1. 5.1.12. Prove or disprove: Every k-chromatic graph G has a proper k-coloring in which some color class has $\alpha(G)$ vertices.

Solution:
True. In a proper coloring, each color class is an independent set. Hence, G is k-colorable if and only if $V(G)$ is the union of k independent sets, $\alpha(G) \geq k$. Let $V(G) = \sqcup_{i=1}^{k} v_i$, where the v_i are independents sets. Let the v_i be ordered such that $|v_i| \geq |v_{i+1}|$. Then $|v_1| = \alpha(G)$. Now, assign each set v_i color i. This coloring is proper. $\square$

2. 5.1.13. Prove or disprove: If $G = F \cup H$, then $\chi(G) \leq \chi(F) + \chi(H)$.

Solution:
True. Let $\chi(F) = k_1$ and $\chi(H) = k_2$. Now color G. We use k_1 and k_2 colors at most because F and H wholly determine G. $\square$

3. 5.1.14. Prove or disprove: For every graph G, $\chi(G) \leq n(G) - \alpha(G) + 1$.

Solution:
True. The result is the same as $\chi(G) + \alpha(G) \leq n(G) + 1$. In the extremal case, $\chi(G) = n(G)$. This means G is complete in this case, so $\alpha(G) = 1$. On the other hand, if $\alpha(G)$ is as large as possible, $\alpha(G) = n(G)$. This means G is totally disconnected, so $\chi(G) = 1$. Hence, since the maximum of $\chi(G) + \alpha(G)$ has been shown to be less than or equal to $n(G) + 1$, we have the desired result. $\square$

4. 5.1.20. Let G be a graph whose odd cycles are pairwise intersecting, meaning that every two odd cycles in G have a common vertex. Prove that $\chi(G) \leq 5$.

Solution:
Any two odd cycles who share a vertex will require at at most 3 colorings. Then two odd cycles must meet in the middle. Now place G in the center, Then $\chi(G)$

5. 5.1.33. Prove that every graph G has a vertex ordering relative to which greedy coloring uses $\chi(G)$ colors.

Solution: