Fixed Income Valuation under Static Credit Risk

P. Caspers

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Abstract

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1 Discounting Curve

1.1 Default free interest rate curve

We assume that there is (at least a proxy) for an interest rate curve which does not include any credit risk. An entity O with $\lambda_O = 0$ i.e. without any risk to default can fund at the rates on this curve. We denote discount factors on this curve by $P(0,T)$.

In the current market situation we take the OIS curves as these proxies, e.g. for EUR this will be the curve stripped from Eonia swaps.

2 Cases

We assume two parties A and B with default termstructures given by default intensities λ_A and λ_B. A third entity O is assumed to have no credit risk, i.e. $\lambda_O = 0$.

2.1 A receives a deterministic cashflow from O

Assume A has the opportunity to enter into a deal where she receives a deterministic cashflow $C(T) > 0$ at time T from O. What amount $C(0)$ is A willing to pay to enter in such a deal? A can fund $P(0,T)\nu_A(T)C(T)$ at time 0 repaying $C(T)$ at time T. This is possible at least if there are investors only requiring to be compensated for the credit risk of A, i.e. which can fund themselves on hte risk free curve.

Therefore $C(0) \geq P(0,T)\nu_A(T)C(T)$, otherwise A would make an instantaneous profit at $t = 0$ and all future cashflows would cancel out. Since on the other hand A has to fund the amount she has to pay $C(0) \leq P(0,T)\nu_A(T)C(T)$, otherwise A has to repay more that she gets out of the deal she enters into. Therefore

$$C(0) = P(0,T)\nu_A(T)C(T) \tag{1}$$

This valuation includes funding costs of A and is therefore dependent on A, i.e. each market participant will have a different valuation of this deal.

2.2 A receives a deterministic cashflow from B

Now we replace O by a counterparty B possibly bearing credit risk. A now funds $P(0,T)\nu_A(T)\nu_B(T)C(T)$ repaying $1_{\tau_B>T}C(T)$ at time T, i.e. the repayment is credit linked to B. It follows with the same arguments as above that

$$C(0) = P(0,T)\nu_A(T)\nu_B(T)C(T) \tag{2}$$

2.3 A receives a collaterized deterministic cashflow from B

Assume that the deal is cash collaterized now, which by definition means that at each time $t, t < T$ the market value of the deal is deposited by B on a cash account. This cash account can freely be used by A. A pays continuously the risk free interest rate on the amount on this account. In a discretized setting at time $T - \Delta t$ it must be the case that the account balance which is the market value of C(T) at $T - \Delta t$ is

$$C(T-\Delta t) = e^{-r(T-\Delta t)\Delta t}C(T) \tag{3}$$

where $r(t)$ denotes the risk free interest rate at time t in order to have a balance of $C(T)$ at time T. Going back one more step we get

$$C(T-2\Delta t) = e^{-r(T-2\Delta t)\Delta t}E(e^{-r(T-\Delta t)\Delta t}C(T)|\mathcal{F}_{T-2\Delta t}) \tag{4}$$

Repeating this argument and letting Δt approach zero we get

$$C(0) = E(e^{-\int_0^T r(t)dt}C(T)) \tag{5}$$

3 Credit Risk Notions

We assume that a credit risk free basis curve is defined such that incoming cashflows not exposed to credit risk are valued in a reasonable way. The choice of this basis curve is discussed in ... (?).

3.1 Default Intensity and Non Default Probability

Credit risk can be represented by a deterministic function $\lambda : [0,T] \to [0,\infty)$ called the default intensity. The probability of non default before and including time t is given by

$$\nu(t) := P(\tau > t) = e^{-\int_0^t \lambda(s)ds} \tag{6}$$

where τ is the (stochastic) default time. A risky cashflow $C(T)$ coming in at time T hat a time t value of

$$1_{\tau > t} P(\tau > T|\tau > t) N(t) E\left(\frac{C(T)}{N(T)}\right) \tag{7}$$

where N is a numeraire and the expectation taken in a suitable martingale measure. This means that the value is 0 if default has already occured and the default free value is weighted by the survival probability at payment time T conditional on survival up to valuation time t. Formula 7 can be rewritten using 6 as

$$1_{\tau > t} e^{-\int_t^T \lambda(s)ds} N(t) E\left(\frac{C(T)}{N(T)}\right) \tag{8}$$

Note that zero recovery is assumed here, i.e. in case of default no payment is received.

