Title

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In this paper we present simulation results comparing the risk, stopping probability, and number of ballots required over multiple rounds of ballot polling risk-limiting audits (RLAs) Minerva, Selection-Ordered (SO) Bravo, and End-of-Round (EoR) Bravo. Bravo is the most commonly used ballot polling RLA and requires the smallest expected number of ballots when ballots are drawn one at a time and the (true) underlying election is as announced. In real audits, multiple ballots are drawn at a time, and Bravo is implemented as SO Bravo or EoR Bravo. Minerva is a recently proposed ballot polling RLA that requires fewer ballots than either implementation of Bravo in a first round with stopping probability 0.9 but requires a predetermined round schedule. It is an open question how these audits compare over multiple rounds and for lower stopping probabilities. Our simulations use stopping probabilities of 0.9 and 0.25. The results are consistent with predictions of the R2B2 open-source library for ballot polling audits. We observe that both Bravo audits are more conservative than Minerva, which stops with fewer ballots, for both first round stopping probabilities. However, the advantage of using Minerva decreases considerably for the smaller first round stopping probability, as one would expect.

Introduction

In this section, we motivate and describe the experiments. We consider a two candidate plurality contest, and assume that ballots are sampled with replacement, as is common in the literature. Note that sampling without replacement is more efficient for large sampling fractions, but Minerva has not been extended for sampling without replacement. We first present relevant definitions.

Definition

An audit $\mathcal{A}$ takes a sample of ballots X as input and gives as output either (1) $Correct$: the audit is complete, or (2) $Uncertain$: continue the audit.

All of the audits discussed in this paper are modeled as binary hypothesis tests. Under the alternative hypothesis, H_a, the announced outcome is correct. In particular, the true underlying ballot distribution is given by the announced ballot tallies. Under the null hypothesis, H_0, a tie is the correct outcome [a]. The maximum risk of an audit is the probability that an audit stops, given that the underlying election is a tie [**?**]. Note that an audit $\mathcal{A}$ includes all audit parameters (maximum risk, round sizes, etc.).

Definition (Risk)

The maximum risk R of audit $\mathcal{A}$ with sample $X \in \{0,1\}^*$ drawn from the true underlying distribution of ballots is $R(\mathcal{A}) = \Pr[\mathcal{A}(X) = Correct \mid H_0]$.

This leads us to the following definition of an α-RLA.

Definition (Risk Limiting Audit (α-RLA))

An audit $\mathcal{A}$ is a Risk Limiting Audit with risk limit α iff $R(\mathcal{A}) \leq \alpha$.

We present measures of stopping probability in the j^{th} round of the audit, given that the underlying election is as announced.

Definition (Stopping Probability)

The stopping probability S_j of an audit $\mathcal{A}$ in round j is

$$S_j(\mathcal{A}) = \Pr[\mathcal{A}(X) = Correct\ in\ round\ j\ \wedge \mathcal{A}(X) \neq Correct\ previously \mid H_a]$$

Experimentally, using our simulations, S_j would be estimated by the fraction of audits that stop in round j. Note that $\sum_j S_j(\mathcal{A}) = 1$. We can also consider the cumulative stopping probability:

Definition (Cumulative Stopping Probability)

The cumulative stopping probability C_j of an audit $\mathcal{A}$ in round j is $C_j(\mathcal{A}) = \sum_{i=1}^{j} S_j$

Experimentally, using our simulations, C_j would be estimated by the fraction of audits that stop in or before round j.

Finally, we are also interested in the probability that an audit will stop in round j given that

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