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Analog and Analog

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When I began this paper (years ago) my concern was with the grand sounding claim that any analog computer can be digitally simulated to any desired degree of precision. Along the way, however, I found the definitions of 'digital' and 'analog' to be tricky and interesting in their own right and they now comprise the bulk of the paper. But the original issue returns at the end, and its resolution involves distinguishing stricter and broader senses of 'analog'—hence my curious title.

Digital

Definitions of terms like 'analog' and 'digital' are guided first by paradigm cases, and intuitions about what these cases have in common. In the final analysis, however, a definition should be more than merely adequate to the intuitive data: it should show that the cases cited are instances of a theoretically interesting general kind, and it should emphasize the fundamental basis of that theoretical interest. An ideal definition makes manifest why the term in question is worth defining. This ideal is easier to approach for 'digital' than for 'analog'.

Standard examples of digital devices include Arabic numerals, abacuses, alphabets, electrical switches, musical notation, poker chips, and (digital) computers.[1] What is important and distinctive about these cases? Several common features stand out:

(1) Flawless copying (and preservation) are quite feasible. For instance, no copy of a Rembrandt painting is aesthetically equal to the original, and the paintings themselves are slowly deteriorating; by contrast, there are millions of perfect copies of (most of) Shakespeare's sonnets, and the sonnets themselves are not deteriorating. The difference is that a sonnet is determined by a sequence of letters , and letters are easy to reproduce—because modest smudges and squiggles don't matter. The same goes for musical scores, stacks of poker chips, and so on.

(2) Interesting cases tend to be complex: composites formed in standard ways from a kit of standard components—like

molecules from atoms. Complexity can also be diachronic, in which case the standard components are actually standard steps or "moves" constituting a sequential pattern. For example, digits of only ten standard kinds suffice, with a sign and decimal point, for writing any Arabic numeral; moreover, a sequence of steps of a few standard sorts will suffice for any multiplication in this notation. Likewise, the most elaborate switching networks can be built with the simplest relays; and all classical symphonies are scored with the same handful of basic symbols.

(3) There can be exactly equivalent structures in different media. Thus, the sonnets could be printed in italics, chiselled in stone, stamped in Braille, or transmitted in Morse code—and nothing would be lost. The same computer program can run on vacuum tube or solid–state hardware; poker chips can be plastic disks, dried beans, or matchsticks.

I call these features *copyability*, *complexity*, and *medium independence*, respectively. The question is: What do they all presuppose? Out of what deep root do they all grow?

All digital devices involve some form of *writing* and then *reading* various *tokens* of various *types*. That is, there are procedures for producing tokens, given the types that they are supposed to be, and procedures for telling or determining the types of given tokens. For example, penciling on white paper a particular inscription of the letter 'A' is a way of writing a token of that alphabetic type, which can then be read by eye. But also, rotating a switch to a specified click–stop is a way of "writing" a token of that setting (type); and that token can then be "read" by determining which of the connected circuits will now conduct electricity. These examples emphasize that the tokens don't have to be symbols (i.e., represent or mean anything), and that writing and reading are here generalized to cover whatever it takes to produce and reidentify the relevant tokens.

But what makes the device digital is something more specific about the write and read procedures: they must be "positive" and "reliable." A *positive procedure* is one which can succeed absolutely and without qualification—that is, not merely to a high degree, with astonishing precision, or almost entirely, but perfectly, one hundred percent! Clearly, whether something is a positive procedure depends on what counts as success. Parking the car in the garage (in the normal manner) is a positive procedure, if getting it all the way in is all it takes to succeed; but if complete success requires getting it exactly centered between the walls, then no parking procedure will be positive. There is no positive procedure for cutting a six–foot board, but there are plenty for cutting boards six feet, plus or minus an inch. The *'can* succeed' means feasibly, and that will depend on the technology and resources available. But we needn't worry about the limits of feasibility, because we care only about procedures that are also *reliable*—ones which, under suitable conditions, can be counted on to succeed virtually every time. With the available technology and resources, reliable procedures are in a sense "easy," or at least established and routine.

