Chapter 10: Dynamic Behavior of Closed-Loop Control Systems

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Chapter Overview

This chapter consists of the following topics:

1. Block Diagram Representation
   - Process
   - Composition Sensor-Transmitter (Analyzer)
   - Controller
   - Current-to-Pressure (I/ P) Transducer
   - Control Valve
2. Block Diagram: Standard Notation
   - Block Diagram Algebra
   - Closed-Loop Transfer Functions

Chapter Overview

This chapter consists of the following topics:

3. Closed-Loop Responses of Simple Control Systems
   - Effect of Proportional Control on Closed-Loop Responses
   - Effect of PI Control on Closed-Loop Responses

Learning Objectives

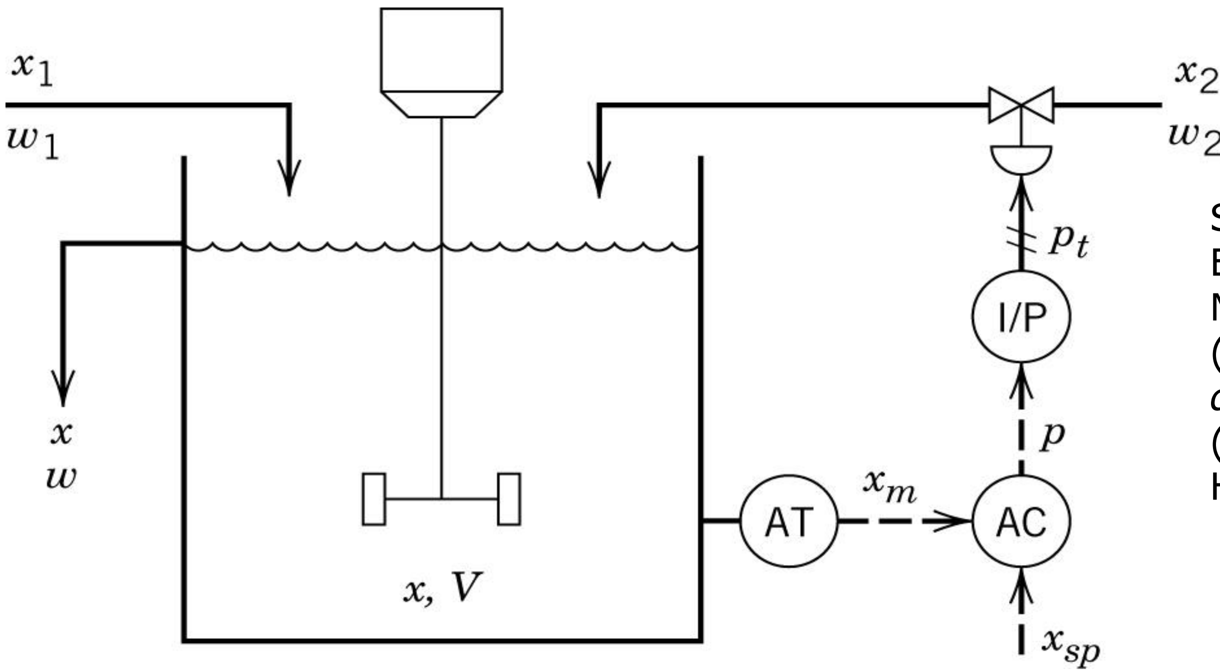
At the end of this chapter, you will be able to:

- Illustrate the development of block diagram
- Analyze closed-loop transfer functions
- Analyze closed-loop responses of simple control systems

1. Feedback Control Loop (or Closed-Loop)

- Feedback control loop:
  - Combination of the process, feedback controller, and the instrumentation.
- Closed-loop system:
  - Denote the controlled process.
- Block diagrams and transfer functions provide a useful description of closed-loop systems.
- We will analyze the dynamic behavior of several simple closed-loop systems.

Block Diagram Representation



Source: Seborg, D. E., Edgar, T. F., & Mellichamp, D. A. (2011). *Process dynamics and control* (2nd ed.)(pp.260). Hoboken, NJ: Wiley.

- Transfer function for each of 5 elements.
- Flow rate w_1 is assumed to be constant and the system is initially operating at the nominal steady state.

1.1 Process

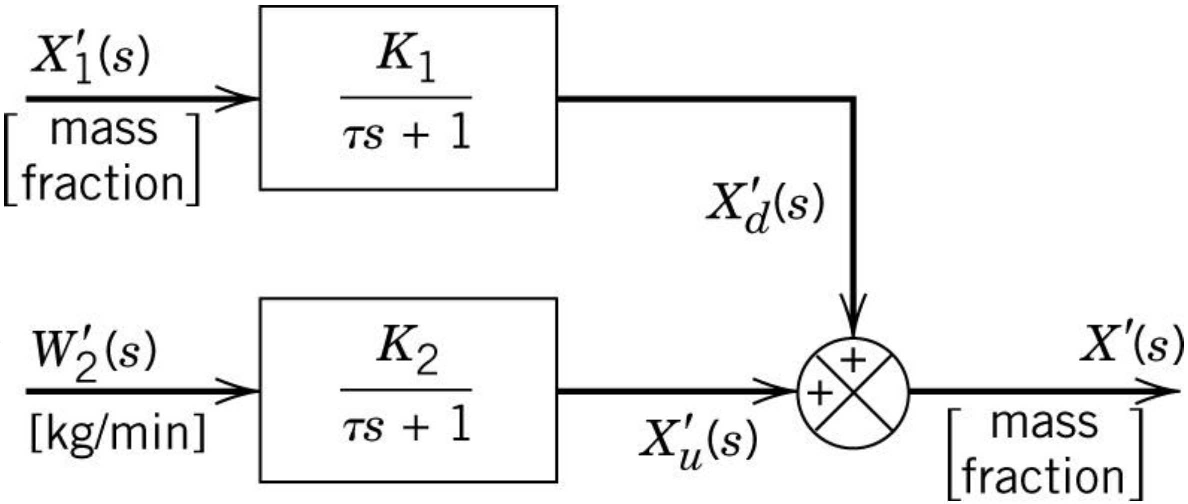
- Process

$$X'(s) = \left(\frac{K_1}{\tau s + 1}\right) X_1'(s) + \left(\frac{K_2}{\tau s + 1}\right) W_2'(s) \qquad (10-1)$$

- Where

$$\tau = \frac{V\rho}{\overline{w}}, \qquad K_1 = \frac{\overline{w_1}}{\overline{w}}, \qquad \text{and} \qquad K_2 = \frac{1-\bar{x}}{\overline{w}} \qquad (10-2)$$

Block Diagram for the Process

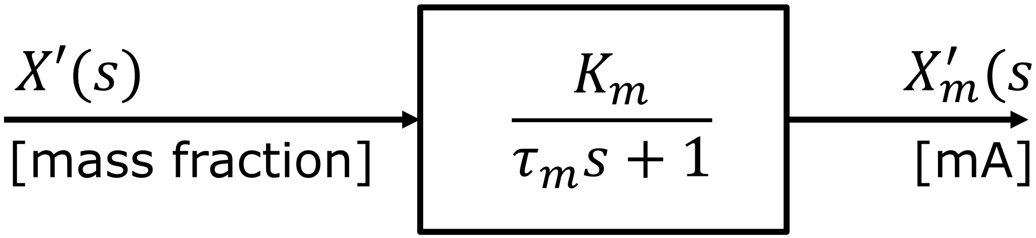


Source: Seborg, D. E., Edgar, T. F., & Mellichamp, D. A. (2011). *Process dynamics and control* (2nd ed.)(pp.261). Hoboken, NJ: Wiley.

