MAE 3128
Biomechanics-I

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Topics covered today:

1. Statics problems in Biomechnanics
2. Building blocks of the musculoskeletal system
3. Mechanics of the elbow
4. Concept of Simple Stress and Simple Strain
5. In-class problems

GW School of Engineering & Applied Science
THE GEORGE WASHINGTON UNIVERSITY

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Biomechanics from a Continuum Mechanics perspective

Reference: Humphrey, J. D., & O'Rourke, S. L. (2015). *An introduction to biomechanics: Solids and fluids, analysis and design* (2nd ed.)

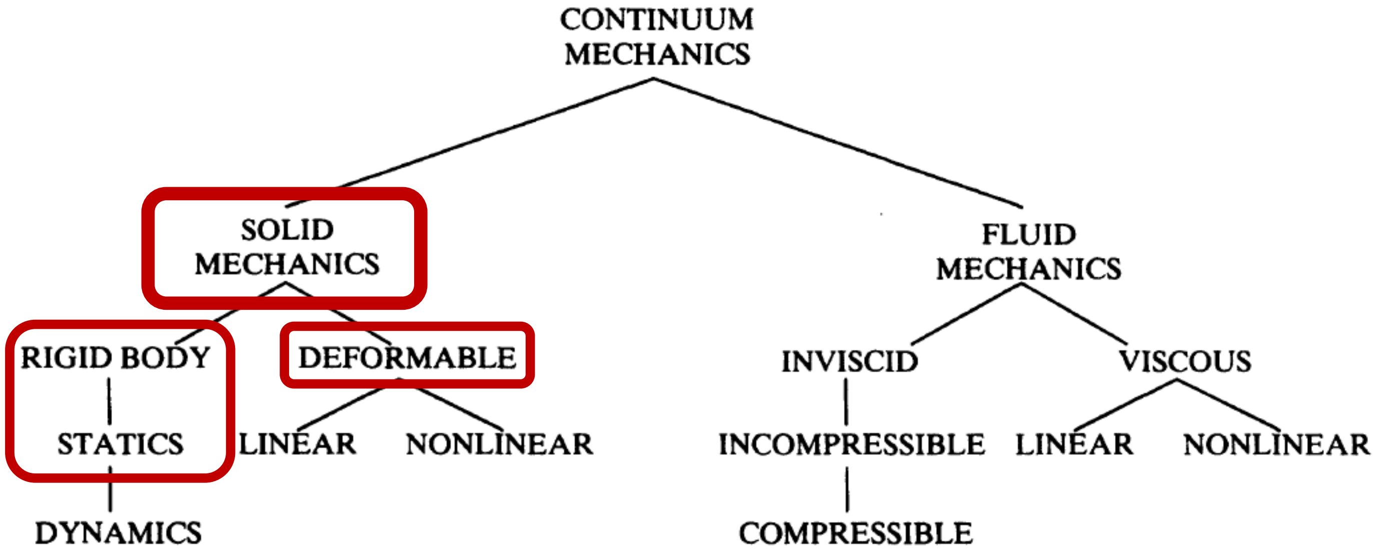


FIGURE 1.4 Flowchart of traditional divisions of study within continuum mechanics. Note that solid mechanics and fluid mechanics focus primarily on solidlike and fluidlike behaviors, not materials in their solid versus fluid/gaseous phases. Note, too, that linear and nonlinear refer to material behaviors, not the governing differential equations of motion. As we shall see in Chap. 11, many materials simultaneously exhibit solidlike (e.g., elastic) and fluidlike (e.g., viscous) behaviors, which gives rise to the study of viscoelasticity and the theory of mixtures, both of which are important areas within continuum biomechanics.

Review of Statics

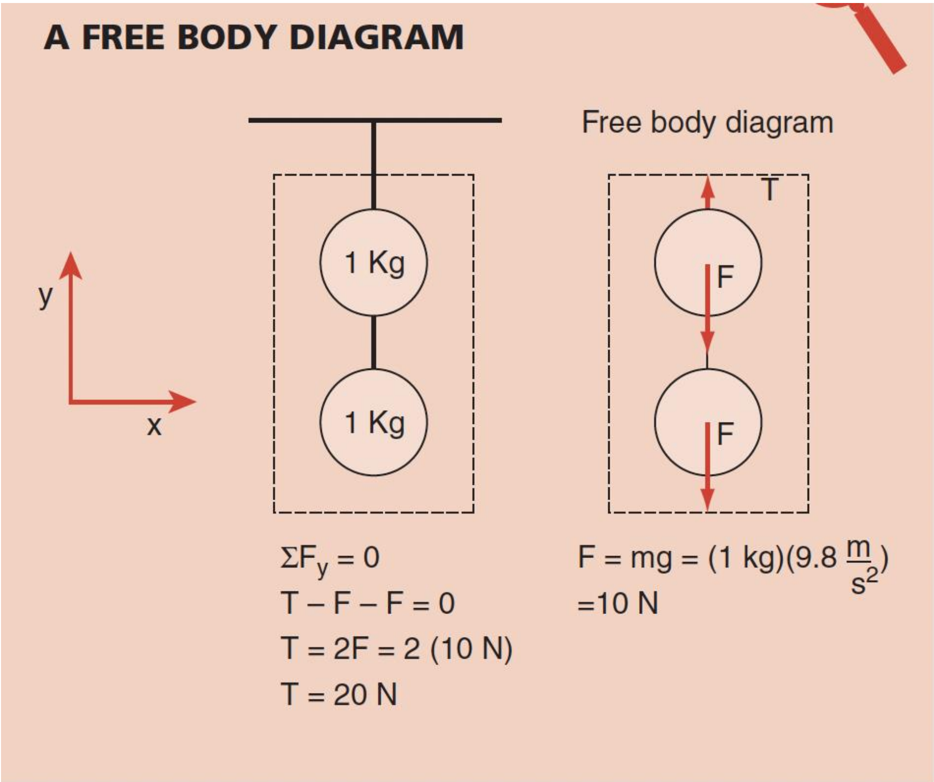
Statics is the study of the forces acting on a body at rest or moving with a constant velocity.

Static equilibrium

- If a body is at rest, there can be no unbalanced external forces acting on it. Therefore, the body is in static equilibrium.
- **All external forces acting on a body must add (in a vector sense) to zero.**
- An extension of this is that the sum of the external moments acting on that body must also be equal to zero for the body to be at rest.

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$

$$\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$$



References: Karduna A: Introduction to Biomechanical Analysis. In: *Kinesiology: Mechanics and Pathomechanics of Human Motion*, Edited by: Carol Oatis, Publisher: Lippincott Williams and Wilkins, 1st edition, 2003, 2nd edition, 2009

Review of Statics

Moment ($\overrightarrow{M}$) is typically caused by a force ($\vec{F}$) acting at a distance ($\vec{r}$) from the center of rotation of a segment.

Static equilibrium

- If a body is at rest, there can be no unbalanced external forces acting on it. Therefore, the body is in static equilibrium.
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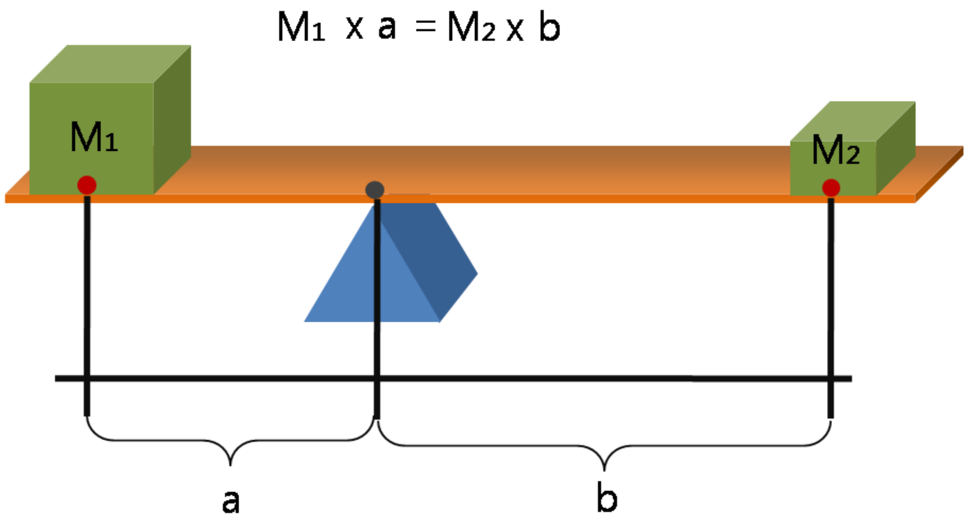
- **An extension of this is that the sum of the external moments acting on that body must also be equal to zero for the body to be at rest.**

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$

$$\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$$

A **moment** tends to cause a rotation and is defined by the cross product: $\vec{M} = \vec{r} \times \vec{F}$



A **lever** is a beam connected by a hinge, or pivot, called a fulcrum.

Reference:

1. **Karduna A**: Introduction to Biomechanical Analysis. In: *Kinesiology: Mechanics and Pathomechanics of Human Motion*, Edited by: Carol Oatis, Publisher: Lippincott Williams and Wilkins, 1st edition, 2003, 2nd edition, 2009
2. By Jjw - Own work, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=12872799

Force as a vector quantity

Reference: Özkaya, Nihat. Fundamentals of Biomechanics : Equilibrium, Motion, and Deformation. 3rd ed. New York: Springer, 2012.

Force may be defined as mechanical disturbance or load that may or may cause motion.

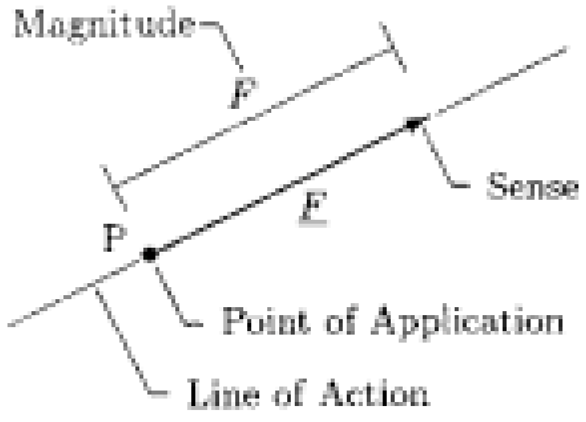
Forces are vector quantities and the principles of vector algebra must be applied to analyze problems involving Forces.

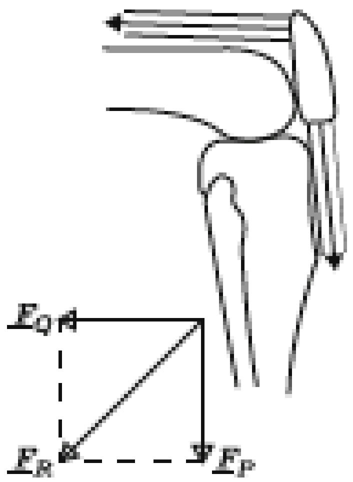
- Forces applied by the quadriceps $\vec{F}_Q$ and patellar tendon $\vec{F}_P$ on the patella are shown.
- The resultant force F_R on the patella due to the forces applied by the quadriceps and patellar tendon can be determined by considering the vector sum of these forces:

$$\vec{F}_R = \vec{F}_Q + \vec{F}_P$$

- If the magnitude of the resultant force needs to be calculated, then the Pythagorean theorem can be utilized:

$$F_R = \sqrt{(F_Q^2 + F_P^2)}$$





Force Systems: External and Internal Forces

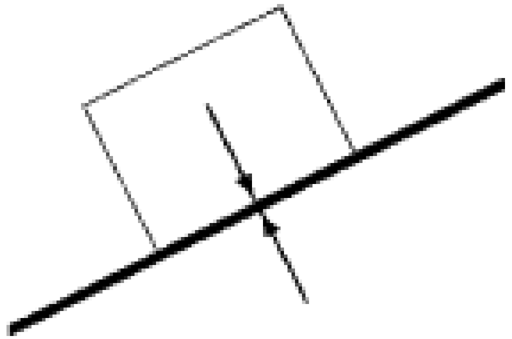
Reference: Özkaya, Nihat. Fundamentals of Biomechanics : Equilibrium, Motion, and Deformation. 3rd ed. New York: Springer, 2012.

