Mechanical modes

$$\frac{\partial}{\partial r_j} T_{ij}(\boldsymbol{r}, t) = \rho_0(\boldsymbol{r}) \frac{\partial^2 U_i}{\partial t^2} - f_i(\boldsymbol{r}, t)$$

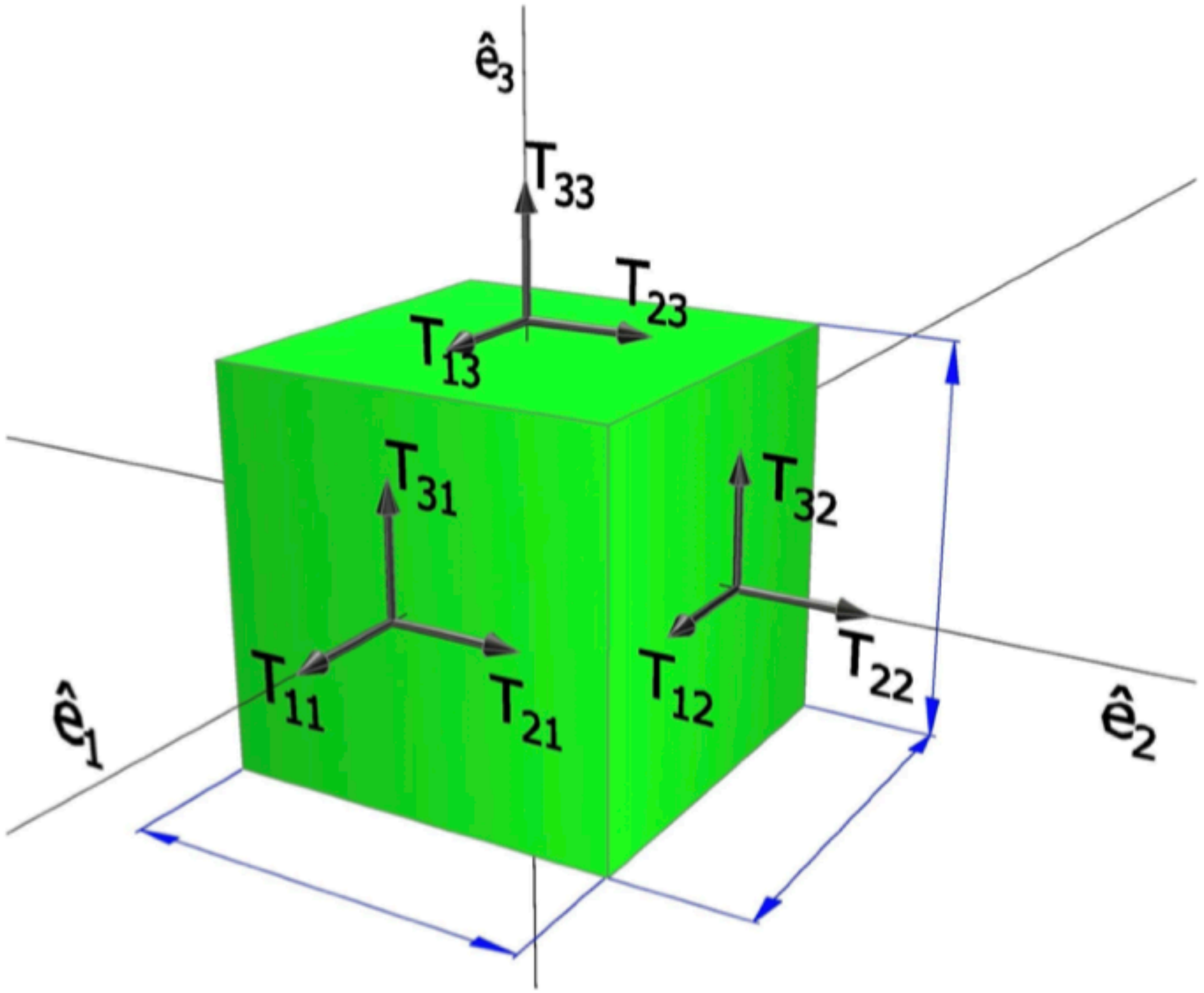
Stress tensor — acceleration — External force

$$T_{ij} = c_{ijkl} S_{kl} + \eta_{ijkl} \frac{\partial S_{kl}}{\partial t}$$

Stiffness (Hooke's law) — Friction

$$S_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial r_j} + \frac{\partial U_j}{\partial r_i}\right)$$

Strain Tensor



$$\left[(\lambda + 2\mu) + \eta_{11}\frac{\partial}{\partial t}\right] \nabla(\nabla \cdot \boldsymbol{U}) - \left[\mu + \eta_{44}\frac{\partial}{\partial t}\right] \nabla \times \nabla \times \boldsymbol{U} = \rho \frac{\partial^2 \boldsymbol{U}}{\partial t^2}$$

longitudinal waves ($\nabla \times \mathbf{u} = 0$) — shear-only ($\nabla \cdot \mathbf{u} = 0$)

