MAE 3128 : BIOMECHANICS - I

Universal Solutions → { material properties were not considered }

GOALS:

① Generalized Equilibrium equations

→ Taylor series expansions

② Navier - Space Equilibrium equations

→ constitutive relationships

$\sigma = f(\epsilon)$

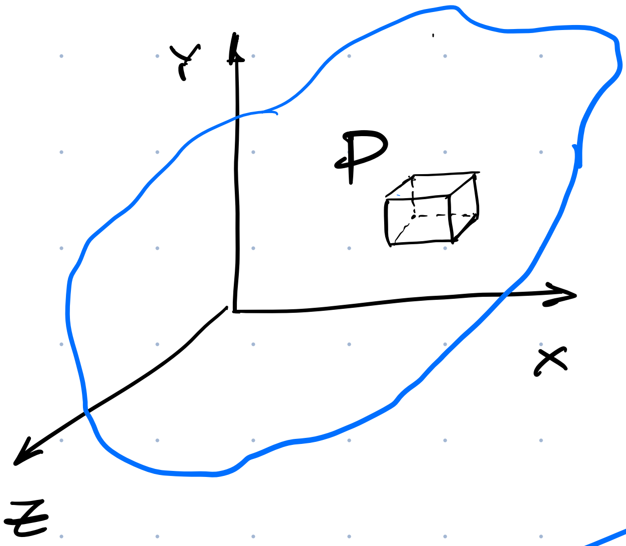
→ Material properties

BIOMECHANICS - I

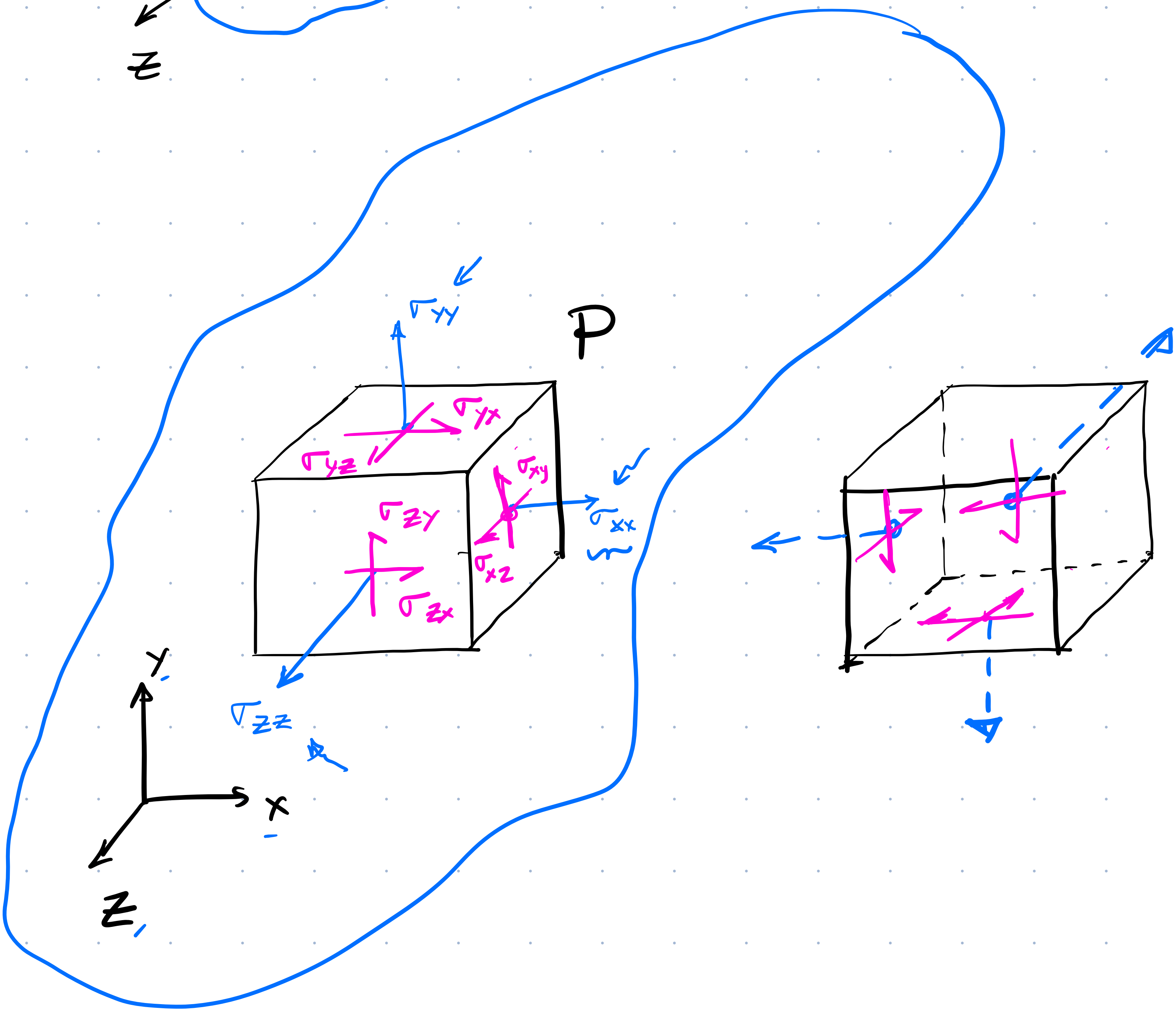
KVB©

(1)

From the previous lecture:



→ We considered a $\boxed{\text{point (P)}}$ with interface parallel to reference planes



normal stresses

$$\sigma_{ij} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$

plane normal to the axis

Direction of reference axis

STRESS TENSOR

2nd order tensor