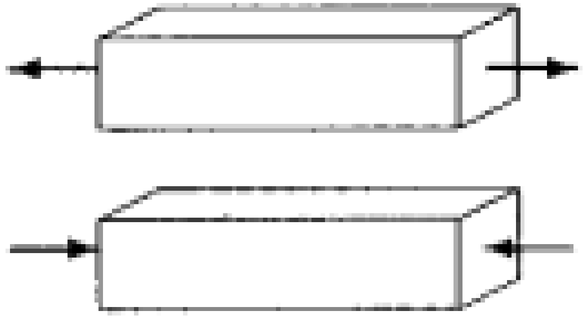
Any two or more forces acting on a single body form a force system.

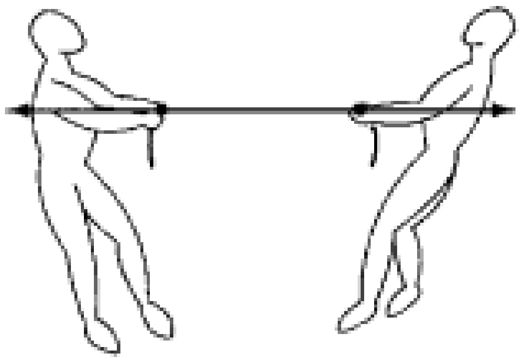
- A force may be broadly classified as **external** or **internal**.
- Almost all commonly known forces are **external forces**.
  - For example, when you push a cart, hammer a nail, sit on a chair, kick a football, or shoot a basketball, you apply an external force on the cart, nail, chair, football, or basketball.
- **Internal forces**, on the other hand, are the ones that hold a body together when the body is under the effect of externally applied forces.
  - For example, a piece of string does not necessarily break when it is pulled from both ends. When a rubber band is stretched, the band elongates to a certain extent.
  - **Forces generated by muscle contractions** are also internal forces. The significance and details of internal forces will be studied by introducing the concept of "**stress**"

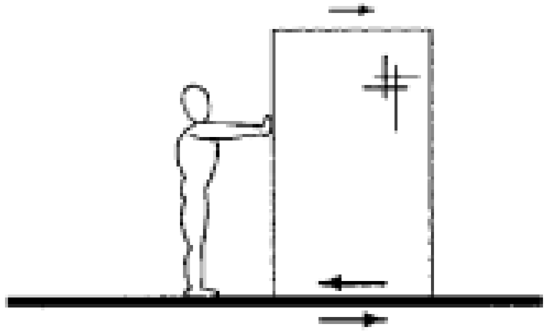
Identify the forces in the force systems

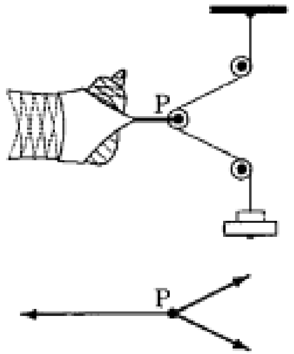
Reference: Özkaya, Nihat. Fundamentals of Biomechanics : Equilibrium, Motion, and Deformation. 3rd ed. New York: Springer, 2012.

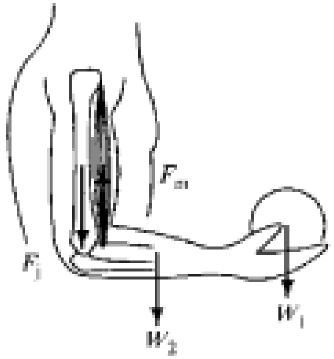




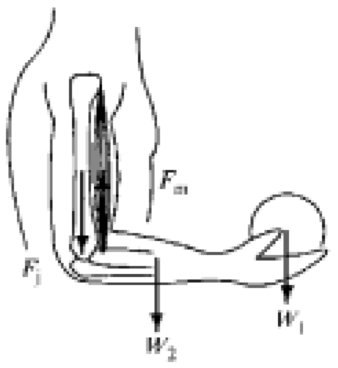


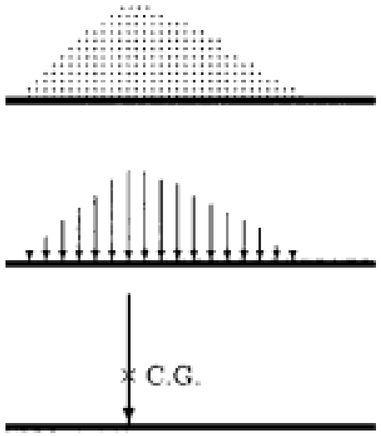


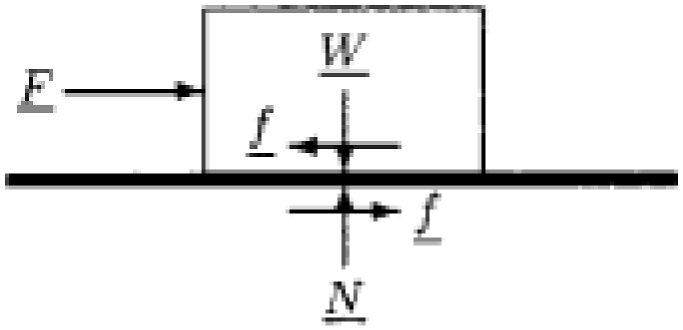


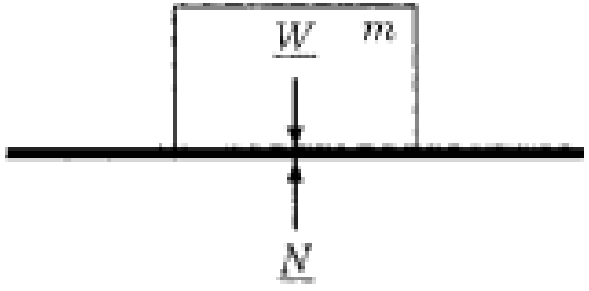


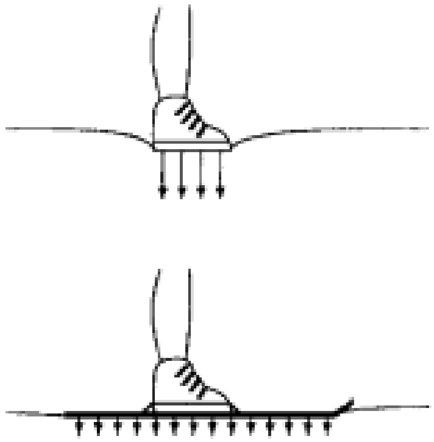
Describe the forces shown in this system

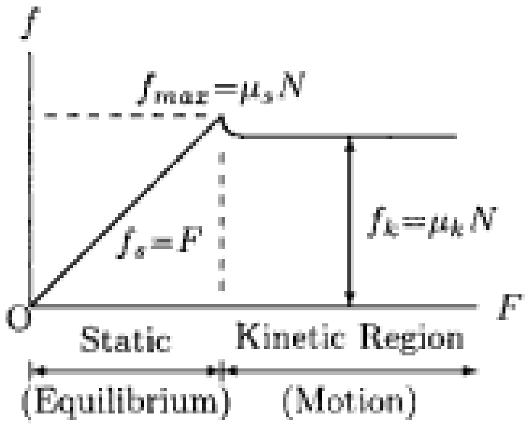










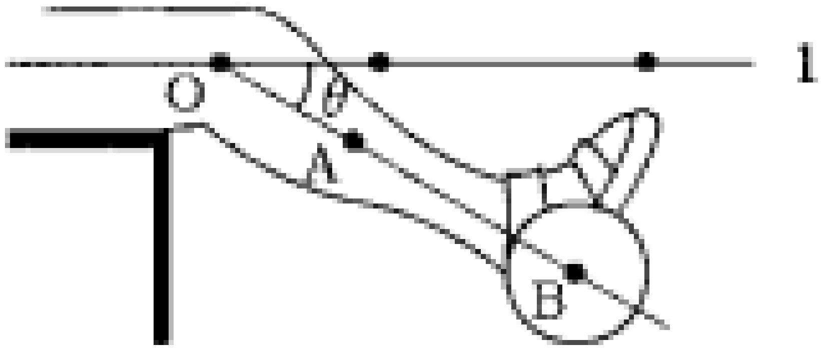


Reference: Özkaya, Nihat. Fundamentals of Biomechanics : Equilibrium, Motion, and Deformation. 3rd ed. New York: Springer, 2012.

Basic Biomechanics Problems: Force and Moment vectors

Worked out example

Consider an athlete wearing a weight boot, and from a sitting position, doing lower leg flexion/extension exercises to strengthen quadriceps muscles.

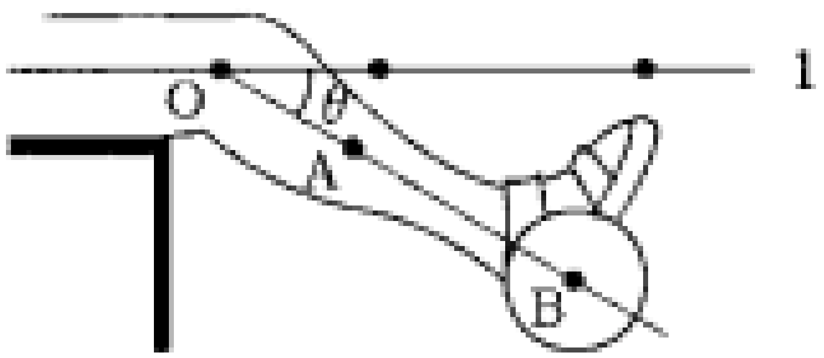


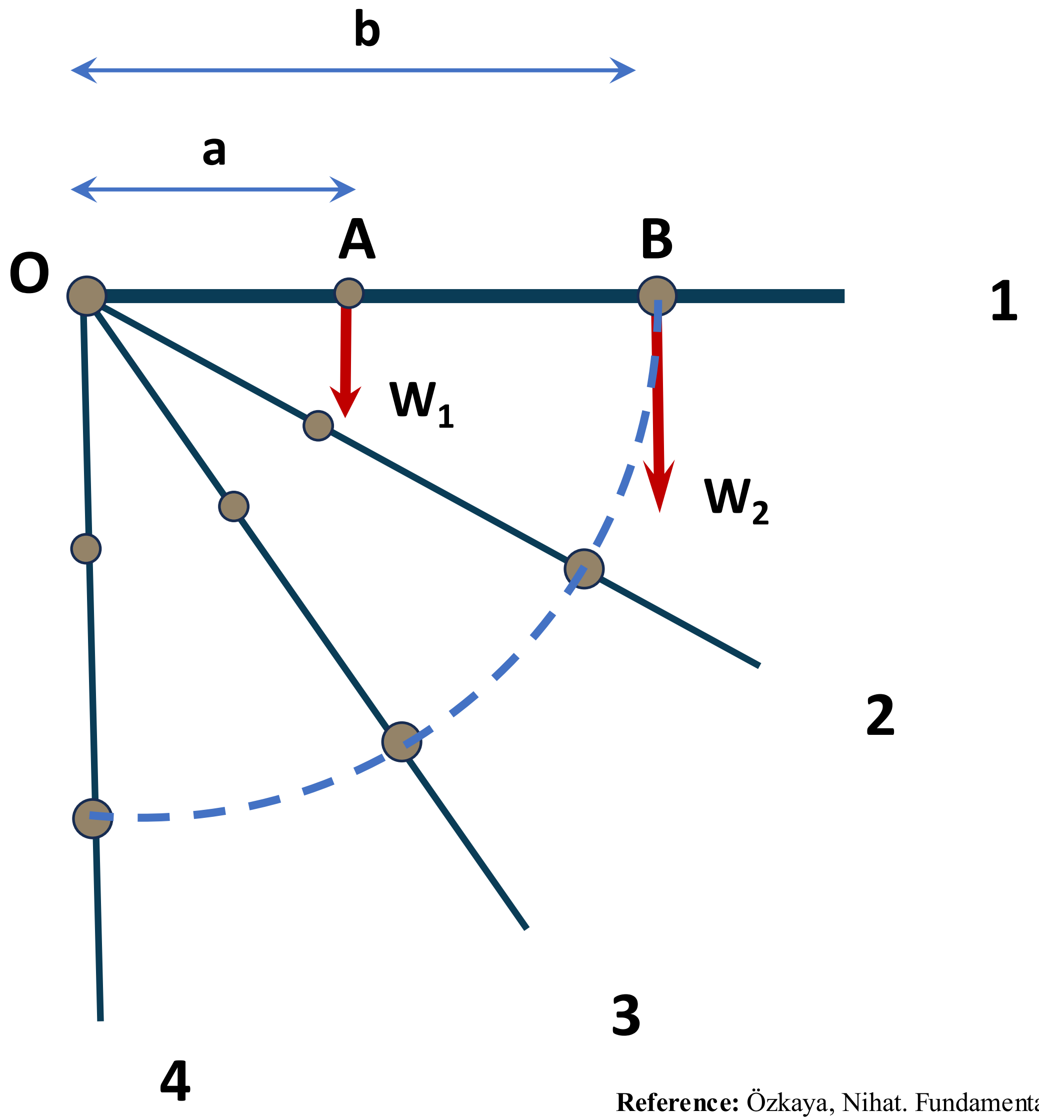
- The weight of the athlete's lower leg is $W_1 = 50\ N$ and the weight of the boot is $W_2 = 100\ N$.
- As measured from the knee joint at point O,
  - The center of gravity (point A) of the lower leg is located at a distance $a = 20\ cm$ and
  - The center of gravity (point B) of the weight boot is located at a distance $b = 50\ cm$.

