

FHR Benchmark Equations

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This document contains the assumptions and equations used for the FHR benchmark excel spreadsheet results.

1 Quantities of Interest

- (a) Effective multiplication factor
- (b) Reactivity coefficients (β_{eff} , fuel Doppler coefficient, FLiBe temperature coefficient, graphite temperature coefficient)
- (c) Tabulated fission source distribution, at several levels of granularity (by fuel plate, by fuel stripe, by 1/5-th fuel stripe). Optional: visualized fission density distribution.
- (d) Neutron flux averaged over the whole model tabulated in 3 coarse energy groups.
- (e) Visualized distribution of the neutron flux distribution, in 3 coarse energy groups
- (f) Neutron spectrum, fuel assembly average. Optional: by region.

1.1 Effective multiplication factor (a)

Assumptions made:

- No. of CPUs = No. of Nodes \times 32 (no. of CPUs in each Blue Waters XE node)
- CPU-time = No. of CPUs \times Total time in simulation (in openmc's results file)
- Wall Clock Time = Total time elapsed (in openmc's results file)

1.2 Reactivity coefficients (b)

We assume 1 energy group and 6 delayed neutron groups for β_{eff} .

$$\beta_{eff} = \sum_k \beta_k$$

Doppler reactivity coefficient (fuel):

$$\frac{\Delta\rho}{\Delta T_f} = \frac{\rho_{1150K} - \rho_{1100K}}{1150 - 1100} \left[\frac{pcm}{K} \right]$$

$$\delta \frac{\Delta\rho}{\Delta T_f} = \frac{\sqrt{\delta(\rho_{1150K})^2 + (\delta\rho_{1100K})^2}}{1150 - 1100} \left[\frac{pcm}{K} \right]$$

Coolant reactivity coefficient (FLiBe):

$$\frac{\Delta\rho}{\Delta T_c} = \frac{\rho_{1150K} - \rho_{1100K}}{1150 - 1100} \left[\frac{pcm}{K} \right]$$

$$\delta \frac{\Delta\rho}{\Delta T_c} = \frac{\sqrt{\delta(\rho_{1150K})^2 + (\delta\rho_{1100K})^2}}{1150 - 1100} \left[\frac{pcm}{K} \right]$$

Graphite reactivity coefficient (graphite):

$$\frac{\Delta\rho}{\Delta T_g} = \frac{\rho_{1150K} - \rho_{1100K}}{1150 - 1100} \left[\frac{pcm}{K} \right]$$

$$\delta \frac{\Delta\rho}{\Delta T_g} = \frac{\sqrt{\delta(\rho_{1150K})^2 + (\delta\rho_{1100K})^2}}{1150 - 1100} \left[\frac{pcm}{K} \right]$$

We assumed all graphite.

1.3 Fission source distribution (c)

Fission density (FD) is calculated by using openmc's 'fission' score (f) divided by the average of all 'fission' scores:

$$FD_i = \frac{f_i}{f_{ave}}$$

where:

$$f_i = \text{Total fission reaction rate [reactions/src]}$$

$$f_{ave} = \text{average of all } f_i \text{ [reactions/src]}$$

The uncertainty calculations for f_{ave} and FD_i :

$$\delta f_{ave} = \frac{1}{N} \sqrt{\sum_i^N f_i^2}$$

$$\delta FD_i = |FD_i| \sqrt{\left(\frac{\delta f_i}{f_i}\right)^2 + \left(\frac{\delta f_{ave}}{f_{ave}}\right)^2}$$

where:

N = No. of fission score values

1.4 Neutron Flux (d, e, f)

Openmc's 'flux' score is given in units of $[\frac{n*cm}{src}]$. For the benchmark, we need to convert it to units of $[\frac{n}{cm^2s}]$. The conversion:

$$\Phi_c = \frac{N * \Phi_o}{V}$$

$$N = \frac{P * \nu}{Q * k}$$

where:

$$\Phi_c = \text{Converted Flux } [\frac{neutrons}{cm^2s}]$$

$$\Phi_o = \text{Original Flux } [\frac{neutrons*cm}{src}]$$

$$N = \text{Normalization factor } [\frac{src}{s}]$$

$$V = \text{Volume of fuel assembly } [cm^3]$$

$$P = \text{Power } [\frac{J}{s}]$$

$$\nu = \frac{\nu_f}{f} [\frac{neutrons}{fission}]$$

$$Q = \text{Energy produced per fission } [\frac{J}{fission}]$$

$$= 3.2044 * 10^{-11} \text{ J per } U_{235} \text{ fission}$$

$$k = k_{eff} [\frac{neutrons}{src}]$$

Flux standard deviation:

$$\delta\Phi_c = \Phi_c * \sqrt{(\frac{\delta\Phi_o}{\Phi_o})^2 + (\frac{\delta\nu_f}{\nu_f})^2 + (\frac{\delta k}{k})^2 + (\frac{\delta f}{f})^2}$$

Reactor power is calculated based on the given reference specific power (P_{sp}) of 200 $\frac{W}{gU}$.

$$P = P_{sp} * V_F * \rho_F * \frac{wt\%U}{100}$$

where:

P = Reactor power [W]

V_F = Volume of fuel [cm^3]

$$= \frac{4}{3} \pi r_1^3 * 101 * 210 * 4 * 2 * 6 * 3$$

ρ_F = density of fuel [g/cc]

$$wt\%_U = \frac{at\%_{U235} * AM_{U235} + at\%_{U238} * AM_{U238}}{\sum (at\%_i * AM_i)} * 100$$

AM = atomic mass