- Effect of changes is additive as a direct consequence of the superposition principle for linear systems.
- Transfer function representation is valid only for linear systems and for nonlinear systems that have been linearized.

1.2 Composition Sensor-Transmitter (Analyzer)

- Approximated by a first-order transfer function:



Source: Seborg, D. E., Edgar, T. F., & Mellichamp, D. A. (2011). *Process dynamics and control* (2nd ed.)(pp.261). Hoboken, NJ: Wiley.

- This instrument has negligible dynamics when $\tau \gg \tau_m$:

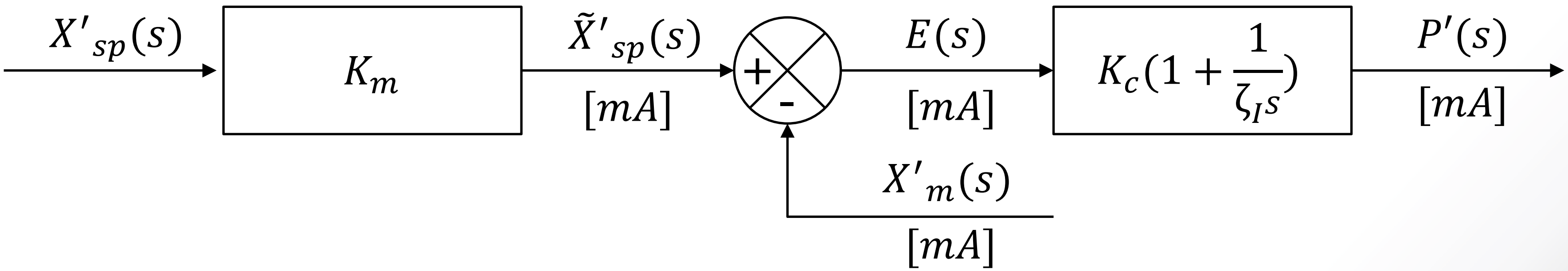
$$\frac{X'_m(s)}{X'(s)} = K_m$$

1.3 Controller

- PI Controller is used

$$\frac{P'(s)}{E(s)} = K_c\left(1 + \frac{1}{\tau_I s}\right) \qquad (10-4)$$

- where $P'(s)$ and $E(s)$ are the Laplace transforms of the controller output $p'(t)$ and the error signal $e(t)$. K_c is dimensionless



Source: Seborg, D. E., Edgar, T. F., & Mellichamp, D. A. (2011). *Process dynamics and control* (2nd ed.)(pp.262). Hoboken, NJ: Wiley.

Controller (Cont'd)

- Error signal:

$$e(t) = \tilde{x}'_{sp}(t) - x'_m(t)$$

- Taking laplace:

$$E(s) = \tilde{X}'_{sp}(s) - X'_m(s)$$

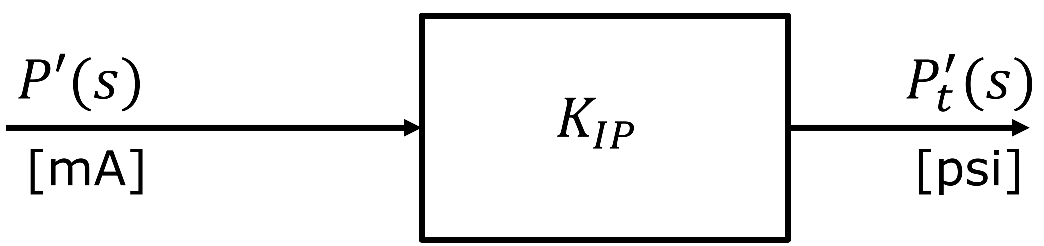
- The symbol $\tilde{x}'_{sp}(t)$ denotes the *internal set-point* composition expressed as an equivalent electrical current signal.
- $\tilde{x}'_{sp}(t)$ is related to the actual composition set point by sensor-transmitter gain K_m:

$$\tilde{x}'_{sp}(t) = K_m x'_{sp}(t)$$

1.4 Current-to-Pressure (I/P) Transducer

- The transducer transfer function merely consists of a steady-state gain K_{IP}

$$\frac{P_t'(s)}{P'(s)} = K_{IP} \qquad (10-9)$$

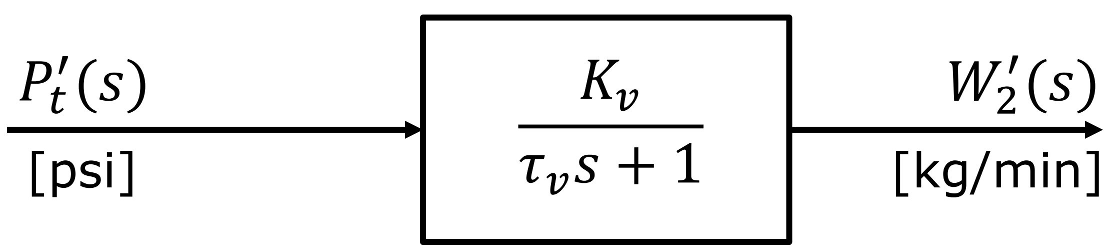


Source: Seborg, D. E., Edgar, T. F., & Mellichamp, D. A. (2011). *Process dynamics and control* (2nd ed.)(pp.262). Hoboken, NJ: Wiley.

