

Statistical Inference with Linear & Logistic Regression



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GW Libraries Workshop
Dan Kerchner ~ February 19, 2021

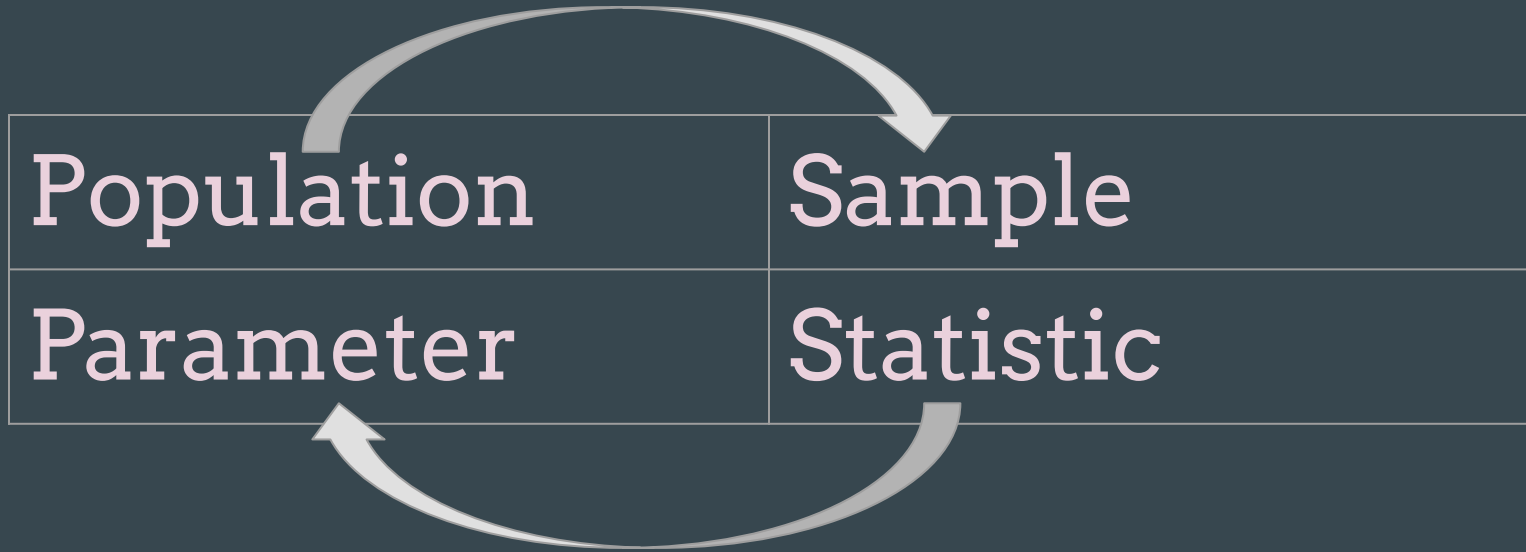
go.gwu.edu/rstats

Logistics

- Just speak up OR use the Zoom chat
- Kiri can provide individual help
- Plan for 1 brief ☕ break

Super-Brief Review of Inference for Regression

High-Level Objective



Regression Model

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_n X_n$$

Interpretation

β_0 = Mean Y when X_i values are 0

β_i = Mean change in Y for a 1-point increase in X_i ,
adjusting for other X variables

Correlation between dependent & independent variables

Pearson correlation (ρ) measures strength and direction (+/-) of linear association between X_i and Y . Ranges from -1 to 1.

If relationship looks non-linear (but monotonic) then **Spearman** correlation should be considered.

$$H_0: \rho = 0$$

$$H_1: \rho \neq 0$$

Valid if joint distribution of X, Y is bivariate normal

Linear Regression Model

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_n X_n$$

where Y is a continuous outcome

Interpretation

β_0 = Mean Y when X_i values are 0

β_i = Mean change in Y for a 1-point increase in X_i ,
adjusting for other X variables

Linear Regression Assumptions

- Observations are independent
- Linearity
- Homoscedasticity
- Normality

GLMs - Generalized Linear Models

$$g(\mu) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_n X_n$$

where $\mu = E(Y)$

Common link functions

$$g(\mu) = \mu$$

$$g(\mu) = \log(\mu)$$

$$g(\mu) = \log(\mu / (1 - \mu))$$

GLMs: Logistic Regression Model

$$\log\left(\overset{\text{odds}}{\frac{p}{1-p}}\right) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_n X_n$$

where p = Probability of $Y = 1$

$1-p$ = Probability of $Y = 0$

Interpretation

β_i = log OR (Odds Ratio) for having $Y = 1$ for a 1-point increase in X_i

e^{β_i} = OR for having $Y = 1$ for a 1-point increase in X_i

Logistic Regression Assumptions

- Binary outcome (0, 1)
- Each predictor is linearly related to the log odds of the outcome

Inference for Regression Modeling

Confidence Interval

95% CI for $\beta = (0.44, 0.49)$

Hypothesis Testing

$H_0: \beta = \beta_0 \leftarrow$ Null Hypothesis

$H_A: \beta \neq \beta_0 \leftarrow$ Alternative Hypothesis

p-value: Chance that we are rejecting H_0 when we should not be

Goals

A photograph of a soccer game on a dirt field. A goalpost is visible on the right side of the field. Several players are on the field, including one in the foreground who is jumping or kicking the ball. The background shows a line of trees under a blue sky with some clouds. The word "Goals" is overlaid in the center of the image in a large, white, serif font.

