1. $A = \{2, 3, 5, 7, 11, 13, 17, 19\}$

$B = \{10, 12, 14, 16, 18, 20\}$

$C = \{1, 3, 5, 7, 9\}$

Gwee Zi Ni   A24CS0078
Lee Jia Yee   A24CS0260
Evelyn Ang   A24CS0068
Leong Jia Ling   A24CS0104

(a) $A \cap C \cup B = \{2,3,5,7,11,13,17,19\} \cap \{1,3,5,7,9\} \cup \{10,12,14,16,18,20\}$

$= \{3,5,7,10,12,14,16,18,20\}$

(b) $A \cap B \cup C = \{2,3,5,7,11,13,17,19\} \cap \{10,12,14,16,18,20\} \cup \{1,3,5,7,9\}$

$= \{1,3,5,7,9\}$

$P(A \cap B \cup C) = \{\phi, \{1\}, \{3\}, \{5\}, \{7\}, \{9\}, \{1,3\}, \{1,5\}, \{1,7\}, \{1,9\},$
$\{3,5\}, \{3,7\}, \{3,9\}, \{5,3\}, \{5,7\}, \{5,9\}, \{7,9\}, \{1,3,5\},$
$\{1,3,7\}, \{1,3,9\}, \{1,5,7\}, \{1,5,9\}, \{1,7,9\}, \{3,5,7\}, \{3,5,9\},$
$\{3,7,9\}, \{5,7,9\}, \{1,3,5,7\}, \{1,3,5,9\}, \{1,5,7,9\}, \{3,5,7,9\},$
$\{1,3,5,7,9\}\}$

(c) $A - C = \{2, 11, 13, 17, 19\}$

(d) $|A| = 8$

$|B| = 6$

$|C| = 5$

(e) $A \cap C = \{3,5,7\}$

$|P(A \cap C)| = 2^3 = 8$

(f) $B = \{10, 12, 14, 16, 18, 20\}$

$C' = \{2, 4, 6, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$

$\therefore B \subset C'$ True

(g) True

2. (a) $(A - C') \cup (B - C) = (A \cap C'') \cup (B \cap C')$ Set difference law

$= (A \cap C) \cup (B \cap C')$

$\therefore (A - C') \cup (B - C)$ is not equal to $A \cup B$.

(b) $(A \cap B) \cup (A - B) = (A \cap B) \cup (A \cap B')$ Set difference law

$= A \cap (B \cup B')$ Distributive law

$= A \cap U$ Complement law

$= A$ Properties of universal set   (shown)

$\therefore (A \cap B) \cup (A - B)$ is equal to A

3. (a) $S = \{a, b, c, d, e, f, g\}$

$T = \{h, i, j, k, l, m, n, p, q\}$

$E = \{p, q, i, s, t, v, w, y, z\}$

(b) $T \cap E = \{p, q\}$

$S \times (T \cap E) = \{a, b, c, d, e, f, g\} \times \{p, q\}$

$= \{(a,p), (a,q), (b,p), (b,q), (c,p), (c,q), (d,p), (d,q), (e,p), (e,q),$
$(f,p), (f,q), (g,p), (g,q)\}$

4. (a) TRUE

(b) TRUE

5. (a)

p	q	r	$\neg r$	$p \wedge r$	$q \vee \neg r$	$p \vee q$	$Q = (p \wedge r) \vee (q \vee \neg r)$	$R = (p \vee q) \vee \neg r$
T	T	T	F	T	T	T	T	T
T	T	F	T	F	T	T	T	T
T	F	T	F	T	F	T	T	T
T	F	F	T	F	T	T	T	T
F	T	T	F	F	T	T	T	T
F	T	F	T	F	T	T	T	T
F	F	T	F	F	F	F	F	F
F	F	F	T	F	T	F	T	T

∴ Since the truth values of Q and truth values of R are the same, $Q \equiv R$.

(b)

p	q	r	$\neg q$	$p \wedge r$	$p \wedge \neg q$	$\neg(p \wedge \neg q)$	$q \vee r$	$Q = (p \wedge r) \vee \neg (p \wedge \neg q)$	$R = (p \wedge r) \rightarrow (q \vee r)$
T	T	T	F	T	F	T	T	T	T
T	T	F	F	F	F	T	T	T	T
T	F	T	T	T	T	F	T	T	T
T	F	F	T	F	T	F	F	F	T
F	T	T	F	F	F	T	T	T	T
F	T	F	F	F	F	T	T	T	T
F	F	T	T	F	F	T	T	T	T
F	F	F	T	F	F	T	F	T	T

∴ Since the truth values of Q and R are different, $Q \not\equiv R$.

Not true. counterexample is 1.

1 is an odd number and less than 7.

(b) Not true. counterexample is 9.

9 is odd number and greater than 7.

7. $\exists x (P(x) \wedge Q(x))$

8. a is odd.

let $a = 2n+1$, where $n \in Z$.

$$a^2 - 3a = (2n+1)^2 - 3(2n+1)$$
$$= 4n^2 + 4n + 1 - (6n+3)$$
$$= 4n^2 + 4n + 1 - 6n - 3$$
$$= 4n^2 - 2n - 2$$
$$= 2(2n^2 - n - 1)$$
$$= 2k \quad (proven)$$

∴ There exists an integer k, such that $a^2 - 3a = 2k$. Therefore, $a^2 - 3a$ is even.

9. Contradiction: Suppose n^2 is an odd integer, then n is not odd. Hence, n is an even.

$$n = 2a, \text{ where } a \in Z$$
$$n^2 = (2a)^2$$
$$= 4a^2$$
$$= 2(2a^2) \text{ where } 2a^2 \text{ is an integer and } k = 2a^2$$
$$= 2k \quad (even)$$

∴ The statement is true.