KVB© | 01/28/2026

BIOMECHANICS - I :

① Deformable bodies ⇝ Stress ⇝ Strain ⇝ Elastic deformations

① STRESS → NORMAL → SHEAR

② LINEAR RELATIONSHIPS → YOUNG'S MODULUS → POISSON'S RATIO → SHEAR MODULUS → HOOKE'S LAW

③ STRESS VS STRAIN

④ EQUALITY OF CROSS-SHEARS

⑤ SIGN CONVENTION

⑥ IN-CLASS PROBLEMS

Will be discussed on 02/04/2026

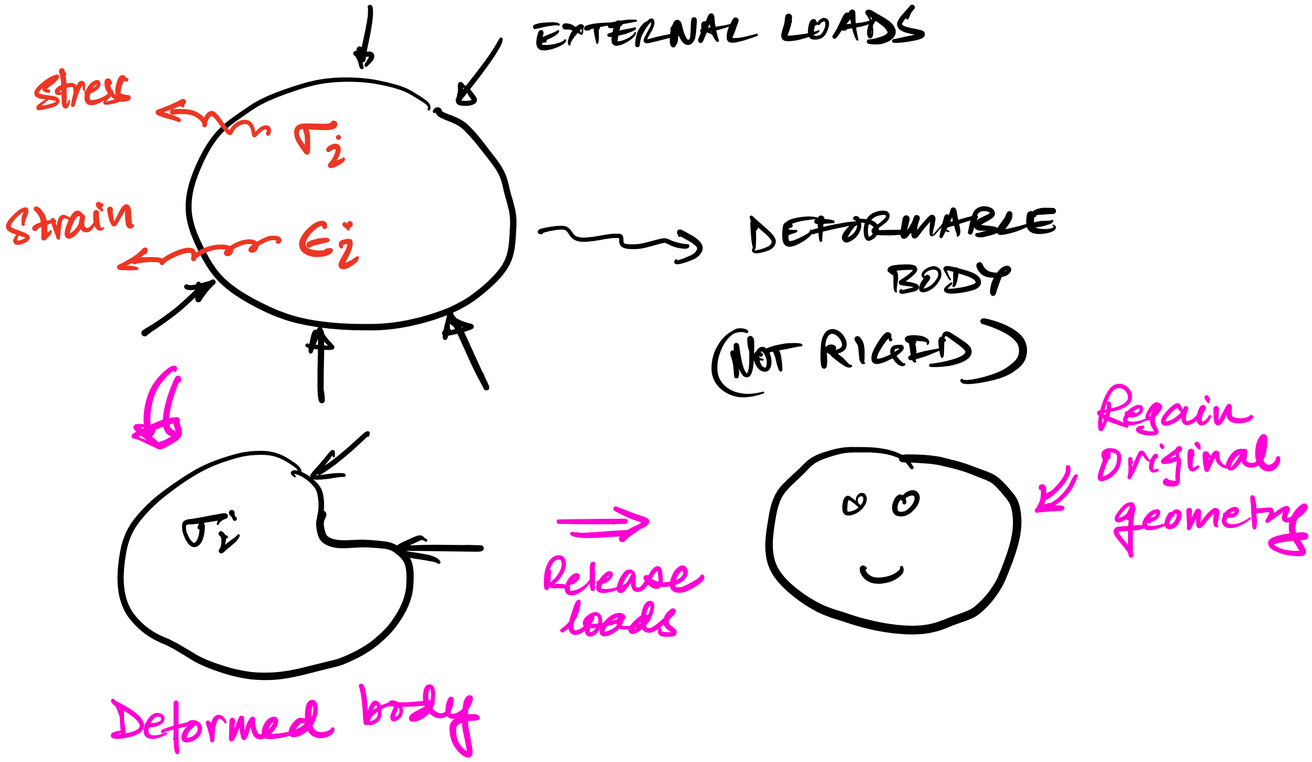
⑦ 2D STRESS FIELDS

⑧ PRINCIPAL STRESS

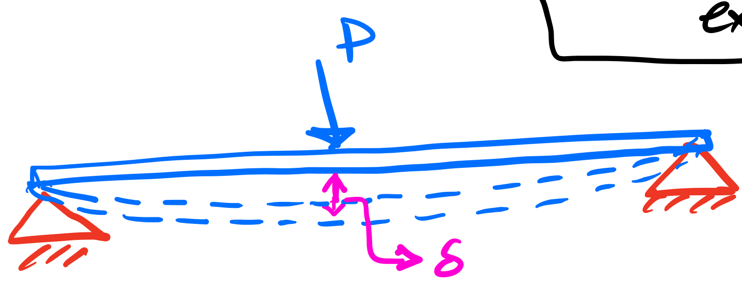
$$\sigma_{n,max,min} = \left(\frac{\sigma_{xx}+\sigma_{yy}}{2}\right) \pm \sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}$$

KEY IDEA - 1 Deformable bodies ⇝ Stress (σ)
⇝ Strain (ϵ)



Civil/Structural Engineering example



Consider a simply supported beam

⇝ Apply a load (P)
⇝ Beam goes through some measurable deformation
⇝ Internal force intensities responding to P ⇝ STRESS (σ)
⇝ External changes in geometry/length-scales (ϵ)

BODY SUBJECTED TO EXTERNAL FORCES/LOADS

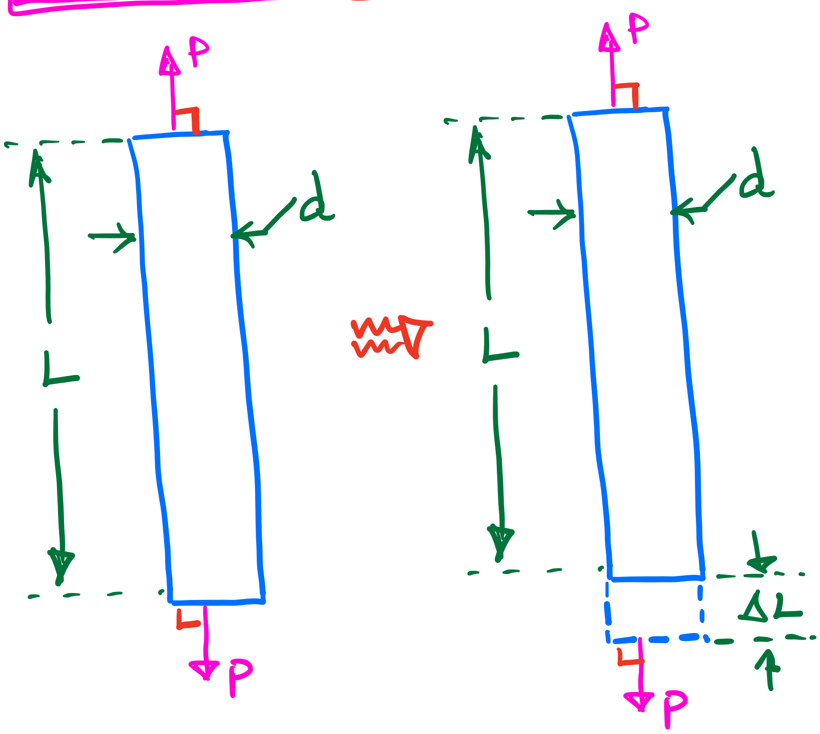
- DEFORMABLE
  - ELASTIC
    - APPLY LOAD: BODY DEFORMS MEASUREABLY
    - RELEASE LOAD: BODY REGAINS ORIGINAL SHAPE
  - PLASTIC
    - APPLY LOAD: DEFORMATION IS MEASUREABLE
- RIGID

KEY DIFFERENCE: THE NATURE OF HOW PERMANENT DEFORMATION HAPPENS

We will focus our attention on deformable bodies, only!