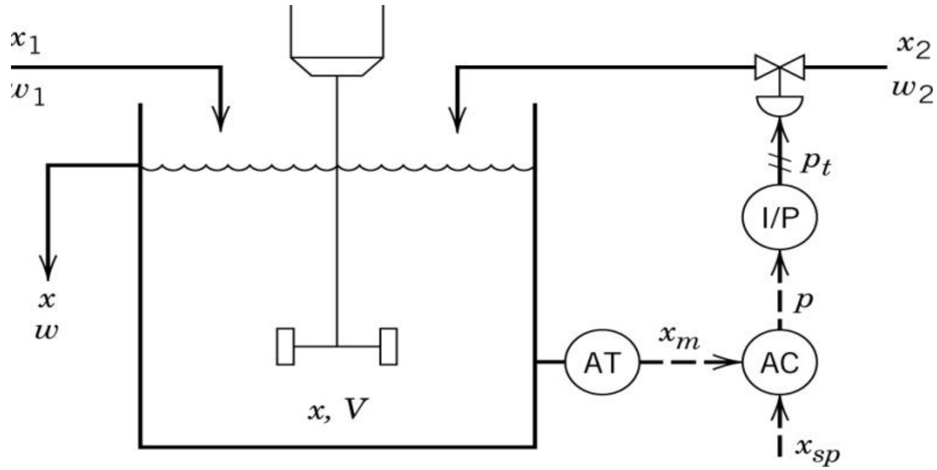
1.5 Control Valve

- Control valves
  - Usually designed such that the flow rate is a nearly linear function of the signal to the valve actuator
  - Therefore, a first-order transfer function is an adequate model:

$$\frac{W_2'(s)}{P_t'(s)} = \frac{K_v}{\tau_v s + 1} \qquad (10-10)$$



Composite Block Diagram of the Controlled System



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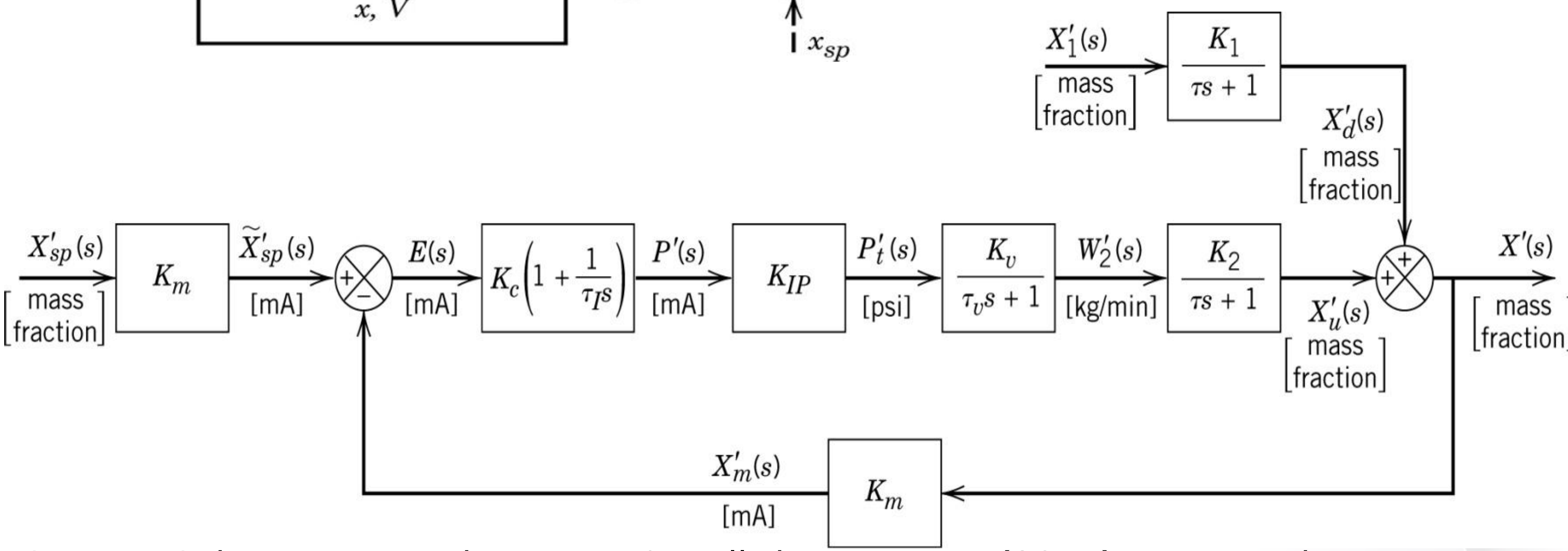
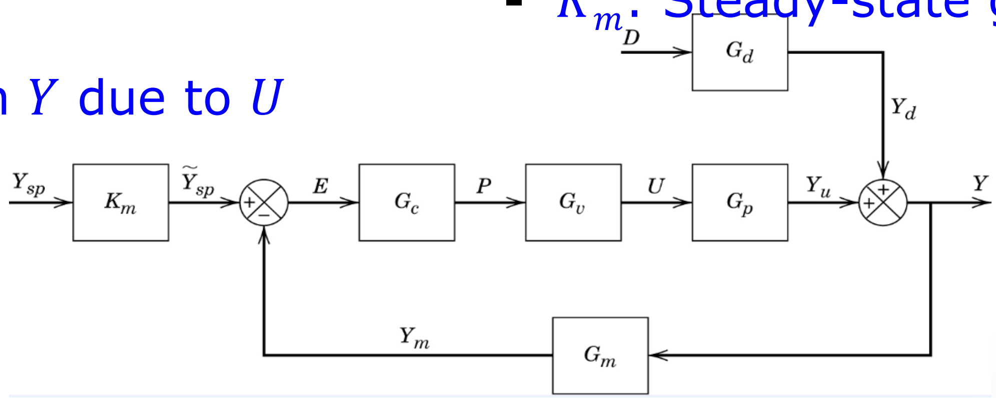


Source: Seborg, D. E., Edgar, T. F., & Mellichamp, D. A. (2011). *Process dynamics and control* (2nd ed.)(pp.263). Hoboken, NJ: Wiley.

2. Block Diagram: Standard Notation

- Y: Controlled variable
- U: Manipulated variable
- D: Disturbance (or load) variable
- P: Controller output
- E: Error signal
- Y_m: Measured value of Y
- Y_{sp}: Set point
- $\tilde{Y}_{sp}$: Internal set point (used by controller)
- Y_u: Change in Y due to U
- Y_d: Change in Y due to D
- G_c: Controller TF
- G_v: TF for the final control element
- G_p: Process TF
- G_d: Disturbance TF
- G_m: TF for sensor add transmitter
- K_m: Steady-state gain for G_m