Determine the net moment generated about the knee joint when the lower leg is

1. Extended horizontally (position 1), and
2. When the lower leg makes an angle of 30 degrees (position 2),
3. Angle of 60 degrees (position 3), and
4. Angle of 90 degrees (position 4) with the horizontal.

Reference: Özkaya, Nihat. Fundamentals of Biomechanics : Equilibrium, Motion, and Deformation. 3rd ed. New York: Springer, 2012.





Given:

$$W_1 = 50\ N$$

$$W_2 = 100\ N$$

$$a = 20\ cm$$

$$b = 50\ cm$$

Need to calculate:

The net moment generated about the knee joint at

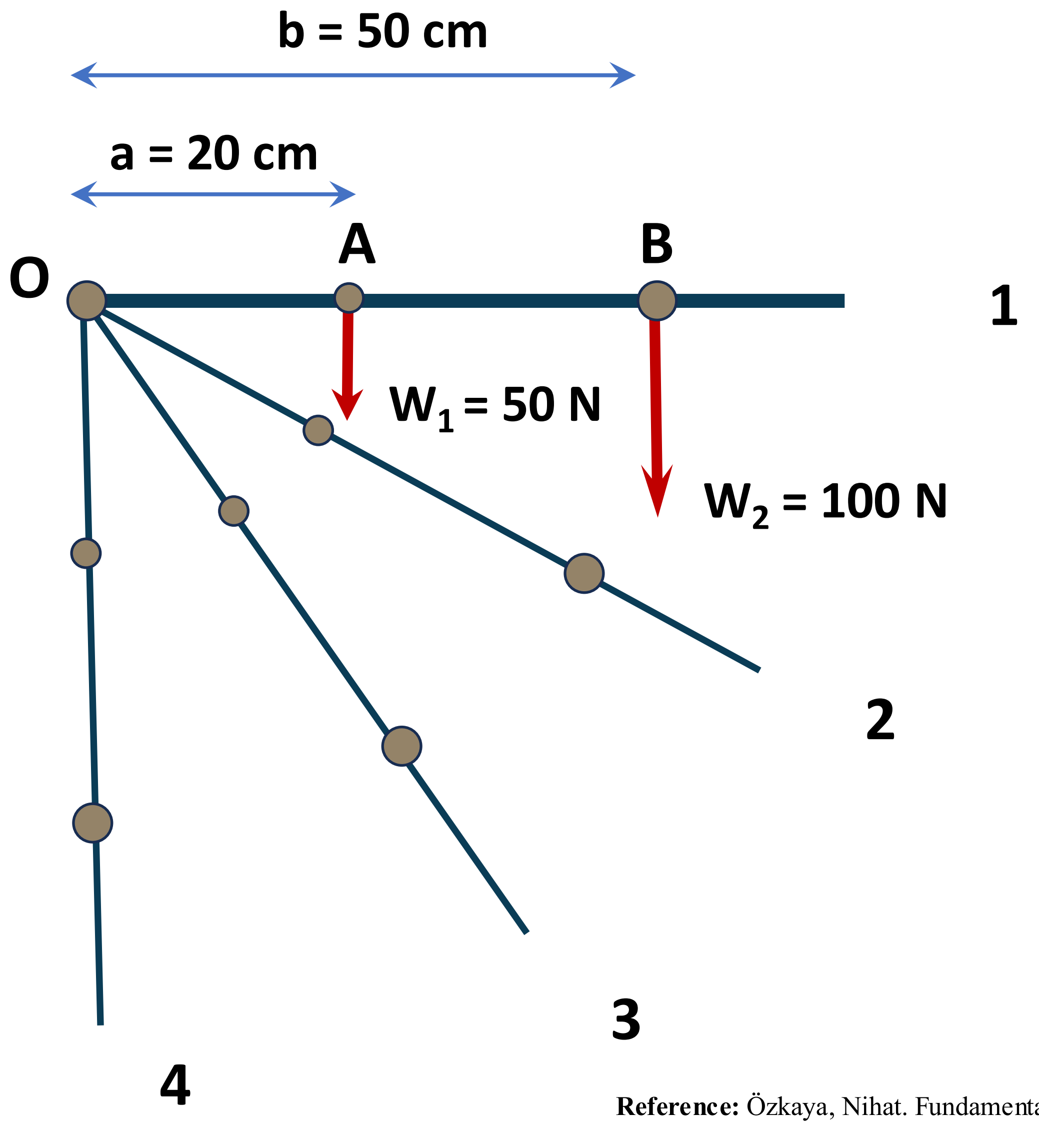
1. Position 1 ($\theta = 0\ degrees$)
2. Position 2 ($\theta = 30\ degrees$)
3. Position 3 ($\theta = 60\ degrees$)
4. Position 4 ($\theta = 90\ degrees$)

Reference: Özkaya, Nihat. Fundamentals of Biomechanics : Equilibrium, Motion, and Deformation. 3rd ed. New York: Springer, 2012.

At position 1 (θ = 0 degrees):

- The lower leg is extended horizontally and the long axis of the **leg is perpendicular to the lines of action of W_1 and W_2.**
- Therefore, **a** and **b** are the lengths of the moment arms for **W_1 and W_2**, respectively.

$$
\begin{aligned}
M_O &= a\,W_1 + bW_2 \\
&= (0.20)(50) + (0.50)(100) \\
&= 60\ Nm\ (clockwise)
\end{aligned}
$$

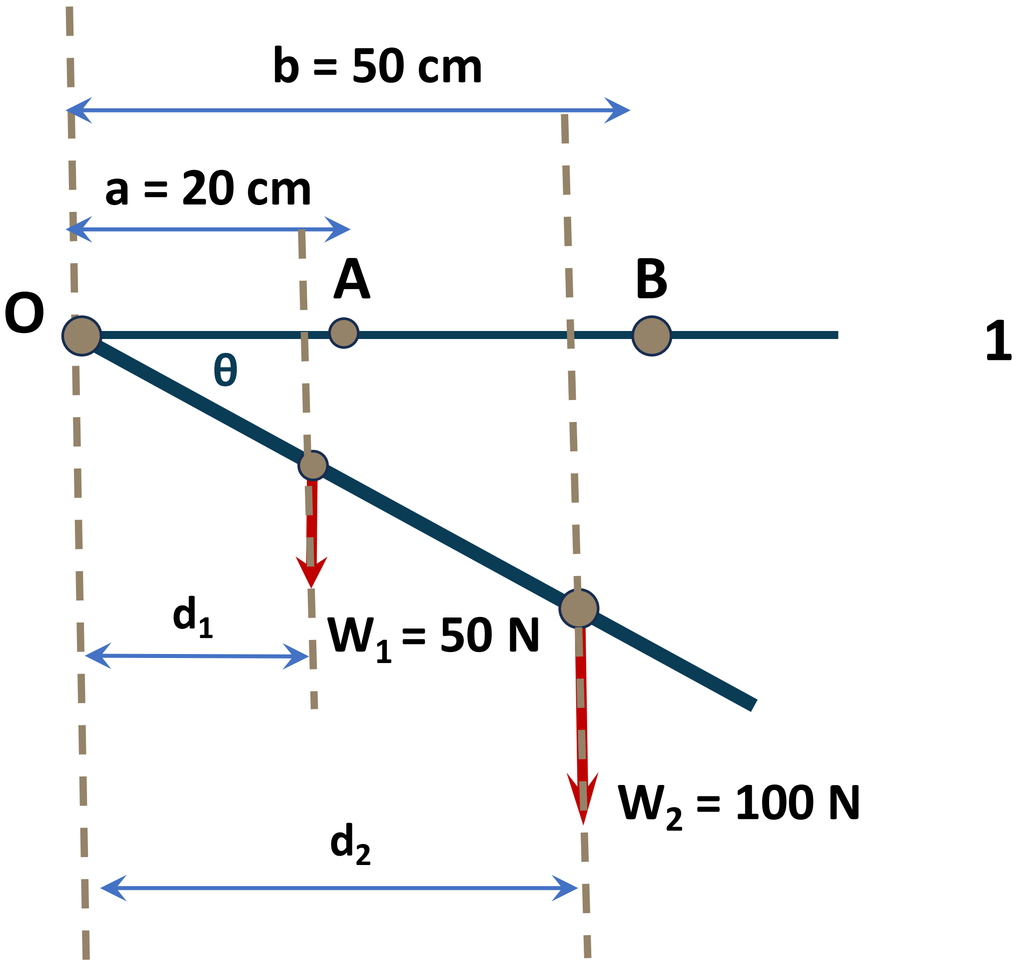


Reference: Özkaya, Nihat. Fundamentals of Biomechanics : Equilibrium, Motion, and Deformation. 3rd ed. New York: Springer, 2012.

At an arbitrary position with (θ degrees):

- The lower leg is making an angle **θ degrees with the** horizontal.
- Forces, **W_1 and W_2** will act at distances **d_1** and **d_2**, respectively.
  - **d_1** and **d_2** are the lengths of the moment arms for **W_1 and W_2**, respectively, at **θ degrees** .

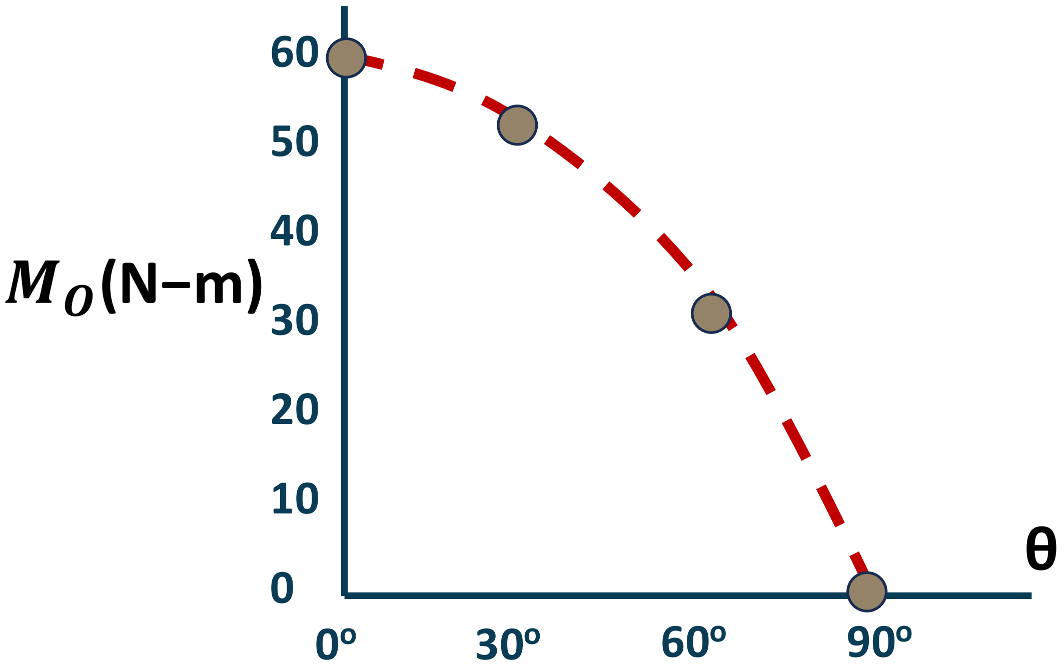
$$
\begin{aligned}
M_O &= d_1\, W_1 + d_2 W_2 \\
&= (a\cos\theta)(W_1) + (b\cos\theta)(W_2) \\
&= (aW_1 + bW_2)\cos\theta \\
&= 60\cos\theta \text{ Nm}
\end{aligned}
$$

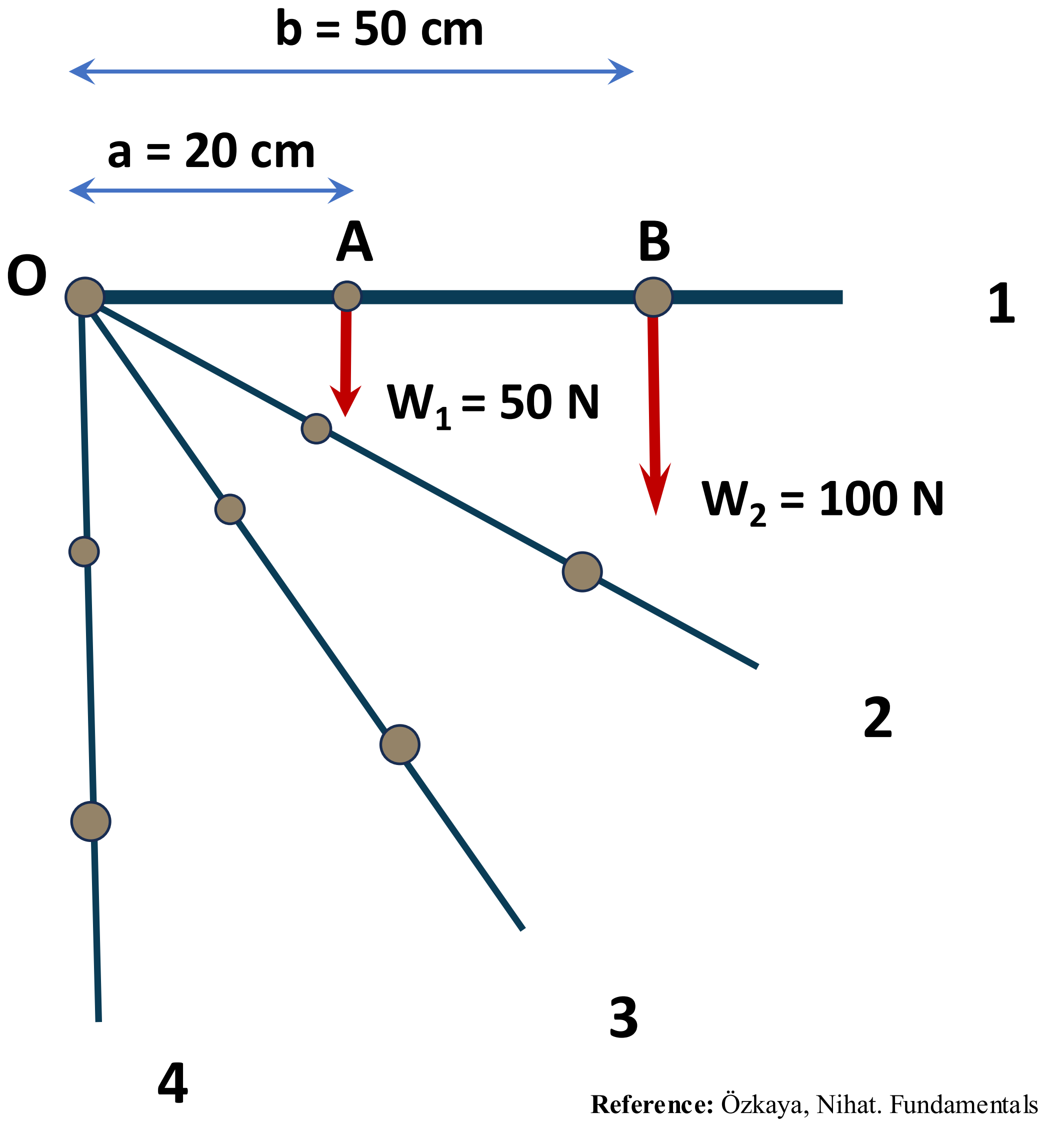


Reference: Özkaya, Nihat. Fundamentals of Biomechanics : Equilibrium, Motion, and Deformation. 3rd ed. New York: Springer, 2012.

$$M_O = 60\cos\theta$$

For position 1	Θ = 0°	$M_O = 60\ Nm\ (\mathbf{cw})$
For position 2	Θ = 30°	$M_O = 52\ Nm\ (\mathbf{cw})$
For position 3	Θ = 60°	$M_O = 30\ Nm\ (\mathbf{cw})$
For position 4	Θ = 90°	$M_O = 0\ Nm\ (\mathbf{cw})$





Reference: Özkaya, Nihat. Fundamentals of Biomechanics : Equilibrium, Motion, and Deformation. 3rd ed. New York: Springer, 2012.

Graded in-class problems

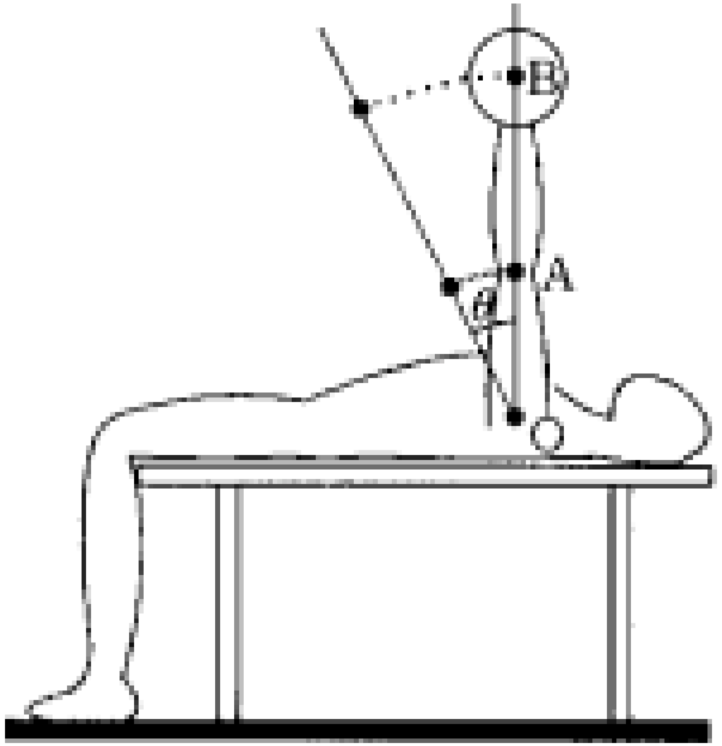
Problem 1

Reference: Özkaya, Nihat. Fundamentals of Biomechanics : Equilibrium, Motion, and Deformation. 3rd ed. New York: Springer, 2012.

Consider an athlete doing shoulder muscle strengthening exercises by lowering and raising a barbell with straight arms.

The position of the arms when they make an angle with the vertical is simplified and shown.

- Point O represents the shoulder joint
- Point A is the center of gravity of one arm
- Point B is a point of intersection of the centerline of the barbell and the extension of line OA.



The distance between

- Points O and A is a = 24 cm and
- Points O and B is b = 60 cm.

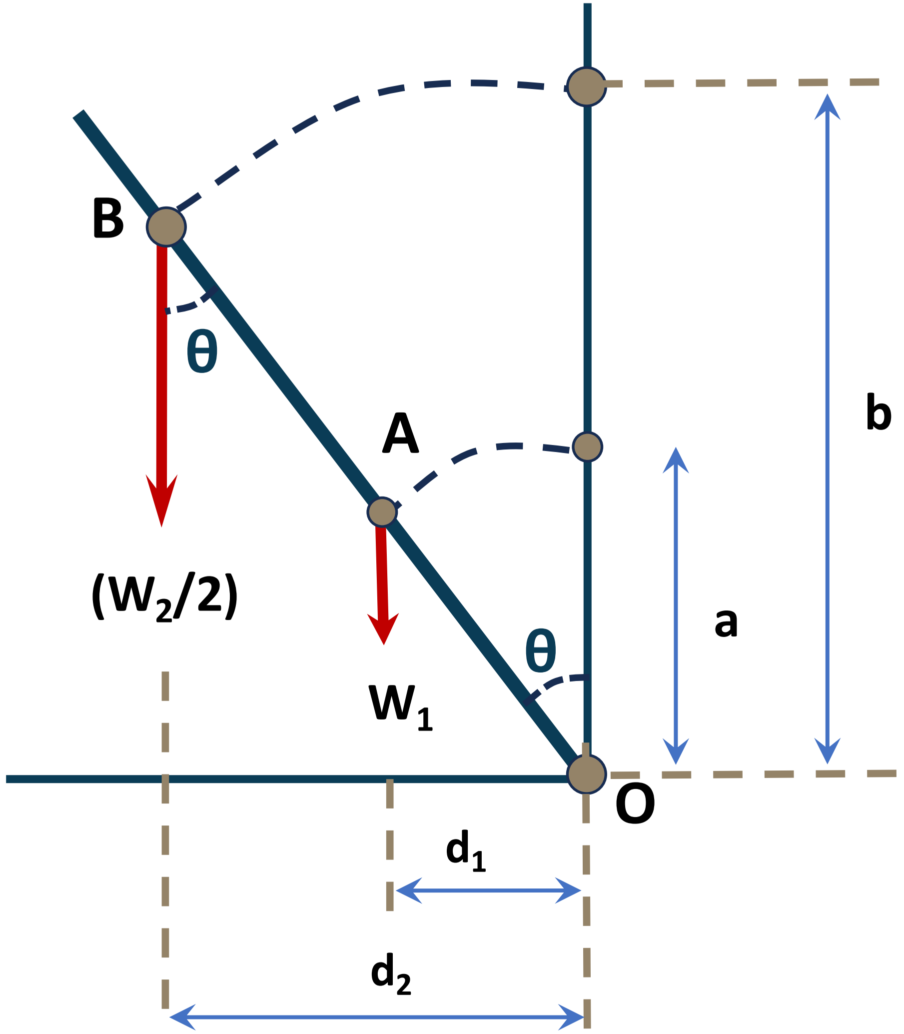
Each arm weighs W_1=50 N

The total weight of the barbell is W_2 = 300 N.

Determine the net moment due to W_1 and W_2 about the shoulder point as a function of θ which is the angle the arm makes with the vertical. Calculate the moments for θ = 0^0, 15^0, 30^0, 45^0, and 60^0.

Hint:

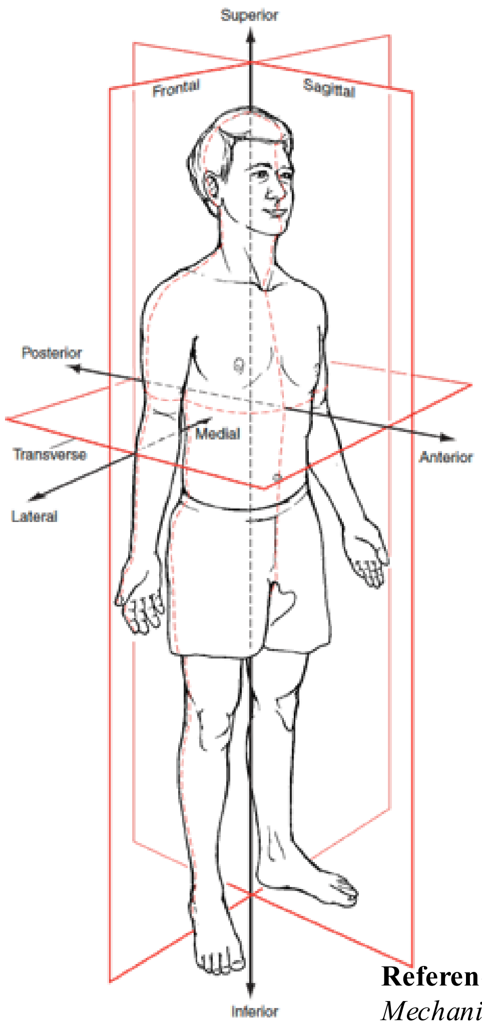
Since the athlete is using both arms, the total weight of the barbell is assumed to be shared equally by each arm.



Building blocks of the Musculoskeletal System

Anatomical position surrounded by three cardinal planes used as reference frames in anatomy and movement analysis:

1. Sagittal,
2. Frontal, and
3. Transverse planes



Planes shown

- Sagittal plane: A vertical plane dividing the body into left and right portions, drawn front to back through the body.
- Frontal (coronal) plane: A vertical plane dividing the body into anterior (front) and posterior (back) portions, drawn side to side.
- Transverse plane: A horizontal plane dividing the body into superior (upper) and inferior (lower) parts at about waist level.

