

REPRESENTATIVE BASKET METHOD APPLIED

P. CASPERS

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ABSTRACT. We apply the representative basket method to different use cases.

1. DESCRIPTION OF THE METHOD

We follow the description in Piterbarg, Interest Rate Modelling. We consider a one factor interest rate model such as the Hull White Model oder Markov Functional Model and denote the driving variable by x . More explicitly in the Hull White case we use $x(t) := r(t) - f(0, t)$ where r is the short rate and $f(0, \cdot)$ the instantaneous forward rate as seen from today ($t = 0$). In the Markov Functional Model x denotes the driver as e.g. often defined by $dx = \sigma e^{at} dW$ with a “Hull White” mean reversion a .

We may use the standardized version y of x instead of x directly, i.e. the affine transformation of x with expectation zero and variance one. In theory the result will not depend on whether we use x or y but in the practical numerical implementation of the procedure it seems more appropriate to work with the standardized variable.

The setting is that an option to exercise into an underlying (which we shall call exotic in the following) is given with expiry time t and underlying value $U(t, x)$, where we stress the dependency of the underlying value on t and x , but omit the (possible) dependency on the rest of the model parameters (like the model volatility, mean reversion, yield term structure or numeraire surface in the case of the markov model).

Now the question arises how to calibrate the model to this specific option. The idea of the representative basket approach is to find a market quoted option whose underlying value mimicks the value of the exotic underlying above at t as close as possible in all states of the model. If we can find such an underlying then the market price of the option to exercise into the underlying should be close to the price of the exotic option. A central assumption here is that the model dynamics is a reasonable framework to price the exotic option and the market option at all.

We specialize the setting to the case where we want to match the exotic option by standard swaptions, i.e. options to exercise into plain vanilla fix versus float swaps. The exotic option may be given for example by an underlying with non constant nominal, rate, indices, start delay, fixing time and some more. We will give several examples later on for what we call exotic here..

To do so we fix a value x_0 of the state variable and match the first three terms of a taylor expansion, i.e. we look for a standard Underlying V with

$$\begin{aligned}
(1.1) \quad & V(x_0) = U(x_0) \\
(1.2) \quad & \frac{\partial V}{\partial x}(x_0) = \frac{\partial U}{\partial x}(x_0) \\
(1.3) \quad & \frac{\partial^2 V}{\partial x^2}(x_0) = \frac{\partial^2 U}{\partial x^2}(x_0)
\end{aligned}$$

The free parameters of a standard swaption's underlying are the nominal, the rate and the maturity. These three variables can be used to minimize the differences of the right and left hand sides of the equations above.

It is not clear that even if we find a V which matches U in the above sense well that the match is globally well as desired originally, i.e. that $U(x) \approx V(x)$ is fulfilled for a wide enough range of values of x . Therefore we should always test the global match before using V for calibration.

The choice of x_0 is not obvious. We will always set x_0 to the expectation of $x(t)$. In terms of y this means that $y_0 = 0$.

2. APPLICATIONS IN THE HULL WHITE MODEL

2.1. Standard Bermudan Swaptions. We start with the easiest case of a standard bermudan swaption. To create a specific case we consider a flat yield termstructure at 3% continuously compounded instantaneous forward rate and day counter Act/365fixed. The swaption volatility is flat at 20% with the same day counter. We consider EUR swaps with standard conventions against Euribor 6m. The delay between exercise date and underlying start is 2 business days.

The exotic underlying is an 10y payer swap with fixed rate 3.5%, to be exercised into on a yearly basis from year 1 to 9. The nominal is 100 EUR.

The evaluation date is June 17th, 2013.

