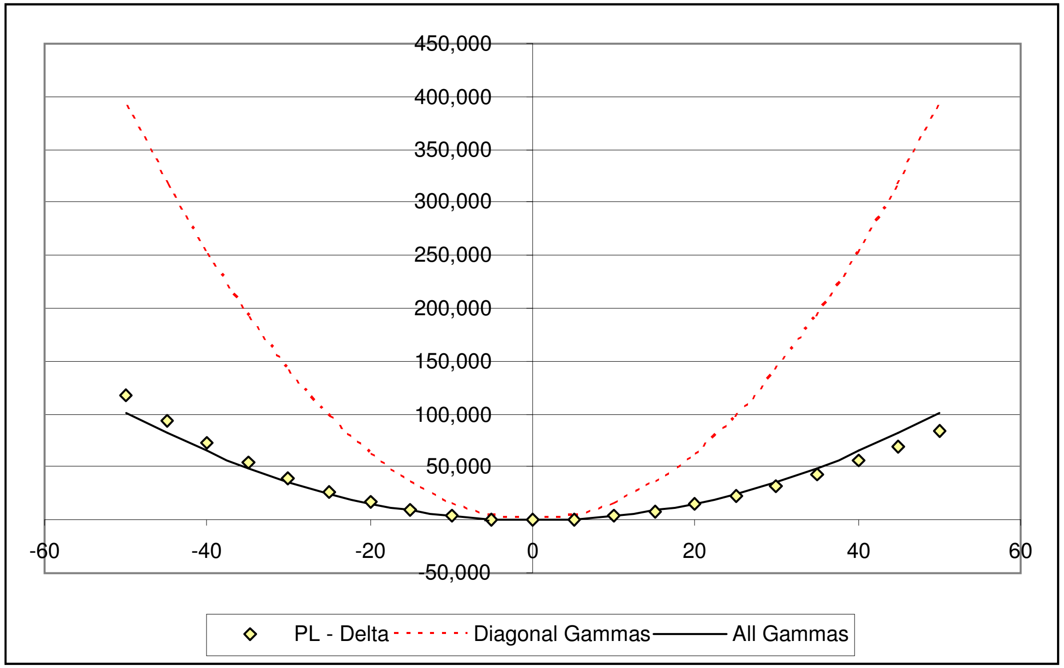
# PL Explanation

Full Second Order Expansion („Delta Gamma Explanation“):

$$\Delta PL = \sum_i \frac{\partial PL}{\partial Rate_i} \Delta Rate_i + \frac{1}{2} \sum_{i,j} \frac{\partial^2 PL}{\partial Rate_i \partial Rate_j} \Delta Rate_i \Delta Rate_j$$

Problem: Limited Computational Ressources might not allow the computation of the whole Gamma Matrix for all deals.

# PL Explanation Diagonal vs. Full




Diagonal Gammas alone give a bad PL Estimation
for parallel shifts of the rate curve

# Task

Given n Buckets, find a numerical Scheme to compute Gammas, such that

- Complexity remains O(n)

- A good PL Estimation at least for important subsets of Curve movements is achieved

In the following we identify „Important Subsets“ with (nearly) parallel moves of the rate curve.