KEY IDEA-2

Elastic Deformations:



Consider a prismatic bar:

- Cross-sectional area of bar is A
- External force applied (P)
- Stress (σ) develops inside the body due to the action of external forces.
- Strain (ϵ) is the measure of deformation

④

$\sigma = \frac{P}{A}$

P → External force axially along the bar

A → Area of cross-section

NORMAL STRESS

Stress develops internally over the entire cross-section, uniformly, and along the same directional-sense as the external force

$\epsilon = \frac{\Delta L}{L}$

ΔL → Measureable change in length

L → Initial length / Original length / Gauge length

NORMAL STRAIN

Simply, the ratio of change in length to original length.

Alternately, TRUE STRAIN is associated with actual or deformed length

KEY IDEA-3

Relationship between σ & ϵ under elastic deformations: (Hooke's Law & Young's Modulus)

$\boxed{\sigma \propto \epsilon}$ → stress, strain proportionality (σ) (ϵ)

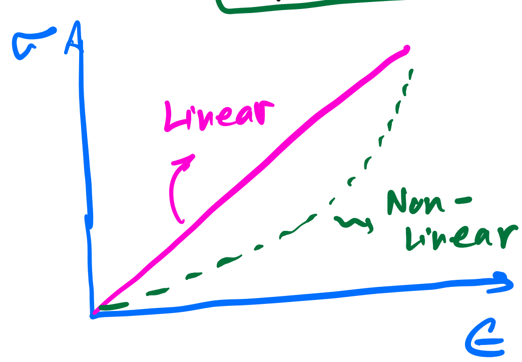
→ How can we define this proportionality?

$\boxed{\sigma = K\epsilon}$

K → Constant → Linear proportionality

→ Function (quadratic, cubic, etc.) → Non-Linear proportionality

→ proportionality constant



Let's focus only on the Linear relationship between σ & ϵ.

HOOKE'S LAW ⇒ $\boxed{\sigma = E\epsilon}$

E → Young's Modulus

→ Linear proportionality constant

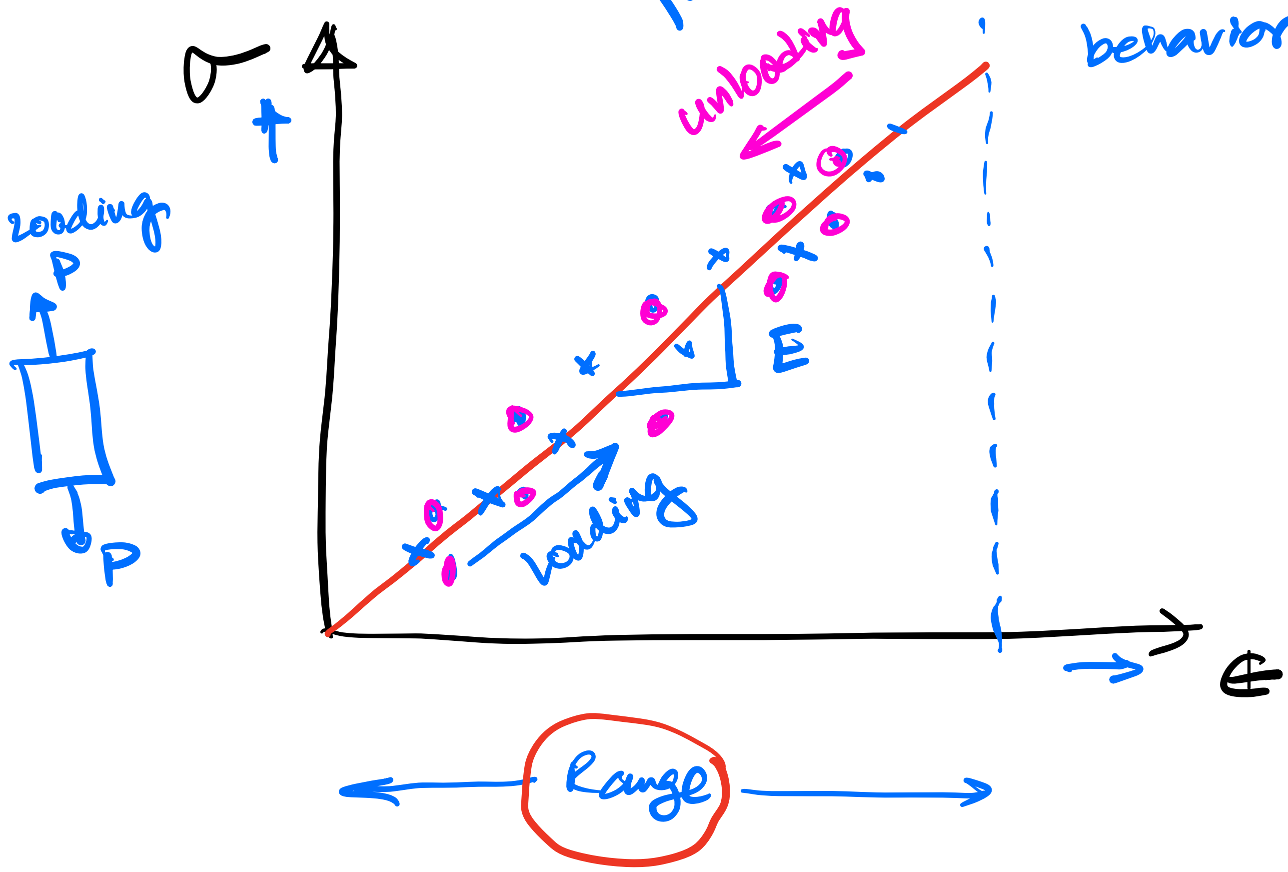
Assumptions in Linear Relationship between σ & ε:

- Material is homogenous → composition of material same through out it's geometry
- Material is elastic → Measureable deformation and ability to regain it's original shape
- Material is isotropic → Material property such as Young's Modulus don't change when direction of stresses change

