Mechanical modes

$$\frac{\partial}{\partial r_j}\underbrace{T_{ij}(\boldsymbol{r},t)}_{\text{Stress tensor}} = \underbrace{\rho_0(\boldsymbol{r})\frac{\partial^2 U_i}{\partial t^2}}_{\text{acceleration}} - \underbrace{f_i(\boldsymbol{r},t)}_{\text{External force}}$$

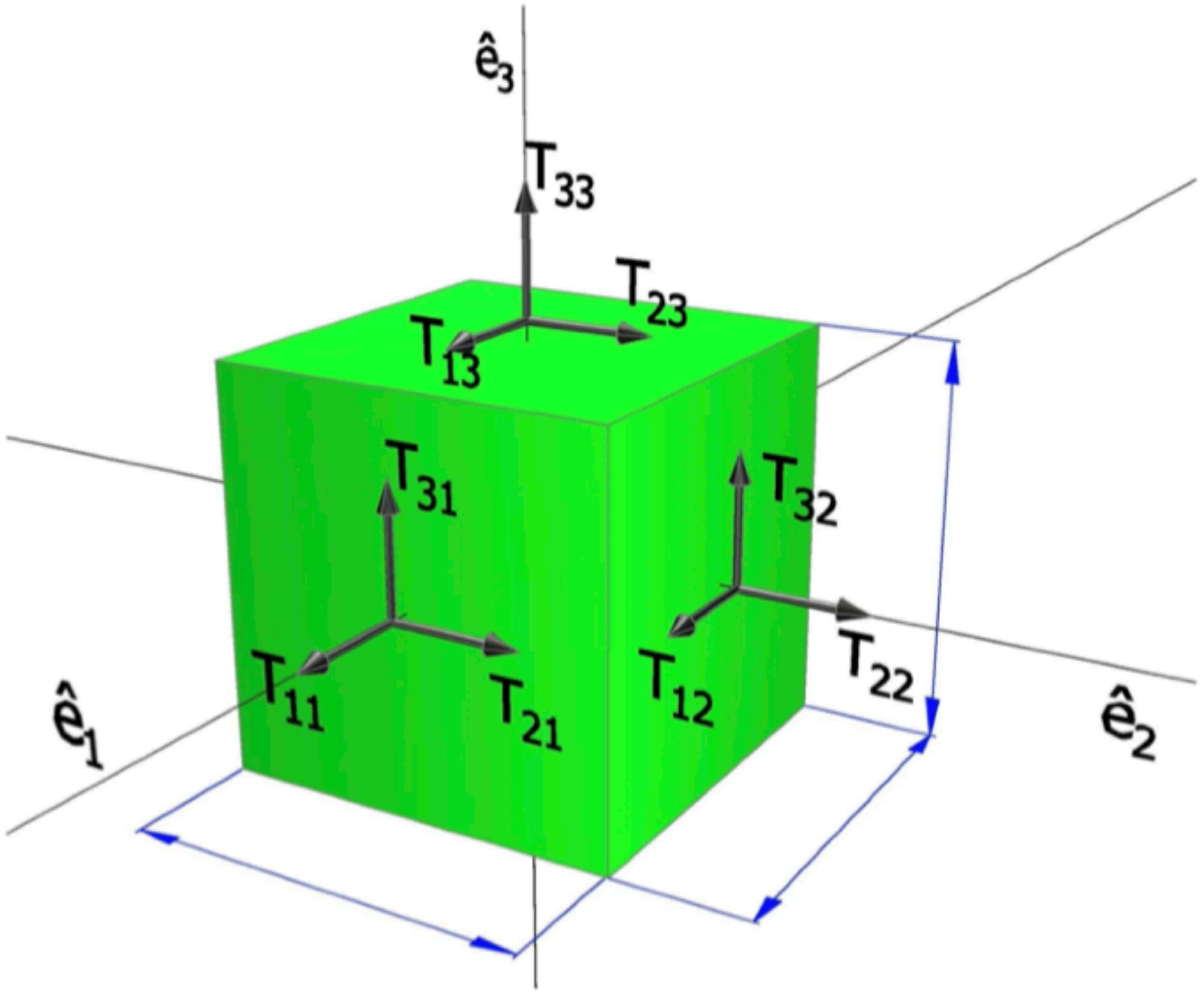
$$T_{ij} = \underbrace{c_{ijkl}S_{kl}}_{\text{Stiffness (Hooke's law)}} + \underbrace{\eta_{ijkl}\frac{\partial S_{kl}}{\partial t}}_{\text{Friction}}$$

$$S_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial r_j} + \frac{\partial U_j}{\partial r_i}\right)$$

