Joe Crandall's PHYS 2161 Midterm 1 Used Heavily Topic SubTopic KNOWTHISMATH Definition break To prepare for the mid-term exam - as I said in class - you should review solutions to your homework assignments 1 through 6 and be able to apply principal definitions and equations in the end of each chapter, Chapter 1 through 4. 1. Newton's Laws of Motion Dot Product $\vec{r}\cdot\vec{s} = |\vec{r}||\vec{s}|cos\theta = r_x s_x = r_y s_y + r_z s_z$ Cross Product $\vec{r}\times\vec{s} =< r_y s_z - r_z s_y, r_z s_x - r_x s_z, r_x s_y - r_y s_x >$ Inertial Frame An inertial frame is any reference frame in which Newton's first law holds, a non-accelerating, non-rotating frame. Unit Vector If (ξ,η,ζ) are an orthogonal system of coordinates, then $\hat{\xi}$ = unit vector in direction of increasing ξ with η and ζ fixed and so on, and any vector $\vec{s}$ can be expanded as $\vec{s} = |\vec{s}|_\xi\hat{\xi} + |\vec{s}|_\eta\hat{\eta} + |\vec{s}|_\zeta\hat{\zeta}$ Newton's Second Law Vector Form $\vec{F} = m\ddot{\vec{r}}$ I Cartesian (x,y,z) $F_x = m\ddot{x}$ $F_y = m\ddot{y}$ $F_z = m\ddot{z}$ I 2D Polar (r,ϕ) $F_r = m(\ddot{r} - r\dot{\phi}^2)$ $F_\phi = m(r\ddot{\phi} + 2\dot{r}\dot{\phi})$ I Cylindrical Polar (ρ,ϕ,z) $F_r = m(\ddot{\rho} - \rho\dot{\phi}^2)$ $F_\phi = m(\rho\ddot{\phi} + 2\dot{\rho}\dot{\phi})$ $F_z = m\ddot{z}$ 2. Projectiles and Charged Particles Linear and Quadratic Drags provided the speed v is well bellow that of sound, the magnitude of the drag force $\vec{f} = -f(v)\hat{v}$ on an object moving through a fluid is usually well approximated as $f(v) = f_{lin} + f_{quad}$ I D denotes diameter of the sphere and coefficients β and γ depend on the nature of the medium I $f_{lin} = bv = \beta Dv$ I $f_{quad} = cv^2 = \gamma D^2 v^2$ I for a sphere in air at STP $\beta = 1.6\times10^{-4} Nsm^{-2}$ and $\gamma = 0.25 Ns^2m^{-4}$ Lorentz Force on a Charged Particle q = charge in coulombs C or ampere seconds As I E = electric field in newtons per coulomb NC^{-1} or volts per meter Vm^{-1} or in SI base units $kgms^{-3}A^{-1}$ I v = velocity in ms^{-1} I B = magnetic field in Teslas T or in newtons per meter per ampere $Nm^{-1}A^{-1}$ or in SI base unites $kgs^{-2}A^{-1}$ It $\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$ q = charge b = unifom magnetic field v = velocity 3. Momentum and Angular Momentum Equation of Motion for a Rocket $m\dot{v} = -\dot{m}v_{ex} + F^{ext}$ The Center of Mass of Several Particles $\vec{R} = \frac{1}{M}\sum_{\alpha=1}^{N} m_\alpha\vec{r}_\alpha = \frac{m_1+\vec{r}_1+...+m_N+\vec{r}_N}{M}$ I M = total mass of all particles $\vec{r}$ = position r relative to an origin O Angular Momentum For a single particle with position $\vec{r}$ relative to and origin O and momentum $\vec{p}$, the angular momentum about O is $\vec{l} = \vec{r}\times\vec{p}$ I For several particles, the total angular momentum is $\vec{L} = \sum_{\alpha=1}^{N}\vec{l}_\alpha = \sum_{\alpha=1}^{N}\vec{r}_\alpha\times\vec{p}_\alpha$ I Provided all the internal forces are central $\dot{\vec{L}} = \vec{\Gamma}^{ext}$ where $\vec{\Gamma}^{ext}$ is the net external torque 4. Energy Work-KE Theorem The change in KE of a particle as it moves from point 1 to point 2 $\Delta T = T_2 - T_1 = \int_1^2 is\vec{F}\cdot d\vec{r} \equiv W(1\rightarrow2)$ where $T = \frac{1}{2}mv^2$ and $W(1\rightarrow2)$ is the work done which by the total force $\vec{F}$ on the particle and is defined by the preceding integral Conservative Forces and Potential Energy A force $\vec{F}$ on a particle is conservative if (i) it depends only on the particle's position, $\vec{F} = \vec{F}(\vec{r})$, and (ii) for any two points 1 and 2, the work $W(1\rightarrow2)$ done by $\vec{F}$ is the same for all paths joining 1 and 2 (or equivalently, $\nabla\times\vec{F} = 0$) If $\vec{F}$ is conservative, we can define a corresponding potential energy so that $U(\vec{r}) = -W(\vec{r_o}\rightarrow\vec{r}) \equiv -\int_{\vec{r_o}}^{\vec{r}}\vec{F}(\vec{r'})\cdot d\vec{r'}$ I and $\vec{F} = -\nabla U$ I If all the forces on a particle are conservative with corresponding potential energies $U_1,...,U_n$ then the total mechanical energy $E = T + U_1 + ... + U_n$ is constant . More Generally if there are also nonconservative forces $\Delta E = W_{nc}$, is the work done by the nonconservative forces. Central Forces A force $\vec{F}(\vec{r})$ is central if it is everywhere directed toward or away from a "force center". If we take the latter to be the origin, $\vec{F}(\vec{r}) = f(\vec{r})\hat{r}$ A central force is spherically symmetric $[f(r) = f(r)]$ if and only if it is conservative Energy of a Multiparticle System If all forces (internal and external) on a multi-particle system are conservative, the total potential energy, $U = U^{int} + U^{ext} = \sum_\alpha\sum_{\beta>\alpha}U_{\alpha\beta} + \sum_\alpha U_\alpha^{ext}$ satisfies (net force on a particle α) $= -\nabla_\alpha U$ and $T + U = constant$