Directional terms labeled

- Superior and inferior arrows indicate toward the head and toward the feet along the body's long axis.
- Anterior/posterior and medial/lateral labels show front vs back and toward vs away from the midline, helping orient structures within these planes.

Reference: Karduna A: Introduction to Biomechanical Analysis. In: *Kinesiology: Mechanics and Pathomechanics of Human Motion*, Edited by: Carol Oatis, Publisher: Lippincott Williams and Wilkins, 1st edition, 2003, 2nd edition, 2009

Bones

- Rigid organs made mostly of mineralized tissue that give the body **structure** and protect internal organs.
- At joints (like the elbow) their shaped surfaces articulate with one another to allow movement. Cartilage covers these surfaces to reduce friction.

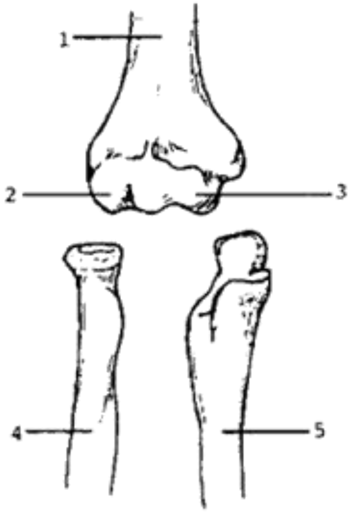


Fig. 5.2 *Bones of the elbow: (1) humerus, (2) capitulum, (3) trochlea, (4) radius, (5) ulna*

Ligaments

- Strong bands or sheets of fibrous connective tissue that connect one bone to another across a joint.
- They stabilize joints by limiting excessive motion and helping maintain proper alignment of the bones.

References:

1. [image source] Özkaya, Nihat. Fundamentals of Biomechanics : Equilibrium, Motion, and Deformation. 3rd ed. New York: Springer, 2012.
2. https://www.kenhub.com/en/library/anatomy/elbow-joint
3. https://my.clevelandclinic.org/health/body/elbow-joint
4. https://www.patrickjostmd.com/normal-anatomy-of-the-elbow.html
5. https://www.jamesrbaileymd.com/elbow-anatomy-orthopedic-surgeon-santa-barbara-ca/
6. https://www.drthorsness.com/elbow-shoulder-elbow-sports-surgeon-joliet-new-lenox-il.html

Muscles

- Soft, contractile tissues made of fibers that can shorten (contract) and lengthen (relax).
- By contracting, muscles pull on tendons and bones to create voluntary movements (like bending the elbow) and also help maintain posture and joint stability.

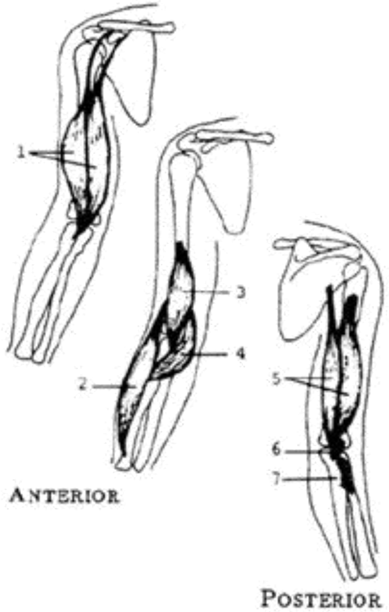


Fig. 5.3 *Muscles of the elbow: (1) biceps, (2) brachioradialis, (3) brachialis, (4) pronator teres, (5) triceps brachii, (6) anconeus, (7) supinator*

Tendons

- Tough, cord-like connective tissues that attach muscle to bone.
- When a muscle contracts, the tendon transmits that force to the bone, producing movement at the joint.

References:

1. Özkaya, Nihat. Fundamentals of Biomechanics : Equilibrium, Motion, and Deformation. 3rd ed. New York: Springer, 2012.
2. https://www.kenhub.com/en/library/anatomy/elbow-joint
3. https://my.clevelandclinic.org/health/body/elbow-joint
4. https://www.patrickjostmd.com/normal-anatomy-of-the-elbow.html
5. https://www.jamesrbaileymd.com/elbow-anatomy-orthopedic-surgeon-santa-barbara-ca/
6. https://www.drthorsness.com/elbow-shoulder-elbow-sports-surgeon-joliet-new-lenox-il.html

Mechanics of the Elbow: Musculoskeltal Structure and Movements

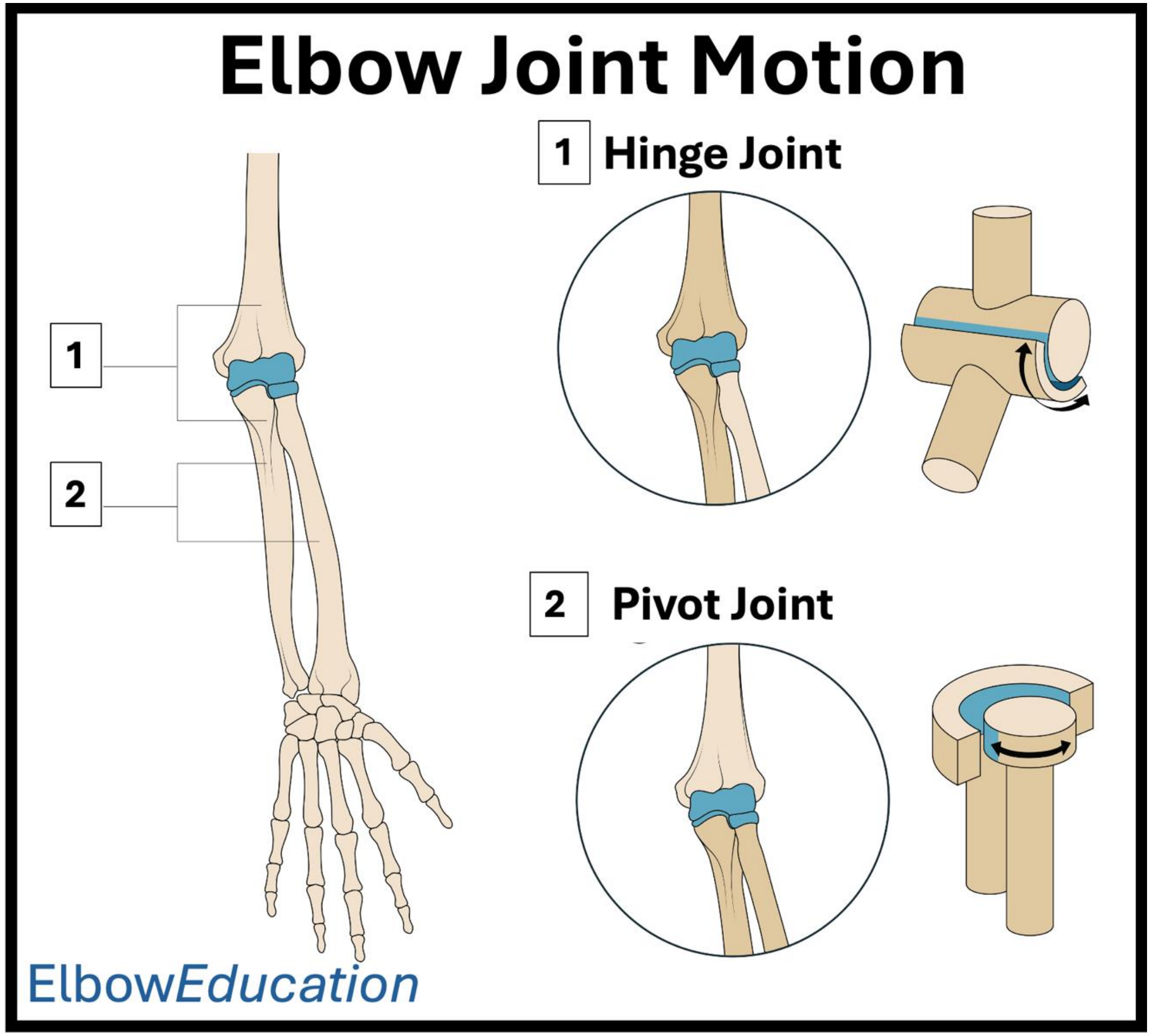
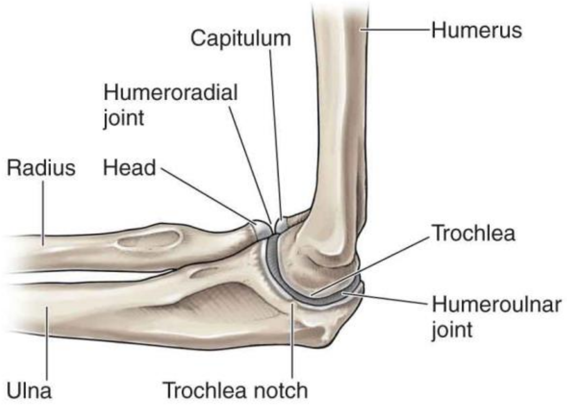


Image source: https://orthoeducation.com/elboweducation/anatomy-of-the-elbow/

Elbow Joint Structure (1/3 Articulations)

Name	Description	Mechanical Equivalent
Humeroulnar Joint	• Trochlea of humerus articulates with trochlear fossa of ulna • Allows only flexion (forearm toward upper arm) and extension (forearm away from upper arm)	Hinge



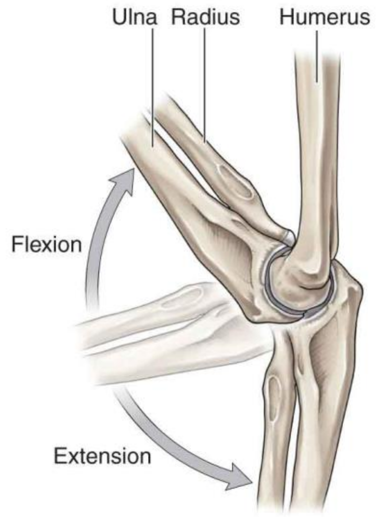


Image source: https://musculoskeletalkey.com/elbow-10/

Elbow Joint Structure (2/3 Articulations)		
Name	**Description**	**Mechanical Equivalent**
Humeroradial Joint	• Capitulum of humerus articulates with radius head • Supports forearm flexion and extension	Hinge

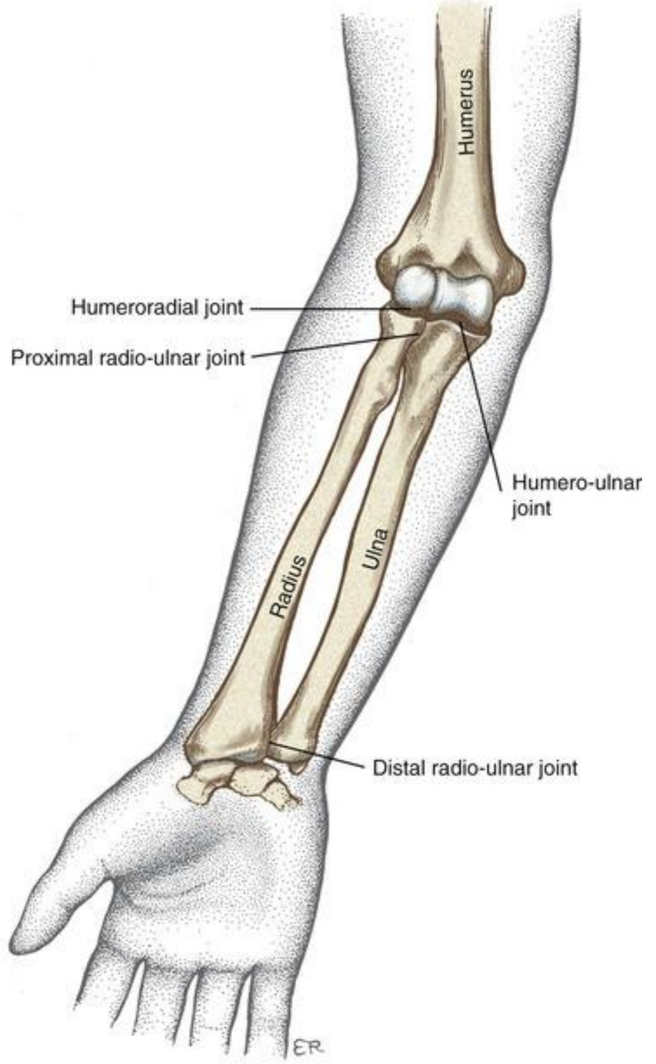
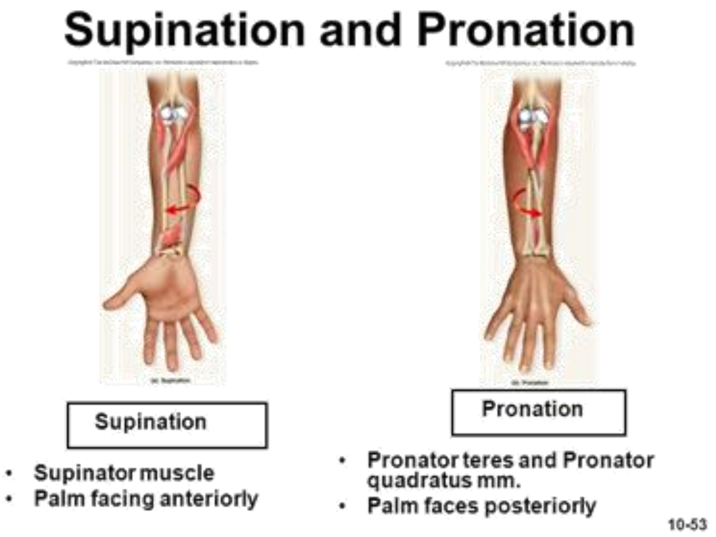