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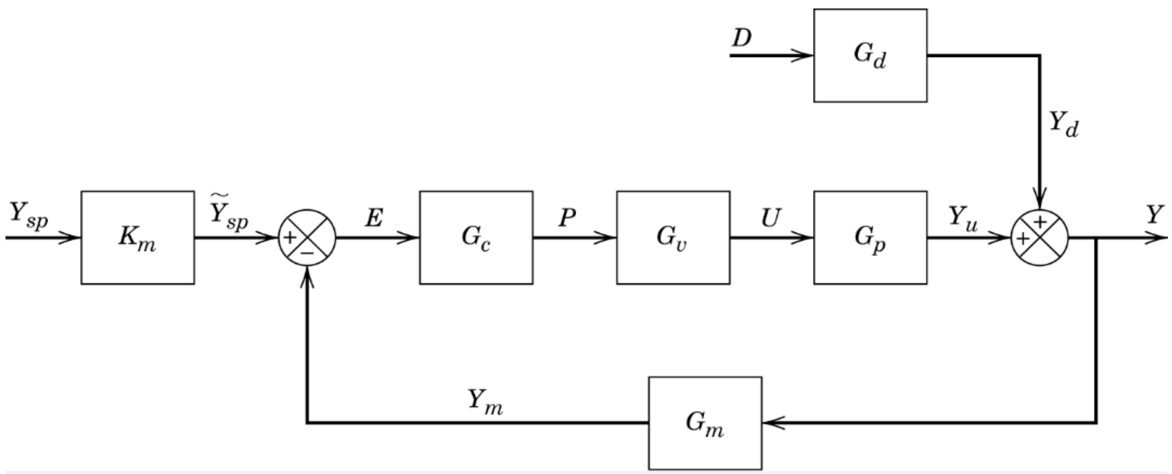
Standard Block Diagram

- General block diagram
  - Variable is the Laplace transform of a deviation variable.
  - Primes and *s* dependence have been omitted.
  - Forward path: Path from E to Y through G_c, G_v and G_p.
  - Feedback path: Path from Y to the comparator through G_m.



Source: Seborg, D. E., Edgar, T. F., & Mellichamp, D. A. (2011). *Process dynamics and control* (2nd ed.)(pp.264). Hoboken, NJ: Wiley.

2.1 Block Diagram Algebra

- Important things to look for:
  - Circle represents algebraic relations.
  - Arrows represent flow of information.
  - Block diagram relates input and output.
- Block diagram reduction
- Block in series

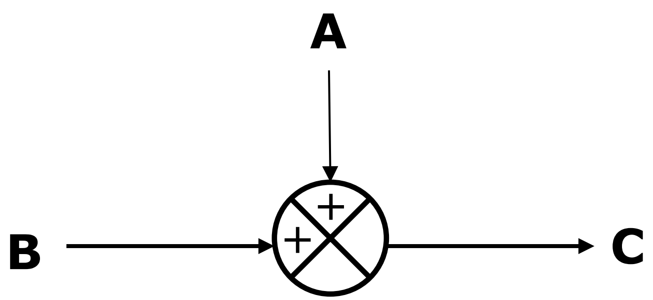
$$X_1 \longrightarrow \boxed{G_1(s)} \xrightarrow{X_2} \boxed{G_2(s)} \longrightarrow \boldsymbol{Y}$$

- Equivalent block diagram

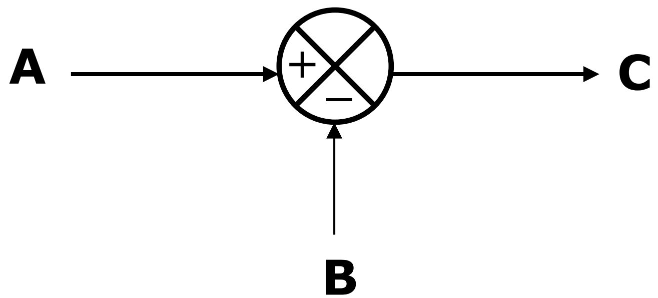
$$X_1 \longrightarrow \boxed{G_1(s)G_2(s)} \longrightarrow$$

Block Diagram Algebra (Cont'd)

- Comparator



$\Rightarrow$ **C = A + B**

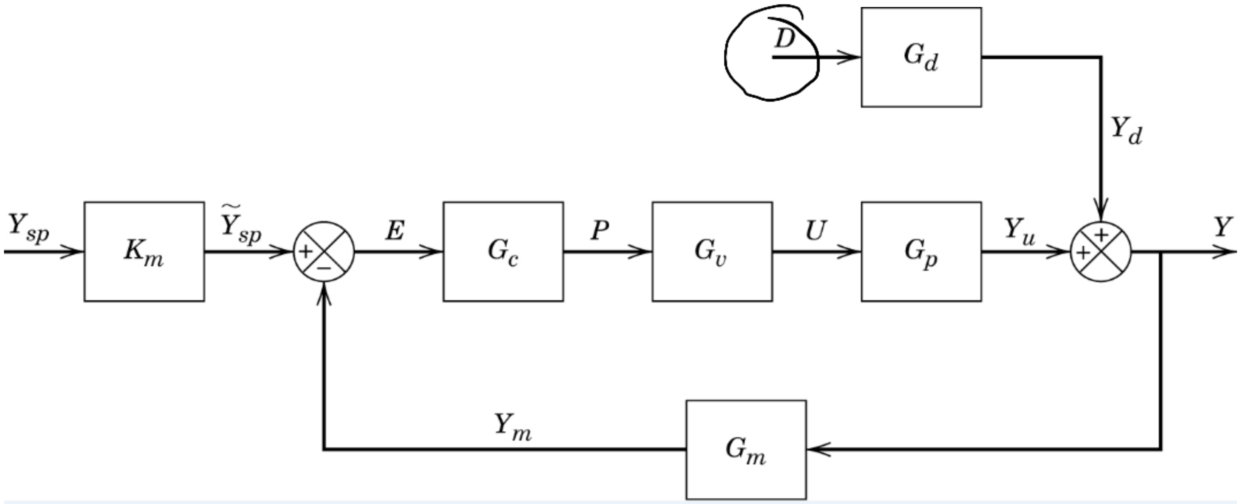


$\Rightarrow$ **C = A − B**

2.2 "Closed-Loop" Transfer Functions

- Closed-loop transfer functions
  - The objective is to find the transfer functions between inputs (Y_{sp} or D) and output (Y) of the closed loop.
  - Indicate the dynamic behavior of the controlled process (i.e., process plus controller, transmitter, valve, etc.).
- Set-point changes ("Servo problem")
  - No disturbance changes.
  - $D = 0$.
- Disturbance changes ("Regulator problem")
  - Process is regulated at a constant set point.

"Closed-Loop" Transfer Functions (Cont'd)



Source: Seborg, D. E., Edgar, T. F., & Mellichamp, D. A. (2011). *Process dynamics and control* (2nd ed.)(pp.264). Hoboken, NJ: Wiley.