What counts as success for write and read procedures? Evidently that the write procedure actually produce a token of the type required, and that the read procedure correctly identify the type of the token supplied. But these are not independent, since usually the procedures themselves jointly function as a working definition of the types—what counts as an inscription of the letter 'A' is determined by what writers produce as one and readers recognize as one. The important constraint is that these be the same, or rather that whatever the writers produce be correctly recognized by the readers, and nothing else be recognized by the readers at all. In other words, the requirement really applies to the composite procedures for the write–read "round–trip," plus a specification of suitable environmental conditions; and there is a kind of trade–off in how stringent the various parts need to be. Thus if the write procedures are very precise, and the suitable conditions provide a very clean "noise–free" environment, then the read procedures can get away with being fairly lax; and so on.

So we can define a *digital device* as:

(i) A set of types;
(ii) A set of feasible procedures for writing and reading tokens of those types; and
(iii) A specification of suitable operating conditions; such that
(iv) Under those conditions, the procedures for the write-read cycle are positive and reliable.

Note that the success condition, that written tokens be correctly read as written (and nothing else be read), indirectly requires that no token in fact be a token of more than one type; that is, the types are disjoint, and hence the relation are 'of–the–same–type' is an equivalence relation.

Now that the definition has been given, I want to make five follow–up points, which should explain it a little further and reveal more of the motivations behind it. First, the copyability feature of digital devices is easily accounted for; indeed, the definition itself is not far from being a fuller statement of what that feature is. The original can be read positively and reliably; and these readings (type identifications) can simply function directly as the specifications to the write procedures for a new round of tokens. The new tokens can also be read positively and reliably, and hence they are exactly the same as (type identical to) the originals—that is, a *perfect* copy.

Second, (non-degenerate) digitalness is not ubiquitous. One often hears that systems are digital only "relative to descriptions"; and too often it is inferred that any object can be construed as any digital system, relative to *some* outlandish (but true) description. Being digital, however, is no more "Relative" than being a fugue or an amplifier. True, the types and procedures (like the musical theme, or the input and output ports) must be specified before the definition applies; but whether there is such a specification according to which the definition is in fact satisfied is not at all relative or trivial or automatic.

Third, our definition differs from Nelson Goodman's (by which it was largely inspired) in two significant ways. He says, in effect, that a disjoint set of types is digital just in case:

> For any candidate token, and for *at least one of* any pair of types (in the set), it is theoretically possible to determine that the candidate is *not* a token of that type.[2]

In other words, for any candidate token, all but at most one of the types can be positively ruled out (by some theoretically possible method)—no token is ever equivocal between two distinct types.

This is most easily explained with examples. Suppose that tokens are penciled line-segments less than a foot long; let Lx be the length of segment x (in inches), and let n be an integer. Then we can specify four different systems in terms of the following four conditions on two line segments being tokens of the same type:

(a) $Lx = Ly$ (any difference in length is a difference in type);
(b) $n<Lx, Ly<n+1$ (segments are of the same type if their lengths fall between the same consecutive inch-marks);
(c) $n+½ <Lx, Ly<n+1$ (as above, except that segments between any inch-mark and the next higher half-inch-mark are "ill-formed"—that is, not tokens of any type); and
(d) $Lx = Ly = n$ (as for (a), except that only segments of exactly integral lengths are acceptable—all others are ill-formed).

The first system is not digital because, no matter what (theoretically possible) method of measurement you used, there would be indefinitely many types to which any given token might belong, for all you could tell. Similarly for (b), except that the problem cases are only the segments very close to integral length, and there are only two types you can't rule out. (c) is a paradigm case of a digital device (assuming you can measure to within a quarter inch).

The last case is the trouble-maker; it is digital by Goodman's criterion, but it doesn't have the copyability feature—and that for two reasons. First, even if there were any tokens, they couldn't be recognized as such, as opposed to ill-formed "scribbles" (noise); and second, duplicate tokens could never be produced at all (except by miraculous accident). Both defects are remedied by requiring that the write–read cycle be positive (and reliable).

