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Εξόρυξη Δεδομένων Άσκηση 2

Ερώτηση 1

α)

$$L(\lambda|x_1 x_2 \dots x_n) = L(\lambda|x_1) L(\lambda|x_2) \dots L(\lambda|x_n)$$
$$= \lambda e^{-\lambda x_1} \cdot \lambda e^{-\lambda x_2} \dots \lambda e^{-\lambda x_n}$$
$$= \lambda^n [e^{\lambda x_1} e^{\lambda x_2} \dots e^{\lambda x_n}]$$
$$= \lambda^n \left[e^{\lambda (x_1 + x_2 + \dots + x_n)} \right]$$

$$\frac{d}{d\lambda} L(\lambda|x_1, x_2, \dots, x_n) = \frac{d}{d\lambda} \left(\lambda^n \left[e^{-\lambda (x_1 + x_2 + \dots + x_n)} \right] \right)$$

$$\frac{d}{d\lambda} \left(\log \left(\lambda^n \left[e^{-\lambda (x_1 + x_2 + \dots + x_n)} \right] \right) \right)$$

$$\frac{d}{d\lambda} \left(\log (\lambda^n) + \log \left[e^{-\lambda (x_1 + x_2 + \dots + x_n)} \right] \right)$$

$$\frac{d}{d\lambda} \left(n \log(\lambda) - \lambda (x_1 + x_2 + \dots + x_n) \right)$$

$$(n)' \log(\lambda) + (n) \log(\lambda)' - (\lambda)' (x_1 + x_2 + \dots + x_n)$$
$$0 + \frac{n}{\lambda} - (x_1 + x_2 + \dots + x_n)$$

$$\frac{n}{\lambda} - (x_1 + x_2 + \dots + x_n) = 0 \Rightarrow \lambda = \frac{n}{x_1 + x_2 + \dots + x_n}$$

$$\lambda = \frac{n}{\sum_{i=1}^{n} x_i}$$

Β) $f(x) = \lambda e^{-\lambda x}$ $X = \{x_1, x_2, \dots, x_n\}$; $x_i \geq 0$, $\vartheta = (\lambda_1, \lambda_2, \pi_1, \pi_2)$

π_1, π_2 πιθανότητες μίξης με $\pi_1 + \pi_2 = 1$

λ_1, λ_2 παράμετροι

$$P(x|\vartheta) = \pi_1 \lambda_1 e^{-\lambda_1 x} + \pi_2 \lambda_2 e^{-\lambda_2 x}$$

$$P(L_k|x_i) = \frac{\pi_k \lambda_k e^{-\lambda_k x_i}}{\pi_1 \lambda_1 e^{-\lambda_1 x_i} + \pi_2 \lambda_2 e^{-\lambda_2 x_2}} \quad \text{για } k = 1, 2$$

Βήμα Ε: (Expectation)

$\gamma_{i1} = P(L_1|x_i)$ $\gamma_{i2} = P(L_2|x_i) = 1 - \gamma_{i1}$

$$\gamma_{i1} = \frac{\pi_1 \lambda_1 e^{\lambda_1 x_i}}{\pi_1 \lambda_1 e^{-\lambda_1 x_i} + \pi_2 \lambda_2 e^{-\lambda_2 x_i}}$$

Βήμα Μ: (Maximization)

$$\pi_1 = \frac{1}{n} \sum_{i=1}^{n} \gamma_{i1} = \frac{1}{n} \sum_{i=1}^{n} \frac{\pi_1 \lambda_1 e^{\lambda_1 x_i}}{\pi_1 \lambda_1 e^{-\lambda x_i} + \pi_2 \lambda_2 e^{-\lambda_2 x_i}}$$

$$\pi_2 = 1 - \pi_1 = \frac{1}{n} - \frac{1}{n} \sum_{i=1}^{n} \frac{\pi_1 \lambda_1 e^{\lambda_1 x_i}}{\pi_1 \lambda_1 e^{-\lambda x_i} + \pi_2 \lambda_2 e^{-\lambda_2 x_i}}$$

Όπου λ_1 και λ_2

$$\lambda_1 = \frac{\sum_{i=1}^{n} \gamma_{i1}}{\sum_{i=1}^{n} \gamma_{i1} x_i}$$

$$\lambda_2 = \frac{\sum_{i=1}^{n} \gamma_{i2}}{\sum_{i=1}^{n} \gamma_{i2} x_i}$$