3.2 Z-Spread

When the numeraire is chosen to be the bank account $B(t) = e^{\int_0^t r(s)ds}$ with a short rate $r(s)$, formula 8 translates directly to

$$1_{\tau > t} E\left(C(T) e^{-\int_t^T (r(s)+\lambda(s))ds}\right) \tag{9}$$

which means, that the default intensity $\lambda(s)$ can also be interpreted as a instantaneous credit spread that should be added to the risk free short rate. We call $\lambda(s)$ the instantaneous z-spread. Both r and λ are expressed with continuous compounding here and must use the same day count convention to be consistent. A risky discount factor can then be written as

$$P(t,T) := E(e^{-\int_t^T (r(s)+\lambda(s))ds} | \mathcal{F}_t) \tag{10}$$

or also as

$$P(t,T) = e^{-(z(t,T)+\Lambda(t,T))T} \tag{11}$$

setting $\Lambda(t,T) := (T-t)^{-1} \int_t^T \lambda(s)ds$ and $z(t,T) = -(T-t)^{-1} \ln P(t,T)$ denoting the usual zero rate for maturity T as seen from t. $\Lambda(t,T)$ is called z-spread for maturity T as seen from t.

The non default probability up to maturity T given survival up to t can be written as

$$P(\tau > T|\tau > t) = e^{-\Lambda(t,T)(T-t)} \tag{12}$$

3.3 Par Spread

Instead of defining a shift in zero rates as e.g. in equation 10 one can also define shifts in par rates (i.e. par spreads) which through the boostrap procedure then implies a shift in the basis curves zero rates, i.e. z-spreads. Par rates here mean market instrument rates used for bootstrapping the curve. Alternatively par rates can also be defined to be swap rates for all maturities of the original curve instruments. This makes a difference e.g. if the basis curve is bootstrapped with FRAs.

3.4 Fair Floater Spread

Assume a floater on a specific Ibor index paying a margin s with maturity T. Requiring that this floater is worth par defines a z-spread for this maturity (we allow either z-spreads for shorter maturities are already defined by some procedure, possibly but not necessarily also by fair floater spreads, we allow also for a flat z-spread to be deduced from the quote).

We assume that the index is estimated on a forward curve different from the basis curve in general (and usually stripped relative to this basis curve). The z-spread then has to be implied.

3.5 CDS Spread

3.6 Asset Swap Spread

The front matter has various entries such as

`\title`, `\author`, `\date`, and `\thanks`

You should replace their arguments with your own.

This text is the body of your article. You may delete everything between the commands

`\begin{document}` . . . `\end{documant}`

in this file to start with a blank document.

4 The Most Important Features

Sectioning commands. The first one is the

`\section{The Most Important Features}`

command. Below you shall find examples for further sectioning commands:

4.1 Subsection

Subsection text.

4.1.1 Subsubsection

Subsubsection text.

Paragraph Paragraph text.

Subparagraph Subparagraph text.

Select a part of the text then click on the button Emphasize (H!), or Bold (Fs), or Italic (Kt), or Slanted (Kt) to typeset *Emphasize*, **Bold**, *Italics*, *Slanted* texts.

You can also typeset Roman, Sans Serif, SMALL CAPS, and `Typewriter` texts.

You can also apply the special, mathematics only commands $\mathbb{BLACKBOARD}$ $\mathbb{BOLD}$, $\mathcal{CALLIGRAPHIC}$, and $\mathfrak{fraktur}$. Note that blackboard bold and calligraphic are correct only when applied to uppercase letters A through Z.

You can apply the size tags – Format menu, Font size submenu – tiny, scriptsize, footnotesize, small, normalsize, large, Large, LARGE, huge and Huge.

You can use the `\begin{quote} etc. \end{quote}` environment for typesetting short quotations. Select the text then click on Insert, Quotations, Short Quotations:

> The buck stops here. *Harry Truman*
>
> Ask not what your country can do for you; ask what you can do for your country. *John F Kennedy*
>
> I am not a crook. *Richard Nixon*
>
> I did not have sexual relations with that woman, Miss Lewinsky. *Bill Clinton*

The Quotation environment is used for quotations of more than one paragraph. Following is the beginning of *The Jungle Books* by Rudyard Kipling. (You should select the text first then click on Insert, Quotations, Quotation):

> It was seven o'clock of a very warm evening in the Seeonee Hills when Father Wolf woke up from his day's rest, scratched himself, yawned and spread out his paws one after the other to get rid of sleepy feeling in their tips. Mother Wolf lay with her big gray nose dropped across her four tumbling, squealing cubs, and the moon shone into the mouth of the cave where they all lived. "*Augrh*" said Father Wolf, "it is time to hunt again." And he was going to spring down hill when a little shadow with a bushy tail crossed the threshold and whined: "Good luck go with you, O Chief of the Wolves; and good luck and strong white teeth go with the noble children, that they may never forget the hungry in this world."
>
> It was the jackal—Tabaqui the Dish-licker—and the wolves of India despise Tabaqui because he runs about making mischief, and telling tales, and eating rags and pieces of leather from the village

> rubbish-heaps. But they are afraid of him too, because Tabaqui, more than any one else in the jungle, is apt to go mad, and then he forgets that he was afraid of anyone, and runs through the forest biting everything in his way.

