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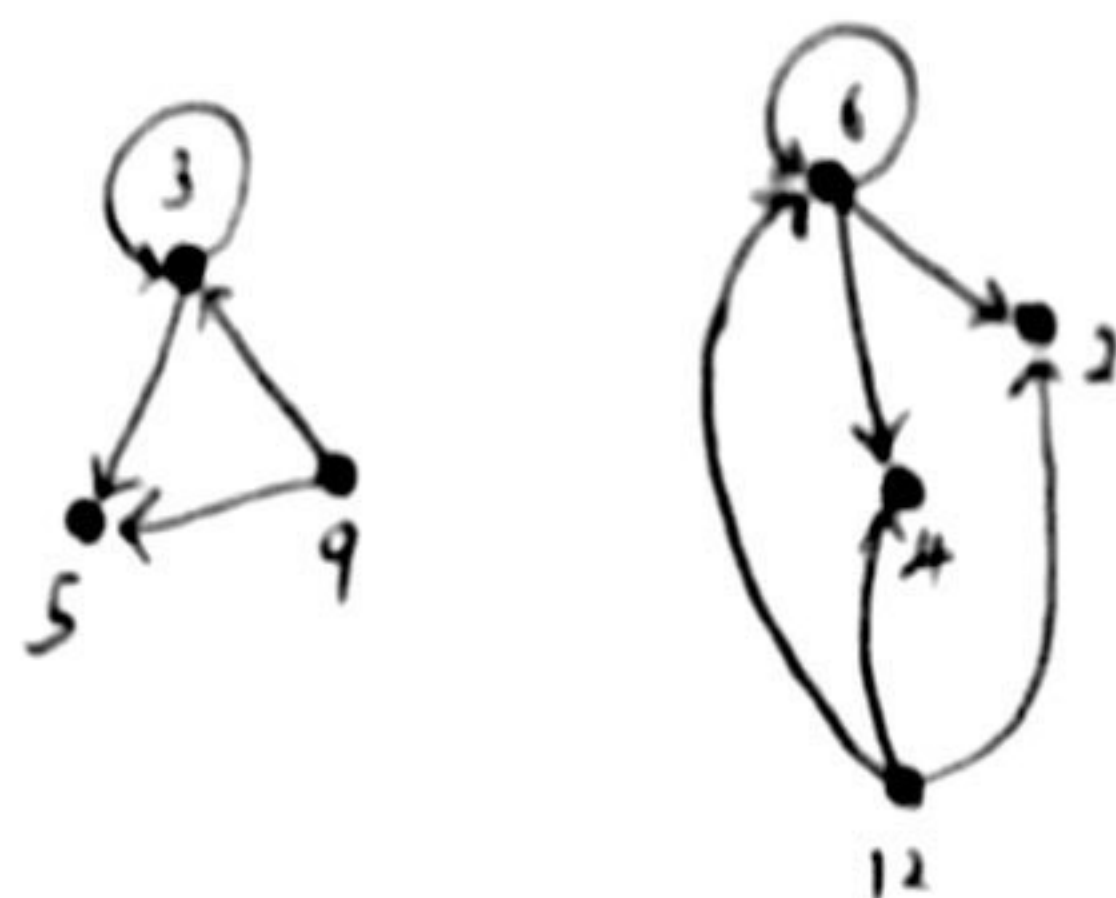
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1. (i) $R = \{(3,3), (3,5), (6,2), (6,4), (6,6), (9,3), (9,5), (12,2), (12,4), (12,6)\}$

(ii)



(iii) domain: $\{3, 6, 9, 12\}$

range: $\{2, 3, 4, 5, 6\}$

2. $D = \{(1,1), (1,8), (1,15), (3,3), (3,10), (8,1), (8,8), (8,15), (10,3), (10,10), (15,1), (15,8), (15,15)\}$

- D is reflexive because all element related to itself, which are $(1,1), (3,3), (8,8), (10,10), (15,15)$

$$M_R = \begin{matrix} & \begin{matrix} 1 & 3 & 8 & 10 & 15 \end{matrix} \\ \begin{matrix} 1 \\ 3 \\ 8 \\ 10 \\ 15 \end{matrix} & \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix} \end{matrix} = M_R^T$$

D is symmetric.

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

D is transitive.

\therefore Since D is reflexive, symmetric and transitive, it is equivalent relation.