shear stress

$$\epsilon_{ij} = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{bmatrix}$$

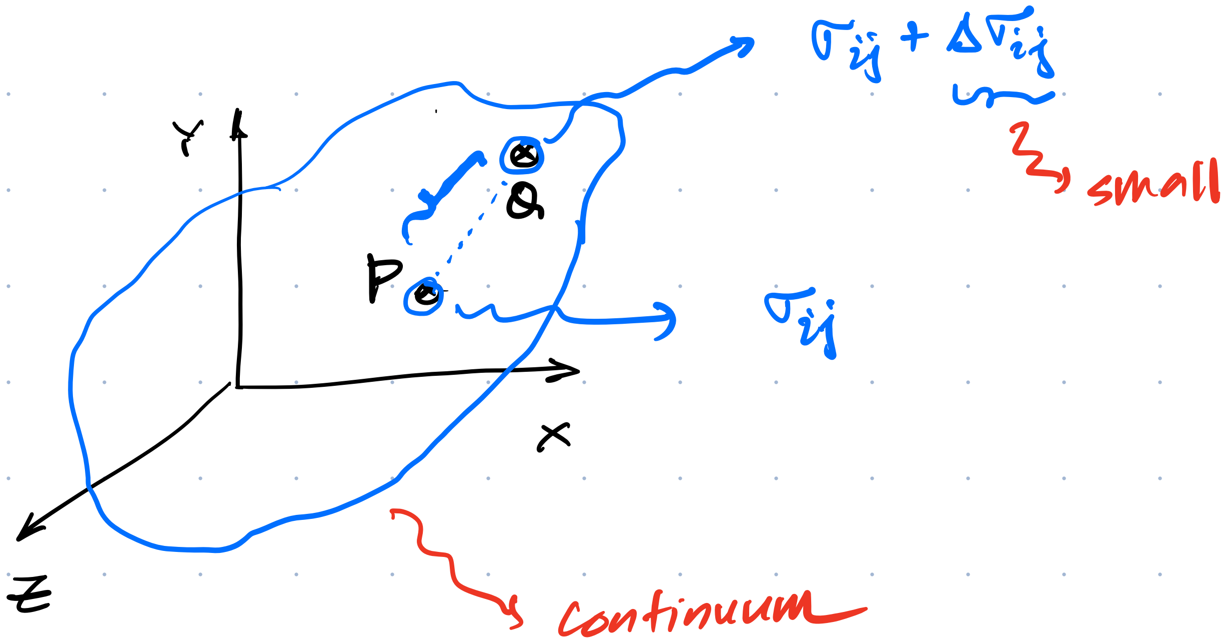
Diagonal terms:

- 3 Normal strains
- Dilation effect
- Change in volume
- No change in shape

Off-diagonal terms:

- 6 symmetric terms
- change in shape
- No change in volume

↳ But stress may vary from point, P to another point (say), Q within the body.



KEY IDEA - 1

How do we come up with a set of Generalized Equations of equilibrium, factoring

(i) Small stress variations

(ii) Within the continuum of the body ?

KEY IDEA – 1.1

(i) Define Taylor series expansion

(ii) Apply it to the stress field in appropriate co-ordinate system.