TABLE 1. Standard Bermudan Swaption Calibration Basket

Option Expiry	End Date	Expiry Time	End Time	Nominal	Rate
June 17th, 2014	June 19th, 2023	1.000000	10.010959	99.999985	0.035000
June 17th, 2015	June 19th, 2023	2.000000	10.010959	100.000000	0.035000
June 16th, 2016	June 20th, 2023	3.000000	10.013699	99.999991	0.034999
June 15th, 2017	June 19th, 2023	3.997260	10.010959	100.000037	0.035000
June 15th, 2018	June 19th, 2023	4.997260	10.010959	100.000000	0.035000
June 17th, 2019	June 19th, 2023	6.002740	10.010959	100.000096	0.035000
June 17th, 2020	June 19th, 2023	7.005479	10.010959	99.999925	0.035000
June 17th, 2021	June 21st, 2023	8.005479	10.016438	100.016358	0.034997
June 16th, 2022	June 20th, 2023	9.002740	10.013699	100.026752	0.034996

Table 1 shows the resulting calibration basket. The swaptions' maturity coincide with the deal maturity, i.e. they constitute a classical coterminal swaption basket. The nominal of the swaptions is also equal to the deal's nominal. The nominal is not relevant for the calibration, but is a nice additional information in cases where the deal is amortizing or accreting. The rate of the calibrating swaption is set to the deal's strike. All this is not very surprising, nevertheless a good test case because maturity, nominal and rate are resulting from a numerical optimization.

The above basket is computed in a Hull White Model with fixed mean reversion $\kappa = 1\%$. Since the model is not calibrated prior to the computation of the calibration basket this basket is computed w.r.t. an initial model volatility of 1% flat.

As a next step we calibrate the Hull White Model volatility to the calibration basket in table 1. For this we use a piecewise constant volatility with step dates equal to the option expiries. Table 2 shows the result of this calibration. The displayed model volatilities are to be understood from the previous date (or today for the first date) until the displayed date.

TABLE 2. Model Volatility calibrated to the Standard Bermudan Swaption Calibration Basket

Option Expiry	Model volatility
June 17th, 2014	0.006619
June 17th, 2015	0.006605
June 16th, 2016	0.006603
June 15th, 2017	0.006574
June 15th, 2018	0.006545
June 17th, 2019	0.006520
June 17th, 2020	0.006524
June 17th, 2021	0.006502

It is rather obvious that the calibration basket if computed w.r.t. the now calibrated model will not change, although the underlying's price depends on the model volatility. Therefore we postpone tests for the stability of the calibration basket when computed in differently calibrated models for later, more interesting cases. The same holds for the check of the global fit of the underlyings' npvs - since the calibration basket essentially consists of the same swaptions as the "exotic" underlying in this example, it is obvious that we have a good global fit.

2.2. Amortizing Bermudan Swaptions. Using the same market data as in section 2.1 we modify the deal given there by letting the nominal amortize linearly to zero.

Again starting with an initial model vol of 1% we get the calibration basket in table 3.

TABLE 3. Amortizing Bermudan Swaption Calibration Basket #1

Option Expiry	End Date	Expiry Time	End Time	Nominal	Rate
June 17th, 2014	September 21st, 2020	1.000000	7.268493	72.303641	0.035004
June 17th, 2015	January 19th, 2021	2.000000	7.597260	64.460264	0.035002
June 16th, 2016	May 20th, 2021	3.000000	7.928767	56.542487	0.035002
June 15th, 2017	October 19th, 2021	3.997260	8.345205	48.727598	0.035005
June 15th, 2018	February 21st, 2022	4.997260	8.687671	41.019669	0.035028
June 17th, 2019	June 20th, 2022	6.002740	9.013699	33.448986	0.034998
June 17th, 2020	October 19th, 2022	7.005479	9.345205	25.606970	0.034991
June 17th, 2021	February 21st, 2023	8.005479	9.687671	17.687368	0.035026
June 16th, 2022	June 20th, 2023	9.002740	10.013699	10.002679	0.034996

We proceed with the calibration to the calibration basket #1 in table 3. This yields the calibrated model volatility as displayed in table 4.

TABLE 4. Model Volatility calibrated to Basket #1

Option Expiry	Model volatility
June 17th, 2014	0.006536
June 17th, 2015	0.006545
June 16th, 2016	0.006549
June 15th, 2017	0.006561
June 15th, 2018	0.006543
June 17th, 2019	0.006529
June 17th, 2020	0.006579
June 17th, 2021	0.006548

We recalculate the calibration basket w.r.t. to the now calibrated model. From this we get the basket in table 5.

