Solutions

Q.1)

a) Overall mass balance:

$$\frac{d(\rho V)}{dt} = w_1 + w_2 - w_3 \tag{1}$$

Energy balance:

$$C\frac{d\left[\rho V(T_3 - T_{ref})\right]}{dt} = w_1 C(T_1 - T_{ref}) + w_2 C(T_2 - T_{ref}) - w_3 C\left(T_3 - T_{ref}\right) \tag{2}$$

Because ρ = constant and $V = \overline{V}$ = constant, Eq. 1 becomes:

$$w_3 = w_1 + w_2 \tag{3}$$

b) From Eq. 2, substituting Eq. 3

$$\rho C\overline{V}\frac{d(T_3 - T_{ref})}{dt} = \rho C\overline{V}\frac{dT_3}{dt} = w_1 C(T_1 - T_{ref}) + w_2 C(T_2 - T_{ref}) - (w_1 + w_2)C\left(T_3 - T_{ref}\right) \tag{4}$$

Constants C and T_{ref} can be cancelled:

$$\rho\overline{V}\frac{dT_3}{dt} = w_1 T_1 + w_2 T_2 - (w_1 + w_2)T_3 \tag{5}$$

Degrees of freedom for the simplified model:

Parameters : ρ, $\overline{V}$

Variables : w_1, w_2, T_1, T_2, T_3
$N_E = 1$
$N_V = 5$

Thus, $N_F = 5 - 1 = 4$

Because w_1, w_2, T_1 and T_2 are determined by upstream units, we assume they are known functions of time:

$w_1 = w_1(t)$
$w_2 = w_2(t)$
$T_1 = T_1(t)$
$T_2 = T_2(t)$

Thus, N_F is reduced to 0.

Q.2)

Energy balance:

$$C_p \frac{d\left[\rho V (T - T_{ref})\right]}{dt} = wC_p(T_i - T_{ref}) - wC_p(T - T_{ref}) - UA_s(T - T_a) + Q$$

Simplifying

$$\rho V C_p \frac{dT}{dt} = wC_pT_i - wC_pT - UA_s(T - T_a) + Q$$
$$\rho V C_p \frac{dT}{dt} = wC_p(T_i - T) - UA_s(T - T_a) + Q$$

Q.3)

Mass Balances:

$$\rho A_1 \frac{dh_1}{dt} = w_1 - w_2 - w_3 \qquad (1)$$

$$\rho A_2 \frac{dh_2}{dt} = w_2 \qquad (2)$$

Flow relations:

Let P_1 be the pressure at the bottom of tank 1.

Let P_2 be the pressure at the bottom of tank 2.

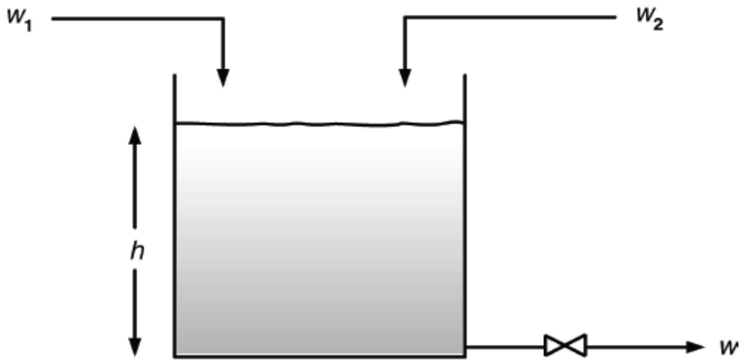
Let P_a be the ambient pressure.

Then
$$w_2 = \frac{P_1 - P_2}{R_2} = \frac{\rho g}{g_c R_2}(h_1 - h_2) \qquad (3)$$

$$w_3 = \frac{P_1 - P_a}{R_3} = \frac{\rho g}{g_c R_3} h_1 \qquad (4)$$

Q.4)

a)



Note that the only conservation equation required to find h is an overall mass balance:

$$\frac{dm}{dt} = \frac{d(\rho A h)}{dt} = \rho A \frac{dh}{dt} = w_1 + w_2 - w \quad (1)$$

Valve equation: $$w = C_v' \sqrt{\frac{\rho g}{g_c} h} = C_v \sqrt{h} \quad (2)$$

where $$C_v = C_v' \sqrt{\frac{\rho g}{g_c}} \quad (3)$$

Substituting the valve equation into the mass balance,

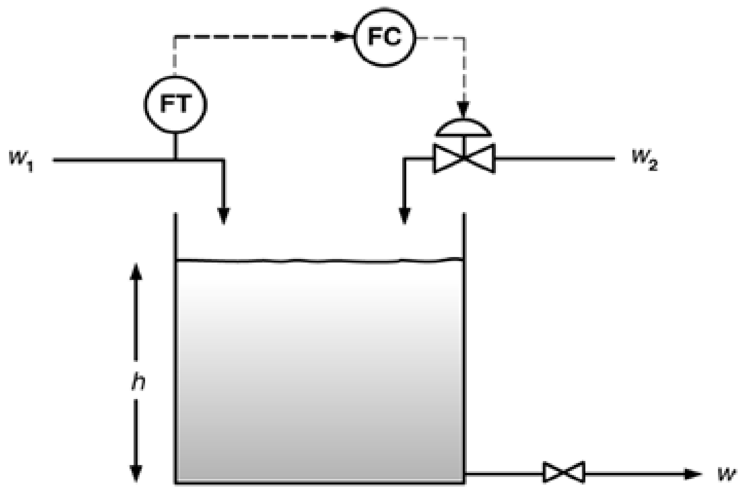
$$\frac{dh}{dt} = \frac{1}{\rho A}(w_1 + w_2 - C_v \sqrt{h}) \quad (4)$$

Steady-state model :

$$0 = \overline{w_1} + \overline{w_2} - C_v \sqrt{\bar{h}} \quad (5)$$

b) $$C_v = \frac{\overline{w_1} + \overline{w_2}}{\sqrt{\bar{h}}} = \frac{2.0 + 1.2}{\sqrt{2.25}} = \frac{3.2}{1.5} = 2.13 \frac{\text{kg/s}}{\text{m}^{1/2}}$$

c) Feedforward control



Rearrange Eq. 5 to get the feedforward (FF) controller relation,

$$w_2 = C_v\sqrt{\overline{h_R}} - w_1 \qquad \text{where } \overline{h_R} = 2.25 \text{ m}$$

$$w_2 = (2.13)(1.5) - w_1 = 3.2 - w_1 \qquad (6)$$

Note that Eq. 6, for a value of $w_1 = 2.0$, gives

$w_2 = 3.2 - 2 = 1.2$ Kg/s which is the desired value.

If the actual FF controller follows the relation, $w_2 = 3.2 - 1.1w_1$ (flow transmitter 10% higher), w_2 will change as soon as the FF controller is turned on,

$w_2 = 3.2 - 1.1\ (2.0) = 3.2 - 2.2 = 1.0$ kg/s

(instead of the correct value, 1.2 kg/s)

Then $C_v\sqrt{\overline{h}} = 2.13\sqrt{\overline{h}} = 2.0 + 1.0$

or $\sqrt{\overline{h}} = \dfrac{3}{2.13} = 1.408$ and $\overline{h} = 1.983$ m (instead of 2.25 m)

Error in desired level $= \dfrac{2.25 - 1.983}{2.25} \times 100\% = 11.9\%$