Joe Crandall's PHYS 3165 Electrodynamics Used Heavily Topic SubTopic KNOWTHISMATH Definition/Constant/Units break Vector Derivatives **Cartesian** $d\vec{l} = dx\hat{x} + dy\hat{y} + dz\hat{z}$ I $d\tau = dxdydz$ I $Gradient: \nabla t = \frac{\partial t}{\partial x}\hat{x} + \frac{\partial t}{\partial y}\hat{y} + \frac{\partial t}{\partial z}\hat{z}$ I $Divergence: \nabla \cdot \vec{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$ I $Curl: \nabla \times \vec{v} = (\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z})\hat{x} + (\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x})\hat{y} + (\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y})\hat{z}$ I $Laplacian: \nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$ I **Spherical** $d\vec{l} = dr\hat{r} + rd\theta\hat{\theta} + r\sin\theta d\phi\hat{\phi}$ I $d\tau = r^2\sin\theta dr d\theta d\phi$ I $Gradient: \nabla t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\phi}$ I $Divergence: \nabla \cdot \vec{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$ I $Curl: \nabla \times \vec{v} = \frac{1}{r\sin\theta}[\frac{\partial}{\partial \theta}(\sin\theta v_\phi) - \frac{\partial v_\theta}{\partial \phi}]\hat{r} + \frac{1}{r}[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{\partial}{\partial r}(r v_\phi)]\hat{\theta} + \frac{1}{r}[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial \theta}]\hat{\phi}$ I $Laplacian: \nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial t}{\partial r}) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\frac{\partial t}{\partial \theta}) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial \phi^2}$ I **Cylindrical** $d\vec{l} = ds\hat{s} + sd\phi\hat{\phi} + dz\hat{z}$ I $d\tau = sdsd\phi dz$ I $Gradient: \nabla t = \frac{\partial t}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial t}{\partial \phi}\hat{\phi} + \frac{\partial t}{\partial z}\hat{z}$ I $Divergence: \nabla \cdot \vec{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$ I $Curl: \nabla \times \vec{v} = [\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}]\hat{s} + (\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s})\hat{\phi} + \frac{1}{s}(\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial \phi})\hat{z}$ I $Laplacian: \nabla^2 t = \frac{1}{s}\frac{\partial}{\partial s}(s\frac{\partial t}{\partial s}) + \frac{1}{s^2}\frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}$

Vector Identities **Triple Products** $\vec{A} \cdot (\vec{B} \times \vec{C}) = \vec{B} \cdot (\vec{C} \times \vec{A}) = \vec{C} \cdot (\vec{A} \times \vec{B})$ I $\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{C} \cdot \vec{A}) - \vec{C}(\vec{A} \cdot \vec{B})$ **Product Rules** $\nabla(fg) = f(\nabla g) + g(\nabla f)$ I $\nabla(\vec{A} \cdot \vec{B}) = \vec{A} \times (\nabla \times \vec{B}) + B \times (\nabla \times \vec{A}) + (\vec{A} \cdot \nabla)\vec{B} + (\vec{B} \cdot \nabla)\vec{A}$ I $\nabla \cdot (f\vec{A}) = f(\nabla \cdot \vec{A}) + \vec{A} \cdot (\nabla f)$ I $\nabla \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\nabla \times \vec{A}) - \vec{A} \cdot (\nabla \times \vec{B})$ I $\nabla \times (f\vec{A}) = f(\nabla \times \vec{A}) - \vec{A} \times (\nabla f)$ I $\nabla \times (\vec{A} \times \vec{B}) = (\vec{B} \cdot \nabla)\vec{A} - (\vec{A} \cdot \nabla)\vec{B} + \vec{A}(\nabla \cdot \vec{B}) - \vec{B}(\nabla \cdot \vec{A})$ **Second Derivates** $\nabla \cdot (\nabla \times \vec{A}) = 0$ I $\nabla \times (\nabla f) = 0$ I $\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$ Fundamental Theorems **Gradient Theorem:** $\int_{\vec{a}}^{\vec{b}} (\nabla f) \cdot d\vec{l} = f(\vec{b}) - f(\vec{a})$ **Divergence Theorem:** $\int (\nabla \cdot \vec{A}) d\tau = \oint \vec{A} \cdot d\vec{a}$ **Curl Theorem:** $\int (\nabla \times \vec{A}) \cdot d\vec{a} = \oint \vec{A} \cdot d\vec{l}$ Basic Equations of Electrodynamics **Potentials** $\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t}$ permittivity of free space $\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/Nm}$ I permeability of free space $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{N/A^2}$ I speed of light $c = 3.00 \times 10^{-8}\ \mathrm{m/s}$ I charge of the electron $e = 1.60 \times 10^{-19}\ \mathrm{C}$ I mass of the electron $m = 9.11 \times 10^{-31}\ \mathrm{kg}$ Spherical and Cylindrical Coordinates **Spherical** $x = r\sin\theta\cos\phi$ I $y = r\sin\theta\sin\phi$ I $z = r\cos\theta$ I $\hat{x} = \sin\theta\cos\phi\hat{r} + \cos\theta\cos\phi\hat{\theta} - \sin\phi\hat{\phi}$ I $\hat{y} = \sin\theta\sin\phi\hat{r} = \cos\theta\sin\phi\hat{r} + \cos\phi\hat{\phi}$ I $\hat{z} = \cos\theta\hat{r} - \sin\theta\hat{\theta}$ I $r = \sqrt{x^2 + y^2 + z^2}$ I $\theta = \tan^{-1}(\frac{\sqrt{x^2+y^2}}{z})$ I $\phi = \tan^{-1}(\frac{y}{x})$ I $\hat{r} = \sin\theta\cos\phi\hat{x} + \sin\theta\sin\phi\hat{y} + \cos\theta\hat{z}$ I $\hat{\theta} = \cos\theta\cos\phi\hat{x} + \cos\theta\sin\phi\hat{y} - \sin\theta\hat{z}$ I $\hat{\phi} = -\sin\phi + \cos\phi\hat{y}$ **Cylindrical** $x = s\cos\phi$ I $y = s\sin\phi$ I $z = z$ I $\hat{x} = \cos\phi\hat{s} - \sin\phi\hat{\phi}$ I $\hat{y} = \sin\phi\hat{s} + \cos\phi\hat{\phi}$ I $\hat{z} = \hat{z}$ I $s = \sqrt{x^2 + y^2}$ I $\phi = \tan^{-1}(y/x)$ I $z = z$ I $\hat{s} = \cos\phi\hat{x} + \sin\phi\hat{y}$ I $\hat{\phi} = -\sin\phi\hat{x} + \cos\phi\hat{y}$ I $\hat{z} = \hat{z}$ Dot Product $\vec{r} \cdot \vec{s} = |\vec{r}||\vec{s}|cos\theta = r_x s_x = r_y s_y + r_z s_z$ Cross Product $\vec{r} \times \vec{s} =< r_y s_z - r_z s_y, r_z s_x - r_x s_z, r_x s_y - r_y s_x >$ 2. Electrostatics charge flows from positive to negative Coulomb's Law $\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{qQ}{|\vec{r}-\vec{r}'|^2}(\vec{r} - \vec{r}')$ I $\epsilon_0 = 8.85 \times 10^{-12}\frac{C^2}{N \cdot m^2}$ Principle of Superposition $\vec{F} = \vec{F_1} + \vec{F_2} + \ldots$ Electric Field $\vec{F} = Q\vec{E}$ I $\vec{E}(r) \equiv \frac{1}{4\pi\epsilon_0}\Sigma_{i=1}^n \frac{q_i}{R_i^2}\hat{R}$ I $\vec{R} = (\vec{r} - \vec{r}')$ I $R = |(\vec{r} - \vec{r}')|$ I $\hat{R} = \frac{\vec{R}}{R}$ Continuous Charge Distribution **Electric field of a line charge:** $\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\lambda(\vec{r}')}{R^2}\hat{R}dl'$ **Electric field of a surface charge:** $\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\sigma(\vec{r}')}{R^2}\hat{R}da'$ **Electric field of a volume charge:** $\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}')}{R^2}\hat{R}d\tau'$ Divergence and Curl of Electrostatic Fields **Gauss's Law:** $\oint \vec{E} \cdot d\vec{a} = \frac{1}{\epsilon_0}Q_{enclosed}$ I $\nabla \cdot \vec{E} = \frac{1}{\epsilon_0}\rho$ **Integral around a closed path:** $\oint \vec{E} \cdot d\vec{l} = 0$ I $\nabla \times \vec{E} = \vec{0}$ Electric Potential $V(\vec{r}) \equiv -\int_O^{\vec{r}} \vec{E} \cdot d\vec{l}$ I O is standard reference point on which we have agreed beforehand I $\vec{E} = -\nabla V$ I $\nabla^2 V = -\frac{\rho}{\epsilon_0}$ **Electric potential of a volume charge:** $\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}')}{R}d\tau'$ I $\oint \vec{E} \cdot d\vec{a} = \frac{1}{\epsilon_0}Q_{enclosed} = \frac{1}{\epsilon_0}\sigma A$ Work and Energy $W = \int_a^b \vec{F} \cdot d\vec{l} = -Q\int_a^b \vec{E} \cdot d\vec{l} = Q[V(\vec{b}) - V(\vec{a})]$ **volume charge density:** $W = \frac{1}{2}\int \rho V d\tau$ electric potential, electric field, charge density triangle **Electric Field**(N/C) $\leftrightarrow$ **Electric potential(Voltage)**(V), (J/C) $-\int_0^{\vec{r}} \vec{E} \cdot d\vec{r} = V(\vec{r})$ I $\vec{E} = -\nabla V$ **Electric Field**(N/C) $\leftrightarrow$ **charge density**(C/m^3) $\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{\hat{R}}{R^2}\rho d\tau$ I $\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$ I $\nabla \times \vec{E} = \vec{0}$ **Electric potential(Voltage)**(V), (J/C) $\leftrightarrow$ **charge density**(C/m^3) $V = \frac{1}{4\pi\epsilon_0}\int \frac{\rho}{R}d\tau$ I $\nabla^2 V = \frac{-\rho}{\epsilon_0}$ Dr. Whites Grid Review **Name of source:** charge q I Gravitational analogies mass m I units coulomb C or ampere in one second As I typical values 1.60×10^{-19} C (charge of an electron) 1.0×10^{-9} C (ping pong ball charge) 1.0×10^{-6} C (van der graaf static charge) I general equation for getting $\vec{E}$ from this quantity, point charge at origin $\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{q}{|\vec{r}|^2}\hat{r}$ I general equation for getting this from $\vec{E}$, Gauss's law $\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enclosed}}{\epsilon_0}$ **Force Constant (Coulombs Constant):** $k = \frac{1}{4\pi\epsilon_0} = 8.987 \times 10^9\ \mathrm{Nm^2C^{-2}}$ I gravitational analogies, $G = 6.674 \times 10^{-11}\ \mathrm{Nm^2kg^{-2}}$ I Units, Newton metre squared per coulomb squared Nm^2/C^2 or meter per farad m/F, F = s^{4}A^{2}m^{-2}kg^{-1} I **Force on q by Q (Coulombs law):** $F = \frac{kqQ}{R^2}\hat{R}$ (electric field $x \to -$) I Gravitational analogies $F = G\frac{m_1 m_2}{r^2}$ force due to gravity I units, Newton N or kilogram meter per second squared kgms^{-2}. typical values $F = \frac{k(1.0 \times 10^{-9})^2}{(0.1)^2} = 8.987 \times 10^{-7}$ N(ping pong ball). general equation for getting $\vec{E}$ from this quantity, $\vec{E} = \frac{\vec{F}}{Q}$. general equation for getting this from $\vec{E}$, $\vec{F} = Q\vec{E}$. long distance behavior of monopoles, $\frac{1}{r^2}$. long distance behavior of dipoles, $F = \frac{k2z}{(z^2+(d/2)^2)^{3/2}}\hat{z} = \frac{1}{z^3}$. long distance behaviors of infinite lines $\frac{1}{r}$. long distance behaviors of infinite planes $F = \frac{gQ}{3\epsilon_0}$ **Electric field by Q:** $\vec{E} = \frac{kq}{R^2}\hat{R}(+ \to -)$ I $g = \frac{Gm}{r^2}$ (gravitational field) I Newton per coulomb N/C or Volts per meter V/m or kilogram meter per seconds cubed ampere kgm/s^{3}A I $E = \frac{k(1.0 \times 10^{-9})}{0.1^2} = 898.7$N/C I monopoles $E = \frac{kQ}{r^2}$ I dipole $E = \frac{kq2z}{[z^2+(d/2)^2]^{3/2}}\hat{z}$ I lines $E = \frac{\lambda}{2\pi r\epsilon_0}$ I planes $E = \frac{\sigma}{3\epsilon}$ **Voltage (electric potential) Q:** $V = \frac{1}{4\pi\epsilon_0}\int \frac{\rho}{R}d\tau$ (potential energy per unit charge) I $\Phi = gh$ (gravitational potential) I Volts V or joule per coulomb J/C, J = N m, C = A s I 1.5 Alkaline battery, 12v typical car battery I $\vec{E} = -\nabla V$ I $V(\vec{r}) = \int_0^{\vec{r}} \vec{E} \cdot d\vec{l}$ I **Potential energy of Q and q when separated by r:** $W = QV(\vec{r})$, $Q = \frac{W}{V(\vec{b})-V(\vec{a})}$ I U = mgh, gravitational potential energy I U_E in joules J , U_E in Coulomb volt squared CV^2 I $\vec{E} = \frac{-\nabla V}{q}$ I $V_r = \int_{ref}^r q\vec{E} \cdot d\vec{s}$ Dr. Whites 4 way intersection Gauss Law $\oint \vec{E} \cdot d\vec{a} = \frac{1}{\epsilon_0}Q_{enclosed}$(should be double integral, cant do in latex right now) I integral around a closed path is evidently zero $\oint \vec{E} \cdot d\vec{l} = 0$ I applying stokes theorem $\nabla \times \vec{E} = 0$ I **Electric Force (N)** Coulombs law $\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{qQ}{R^2}\hat{R}$ I **Electric Force**(N) $\leftrightarrow$ **Electric Field**(N/C) $\vec{F} = Q\vec{E}$ I **Electric Field**(N/C) $\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}')}{R^2}\hat{R}d\tau$ I **Electric Field**(N/C) $\leftrightarrow$ **Electric potential(Voltage)**(V), (J/C) $-\int_0^{\vec{r}} \vec{E} \cdot d\vec{r} = V(\vec{r})$ I $\vec{E} = -\nabla V$ **Electric potential(Voltage)**(V), (J/C) $\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}')}{R}\hat{R}d\tau$ I $\nabla^2 V = \frac{-\rho}{\epsilon_0}$ **Electric potential(Voltage)**(V), (J/C) $\leftrightarrow$ **potential energy (work done)**(J), (N m), (W s), (C V) I $W = \frac{1}{2}\int \rho V d\tau$ I W = work in joules **potential energy (work done)**(J), (N m), (W s), (C V) $U = \frac{kqQ}{r}$???? $V(b) - V(a) = \frac{W}{Q}$ I $W = \frac{\epsilon_0}{2}\int_V E^2 d\tau = \oint_S V\vec{E} \cdot d\vec{a}$ Midterm II 5. Magnetostatics 5.1 The Lorentz Force Law $\vec{F_{mag}} = Q(\vec{v} \times \vec{B})$ I $\vec{F} = Q(\vec{E} + (\vec{v} \times \vec{B}))$ I **cyclotron motion** $QvB = m\frac{v^2}{R}$ or $p = QBR$ where $p = mv$ I **magnetic forces do no work** $dW_{mag} = \vec{F_{mag}} \cdot d\vec{l} = Q(\vec{v} \times \vec{B}) \cdot \vec{v}dt0$ I **current** $\vec{F_{mag}} = \int I(d\vec{l} \times \vec{B})$ I **surface current density** $\vec{K}$ I $\vec{K} = \frac{d\vec{I}}{dl_\perp}$ I **surface current densitt** σ $\vec{K} = \sigma\vec{v}$ I $\vec{F_{mag}} = \int(\vec{v} \times \vec{B})\sigma da = \int(\vec{K} \times \vec{B})da$ I **volume current density** $\vec{J}$ I $\vec{J} = \frac{d\vec{I}}{da_\perp}$ I **charge density density** ρ I $\vec{J} = \rho\vec{v}$ I $\vec{F_{mag}} = \int(\vec{v} \times \vec{B})\rho d\tau = \int(\vec{J} \times \vec{B})d\tau$ I $\nabla \cdot \vec{J} = -\frac{\partial\rho}{\partial t}$ I **Cyclotron motion** $QvB = m\frac{v^2}{R}$ or $p = mv = QBR$ I $\nabla \cdot = \frac{\partial\rho}{\partial t}$ 5.2 The Biot-Savart Law Stationary charges -¿ constant electric fields; electrostatics I Steady currents -¿ constant magnetic fields: magnetostatics I electro/magnetostatics is the regime $\frac{\partial\rho}{\partial t} = 0$ and $\frac{\partial\vec{J}}{\partial t} = \vec{0}$ I $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{I} \times \hat{R}}{R^2}dl' = \frac{\mu_0}{4\pi}I\int \frac{d\vec{l}' \times \hat{R}}{R^2}$ I permeability of free space $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{NA^{-2}}$ I **surface current** $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{K}(\vec{r}') \times \hat{R}}{r^2}da'$ I **volume current** $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}') \times \hat{r}}{r^2}d\tau'$ 5.3 The divergence and Curl of B the integral of $\vec{B}$ around a circular path of radius s, centered at the wire is $\oint \vec{B} \cdot d\vec{l} = \oint \frac{\mu_0 I}{2\pi s}dl = \frac{\mu_0 I}{2\pi s}\oint dl = \mu_0 I$ I $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$ I $I_{enc} = \int \vec{J} \cdot d\vec{a}$ I $\int(\nabla \times \vec{B}) \cdot d\vec{a} = \mu_0\int \vec{J} \cdot d\vec{a}$ I $\nabla \times \vec{B} = \mu_0\vec{J}$ I $\nabla \cdot \vec{B} = 0$ I Ampere's Law I $\nabla \times \vec{B} = \mu_0\vec{J}$ I $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$ I Electrostatics: Coulomb $\to$ Gauss, Magnetostatics: Biot-Savart $\to$ Ampered Solenoid $\oint \vec{B} \cdot d\vec{l} = B_\phi(2\pi s) = \mu_0 I_{enclosed}$ for a loop that is half inside and half outside of the solenoid $\oint \vec{B} \cdot d\vec{l} = BL = \mu_{naught}I_{enc} = \mu 0IL$ I $\vec{B} = \mu_0 nI\hat{Z}$ inside the solenoid = 0 outside the solenoid I B field for Toroid$\vec{B}(\vec{r}) = \frac{\mu_0 NI}{2\pi s}\hat{\phi}$for points inside the toroid = 0for points outside the coil I **Maxwell's equations for electrostatics** $\nabla \cdot \vec{E} = \frac{1}{\epsilon_0}\rho$(Gauss's law) $\nabla \times \vec{E} = \vec{0}$(no name) I **Maxwell's equations for magnetostatics** $\nabla \cdot \vec{B} = 0$(no name) $\nabla \times \vec{B} = \mu_0\vec{J}$(Ampere's law) **Maxwell's equations and the force law** $\vec{F} = Q(\vec{E} + \vec{v} \times \vec{B})$ 5.4 Magnetic Vector Potential just as $\nabla \times \vec{E} = \vec{0}$ permits the introduction of a scalar potential V in electrostatics $\vec{E} = \nabla V$ so $\nabla \cdot \vec{B} = 0$ invites the introduction of a vector potential A in magnetostatics $\vec{B} = \nabla \times \vec{A}$ I $\nabla \cdot \vec{A} = 0$ I $\nabla^2\vec{A} = -\mu_0\vec{J}$ I $\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}')}{R}d\tau'$ I Vector potential for line current$\vec{A} = \frac{\mu_0}{4\pi}\int \frac{\vec{I}}{R}dl' = \frac{\mu_0 I}{4\pi}\int \frac{1}{R}d\vec{l}'$ Vector potential