3. (i)

$$\begin{matrix} & \begin{matrix} s & t & u & v \end{matrix} \\ \begin{matrix} s \\ t \\ u \\ v \end{matrix} & \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

(ii)

	s	t	u	v
in-degree	1	2	3	1
out-degree	3	3	2	0

(iii) - The relation of R is not reflexive because not all element is related to itself.
 $(v, v) \notin R$.

- The relation of R is not antisymmetric because there is $(s, u) \in R$ and $(u, s) \in R$.

$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

The relation of R is not transitive.

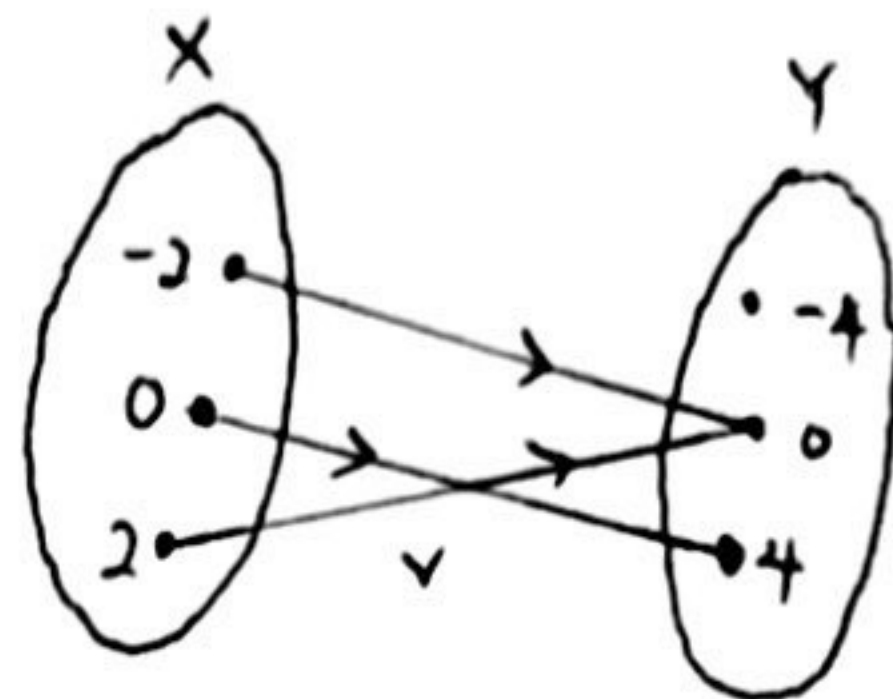
\therefore A partial order must be reflexive, antisymmetric and transitive. Since R is reflexive, not antisymmetric and not transitive, the relation of R is not a partial order.

4. $V(x) = 4 - x^2$

$$V(-2) = 4 - (-2)^2 = 0$$

$$V(0) = 4 - (0)^2 = 4$$

$$V(2) = 4 - (2)^2 = 0$$



$$V(x) = \{(-2, 0), (0, 4), (2, 0)\}$$

$V(x)$ is not one-to-one because both of -2 and 2 on X are pointing to 0 on Y .

$V(x)$ is not onto because there is no element pointing to -4 on Y .

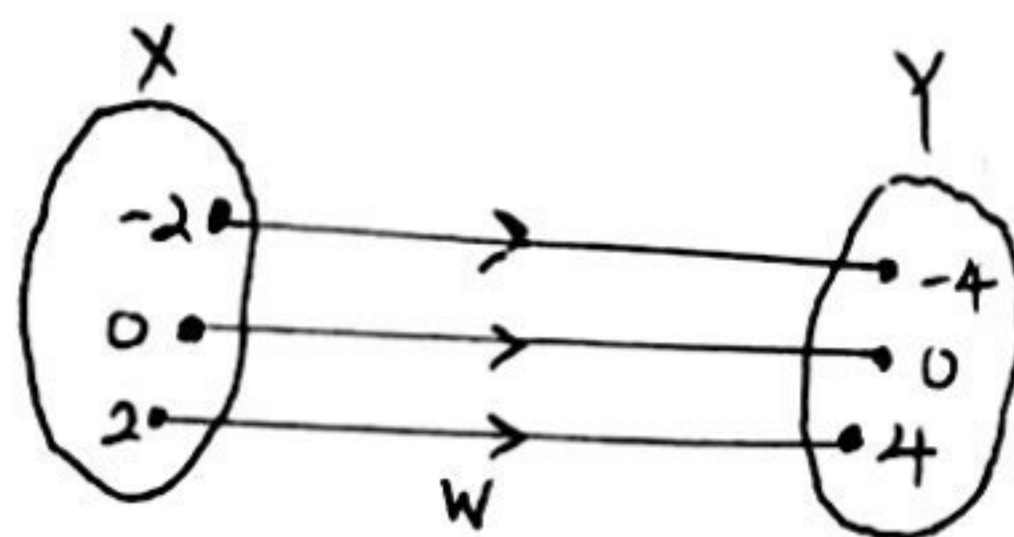
$\therefore V(x)$ is not bijection because it is not one-to-one and not onto.