- $Y = Y_d + Y_u$ where $Y_d = G_d D$ & $Y_u = G_p U$
- Combining $Y = G_p U + G_d D$ where $U = G_v P = G_v G_c E$
- $E = \widetilde{Y_{sp}} - Y_m$ where $\widetilde{Y_{sp}} = Y_{sp} K_m$ & $Y_m = G_m Y$
- $U = (Y_{sp} K_m - G_m Y) G_v G_c$
- $Y = \left(Y_{sp} K_m - G_m Y\right) G_v G_c G_p + G_d D$

"Closed-Loop" Transfer Functions (Cont'd)

- Rearranging
- $Y = \frac{K_m G_p G_c G_v}{(1+G_m G_p G_c G_v)} Y_{sp} + \frac{G_d}{(1+G_m G_p G_c G_v)} D$

- Set-point changes "Servo"
  - Assume $Y_{sp} \neq 0$ and $D = 0$ (set-point changes while disturbance change is zero).

$$\frac{Y(s)}{Y_{sp}(s)} = \frac{K_m G_c G_v G_p}{1 + G_c G_v G_p G_m} \qquad (10-26)$$

"Closed-Loop" Transfer Functions (Cont'd)

- Disturbance changes "Regulator"

$$\frac{Y(s)}{D(s)} = \frac{G_d}{1 + G_c G_v G_p G_m} \qquad (10-29)$$

- Note that both TF $(10-26 \,\&\, 10-29)$ have the same denominator $(1 + G_c G_v G_p G_m)$.
- Denominator is often written as $(1 + G_{OL})$.
- G_{OL}: Open loop transfer function

Short-Cut Method for Closed-Loop TF

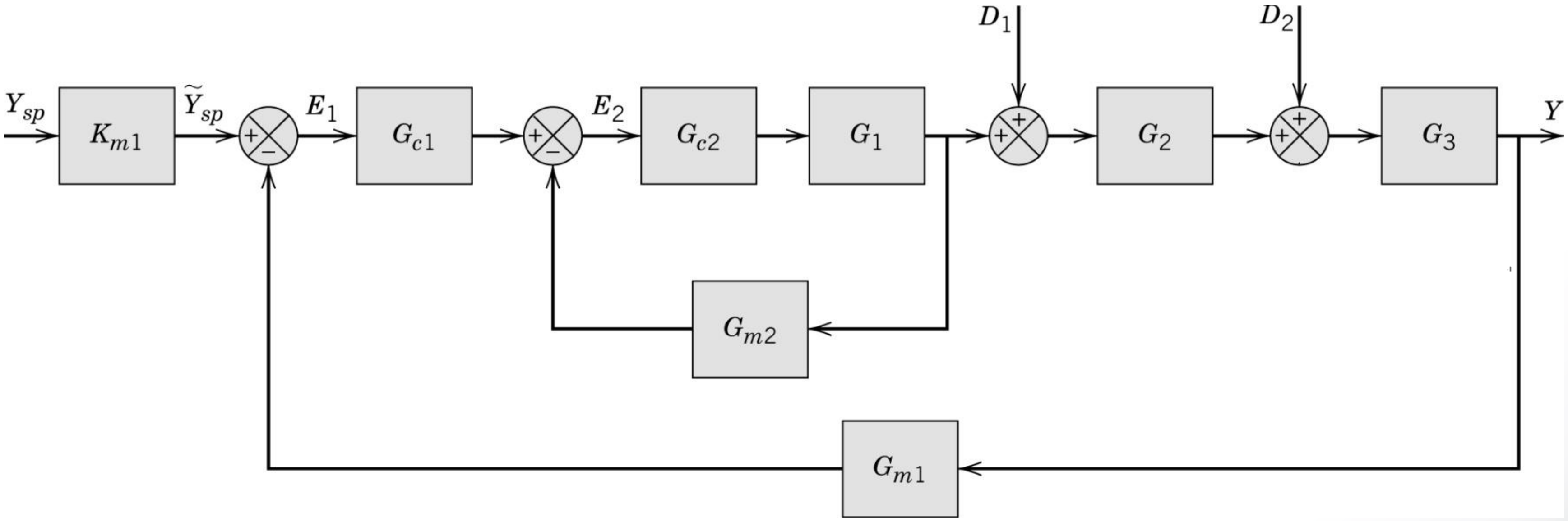
- Closed-loop transfer functions for block diagrams:

$$\frac{Z}{Z_i} = \frac{\pi_f}{1 + \pi_e} \qquad (10 - 31)$$

- Z: Output variable or any internal variable within the control loop.
- Z_i : Input variable (e.g., Y_{sp} or D).
- π_f : Product of the transfer functions in the forward path from Z_i to Z.
- π_e : Product of every transfer function in the feedback loop.
- Applicable only to portions of a block diagram that includes a feedback loop with a negative sign in the comparator.

Exercise 10.1

- Find Y/Y_{sp}



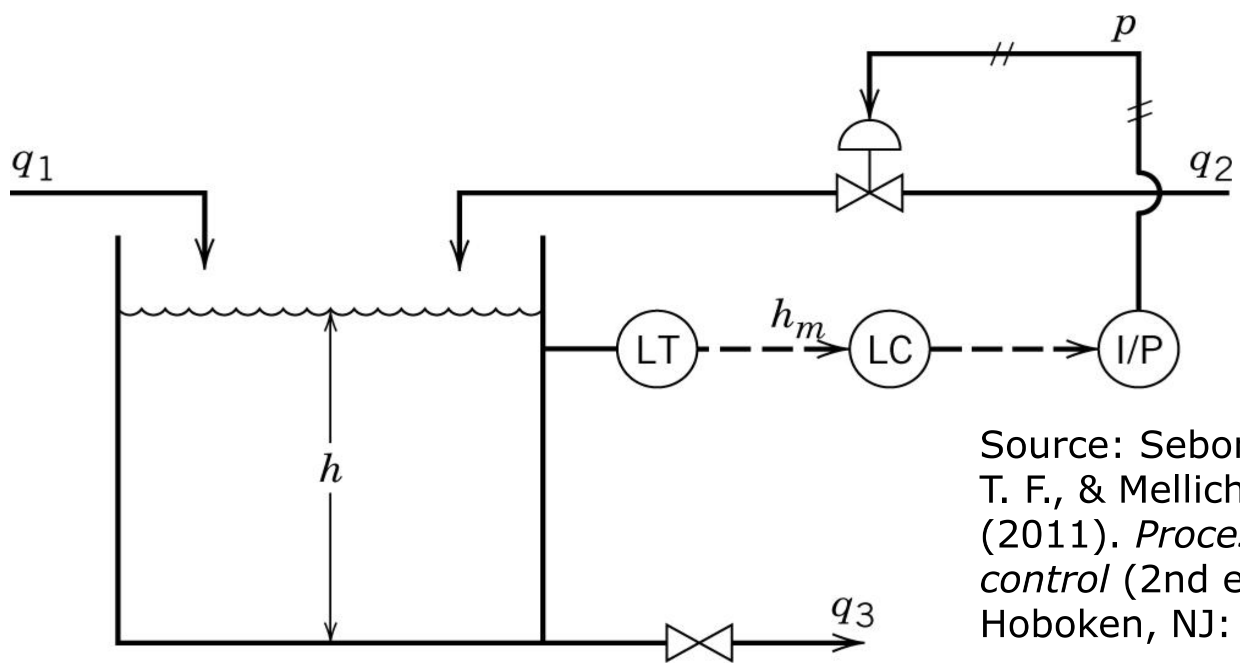
Source: Seborg, D. E., Edgar, T. F., & Mellichamp, D. A. (2011). *Process dynamics and control* (2nd ed.)(pp.267). Hoboken, NJ: Wiley.