TABLE 5. Amortizing Bermudan Swaption Calibration Basket #2

Option Expiry	End Date	Expiry Time	End Time	Nominal	Rate
June 17th, 2014	August 19th, 2020	1.000000	7.178082	72.407634	0.035002
June 17th, 2015	January 19th, 2021	2.000000	7.597260	64.644593	0.034995
June 16th, 2016	May 20th, 2021	3.000000	7.928767	56.710404	0.034993
June 15th, 2017	October 19th, 2021	3.997260	8.345205	48.892540	0.034999
June 15th, 2018	February 21st, 2022	4.997260	8.687671	41.176970	0.035014
June 17th, 2019	June 20th, 2022	6.002740	9.013699	33.557383	0.034989
June 17th, 2020	October 19th, 2022	7.005479	9.345205	25.659932	0.034978
June 17th, 2021	February 21st, 2023	8.005479	9.687671	17.700507	0.035012
June 16th, 2022	June 20th, 2023	9.002740	10.013699	10.003736	0.034996

The baskets in tables 3 and 5 are very similar, and so is a recalibrated model to the last basket #2. This is shown in comparison to the prior calibration to basket #1 in table 6.

TABLE 6. Model volatility for Basket #1 and #2

Option Expiry	Model volatility #1	Model volatility #2
June 17th, 2014	0.006536	0.006533
June 17th, 2015	0.006545	0.006546
June 16th, 2016	0.006549	0.006549
June 15th, 2017	0.006561	0.006561
June 15th, 2018	0.006543	0.006539
June 17th, 2019	0.006529	0.006530
June 17th, 2020	0.006579	0.006576
June 17th, 2021	0.006548	0.006545

2.3. DV01 and DV02 and Vega for Amortizing Bermudan Swaptions.

We compute the standard sensitivities in the setup of section 2.2. We do this using different strategies. The first strategy is just to shift the rate curve and volatility without recalibrating the model. Obviously the vega will be zero in this case. The DV01 and DV02 will reflect the shift in the yield curve, but the model volatility will stay unchanged. The second strategy is to shift the market parameters and then recalibrate the model to the initial calibration basket, i.e. the instruments in the basket remain the same w.r.t. strike, maturity and nominal. The third strategy is additionally to the recalibration of the model to rebuild the calibration basket via the matching process described above and then recalibrate the model to this new basket.

The rate shift is 10bp up and down, the DV01 is the central DV01 coming from these shifts. The volatility shift is 1% up.

The results are displayed in table 7.

TABLE 7. Sensitivities under different recalibration strategies

Strategy	DV01	DV02	Vega
No recalibration	0.010902	0.000125	0.000000
Model recalibration	0.013097	0.00153	0.069963
Basket & model recalibration	0.013094	0.00153	0.069967

2.4. Global Stability of Underlying NPV Match. We take the example from the previous section 2.2 to investigate how well the exotic underlying NPV is matched by the standard underlying on a global scale. For this we plot both NPVs in the range of $-5...5$ standard deviations of the state variable. We choose the expiry date June 14th, 2019 for this test.

Figure 1 shows the result, which looks very satisfactory in this case.

2.5. Step Up / Down Coupons. Again we start with the setup in 2.1. This time we modify the coupon to step up linearly from 3.5% to 8.5% in steps of 0.5%. Table 8 shows the result (which is the initial basket, we do not show the (very similar) basket computed w.r.t. the calibrated model).

TABLE 8. Step up Bermudan Swaption Calibration Basket

Option Expiry	End Date	Expiry Time	End Time	Nominal	Rate
June 17th, 2014	July 19th, 2023	1.000000	10.093151	102.732548	0.058106
June 17th, 2015	June 19th, 2023	2.000000	10.010959	102.138877	0.060935
June 16th, 2016	June 20th, 2023	3.000000	10.013699	101.608610	0.063760
June 15th, 2017	June 19th, 2023	3.997260	10.010959	101.147064	0.066548
June 15th, 2018	June 19th, 2023	4.997260	10.010959	100.757428	0.069317
June 17th, 2019	June 19th, 2023	6.002740	10.010959	100.444229	0.072051
June 17th, 2020	June 19th, 2023	7.005479	10.010959	100.208110	0.074743
June 17th, 2021	June 21st, 2023	8.005479	10.016438	100.087119	0.077400
June 16th, 2022	June 20th, 2023	9.002740	10.013699	100.027370	0.079992

The sensitivities under the different strategies are shown in 9.

