The full Brillouin gain calculation

$$\left(v_p \partial_z + \partial_t + v_p \alpha_p / 2 \right) \tilde{a}_p = - i \tilde{g}_0 \tilde{a}_s \tilde{b}$$

$$\left(\pm v_s \partial_z + \partial_t + v_s \alpha_s / 2 \right) \tilde{a}_s = - i \tilde{g}_0^* \tilde{b}^* \tilde{a}_p$$

$$\left[v_m \partial_z + \partial_t + \left(i \Delta_m + \gamma_m / 2 \right) \right] \tilde{b} = - i \tilde{g}_0^* \tilde{a}_s^* \tilde{a}_p,$$
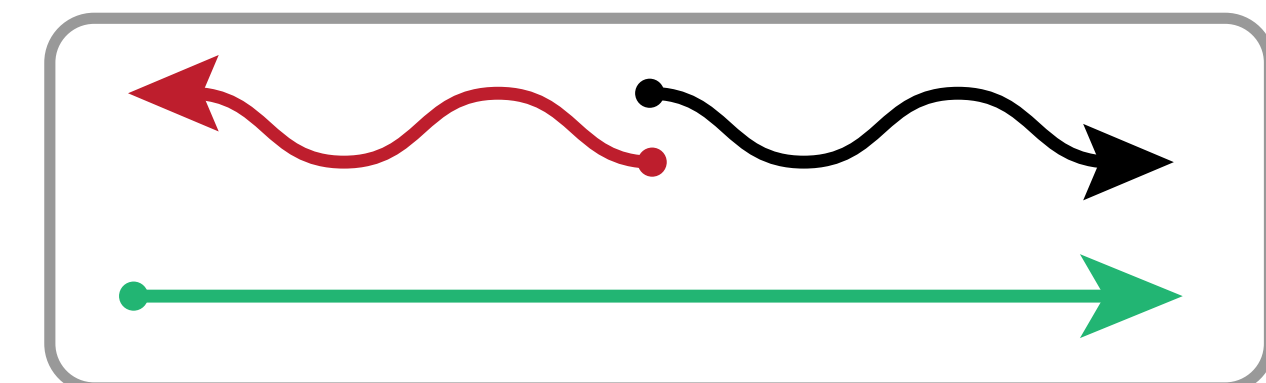
Propagation   **Detuning**   **Loss**

Polarization components: $\propto U(\mathbf{r}) E(r)$

$$
\begin{pmatrix}
b(a_1)^* & zk_2 + t(-\Omega + \omega_1) \\
(b)^*(a_1)^* & z(-2k_1 - k_2) + t(\Omega + \omega_1) \\
b(a_2)^* & z(k_1 + 2k_2) + t(-2\Omega + \omega_1) \\
(b)^*(a_2)^* & -zk_1 + t\omega_1 \\
ba_1 & z(2k_1 + k_2) + t(-\Omega - \omega_1) \\
(b)^*a_1 & -zk_2 - t(\omega_1 - \Omega) \\
ba_2 & zk_1 - t\omega_1 \\
(b)^*a_2 & z(-k_1 - 2k_2) + t(2\Omega - \omega_1)
\end{pmatrix}
$$

1. Wiederhecker, G. S., Dainese, P. & Alegre, T. P. M. Brillouin optomechanics in nanophotonic structures. APL Photonics 4, (2019).

The full Brillouin gain calculation

$$\left(v_p \partial_z + \partial_t + v_p \alpha_p/2 \right) \tilde{a}_p = -i\tilde{g}_0 \tilde{a}_s \tilde{b}$$

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Propagation   **Detuning**   **Loss**

Acoustic drive: $\propto \nabla^2 E^2(r)$

$$\begin{pmatrix} -2q^2 & \left(a_2\right)^* & a_1 & qz - t\Omega \\ -2q^2 & \left(a_1\right)^* & a_2 & -qz + t\Omega \end{pmatrix}$$

Polarization components: $\propto U(\mathbf{r})E(r)$

$$\begin{pmatrix} b(a_1)^* & zk_2 + t(-\Omega + \omega_1) \\ (b)^*(a_1)^* & z(-2k_1 - k_2) + t(\Omega + \omega_1) \\ b(a_2)^* & z(k_1 + 2k_2) + t(-2\Omega + \omega_1) \\ (b)^*(a_2)^* & -zk_1 + t\omega_1 \\ ba_1 & z(2k_1 + k_2) + t(-\Omega - \omega_1) \\ (b)^*a_1 & -zk_2 - t(\omega_1 - \Omega) \\ ba_2 & zk_1 - t\omega_1 \\ (b)^*a_2 & z(-k_1 - 2k_2) + t(2\Omega - \omega_1) \end{pmatrix}$$

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