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Birkhoff Duality

Key sources: nLab:Birkhoff duality

Summary. Finite distributive lattices are dual to finite posets via ideals and sets of join-irreducibles.

Idea

Birkhoff duality identifies a finite distributive lattice with the poset of its join-irreducible elements, while the lattice itself is recovered as the lattice of order ideals of that poset. Morphisms become order-preserving maps in the opposite direction.

Definition 1 (Duality statement)**.** Let FDLat be the category of finite distributive lattices and FPos the category of finite posets. The functor sending L to its poset of join-irreducibles $J(L)$ and P to its lattice of ideals $\mathrm{Idl}(P)$ yields an equivalence $\mathsf{FDLat} \simeq \mathsf{FPos}^{\mathrm{op}}$.

See also:

- *category*
- *functor*
- *natural transformation*

Incoming links:

- *dg algebra*

References

- nLab:Birkhoff duality|nLab:Birkhoff duality|