MAE 3128
Biomechanics-I

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Topics covered today:

1. Fundamental notions and definitions
2. Point of Departure + Examples
3. Continuum mechanics + history
4. Natural and Extreme Length Scales of Observation in Biomechanics
5. Review of Statics
6. In-class problems

GW School of Engineering & Applied Science
THE GEORGE WASHINGTON UNIVERSITY

Spring 2026

Photo: Kartik Bulusu

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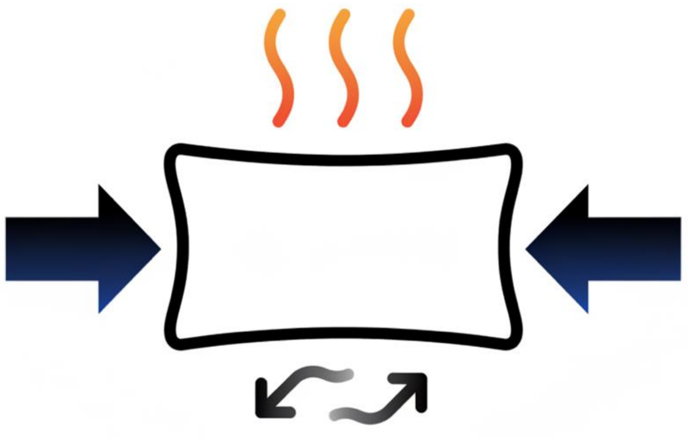
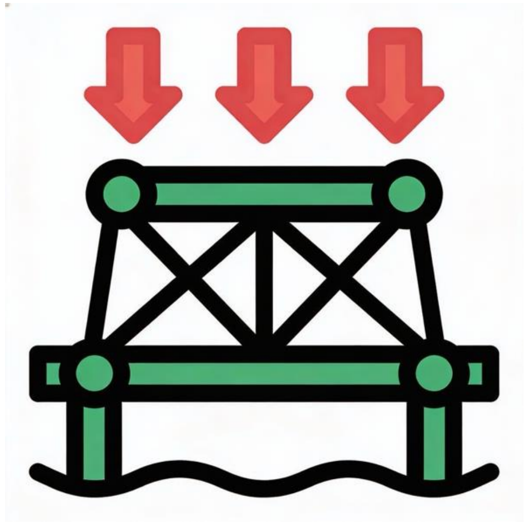
Preliminaries

Mechanics is the physical science concerned with the movement and deformation of a body acted on by mechanical, thermal, or other loads.



A **structure** is a set of components needed to support loads and keep deformation within accepted limits.



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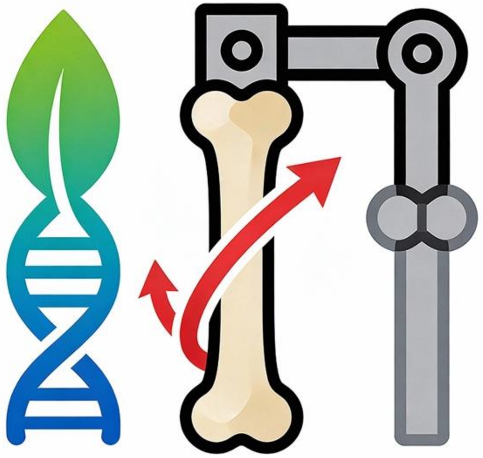
Point of Departure

Biology is the study of living things

Mechanics is the study of motions and the applied loads that cause them.

Biomechanics
- is the development, extension, and application of mechanics
- for the purposes of understanding better
    - the influence of applied loads on the structure, properties, and function of living things and the structures with which they interact.



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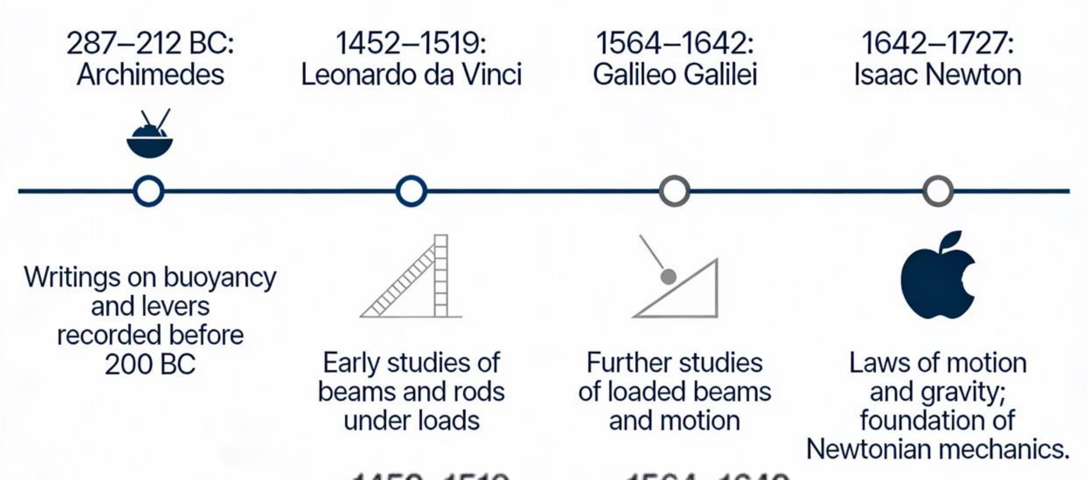
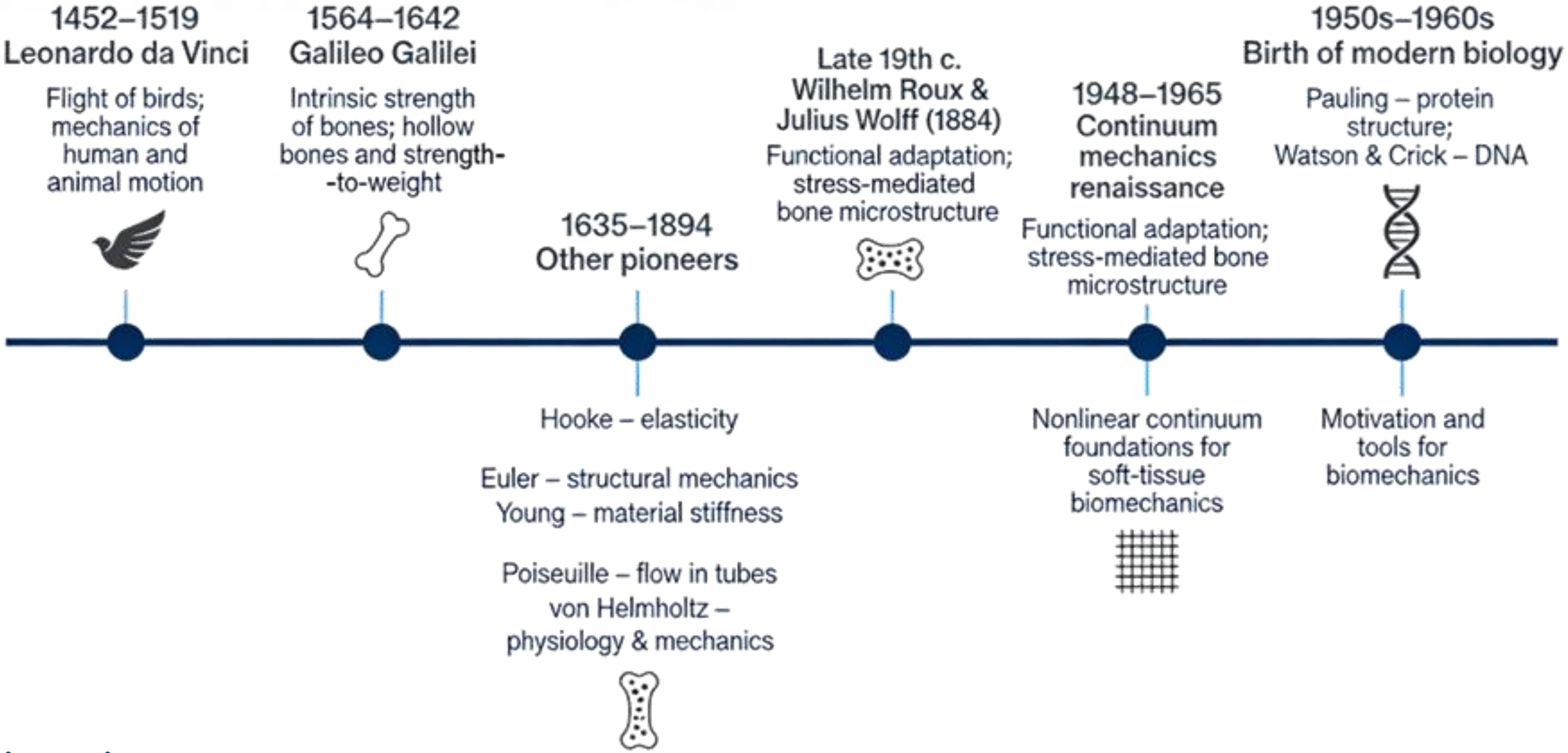
Historical Timeline from Classical Mechanics to Biomechanics





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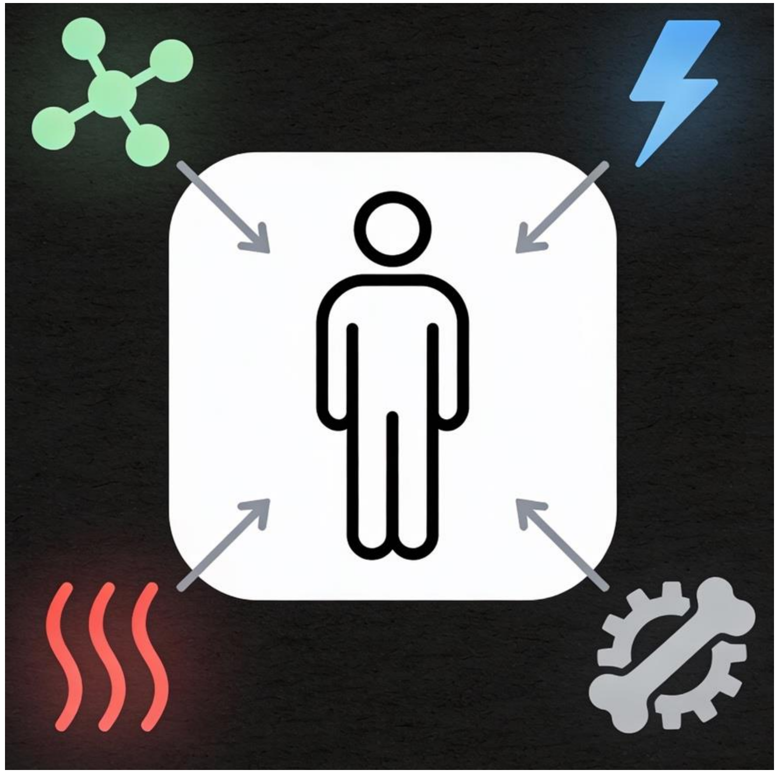
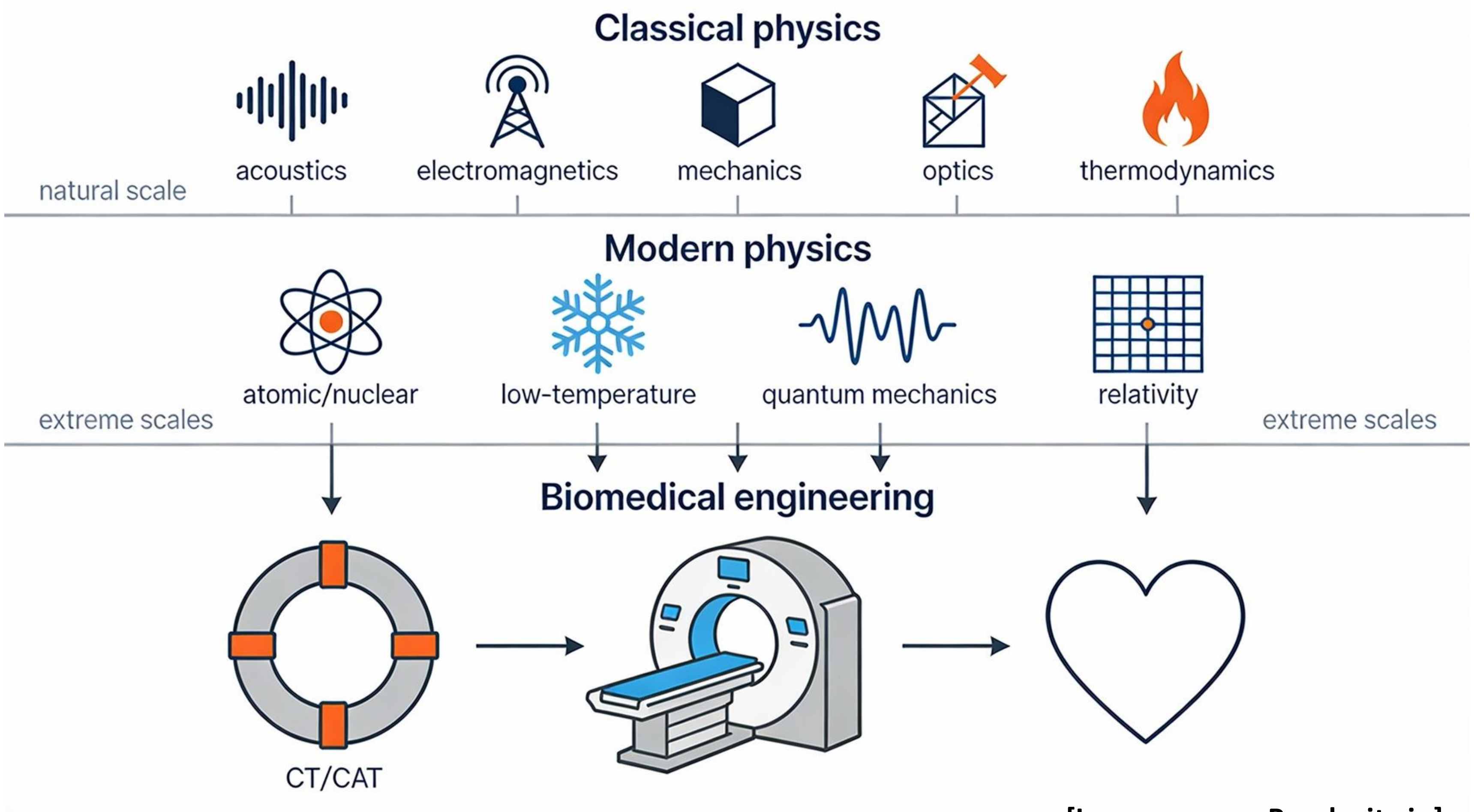
Biomechanics at various length scales

Natural and Extreme (length) scale of observation or experience





[Image source: Perplexity.io]

Extreme (length) scale of observation or experience

A Brief on Cell Biology

Reference: Humphrey, J. D., & O'Rourke, S. L. (2015). *An introduction to biomechanics: Solids and fluids, analysis and design* (2nd ed.)

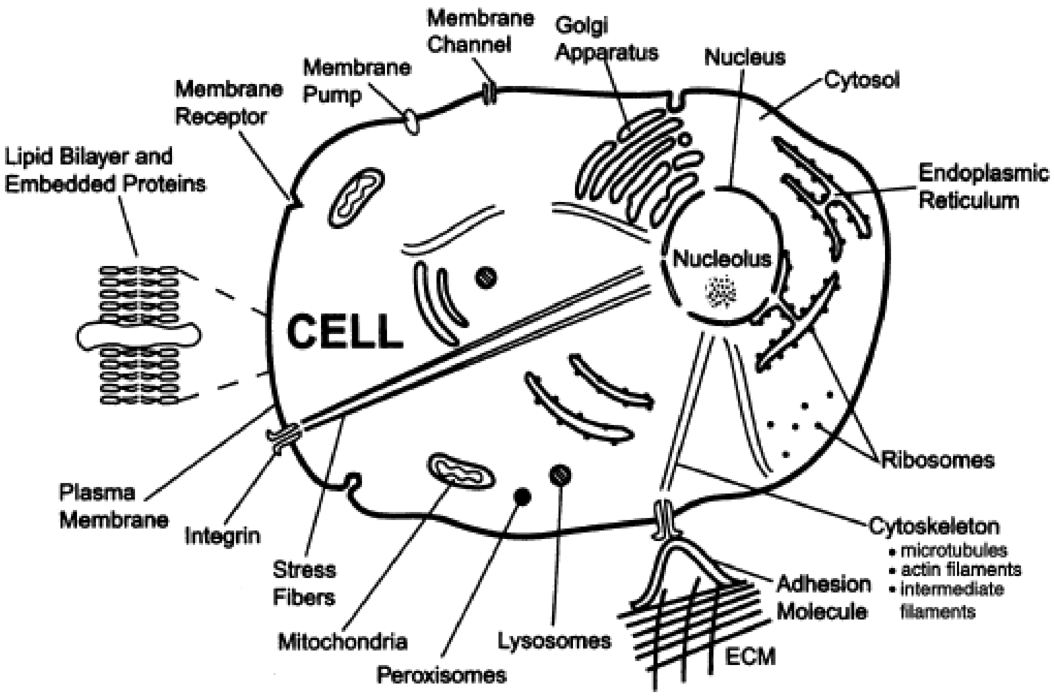
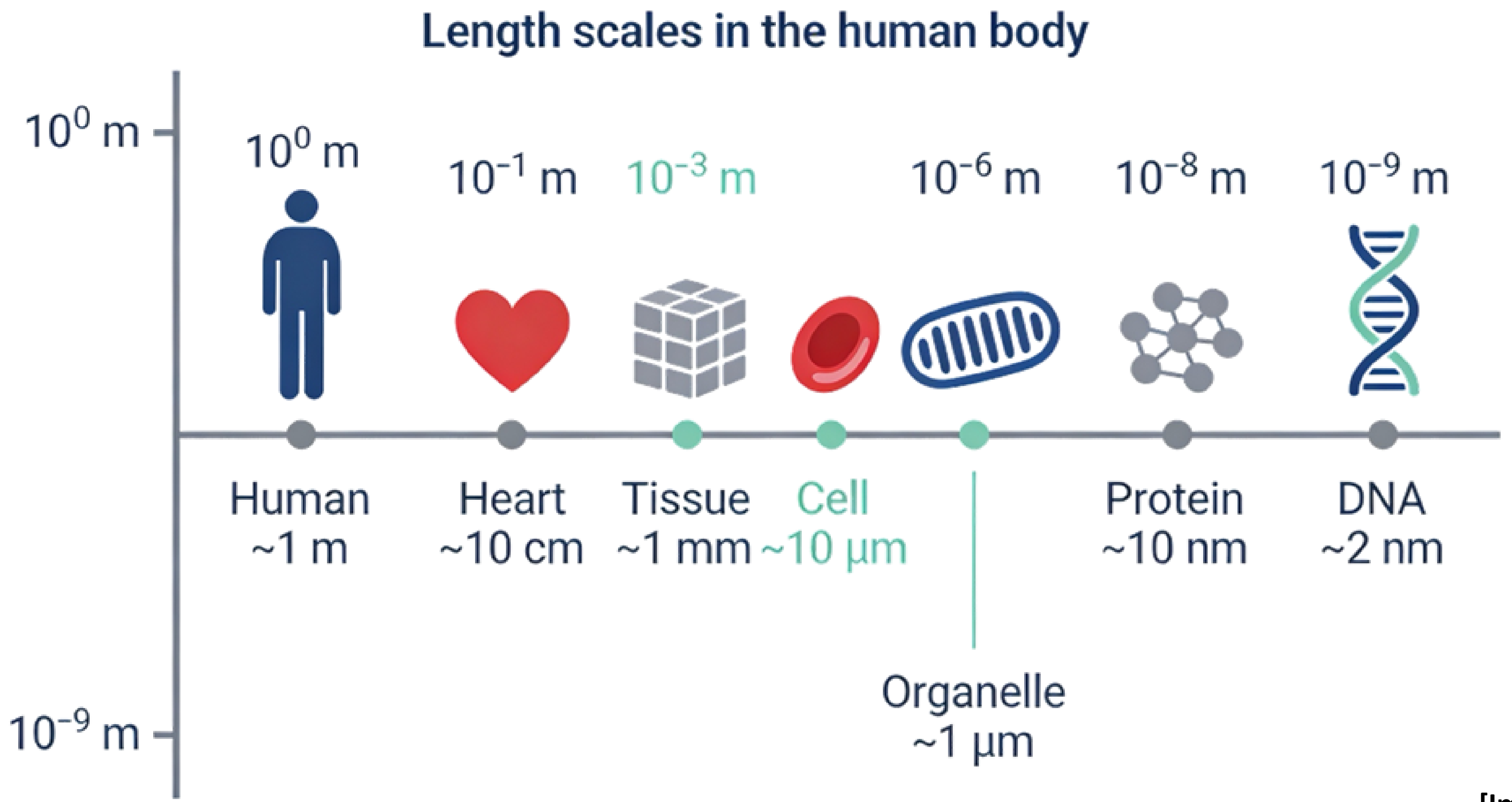