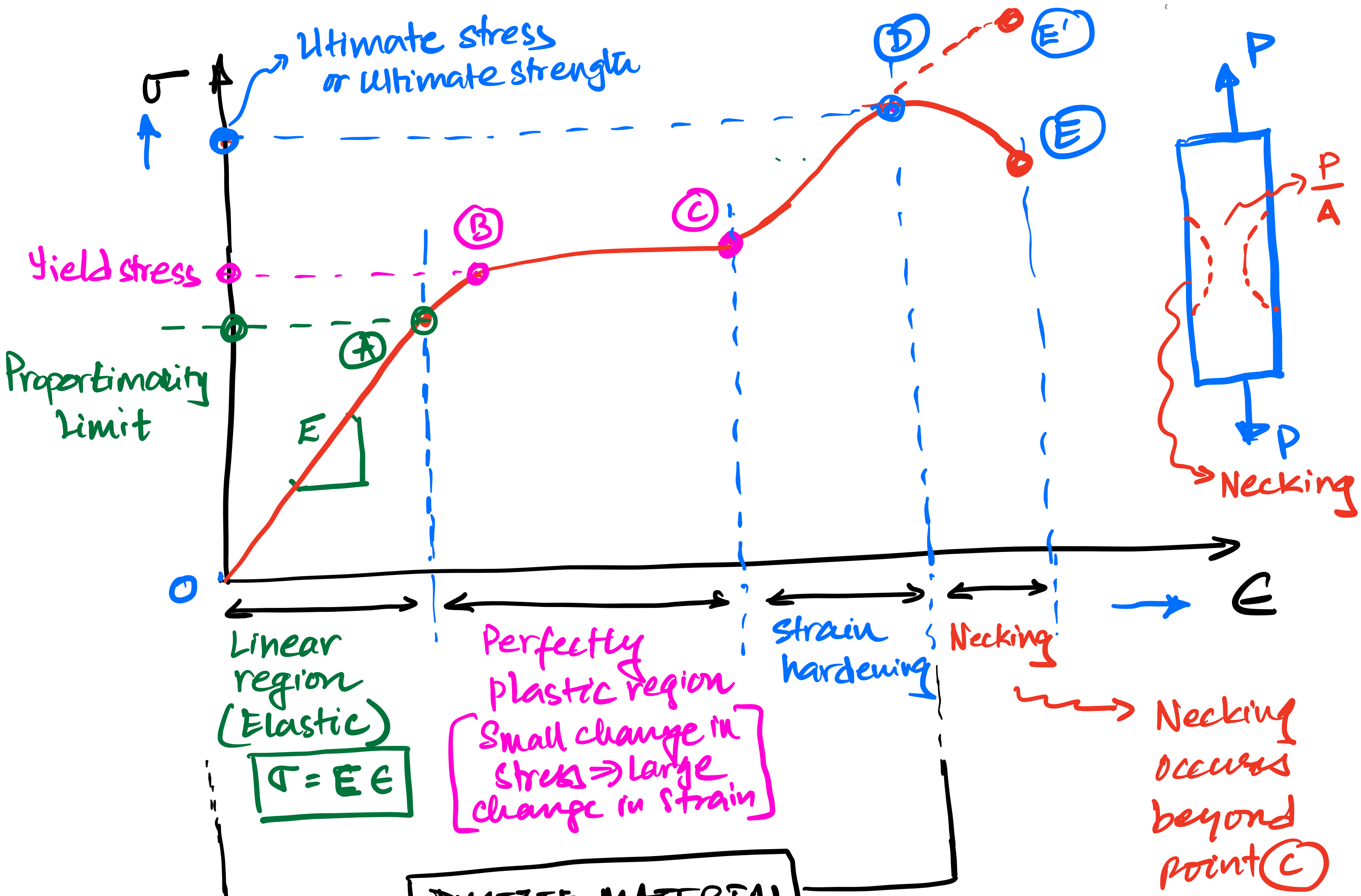
$$\boxed{\sigma = E\epsilon}$$

(sigma) (epsilon)

Linear elastic material behavior



STRESS VS STRAIN : (EXAMPLE: STEEL)



DUCTILE MATERIAL

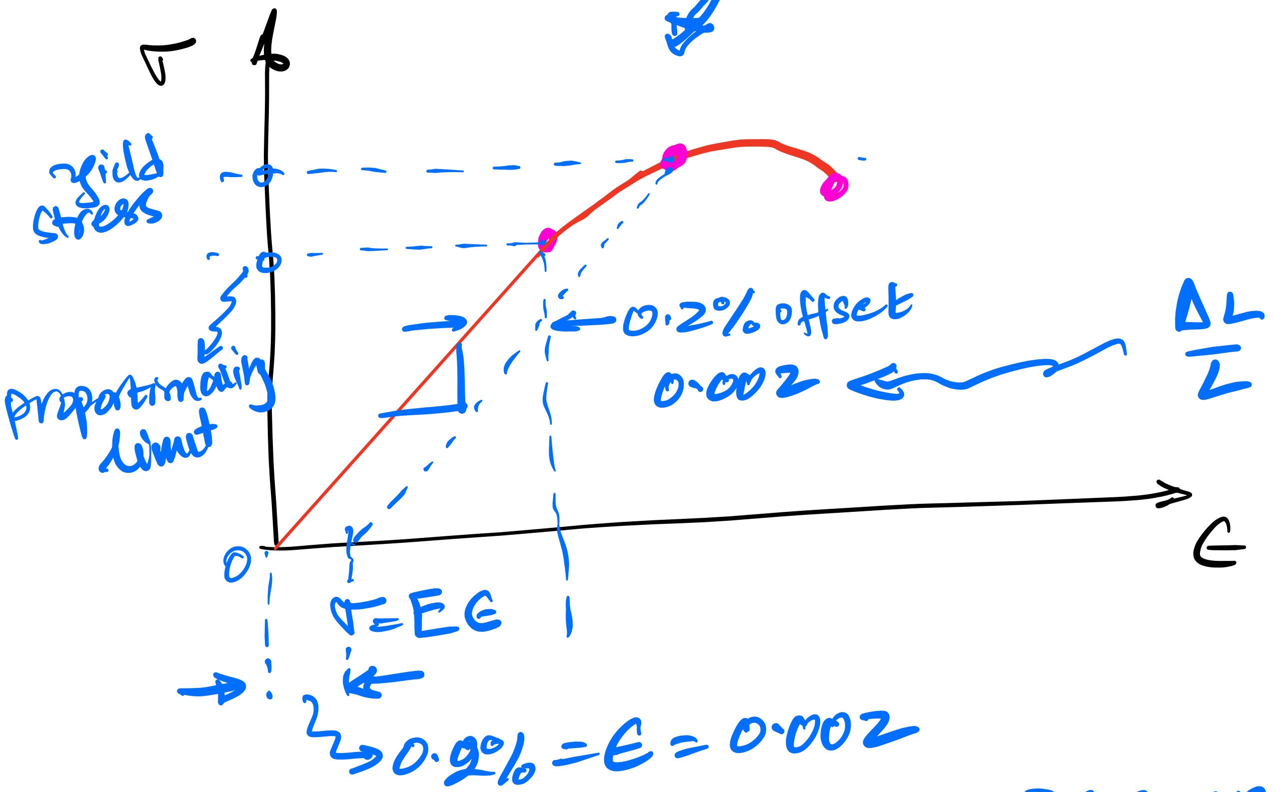
Material deforms significantly before reaching the ultimate stress and consequently, failing completely