$$W(x) = 2x$$

$$W(-2) = 2(-2) = -4$$

$$W(0) = 2(0) = 0$$

$$W(2) = 2(2) = 4$$



$$W(x) = \{(-2, -4), (0, 0), (2, 4)\}$$

$W(x)$ is one-to-one because each element in Y has at most one arrow.

$W(x)$ is onto because each element in Y has at least one arrow pointing to it.

$\therefore W(x)$ is bijection because it is one-to-one and onto.

5. (i) $g(x) = \frac{2}{3}x$

$$y = \frac{2}{3}x$$

$$3y = 2x$$

$$x = \frac{3}{2}y$$

$$g^{-1}(y) = \frac{3}{2}y$$

\therefore Inverse function of $g(x)$

$$\text{is } g^{-1}(y) = \frac{3}{2}y$$

(ii) $(g \circ g \circ f)(x) = gg f(x)$

$$= gg[f(x)]$$

$$= gg(7x-2)$$

$$= g[g(7x-2)]$$

$$= g\left[\frac{2}{3}(7x-2)\right]$$

$$= \frac{2}{3}\left[\frac{2}{3}(7x-2)\right]$$

$$= \frac{2}{3}\left(\frac{14}{3}x - \frac{4}{3}\right)$$

$$(g \circ g \circ f)(x) = \frac{28}{9}x - \frac{8}{9}$$

$$\therefore \text{Composition } (g \circ g \circ f)(x) = \frac{28}{9}x - \frac{8}{9}$$

$$(i) F_0 = 5.0$$

$$F_1 = 4.5$$

$$F_t = F_{t-1} + \frac{1}{5} F_{t-2}, t \geq 2$$

$$(ii) F_0 = 5.0$$

$$F_1 = 4.5$$

$$F_2 = F_{(2-1)} + \frac{1}{5} F_{(2-2)}$$

$$= F_1 + \frac{1}{5} F_0$$

$$= 4.5 + \frac{1}{5}(5.0)$$

$$F_2 = 5.5$$

$$F_3 = F_{(3-1)} + \frac{1}{5} F_{(3-2)}$$

$$= F_2 + \frac{1}{5} F_1$$

$$= 5.5 + \frac{1}{5}(4.5)$$

$$F_3 = 6.4$$

$$F_4 = F_{(4-1)} + \frac{1}{5} F_{(4-2)}$$

$$= F_3 + \frac{1}{5} F_2$$

$$= 6.4 + \frac{1}{5}(5.5)$$

$$F_4 = 7.5$$

$$F_5 = F_{(5-1)} + \frac{1}{5} F_{(5-2)}$$

$$= F_4 + \frac{1}{5} F_3$$

$$= 7.5 + \frac{1}{5}(6.4)$$

$$F_5 = 8.78$$

$$7. W_0 = 5$$

$$W_1 = 7$$

input: n

output: w(n)

w(n)

{

if (n=0)

return 5

else if (n=1)

return 7

else

return (2 * w(n-1) + w(n-2))

}

When $n \geq 2$,

$$w(n) = 2w(n-1) + w(n-2)$$

$$w(2) = 2w(2-1) + w(2-2)$$

$$= 2w(1) + w(0)$$

$$= 2(7) + 5$$

$$w(2) = 19$$

$$w(3) = 2w(3-1) + w(3-2)$$

$$= 2w(2) + w(1)$$

$$= 2(19) + 7$$

$$w(3) = 45$$

$$w(4) = 2w(4-1) + w(4-2)$$

$$= 2w(3) + w(2)$$

$$= 2(45) + 19$$

$$w(4) = 109$$