FIGURE 1.5 Schema of a mammalian cell showing its three primary constituents: the cell membrane (with various receptors, pumps, channels, and transmembrane proteins), the cytoplasm (including many different types of organelles, the cytoskeleton, and the cytosol), and the nucleus. From a mechanics perspective, the three primary proteins of the cytoskeleton (actin, intermediate filaments, and microtubules) are of particular importance. [From Humphrey (2002), with permission.].

Spectrum of length scales in the human body

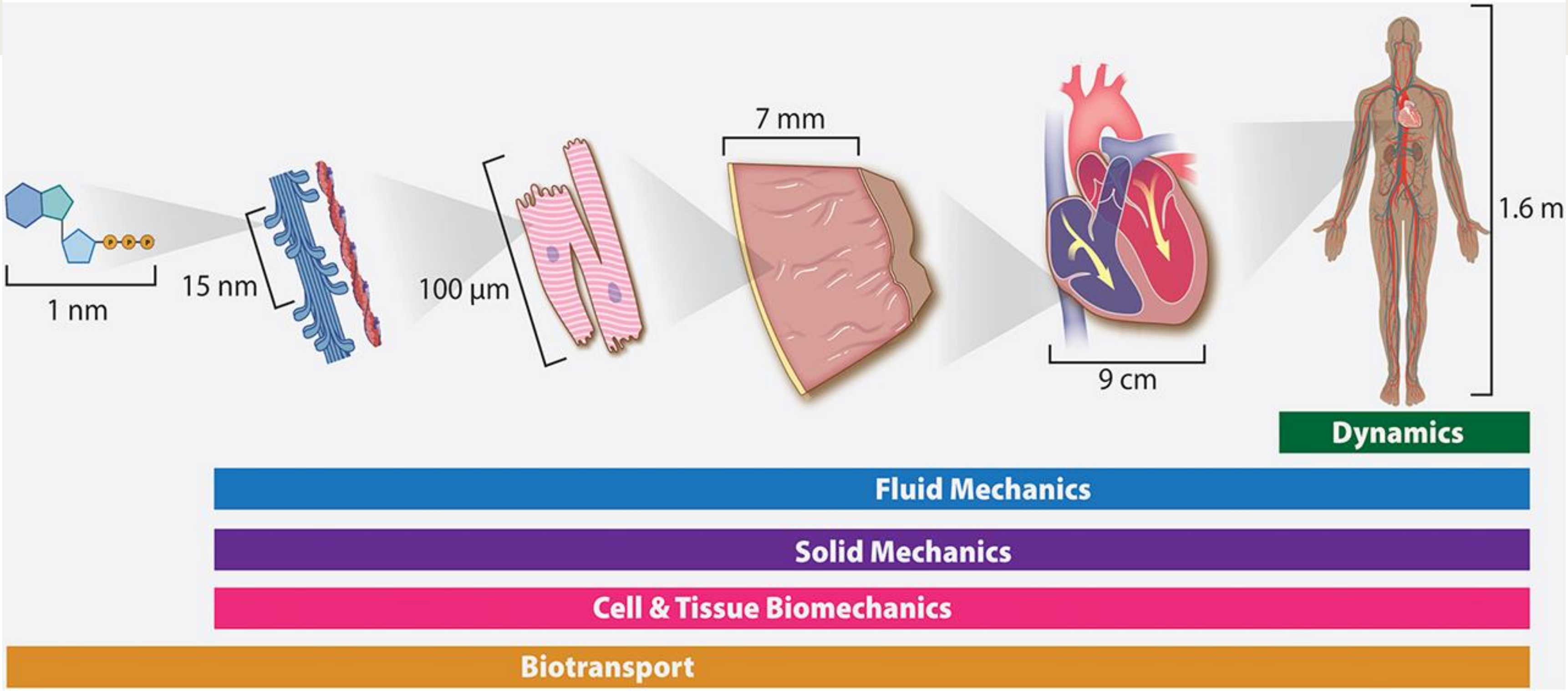


[Image source: Perplexity.io]

How do all the length scales interact to produce a heartbeat?

From: A Systems Approach to Biomechanics, Mechanobiology, and Biotransport

J Biomech Eng. 2024;146(4). doi:10.1115/1.4064547

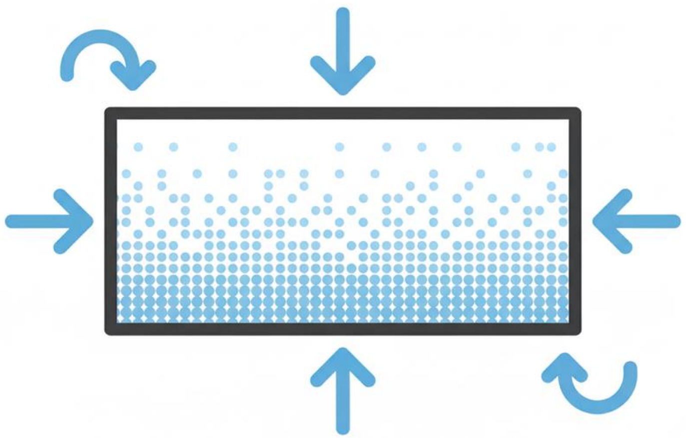


Toward Engineering problem solving in Biomechanics

Fundamental idealization in Biomechanics

The continuum.

- Continuous distribution of matter
- We are interested only in the average measurable effects on the body
  - Example: Pressure, density, and temperature are the gross effects of the actions of the many molecules and atoms.
- Without such simplification we would need to consider molecular or other elementary body interactions.