Alternately, BRITTLE MATERIALS, fail drastically without showing any prolonged deformation characteristics

\# Generally, universal testing machines or Uniaxial testing machines (UTMs) are used to provide forces to generate this type of data & curves

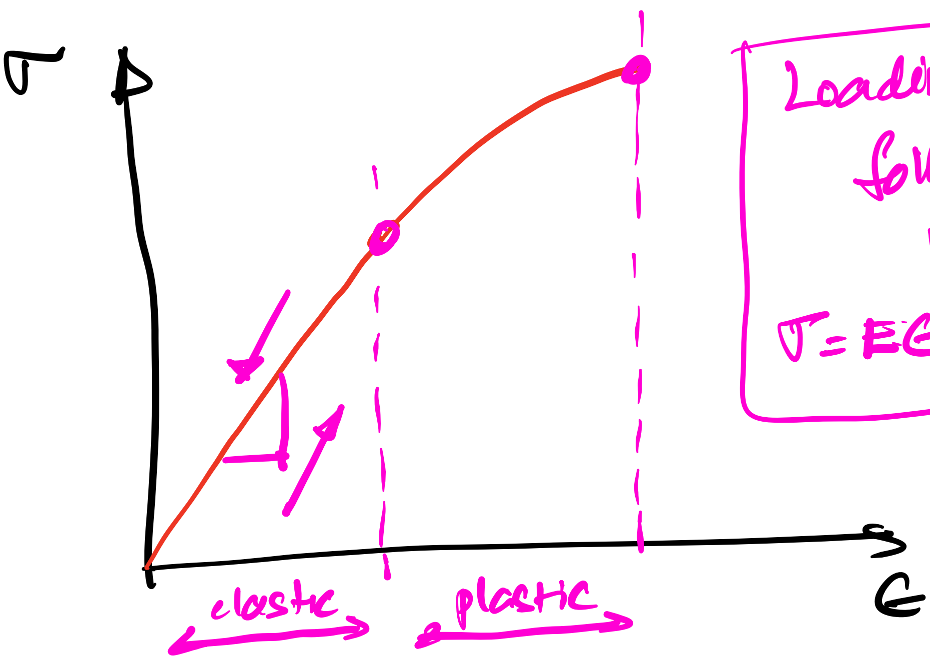
MATERIALS → DUCTILE

MATERIALS → BRITTLE

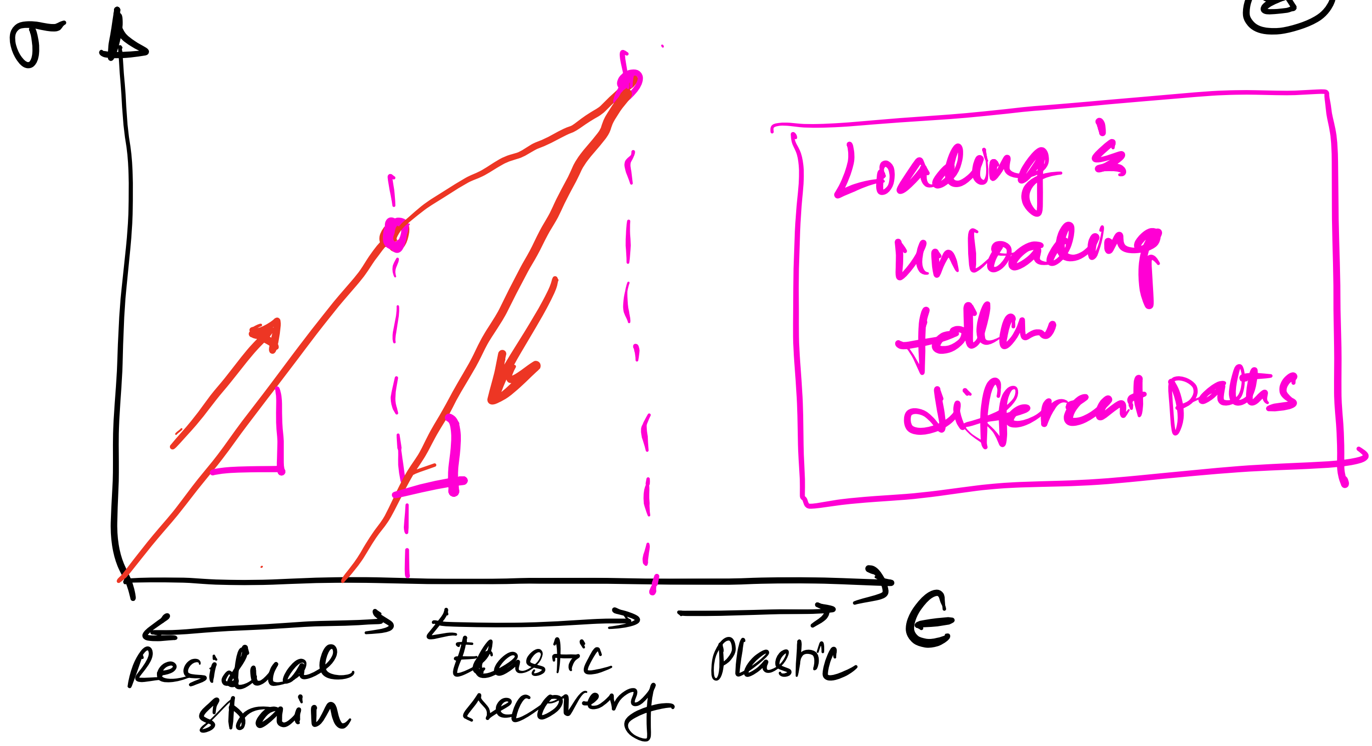


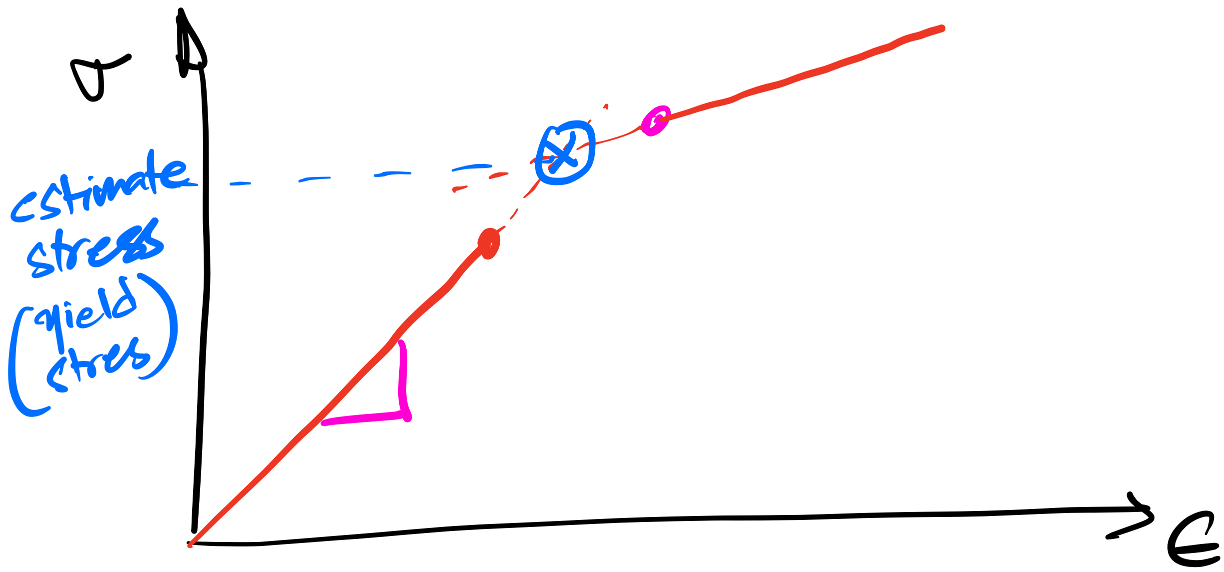
Draw a line parallel to $\sigma = E\epsilon$ curve

Other case scenarios:



Loading & unloading follow same path on the $\sigma = E\epsilon$, curve



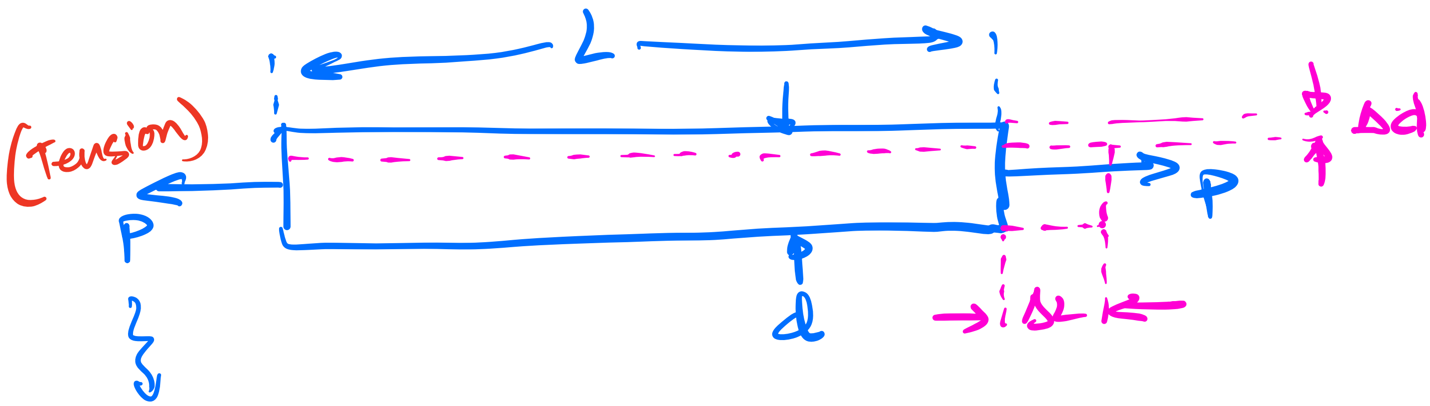


KEY TAKEAWAY:

* Depending on stress-strain characteristics material can be ductile or brittle.

KEY IDEA-4 Lateral strain & Poisson's ratio:

Consider (again) a prismatic bar:



Assumptions:
- acts uniformly over the entire section
- homogenous
- elastic

Recall:

$$\boxed{\text{Axial strain: } \epsilon_A = \frac{\Delta L}{L}}$$

$$\boxed{\text{Lateral strain: } \epsilon_L = \frac{\Delta d}{d}}$$

(*) We can quantify strain in the orthogonal direction w.r.t applied force

Defn:

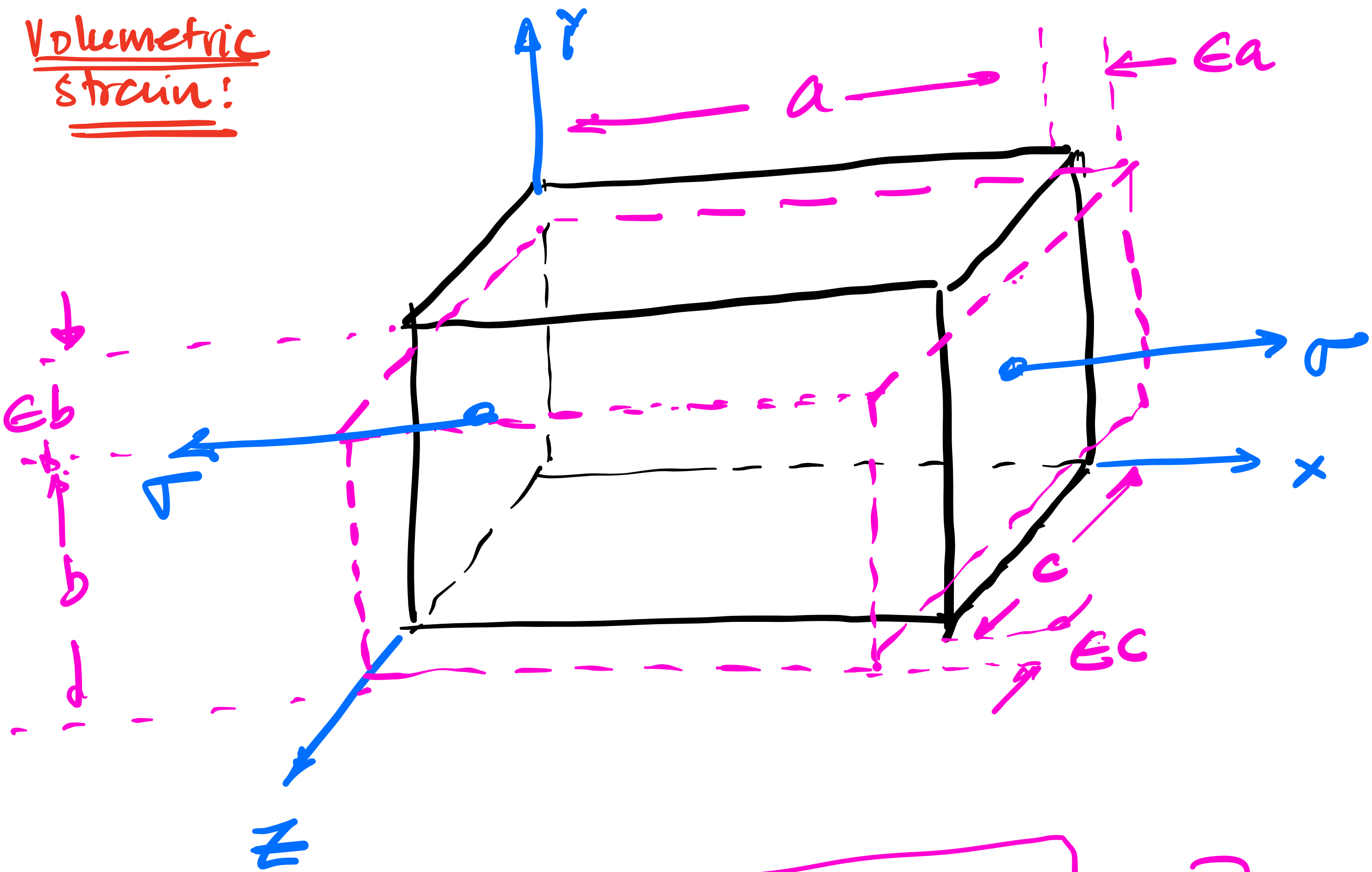
$$\boxed{\text{Poisson's ratio: } \nu = \frac{\text{Lateral strain}}{\text{Axial strain}}}$$

[So far we have two material properties viz.,
1. Young's modulus (E)
2. Poisson's Ratio (ν)]

KEY IDEA-5

Volumetric strain:

Volume change under uniaxial tension: (σ)



Initial Volume: $\boxed{abc = V_{in}}$ → (1)

Final Volume: $V_{fin} =$

$$a(1+\epsilon) \cdot b(1-\gamma\epsilon) \cdot c(1-\gamma\epsilon)$$

$$= abc(1+\epsilon)(1-\gamma\epsilon)^2$$

$$= abc(1+\epsilon)(1-2\gamma\epsilon+\cancel{\epsilon^2}) \to 0$$

Assume: $\epsilon \ll 1$ $\therefore$ $\epsilon^2 \rightsquigarrow$ negligible

11

$$\boxed{V_{fin} = abc(1+\epsilon-2\nu\epsilon)} \rightarrow 2$$

$$\Delta V = V_{fin} - V_{in}$$

$$= \{(abc)(1+\epsilon-2\nu\epsilon)\} - (abc)$$

$$\boxed{\Delta V = abc\,\underbrace{\epsilon}_{\text{axial strain}}(1-2\nu)} \rightarrow 3$$

$\nu \rightarrow$ Poisson's ratio

Volumetric Strain : $\boxed{e = \dfrac{\Delta V}{V_{in}}}$

$$e = \frac{\cancel{abc}\,\epsilon(1-2\nu)}{\cancel{abc}}$$

$$\boxed{e = \epsilon(1-2\nu)}$$

$$\boxed{\because\ \sigma = E\epsilon}$$

Stress

$$\boxed{e = \frac{\sigma}{E}(1-2\nu)}$$

Volumetric Strain

$\nu \rightarrow$ Poisson's ratio

$E \rightarrow$ Young's Modulus

Summarizing so far:

① Stress ⇝ Normal stress (σ)

② Strain ⇝ Axial strain (ϵ_A)
⇝ Lateral strain (ϵ_L)
⇝ Volumetric strain (e)

③ Hooke's Law: $\boxed{\sigma = E\,\epsilon}$
ϵ → Axial, Lateral or volumetric strain
E → Young's modulus

④ **Material Properties:**

E → Young's Modulus $\boxed{E = \frac{\sigma}{\epsilon}}$

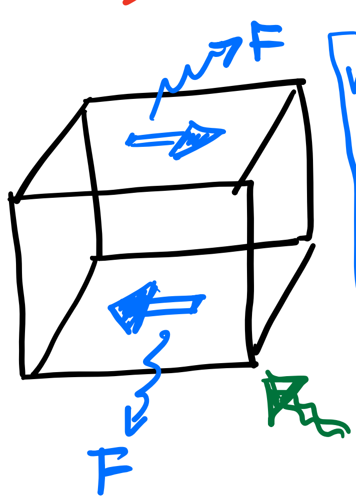
γ → Poisson's Ratio $\boxed{\gamma = \epsilon_L / \epsilon_A}$

KEY IDEA-6 **Shear Stress:**

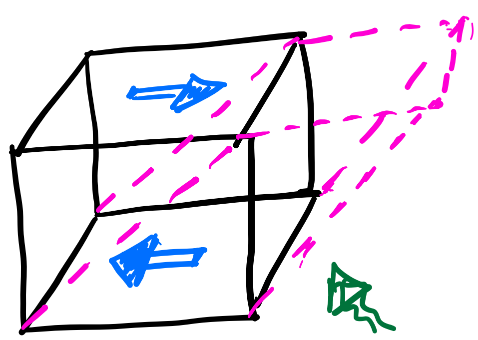
↳ So far we considered normal stress and strain(s) under uniaxial tension.

↳ Now we consider the stress developed along two planes parallel to each other under uniaxial tension.

↳ SHEAR STRESS



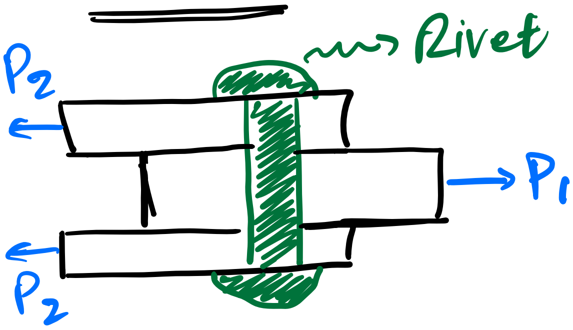
↳ Consider the volume shown
↳ Two forces are acting on the opposite surfaces
↳ Along the surface
↳ Over its entire surface area



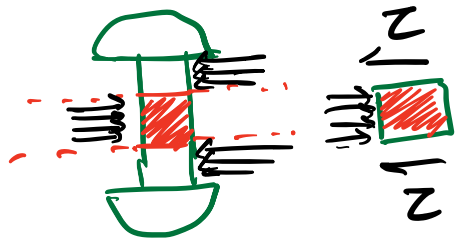
See how it deforms as shown with dotted lines

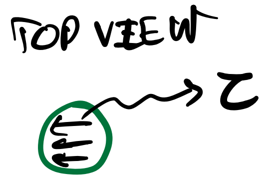
$$\boxed{\tau_{avg,\,shear} = \frac{F}{A}}$$