Image source: https://clinicalgate.com/elbow-and-forearm-3/

Elbow Joint Structure (3/3 Articulations)		
Name	**Description**	**Mechanical Equivalent**
Proximal Radioulnar Joint	• Allows radius and ulna to rotate relative to each other • Enables pronation (palm down) and supination (palm up)	Pivot



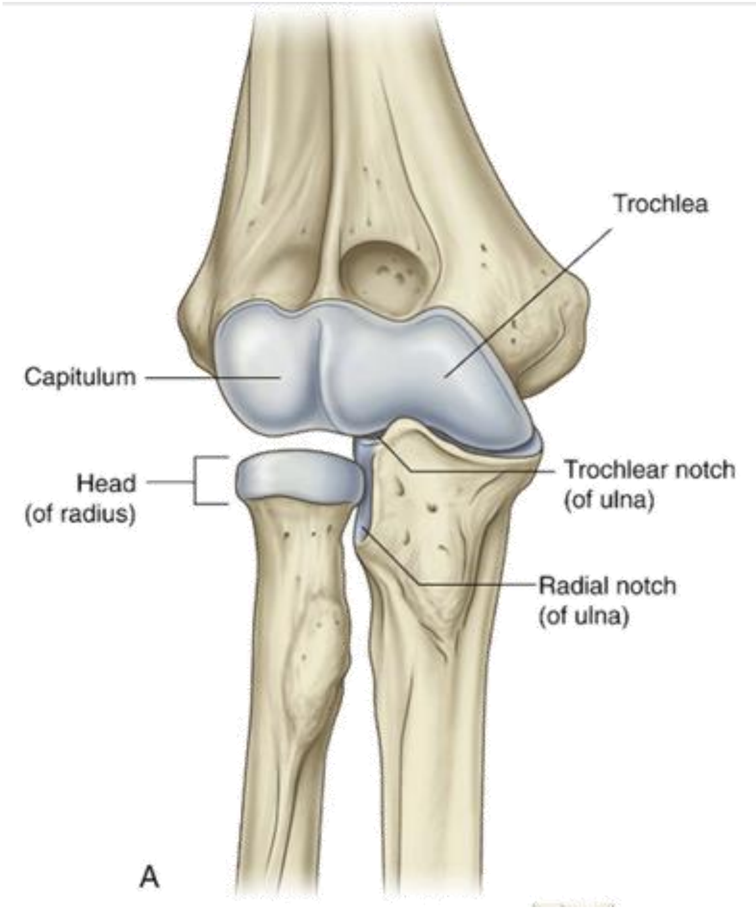


Image source: https://www.mickeymed.com/article/proximalradioulnarjoint

Major Elbow Muscles

- **Flexors (Bending the Elbow)**
  - **Biceps brachii**: Powerful flexor; best when forearm is supinated; strongest supinator
  - **Brachialis**: Strongest overall flexor; works regardless of forearm position (doesn't attach to radius)
- **Extensor (Straightening the Elbow)**
  - **Triceps brachii**: Primary extensor; largest muscle group at elbow
- **Rotation Muscles**
  - **Pronator teres** — Rotates forearm to palm-down position
  - **Supinator** — Rotates forearm to palm-up position

These muscles attach via tendons (like the biceps and triceps tendons) to the bones (humerus, radius, ulna) and work in opposing pairs: flexors bend, extensors straighten, and pronators/supinators rotate the forearm, all coordinated by ligaments that stabilize the joint.

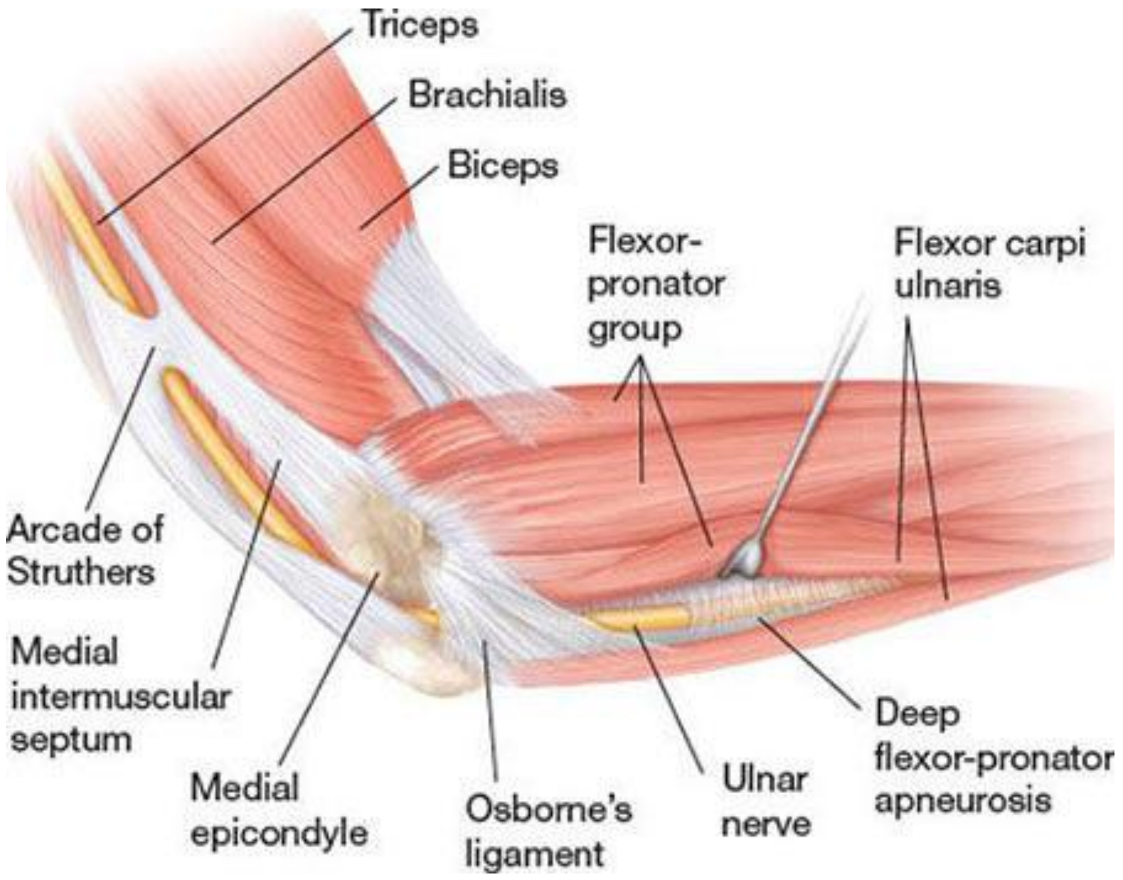
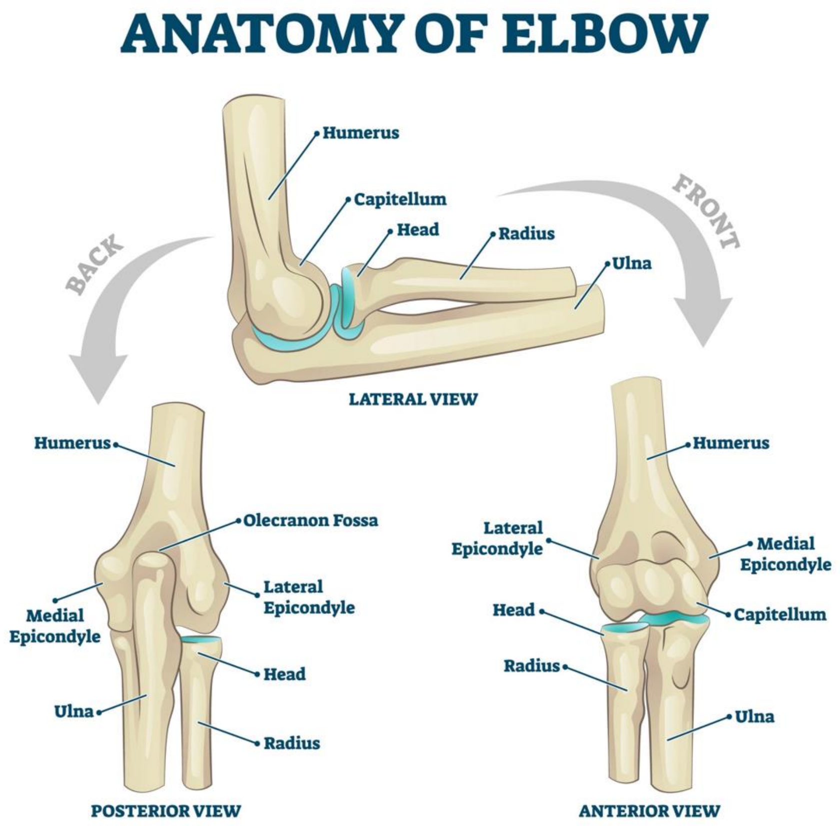


Image Source: https://www.sportspainmanagementnyc.com/elbow-injuries/

Common Elbow Injuries

- Fractures at epicondyles of humerus and olecranon process of ulna
- Overuse injuries: Tennis elbow and golfer's elbow (inflammatory tendon damage from repetitive motions)



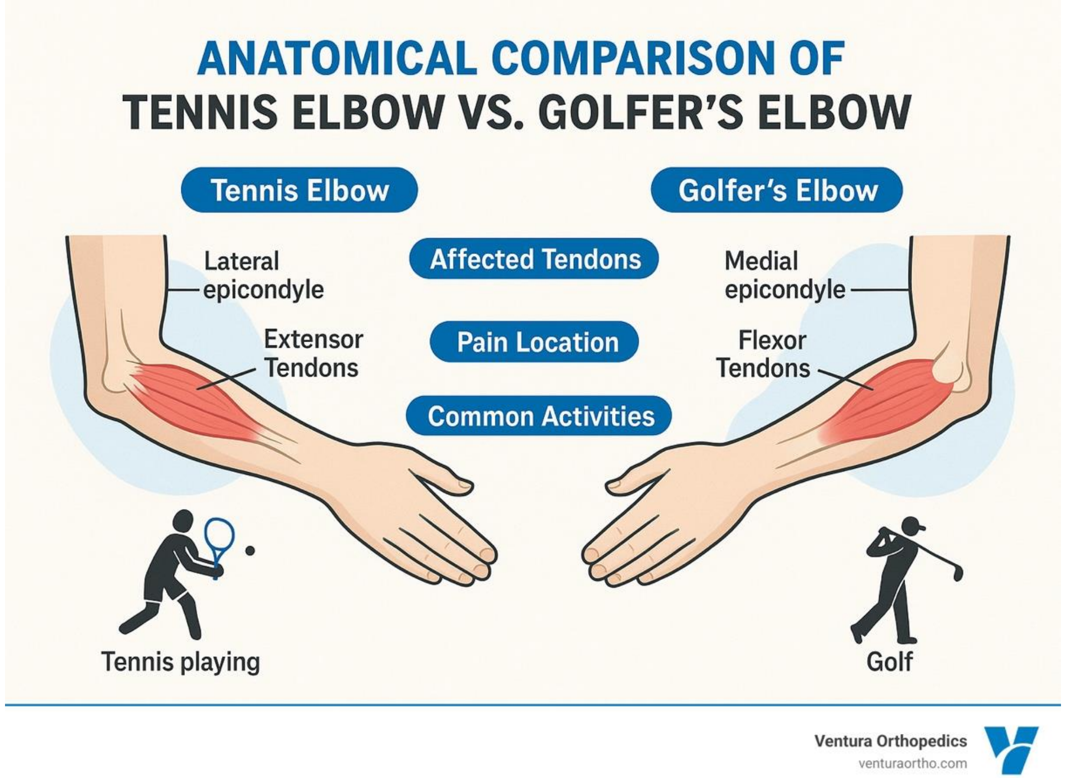


Image source:
https://physioflowpt.com/adult-elbow-fractures/
https://venturaortho.com/golfer-and-tennis-elbow-when-is-surgery-recommended/

Force Systems in Biomechanics: External and Internal Forces

Reference: Özkaya, Nihat. Fundamentals of Biomechanics : Equilibrium, Motion, and Deformation. 3rd ed. New York: Springer, 2012.

