Solutions

Q.1

For tank 1

$$A_1 \frac{dh_1}{dt} = q_i - q_1 \qquad (1)$$

$$q_1 = \frac{h_1}{R_1} \qquad (2)$$

$$A_1 \frac{dh_1}{dt} = q_i - \frac{h_1}{R_1}$$

$$0 = \overline{q_i} - \frac{\overline{h_1}}{R_1}$$

$$A_1 \frac{dh_1{}'}{dt} = q_i{}' - \frac{h_1{}'}{R_1} \qquad (3)$$

Similarly, $q_1{}' = \frac{h_1{}'}{R_1}$ (4)

Taking Laplace of (3)

$$A_1 S H_1'(S) = Q_i'(S) - \frac{H_1'(S)}{R_1}$$

$$A_1 R_1 S H_1'(S) = R_1 Q_i'(S) - H_1'(S)$$

$$H_1'(S)[A_1 R_1 S + 1] = R_1 Q_i'(S)$$

$$\frac{H_1'(S)}{Q_i'(S)} = \frac{R_1}{A_1 R_1 S + 1} = \frac{\kappa_1}{\tau_1 S + 1} \qquad (5)$$

$$\kappa_1 = R_1 \quad \tau_1 = A_1 R_1$$

Similarly, transfer function relating $Q_1'(S)$ and $H_1'(S)$ can be attained by transforming (4),

$$\frac{Q_1'(S)}{H_1'(S)} = \frac{1}{R_1} = \frac{1}{\kappa_1} \qquad (6)$$

For tank 2

$$A_2 \frac{dh_2}{dt} = q_1 - q_2$$

$$A_2 \frac{dh_2}{dt} = q_1 - \frac{h_2}{R_2}$$

$$0 = \overline{q_i} - \frac{\overline{h_2}}{R_2}$$

$$A_2 \frac{dh_2{}'}{dt} = q_1{}' - \frac{h_2{}'}{R_2}$$

$$A_2 S H_2'(S) = Q_1'(S) - \frac{H_2'}{R_2}$$

$$A_2 R_2 S H_2'(S) = R_2 Q_1'(S) - H_2'(S)$$

$$\frac{H_2'(S)}{Q_1'(S)} = \frac{R_2}{A_2 R_2 S + 1} = \frac{\kappa_2}{\tau_2 S + 1} \qquad (7)$$

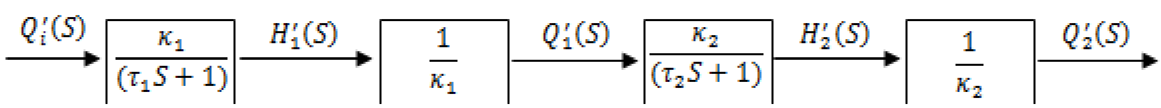
$$\kappa_2 = R_2 \quad \tau_2 = A_2 R_2$$

$$q_2 = \frac{h_2}{R_2}$$

$$Q_2'(S) = \frac{H_2'(S)}{R_2} \qquad \frac{Q_2'(S)}{H_2'(S)} = \frac{1}{\kappa_2}$$

$$\frac{Q_2'(S)}{Q_i'(S)} = \frac{Q_2'(S)}{H_2'(S)} \frac{H_2'(S)}{Q_1'(S)} \frac{Q_1'(S)}{H_1'(S)} \frac{H_1'(S)}{Q_i'(S)} = \frac{1}{\kappa_2} \frac{\kappa_2}{(\tau_2 S + 1)} \frac{1}{\kappa_1} \frac{\kappa_1}{(\tau_1 S + 1)} = \frac{1}{(\tau_1 S + 1)(\tau_2 S + 1)}$$

The figure below is a block diagram showing the information flow for this system



Q.2)

a) $$2\frac{dy_1}{dt} = -2y_1 - 3y_2 + 2u_1 \qquad (1)$$

$$\frac{dy_2}{dt} = 4y_1 - 6y_2 + 2u_1 + 4u_2 \qquad (2)$$

Taking Laplace transform of the above equations and rearranging,

$$(2s+2)Y_1(s) + 3Y_2(s) = 2U_1(s) \qquad (3)$$

$$-4\,Y_1(s) + (s+6)Y_2(s) = 2U_1(s) + 4U_2(s) \qquad (4)$$

Solving Eqs. 3 and 4 simultaneously for $Y_1(s)$ and $Y_2(s)$,

$$Y_2(s) = \frac{2(s+3)U_1(s) + 4(s+1)U_2(s)}{(s+3)(s+4)}$$

$$Y_1(s) = \frac{(s+3)U_1 - 6U_2(s)}{(s+3)(s+4)}$$

Therefore,

$$\frac{Y_1}{U_1} = \frac{1}{(s+4)}$$

$$\frac{Y_1}{U_2} = \frac{-6}{(s+3)(s+4)}$$

$$\frac{Y_2}{U_1} = \frac{2}{(s+4)}$$

$$\frac{Y_2}{U_2} = \frac{4(s+1)}{(s+3)(s+4)}$$

Q.3)

a) K = 5

b) y(t) = 10

c) y(10) = 6.32; 63.2% of the final value

d) y(t) = 0