# The full Brillouin gain calculation

$$\left(v_p\partial_z + \partial_t + v_p\alpha_p/2\right)\tilde{a}_p = -i\tilde{g}_0\tilde{a}_s\tilde{b}$$

$$\left(\pm v_s\partial_z + \partial_t + v_s\alpha_s/2\right)\tilde{a}_s = -i\tilde{g}_0^*\tilde{b}^*\tilde{a}_p$$

$$\left[v_m\partial_z + \partial_t + \left(i\Delta_m + \gamma_m/2\right)\right]\tilde{b} = -i\tilde{g}_0^*\tilde{a}_s^*\tilde{a}_p,$$

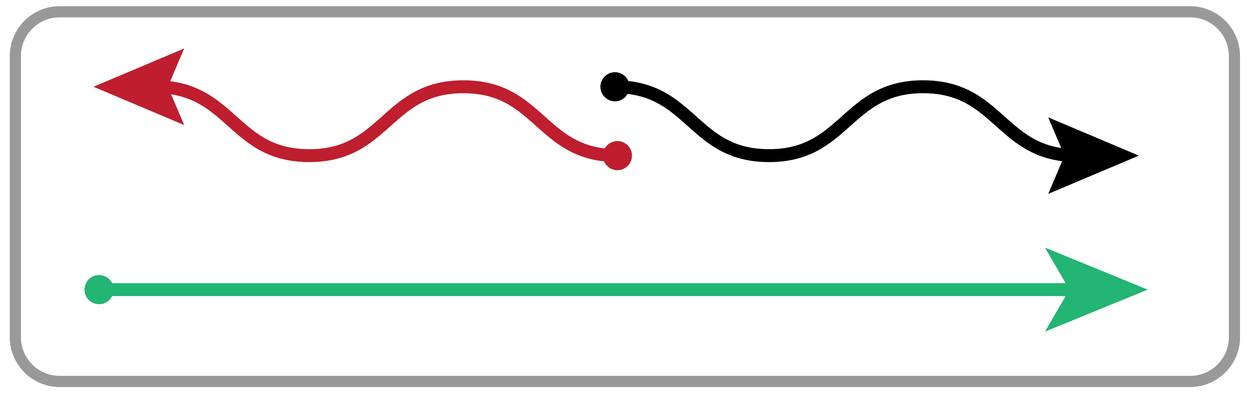
**Propagation** **Detuning** **Loss**

Acoustic drive: $\propto \nabla^2 E^2(r)$

$$\begin{pmatrix} -2q^2 & (a_2)^* & a_1 & qz - t\Omega \\ -2q^2 & (a_1)^* & a_2 & -qz + t\Omega \end{pmatrix}$$

Polarization components: $\propto U(\mathbf{r})E(r)$

$$\begin{pmatrix} b(a_1)^* & zk_2 + t(-\Omega+\omega_1) \\ (b)^*(a_1)^* & z(-2k_1-k_2) + t(\Omega+\omega_1) \\ b(a_2)^* & z(k_1+2k_2) + t(-2\Omega+\omega_1) \\ (b)^*(a_2)^* & -zk_1 + t\omega_1 \\ ba_1 & z(2k_1+k_2) + t(-\Omega-\omega_1) \\ (b)^*a_1 & -zk_2 - t(\omega_1-\Omega) \\ ba_2 & zk_1 - t\omega_1 \\ \text{(b)}^*a_2 & z(-k_1-2k_2) + t(2\Omega-\omega_1) \end{pmatrix}$$



1. Wiederhecker, G. S., Dainese, P. & Alegre, T. P. M. Brillouin optomechanics in nanophotonic structures. APL Photonics 4, (2019).

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$$\left(v_p\partial_z + \boxed{\partial_t} + \boxed{v_p\alpha_p/2}\right)\tilde{a}_p = \boxed{-i\tilde{g}_0\tilde{a}_s\tilde{b}}$$

$$\left(\pm v_s\partial_z + \boxed{\partial_t} + \boxed{v_s\alpha_s/2}\right)\tilde{a}_s = \boxed{-i\tilde{g}_0^*\tilde{b}^*\tilde{a}_p}$$

$$\left[\boxed{v_m\partial_z + \partial_t} + \left(\boxed{i\Delta_m} + \boxed{\gamma_m/2}\right)\right]\tilde{b} = \boxed{-i\tilde{g}_0^*\tilde{a}_s^*\tilde{a}_p},$$

- Steady state: $\partial_t = 0$
- Lossy mechanical wave (large $\gamma_m/v_m$)

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