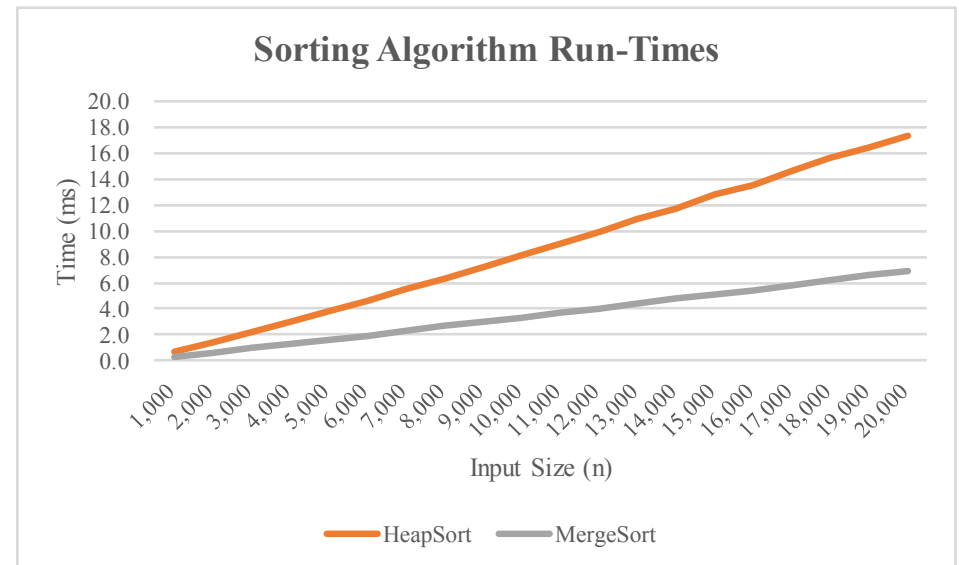
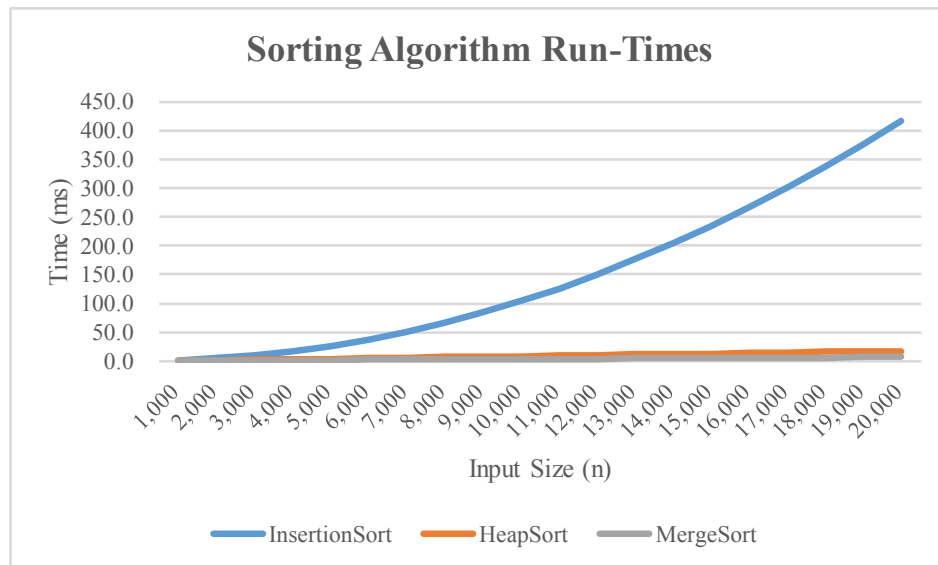


The graphs below show the run-time in milliseconds over a range of input sizes from 1,000 to 20,000. Note that the for each input size,  $n$ , the time is calculated as the average run-time for 10 separate executions of each algorithm.

The graph below on the left below shows the run-times for InsertionSort, HeapSort and MergeSort. The graph below on the right shows the run-times for HeapSort and MergeSort only. Because the worst-case run-time of InsertionSort is  $O(n^2)$  and the worst-case run-time of HeapSort and MergeSort is  $O(n \lg n)$ , the run-time for InsertionSort grows at a much faster rate than it does for HeapSort and MergeSort as the input size,  $n$ , increases. The growth of HeapSort and MergeSort cannot be seen clearly on the same scale as InsertionSort, so I included an additional graph showing just HeapSort and MergeSort.