for surface current$\vec{A} = \frac{\mu_0}{4\pi}\int \frac{\vec{K}(\vec{r}')}{R}da'$ I Magnetic Vector Potential, current density, and Magnetic field triangle relationships $\vec{A} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}}{R}d\tau$ I $\nabla^2\vec{A} = -\mu_0\vec{J}$ I $\vec{B} = \frac{\mu_0}{4\pi}\int \frac{\vec{J} \times \hat{R}}{R^2}d\tau$ I $\nabla \times \vec{B} = \mu_0\vec{J}$; $\nabla \cdot \vec{B} = 0$ I $\vec{B} = \nabla \times \vec{A}$; $\nabla \cdot \vec{A} = 0$ I $\vec{A}_{dip}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{\vec{m} \times \hat{r}}{r^2}$ were $\vec{m}$ is the magnetic dipole moment I $\vec{m} = I\int d\vec{a} = I\vec{a}$ where $\vec{a}$ is the vector area of the loop I $\vec{B}_{dip}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{1}{r^3}[3(\vec{m} \cdot \hat{R})\hat{R} - \vec{m}]$ 6. Magnetic Fields in Matter 6.1 Magnetization Torque $\vec{N} = \vec{m} \times \vec{B}$ where $m = Iab$ (square) is the magnetic dipole moment of the loop I In particular, the torque is again in such direction as to line the dipole up parallel to the field. It is this torque that accounts for paramagnetism. Since every electron constitutes a magnetic dipole you might expect paramagnetism to be a universal phenomenon. Actually, quantum mechanics tends to lock the electrons within a given atom together in pairs with opposite spin. I For an infinitesimal loop $\vec{F} = \nabla(\vec{m} \cdot \vec{B})$ I In the presence of a magnetic field, each atom picks up a little "extra dipole moment, and these increments are all antiparallel to the field. This is the mechanism for diamagnetism. I $\vec{M}$ is call the magnetization, it plays a role analogous to the polarization $\vec{P}$ in electrostatics 6.2 The field of a Magnetized Object $\vec{J_b} = \nabla \times \vec{M}$ I $\vec{K_b} = \vec{M} \times \hat{n}$ 6.3 The Auxiliary Field H not covered 6.4 Linear and Non-Linear Media **Ferromagnetism** In a linear medium, the alignment of atomic dipoles is maintained by a magnetic field imposed from the outside. Ferromagnets - which are emphatically not linear - require no external fields to sustain the magnetization; the alignment is frozen in. In Ferromagnets, each dipole likes to point in the same direction as its neighbors. The reason is quantum mechanical. Alignment occurs in relatively small patches called domains. The net effect of the magnetic field is to move the domain boundaries. If the B field is strong enough one domain takes over entirely, and the iron is said to be saturated. Shifting domain boundaries is not entirely reversible. The path is traced out in the hysteresis loop. Chapter 7 7.1 Ohm's Law $\vec{J}$ the current density is proportional to the force per unit charge $\vec{f}$ I $\vec{J} = \sigma\vec{f}$ where σ is the proportionality factor called the conductivity of the material I $\rho = \frac{1}{\sigma}$ where ρ is the resistivity of the material I $\vec{J} = \sigma(\vec{E} + \vec{v} \times \vec{B})$ I $\vec{J} = \sigma\vec{E}$ I $V = IR$ I $P = VI = I^2R$ I ε is the electromotive force or emf, the integral of a force per unit charge I $\varepsilon = \oint \vec{f} \cdot d\vec{l} = \oint \vec{s}_s \cdot d\vec{l}$ I there are two forces involved in driving current around a circuit: the source, $\vec{f_s}$ with is ordinarily confined to one portion of the loop (a battery) and an electrostaic force, which serves to motth out the flow and communicate the influence of the source to distance parts of the circuit I $\vec{f} = \vec{f_s} + \vec{E}$ I The flux rule for motional emf $\varepsilon = \frac{-d\Phi}{dt}$ I whenever the magnetic flux through a loop changes, an emf will appear in the loop 7.2 Electromagnetic Induction a changing magnetic field induces an electric field I $\varepsilon = \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi}{dt} = -\int \frac{\partial\vec{B}}{\partial t} \cdot \partial\vec{a}$ I we can convert Faradays law from integral form into differential from by applying Stokes theorem I $\nabla \times \vec{E} = -\frac{\partial\vec{B}}{\partial t}$ I The work does on a unit charge against the back emf, in one trip around the circuit is -ε I $\frac{dW}{dt} = -\varepsilon I = LI\frac{dI}{dt}$ I If we start with zero current abd build it up to a final value I, the work done $W = \frac{1}{2}LI^2$ I $W = \frac{1}{2\mu_0}\int_{allspace} B^2 d\tau$ 7.3 Maxwells Equations $\nabla \times \vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$ I a changing electric field induces a magnetic field I Electric Field E in NC^{-1} or Vm^{-1} or kgms^{-3}A^{-1} 1.0 NC^{-1} is small, like the field produced by a 1.0 V battery between its ports if were separated by a meter, 3×10^6 NC^{-1} makes sparks in the air, $\vec{F} = q\vec{E}$, and volume charge $\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(r')}{R^2}\hat{R}d\tau'$ I magnetic dipole moment $\vec{m}$ in NmT^{-1} or Am^2 or JT^{-1} 1 amp (2.1 amp in high power led) (9 amp in toster) 1 meter circumference so r=.5 so $m = .5^2\pi(1) = .25\pi$, $\vec{m} = I\int d\vec{a} = I\vec{a}$, $\vec{a}$ is the area vector of the loop, if the loop is flat, $\vec{a}$ is the ordinary area enclosed magnetic force constant $K_a = \frac{\mu_0}{4\pi} = 1 \times 10^{-7}$ NA^{-2} or kgms^{-2}A^{-2} I Current I in A, 5 amp on one