FIGURE 1. Global fit of amortizing swap underlying NPV (solid exotic npv, dotted standard underlying npv)

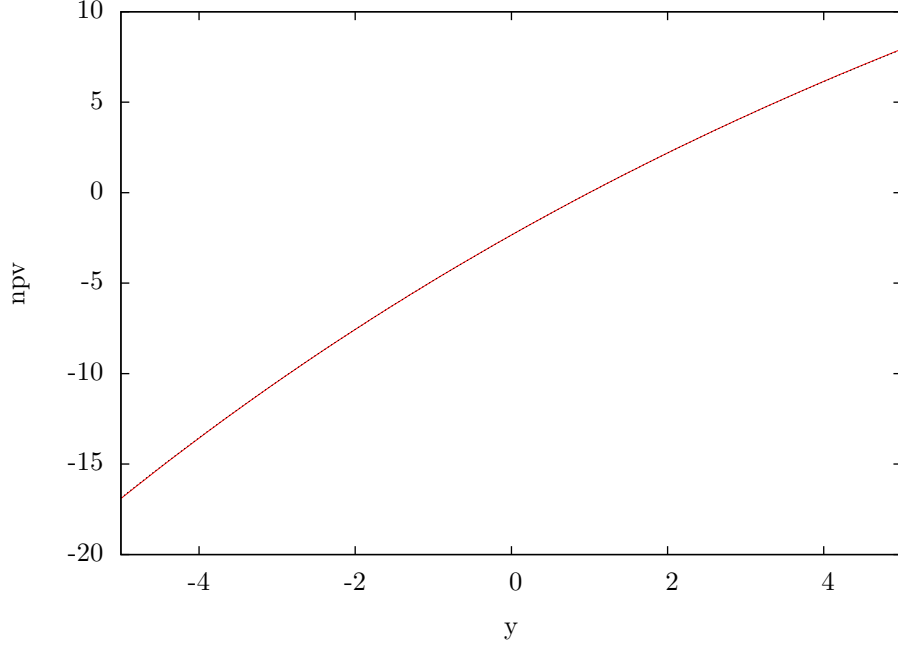


TABLE 9. Sensitivities under different recalibration strategies

Strategy	DV01	DV02	Vega
No recalibration	0.002292	0.000020	0.000000
Model recalibration	0.003904	0.000057	0.060656
Basket & model recalibration	0.003904	0.000057	0.060654

2.6. Fixed rate callable bonds. We aim to apply the method above to fixed rate bonds with one or several call rights. The set up concerning the market data is the same as in 2.1. We consider a long position in a 10y bond paying a coupon of 3.5% yearly, though not entirely realistic using the same conventions as in the fixed leg of a vanilla Euribor Swap. The bond is yearly callable by the issuer with a notice period of 2 days, again just adopting the swaption conventions for simplicity here. The notional of the bond and the redemption value on each call date is 100.0.

The credit risk of the bond is expressed as a z-Spread of 100bp (w.r.t. continuous compounding and an Act/365 Fixed day counter).

With that the dirty fair value (as of the evaluation date) is 95.2669. The calibration basket computed w.r.t. an initial model volatility of 1% is displayed in table 10.

What can be seen is that the strike of the calibrating swaption is the deal strike adjusted by a quantity roughly corresponding to the z-Spread (note that this has different conventions compared to the swaption fixed rate of course).

