1. Suppose $r \geq 2$ is even. Show that no r-regular graph contains a bridge.

Solution:
Suppose G is r-regular.

2. Given integers m and k where $2 \leq m \leq k$, show that there exists a graph G where G has a clique number of m and a chromatic number of k.

Solution:

3. Suppose G is a graph. Show that G or the complement of G is connected.

Solution:
Define G^C to be the complement of G. Suppose G is not connected. Let $\{C_k\}_{k=1}^n$ be the connected components of G where $n \geq 2$. Choose $u, v \in V(G)$. We show there exists a uv-path in G^C. If $uv \notin E(G)$, then $uv \in E(G^C)$, which is a uv-path. Suppose that $uv \in E(G)$. Without loss of generality, $u, v \in V(C_1)$. Choose component C_k where $1 < k < n$ and $w \in V(C_k)$. Since $uw, wv \notin E(G)$, $uw, wv \in E(G^C)$. Hence, (uw, wv) is a uv-path in G^C. Therefore, G^C is connected.

4. Let M be a matching in a graph G and let u be an M-unsaturated vertex of G. Prove that if G has no M-augmenting path that starts at u, then u is unsaturated in some maximum matching in G.

Solution:
Without loss of generality, suppose G is connected. Suppose u is saturated in every maximum matching in G. We show that G has an M-augmenting path that starts at u. Since u is not M-saturated, M is not a maximum matching. There exist an M-augmenting