typical 12 volt motor vehicle headlight, used in ampere's law $\nabla \times \vec{B} = \mu_0\vec{J}$ or $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed}$ where $\vec{J}$ is volume current density I Magnetic Flux Φ_B in V s or kgm^{2}s^{-3}A^{-1} if the magnetic field is perpendicular to the area and B field is 1 Tesla and the area had a radius 1. The magnetic flux is π. Magnetic flux through a closed surface $\Phi_B = \oiint_s \vec{B} \cdot d\vec{a} = 0$ sometimes faradays law is useful $\Phi_B = \cos\theta AB$ where theta is the angle off of perpendicular to the surface, A is the area and B is the magnetic field I Resistance in ohms Ω or kgm^{2}s^{-3}A^{-2} or VA^{-1} 1.5 volt alkaline battery and 2.7 high power led current you get 1.5/2.7 = 0.55 ohms. $V = IR$ and $P = VI = \frac{V^2}{R} = I^2R$ I Power in watts W or kgm^{2}s^{-3} or Js^{-1} I Volume current density $\vec{J}$ in $\frac{A}{m^2}$, used in $\nabla \times \vec{B} = \mu_0\vec{J}$, $\int(\nabla \times \vec{B}) \cdot d\vec{a} = \oint \vec{B} \cdot d\vec{l} = \mu_0\int \vec{J} \cdot d\vec{a}$, $\nabla^2\vec{A} = -\mu_0\vec{J}$ 2.7 amps in high power led is passing through a area of 1meter by 1 meter you get $\vec{J} = 2.7$ Am^{-2} I Torque $\vec{N}$ in J or N m or kgm^{2}s^{-2} , $\vec{N} = \vec{p} \times \vec{E}$ where dipole $\vec{p} = q\vec{d}$ in a uniform field $\vec{E}$ I Magnetic Field in units of Tesla T flows from North to South, typical refrigerator magnet 5×10^{-3} T earths magnetic field flows from the geographic south pole to the geographic north pole, magnetic fields in the same direction = attraction, magnetic fields in opposite directions = repulsion I magnetic vector potential $\vec{A}$ in Vsm^{-1} or kgms^{-2}A^{-1} or NA^{-1} , $\vec{B} = \nabla \times \vec{A}$, $\vec{A} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}}{R}d\tau$ I Farad unit of electrical capacitance in s^{4}A^{2}m^{-2}kg^{-1} Midterm II test corrections Biot-Savart Law $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{I} \times \hat{R}}{R^2}dl' = \frac{\mu_0}{4\pi}I\int \frac{dl' \times \hat{R}}{R^2}$ I Amperian Loop $\nabla \times \vec{B} = \mu_0\vec{J}$ use stokes theorem to convert from differential form to integral form $\int(\nabla \times \vec{B}) \cdot d\vec{a} = \oint \vec{B} \cdot d\vec{l} = \mu_0\int \vec{J} \cdot d\vec{a}$, therefor the current enclosed by the amperian loop $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed}$ I Lorentz force law $\vec{F}_{mag} = Q[\vec{E} + (\vec{v} \times \vec{B})]$ I Electric field for large values of r point charge $\frac{1}{r^2}$, line charge $\frac{1}{r}$, dipole $\frac{1}{r^3}$, plane of charge constant I integration trig substitiution $\int(x^2 + z^2)^{-3/2}$ I $x = z\tan(u)$ I $dx = z\sec^2(u)du$ I plug into $(x^2 + z^2)^{-3/2}$ I $z^2\tan^2(u) + z^2$ I $z^2(\tan^2(u) + 1)^{-3/2}$ I magnetic field of a dipole $\vec{B}_{dipole}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{1}{r^3}[3(\vec{m} \cdot \hat{r})\hat{r} - \vec{m}]$ Volt V unit of voltage in kgm^{2}A^{-1}s^{-3} Q3 Two very long wires two wires with current running in opposite directions, repulsive I B field $\frac{1}{s}forawire$ I field approximately $\frac{2K_m I}{s}$ where $K_m = 1 \times 10^{-7}$ NA^2 I Q4 Two very long wires different current if current changed to 10 amp out of page, magnetic field at A now zero I the torque is in the direction perpendicular to the one of motion Q5 Long cylinder perpendicular across the equator m mass. b radius, q charge, distance D $\vec{V_0} = v_0(-\hat{z})$ I $F_{total} = F_g + F_{electrostaticsandmagnetostatics}$ I $\vec{F} = Q(\vec{E} + \vec{v} \times \vec{B})$ I $\vec{B} = B\hat{z}$ I $\vec{J} = J(-\hat{z})$ I current density varies linearly $J(r) = J_0(1 - \frac{s}{R}) + J_R(\frac{s}{R})$ I $\langle s, \phi, z\rangle$ I earths magnetic field is parallel to the velocity vector of the particle, has no effect on velocity I $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J} \times \hat{R}}{R^2}d\tau'$ DO NOT USE I $\int_0^{2\pi} \vec{B}d\vec{l} = \mu_0\int \vec{J} \cdot d\vec{a}$ I $\vec{V} = -v\hat{z}$, $\vec{B} = -B\hat{\phi}$, $\vec{F_b} = -F\hat{s}$, $\vec{F_g} = -F\hat{s}$ I $F_t = F_g + QVB$ I $d\vec{l} = sd\theta$ I $2\pi SB = \mu_0\int_0^{2\pi}\int_0^s J(r) = J_0(1 - \frac{s}{R}) + J_R(\frac{s}{R})dssd\theta$ I $F = ma$ I Q6 constant and proportional surface current densities at origin B field $\hat{z}$ I B field very far on xy plane $-\hat{z}$ I very far on Z axis $\hat{z}$ I spiraling steady current is dipole magnetic field in magnetostatics $\frac{1}{r^3}$ I side note $\vec{J} = \rho\vec{v}$ volume current density equals volume charge density times velocity, also true for $\vec{K} = \sigma\vec{v}$ sigma in this case is charge density NOT conductivity, and $\vec{I} = \lambda\vec{v}$ I the phonograph record is best described by the current density proportional to radius I the u tape is not described well by wither the constant magnitude current density of proportional to radius. Modeled well as the limit as constant and proportional approach each other, also has a depth element I the single wire wrapped in eye of a stove is modeled by a current density with constant magnitude, current in a wire is same velocity where ever you are I $\langle\hat{s}, \hat{\phi}, \hat{z}\rangle$ I $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{K}(\vec{r}) \times \hat{R}}{R^2}dl'$ I $r = 0$, $r' = ss'$, $\vec{R} = -s'\hat{s}$, $R = s'$, $\hat{R} = \frac{\vec{R}}{R} = -\hat{s}$, $da' = dssd\phi$, $\vec{K}(\vec{r}) = K_B(\frac{s'}{b})\hat{\phi}$ I $B = \frac{\mu_0}{4\pi}\int_0^{2\pi}\int_a^b \frac{K_b(\frac{s'}{b})\hat{z}}{s'^2}dss'd\phi$ Q7 cylindrical curl of magnetic vector potential $\vec{B} = \nabla \times \vec{A}$ I $\vec{A} = C(\frac{s'}{a})^2\hat{z}$ for $s' < a$ I $\vec{A} = 2k_m I\ln(s')\hat{z}$ for $s' > a$ I $\vec{B_1} = -2c\frac{s'}{a^2}\phi$ for $s' < a$ I $\vec{B_2} = -2k_m I\frac{1}{s'}$ for $s' > a$ I $\frac{1}{r}$ wire of charge relationship for E field generated, and $\frac{1}{r}$ wire of charge relationship for B field generated I chose C to make sure B is in tesla's, to be continuous equations must be equal at $s = a'$ I Q8 infinite cylinder vs fixed length Case I I $\oint \vec{B} \cdot d\vec{a} = I_{enclosed}\mu_0$ I $I_{enclosed} = \int J(s')da$ I If $\vec{I} = \hat{z}$ and $\vec{B} = \hat{\phi}$ I B goes to $\frac{1}{s}$ I $\int_0^{2\pi} B\hat{\phi} \cdot sd\theta\hat{\phi} = \mu_0\int_0^{2\pi}\int_0^R \frac{I_0}{\pi R^2}(\frac{s'}{R})dss'd\theta$ I $B2\pi s = \frac{2I_0\mu_0}{3}$ for $s > R$ and $= \frac{2I_0\mu_0 s^3}{3R^3}$ for $s < R$ I Case 2 I $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}) \times \hat{R}}{R^2}d\tau'$ I If $\vec{I} = \hat{\phi}$ then $\hat{B} = \hat{z}$ I Spinning loop of charge, this is a dipole, had a $\frac{1}{r^3}$ relationship as $r \to \infty$ therefore $B \to \frac{1}{r^3}$ I Q9 orbiting charge, earths magnetic field behaves like a dipole $\langle s, \phi, z\rangle$ I $a = \frac{V^2}{R}$ centripetal acceleration I $F = ma$ I $\vec{F_g} = F_g(-\hat{s})$, $\vec{I} = I\hat{\phi}$, $\vec{a} = a(-\hat{s})$, $\vec{B} = B(\hat{z})$, $\vec{F}F\hat{s}$ I Magnetic field acts in the opposite direction to the gravitational field I $B = \frac{k_m M}{r^3}$ m is magnetic dipole moment $\frac{B_0}{B(r)} = \frac{R^3}{R^3_{earth}}$ I $\frac{G_0}{G(r)} = \frac{R^2}{R^2}_{earth}$ (9.8) I $\hat{s}$ only, get rid of vectors I $F = m(-a) = \frac{-mv^2}{R} = F_{mag} - F_g$ I $F_{mag} = QvB$ and $F_g = mg$ I BIG R is Radius not Source field point vector minus source vector $\frac{-mv^2}{R} = qv\frac{B_0 R^3_{earth}}{R^3} - \frac{mg_0 R^2_{earth}}{R^2}$ I convert with period $T = \frac{d}{v} = \frac{2\pi R}{v}$ therefore $v = \frac{2\pi R}{T}$ I $R^3 = \frac{-q2\pi TR^3_{earth}}{m4\pi} + \frac{mg_0 T^2 R^2_{earth}}{m4\pi}$

Final Maxwells Equaition **Gauss's Law** Differential $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$ and Integral $\oiint \vec{E} \cdot d\vec{a} = \frac{Q_{enclosed}}{\epsilon_0}$ I **no name** Differential $\vec{\nabla} \cdot \vec{B} = 0$ and integral $\oiint \vec{B} \cdot d\vec{a} = 0$ I **Faraday's Law** Differential $\vec{\nabla} \times \vec{E} = -\frac{\partial\vec{B}}{\partial t}$ and Integral $\oint \vec{E} \cdot d\vec{l} = \frac{-\partial\Phi_{mag}}{\partial t}$ I **Ampere's law with Maxell's correction** $\nabla \times \vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$ and Integral $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed} + \mu_0\epsilon_0\int \frac{\partial\vec{E}}{\partial t} \cdot d\vec{a}$ Symbols and Units and Common Equations Electric field E in NC^{-1} or Vm^{-1} or Vm^{-1} or kgms^{-3}A^{-1}. 3×10^6 Vm^{-1} Dielectric breakdown of air, or 3×10^3 Vmm^{-1} could be in the lab. Therefor 1 Vm^{-1} is small. Coulomb's Law $\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{qQ}{R^2}\hat{R}$, $\vec{F} = Q\vec{E}$, for volume charge $\vec{F} = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}')}{R^2}\hat{R}d\tau'$, $\vec{E} = -\nabla V$ I Voltage V in V or NmC^{-1} or WA^{-1} or JC^{-1} or kgm^{2}A^{-1}s^{-3} I Energy E in J or kgm^{2}s^{-2} I Force F in N or kgms^{-2} I Electric Charge Q in C or A s I Gravitational field g in Nkg^{-1} or ms^{-2}. 1.0 Nkg^{-1} is small, about 1/10 as big as g on earths surface, $Weight = mg$ I Magnetic Field B in T or NA^{-1}m^{-1} or kgA^{-1}s^{-2}. 5×10^{-3} T is the strength of a typical refrigerator magnet. 1 T is therefor large. Magnetic Forces $\vec{F}_{mag} = Q(\vec{v} \times \vec{B})$ Lorentz Force Law $\vec{F} = Q[\vec{E} + (\vec{v} \times \vec{B})]$ Biot-Savart law $B(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{I} \times \hat{R}}{R^2} dl' = \frac{\mu_0}{4\pi} I \int \frac{d\vec{l} \times \hat{R}}{R^2}$ I Electric Potential V_E in V or JC^{-1} or kgm^{2}A^{-1}C^{-3}. 1.5 V in an alkaline battery. Therefor 1.0 V is of average size. Poisson's equation $\nabla^2 V = -\frac{\rho}{\epsilon_0}$, Laplace's equation $\nabla^2 V = 0$, Voltage for volume change density $V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r'})}{R} d\tau'$, $V = -\int_0^{\vec{r}} \vec{E} \cdot d\vec{l}$ (reference point for zero potential is at infinity) Magnetic Force Constant k_A in Nm^{-1} or kgs^{-2}, Where $K_A = 1 \times 10^{-7}$ Nm^{-1} $= \frac{\mu_0}{4\pi}$ where ($\mu_0 = 4\pi \times 10^{-7}$ NA^{-2}) is the magnetic constant, or vacuum permeability. I Current I in Cs^{-1} or A. 2.0×10^{-1} A A constant current in a common light emitting diode. 