TABLE 10. Bermudan Callable Fixed Rate Bond Calibration Basket

Option Expiry	End Date	Expiry Time	End Time	Nominal	Rate
June 17th, 2014	April 19th, 2023	1.000000	9.843836	97.656084	0.024691
June 17th, 2015	May 19th, 2023	2.000000	9.926027	97.907645	0.024690
June 16th, 2016	May 22nd, 2023	3.000000	9.934247	98.174928	0.024692
June 15th, 2017	May 19th, 2023	3.997260	9.926027	98.473170	0.024693
June 15th, 2018	June 19th, 2023	4.997260	10.010959	98.774638	0.024693
June 17th, 2019	June 19th, 2023	6.002740	10.010959	99.085436	0.024693
June 17th, 2020	June 19th, 2023	7.005479	10.010959	99.378093	0.024701
June 17th, 2021	June 21st, 2023	8.005479	10.016438	99.662271	0.024701
June 16th, 2022	June 20th, 2023	9.002740	10.013699	99.915250	0.024701

The model calibration w.r.t. the basket in table 10 can be seen in table 11. The price of the call right in this model is -4.630688 . Finally we can compute sensitivities with the different recomputation strategies mentioned in the preceding sections. The results are displayed in table 12.

TABLE 11. Model Volatility Fixed Rate Bond

Option Expiry	Model volatility
June 17th, 2014	0.005600
June 17th, 2015	0.005581
June 16th, 2016	0.005573
June 15th, 2017	0.005554
June 15th, 2018	0.005534
June 17th, 2019	0.005542
June 17th, 2020	0.005498
June 17th, 2021	0.005503

TABLE 12. Sensitivities Callable Fix Rate Bond under different recalibration strategies

Strategy	DV01	DV02	Vega
No recalibration	-0.052415	0.000427	0.000000
Model recalibration	-0.048332	0.000481	0.121620
Basket & model recalibration	-0.048332	0.000481	0.121619

2.7. **Callable Zero Bonds.** ...todo...

2.8. **Switchable Bonds.** ...todo...

2.9. **Non Standard Tenor Swaptions.** Consider an EUR swaption (5y into 5y, to fix an example) on an underlying Euribor 3m swap. There are no direct market quotes for implied volatilities for such a swaption, because the floating tenor is not the standard 6m tenor.

We assume the relevant 3m forward swap rate to be 3% and the 6m rate to be 4% and apply the representative basket approach to find a Euribor 6m swaption that matches the non standard 3m swaption.

The evaluation date is June 17th, 2013. The model parameters are 1% for both volatility and reversion. We set the nominal of the 3m swaption to 100.

Table 13 summarizes the results: Roughly the suggestion is to price a 6m swaption instead of the 3m swaption with a strike shifted by the basis between the two swap rates. This recipe is fine tuned by a slightly longer maturity (3 months), lower nominal (by around 5 percent) and higher strike (by around 5 basispoints).

TABLE 13. 3m 6m swaption volatility conversion

Strike 3m Swaption	Expiry	Maturity	Nominal	Rate
2%	June 19th, 2018	September 21st, 2023	94.4664	0.0304037
3%	June 19th, 2018	September 21st, 2023	94.4354	0.0405083
4%	June 19th, 2018	September 21st, 2023	94.4056	0.0506342

3. COUNTEREXAMPLES

3.1. CMS Swaptions. We consider a bermudan option with yearly rights to exercise into a swap exchanging (also yearly) CMS10Y coupons with EUR EURIBOR 6M. The valuation of such a deal can e.g. be done in a Hull White 1F model, or a Markov Functional 1F model. The question is if a reasonable representative basket can be computed and if yes, whether it is close to a basket of (atm) coterminial swaptions (which seems to be done often in practice in case of the Markov model).

The nominal of the deal is 100,000 EUR.

3.1.1. Hull White Model. The yield term structure is considered to be flat at 3%, the swaption volatility structure flat at 30%. We start with a Hull White Model with reversion 2% and model volatility at 1%. Table 14 shows the calibration basket computed in this model as well as the model volatility calibrated to this basket. We recompute the basket on the calibrated model and calibrate the model to this new basket. The result is shown in table 15. After 4 of such iterations we arrive at the basket and model volatility in table 16. Clearly the computed calibration basket depends on the model parameters much more than in the previous examples. Even worse, iterating the procedure does not converge to a “fixed point” basket.

