1 Bayesian Inference

Probability Theory is used to quantify uncertainties.

Probability Distributions are associated with uncertain
measurements.

The specification of probability distributions to the
uncertain deterministic relations between some of them
defines a statistical model.

y = percentage of people who remembered having watched the ad
x = advertising expenditure
/pi = awareness probability

Simplest link between advertising expenditure and awareness
is given by the linear relation

\pi = \alpha + \Beta x

2 Markov Chain Monte Carlo

3 General Overview

3.1 Stochastic simulation

3.2 Bayesian inference

3.3 Approximate methods of inference

3.4 Markov Chains

3.5 Gibbs Sampling

3.6 Metropolis-Hastings algorithms

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############################################################################################
#
# Example 6.1 (pages 192-3) âĂŞ Figures 6.1 (page 194)  - Random walk Metropolis-Hastings al
#
# y[i] : velocity of an enzymatic reacton (in counts/min/min) as a function of
#        substrate concentration
# x[i] : (in ppm) where the enzyme has been treated with Puromycin
#
#
```

```
# Referencia: Carlin and Louis (1998) - Bayes and Empirical
# Bayes Methods for Data Analysis - Chapman & Hall/CRC
#
# paginas: 233-234.
#
############################################################################################
#
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#
############################################################################################
mu      = function(th,x){gamma+alpha*x/(th+x)}
like    = function(th){prod(dnorm(y,mu(th,x),sqrt(tau2)))}
loglike = function(th){sum(dnorm(y,mu(th,x),sqrt(tau2),log=TRUE))}
logpost = function(th){dnorm(th,theta0,Ctheta,log=TRUE)+loglike(th)}
n       = 12
x       = c(0.02,0.02,0.06,0.06,0.11,0.11,0.22,0.22,0.56,0.56,1.10,1.10)
y       = c(76,47,97,107,123,139,159,152,191,201,207,200)
gamma   = 50
alpha   = 170
tau2    = 126
theta0  = 0.0
Ctheta  = 100
thetas  = seq(0.08,0.20,length=1000)
likes   = NULL
for (i in 1:1000)
  likes = c(likes,like(thetas[i]))
thetamle = thetas[likes==max(likes)]
xs       = seq(min(x),max(x),length=1000)
mus      = NULL
for (i in 1:1000)
  mus = c(mus,mu(thetamle,xs[i]))

par(mfrow=c(1,2))
plot(thetas,likes,xlab="theta",ylab="likelihood",type="l")
plot(x,y)
lines(xs,mus,col=2)

##############################################################################
# Random Walk Metropolis-Hastings
##############################################################################
set.seed(192796)
```

```
Vtheta = 0.1
theta  = 0.4
burnin = 0
M      = 1000
sample = theta
ind    = 0
for (iter in 1:(burnin+M-1)){
  th1 = rnorm(1,theta,Vtheta)
  prob = exp(logpost(th1)-logpost(theta))
  if (runif(1) < prob){
    theta = th1
    ind   = ind + 1
  }
  sample = c(sample,theta)
}
sample = sample[(burnin+1):length(sample)]
acceptancerate = length(unique(sample))/(M+1)
mean=mean(sample)
sd=sqrt(var(sample))
L=quantile(sample,0.025)
U=quantile(sample,0.975)
c(mean,sd,L,U)


# Figure 6.1
# ----------
par(mfrow=c(1,1))
plot(sample,xlab="iteration",ylab=expression(theta),main="",type="l",cex=3,axes=F)
axis(1)
axis(2)
```

4 MCMCpack

```
install.packages("MCMCpack")
data(swiss)
posterior1 <- MCMCregress(Fertility ~ Agriculture + Examination + Eduction + Catholic + Infa
summary(posterior1)
```

source:

Markov Chain Monte Carlo. Dani Gamerman and Hedibert F. Lopes. ChapmanHall CRC Boca Raton Florida. 2006.