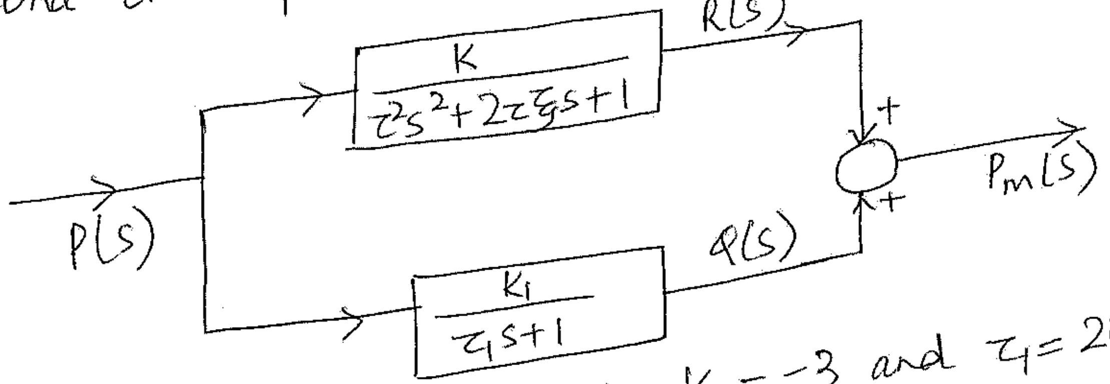
Additional Question

Ques: Consider the following model structure where second order process is in parallel with first order process.



Preliminary test shows that $K_1 = -3$ and $\tau_1 = 20$.
An additional test is made by giving a step change of P from 2 to 4 and the results are tabulated below:

Time:	0	5	10	15	20	30	40	50	60	70	80	90
Pm :	12	15	24	28	26	16	18	20	18	17	18	18.

Estimate K, τ and ξ.

Answer

$$P(t) = 4 - 2 = 2.$$

$$P(s) = 2/s.$$

$$\frac{Q(s)}{P(s)} = \frac{K_1}{\tau_1 s + 1} \Rightarrow Q_1(s) = \frac{K_1}{\tau_1 s + 1} \times P_1(s)$$

$$Q_1(s) = \frac{K_1}{\tau_1 s + 1} \times \frac{2}{s} \Rightarrow q'(t) = 2K_1(1 - e^{-t/\tau_1})$$

$$= 2 \times (-3)[1 - e^{-t/20})$$

$$= -6(1 - e^{-t/20})$$

$$P_m(s) = R(s) + Q(s)$$

$$P_m'(t) = R'(t) + q'(t)$$

$$p_m(t) - p_m(0) = r'(t) + q'(t)$$

$$p_m(t) = r'(t) + q'(t) + p_m(0).$$

or

$$r'(t) = p_m(t) - p_m(0) - q'(t)$$

$$r'(t) = P_m(t) - 12 + 6(1 - e^{-t/20})$$

① $$K = \frac{r'(t\to\infty)}{p(t\to\infty) - p(t\to 0)} = \frac{18 - 12 + 6(1 - 0)}{4 - 2} = 6.$$

$$\boxed{K = 6}$$

② $$\text{Overshoot} = \frac{r'(t=15) - r'(t\to\infty)}{r'(t\to\infty)}$$

$$= \frac{28 - 6 + 6(1 - e^{-15/20}) - 12}{12}$$

$$= 0.597$$

$$\text{Overshoot} = \exp\left(\frac{-\pi \xi}{\sqrt{1-\xi^2}}\right)$$

$$\exp\left(\frac{-\pi \xi}{\sqrt{1-\xi^2}}\right) = 0.597$$

$$\boxed{\xi = 0.17}$$

③ Period for $r'(t)$ = Period for $P_m(t)$

$$P = 50 - 15 = 35$$

$$P = \frac{2\pi\tau}{\sqrt{1-\xi^2}} = 35$$

$$\boxed{\tau = 5.45}$$

Ans