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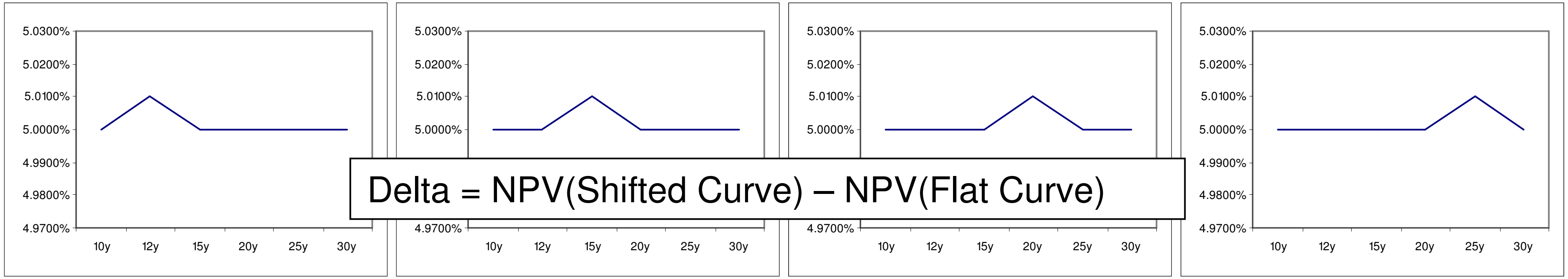
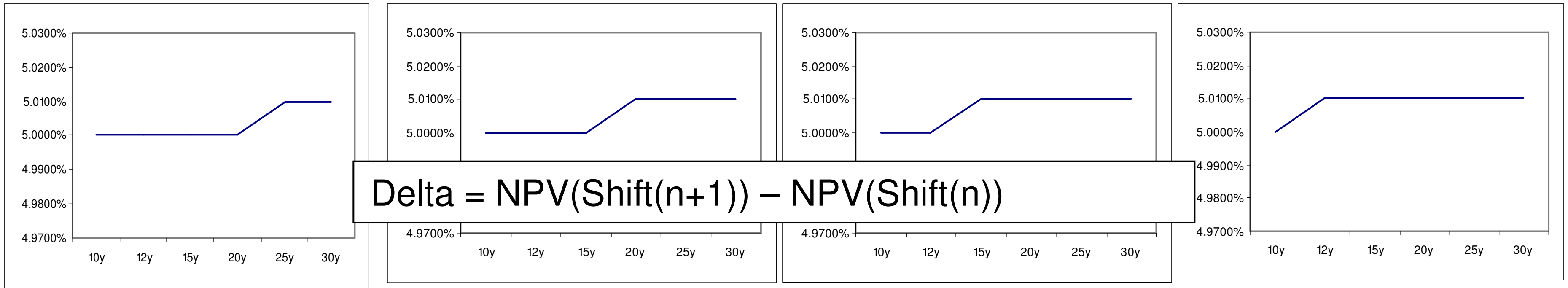
# Approach

Instead of shifting the rate curve buckets „one by one“




Delta = NPV(Shifted Curve) – NPV(Flat Curve)

... begin at the highest maturity and leave shifted buckets unchanged („backward shift“).




Delta = NPV(Shift(n+1)) – NPV(Shift(n))

# Approach 1 vs. 2

Shifting „one by one“:

$\Gamma_1 = PL_{10y,10y}$

$\Gamma_2 = PL_{12y,12y}$

$\Gamma_3 = PL_{15y,15y}$

$\Gamma_4 = PL_{20y,20y}$

| | 10y | 12y | 15y | 20y |
|---|---|---|---|---|
| 10y | 1 | | | |
| 12y | | 2 | | |
| 15y | | | 3 | |
| 20y | | | | 4 |

„Backward Shift“:

$\Gamma_1 = PL_{10y,10y} + 2\,(PL_{10y,12y} + PL_{10y,15y} + PL_{10y,20y})$

$\Gamma_1 = PL_{12y,12y} + 2\,(PL_{12y,15y} + PL_{12y,20y})$

$\Gamma_1 = PL_{15y,15y} + 2\,PL_{15y,20y}$

$\Gamma_1 = PL_{20y,20y}$

| | 10y | 12y | 15y | 20y |
|---|---|---|---|---|
| 10y | 1 | 1 | 1 | 1 |
| 12y | 1 | 2 | 2 | 2 |
| 15y | 1 | 2 | 3 | 3 |
| 20y | 1 | 2 | 3 | 4 |