Defn:

Taylor series:

The Taylor series of a

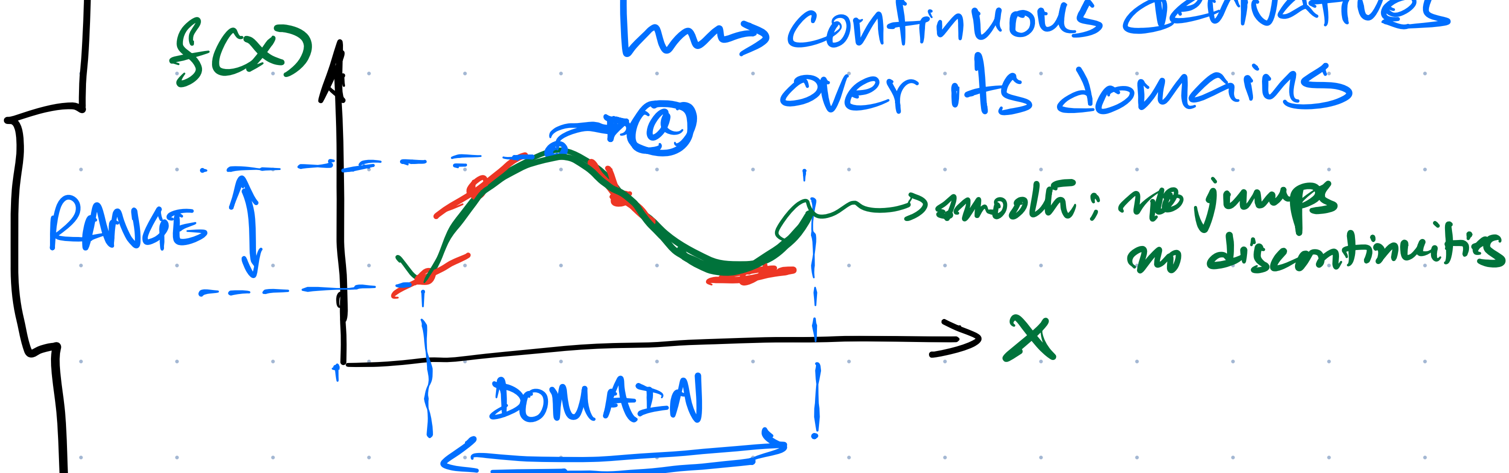
(i) REAL- OR COMPLEX-VALUED function

$f: \mathbb{R} \to \mathbb{R}$ $\qquad$ $g: \mathbb{C} \to \mathbb{C}$

(ii) That is infinitely divisible

→ smooth

→ continuous derivatives over its domains



(iii) at a real or complex number, "a" is a power series or infinite series

④

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots \to \infty$$

you want to know this

you know this

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

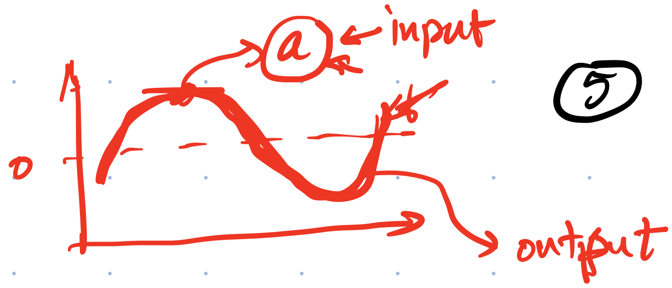
where

- $n!$ ⇝ factorial of n
- $0! = 1$
- $(x-a)^0 = 1$
- $f^{(0)}(a) = f(a)$ ⇝ Derivative of order-zero is f, itself.
- $(x-a) << 1$

$$f'(a) = f^{(1)}(a)$$
$$f''(a) = f^{(2)}(a)$$
$$f'''(a) = f^{(3)}(a)$$

Example

$$\boxed{f(x) = \sin(x)}$$



0th derivative   1st derivative   2nd derivative

(I) $$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots$$

(II) $\sin(x) \approx x$
for very small x: $\boxed{\sin(0) = 0}$
$\Rightarrow \boxed{a = 0}$

known point

$\boxed{f(x) = \sin(x)}$

(III)

$f'(x)$	$\cos(x)$	$f'(a) = \cos(0)$	$f'(a) = 1$ ✓
$f''(x)$	$-\sin(x)$	$f''(a) = -\sin(0)$	$f''(a) = 0$ ✓
3rd derivative $f'''(x)$	$-\cos(x)$	$f'''(a) = -\cos(0)$	$f'''(a) = -1$ ✓
4th derivative $f''''(x)$	$\sin(x)$	$f''''(a) = \sin(0)$	$f''''(a) = 0$ ✓

IV

$$\underbrace{\sin(x)}_{f(x)} = \cancelto{0}{\sin(0)} + \frac{\cancelto{1}{\cos(0)}}{1!}(x-0) + \left[\frac{\cancelto{0}{-\sin(0)}}{2!}\right](x-0)^2$$