Elastic wave equation

$$\boldsymbol{U}(\boldsymbol{r}, t) = \tilde{\boldsymbol{u}}^{(n)}(\boldsymbol{r}) e^{-\mathrm{i}\Omega_n t} + \mathbf{c.c.}$$
$$= \boldsymbol{u}^{(n)}(x, y) e^{\mathrm{i}[q_n z - \Omega_n t]} + \mathbf{c.c.}$$

Waveguide mode ansatz

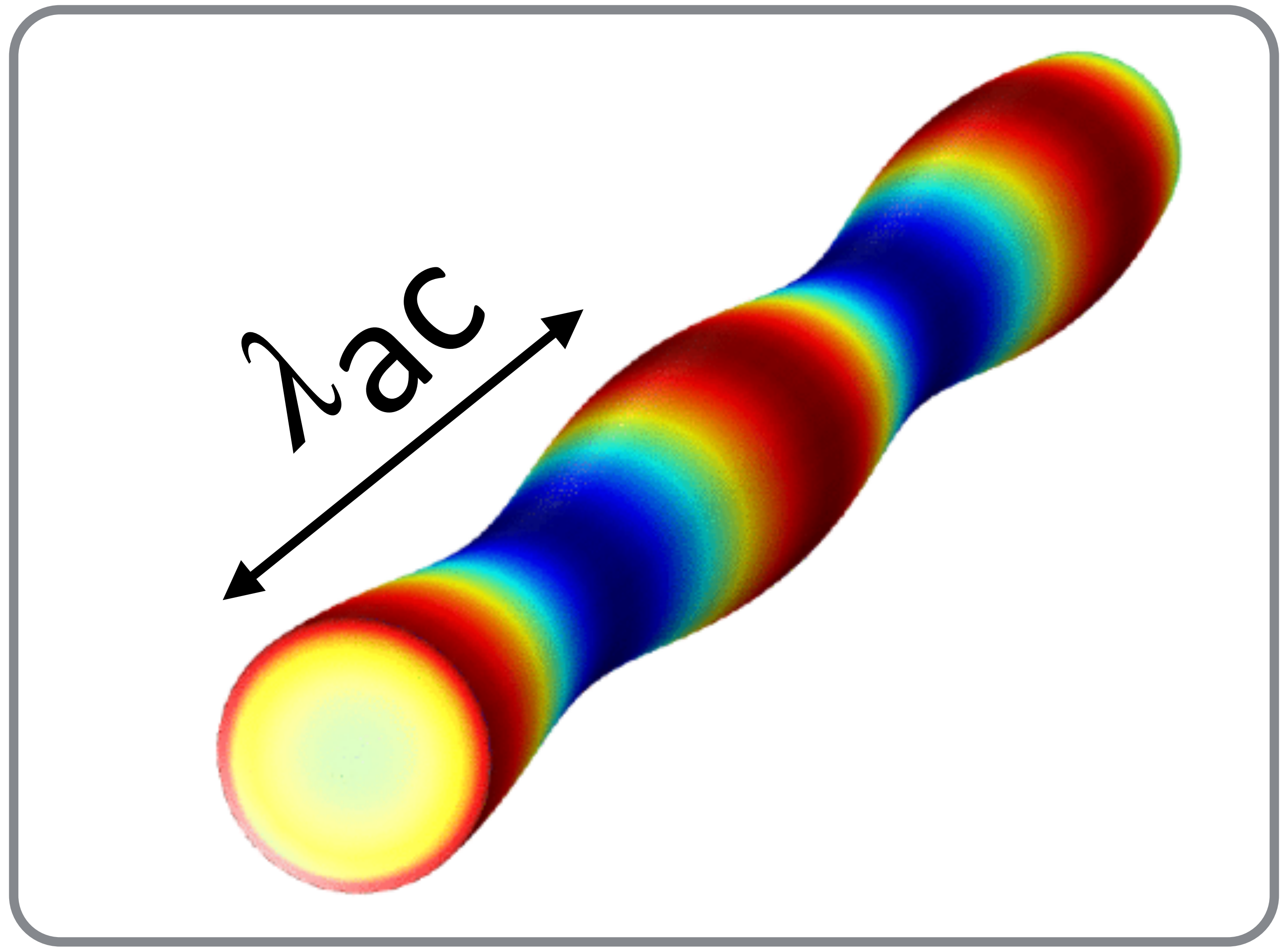
Mechanical modes

$$\left[(\lambda+2\mu)+\eta_{11}\frac{\partial}{\partial t}\right]\nabla(\nabla\cdot\boldsymbol{U})-\left[\mu+\eta_{44}\frac{\partial}{\partial t}\right]\nabla\times\nabla\times\boldsymbol{U}=\rho\frac{\partial^2\boldsymbol{U}}{\partial t^2}$$

longitudinal waves ($\nabla\times\mathbf{u}=0$) shear-only ($\nabla\cdot\mathbf{u}=0$)

Elastic wave equation

Radial mode



Torsional mode