This is despite the fact that the underlying is well matched globally as can be seen in table 2 for the (example) expiry date November 13th, 2014. However since this can not be made consistent with the model parameters, it is not of much use.

TABLE 14. Initial basket for bermudan cms swaption and model calibration

option date	maturity date	nominal	strike	model vol
November 13th, 2014	January 18th, 2027	6766.21	0.0171256	0.00765986
November 12th, 2015	October 16th, 2026	6612.48	0.0189743	0.00827473
November 11th, 2016	September 15th, 2026	6363.61	0.0208866	0.00890572
November 13th, 2017	August 17th, 2026	6124.51	0.0226689	0.00935093
November 13th, 2018	June 15th, 2026	5832.46	0.0243944	0.00973056
November 13th, 2019	May 15th, 2026	5457.37	0.0260259	0.0100563
November 12th, 2020	April 16th, 2026	4806.48	0.0279102	0.0106603
November 11th, 2021	April 15th, 2026	3965.47	0.0295338	0.0108335
November 11th, 2022	April 15th, 2026	2568.67	0.0311619	0.0111066

TABLE 15. Basket for bermudan cms swaption and model calibration after 1 iteration

option date	maturity date	nominal	strike	model vol
November 13th, 2014	January 18th, 2027	6754.08	0.0141528	0.00705231
November 12th, 2015	November 16th, 2026	6592.62	0.0158008	0.00764825
November 11th, 2016	October 15th, 2026	6342.6	0.0176291	0.00830588
November 13th, 2017	August 17th, 2026	6098	0.0194733	0.00882439
November 13th, 2018	July 15th, 2026	5799.75	0.0213665	0.00940993
November 13th, 2019	May 15th, 2026	5420.18	0.0232939	0.00984241
November 12th, 2020	May 18th, 2026	4782.37	0.0255655	0.0106748
November 11th, 2021	April 15th, 2026	3944.7	0.0276639	0.0108926
November 11th, 2022	March 16th, 2026	2583.39	0.0298118	0.0114614

TABLE 16. Basket for bermudan cms swaption and model calibration after 4 iterations

option date	maturity date	nominal	strike	model vol
November 13th, 2014	February 17th, 2027	6733.68	0.0114741	0.00646804
November 12th, 2015	December 16th, 2026	6569.9	0.0128298	0.00699711
November 11th, 2016	November 16th, 2026	6322.62	0.0143681	0.00759651
November 13th, 2017	September 15th, 2026	6071.45	0.0159748	0.00811652
November 13th, 2018	August 17th, 2026	5768.42	0.0176925	0.00869546
November 13th, 2019	June 15th, 2026	5388.46	0.019509	0.00915953
November 12th, 2020	June 16th, 2026	4732.83	0.0217088	0.0100562
November 11th, 2021	May 15th, 2026	3906.26	0.0238182	0.0104082
November 11th, 2022	April 15th, 2026	2557.7	0.0261311	0.0110801

3.1.2. *Markov Model.* In the markov functional model the situation even gets worse in the sense that the approximation to the exotic underlying NPV function is - depending on the smile the model is calibrated to - only locally good. In addition the same self consistency problems as for the Hull White Model occur, as described in the previous section. Figures 3 and 4 demonstrates this for option expiry November 13, 2014, for a calibration to a flat resp. SABR smile (the latter with parameters $\alpha = 0.15$, $\beta = 0.80$, $\rho = -0.30$, $\nu = 0.20$), the rest of the setup being the same as in the previous section.

