7

Dividing $(1-\nu^2)$ on both sides of equation (14)

$$\frac{d}{dr}\big(\underbrace{\sigma_{\theta\theta}+\sigma_{rr}}_{\text{constant}}\big) = 0$$

$$\boxed{(\sigma_{\theta\theta}+\sigma_{rr}) = C} \longrightarrow (15)$$

Recall equation 10 :

$$\boxed{\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}(\sigma_{rr}-\sigma_{\theta\theta}) = 0} \longrightarrow (10)$$

$$\boxed{\frac{d}{dr}(r\sigma_{rr}) = \sigma_{\theta\theta}} \longrightarrow (16)$$

Aside :

$$\frac{d}{dr}(r\sigma_{rr}) = \sigma_{rr} + r\frac{d\sigma_{rr}}{dr} = \sigma_{\theta\theta}$$

$$\sigma_{rr} + r\frac{d\sigma_{rr}}{dr} = \sigma_{\theta\theta}$$

$$\Rightarrow \frac{d\sigma_{rr}}{dr} + \frac{(\sigma_{rr}-\sigma_{\theta\theta})}{r} = 0$$

$$\boxed{\frac{d}{dr}(r\sigma_{rr}) = \sigma_{\theta\theta}} \rightarrow 16$$

$$\boxed{\sigma_{\theta\theta} + \sigma_{rr} = C} \rightarrow 15$$

Substitute equation 16 into Equation 15

$$\boxed{\frac{d}{dr}(r\sigma_{rr}) + \sigma_{rr} = C}$$

$$\Rightarrow \quad r\frac{d\sigma_{rr}}{dr} + \sigma_{rr} + \sigma_{rr} = C$$

$$\Rightarrow \quad \boxed{r\frac{d\sigma_{rr}}{dr} + 2\sigma_{rr} = C} \rightarrow 17$$

Equation 17 ~~ Solution for σ_{rr}

{ Two stress to solve for $\sigma_{rr}, \sigma_{\theta\theta}$ }

Solution of equation 17 : σ_{rr}

step 1 : Rewrite equation 17 in a more compact form

$$\boxed{r\frac{d\sigma_{rr}}{dr} + 2\sigma_{rr} = C} \longrightarrow (17)$$

same!

$$\Rightarrow \boxed{\frac{1}{r}\frac{d}{dr}\left(r^2\sigma_{rr}\right) = C} \longrightarrow (18)$$

step 2: Integrate on both sides of Eq 18

$$\Rightarrow \int d\left(r^2\sigma_{rr}\right) = \int Cr\,dr$$

$$\Rightarrow \boxed{r^2\sigma_{rr} = \frac{Cr^2}{2} + C_1}$$

Aside:

$$\frac{1}{r}\frac{d}{dr}\left(r^2\sigma_{rr}\right) = C \rightarrow (18)$$

$$\Rightarrow \frac{1}{r}\left(r^2\frac{d\sigma_{rr}}{dr} + 2r\sigma_{rr}\right) = C$$

$$\Rightarrow \frac{1}{\cancel{r}}r^{\cancel{2}}\frac{d\sigma_{rr}}{dr} + \frac{2\cancel{r}}{\cancel{r}}\sigma_{rr} = C$$

$$\Rightarrow r\frac{d\sigma_{rr}}{dr} + 2\sigma_{rr} = C$$

$\hookrightarrow$ (17)

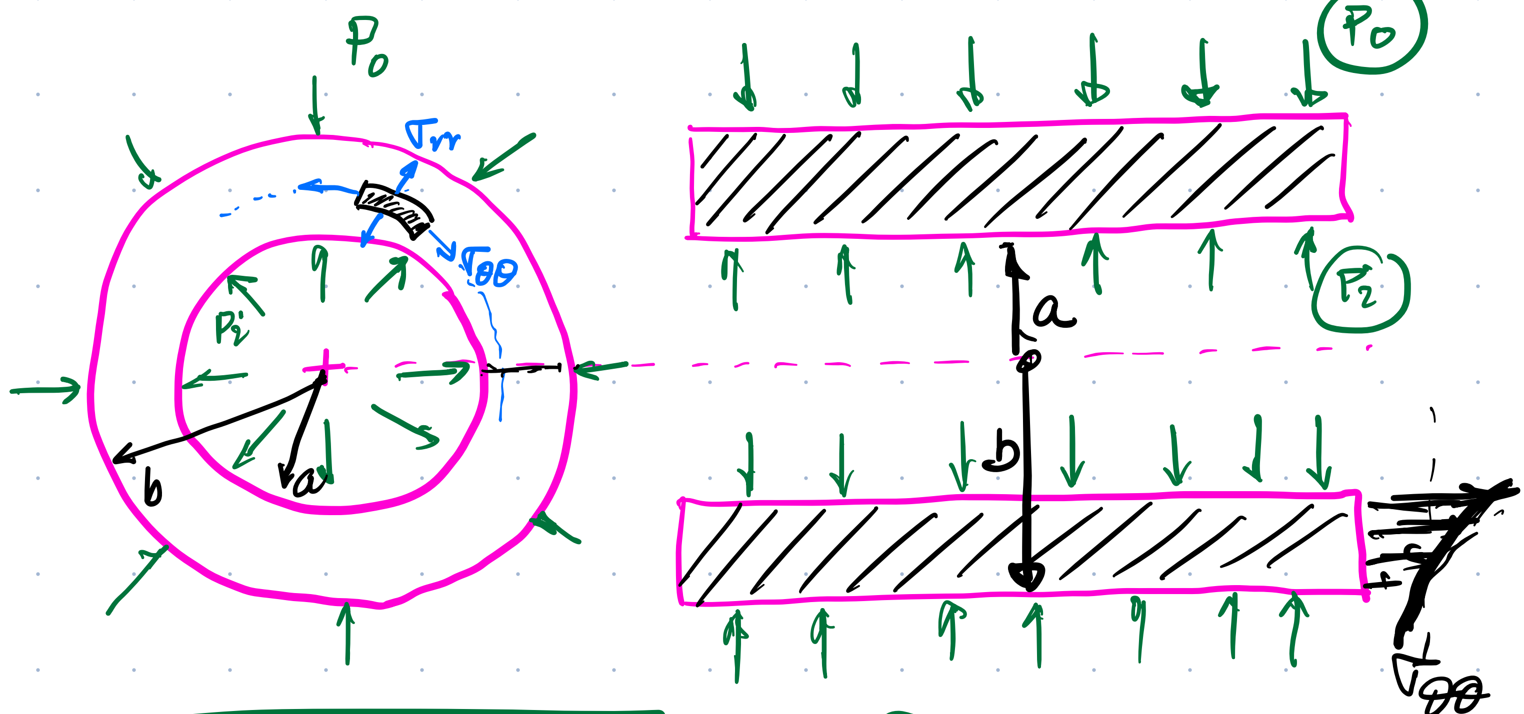
$$\Rightarrow \quad \sigma_{rr} = \frac{Cr^2}{2}\frac{1}{r^2} + C_1\left(\frac{1}{r^2}\right)$$

{Multiplying both sides with $\frac{1}{r^2}$}

$$\Rightarrow \quad \boxed{\sigma_{rr} = \frac{C}{2} + \frac{C_1}{r^2}} \longrightarrow (19)$$

Solution for σ_{rr} But we still need to eliminate C & C_1

Step 3: Apply Boundary conditions



$$\boxed{\sigma_{rr}(r=a) = -P_i} \longrightarrow (20)$$

$$\boxed{\sigma_{rr}(r=b) = -P_o} \longrightarrow (21)$$