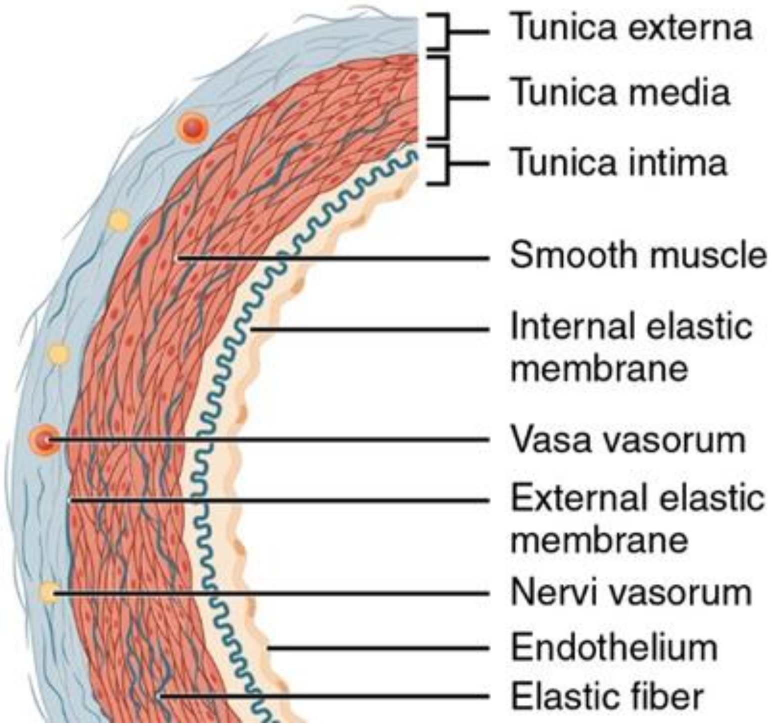


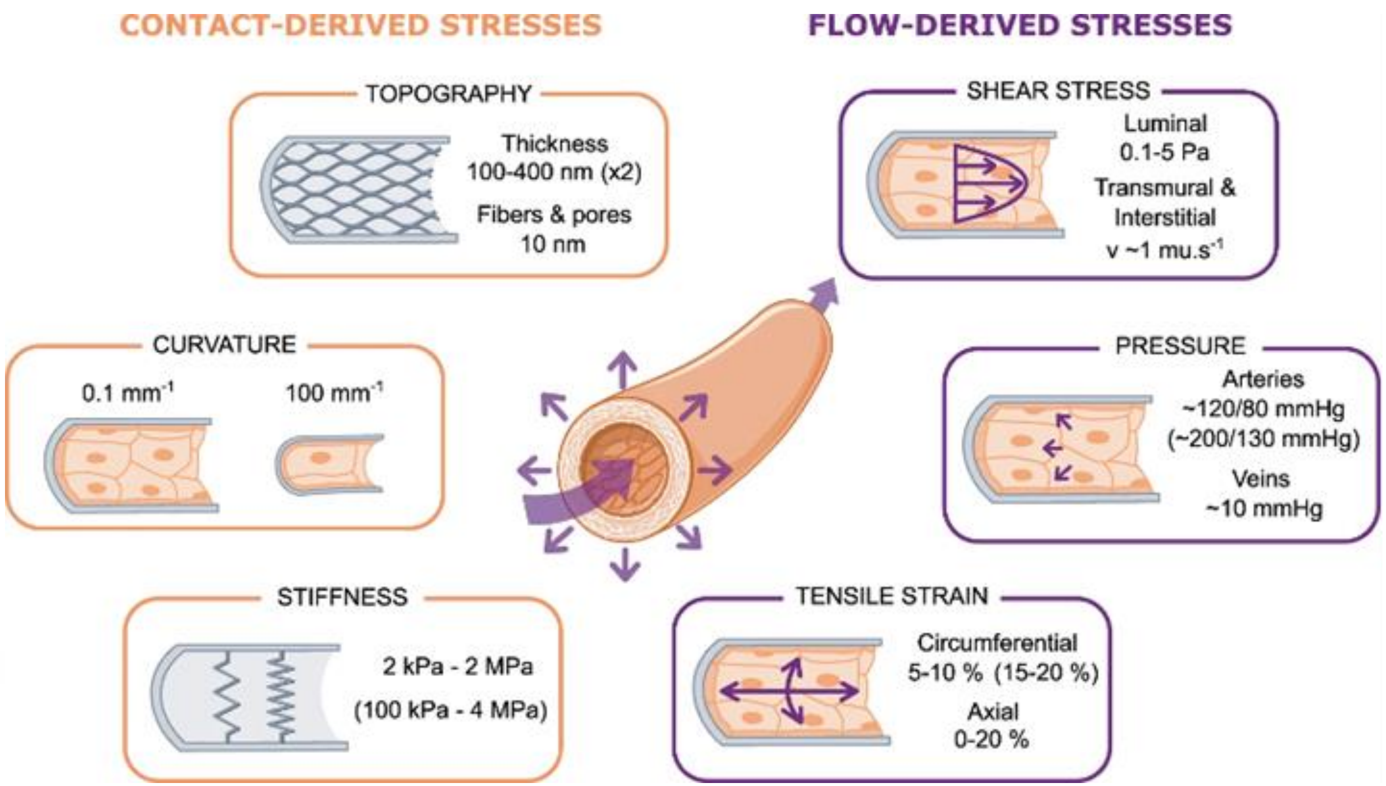
What is the behavior of matter at the natural length scale of observation or experience under continuum assumption ?

[Image source: Perplexity.io]

Forces felt by cells (on average) within the wall of an artery:
Natural (length) scale of observation or experience

The Structure of An Artery Wall

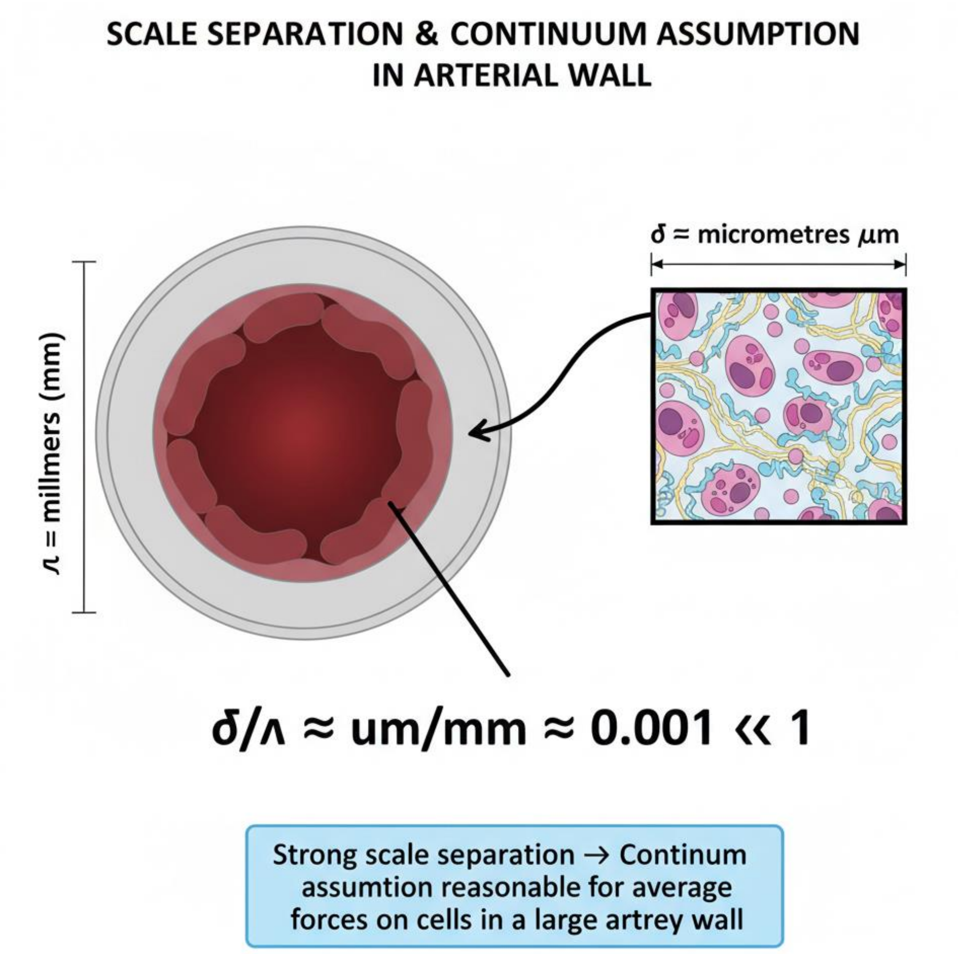




Reference: https://commons.wikimedia.org/wiki/File:Art%C3%A9re_membrane.png

Reference: Dessalles, C.A., Leclech, C., Castagnino, A. *et al.* Integration of substrate- and flow-derived stresses in endothelial cell mechanobiology. *Commun Biol* **4,** 764 (2021). https://doi.org/10.1038/s42003-021-02285-w

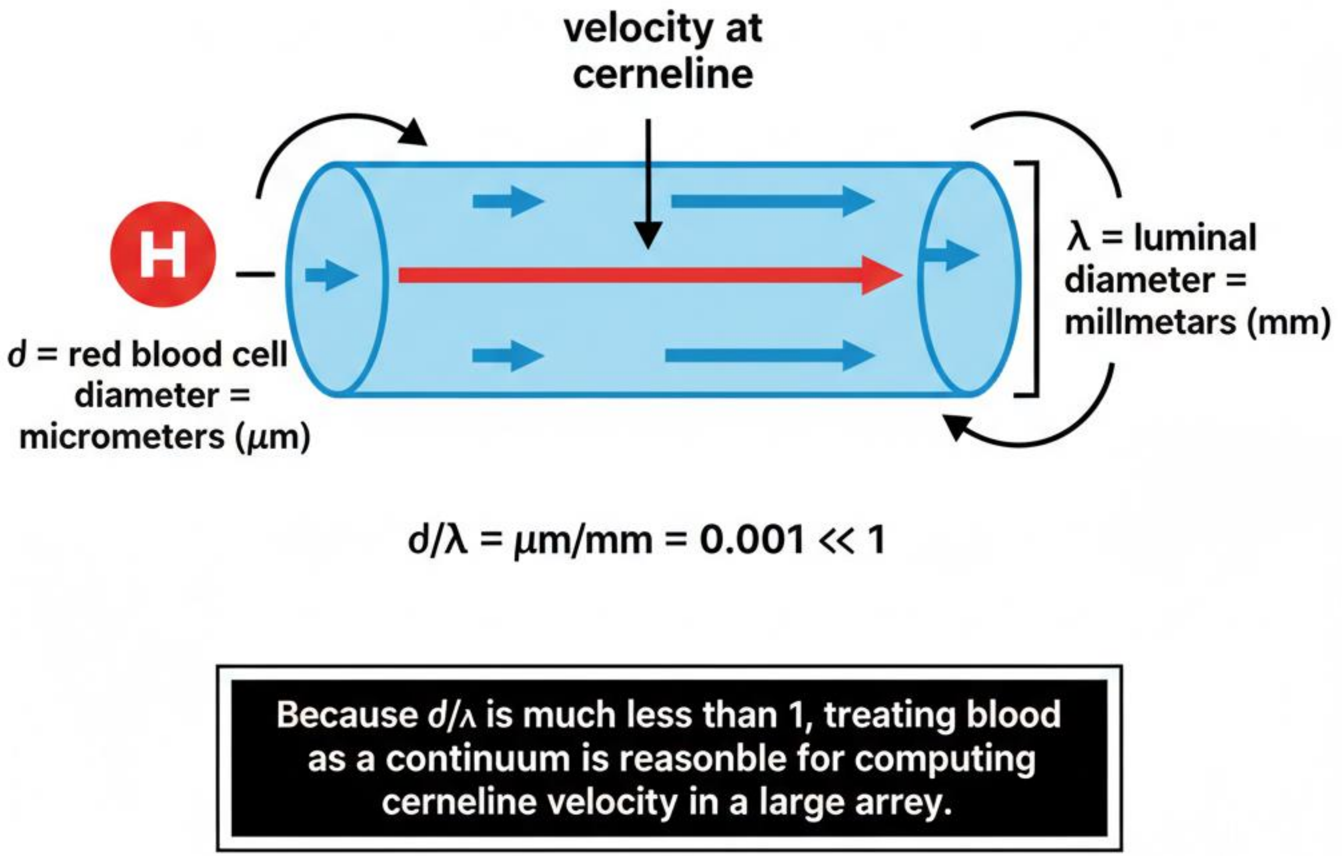
Forces felt by cells (on average) within the wall of a large artery:
Natural (length) scale of observation or experience