EXAMPLE: Three plates connected by a rivet

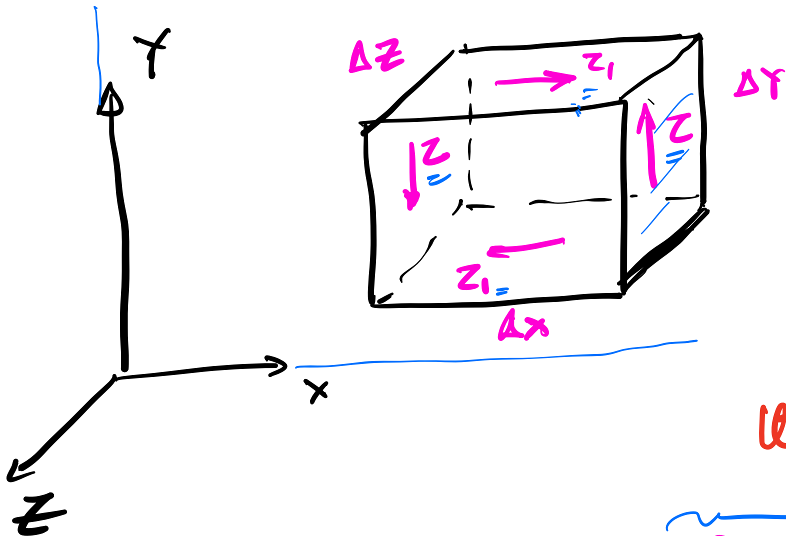


FBD of a section:





KEY IDEA - F Concept of the equality of cross-shears



Shear stress (τ) separated by Δx forms a couple!

We therefore calculate the moment:

$$M = \underbrace{(\tau)(\text{Surface area})}_{\text{Force}}\ \underbrace{(\text{distance})}_{\text{Lever arm}}$$

$$M = (\tau)(\Delta Y)(\Delta Z)(\Delta x)$$

$$M = \tau(\Delta x)(\Delta Y)(\Delta Z)$$

|||ly $M_1 = \tau_1 (\Delta x)(\Delta Y)(\Delta Z)$

Moment balance $\boxed{\Sigma M = 0} \Rightarrow M_1 = M$

$\boxed{\tau = \tau_1}$ → **Equality of cross-shears** ⇒ **No Rotation!**

x plane   y plane

KEY IDEA-8

Sign convention & Stress notation:

(σ)

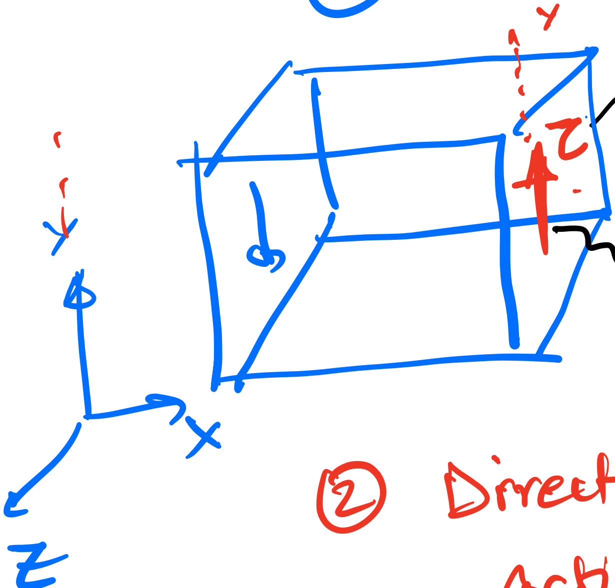
Axial stress (Sign convention)

Tension → +ve

Compression → −ve

Shear stress notation (τ)

① Plain on which it is acting



n ∥ x-axis

τ_{xy}

Shear stress τ_{xy}

② Direction along the axis it is acting

Axial or Normal stress notation

σ_{xx}

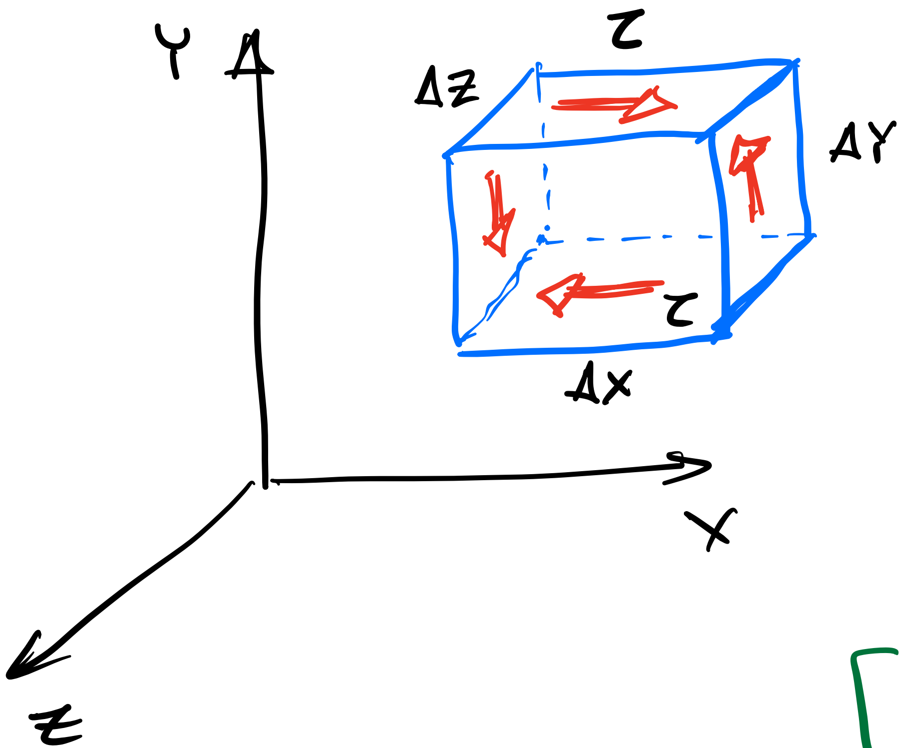
σ_{yy}

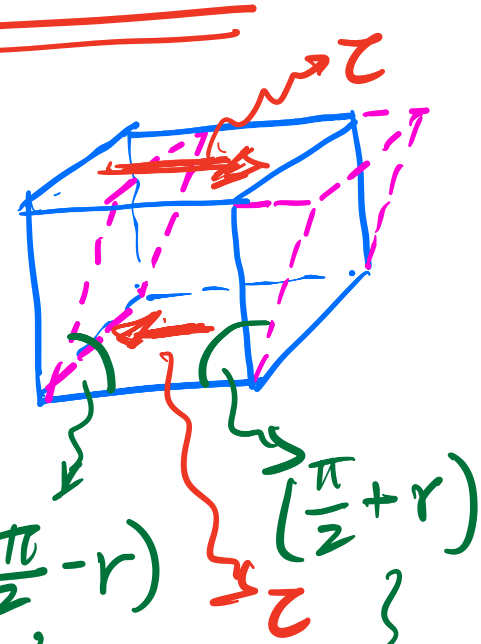
Plane on which it is acting

Direction along the axis its is acting.