NOT ON MIDTERM PAST THIS POINT

5. Oscillations 5.1 Hooke's Law $F = -kx \Leftrightarrow U = \frac{1}{2}kx^2$ k is the force constant Taylor Series $\frac{1}{1-x} = \sum_{n=0}^{\infty}x^n = 1 + x + x^2 + x^3 + \cdots$ (when $-1 < x < 1$) I $e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$ I $\sin x = \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}x^{2n+1} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$ I $\cos x = \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}x^{2n} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$ I 5.2 Simple Harmonic Motion E=total energy, T=K=kinetic energy, U=potential energy, k=force constant, ω=angular velocity, δ=phase shift, A=amplitude, t=time I $x(t) = Acos(\omega t - \delta)$ I $U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2cos^2(\omega t - \delta)$ I $K = T = \frac{1}{2}m\dot{x}^2 = \frac{1}{2}mA^2\omega^2sin^2(\omega t - \delta) = \frac{1}{2}kA^2sin^2(\omega t - \delta)$ I where $\omega^2 = \frac{k}{m}$ I $E = T + U = \frac{1}{2}KA^2$ I $\ddot{x} = -\omega^2 x \Leftrightarrow x(t) = Acos(\omega t - \delta)$ AngleSum and Angle Difference Identities $sin(\alpha+\beta) = sin(\alpha)cos(\beta) + cos(\alpha)sin(\beta)$ I $sin(\alpha-\beta) = sin(\alpha)cos(\beta) - cos(\alpha)sin(\beta)$ I $cos(\alpha+\beta) = cos(\alpha)cos(\beta) - sin(\alpha)sin(\beta)$ I $cos(\alpha-\beta) = cos(\alpha)cos(\beta) + sin(\alpha)sin(\beta)$ Euler's formula $e^{ix} = cosx + isinx$ I $e^{-ix} = cosx - isinx$ Half Angle Formulas $sin^2x = \frac{1-cos(2x)}{2}$ I $cos^2x = \frac{1+cos(2x)}{2}$ 5.3 Two-Dimensional Oscillators $x(t) = A_xcos(\omega t)$ I $y(t) = A_ycos(\omega t - \delta)$ I $\delta = \delta_y - \delta_x$ relative phase of the y and x oscillations 5.4 Damped Oscillations $m\ddot{x} + b\dot{x} + kx = 0$ I $\frac{b}{m} = 2\beta$ Damping constant $= \beta$ I $\omega_0 = \sqrt{\frac{k}{m}}$ Natural frequency $= \omega_o$ I decay parameter $=\beta - \sqrt{\beta^2 - \omega_o^2}$ 5.6 Resonance