1 A is on larger side but still reasonable. $\vec{F}_{mag} = \int I(d\vec{l} \times \vec{B}) = \int(\vec{I} \times \vec{B})dl$, $\vec{I} = \lambda\vec{v}$, Surface current density $\vec{K} \equiv \frac{d\vec{I}}{dl}$ where $\vec{K} = \sigma\vec{v}$. $\vec{F}_{mag} = \int(\vec{v} \times \vec{B})\sigma da = \int(\vec{K} \times \vec{B})da$, volume current density $\vec{J} \equiv \frac{d\vec{I}}{da}$ where $\vec{J} = \rho\vec{v}$ $\vec{F}_{mag} = \int(\vec{v} \times \vec{B})\rho da = \int(\vec{J} \times \vec{B})d\tau$. $I = \int \vec{J} da$. $V = IR$. $P = VI = I^2 R$ I Magnetic Flux Φ in Wb (webber) or V s or JA^{-1} or JA^{-1} or Tm^{-2} or kgm^{2}s^{-2}A Bar magnet 1×10^{-4} T and coil with radius 1×10^{-2} m therefor $\Phi = 1 \times 10^{-2^2}\pi(1 \times 10^{-4}\text{ T}) = 3.14 \times 10^{-8}$ Wb. Therefor 1 Wb is larger. for motional emf (electromotive force)$\varepsilon \equiv \oint \vec{f} \cdot d\vec{l}$. megnetic flux through a loop changes $\varepsilon = -\frac{d\Phi}{dt}$. Also Faradays law in integral form I Resistance R in Ω or VA^{-1} or S^{-1} (Siemens unit of conductivity) or or kgm^{2}s^{-3}A^{-2}. Small light bulbs or 50 Ω or resistance of small copper wire 1×10^{-1} Ω. Therefore or 1 Ω is on the small size but about average. $V = IR$ and $P = VI = I^2R$. resistivity $\rho = \frac{1}{\sigma}$ where σ is conductivity. Resistivity or copper $rho = 1.68 \times 10^{-8}$ Ω m. Current density $\vec{J} = \sigma\vec{f}$ where $\vec{f}$ is the force per unit charge. $\vec{J} = \sigma(\vec{E} + \vec{v} \times \vec{B}) = \sigma\vec{E}$ I Power P in W or kgm^{2}s^{-3} I Torque N in N or kgms^{-2}. With 1 A and area of 1 m^2 creates a magnetic dipole moment m of 1 Am^2 crossed with 1 T results with 1 Am^{2}kgA^{-1}s^{-2} or 1 J. $\vec{N} = \vec{m} \times \vec{B}$ and magnetic dipole moment $m = Iab$ I electric dipole moment $\vec{p}$ in C m or A s m. Two ping pong balls of $q = 1 \times 10^{-8}$ C separated by a distance of 1×10^{-2} m which results in a electric dipole moment of 1×10^{-10} A s m. Therefore 1 A s m is large. $r\vec{h}o \equiv \vec{r}'\rho(\vec{r}')d\tau'$. The dipole contribution to the potential $V_{dipole}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}$. For a physical dipole $\vec{p} = qr'_+ - q\vec{r}'_- = q(\vec{r}^{\,n}_+ nd\prime - \vec{r}'_-)q\vec{d}$. $\vec{p} = \sum_{i=1}^{n} q_i\vec{r}'_i$. $\vec{E}_{dip}(r,\theta) = \frac{p}{4\pi\epsilon_0 r^3}(2\cos\theta\hat{r} + \sin\theta\hat{\theta})$ I Magnetic vector potential $\vec{A}$ in Vsm^{-1} or kgm^{2}A^{-1}s^{-3}sm^{-1} or kgmA^{-1}s^{-2} or NA^{-1}. $\vec{E} = -\vec{\nabla}V$. $\vec{B} = \vec{\nabla} \times \vec{A}$. $\vec{\nabla} \cdot \vec{A} = 0$. $\nabla^2\vec{A} = -\mu_0\vec{J}$. $\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int\frac{\vec{J}(\vec{r}')}{R}d\tau$ (modify for line and surface currents) I levitate a square loop of wire $< s, \phi, z >$ I $\vec{F}_T = -F\hat{s}$, $\vec{F}_B = F\hat{s}$, $\vec{F}_L = F\hat{z}$, $\vec{F}_R = -F\hat{z}$ I assume E = 0 in amperes law $\int_0^{2\pi} B\phi \cdot rd\phi = \mu_0 I$ I $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$ I Full form Lorentz force law $\vec{F} = \iiint(\rho\vec{E} + \vec{J} \times \vec{B})dv$ I applied to problem $qv \times \vec{B} = F_{magneticforce}$ I $I \times B = F_m$ I $F_g = ma$ I $F_T = F_g + F_{mag}$ I $0 = mg(\hat{s}) + \frac{\mu_o}{2\pi(s-\frac{a}{2})}(-\hat{s}) + \frac{\mu_0 I^2}{2\pi(s+\frac{a}{2})}(\hat{s})$ I $\therefore I = \sqrt{\frac{mg}{(\frac{\mu_u}{2\pi(s-\frac{a}{2})} - \frac{\mu_0}{2\pi(s+\frac{a}{2})})}}$ wire with current density $< s, \phi, z >$ I $\vec{J}(s) = Ae^{-as^2}$ I $a = \text{m}^{-2}$ and $A = \text{A}^1\text{m}^{-2}$ I $J = \frac{dI}{da}$ I $\int_0^{2\pi} Ae^{-ands^2}dssd\phi = \vec{I}\hat{z}$ I $\vec{I} = \frac{A\pi}{a}(1 - e^{-aR^2})\hat{z}$ I For a light bulb 100 watt, 120V, 144 ohm, 0.8333 amps I Amperes law (assume $E = 0$) $\int_0^{2\pi} Bd\phi s = \mu_0\frac{A\pi}{a}(1 - e^{-aR^2})$ I $B = \frac{\mu_0\frac{A\pi}{a}(1-e^{-aR^2})}{2\pi s}$ I units $\frac{\frac{N}{A^2}(\frac{A}{m^2}(\frac{1}{\frac{1}{m^2}}))}{m} = \frac{kg}{s^2} = T$ values $B = \frac{4\pi \times 10^{-7}\frac{2.786\pi}{1}(1-e^{-1(1\times10^{-3})^2})}{2\pi(1)} = 1.7 \times 10^{-12}$ T makes sense since we are very far away with respect to the wire which of Maxwell's equations electric field for charge distribution that increases from the origin to radius R and zero outside $< s, \theta, \phi >$ I Gauss's law, Gaussian surface on the left hand side $\int_0^{2\pi}\int_0^{\pi} E\hat{s} \cdot rd\theta r\sin\theta = \frac{Q(r)}{\epsilon_0} = \frac{qr}{R\epsilon_0}$ for $r < R$ Magnetic field of wire current I surrounded by cylindrical shell with opposite current 4I amperes law in cylindrical coordinates $\int_0^{2\pi} Brd\phi = \mu_0 I$ for $r < R$ I center $I\hat{Z}$, inside $-I\hat{z}$, outside $-3I\hat{z}$ Voltage and current generated when a gold ring is spun on a table **no maxwell equation** $\lambda\vec{v} = \vec{I}$ and $\vec{k} = \sigma\vec{v}$ and $\vec{J} = \rho\vec{v}$ I $\omega = \frac{v}{R}$ I $\vec{v} = \omega R\hat{\phi}$ I $\vec{I} = \lambda\omega R\hat{\phi}$ I $V = \frac{1}{4\pi}\int\frac{\lambda(r')}{R}dl$ (R in this case is curly R) I $r = z\hat{z}$ and $r' = R\hat{s}$ (R radius, not curly R), $\vec{R} = z\hat{z} - R\hat{s}$ (first curly R, second Radius R) I $V = \frac{1}{4\pi}\int_0^{2\pi}\frac{\lambda}{\sqrt{z+R^2}}rd\phi$ Voltage of a