Use the Verbatim environment if you want LaTeX to preserve spacing, perhaps when including a fragment from a program such as:

```
#include <iostream>          // < > is used for standard libraries.
void main(void)              // ''main'' method always called first.
{
 cout << ''This is a message.'';
                             // Send to output stream.
}
```

(After selecting the text click on Insert, Code Environments, Code.)

4.2 Mathematics and Text

It holds [1] the following

Theorem 1 *(The Currant minimax principle.) Let T be completely continuous selfadjoint operator in a Hilbert space H. Let n be an arbitrary integer and let $u_1, \ldots, u_{n-1}$ be an arbitrary system of $n-1$ linearly independent elements of H. Denote*

$$\max_{\substack{v \in H, v \neq 0 \\ (v,u_1)=0,\ldots,(v,u_n)=0}} \frac{(Tv, v)}{(v, v)} = m(u_1, \ldots, u_{n-1}) \tag{13}$$

Then the n-th eigenvalue of T is equal to the minimum of these maxima, when minimizing over all linearly independent systems $u_1, \ldots u_{n-1}$ in H,

$$\mu_n = \min_{u_1,\ldots,u_{n-1} \in H} m(u_1, \ldots, u_{n-1}) \tag{14}$$

The above equations are automatically numbered as equation (13) and (14).

4.3 List Environments

You can create numbered, bulleted, and description lists using the tag popup at the bottom left of the screen.

1. List item 1
2. List item 2
   (a) A list item under a list item.
   The typeset style for this level is different than the screen style. The screen shows a lower case alphabetic character followed by a period while the typeset style uses a lower case alphabetic character surrounded by parentheses.

    (b) Just another list item under a list item.
        i. Third level list item under a list item.
            A. Fourth and final level of list items allowed.

- Bullet item 1

- Bullet item 2

    – Second level bullet item.

        * Third level bullet item.

            · Fourth (and final) level bullet item.

Description List Each description list item has a term followed by the description of that term. Double click the term box to enter the term, or to change it.

Bunyip Mythical beast of Australian Aboriginal legends.

4.4 Theorem-like Environments

The following theorem-like environments (in alphabetical order) are available in this style.

Acknowledgement 2 *This is an acknowledgement*

Algorithm 3 *This is an algorithm*

Axiom 4 *This is an axiom*

Case 5 *This is a case*

Claim 6 *This is a claim*

Conclusion 7 *This is a conclusion*

Condition 8 *This is a condition*

Conjecture 9 *This is a conjecture*

Corollary 10 *This is a corollary*

Criterion 11 *This is a criterion*

Definition 12 *This is a definition*

Example 13 *This is an example*

Exercise 14 *This is an exercise*

Lemma 15 *This is a lemma*

Proof. This is the proof of the lemma. ■

Notation 16 *This is notation*

Problem 17 *This is a problem*

Proposition 18 *This is a proposition*

Remark 19 *This is a remark*

Solution 20 *This is a solution*

Summary 21 *This is a summary*

Theorem 22 *This is a theorem*

Proof of the Main Theorem. This is the proof. ■

This text is a sample for a short bibliography. You can cite a book by making use of the command `\cite{KarelRektorys}`: [1]. Papers can be cited similarly: [2]. If you want multiple citations to appear in a single set of square brackets you must type all of the citation keys inside a single citation, separating each with a comma. Here is an example: [2, 3, 4].

References

[1] Rektorys, K., *Variational methods in Mathematics, Science and Engineering*, D. Reidel Publishing Company, Dordrecht-Hollanf/Boston-U.S.A., 2th edition, 1975

[2] Bertóti, E.: *On mixed variational formulation of linear elasticity using nonsymmetric stresses and displacements*, International Journal for Numerical Methods in Engineering., **42**, (1997), 561-578.

[3] Szeidl, G.: *Boundary integral equations for plane problems in terms of stress functions of order one*, Journal of Computational and Applied Mechanics, **2**(2), (2001), 237-261.

[4] Carlson D. E.: *On Günther's stress functions for couple stresses*, Quart. Appl. Math., **25**, (1967), 139-146.

A The First Appendix

The appendix fragment is used only once. Subsequent appendices can be created using the Section Section/Body Tag.