KEY IDEA - 9

Hooke's Law (revisited):





[shear strain under the action of shear stress (τ)]

Recall: Hooke's Law for axial stress

$$\boxed{\sigma = E\epsilon}$$

σ → Axial or Normal stress

E → Young's modulus or Elastic modulus

ϵ → Volumetric strain

Hooke's Law for shear stress:

$$\boxed{\tau = G\gamma}$$

τ → shear stress

G → shear modulus

γ → shear strain

Summarizing so far:

① Stress
- Normal stress (σ)
- Shear stress (τ)

② Strain
- Axial strain (ϵ_A)
- Lateral strain (ϵ_L)
- Volumetric strain (e)
- Shear strain (γ)

③ Hooke's Law: $\boxed{\sigma = E\,\epsilon}$

σ → Normal stress
E → Young's modulus
ϵ → Axial, lateral or volumetric strain

$\boxed{\tau = G\gamma}$

τ → Shear stress
G → Shear modulus
γ → Shear strain

④ Material Properties:

E → Young's Modulus $\boxed{E = \frac{\sigma}{\epsilon}}$

ν → Poisson's ratio $\boxed{\nu = \epsilon_L / \epsilon_A}$

G → Shear modulus

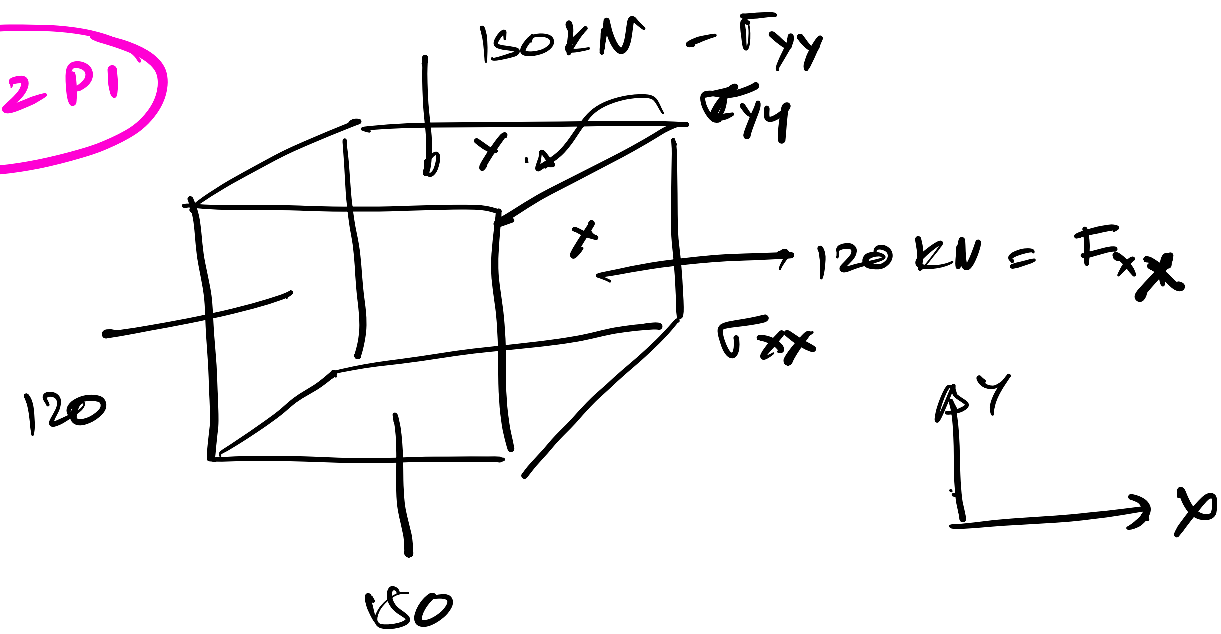
$$\boxed{G = \frac{E}{2(1+\nu)}}$$

E → Young's modulus
ν → Poisson's ratio

HINTS FOR IN CLASS PROBLEM SET-2

NOTE: ① σ_{xx} & τ_{xy} are presented as normal & shear stress in the written notes
② They are same as σ_{xx} & σ_{xy} in text book or in-class problem set

CW2 P1

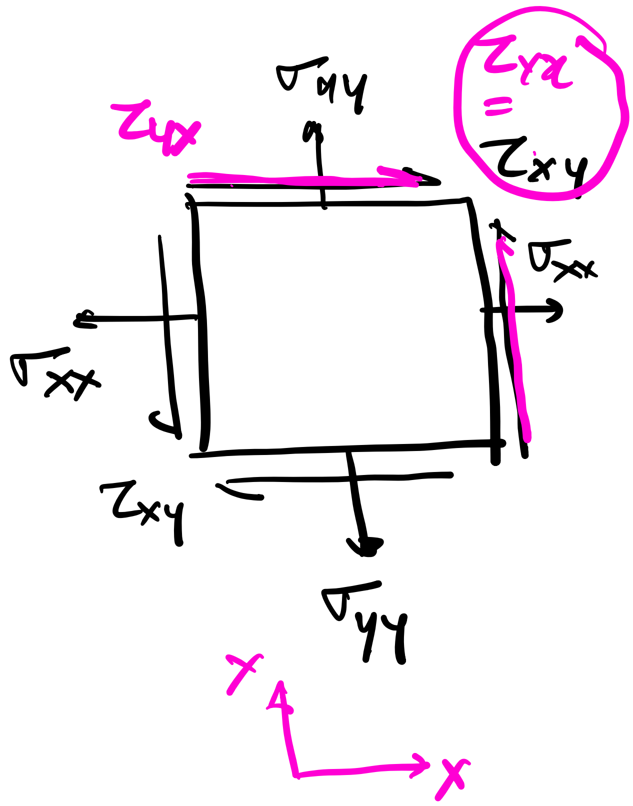


$120kPa = \sigma_{xx} = \dfrac{F_{xx}}{A}$

$150kPa = \sigma_{yy} = \dfrac{F_{yy}}{A}$

$$\boxed{\sigma_{xy} = \sigma_{yx} = 0}$$

$$\boxed{\tau_{xy} = \tau_{yx} = 0}$$



CW2 P3

$$\sigma_{xx}' = \sigma_{xx} \cos^2\alpha + 2\sigma_{xy} \sin\alpha \cos\alpha + \sigma_{yy} \sin^2\alpha$$

$$\sigma_{yy}' = \sigma_{xx} \sin^2\alpha - 2\sigma_{xy} \sin\alpha \cos\alpha + \sigma_{yy} \cos^2\alpha$$

$$\sigma_{xy}' = 2 \sin\alpha \cos\alpha \left(\frac{\sigma_{yy} - \sigma_{xx}}{2}\right) + \left(\cos^2\alpha - \sin^2\alpha\right) \sigma_{xy}$$

CW2 P3

$$\alpha_p = \frac{1}{\alpha} \tan^{-1}\left(\frac{\sigma_{xy}}{(\sigma_{xx} - \sigma_{yy})/2}\right)$$

$$\alpha_s = \frac{1}{\alpha} \tan^{-1}\left(\frac{\sigma_{yy} - \sigma_{xx}}{2\sigma_{xy}}\right)$$