Mechanical modes (Photo-elastic effect)

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$$S_I = \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{bmatrix} = \begin{bmatrix} S_{xx} \\ S_{yy} \\ S_{zz} \\ 2S_{yz} \\ 2S_{xz} \\ 2S_{xy} \end{bmatrix}$$

$$[\Delta\beta(\boldsymbol{r};\overleftrightarrow{S})]_{ij} = p_{ijkl}(\boldsymbol{r})S_{kl}(\boldsymbol{r})$$

terms of the "impermeability tensor"

Photo-elastic effect is described in

Voigt notation: Strain is a

symmetric tensor

$$\Rightarrow \Delta\epsilon_{im}\beta_{mn} = -\epsilon_{im}\Delta\beta_{mn}$$

$$\Rightarrow \Delta\epsilon_{im}\beta_{mn}\epsilon_{nj} = -\epsilon_{im}\Delta\beta_{mn}\epsilon_{nj}$$

$$\Rightarrow \Delta\epsilon_{ij} = -\epsilon_{im}\Delta\beta_{mn}\epsilon_{nj}$$

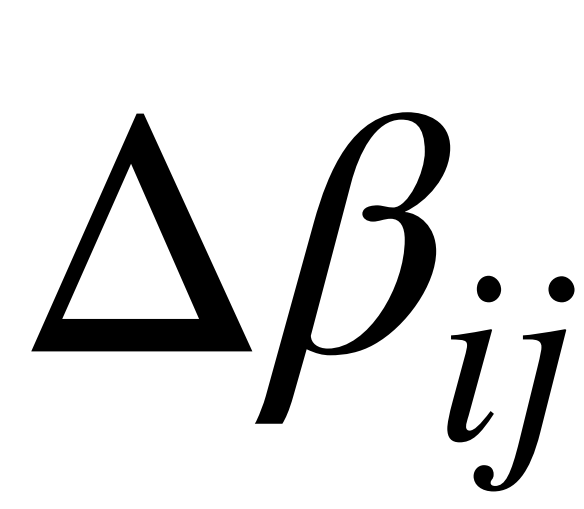
$$\Rightarrow \Delta\epsilon_{ij} = -\epsilon_{im}\left(p_{mnrs}S_{rs}\right)\epsilon_{nj}$$

$$\Rightarrow \Delta\epsilon_{ij} = -\epsilon^2 \left(p_{ijrs} S_{rs} \right)$$

$$\beta_{ij}\epsilon_{jk} = \delta_{ik}$$

$$\beta_{ij} x_i x_j = 1$$

Perturbation



$$\Rightarrow \Delta\epsilon_I = -\epsilon^2 p_{IJ} S_j$$

$$\begin{bmatrix} p_{11} & p_{12} & p_{12} & 0 & 0 & 0 \\ p_{12} & p_{11} & p_{12} & 0 & 0 & 0 \\ p_{12} & p_{12} & p_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & p_{44} \end{bmatrix}$$


S_1
S_2
S_3
S_4
S_5
S_6

$$\Delta \epsilon_I = -\epsilon^2$$

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$$\Rightarrow \Delta\epsilon_{ij} = -\epsilon^2 \left(p_{ijrs} S_{rs} \right)$$