The second difference between our definition and Goodman's was hinted at in the remark about a stringency trade-off. It is common digital electronics practice to build pulse detectors that flip "high" on signals over about two and a half volts, flopping "low" on smaller signals. Since this is a sharp threshhold, and not even very consistent from unit to unit or moment to moment, these detectors cannot define a digital token-scheme, by Goodman's lights. What saves the day for engineers is that pulse *generators* produce only signals very close to zero and five volts respectively, and the whole apparatus can be well shielded against "static," so the detectors never actually get confused. Again focusing on the write–read cycle in the ambient conditions is the definitional remedy. But I think a broader point can be made. In making his determinations "theoretically possible," without mentioning the determination procedures, let alone the production procedures or the working conditions, Goodman betrays a mathematician's distaste for the nitty-gritty of *practical* devices. But *digital*, like *accurate, economical,* or *heavy-duty*, is a mundane engineering notion, root and branch. It only makes sense as a practical means to cope with the vagaries and vicissitudes, the noise and drift, of earthly existence. The definition should reflect this character.

My fourth follow-up point is a reply to David Lewis, and some comments on complexity. Lewis offers a counter-example to Goodman, which, if it worked at all, would work against us as well.[3] Imagine representing numerical values with a variable resistence—a setting of 137 ohms represents 137, and so on. And suppose the variable resistor is constructed with a rotary switch and a lot of discrete one-ohm resistors, such that the 137th switch position connects 137 resistors in series, for a 137–ohm total. This, says Lewis, is analog representation (hence, not digital), just as if the variable resistor were a sliding contact moving smoothly along a wire with uniform resistance per unit length. But, since the switch has click–stops, he claims it would be (mis–) classified as digital by Goodman.

I think the case is underdescribed. Assuming the representations are to be read with an ohmmeter, then we need to know how accurate the meter is, how precise and stable the one–ohm resistors are, and the total number of switch positions. If there are a thousand positions, and the meter and resistors are good only to one percent, then (whether or not it's analog) the device surely isn't digital; but it satisfies neither Goodman's conditions nor ours. On the other hand, if there are only two hundred positions, and the meter and resistors are good to one part in a thousand, then it satisfies both Goodman's conditions and ours. But I think it's clearly digital—just as digital as a stack of silver dollars, even when the croupier "counts" them by height.

Lewis, however, has another point in mind. He notes (p. 326) "the many combinations of values" that are possible when several parameters are used together. For instance, a bank of six switches, each with only ten positions, could represent any integer up to a million, even with crude equipment. This is a special case of the complexity feature; Lewis calls it 'multidigitality', and proposes it as an additional condition on digital representation. To evaluate this proposal, we should see how the multidigitality (complexity) condition relates to the other conditions, whether Goodman's or ours.

Consider two similar systems for representing wagers in a poker game. Each uses different colored tokens for different denominations, red and blue being worth ten and a hundred times more than white, respectively. But in one system the tokens are standard colored disks ("chips"), while in the other they are measured volumes of colored sand—one tablespoon corresponding to one chip, say. Though both systems are multidigital in Lewis' sense, the complexity is silly and useless in the sand case. For suppose players can measure volumes to within two percent, and imagine trying to bet 325 units. It's crazy! The expected error on the blue sand (the grains that stick to the spoon) is more valuable than the entire five spoonfuls of white sand. A stack of three blue chips, on the other hand, can be counted positively and reliably (no residual error at all); so the white chips are not overwhelmed, and remain perfectly significant.

Lewis, of course, would not deem poker sand digital, any more than we would: his multidigitality is a *further* condition, not an alternative. What the example shows is rather that multidigitality only "pays off" in systems that are already digital in our sense—the sense in which poker chips would still be digital even if they were all one color. I take the lesson to be that our definition has already captured the basic phenomenon, and that complex systems are just an important special case, which the underlying digitalness makes feasible. We see the complexity feature not as essential to being digital, but (if anything) vice versa.

In considering complex types, it is essential that not only their constituent atomic types be digital, but also their modes of combination. For instance, the power of Arabic numerals depends not only on the reliable positive procedures for (writing and reading) individual digit tokens, but just as much on the reliable positive procedures for concatenating them left to right, and so on. In effect, syntactical structures must themselves be digital types. And this point applies equally to diachronic complexity: each individual "step" (transformation, move) must be a token of a type in a set whose members' tokens can be produced and recognized reliably and positively. The types in this set might be identified with executable instructions, as in computer "languages," or with permissive rules, as in logical deductions or formal games like chess and checkers. It should be clear that complexity in digital devices goes far beyond arithmetic (or poker chip) multidigitality, and that the crucial dependence on reliable positive procedures rises dramatically with intricacy and elaborateness.