[Image source: Perplexity.io]

Centerline velocity within a large artery:

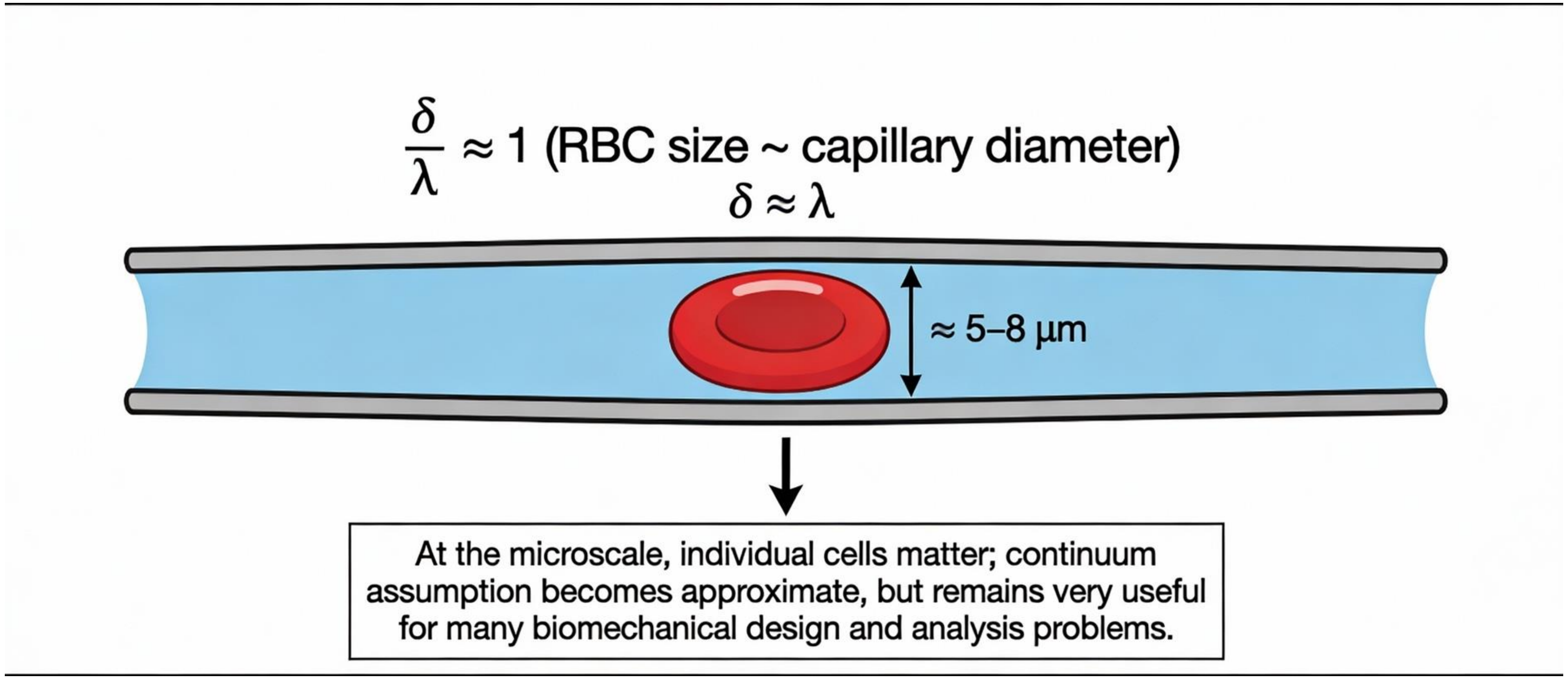
Natural (length) scale of observation or experience



[Image source: Perplexity.io]

Forces felt by cells (on average) within a capillary:

Natural scale of observation or experience



[Image source: Perplexity.io]

Biomechanics from a Continuum Mechanics perspective

Reference: Humphrey, J. D., & O'Rourke, S. L. (2015). *An introduction to biomechanics: Solids and fluids, analysis and design* (2nd ed.)

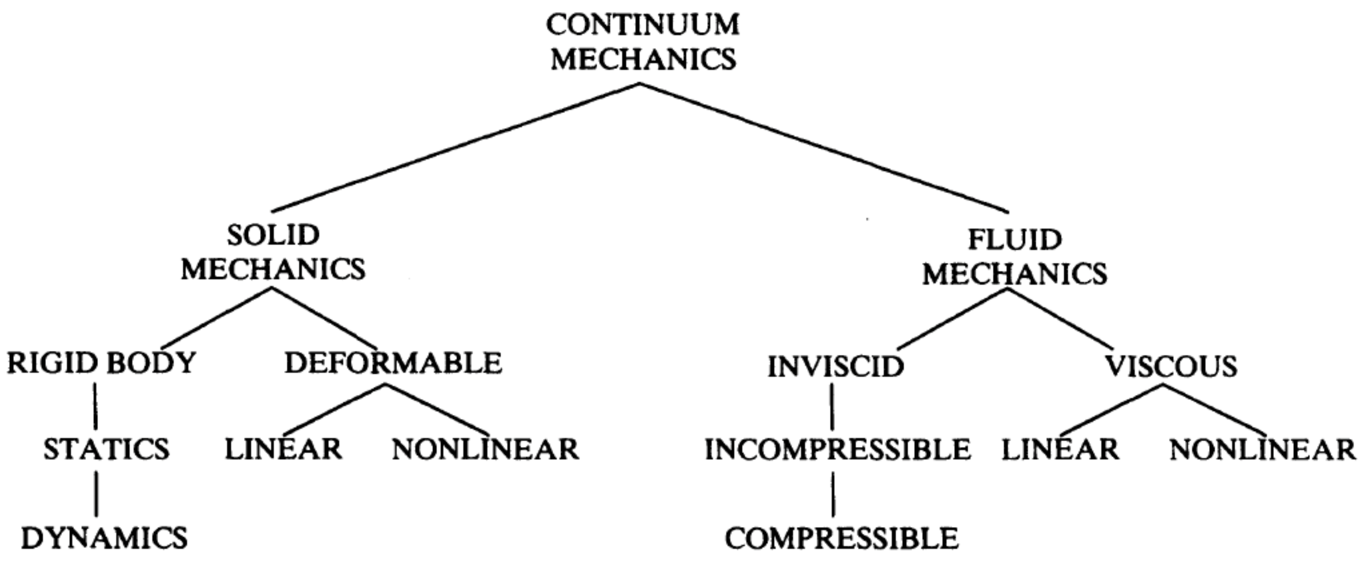


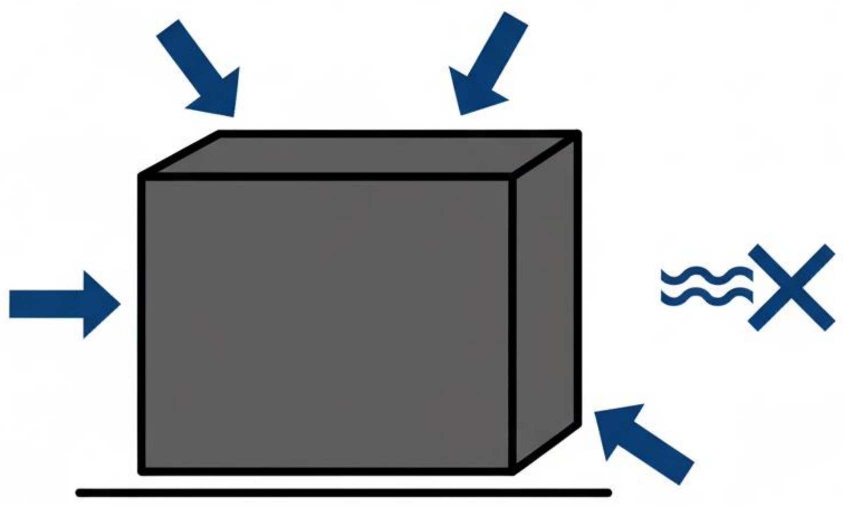
FIGURE 1.4 Flowchart of traditional divisions of study within continuum mechanics. Note that solid mechanics and fluid mechanics focus primarily on solidlike and fluidlike behaviors, not materials in their solid versus fluid/gaseous phases. Note, too, that linear and nonlinear refer to material behaviors, not the governing differential equations of motion. As we shall see in Chap. 11, many materials simultaneously exhibit solidlike (e.g., elastic) and fluidlike (e.g., viscous) behaviors, which gives rise to the study of viscoelasticity and the theory of mixtures, both of which are important areas within continuum biomechanics.

Review of Fundamental Notions:

Building blocks of Biomechanics problem solving

The Rigid Body.

- A piece of continuum with deformations too small to affect analysis when acted on by forces



Forces felt by cells (on average):

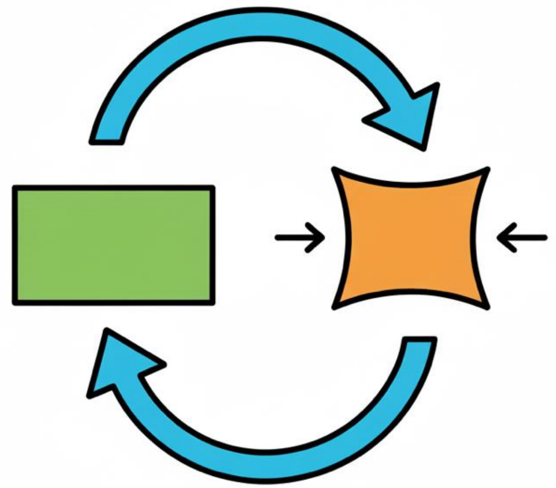
Natural scale of observation or experience as a Rigid body

- **Very small deformations relative to cell size**
  - Flow or loading conditions causing minor shape changes; RBCs mainly translate and tumble,
  - Small oscillatory strains on endothelial cells, can be treated as rigid particles
  - The model applies when focusing on trajectories, orientation, or collisions, not cell stresses.
- **Short time scales compared with relaxation times**
  - If loading is impulsive or changes too fast for the cell to deform, inertia dominates, allowing for a local rigid-body approximation.
- **Pathological or artificial stiffening**
  - In diseases or experiments with markedly stiffened cells (e.g., malaria-infected or chemically fixed RBCs, or a heavily cross-linked endothelial cytoskeleton),

[Image source: Perplexity.io]

The elastic body.