Strain Tensor

$$\underbrace{\left[(\lambda + 2\mu) + \eta_{11}\frac{\partial}{\partial t}\right]\nabla(\nabla\cdot\boldsymbol{U})}_{\text{longitudinal waves } (\nabla\times\mathbf{u}=0)} - \underbrace{\left[\mu + \eta_{44}\frac{\partial}{\partial t}\right]\nabla\times\nabla\times\boldsymbol{U}}_{\text{shear-only } (\nabla\cdot\mathbf{u}=0)} = \rho\frac{\partial^2\boldsymbol{U}}{\partial t^2}$$

Elastic wave equation



Mechanical modes

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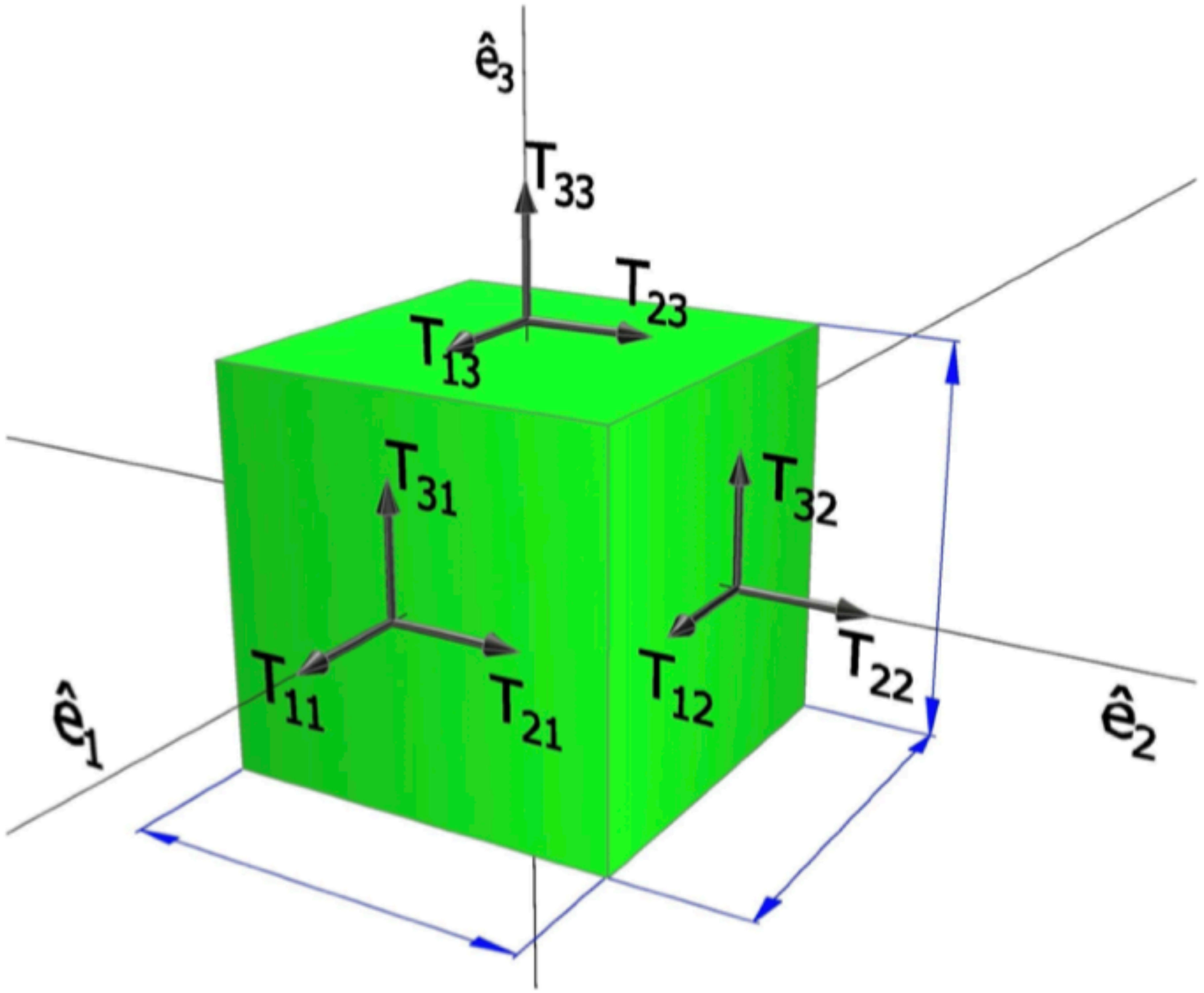
Strain Tensor



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Elastic wave equation

$$\begin{aligned}\boldsymbol{U}(\boldsymbol{r},t) &= \tilde{\boldsymbol{u}}^{(n)}(\boldsymbol{r})e^{-\mathrm{i}\Omega_n t} + \mathbf{c.c.}\\ &= \boldsymbol{u}^{(n)}(x,y)e^{\mathrm{i}[q_n z - \Omega_n t]} + \mathbf{c.c.}\end{aligned}$$

Waveguide mode ansatz