

# Statistical Inference with Linear & Logistic Regression



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GW Libraries Workshop  
Dan Kerchner ~ November 5, 2021

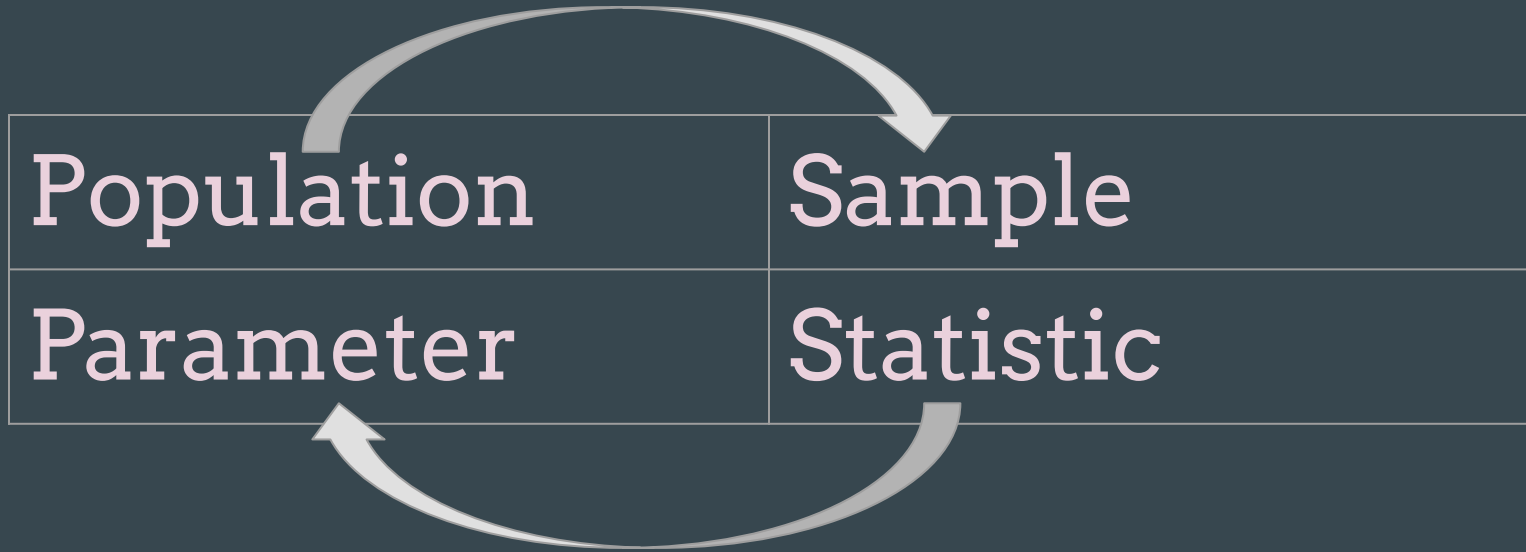
[go.gwu.edu/rstats](https://go.gwu.edu/rstats)

# Logistics

- Just speak up OR use the Zoom chat
- Plan for 1 brief  break

# Super-Brief Review of Inference for Regression

# High-Level Objective



# Regression Model

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_n X_n$$

(and an interaction term might look like  $\beta_{12} X_1 X_2$ )

## Interpretation

$\beta_0$  = Mean  $Y$  when  $X_i$  values are 0

$\beta_i$  = Mean change in  $Y$  for a 1-point increase in  $X_i$ ,  
adjusting for other  $X$  variables

# Correlation between dependent & independent variables

**Pearson** correlation ( $\rho$ ) measures strength and direction (+/-) of linear association between  $X_i$  and  $Y$ . Ranges from -1 to 1.

If relationship looks non-linear (but monotonic) then **Spearman** correlation should be considered.

$$H_0: \rho = 0$$

$$H_1: \rho \neq 0$$

Valid if joint distribution of  $X, Y$  is bivariate normal

# Linear Regression Model

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_n X_n$$

where  $Y$  is a continuous outcome

## Interpretation

$\beta_0$  = Mean  $Y$  when  $X_i$  values are 0

$\beta_i$  = Mean change in  $Y$  for a 1-point increase in  $X_i$ ,  
adjusting for other  $X$  variables

# Linear Regression Assumptions

- Observations are independent
- Linearity
- Homoscedasticity
- Normality



# GLMs - Generalized Linear Models

$$g(\mu) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_n X_n$$

where  $\mu = E(Y)$

## Common link functions

$$g(\mu) = \mu$$

$$g(\mu) = \log(\mu)$$

$$g(\mu) = \log(\mu / (1 - \mu)) = \text{logit}(\mu)$$

# GLMs: Logistic Regression Model

$$\log\left(\overset{\text{odds}}{\frac{p}{1-p}}\right) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_n X_n$$

where  $p$  = Probability of  $Y = 1$

$1-p$  = Probability of  $Y = 0$

## Interpretation

$\beta_i$  = log OR (Odds Ratio) for having  $Y = 1$  for a 1-point increase in  $X_i$ ,  
*adjusting for other predictors*