3. Closed-Loop Responses of Simple Control Systems

- Given
  - Process
  - Type of controller
  - Other dynamics such as valve, transmitter, etc.

- Analyze responses
  - Set-point changes
  - Disturbance

Closed-Loop Responses of Simple Control Systems (Cont'd)



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- Liquid-level control system
  - Controlled variable h; manipulated variable q_2; disturbance variable $q_{1.}$
  - Flow-head relation is linear $q_3 = h/R$.

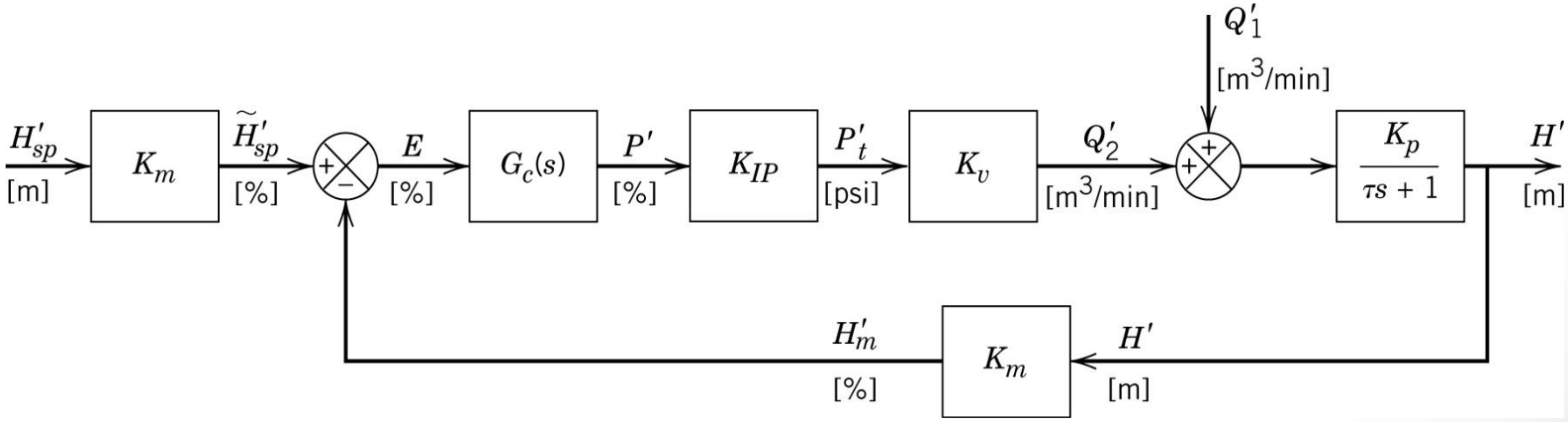
Dynamics

- Process Dynamics
- $H(s) = \frac{R}{(ARs+1)} Q_1(s) + \frac{R}{(ARs+1)} Q_2(s)$
- $G_p = \frac{K_p}{(\tau s+1)} = \frac{H(s)}{Q_2(s)}$ $\quad (10-34)$
- $G_d = \frac{K_p}{(\tau s+1)} = \frac{H(s)}{Q_1(s)}$ $\quad (10-35)$

- Note that G_p & G_d are identical, because q_1 and q_2 are both inlet flow rate and thus have the same effect on h.
- $K_p = R;\ \tau = AR$

Other Dynamics

- For simplicity, assume negligible dynamics for level transmitter, transducer, and control valve
- Level transmitter $G_m(s) = Km$
- I/P transducer $G_{IP}(s) = KIP$
- Control valve $G_v(s) = K_v$



Source: Seborg, D. E., Edgar, T. F., & Mellichamp, D. A. (2011). *Process dynamics and control* (2nd ed.)(pp.268). Hoboken, NJ: Wiley.

3.1 Effect of Proportional Control on Closed-Loop Responses

- Proportional controller $G_c(s) = Kc$
- Closed-loop TF
- $H = \frac{K_m G_p K_c K_{IP} K_v}{(1+G_p K_c K_{IP} K_v K_m)} H_{sp} + \frac{G_p}{(1+G_p K_c K_{IP} K_v K_m)} Q_1$
- Plugging G_p
- $H = \frac{K_1}{(1+\tau_1 s)} H_{sp} + \frac{K_2}{(1+\tau_1 s)} Q_1$
- $K_1 = \frac{K_{OL}}{1+K_{OL}}$ $\quad K_2 = \frac{K_p}{1+K_{OL}}$
- $\tau_1 = \frac{\tau}{1+K_{OL}}$ $\quad K_{OL} = K_c K_{IP} K_v K_p K_m$

Effect of Proportional Control on Closed-Loop Responses

$$H = \frac{K_1}{(1+\tau_1 s)} H_{sp} + \frac{K_2}{(1+\tau_1 s)} Q_1$$

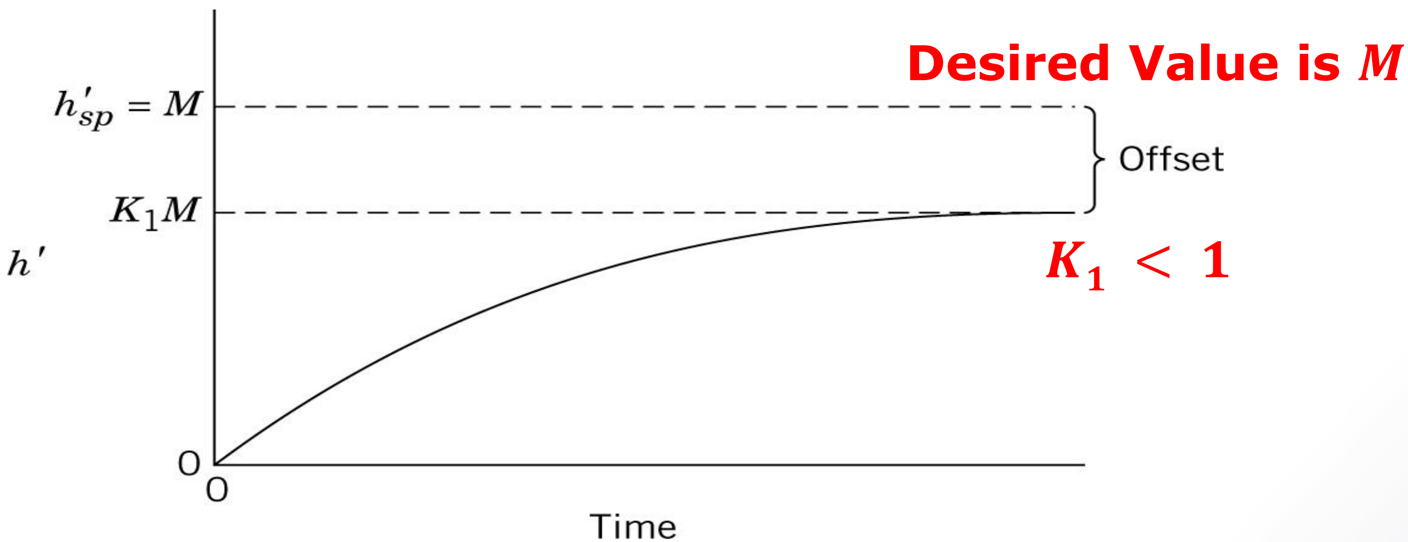
- Closed-loop (CL) process has first order dynamics with time constant τ_1.
- For both TFs, time constant is the same (Characteristics equation), but gain is different.
- Which response is faster? Open loop or closed loop?
- $\tau_1 = \frac{\tau}{1+K_c K_{IP} K_v K_p K_m}$
- Decreases with increasing K_c always < τ, i.e., CL response is faster than OL response.