E-mail address, P. Caspers: pcaspers1973@googlemail.com

FIGURE 2. Global fit of CMS swap underlying NPV in Hull White
(solid exotic npv, dotted standard underlying npv)

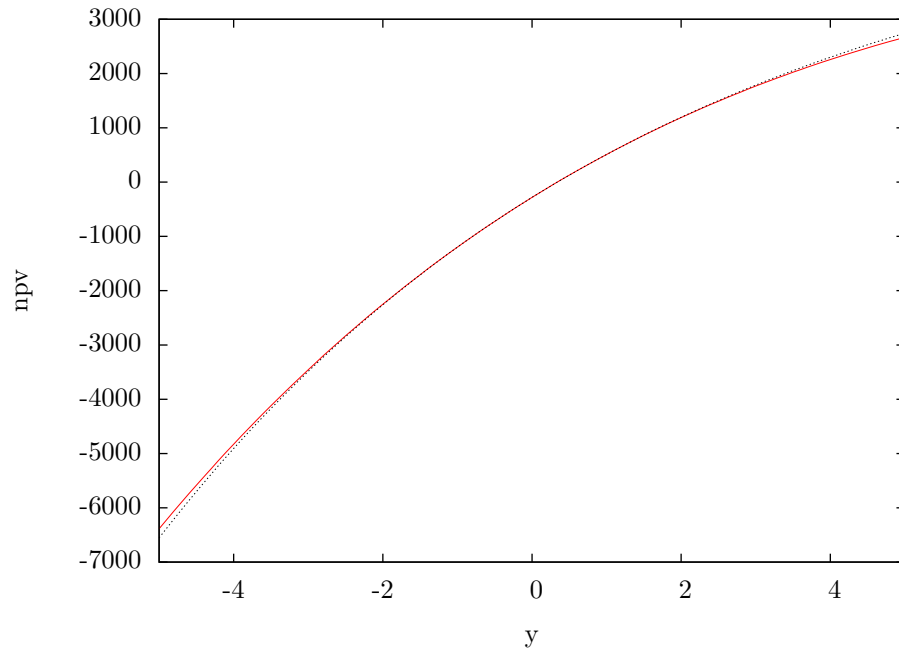


FIGURE 3. Global fit of CMS swap underlying NPV in Markov (Flat Smile, solid exotic npv, dotted standard underlying npv)

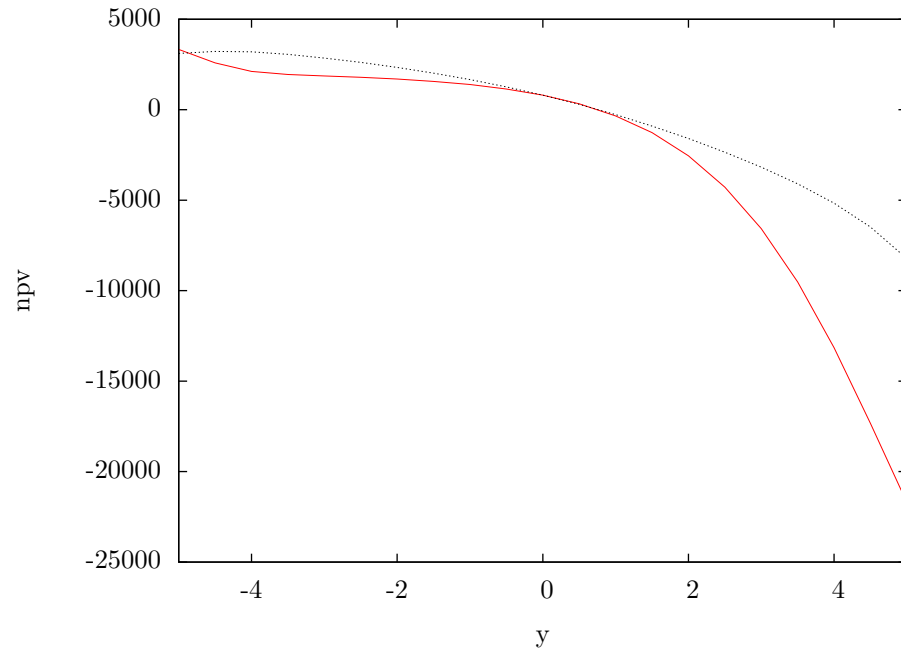


FIGURE 4. Global fit of CMS swap underlying NPV in Markov (SABR Smile, solid exotic npv, dotted standard underlying npv)