$e^{\beta_i}$  = OR for having  $Y = 1$  for a 1-point increase in  $X_i$

# Logistic Regression Assumptions

- Binary outcome (0, 1)
- Each predictor is linearly related to the log odds of the outcome

# Inference for Regression Modeling

## Confidence Interval

95% CI for  $\beta = (0.44, 0.49)$

## Hypothesis Testing

$H_0: \beta = \beta_0 \leftarrow$  Null Hypothesis

$H_A: \beta \neq \beta_0 \leftarrow$  Alternative Hypothesis

p-value: Chance that we are rejecting  $H_0$  when we should not be

# Goals

A photograph of a soccer game on a dirt field. A goalpost is visible on the right side of the field. Several players are on the field, including one in the foreground who is jumping or kicking the ball. The background shows a line of trees under a blue sky with some clouds. The word "Goals" is overlaid in the center of the image.

# Today's Goal

- Learn to use R to read in data and conduct regression analysis and associated inference tests
  - Checking assumptions
  - Visualizing
  - Computing p-values, regression coefficients, confidence intervals, and odds ratios (for logistic models)

# Today: 2 Scenarios

- Linear Regression (continuous outcome)
  - Single variable (1 continuous)
  - Multivariable (1 continuous, 1 categorical)
- Logistic Regression (categorical outcome)
  - Multivariable (continuous and categorical)

# Today's Data Set: Siddarth et al., 2018

Siddarth P, Burggren AC, Eyre HA, Small GW, Merrill DA (2018) Sedentary behavior associated with reduced medial temporal lobe thickness in middle-aged and older adults. PLoS ONE 13(4): e0195549.

[doi.org/10.1371/journal.pone.0195549](https://doi.org/10.1371/journal.pone.0195549)

- Examined associations between sedentary behavior and medial temporal lobe (MTL) subregion integrity
- 35 non-demented middle-aged and older adults
- Measured physical activity levels w/questionnaire
- Measured MTL thickness w/MRI scan
- Adjusted for age



# Today's Data Set: Framingham Heart Study

- [framinghamheartstudy.org](https://www.framinghamheartstudy.org)
- Long-term prospective study of the etiology of cardiovascular disease among a population of subjects in Framingham, MA
- Began in 1948 with 5,209 subjects
- Is the source of the term "risk factor"
- Over 3,000 peer-reviewed papers published based on this study
- Participants were each followed for a total of 24 years for cardiovascular events (heart attack, stroke, death, etc.)

## Some Handy R Links

## Some R packages & functions for regression++

- **lme4** - Linear mixed-effects models
- **MASS** - Model selection
- **ts()**, **arima()** - Time series object, model (part of **stats**)
- **forecast** - Forecasting for time series and linear models
- **mice** - imputation of missing data
- **medflex** - mediation analysis
- **splines**
- **survival** - survival analysis; JM - joint modeling
- AND MANY, MANY, MANY MORE

# Tutorials

- RStudio R paths: [education.rstudio.com/learn/](https://education.rstudio.com/learn/)
- Data Carpentry & Software Carpentry:
  - [datacarpentry.org](https://datacarpentry.org) and [software-carpentry.org](https://software-carpentry.org)
- LinkedIn Learning @ GW: [go.gwu.edu/linkedinlearning](https://go.gwu.edu/linkedinlearning)
- [r-tutor.com/r-introduction](https://r-tutor.com/r-introduction) & [r-tutor.com/elementary-statistics](https://r-tutor.com/elementary-statistics)
- UCLA Data Analysis Examples: [stats.idre.ucla.edu/other/dae/](https://stats.idre.ucla.edu/other/dae/)
- Visualizing regression results:  
[worldbank.github.io/r-econ-visual-library/RegressionCoef.html](https://worldbank.github.io/r-econ-visual-library/RegressionCoef.html)
- R Graph Gallery (w/code): [r-graph-gallery.com](https://r-graph-gallery.com)

# Books you can access for free

- Free books online - Hadley Wickham:
  - R for Data Science [r4ds.had.co.nz](https://r4ds.had.co.nz)
  - Advanced R [adv-r.hadley.nz/](https://adv-r.hadley.nz/)
- Through your GW library privileges:

ADVANCED SEARCH

Search for: ☐ Catalog + Articles ☒ Catalog ☐ Articles

Subject ▼ contains ▼ R (Computer programming language)

## Reference Links

- R language (CRAN): [r-project.org](https://r-project.org)
- R search engine: [rseek.org](https://rseek.org)
- [rstudio.com](https://rstudio.com)
  - Cheat Sheets! [rstudio.com/resources/cheatsheets](https://rstudio.com/resources/cheatsheets)
- [stackoverflow.com](https://stackoverflow.com)

# Statistics+R help @ GW

R-Statistics Appointments:

[academiccommons.gwu.edu/statistical-consulting](https://academiccommons.gwu.edu/statistical-consulting)

Also...

Appointments with me:

[calendly.com/kerchner](https://calendly.com/kerchner)

Coding consultations (Python, R, git, etc.):

[calendly.com/gwul-coding/](https://calendly.com/gwul-coding/)

Thanks!

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