# Theoretical Foundation

Let $U \subseteq IR^n$ be an open subset, $f \in C^2(U)$, $x \in U$, $f(x) = 0$. Let $t_i \in IR^n$ represent the „backward" shifts $t_1=(1,1,...,1)$, $t_2=(0,1,....,1,1)$, ..., $t_n=(0,0,....0,1)$ and $t_{n+1} = 0$. Then the following relations hold for all i = 1, ... , n:

$$\lim_{h\to 0}\frac{(f(x+ht_i)-f(x+ht_{i+1}))-(f(x-ht_i)-f(x-ht_{i+1}))}{2h}=\frac{\partial f}{\partial x_i}(x)$$

$$\lim_{h\to 0}\frac{(f(x+ht_i)-f(x+ht_{i+1}))+(f(x-ht_i)-f(x-ht_{i+1}))}{h^2}=\frac{\partial^2 f}{\partial {x_i}^2}(x)+\sum_{j=1}^{n-i}\left(\frac{\partial^2 f}{\partial x_i \partial x_{i+j}}(x)+\frac{\partial^2 f}{\partial x_{i+j}\partial x_i}(x)\right)$$

# Example: CMS Spread Floor

CMS Spread Floor at 0%, 10y

| Maturity | Delta old | Delta new | Gamma old | Gamma new |
|---|---|---|---|---|
| 1y | -175 | -175 | 3 | -6 |
| 2y | -596 | -596 | 17 | -26 |
| 3y | -1,833 | -1,833 | 91 | -68 |
| 4y | -2,539 | -2,540 | 198 | -105 |
| 5y | -1,064 | -1,065 | 516 | -87 |
| 7y | 1,946 | 1,946 | 898 | -147 |
| 10y | 28,472 | 28,472 | 991 | 45 |
| 15y | -18,568 | -18,568 | 314 | 383 |
| 30y | -2,738 | -2,738 | 12 | 12 |
| Total | **2,906** | **2,904** | **3,039** | **1** |

PL Estimation Error for parallel rate curve shifts:

| | **-60** | **-30** | **0** | **30** | **60** |
|---|---|---|---|---|---|
| **old** | 546.94 | 136.74 | - | 136.76 | 547.04 |
| **new** | 0.09 | 0.03 | - | 0.04 | 0.17 |

(shifts and errors in basispoints of reference notional)

# Real life examples

The new backward shift method is – of course – not always better than the old method.

Here are Delta Gamma PL Estimations for real rate curve movements and real deals:

| PL 12.06. to | PL | Delta Gamma old | Delta Gamma new | Error old | Error new |
|---|---|---|---|---|---|
| 25.06. | -112,369.15 | -105,287.22 | -110,405.14 | 7,081.93 | 1,964.01 |
| 24.06. | -78,775.85 | -73,667.02 | -76,888.15 | 5,108.84 | 1,887.70 |
| 23.06. | -64,707.09 | -63,474.97 | -65,578.34 | 1,232.12 | -871.25 |
| 15.06. | -74,327.82 | -72,785.35 | -74,129.31 | 1,542.47 | 198.51 |

New method is better than old

| PL 12.06. to | PL | Delta Gamma old | Delta Gamma new | Error old | Error new |
|---|---|---|---|---|---|
| 25.06. | -9,017,552.58 | -8,457,076.56 | -8,374,980.11 | 560,476.02 | 642,572.47 |
| 24.06. | -7,011,072.48 | -6,567,190.27 | -6,505,016.19 | 443,882.21 | 506,056.29 |
| 23.06. | -5,854,888.71 | -5,477,069.85 | -5,437,135.71 | 377,818.86 | 417,752.99 |
| 15.06. | -4,481,389.50 | -4,322,164.71 | -4,302,556.99 | 159,224.79 | 178,832.51 |

New method is worse than old

# Conclusion

The new method may be better, but also worse than the old method. This depends on the deal (its gamma matrix) and how the rate curve movement fits (are significant cross gammas multiplied by an approximately correct shift?).

However, for (nearly) parallel rate curve movements, the new method is always better than the old.

Believing in the „general result“ that the first principal component of rate curve movements is nearly a parallel shift, the new method is preferable in this statistical sense.