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1\. (a) Ways from Presint 4 to Presint 5 = 3 car routes + 4 walking routes
= 7 routes

Ways from Presint 5 to Presint 6 = 4 car routes + 5 walking routes
= 9 routes

∴ Total ways going from Presint 4 to Presint 6
= 7 routes × 9 routes
= 63 routes

(b) (i) Order is important ⇒ Permutation

$$P(8,8) = \frac{8!}{(8-8)!}$$
$$= 8!$$
$$= 40320$$

∴ Ways can arrange all the letters = 40320 ways

(ii) Order is important ⇒ Permutation

$$P(8,5) = \frac{8!}{(8-5)!}$$
$$= \frac{8!}{3!}$$
$$= 6720$$

∴ Ways can arrange 5 strings of letters = 6720 ways

(iii)
$$\underset{S}{\underline{1}} \times \underline{6} \times \underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} \times \underset{E}{\underline{1}}$$
$$6 \times 5 \times 4 \times 3 \times 2 \times 1 = 6!$$
$$= 720$$

∴ Ways can arrange the strings that always start with S and end with E = 720 ways

1. (c) (i) Men $= (5-1)!$

$= 4!$

$= 24$

Women $= 5!$

$= 120$

$\therefore$ Ways that 5 men and 5 women set around a circular table so that the gender alternate $= 120 \times 24$

$= 2880$

(ii) $\boxed{- -}\ \underset{2}{-}\ \underset{3}{-}\ \underset{4}{-}\ \underset{5}{-}\ \underset{6}{-}\ \underset{7}{-}\ \underset{8}{-}\ \underset{9}{-}$
$\ \ 1$

$(9-1)! \times 2! = 80640$

$\therefore$ Ways that among the 10 people, 2 are couple and need to be seat next to each other

$= 80640$

(iii) $\boxed{M\ M\ M\ M\ M}\ \boxed{W\ W\ W\ W\ W}$

$2! \times 5! \times 5! = 14400$

$\therefore$ Ways that people were seated in groups between the men and women

$= 28800$

2. (a) Select $\Rightarrow$ Order is not important

$\Rightarrow$ Combination

Case 1 : 3 women and 2 men

$C(6,3) \times C(8,2)$

$= \frac{6!}{3!(6-3)!} \times \frac{8!}{2!(8-2)!}$

$= 20 \times 28$

$= 560$

Case 2 : 4 women and 1 men

$C(6,4) \times C(8,1)$

$= \frac{6!}{4!(6-4)!} \times \frac{8!}{1!(8-1)!}$

$= 15 \times 8$

$= 120$

Case 3 : 5 women and 0 men

$C(6,5) \times C(8,0)$

$= \frac{6!}{5!(6-5)!} \times \frac{8!}{0!(8-0)!}$

$= 6 \times 1$

$= 6$

$\therefore$ Ways forming a Committee that at least 3 women are on the Committee

$=$ case 1 + case 2 + case 3

$= 560 + 120 + 6$

$= 686$

20 students are there.

One group = 4 students.

Half of it is girls → 10 girls, 10 boys.

Case select 4 students with no restriction:

$$C(20,4) = \frac{20!}{4!(20-4)!} = 4845$$

Case that the group consists of girls only.

$$C(10,4) = \frac{10!}{4!(10-4)!} = 210$$

∴ Ways that a group be selected if at least one boy must be there in the team

= Case select 4 students with no restriction − Case that the group consists of girls only

= 4845 − 210

= 4635 ways

3. (a) (i) $(5-1)! = 4! = 24$ ∴ ways for these people to be seated around the table = 24 ways

(ii) $\boxed{C \ \underline{V} \ \underline{V}}_1 \ \underline{\ \ }_2 \ \underline{\ \ }_3$

$(3-1)! \times 3! = 2! \times 3!$
$= 12$

∴ ways if the captain and both vice-captains should be seated next to each other = 12 ways

