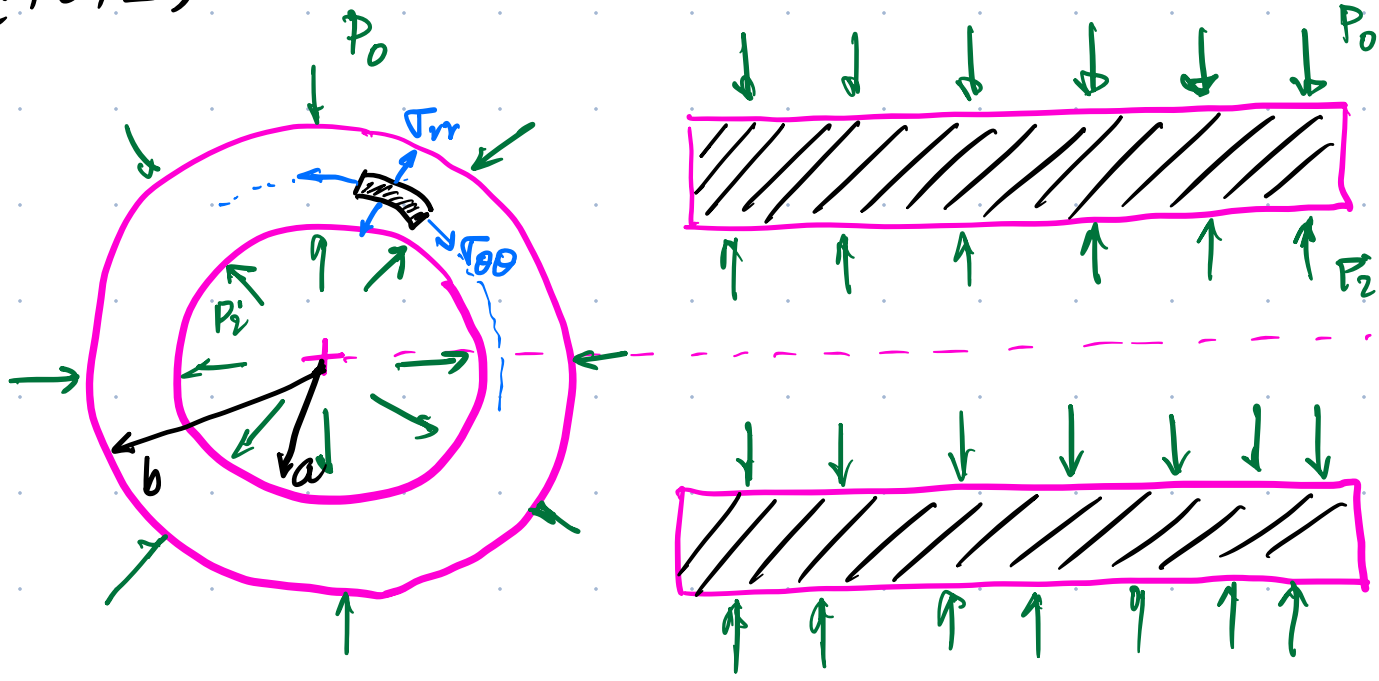
THICK-WALLED, PRESSURIZED CYLINDER INFLATION PROBLEM (LAMÉS PROBLEM)

(r, θ, z)



ASSUMPTIONS

(1) MATERIAL: (a) LEHI

(4) $\epsilon_{rr} = \frac{1}{E}[\sigma_{rr} - \nu(\sigma_{\theta\theta} + \sigma_{zz})]$

(5) $\epsilon_{\theta\theta} = \frac{1}{E}[\sigma_{\theta\theta} - \nu(\sigma_{rr} + \sigma_{zz})]$

(6) $\epsilon_{zz} = \frac{1}{E}[\sigma_{zz} - \nu(\sigma_{rr} + \sigma_{\theta\theta})]$

where

(7) $\epsilon_{rr} = \frac{\partial u_r}{\partial r}$, (8) $\epsilon_{\theta\theta} = \frac{u_r}{r}$

(9) $\epsilon_{zz} = \frac{\partial u_z}{\partial z}$

(b) small strain

(c) Axial strain: $\epsilon_{zz} = 0$ or constant

(2) STRESSES:

Axisymmetric conditions

(a) $\frac{\partial}{\partial \theta} = 0$ (c) $\tau_{rz} = 0$ (d) $\tau_{r\theta} = 0$

(b) $\frac{\partial}{\partial z}\tau_{(face)(direction)} = 0$

(3) PRESSURE:

(a) Uniformly distributed

(4) Body forces:

No body forces

IN 3D Cylindrical Co-ordinates:

(r, θ, z)

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta r}}{\partial \theta} + \left(\frac{\partial \sigma_{zr}}{\partial z} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r}\right) + \rho g_r = 0 \quad \hookrightarrow (1)$$

$$\frac{\partial \sigma_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta\theta}}{\partial \theta} + \left(\frac{\partial \sigma_{z\theta}}{\partial z} + \frac{2\sigma_{r\theta}}{r}\right) + \rho g_\theta = 0 \quad \hookrightarrow (2)$$

$$\left(\frac{\partial \sigma_{rz}}{\partial r} + \frac{\sigma_{rz}}{r}\right) + \frac{1}{r}\frac{\partial \sigma_{\theta z}}{\partial \theta} + \frac{\partial \sigma_{zz}}{\partial z} + \rho g_z = 0 \quad \hookrightarrow (3)$$

$$\boxed{\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}(\sigma_{rr} - \sigma_{\theta\theta}) = 0} \longrightarrow (10)$$

σ_{rr}, $\sigma_{\theta\theta}$ $\rightsquigarrow$ normal directions