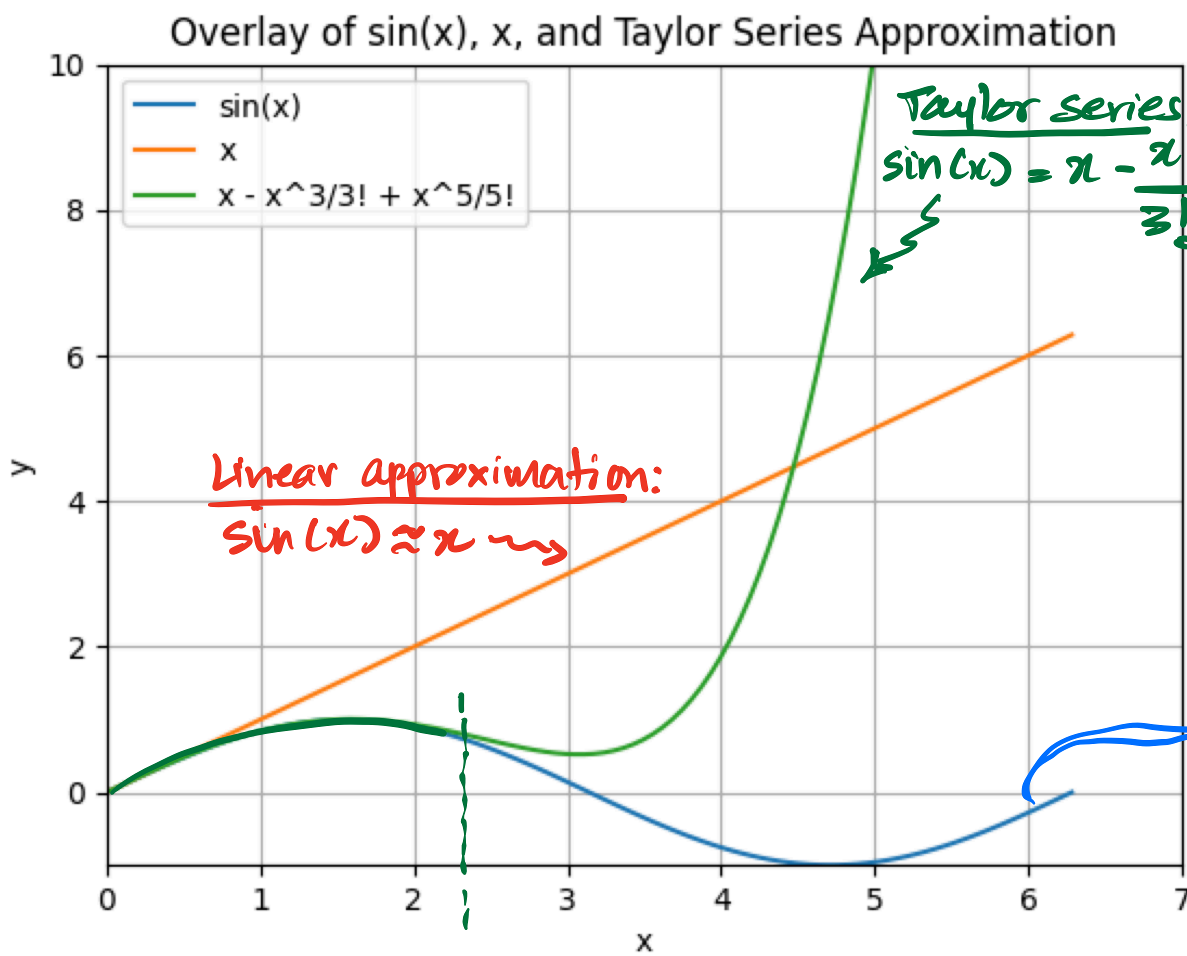
$$+ \left\{\frac{\cancelto{1}{-\cos(0)}}{3!}\right\}(x-0)^3 + \frac{\cancelto{0}{\sin(0)}}{4!}(x-0)^4$$

$+ \dots\dots$ HIGHER ORDER TERMS (HOT)

$$\boxed{\Rightarrow \sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots \text{HOT}}$$

approximated at a point $a=0$

We picked $a=0$
Recreated $\sin(x)$ w.r.t to $a=0$



Taylor series
$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!}$
$x \in [0, 2\pi]$