- A force may be broadly classified as **external** or **internal**.
  - **External forces:** gravity on the body or its segments, manual or machine forces during exercise/stretching, and forces from prostheses or implements.
  - **Internal forces:** produced by muscles, ligaments, tendons, and joint interactions.
- **Main unknowns in static musculoskeletal problems**
  - Joint reaction forces.
  - Muscle forces.
- **Information needed for joint mechanical analysis**
  - Direction and magnitude (vector characteristics) of muscle tension and exact muscle attachment sites.
  - Mass/weight of each body segment and their centers of gravity.
  - Anatomical axis of rotation for the joint being analyzed.

To apply the principles of statics to analyze the mechanics of human joints, we shall adopt the following assumptions and limitations:

Reference: Özkaya, Nihat. Fundamentals of Biomechanics : Equilibrium, Motion, and Deformation. 3rd ed. New York: Springer, 2012.

- The anatomical axes of rotation of joints are known.
- The locations of muscle attachments are known.
- The line of action of muscle force is known.
- Segmental weights and their centers of gravity are known.
- Frictional factors at the joints are negligible.
- Dynamic aspects of the problems will be ignored.
- Only two-dimensional problems will be considered.

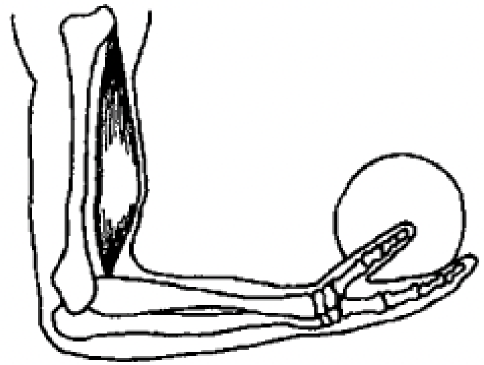
Graded in-class problems

Problem 2

Reference: Özkaya, Nihat. Fundamentals of Biomechanics : Equilibrium, Motion, and Deformation. 3rd ed. New York: Springer, 2012.

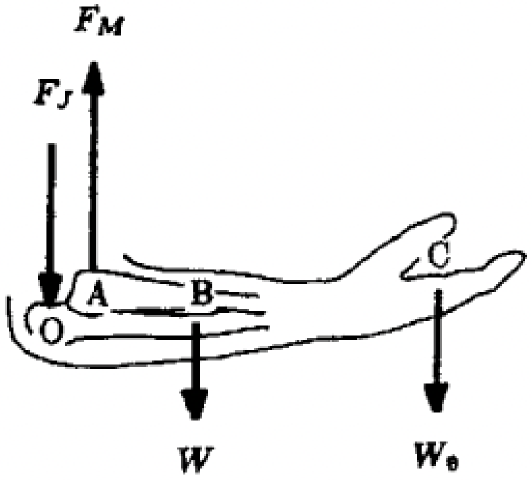
Consider the arm shown in the figure.

- The elbow is flexed to a right angle and an object is held in the hand.
- The forces acting on the forearm are shown in this figure.



The free-body diagram of the forearm is shown on a mechanical model in this figure.

- This model assumes that the biceps is the major flexor and that the line of action (of force vector) in the biceps is vertical.
- Assume that the parameters are given as follows:
  - Distance OA: a = 4 cm,
  - Distance OB: b = 15 cm,
  - Distance OC: c = 35 cm
  - W = 20 N
  - W_O = 80 N

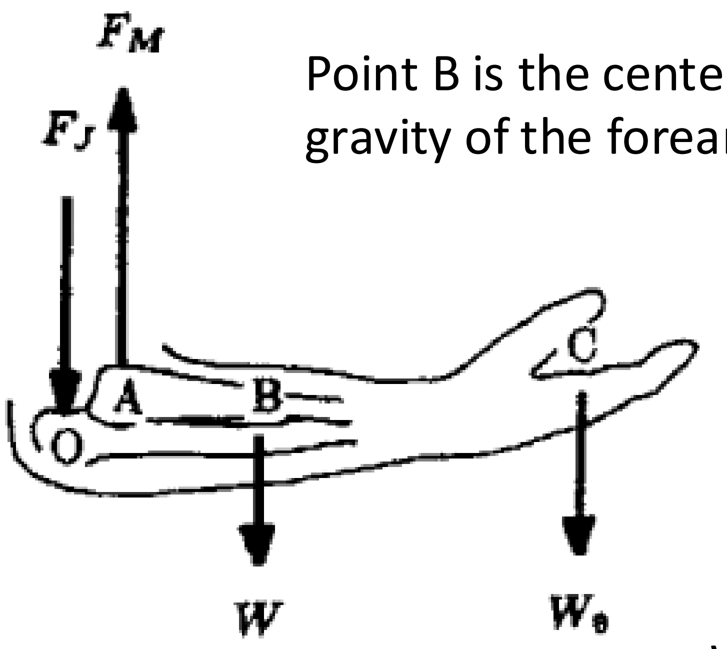


Determine the magnitudes of the muscle force, and the joint reaction force at the elbow.

F_M is the magnitude of the force exerted by the biceps on the radius

F_J is the magnitude of the reaction force at the elbow joint.

Point B is the center of gravity of the forearm



Point C is a point on the forearm that lies along a vertical line passing through the center of gravity of the weight in the hand.

Point O designates the axis of rotation of the elbow joint, which is assumed to be fixed for practical purposes.

W_O is the weight of the object held in the hand

W is the total weight of the forearm.

Point A is the attachment of the biceps muscle on the radius

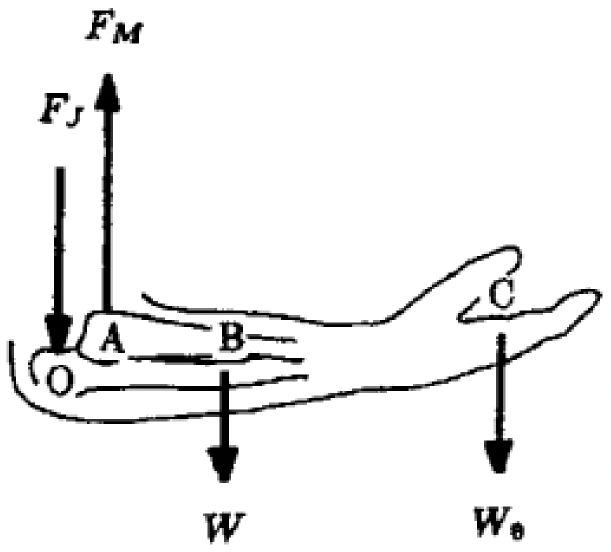
Determine the magnitudes of the muscle force, and the joint reaction force at the elbow.

Reference: Özkaya, Nihat. Fundamentals of Biomechanics : Equilibrium, Motion, and Deformation. 3rd ed. New York: Springer, 2012.

Step 0: State the assumptions and identify the unknowns

Reference: Özkaya, Nihat. Fundamentals of Biomechanics : Equilibrium, Motion, and Deformation. 3rd ed. New York: Springer, 2012.

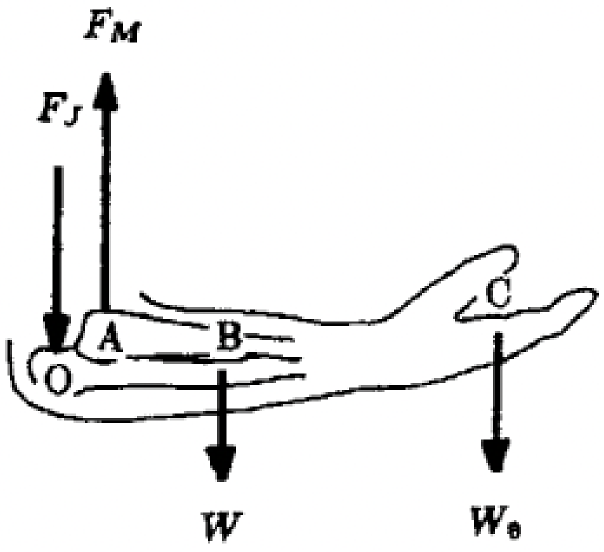
- ✓ The anatomical axes of rotation of joints are known.
- ✓ The locations of muscle attachments are known.
- ✓ The line of action of muscle force is known.
- ✓ Segmental weights and their centers of gravity are known.
- ✓ Frictional factors at the joints are negligible.
- ✓ Dynamic aspects of the problems will be ignored.
- ✓ Only two-dimensional problems will be considered.

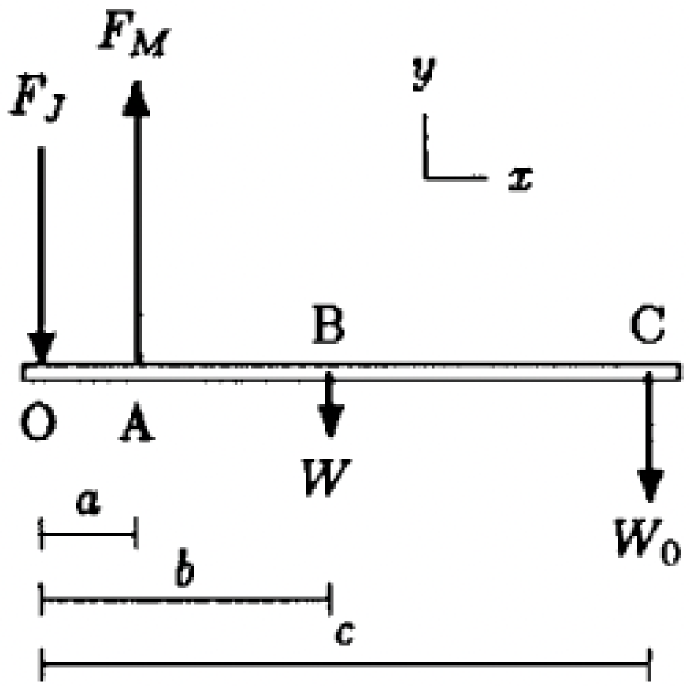


The unknowns in the problem are the magnitudes and directions of F_M and F_J ; the muscle and joint reaction forces, respectively.

Step 1: Draw a free body diagram

Reference: Özkaya, Nihat. Fundamentals of Biomechanics : Equilibrium, Motion, and Deformation. 3rd ed. New York: Springer, 2012.





Note:
1. The distances between point O and points A, B, and C are measured as a, b, and c, respectively.
2. The line of action of the muscle force is assumed to be vertical.
3. The gravitational forces are vertical as well. Therefore, for the equilibrium of the lower arm, the line of action of the joint reaction force must also be vertical (a parallel force system).

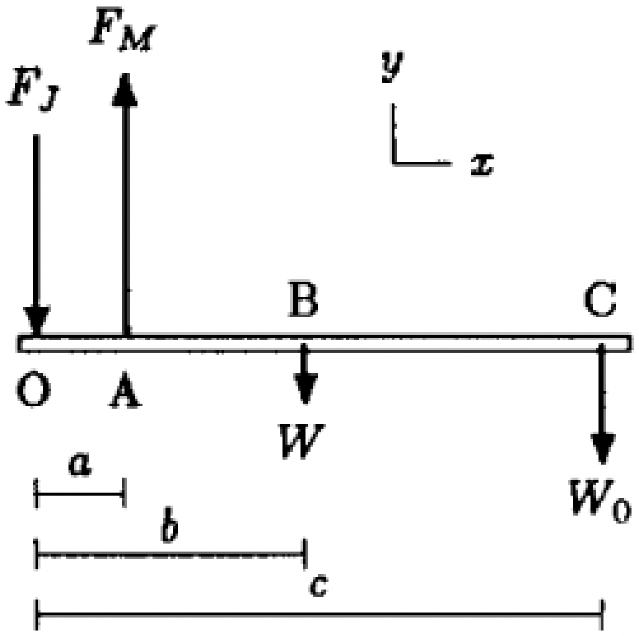
Step 2: Apply Static Equilibrium Conditions

Reference: Özkaya, Nihat. Fundamentals of Biomechanics : Equilibrium, Motion, and Deformation. 3rd ed. New York: Springer, 2012.