(b) Case for allocating the beds with no restriction: $5! = 120$ ways

Case for allocating the beds with the head of camp sit next to the assistant:

$\boxed{\underline{\ \ } \ \underline{\ \ }}_1 \ \underline{\ \ }_2 \ \underline{\ \ }_3 \ \underline{\ \ }_4$

$4! \times 2! = 48$ ways

∴ Case that the head of camp cannot sit next to the assistant

= case for allocating the bed with no restriction − case for allocating the bead with the head of camp sit next to the assistant

= 120 − 48

= 72 ways

(c) Half a dozen = 6

(i) Choose 6 chocolates ⇒ Order is not important

⇒ Combination

Same types of chocolates is allowed ⇒ Combination with repetition is allowed

$$\binom{n+r-1}{r} = \frac{(n+r-1)!}{r!(n-1)!}$$

$n=10$, $r=6$

$$\binom{10+6-1}{6} = \frac{(10+6-1)!}{6!(10-1)!}$$

$$= \frac{15!}{6!9!}$$

$$= 5005$$

∴ ways for buying half a dozen chocolate with no restriction = 5005 ways

(ii) Case 1 : 4 hazelnut flavoured chocolate and 2 the other types of chocalate

Types of chocolate = 9 ( excluding hazelnut flavoured chocolate)

$n=9$, $r=2$

$$\binom{9+2-1}{2} = \frac{(9+2-1)!}{2!(9-1)!} = \frac{10!}{2!8!} = 45 \text{ ways}$$

Case 2 : 5 hazelnut flavoured chocolate and 1 other types of chocolate

$n=9$, $r=1$

$$\binom{9+1-1}{1} = \frac{(9+1-1)!}{1!(9-1)!} = \frac{9!}{1!8!} = 9 \text{ ways}$$

Case 3 : 6 hazelnut flavoured chocolate and no other types of chocolate

$n=9$, $r=0$

$$\binom{9+0-1}{0} = \frac{(9+0-1)!}{0!(9-1)!} = \frac{8!}{0!8!} = 1 \text{ ways}$$

∴ ways for buying half a dozen chocolate with at least 4 hazelnut flavoured chocolate = Case1 + Case 2 + Case 3

$= 45 + 9 + 1$

$= 55$ ways ✱

(iii) No two chocolates for the same type ⇒ Repetition of types of chocolate is not allowed

$$C(10,6) = \frac{10!}{6!(10-6)!}$$

$$= 210$$

∴ Ways for buying half a dozen chocolate with no two chocolates of the same type = 210

(d) (i) Select 11 player ⇒ Combination

$$C(13,11) = \frac{13!}{11!(13-11)!}$$

$$= 78 \text{ ways}$$

∴ Ways are there to chose 11 players to take start the game = 78 ways

(ii) Assign 11 positions ⇒ Order is important ⇒ Permutation

$$P(13,11) = \frac{13!}{(13-11)!}$$

$$= \frac{13!}{2!}$$

$$= 3113510400$$

∴ Ways are there to assign 11 positions from the pool of 13 players = 3113510400 ways.

(iii) Case 1 : 1 woman and 10 men

$$C(3,1) \times C(10,10) = \frac{3!}{1!(3-1)!} \times \frac{10!}{10!(10-10)!}$$

$$= 3 \times 1$$

$$= 3 \text{ ways}$$

Case 2 : 2 women and 9 men

$$C(3,2) \times C(10,9) = \frac{3!}{2!(3-2)!} \times \frac{10!}{9!(10-9)!}$$

$$= 3 \times 10$$

$$= 30 \text{ ways}$$

Case 3 : 3 women and 8 men

$$C(3,3) \times C(10,8) = \frac{3!}{3!(3-3)!} \times \frac{10!}{8!(10-8)!}$$

$$= 1 \times 45$$

$$= 45 \text{ ways}$$

$\therefore$ Ways to chose 11 players to start the game ... must be a women = case 1 + case 2 + case 3

$$= 3 + 30 + 45$$

$$= 78 \text{ ways}$$

4. (a) To get two balls of the same colour from 3 colour balls, at least 4 balls must be taken.