Linear approximation:
$\sin(x) \approx x$

$\sin(x)$
$x \in [0, 2\pi]$

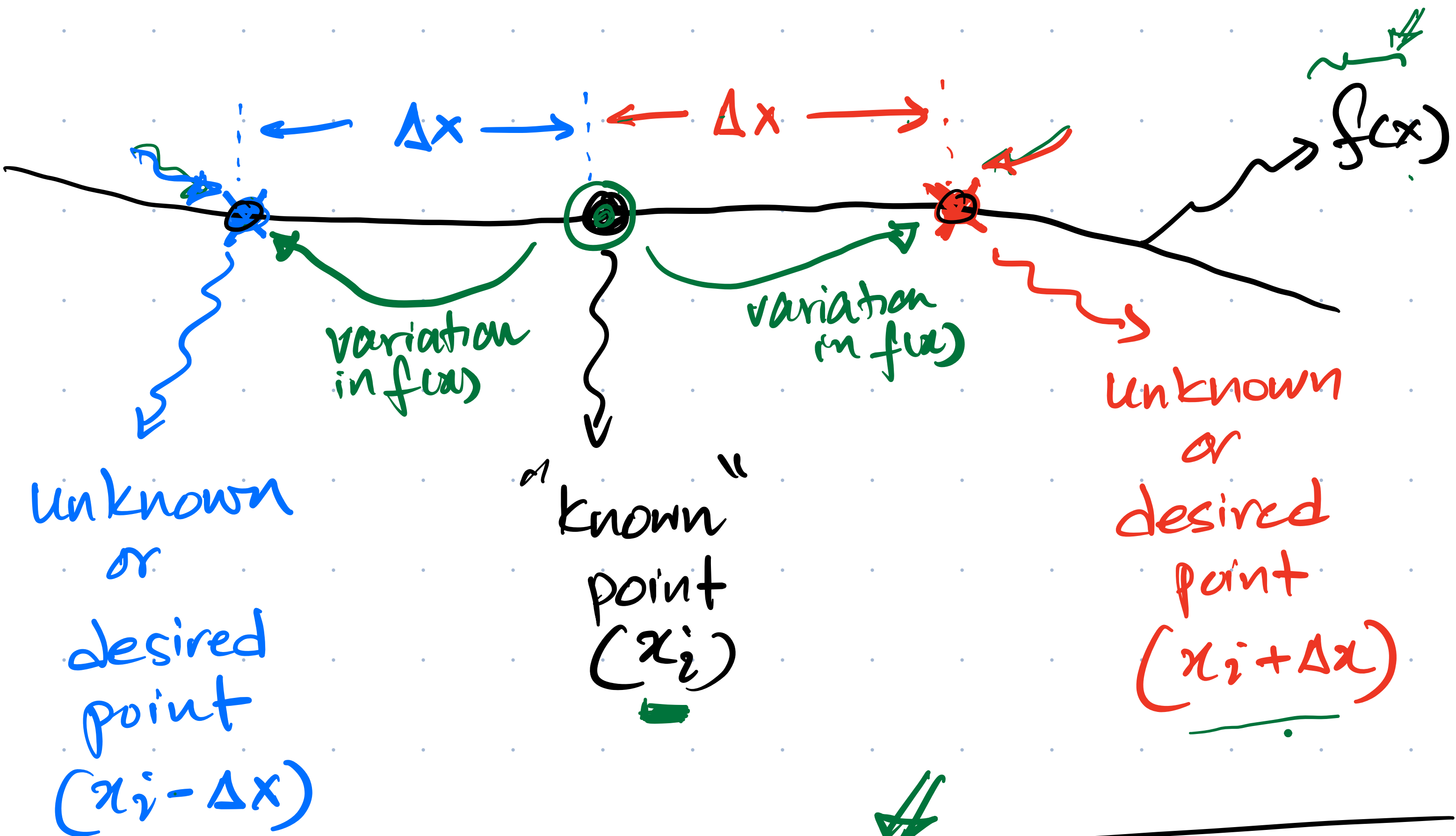
$x \approx 2.5$ (Not 2.5 radians!)

Taylor series in a spatial context:

$$f(x_i+\Delta x) = f(x_i) + \frac{\Delta x}{1!}f'(x_i) + \frac{(\Delta x)^2}{2!}f''(x_i) + \frac{(\Delta x)^3}{3!}f'''(x_i) + O(\Delta x)^4$$

$$f(x_i-\Delta x) = f(x_i) - \frac{\Delta x}{1!}f'(x_i) + \frac{(\Delta x)^2}{2!}f''(x_i) - \frac{(\Delta x)^3}{3!}f'''(x_i) + O(\Delta x)^4$$

where Δx = Distance between the "known" point, x_i, & the "desired" point, $x_i \pm \Delta x$



GOAL: We want to what's happening in the function f(x) at some ($x_i \pm \Delta x$) w.r.t (x_i) where know $f(x_i)$