3.1 Effect of Proportional Control on Closed-Loop Responses

- Gain?
- $K_1 = \frac{K_p K_c K_{IP} K_v K_m}{1+K_p K_c K_{IP} K_v K_m}$
- $K_2 = \frac{K_p}{1+K_p K_c K_{IP} K_v K_m}$
- K_1 not equal to 1 unless K_c = infinity, K_1 always < 1
- K_2 not equal to 0 unless K_c = infinity, K_2 always < K_p

Set-Point Change (Servo Problem)

- CL Response of step change of magnitude *M* in set point.
- $H_{sp} = \frac{M}{s} \quad (D = 0 \text{ or } Q_1 = 0)$
- $h(t) = K_1 M(1 - e^{-t/\tau_1})$ $\qquad (10-41)$

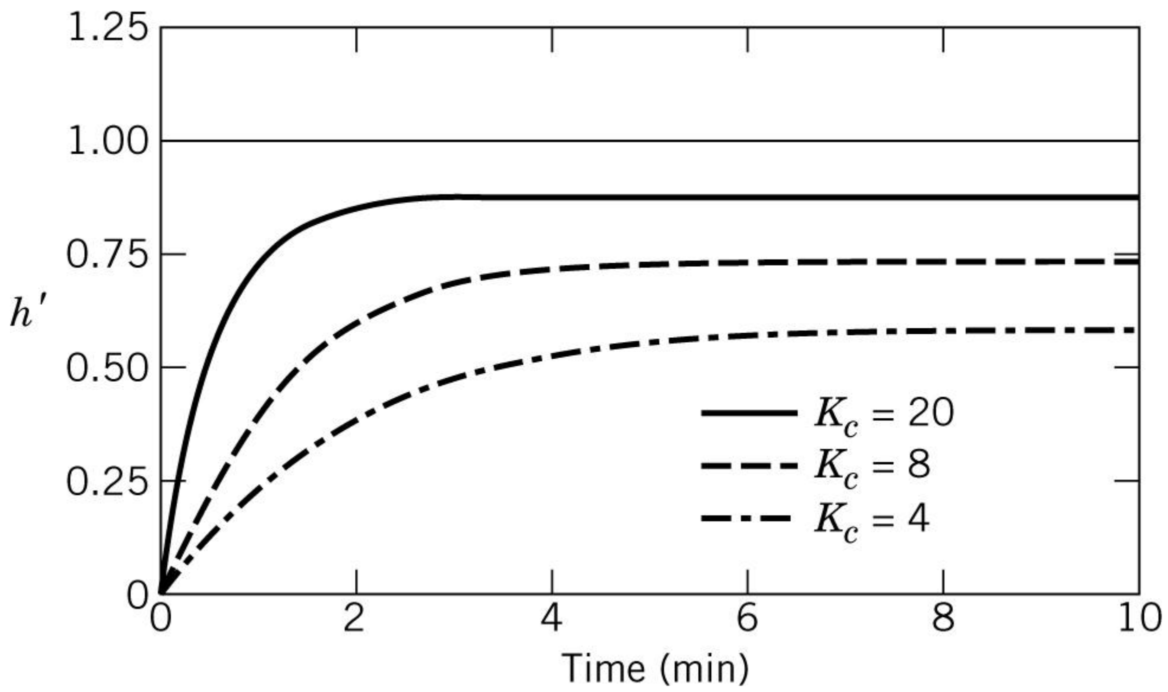


Source: Seborg, D. E., Edgar, T. F., & Mellichamp, D. A. (2011). *Process dynamics and control* (2nd ed.)(pp.270). Hoboken, NJ: Wiley.

Set-Point Change (Cont'd)

- Offset
  - Steady state error
  - Difference between the steady state value of set point and the height
  - $h'_{sp}(\infty) - h'(\infty)$
  - $\frac{M}{1+K_p K_c K_{IP} K_v K_m}$
- How the offset can be reduced?

Set-Point Change (Cont'd)



Source: Seborg, D. E., Edgar, T. F., & Mellichamp, D. A. (2011). *Process dynamics and control* (2nd ed.)(pp.272). Hoboken, NJ: Wiley.

- Offset can be reduced by increasing gain of controller.
- Making K_c too large can cause oscillatory or unstable responses.

Disturbance Changes (Regulator)

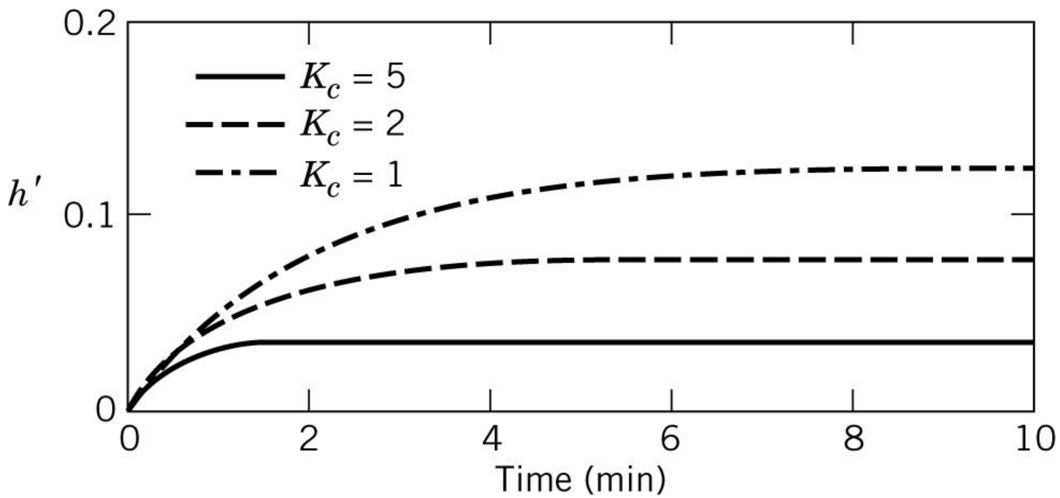
- $H = \frac{K_1}{(1+\tau_1 s)} H_{sp} + \frac{K_2}{(1+\tau_1 s)} Q_1$
- $H_{sp} = 0$
- $\frac{H}{Q_1} = \frac{K_2}{1+\tau_1 s}$
- CL response of step change of magnitude M in disturbance
- $Q_1 = \frac{M}{s} \quad (H_{sp} = 0)$
- $h(t) = K_2 M(1 - e^{-t/\tau_1}) \qquad (10-56)$