My fifth follow-up point is really just a footnote to the preceding. In digital devices, the main thing is eliminating confusion over the type of any token; and a primary motivation ("payoff") for this is the ability to keep great complexity manageable and reliable. In such cases it is generally the complexity itself—that is, the structure, form or pattern of the complex tokens and processes—which really matters; the digital atomic tokens are merely means to this larger end. Hence any digital atomic tokens which will admit of a corresponding variety of digital combinations and transformations would do as well. Since all the relevant structure is digital, the substitute will be not just similar in form but exactly isomorphic. (The sonnets can be spelled perfectly in Morse code, and so on.) The very features which make reproduction reliable and positive also make formal transubstantiation reliable and positive. Digital devices are precisely those in which complex form is reliably and positively abstractable from matter—hence the medium-independence feature.

Analog

'Analog' can be understood in broader and narrower senses; but even in the latter, analog devices comprise a motley crew. I am not at all confident that a satisfactory general definition is possible—which amounts, I suppose, to doubting whether they are a well-defined natural kind. Standard examples of analog devices include slide rules, scale models, rheostats, photographs, linear amplifiers, string models of railroad networks, loudspeakers, and electronic analog computers. As before, we can try to extract some salient common features from these varied cases; three stand out:

(1) Variations are smooth or continuous, without "gaps." There are no click-stops, or forbidden intermediate positions on a slide rule or a rheostat; photographs have (in principle) a continuous gray-scale, varying with two continuous position dimensions. This is everybody's aboriginal intuitive idea of analog systems; unlike switches, abacusses, or alphabetic inscriptions, every (relevant) setting or shape is allowed—nothing is ill-formed.

(2) Within the relevant variations, every difference makes a difference. The smallest rotation of a rheostat counts as changing the setting (a little); slight bending alters loudspeaker output; a photographic copy isn't perfect if it's slightly fuzzier, slightly darker, or slightly more contrasty. This is the complement of the previous feature: not only are all variations allowed, but they all matter (again unlike switches, abacusses and letters).

(3) Nevertheless, only certain "dimensions" of variation are relevant. Slide rules can be made indifferently of metal or bamboo, and their color and weight don't (strictly) matter. The thickness of the paper and even the chemistry of the emulsion are irrelevant to a photograph as such—assuming they don't affect the distribution of gray levels.

We call these the *smoothness*, *sensitivity*, and *dimensionality* features, respectively.

Though it need not (for any theoretical reason) be the best approach, we can pattern the definition of 'analog' after that of 'digital'. That is, we start with a set of types, and consider the procedures for producing and reidentifying tokens of those types.

For analog devices, the procedures for the write–read cycle are *approximation procedures*—that is, ones which can "come close" to perfect success. More specifically, there is some notion of margin of error (degree of deviation from perfect success) such that:

(i) the smaller this margin is set, the harder it is to stay within it;
(ii) available procedures can (reliably) stay within a pretty small margin;
(iii) there is no limit to how small a margin better (future, more expensive) procedures may be able to stay within;[4] but
(iv) the margin can never be zero—perfect procedures are impossible.

So all ordinary (and extraordinary) procedures for parking the car right in the center of the garage, cutting six–foot boards, measuring out three tablespoons of blue sand, and copying photographs, are approximation procedures. But there are no approximation procedures for raising the dead, writing poetry, winning at roulette, or counting small piles of poker chips. Approximation procedures are, in a clear sense, the antithesis of positive procedures; the two are exclusive, but of course not exhaustive. There is no need to write out the definition of *analog device*—it is the same as for digital, except with 'approximation' substituted for 'positive' (and a margin of error included in the specified conditions).