- A piece of continuum that
  - Deforms when acted on by forces and
  - Resumes original unloaded share when loads are removed from the body



Forces felt by cells (on average):

Natural scale of observation or experience as an Elastic body

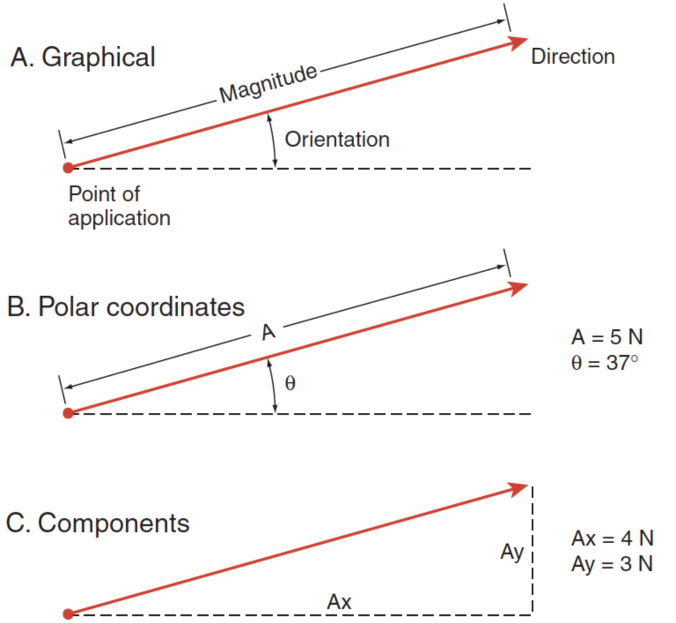
- **Moderate deformations where shape changes but material response is mostly elastic**
  - RBCs in typical arterial or venous shear flows undergo tank-treading and elongation; their behavior is well captured by elastic-membrane or shell models characterized by shear modulus and bending rigidity.
  - Endothelial cells indented over milliseconds to seconds (e.g., AFM or micropipette tests) to estimate effective moduli of the cortex, cytoskeleton, and nucleus.
- **Need for internal stress/strain fields**
  - Whenever questions involve mechanotransduction, membrane tension, cytoskeletal stresses, or nucleus deformation, rigid-body models are inadequate and continuum elastic or viscoelastic models of the cell are required.

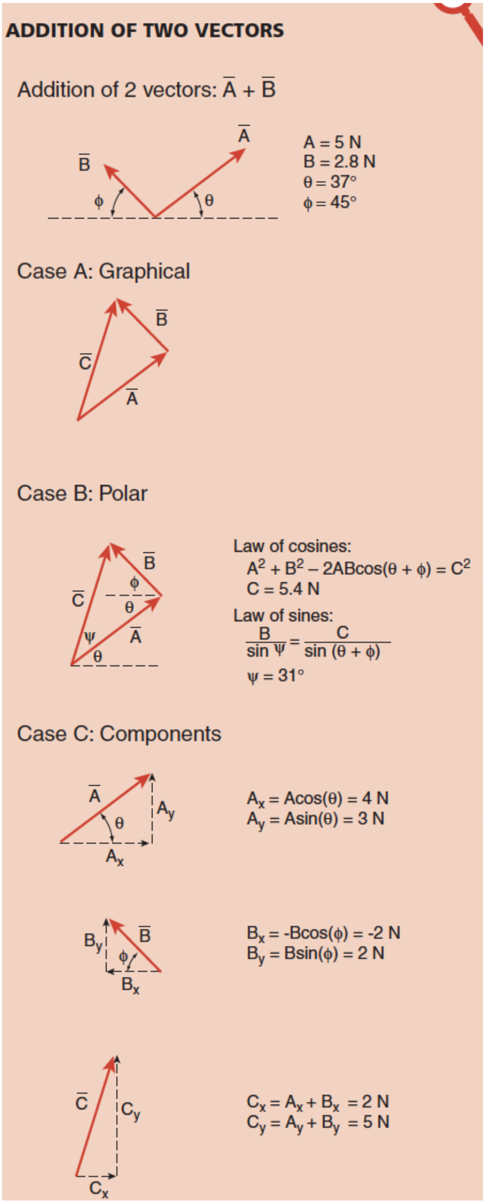
[Image source: Perplexity.io]

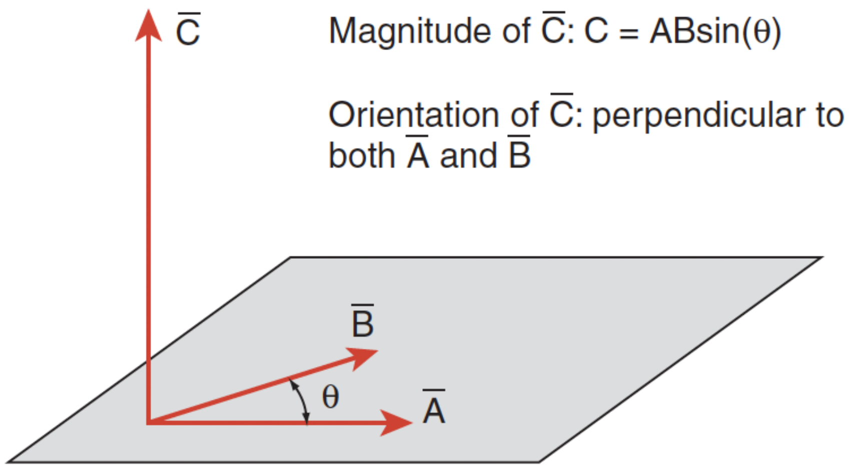
Review of Vectors

A vector.

- A vector is represented by
  - Three orthogonal rectilinear components
  - Along the axes of a Cartesian reference







For vectors

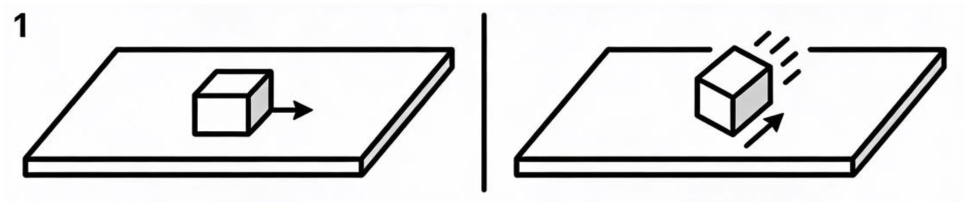
$\vec{a}(a_1, a_2, a_3)$ and $\vec{b}(b_1, b_2, b_3)$

$$\vec{a} \times \vec{b} = (a_2 b_3 - a_2 b_3)\hat{\imath} - (a_1 b_3 - a_3 b_2)\hat{\jmath} + (a_1 b_2 - a_2 b_1)\hat{k}$$

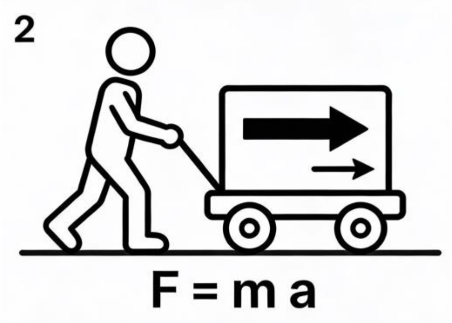
Reference: Karduna A: Introduction to Biomechanical Analysis. In: *Kinesiology: Mechanics and Pathomechanics of Human Motion*, Edited by: Carol Oatis, Publisher: Lippincott Williams and Wilkins, 1st edition, 2003, 2nd edition, 2009

Newton's Laws

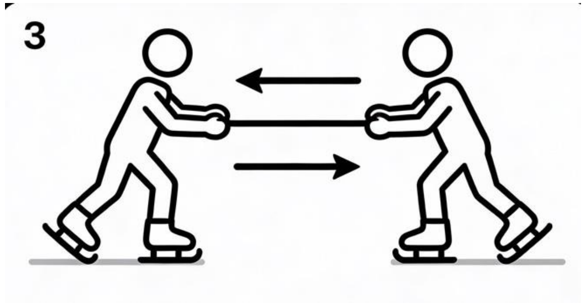
First law (inertia)
An object at rest stays at rest, and an object in motion stays in motion at constant speed in a straight line unless acted on by a net external force.



Second law (force and acceleration)
The acceleration of an object is directly proportional to the net force on it and inversely proportional to its mass; in compact form F=ma*F*=*ma* for constant mass.



Third law (action–reaction)
For every action, there is an equal and opposite reaction; when two objects interact, they exert forces on each other that are equal in magnitude and opposite in direction.



