



The full Brillouin gain calculation

$$\left(v_p \partial_z + \partial_t + v_p \alpha_p / 2 \right) \tilde{a}_p = -i \tilde{g}_0 \tilde{a}_s \tilde{b}$$

$$\left(\pm v_s \partial_z + \partial_t + v_s \alpha_s / 2 \right) \tilde{a}_s = -i \tilde{g}_0^* \tilde{b}^* \tilde{a}_p$$

$$\left[v_m \partial_z + \partial_t + \left(i \Delta_m + \gamma_m / 2 \right) \right] \tilde{b} = -i \tilde{g}_0^* \tilde{a}_s^* \tilde{a}_p,$$

Propagation

Detuning

Loss

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The full Brillouin gain calculation

$$\left(\underbrace{v_p \partial_z + \partial_t}_{\text{Propagation}} + \underbrace{v_p \alpha_p / 2}_{\text{Loss}} \right) \tilde{a}_p = \underbrace{-i \tilde{g}_0 \tilde{a}_s \tilde{b}}_{\text{Detuning}}$$

$$\left(\underbrace{\pm v_s \partial_z + \partial_t}_{\text{Propagation}} + \underbrace{v_s \alpha_s / 2}_{\text{Loss}} \right) \tilde{a}_s = \underbrace{-i \tilde{g}_0^* \tilde{b}^* \tilde{a}_p}_{\text{Detuning}}$$

$$\left[\underbrace{v_m \partial_z + \partial_t}_{\text{Propagation}} + \underbrace{(i \Delta_m + \gamma_m / 2)}_{\text{Detuning}} \right] \tilde{b} = \underbrace{-i \tilde{g}_0^* \tilde{a}_s^* \tilde{a}_p}_{\text{Loss}}$$

Propagation

Detuning

Loss

Polarization components: $\propto U(\mathbf{r})E(r)$

$$\begin{pmatrix} b(a_1)^* & zk_2 + t(-\Omega + \omega_1) \\ (b)^*(a_1)^* & z(-2k_1 - k_2) + t(\Omega + \omega_1) \\ b(a_2)^* & z(k_1 + 2k_2) + t(-2\Omega + \omega_1) \\ (b)^*(a_2)^* & -zk_1 + t\omega_1 \\ ba_1 & z(2k_1 + k_2) + t(-\Omega - \omega_1) \\ \underline{(b)^*a_1} & \underline{-zk_2 - t(\omega_1 - \Omega)} \\ \underline{ba_2} & \underline{zk_1 - t\omega_1} \\ (b)^*a_2 & z(-k_1 - 2k_2) + t(2\Omega - \omega_1) \end{pmatrix}$$