Recall: Static equilibrium

- If a body is at rest, there can be no unbalanced external forces acting on it. Therefore, the body is in static equilibrium.
- **All external forces acting on a body must add (in a vector sense) to zero.**
- An extension of this is that the sum of the external moments acting on that body must also be equal to zero for the body to be at rest.

$$\sum F_y = 0 \qquad \sum M_O = 0$$

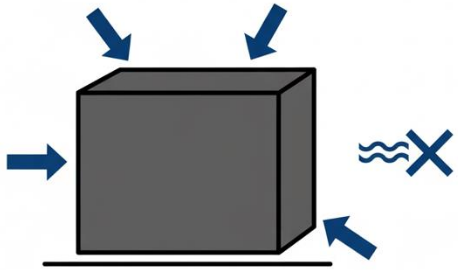


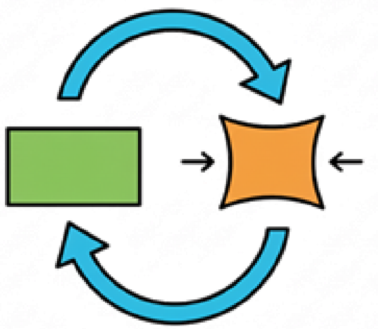
Concept of Simple Stress and Simple Strain

Idea of Deformation: Departure from Rigid bodies

- An object with no constraints moves in the **same direction** as an external force applied to it.
- If motion in that direction is constrained, the object **deforms** instead of freely moving.
- Deformation means there is **relative displacement between points** within the object.
- The **amount of deformation** depends on:
  - Magnitude, direction, and duration of the applied force.
  - Material properties (how stiff or flexible it is).
  - Geometry or shape of the object.
  - Environmental conditions such as temperature and humidity.

Rigid body	Deformable body
Distances between any two points in the body are assumed to remain **constant**, no matter what forces act on it.	Distances between points can **change** when forces are applied, so the body can stretch, compress, bend, or shear
Used mainly in basic statics and dynamics, where small real deformations are neglected to simplify analysis.	Studied in strength of materials/solid mechanics, where stresses, strains, and material properties (stiffness, failure) are important.

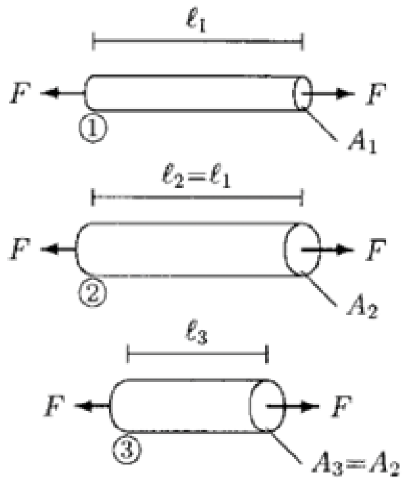




[Image source: Perplexity.io]

Thought experiment

Reference: Özkaya, Nihat. Fundamentals of Biomechanics : Equilibrium, Motion, and Deformation. 3rd ed. New York: Springer, 2012.



Specimen Comparison

Three bars made of the same material are tested under uniaxial tension with different geometric parameters.

- Bar 1 & 2: same length but different cross-sectional areas ($A_1 \neq A_2$).
- Bar 2 & 3: same cross-sectional area but different lengths ($\ell_2 \neq \ell_3$).

Normalized Representation

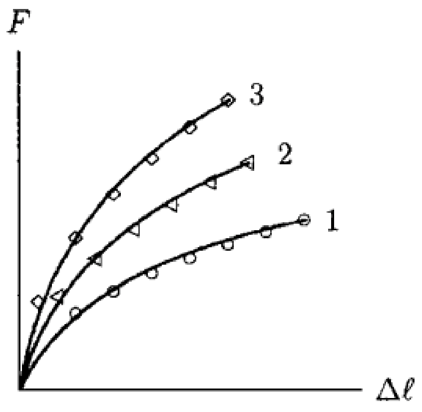
Stress-Strain Diagram

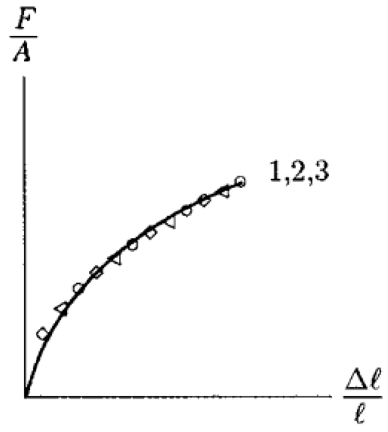
- Divides force by cross-sectional area:
  - **F/A (stress)**.
- Divides elongation by original length:
  - **Δℓ/ℓ (strain)**.
- **Single curve** represents material properties independent of geometry.
- Eliminates specimen geometry effects, enabling material comparison.

Load versus Elongation Diagram

Applied force (F) is plotted against elongation (Δℓ) for each specimen.

- Larger cross-sectional area → **more difficult to deform** (curves 1 vs 2).
- Longer specimen → **larger deformation** under same force (curves 2 vs 3).



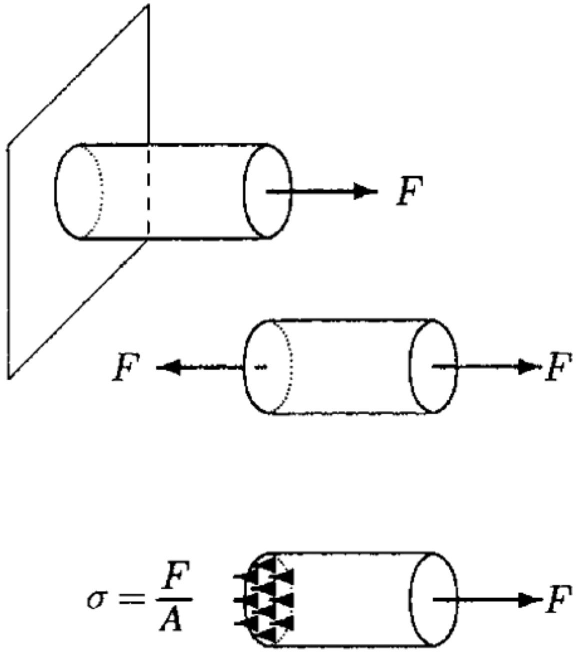


Stress: Internal Force Intensity

Reference: Özkaya, Nihat. Fundamentals of Biomechanics : Equilibrium, Motion, and Deformation. 3rd ed. New York: Springer, 2012.

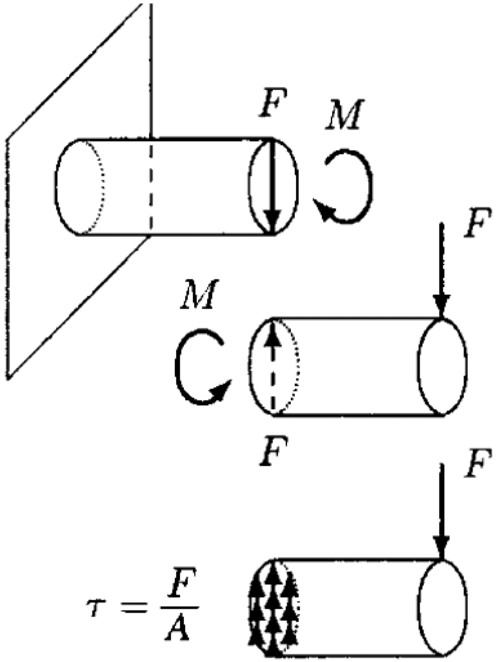
Normal Stress

- **Defined as: $\sigma = F / A$** (force / cross-sectional area)
- **Intensity of internal force perpendicular to the cut section**
  - **Tensile stress**: pulling forces
  - **Compressive stress**: pushing forces



Shear Stress

- **Defined as: $\tau = F / A$** (force / area)
- **Intensity of internal force acting parallel or tangent to a plane**
  - Represents stress distributed over the cut surface



Stress Units

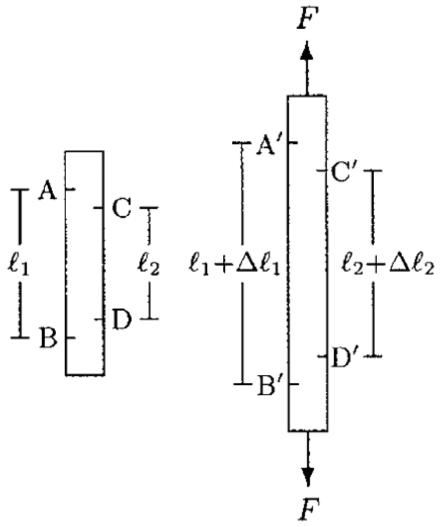
- **SI**: N/m^2 (Pascal or Pa)
- **CGS:** dyn/cm^2
- **British**: lb/ft^2 or psi
- **Same dimension and units as pressure**

Strain: Deformation

Reference: Özkaya, Nihat. Fundamentals of Biomechanics : Equilibrium, Motion, and Deformation. 3rd ed. New York: Springer, 2012.

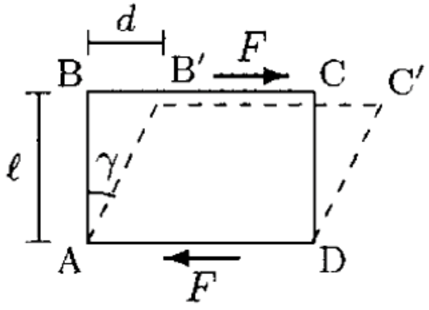
Normal Strain (axial/linear strain)

- **Defined as: $\varepsilon = \Delta\ell / \ell$** (change in length / original length)
- Dimensionless quantity (no units, though sometimes expressed as cm/cm or %)
    - **Tension**: length increases → $\Delta\ell$ and ε are positive
    - **Compression**: length decreases → $\Delta\ell$ and ε are negative



Shear Strain

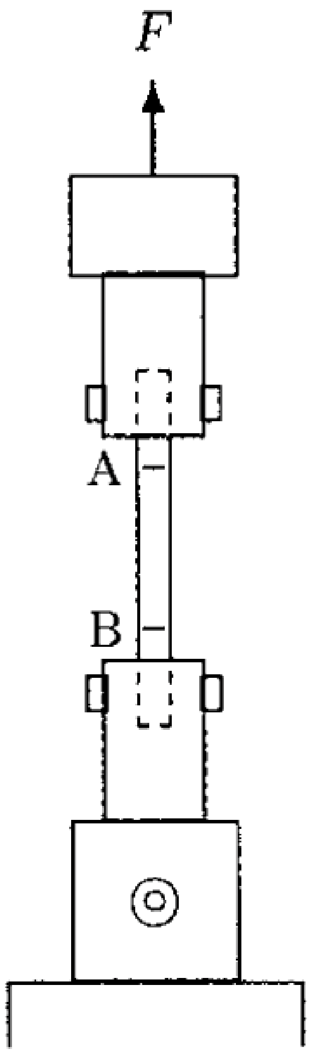
- Change in angle between two initially perpendicular lines
- **Defined as: $\gamma \approx \tan(\gamma) \approx d/\ell$** (horizontal displacement / height)
    - Rectangle deforms into a parallelogram under shear forces



Graded in-class problems

Problem 3

Reference: Özkaya, Nihat. Fundamentals of Biomechanics : Equilibrium, Motion, and Deformation. 3rd ed. New York: Springer, 2012.



A circular cylindrical rod with radius r =1.26 cm is tested in a uniaxial tension test as shown in the Figure.

- **Before applying a tensile force of F = 1000 N, two points, A and B, spaced by l_0 = 30 cm (gage length), are marked on the rod.**
- **After the force is applied, the distance between A and B is measured as l_1 = 31.5 cm.**

Determine the tensile strain and average tensile stress generated in the rod.