The follow–up points again tell the story. First, Goodman says a scheme is analog if dense—that is, if between any two types there is a third (see pp. 160 and 136). The main difficulty is that "between" is not well–defined for all cases that seem clearly analog. What, for instance, is "between" a photograph of Carter and one of Reagan? Yet it is easy to set resolution and linearity limits such that copying photographs is an approximation procedure. Similar observations apply to scale models.

Second, Lewis suggests that analog representation is representation in terms of magnitudes that are primitive or almost primitive in some good reconstruction of the language of physics (see pp. 324–25). He mentions only representations of numbers, and it isn't clear how he would generalize his criterion to non–numerical representations (e.g., portraits), or to non–representational analog devices. But more to the point, I see no reason why we could not have analog representation of numbers by, say, hue (as in multiple pH paper or various flame–tests) or even by bacterial growth rate (e.g., in a model of a resource–limited chain reaction); yet surely neither of these is "almost primitive" (whatever exactly that means).[5]

Third, it seems to me that there is an important digital-like character to all the standard analog devices—specifically in the dimensionality feature. Speaking freely for a moment, the essential point about (atomic) digital types is that there tend to be relatively few of them, and they are all clearly distinct and separated. Though the types of analog schemes are themselves not like this (they "blend smoothly" into one another), the *dimensions* along which they vary *are* relatively few and clearly distinct. Thus for photographs there are exactly three orthogonal dimensions: horizontal, vertical, and gray scale. A string model of a rail network has exactly one string piece for each rail link and exactly one knot for each junction (none of which blend together). But the best example is a regular analog computer with its electronic adders, integrators, multipliers, inverters, and the like, each as discrete and determinate in type as any mathematical symbol, and their circuit connections as well-defined as the formation of any equation. Indeed, though the state and adjustment types of an analog computer are analog, the *set-up* types are perfectly digital—the component identifications and interconnections are positive and reliable.

This "second-order" digitalness of analog devices is important in two ways. First, it is, at least roughly, a necessary condition for the write and read procedures to be approximation procedures in complex systems. In one-dimensional devices, like rheostats and slide rules, it suffices to say that between any distinct types there lies a third. But in multidimensional cases, where betweenness is not in general well defined, it is crucial to have the determinate set of independent dimensions such that a copy which is close on each dimension is ipso facto close overall—hence the intelligible importance of resolution in photocopying, of precision components in analog computers, and so on. This is what gives *approximation* its sense.

It is also what gives digital simulation its grip, and for essentially the same reason. Everybody knows that photographs can be "digitized" by dividing the area into equally spaced dots, and the gray scale into equally spaced shades; the fineness of the spacings determines the quality of the digitizing, just as the smallness of the error margins determines the closeness of the approximation. Likewise when a digital computer program simulates an analog computer, the values of all the fixed components and the initial values of all the variables are stored in specified registers, and then successive variable values are computed incrementally, using interaction equations determined by the circuit structure; and the accuracy of the simulation is controlled (primarily) by the number of bits in all those registers, and the fineness of the time increment. If the system were not second–order digital, such simulation could not get off the ground: there would be no particular set of parameters to digitize in specified registers (let alone equations for computing their interactions).

My fourth follow–up point, then, is the original sixty–four dollar question: Is *every* analog device second–order digital? Or: Is it really true that any analog device can in principle be digitally simulated to any desired degree of precision? To the extent that the suggestions in the previous three paragraphs hold up, the answers would appear to be: 'Yes'. Being second–order digital is equally the general condition for the possibility of write–read approximation procedures and of digital simulation techniques; and approximation procedures for the write–read cycle are criterial for analog devices.

They are criterial, that is, for analog devices in the *narrow* sense—that's all we've discussed so far. But the universal digital simulability claim is often made in a more sweeping tone, as if it applied to *everything*. Are there systems, perhaps "analog" in some broader sense, which are not second–order digital, and not necessarily digitally simulable (to whatever desired precision, etc.)? There are, of course, all manner of mongrel devices, analog in some respects or parts, digital in others; but these present no greater obstacle to simulation than purebred devices. If we think of digital devices as clean and resilient, and analog ones as messy and touchy (with the mongrels in between?), then the question becomes: Can there be systems even messier and touchier than pure analog—second-order messy, as it were?