[Image source: Perplexity.io]

Review of Statics

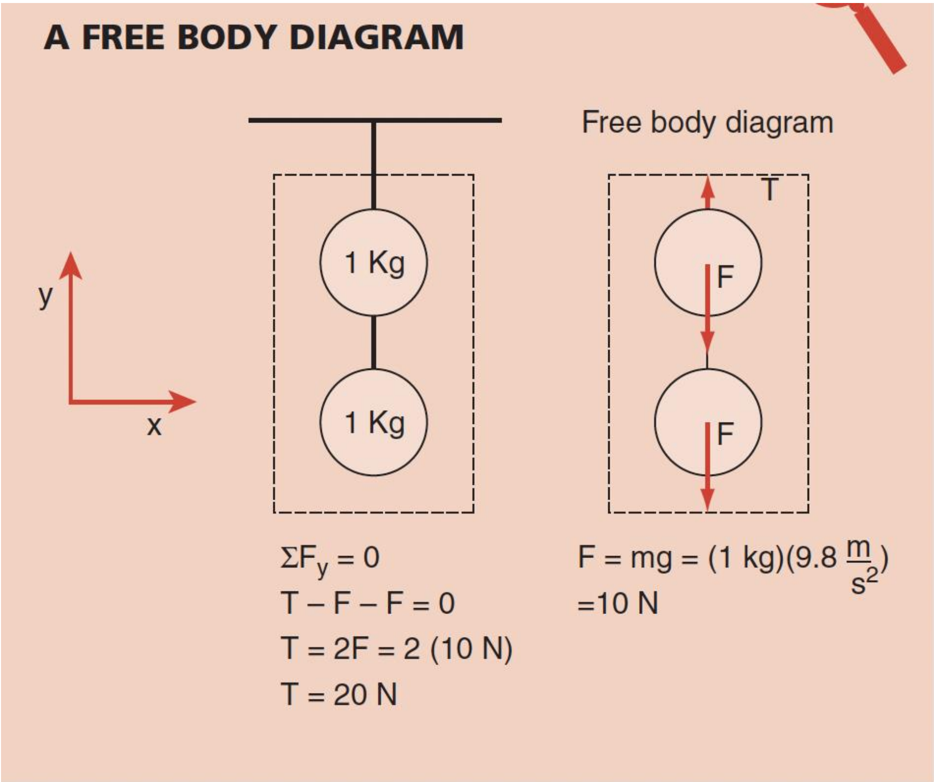
Statics is the study of the forces acting on a body at rest or moving with a constant velocity.

Static equilibrium

- If a body is at rest, there can be no unbalanced external forces acting on it. Therefore, the body is in static equilibrium.
- **All external forces acting on a body must add (in a vector sense) to zero.**
- An extension of this is that the sum of the external moments acting on that body must also be equal to zero for the body to be at rest.

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$

$$\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$$



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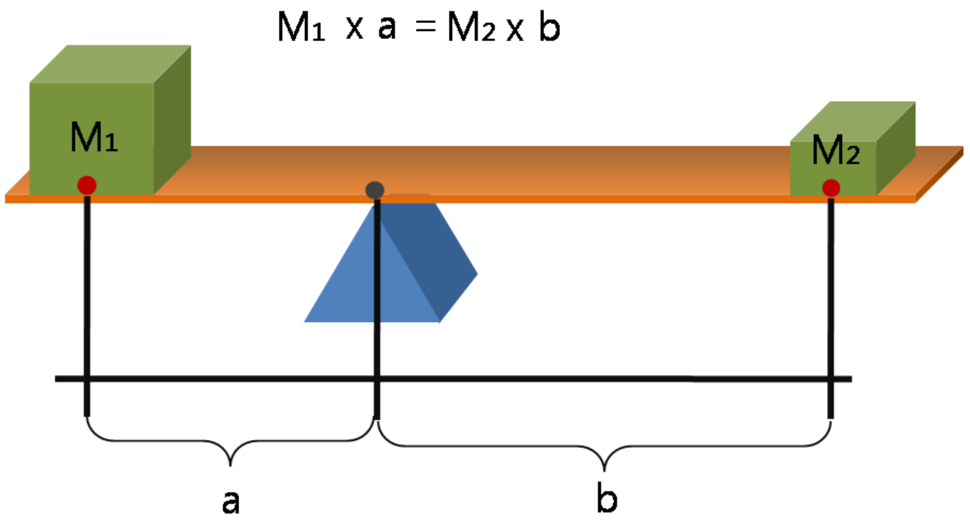
Review of Statics

Moment ($\vec{M}$) is typically caused by a force ($\vec{F}$) acting at a distance ($\vec{r}$) from the center of rotation of a segment.

A **moment** tends to cause a rotation and is defined by the cross product: $\vec{M} = \vec{r} \times \vec{F}$



A **lever** is a beam connected by a hinge, or pivot, called a fulcrum.

Static equilibrium

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Reference:

1. **Karduna A**: Introduction to Biomechanical Analysis. In: *Kinesiology: Mechanics and Pathomechanics of Human Motion*, Edited by: Carol Oatis, Publisher: Lippincott Williams and Wilkins, 1st edition, 2003, 2nd edition, 2009
2. By Jjw - Own work, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=12872799

Musculoskeletal examples involving static equilibrium

Force vector balance in gait or quiet standing

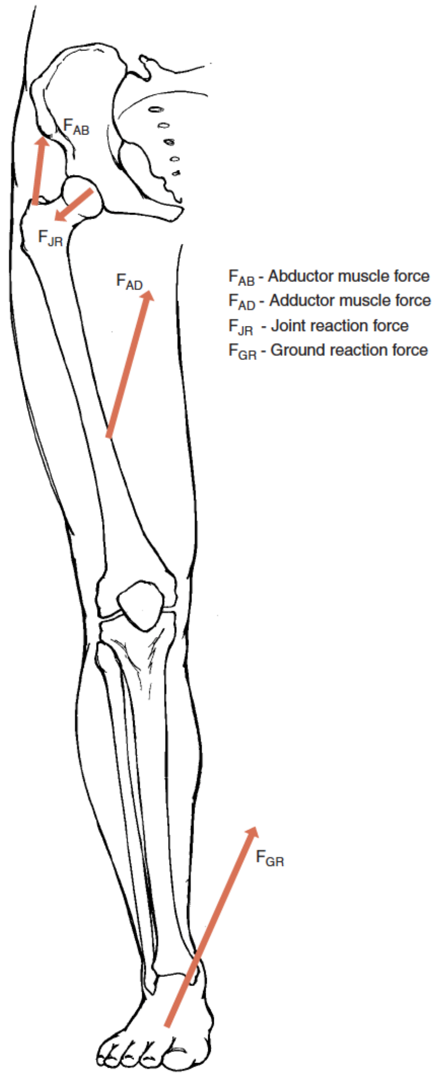
- Frontal-plane hip joint loading
- During single-leg stance and
- The muscle-joint-ground.

Guiding questions for graded notes:

1. How do F_{AB} and F_{AD} work together to counteract the external moment created by body weight and F_{GR} about the hip joint center?

2. Why is the direction and point of application of F_{GR} critical for overall lower-limb alignment and joint loading (hip, knee, ankle)?

3. How would changes in muscle strength, use of assistive devices, or altered foot placement modify the magnitudes and directions of these four forces and the resulting joint reaction?



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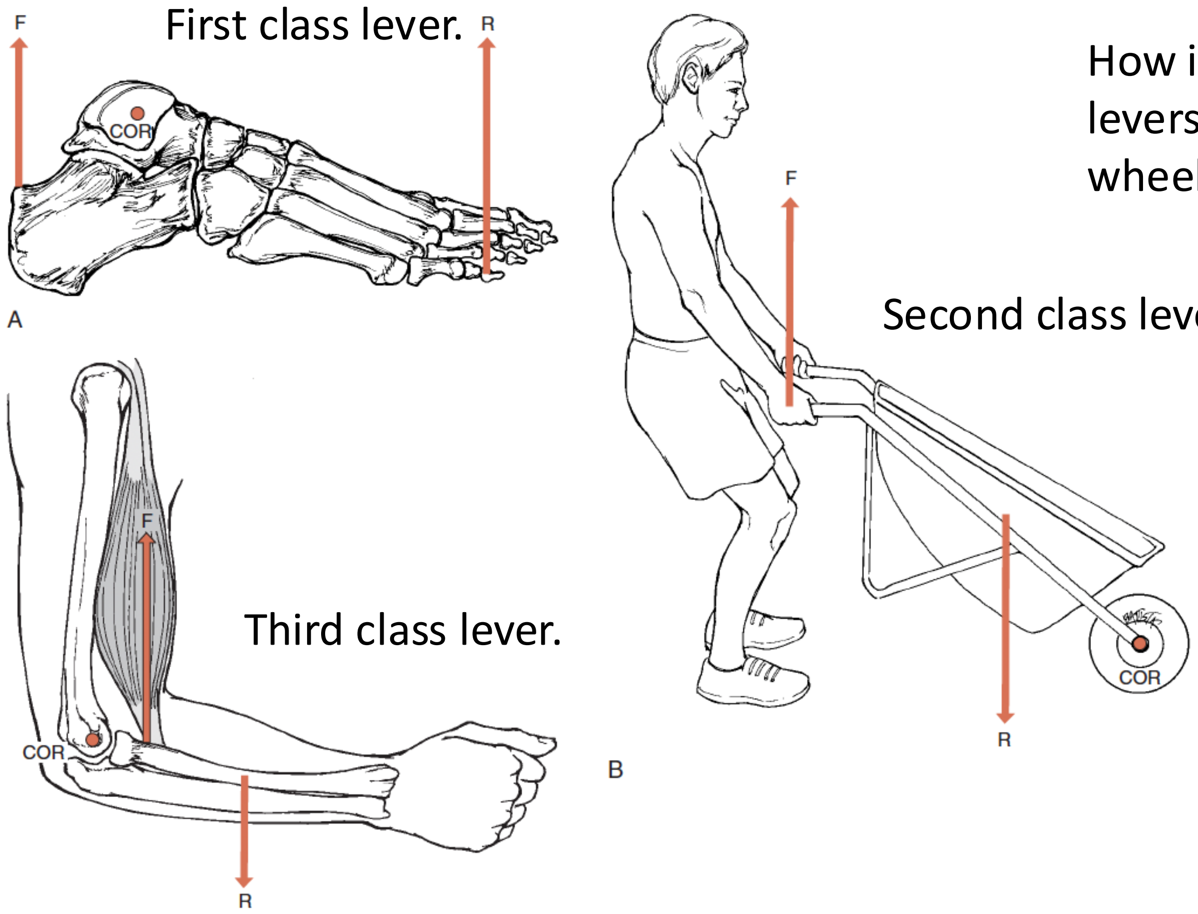
Musculoskeletal examples involving static equilibrium