point charge Q centered with in a metal sphere with charge -3Q $< s, \phi, \theta >$ use Gauss law to get E then solve for potential I $\oiint\vec{E} \cdot d\vec{a} = \frac{Q_{enclosed}}{\epsilon_0}$ I $\int_0^{2\pi}\int_0^{4} Erd\theta r\sin\theta d\phi$ where $E_{out} = \frac{-2Q}{4\pi r^2\epsilon_0}$ and $E_{in} = \frac{Q}{4\pi r^2\epsilon_0}$ I solve for potential coming in from infinity $V(\vec{r}) = -\int_{\infty}^{R}\vec{E}_{out} \cdot dl - \int_R^r \vec{E}_{in} \cdot dl$ I $V_{out} = \int_{\infty}^{R}\frac{-2Q}{4\pi r^2\epsilon_0}dr$ I $V_{in} = -\int_R^r = \frac{Q}{4\pi r^2\epsilon_0}$ I $\therefore V_{inside} = \frac{Q}{4\pi\epsilon_0}(\frac{R-3r}{rR})$ where $r < R$ and $V_{outside}(r) = \frac{-2Q}{4\pi\epsilon_0 r}$ for $r > R$ (no R is curly) Electric field of a point charge a distance D from an infinite metal sheet **no maxwell equation** $< s, \theta, \phi >$ $\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}\hat{R}$ (R is curly) and $\vec{E} = 0$ where $s < 0$ I $r = s\hat{s} + \theta\hat{\theta} + \phi\hat{\phi}$ I $r' = (R + d)\hat{s}$ I $\vec{R} = (s - R - d)\hat{s} + \theta\hat{\theta} + \phi\hat{\phi}$ Electric field of a point charge located a distance D from an infinite metal sheet **no maxwell equation** $V(x, y, z) = \frac{1}{4\pi\epsilon_0}[\frac{q}{\sqrt{x^2+y^2+(z-d)^2}} - \frac{q}{\sqrt{x^2+y^2+(z+d)^2}}]$ (-d for +Q and +d for -Q) I when v=0 and z=0 I $\vec{E} = -\nabla V = -(\frac{dV}{dx} + \frac{dV}{dy} + \frac{dV}{dz})$)however only true for $z \geq 0$ but that is all we care about for classic image problem Electric field of three parallel sheets of charge, each with charge density σ_0 **Gauss law (pillbox)**¡x,y,z¿ $\oiint\vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$ I $\int_0^L\int_0^L Edxdy + \int_0^L\int_0^L Edxdy = int_0^L\int_0^L \sigma dxdy$ I $2EL^2 = \frac{\sigma xy}{\epsilon_0}$ I $E = \frac{\sigma}{2\epsilon_0}$ sheet of charge for infinite sheet of charge I do we multiply by 3 potential of line of charge with density λ, surrounded by a cylindrical shell with density -4λ **gauss law** $< s, \phi, z >$ I $\int_0^L\int_0^{2\pi} Esd\phi dz = \frac{\int_0^L \lambda dz}{\epsilon_0}$ I $E = \frac{\lambda}{2\pi s\epsilon_0}$ I then solve for potential working from the outside in I remembering that $E = \frac{-\lambda}{2\pi s\epsilon_0}$ for R¿s I $V = -\int_{\infty}^{R}\vec{E} \cdot d\vec{l} - \int_R^r\vec{E} \cdot d\vec{l}$

long solid metal cylinder along the equator $< r, \theta, \phi >$ I Lorentz force law $\vec{F} = Q[\vec{E} + (\vec{v} \times \vec{B})]$ $\vec{F}_g = -F\hat{s}$ I $\vec{F}_{mag} = F\hat{s}$ forces and directions $J(r) = J_0(1 - \frac{s}{R}) + J_R(\frac{s}{R})$ where $s \leq R$ current density $\int_0^{2\pi}\int_0^R \vec{J}dssd\theta = \iint\frac{dI}{da}da$ I $R^2\pi(\frac{J_0}{3} + \frac{2J_r}{3}) = I_{enclosed}$ total current flowing and 3 way check assume $E = 0$ and $B_{earth} = 5 \times 10^{-5}$ T acceleration of particle after released $F_t = F_g + F_{magearth} + F_{magpipe}$ I $ma = -mg + qvB_e + qv\frac{mu_0 I_{enc}}{2\pi(R+D)}$ (no B for pipe) parameters so different forces dominate $q- > \infty$ and $a- > \infty$

charge distributions i) plane, line, dipole, proton for E field, $C, \frac{1}{r}, \frac{1}{r^3}, \frac{1}{r^2}$ and for potential V r, $ln(r), \frac{1}{r^2}, \frac{1}{r}$ ii) force a distance $\vec{F} = q(\vec{E} \cdot \vec{v} \times \vec{B})$ I so$F = qE$ therefor for plane, line, dipole, and proton $qC, \frac{q}{r}, \frac{q}{r^3}, \frac{q}{r^2}$

A through M placed on a line $E = \frac{1}{r^2}$ I $E = \frac{1}{4\pi\epsilon_0}\int\frac{\rho(\vec{r}')}{R^2}\hat{R}d\tau'$ I e field x-¿- I $\vec{E}_p = k4q(\frac{1}{1^2})\hat{x} + \frac{1}{2^2}\hat{y}$ I $\vec{E}_e = k(-q)(\frac{1}{1^6})\hat{-}x + \frac{1}{2^2}\hat{y}$ I $\therefore E_* = kq[(\frac{-143}{36})\hat{x} + \frac{5}{4}\hat{y}]$ I same process for potential only with $\frac{1}{r}$ expression for plane wave electric field $\widetilde{\hat{E}}(\vec{r}, t) = \widetilde{E}_0 e^{i(\vec{k}\cdot\vec{r} - \omega t}\hat{n}$ I polarization is in direction of amplitude of electric field I $< x, y, z >$ I wave number vector is in direction of propagation I $\vec{k} = k\hat{x}$ and angular frequency $\omega = 2\pi f$ and $k = \frac{2\pi f}{c}$ and $c = f\lambda$ I If you only look at the real term $E(r, t) = E_0\cos(\vec{k} \cdot \vec{r} - \omega t)\hat{n}$ I $\vec{E}(\vec{r}, t) = 100\cos[\frac{2\pi(1.02 \times 10^8)}{c}\hat{x} \cdot \vec{r} - (s\pi(1.02 \times 10^8)t)]\hat{z}$ I $\hat{x}$ propagation direction $\hat{z}$ polarization direction I $\widetilde{\hat{B}}(\vec{r}, t) = \frac{1}{c}\widetilde{E}_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)}(\hat{\vec{k}} \times \hat{\vec{n}}) = \frac{1}{c}\hat{\vec{k}} \times \widetilde{\vec{E}}$ I $B = \frac{1}{c}\hat{x} \times E\hat{z} = -\frac{1}{c}E\hat{Y}$ I Power is J/s or watt I The energy flux density (energy per unit area, per unit time) is the Poynting vector $\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B})$ I multiply the pointing vector by area to get the powering since pointing vector is (energy per unit area, per unit time) I Solve maxwells equation using E and B

Sphere with Uniform Density, Poisson compared to Gauss

distance behavior of point charge and conducting metal sheet

separation of variables