The full Brillouin gain calculation

- Steady state: $\partial_t = 0$
- Lossy mechanical wave (large γ_m / ν_m)

$$\partial_z P_p = -G_B P_p P_s - \alpha_p P_p$$
$$\partial_z P_s = \pm G_B P_p P_s \mp \alpha_s P_s$$

Lorentzian lineshape

$$G_B(\Omega) = Q_m \frac{2\omega_p \mathscr{L}(\Omega)}{\bar{m}_{\mathrm{eff}} \Omega_m^2} \left| \int f_{\mathrm{mb}}^{\mathrm{wg}} dl + \int f_{\mathrm{pe}}^{\mathrm{wg}} dA \right|^2$$

Effective mass — Overlap (MB) — Overlap (PE)

Overlap (MB)

$$f_{\mathrm{mb}}^{(\mathrm{wg})} = \frac{\mathbf{u}^* \cdot \hat{n} \left(\delta\varepsilon_{\mathrm{mb}} \mathbf{E}_{\mathrm{p},\parallel}^* \cdot \mathbf{E}_{\mathrm{s},\parallel} - \delta\varepsilon_{\mathrm{mb}}^{-1} \mathbf{D}_{\mathrm{p},\perp}^* \cdot \mathbf{D}_{\mathrm{s},\perp} \right)}{\max(|\mathbf{u}|) N_{\mathrm{p}}^{(\mathrm{wg})} N_{\mathrm{s}}^{(\mathrm{wg})}}$$

Overlap (PE)

$$f_{\mathrm{pe}}^{(\mathrm{wg})} = \frac{\mathbf{E}_{\mathrm{p}}^* \cdot \delta\boldsymbol{\varepsilon}_{\mathrm{pe}}^* \cdot \mathbf{E}_{\mathrm{s}}}{\max(|\mathbf{u}|) N_{\mathrm{p}}^{(\mathrm{wg})} N_{\mathrm{s}}^{(\mathrm{wg})}}$$

$$\left[f_{\mathrm{pe}}^{\mathrm{wg}} \right] = \mathrm{N/W/m^3}$$
$$\left[f_{\mathrm{mb}}^{\mathrm{wg}} \right] = \mathrm{N/W/m^2}$$

$$\bar{m}_{\mathrm{eff}} = \frac{1}{\max |\mathbf{u}_m|^2} \int \rho |\mathbf{u}_m|^2 dA$$