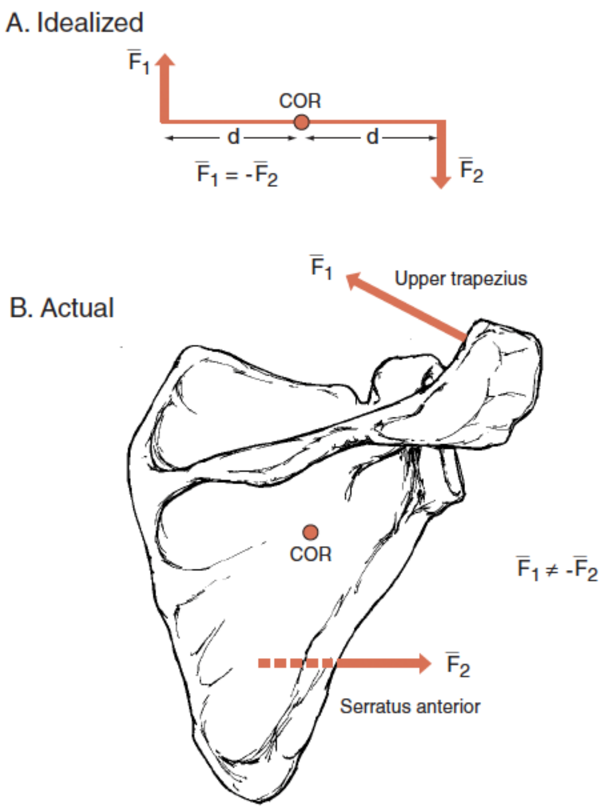


How internal muscle forces and external reaction forces act as levers about a **center of rotation (COR)** at the ankle, elbow, and wheelbarrow wheel

- There are several cases of first class levers; however, most joints in the human body behave as third class levers
- Second class levers are almost never observed within the body

Reference: Karduna A: Introduction to Biomechanical Analysis. In: *Kinesiology: Mechanics and Pathomechanics of Human Motion*, Edited by: Carol Oatis, Publisher: Lippincott Williams and Wilkins, 1st edition, 2003, 2nd edition, 2009

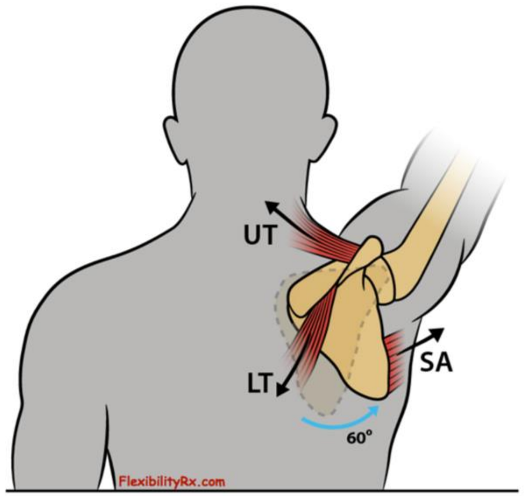
Musculoskeletal examples involving static equilibrium



Force couples. Distinction between an idealized force couple (A) and a more realistic one (B). Even though the scapular example given is not a true force couple, it is typically referred to as one. COR, center of rotation.

Force couple

- The two applied forces create a moment, they have the same magnitude and orientation but opposite directions.
- Therefore, their vector sum is zero.
- A pure force couple results in rotational motion only, since there are no unbalanced forces.



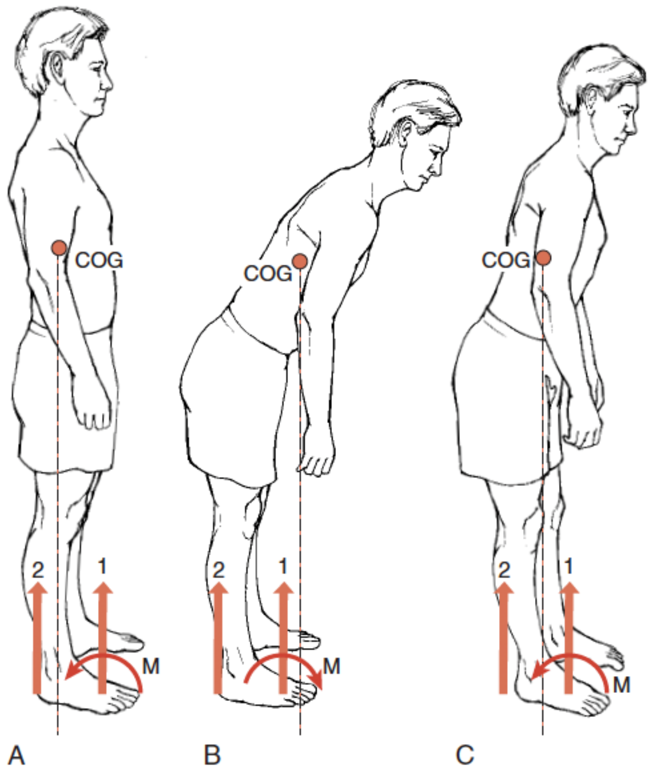
upward rotation: *(UT) Upper Trap: (LT) Lower Trap: (SA) Serratus Anterior*

Reference:

1. **Karduna A**: Introduction to Biomechanical Analysis. In: *Kinesiology: Mechanics and Pathomechanics of Human Motion*, Edited by: Carol Oatis, Publisher: Lippincott Williams and Wilkins, 1st edition, 2003, 2nd edition, 2009
2. https://www.flexibilityrx.com/wp-content/uploads/2014/09/Scapular-Upward-Rotation.jpg

Musculoskeletal examples involving static equilibrium

How forward trunk lean changes lower-limb joint loading and balance in quiet standing

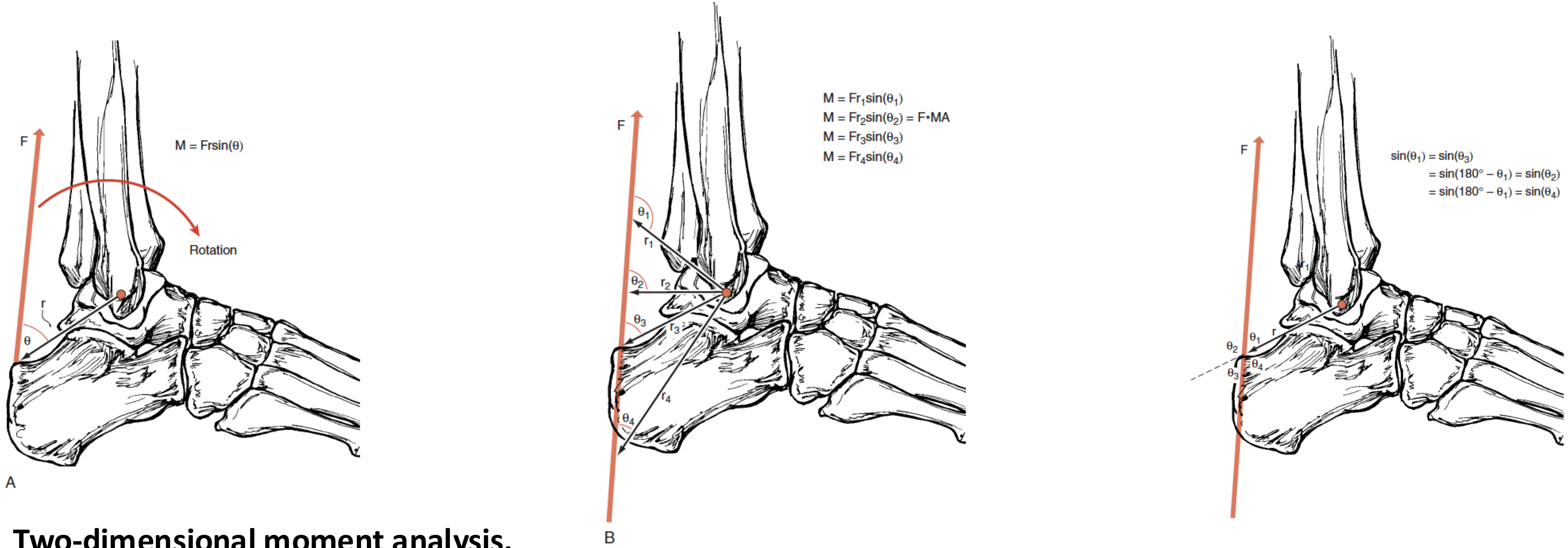


Guided questions for graded notes:

- As the trunk leans forward, how does the moment arm of the COG about the ankle change, and what does that imply about the required muscle activity at the ankle and hip?
- Why does the relative magnitude of the ground reaction forces at sites 1 and 2 need to change between A, B, and C, and how might this relate to balance strategies or risk of falls in older adults or patients with weakness?

Reference: Karduna A: Introduction to Biomechanical Analysis. In: *Kinesiology: Mechanics and Pathomechanics of Human Motion*, Edited by: Carol Oatis, Publisher: Lippincott Williams and Wilkins, 1st edition, 2003, 2nd edition, 2009

Musculoskeletal examples involving static equilibrium



Two-dimensional moment analysis.

A. Plantar flexion moment created by force at the Achilles tendon.

B. Note that no matter which distance vector is chosen, the value for the moment is the same.

C. Also, no matter which angle is chosen, the value for the sine of the angle is the same, so the moment is the same.

Reference: Karduna A: Introduction to Biomechanical Analysis. In: *Kinesiology: Mechanics and Pathomechanics of Human Motion*, Edited by: Carol Oatis, Publisher: Lippincott Williams and Wilkins, 1st edition, 2003, 2nd edition, 2009

Graded In-class problems