Today's Goal

- Learn to use R to read in data and conduct regression analysis and associated inference tests
 - Checking assumptions
 - Visualizing
 - Computing p-values, regression coefficients, confidence intervals, and odds ratios (for logistic models)

Today: 2 Scenarios

- Linear Regression (continuous outcome)
 - Single variable (1 continuous)
 - Multivariable (1 continuous, 1 categorical)
- Logistic Regression (categorical outcome)
 - Multivariable (continuous and categorical)

Today's Data Set: Framingham Heart Study

- [framinghamheartstudy.org](https://www.framinghamheartstudy.org)
- Long-term prospective study of the etiology of cardiovascular disease among a population of subjects in Framingham, MA
- Began in 1948 with 5,209 subjects
- Is the source of the term "risk factor"
- Over 3,000 peer-reviewed papers published based on this study
- Participants were each followed for a total of 24 years for cardiovascular events (heart attack, stroke, death, etc.)

Some Handy R Links

Tutorials

- RStudio R paths: education.rstudio.com/learn/
- Data Carpentry & Software Carpentry:
 - datacarpentry.org
 - software-carpentry.org
- LinkedIn Learning @ GW: go.gwu.edu/linkedinlearning
- r-tutor.com/r-introduction & r-tutor.com/elementary-statistics
- UCLA Data Analysis Examples: stats.idre.ucla.edu/other/dae/
- R Graph Gallery (w/code): r-graph-gallery.com

Books you can access for free

- Free books online - Hadley Wickham:
 - R for Data Science r4ds.had.co.nz
 - Advanced R adv-r.hadley.nz/
- Through your GW library privileges:

ADVANCED SEARCH

Search for: ☐ Catalog + Articles ☒ Catalog ☐ Articles

Subject ▼ contains ▼ R (Computer programming language)

Reference Links

- R language (CRAN): r-project.org
- R search engine: rseek.org
- rstudio.com
 - Cheat Sheets! rstudio.com/resources/cheatsheets
- stackoverflow.com

Statistics+R help @ GW

R-Statistics Appointments:

calendly.com/statistical-consulting-gw

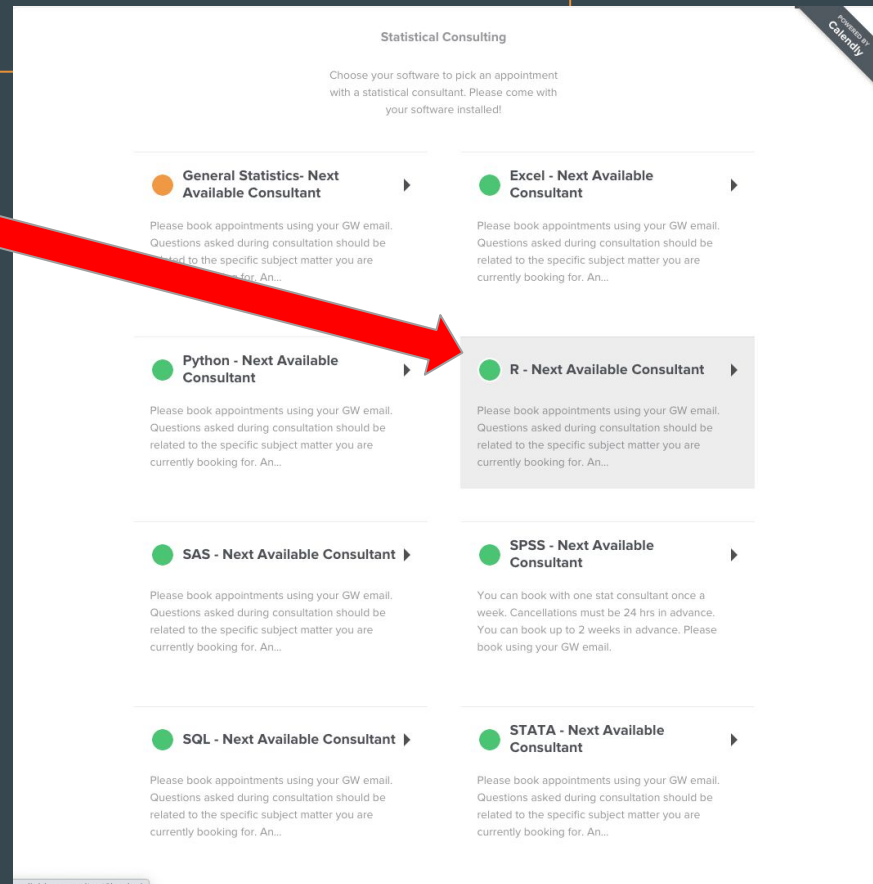
Also...

Appointments with me:

calendly.com/kerchner

Coding consultations (Python, git, etc.):

calendly.com/gwul-coding/



Thanks!

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