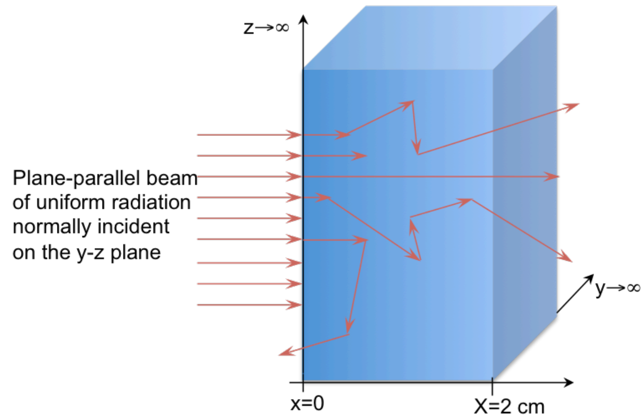
RADI.6060 Monte Carlo Simulation of Radiation Transport
Homework 3, Due April 15, 2024

1. Consider the geometry shown in the figure. A very small diameter plane-parallel pencil-beam of 100 keV x-rays is normally incident on the left side of a semi-infinite water-equivalent parallelepiped. The dimension of the slab is infinite in the y and z directions, but not in the x-direction. The interaction cross-sections of water are given below.

   Write a Monte Carlo computer code to estimate the fraction of the particles that are:

   a.) Absorbed;
   b.) Transmitted through the x=2 cm plane;
      1. Scattered and transmitted
      2. Uncollided and transmitted
   c.) Backscattered through the x=0 cm plane
   d.) Compare your result in b.2) with the analytical uncollided formula.



Incident Photon Energy	Photoelectric absorption XS	Scatter XS
50 keV	2.725E-02 cm^{-1}	1.803E-01 cm^{-1}
100 keV	2.763E-03 cm^{-1}	1.626E-01 cm^{-1}

Assume that the polar angular scatter PDF is $f(\mu) = 1 + \mu^2$, while there is a uniform scatter in the azimuthal direction, such that the scatter cross-section is $\Sigma_s(E,\mu) = 2\pi\,\Sigma_s(E)f(\mu)$, where Σ_s is given in the table above and μ is the cosine of the polar scatter angle.

Use adequate number of histories to make the relative uncertainties below 1% and report the MC estimate as $\hat{x} \pm \hat{\sigma}$.

Hints

A. Because of the very small beam diameter, you may assume that the "pencil" beam is incident effectively at a point on the surface of the x=0 plane. You can conveniently take its coordinate as (0,0,0), if you wish. Therefore, every new history starts at this point.

B. We have shown in class that the collisional PDF is written as $f(x) = \Sigma_t e^{-\Sigma_t x}$. Here, $f(x)dx$ represents the probability that the incident particle will have its interaction with the material within *dx* about *x*. In this problem, the following steps are executed:
   (1) Select the source energy (if the beam is not monoenergetic.) Note: this HW specifies a monoenergetic beam.
   (2) Sample the collisional PDF to determine where the first collision will occur. Note that $\Sigma_t = \Sigma_a + \Sigma_s$.
   (3) Test weather the first collision is beyond 2 cm. Beyond 2 cm means that the particle did not suffer collision (uncollided). In this case, record the uncollided score and start a new history at (1).

(4) If the particle suffers collision before it reaches the boundary of the slab, we sample the non-absorption probability to find whether the particle is absorbed or scattered.
(5) If absorbed, record the appropriate score, then start a new history, starting at (1).
(6) If scattered, sample the angular distribution, polar angle, $f(\mu)$, and the azimuthal angle (uniform within 2π). This gives the new direction for the particle in the directional coordinate system, which must be converted to the spatial coordinate system to determine the equation of this line. Use the Ω_x=… , Ω_y=… , Ω_z=… equations given earlier.
(7) Now sample the collisional PDF along the new direction of the particle and check whether the collision will be within the boundary of the slab. This simply means that you calculate the distance along the line between the collision point and the intersection point with the slab boundary, and check whether the distance to the next collision is shorter or longer than this. Note that if the sampled polar cosine $\mu < 0$ in step (6), the particle direction will be towards the x=0 plane (reverse) while when $\mu > 0$, it will go towards the x=2 cm plane (forward).
(8) If the next collision along the path of the particle is beyond the intersection point with the x=0 or x=2 plane, it means that is has been backscattered or transmitted, respectively. Make sure to record whether the transmission was forward (passed the x=2 plane) or reverse (passed the x=0 plane). Start a new history (1).
(9) If the collision occurred inside the slab, sample the non-absorption probability to find whether the particle is absorbed or scattered. Continue from (4) on.