The tables below show a comparison of the actual and theoretical run-times for each algorithm. O-notation gives an upper bound on a function to within a constant factor. Such constant factors can be calculated for the three sorting algorithms by dividing the simulated run-time by the theoretical run-time. Since O-notation gives an upper bound, the maximum hidden constant is calculated over all input sizes. For InsertionSort, the maximum hidden constant is 1.1. For HeapSort, the maximum hidden constant is 64.6. For MergeSort, the maximum hidden constant is 32.1.

**InsertionSort Run-Time (RT)**

n	Theoretical RT $O(n^2)$	Simulated RT (ms)	Hidden Constant
1,000	$1 \times 10^6$	1.1	1.1
2,000	$4 \times 10^6$	4.3	1.1
3,000	$9 \times 10^6$	9.6	1.1
4,000	$16 \times 10^6$	16.8	1.1
5,000	$25 \times 10^6$	26.3	1.1
6,000	$36 \times 10^6$	37.6	1.0
7,000	$49 \times 10^6$	51.0	1.0
8,000	$64 \times 10^6$	66.6	1.0
9,000	$81 \times 10^6$	84.8	1.0
10,000	$100 \times 10^6$	103.9	1.0
11,000	$121 \times 10^6$	125.9	1.0
12,000	$144 \times 10^6$	149.8	1.0
13,000	$169 \times 10^6$	176.3	1.0
14,000	$196 \times 10^6$	203.7	1.0
15,000	$225 \times 10^6$	234.1	1.0
16,000	$256 \times 10^6$	266.3	1.0
17,000	$289 \times 10^6$	300.8	1.0
18,000	$324 \times 10^6$	336.9	1.0
19,000	$361 \times 10^6$	375.1	1.0
20,000	$400 \times 10^6$	416.4	1.0
Maximum Hidden Constant Value			1.1

**HeapSort Run-Time (RT)**

n	Theoretical RT $O(n \lg n)$	Simulated RT (ms)	Hidden Constant
1,000	$10 \times 10^3$	0.6	64.6
2,000	$22 \times 10^3$	1.4	63.4
3,000	$35 \times 10^3$	2.2	62.9
4,000	$48 \times 10^3$	3.0	61.8
5,000	$61 \times 10^3$	3.8	61.8
6,000	$75 \times 10^3$	4.6	61.1
7,000	$89 \times 10^3$	5.5	61.4
8,000	$104 \times 10^3$	6.3	61.1
9,000	$118 \times 10^3$	7.2	60.9
10,000	$133 \times 10^3$	8.2	61.4
11,000	$148 \times 10^3$	9.0	60.9
12,000	$163 \times 10^3$	9.9	60.9
13,000	$178 \times 10^3$	10.9	61.3
14,000	$193 \times 10^3$	11.7	60.7
15,000	$208 \times 10^3$	12.8	61.6
16,000	$223 \times 10^3$	13.5	60.5
17,000	$239 \times 10^3$	14.6	61.1
18,000	$254 \times 10^3$	15.6	61.3
19,000	$270 \times 10^3$	16.5	61.1
20,000	$286 \times 10^3$	17.4	60.8
Maximum Hidden Constant Value			64.6

**MergeSort Run-Time (RT)**

n	Theoretical RT $O(n \lg n)$	Simulated RT (ms)	Hidden Constant
1,000	$10 \times 10^3$	0.3	32.1
2,000	$22 \times 10^3$	0.6	28.5
3,000	$35 \times 10^3$	0.9	27.4
4,000	$48 \times 10^3$	1.3	27.2
5,000	$61 \times 10^3$	1.6	26.6
6,000	$75 \times 10^3$	1.9	25.6
7,000	$89 \times 10^3$	2.3	26.0
8,000	$104 \times 10^3$	2.7	25.6
9,000	$118 \times 10^3$	3.0	25.2
10,000	$133 \times 10^3$	3.3	25.2
11,000	$148 \times 10^3$	3.7	25.1
12,000	$163 \times 10^3$	4.0	24.9
13,000	$178 \times 10^3$	4.4	24.7
14,000	$193 \times 10^3$	4.8	25.1
15,000	$208 \times 10^3$	5.1	24.6
16,000	$223 \times 10^3$	5.4	24.3
17,000	$239 \times 10^3$	5.8	24.5
18,000	$254 \times 10^3$	6.2	24.4
19,000	$270 \times 10^3$	6.6	24.5
20,000	$286 \times 10^3$	6.9	24.2
Maximum Hidden Constant Value			32.1