6. Calculus of Variations The Euler-Lagrange Equation $S = \int_{x_1}^{x_2} f[y(x), y'(x), x]dx$ taken along y=y(x) with respect to variations of that path if and only if y(x) satisfies the Euler-Lagrange equation $\frac{df}{dy} - \frac{d}{dx}\frac{df}{dy'} = 0$ Several Variables If there are n dependent variables in the original integral, there are n Euler-Lagrange equations. For instance, an integral of the form $S = \int_{u_1}^{u_2} f[x(u), y(u), x'(u), y'(u), u]du$ with two dependent variables [x(u) and y(u)], is stationary with respect to variations of x(u) and y(u) if and only if these two functions satisfy the two equations $\frac{df}{dx} = \frac{d}{du}\frac{df}{dx'}$ and $\frac{df}{dy} = \frac{d}{du}\frac{df}{dy'}$

7. Lagrange's Equations The Lagrangian $\mathcal{L} = T - U$ The Lagrange's Equation For any holonomic system, Newton's second law is equivalent to the n Lagrange equations $\frac{d\mathcal{L}}{dq_i} = \frac{d}{dt}\frac{d\mathcal{L}}{d\dot{q}_i}$ $[i = 1,...,n]$ Generalized Momenta and Ignorable Coordinates The ith generalized momentum p_i is defined to be the derivative $p_i = \frac{d\mathcal{L}}{d\dot{q}_i}$ If $\frac{d\mathcal{L}}{dq_i} = 0$, then we say the coordinate q_i is ignorable and the corresponding generalized momentum is constant.

The Hamiltonian ...

8. Two-Body Central-Force Problems relative coordinate $\vec{r} = \vec{r_1} - \vec{r_2}$ I reduced mass $\mu = \frac{m_1 m_2}{m_1 + m_2}$ The Equivalent One-Dimensional Problem $U_{eff}(r) = U(r) + U_{cf}(r) = U(r) + \frac{l^2}{2\mu r^2}$ I U_{cf} is called the centrifugal potential energy The Transformed Radial Equation ... The Kepler Orbits ... 10. Rotational Motion of Rigid Bodies $\vec{L} = \vec{L}$(motion of CM) + $\vec{L}$(motion relative to CM) I T = T(motion of CM) + T(motion relative to CM) Moment of Inertia Tensor angular momentum $\vec{L}$ and angular velocity ω of a ridge body are related by $\vec{L} = \vec{I}\omega$ I diagonal and off-diagonal elements $I_{xx} = \sum_\alpha m_\alpha(y_\alpha^2 + z_\alpha^2)$ I $I_{xy} = -\sum_\alpha m_\alpha x_\alpha y_\alpha$

Principal Axes $\vec{L} = \lambda\omega$ I $I' = \lambda_1$... in central diagonal of matrix Euler's Equations ... Euler's Angles ...