I don't see why not. Consider the metabolic system of the rat, tokens of which are often used as "analogs" of our own metabolism, to predict the effects of fancy drugs, and so on. Now, some general metabolic relationships are known, and quite a few more specific local mechanisms are understood. But these by no means provide a complete description, in terms of which responses to strange chemicals can confidently be predicted. The millions of delicate hormonal balances, catalytic reactions, surface effects, and immunological responses, all interdependent in a bio–chemical frenzy of staggering proportions, can be catastrophically disrupted by the bizarrest of "side–effects." A minute occurrence on one side of a tiny membrane can have vastly different consequences from the same occurrence on the other side—and every rat contains billions of membranes.

There is essentially no way to gain detailed, quantitative control over such a mess—no hope of delineating a set of "state-variables" which fully characterize it at a time. Risky as long-term predictions are, I think it safe to announce that there will *never* be a digital simulation of human physiology reliable enough to supplant (or even challenge) biological and clinical testing of new drugs. And the reason, basically, is that metabolic systems are not second-order digital. Accordingly, there is also no specifying the "grain"or "resolution" of an approximation procedure for bio-chemical duplication of, say, healthy rats; there are no relevant dimensions along which such specification could make sense.

Fifth and final follow-up point: "But isn't physics second-order digital? What about digital simulation at the level of atoms and molecules (quarks and leptons, . . . whatever)?" I have two different replies to this question. In the first place, the idea is absolutely preposterous. Remember how impressed you were when you first heard that a computer with the capacity of the human brain would be the size of Texas and twenty stories high?[6] Well, the fastest and largest state-of-the-art computers today can be overwhelmed by the problem of digitally simulating a single large organic molecule (atom for atom); and there are more molecules in a human body than there would be pocket calculators, if the entire Earth were packed solid with them.[7] But simulating the molecules individually wouldn't scratch the surface: when their interactions are included, the required computations go up combinatorially!

But second, and more to the point, switching to the atomic level changes the subject. When the claim is made that photographs, linear amplifiers, and analog computers can be digitally simulated to any desired precision, that has nothing to do with fundamental physics; it would not matter if physicists had found swirling vortices in the plenum, or infinitely many infinitessimal monads. The digital simulability of analog devices is a claim about *macro*scopic phenomena. The range and variety of circumstances in which it holds is truly astonishing and important (the scientific revolution depended on it); but it is also important that such simulability (i.e., second-order digitalness) is *not* universal. This second important fact is completely missed and covered up by the careless shift in topic to micro-physics.

Conclusion

'Digital' and 'analog' (in the narrow or strict sense) are both best understood in terms of the kind of practical procedures employed for the writing and reading of tokens of allowed types—these being positive and approximation procedures, respectively. And, sticking to the narrow sense, it is plausibly the case that any analog device can (in principle) be digitally simulated to any desired precision. But there are other cases which, though they do not fit this strict mold, still seem to be "analog" in some broader sense; and for at least some of these, the digital simulability claim is wildly implausible.

NOTES

1. I resort to the non–committal "devices" because anything more specific seems wrong; thus (as the above list shows) not everything digital is a representation, a process, a computer, a machine, or what have you. Indeed, even the implication of plan or contrivance in 'device' should be ignored, for some biological systems might be digital.

2. See Goodman (1968) pp. 136–37 and 161. This is a paraphrase into our terminology of his definition of "syntactic finite differentiation," which is his essential condition for being a digital *scheme*. He also has a more stringent notion of a digital *system*, which has similar conditions imposed on its semantics (see pp. 152 and 161). I ignore the latter because in my view digital devices are not necessarily representational or symbolic.

3. See Lewis (1971), p. 322. He actually offers two counter–examples, but they are based on the same idea; so we consider only the simpler one.

4. Quantum–mechanical limits are almost always distracting and boring; so let's ignore them.

5. Ned Block and Jerry Fodor make essentially this point in an old (ca. 1971), manuscript which, so far as I know, has never been published.

6. I've forgotten where I heard this, or how long ago; and Lord knows how the calculation was made. But I've never forgotten the image.

7. 180 lb = 5×10^{28} hydrogen masses; the volume of the Earth is 7×10^{25} cubic inches.

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