

1. 5. Let G be a k -connected graph. Let $A = \{a_1, a_2, \dots, a_k\}$ and $B = \{b_1, b_2, \dots, b_k\}$ be disjoint subsets of $V(G)$ such that $|A| = |B| = k$. Prove that G contains k pairwise disjoint A, B -paths. (That is, G contains k paths each of which start with a vertex from the set A , end with a vertex from the set B and share no vertices.)

Solution:

Create G' by adding two vertices, v_A and v_B and $2k$ edges to G such that v_A and v_B both have k edges incident to them such that $v_A \leftrightarrow a_i$ and $v_B \leftrightarrow b_i$ for every $1 \leq i \leq k$. Since G' only adds edges and vertices to G , G' is k -connected as well. Hence, there exists k pairwise disjoint A, B -paths in G' from v_A to v_B .