$$K_2 = \frac{K_p}{1 + K_p K_c K_{IP} K_v K_m}$$

$$\tau_1 = \frac{\tau}{1+K_p K_c K_{IP} K_v K_m}$$

Disturbance Changes (Regulator)

- Offset
  - $h'_{sp}(\infty) - h'(\infty)$
  - Offset $= 0 - K_2 M = -\mathrm{K_2 M} = -\dfrac{K_p M}{1+K_p K_c K_{IP} K_v K_m}$
- Increasing K_c reduces the amount of offset (same as for servo problem).



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3.2 Effect of PI Control on Closed-Loop Responses

- $H = \frac{K_m G_p G_c K_{IP} K_v}{(1+G_p G_c K_{IP} K_v K_m)} H_{sp} + \frac{G_p}{(1+G_p G_c K_{IP} K_v K_m)} Q_1$

- Plug G_c and G_p

- $G_p = \frac{K_p}{(\tau s+1)} = \frac{H(s)}{Q_2(s)}$

- $G_c = K_c(1 + \frac{1}{\tau_I s})$

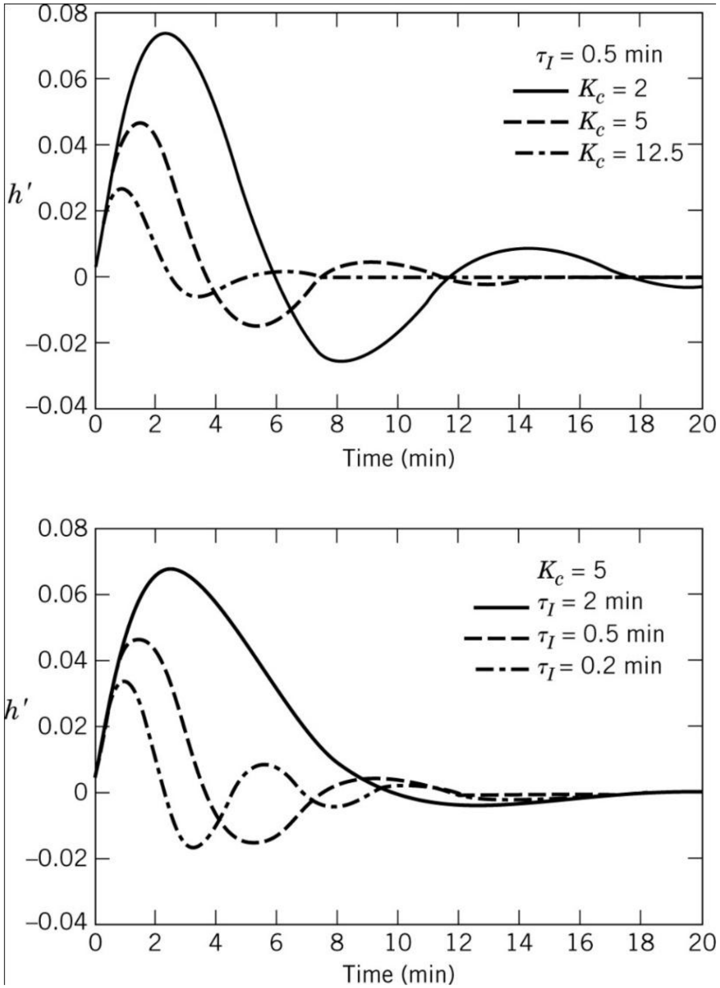
Disturbance Changes (Regulator)

- $\frac{H}{Q_1} = \frac{G_p}{(1+G_p G_c K_{IP} K_v K_m)} = \frac{K_p \tau_I s}{\tau_I s(\tau s+1)+K_{OL}(\tau_I s+1)}$ (10 – 59)
- $\frac{H}{Q_1} = \frac{K_3 s}{\tau_3{}^2 s^2+2\zeta_3\tau_3 s+1}$ (10 – 60)
- $K_3 = \frac{\tau_I}{K_c K_{IP} K_v K_m}$ (10 – 61)
- $\zeta_3 = \frac{1}{2}\left(\frac{1+K_{OL}}{\sqrt{K_{OL}}}\right)\sqrt{\frac{\tau_I}{\tau}}$ (10 – 62)
- $\tau_3 = \sqrt{\frac{\tau\tau_I}{K_{OL}}}$ (10 – 63)

Disturbance Changes (Regulator)

- CL response of unit step change in disturbance
- $H(s) = \frac{K_3}{\tau_3{}^2 s^2 + 2\zeta_3 \tau_3 s + 1}$ $\quad (10-64)$
- Response is a damped oscillation for $0 < \zeta_3 < 1$
- $h(t) = \frac{K_3}{\tau_3 \sqrt{1-\zeta_3{}^2}} e^{-\zeta_3 t/\tau_3} \sin[\sqrt{1-\zeta_3{}^2}\ \ t/\tau_3]$ $\quad (10-65)$
- Offset?

Disturbance Changes (Regulator)



- Integral action eliminates offset.
- $\uparrow K_c$ or $\tau_I \downarrow$ speeds up the response.
- Response is oscillatory as either $K_c \downarrow$ (unexpected) or $\tau_I \downarrow$ (expected).
- In general, closed loop response becomes more oscillatory as $K_c\uparrow$.
- Unexpected result because of the dynamic lags associated with the control valve and transmitter are neglected.

Source: Seborg, D. E., Edgar, T. F., & Mellichamp, D. A. (2011). *Process dynamics and control* (2nd ed.)(pp.275). Hoboken, NJ: Wiley.

Summary

- We consider the dynamic behavior of processes that are operated using feedback control.
- Block diagrams and transfer functions provide a useful description of closed-loop systems.
- We analyze the dynamic behavior of several simple closed-loop systems.
- We consider the dynamic behavior of several elementary control problems for disturbance variable and set-point changes. The transient responses can be determined in a straightforward manner if the CLTFs are available.
- Suggested Reading: Chapter 10 of Seborg

Review Questions

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Chapter 10: Dynamic Behavior of Closed-Loop Control Systems

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