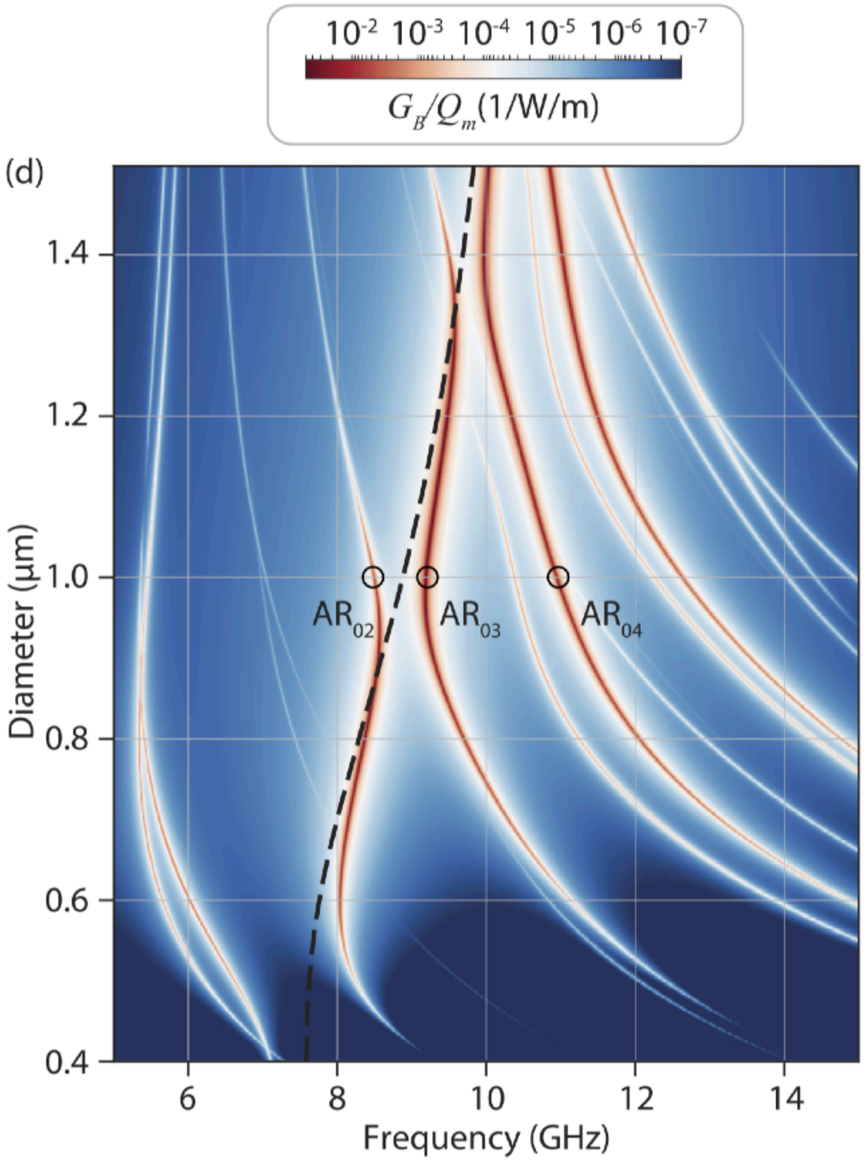
Effective mass

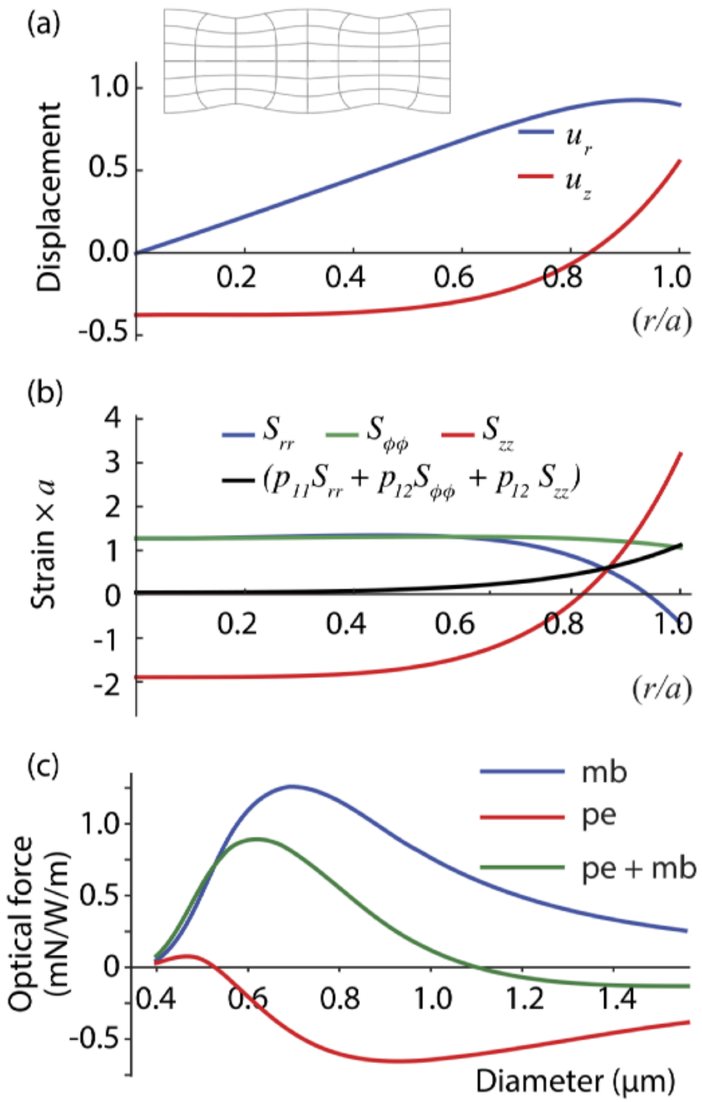
$$N_{\mathrm{i}}^{\mathrm{wg}} = \left(2\Re \left(\int \mathbf{E}_{\mathrm{i}} \times \mathbf{H}_{\mathrm{i}}^* \cdot \hat{z} dA \right) \right)^{1/2} \qquad N_{\mathrm{i}}^{\mathrm{cav}} = \left(\varepsilon_0 \int \varepsilon |\mathbf{E}_{\mathrm{i}}|^2 dV \right)^{1/2}$$

Mode normalization

1. Wiederhecker, G. S., Dainese, P. & Alegre, T. P. M. Brillouin optomechanics in nanophotonic structures. APL Photonics 4, (2019).

Silica nanowire revisited





$$f_{\mathrm{mb}}^{(\mathrm{wg})} = \frac{\mathbf{u}^* \cdot \hat{n}\left(\delta\varepsilon_{\mathrm{mb}}\mathbf{E}_{\mathrm{p},\parallel}^* \cdot \mathbf{E}_{\mathrm{s},\parallel} - \delta\varepsilon_{\mathrm{mb}}^{-1}\mathbf{D}_{\mathrm{p},\perp}^* \cdot \mathbf{D}_{\mathrm{s},\perp}\right)}{\max(|\mathbf{u}|)N_{\mathrm{p}}^{(\mathrm{wg})}N_{\mathrm{s}}^{(\mathrm{wg})}}$$

$$f_{\mathrm{pe}}^{(\mathrm{wg})} = \frac{\mathbf{E}_{\mathrm{p}}^* \cdot \delta\boldsymbol{\varepsilon}_{\mathrm{pe}}^* \cdot \mathbf{E}_{\mathrm{s}}}{\max(|\mathbf{u}|)N_{\mathrm{p}}^{(\mathrm{wg})}N_{\mathrm{s}}^{(\mathrm{wg})}}$$

$$\delta\varepsilon_{\mathrm{pe}} = -\varepsilon_0 n^4 \mathbf{p} : \mathbf{S}$$