$\therefore$ At least 4 balls

(b) pigeon = total number of cheesecakes ( $n = 80$ )
pigeonholes : people which included thirty students and two teachers ( $m = 32$ )

$k = \frac{n}{m}$ , where $k$ is the minimum number of cheese cake per people.

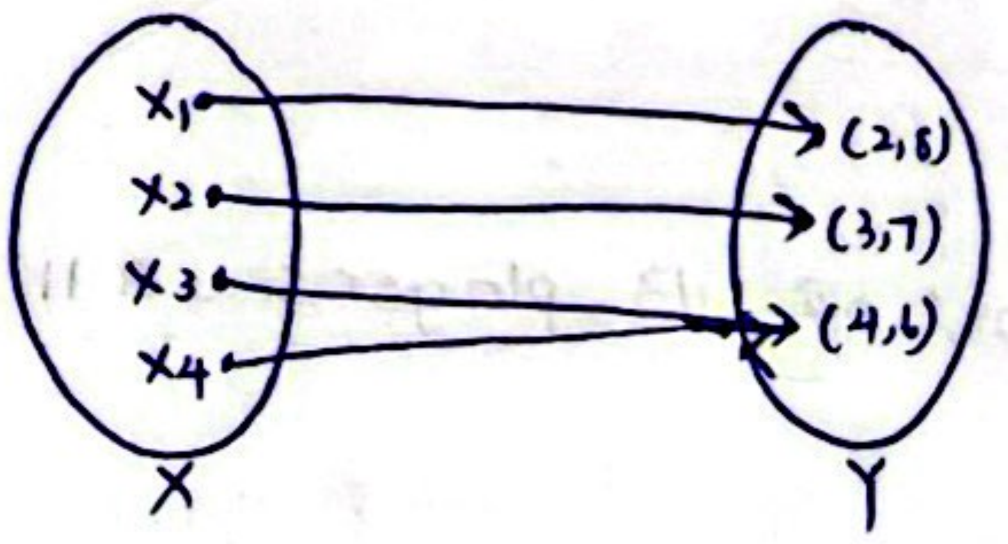
$= \frac{80}{32}$

$= 2.5$

$k = 3$ (shown)

$\therefore$ Each people can have at least three pieces of cheesecake.

(c)




The pairs that can has sum of 10 is the pigeonholes, Y.

$Y = \{(2,8), (3,7), (4,6)\}$

$|Y| = 3$

The integers that we choose from the set X is the pigeon,
Since there may occurs when we chose 3 integers from set X that has no sum of 10, which is $\{2, 3, 4\}$.
So that X should be at least 4.

$|X| = 4$

$\therefore$ Since $|X| > |Y|$, at least one pair has a sum of 10 if at least 4 integers be choose.

(d) By using the third form of Pigeonhole Principle,

$k = \frac{n}{m}$ , where $k$ = minimum number of student per same grade ( $k = 6$ )
$n$ = number of student (pigeon)
$m$ = number of grades (pigeonholes, $m = 5$)

$6 = \frac{n}{5}$ , for the $\frac{n}{5}$ , it must be $>$ 5 (which can be round up)

Here,

$$\frac{n}{5} > 5$$

$$n > 5 \times 5$$

$$n > 25$$

$$n = 26$$

$\therefore$ There is minimum 26 of students required in a discrete mathematics class to be sure that at least six will receive the same grade.

Pigeon : number of computers ($n=6$)

The computer can connect to the other computers, which 0 or more. But it cannot connected to iteself.

Pigeonhole : number of computer connected ($m=5$)

$$k = \frac{n}{m}$$

$$= \frac{6}{5}$$

$$= 1.2$$

$$k = 2$$ (shown)