# Problem Set-5

**PH11001** (Spring 2019-20)

*Electrodynamics, Electromagnetic waves.*

February 10, 2020

1. *Flux through a given surface*

   (a) Consider a tiny (point) dipole sitting at the origin, pointing towards the $z$ axis. Consider the distance between the positive and negative charge $d \ll 1$ units. The dipole moment of the dipole is given by $\vec{p} = p\hat{k}$. Its Electric field is given by

   $$\vec{E}(x,y,z) = \frac{1}{4\pi\epsilon_0}\left(\frac{3(\vec{p}\cdot\hat{r})\hat{r}}{r^3} - \frac{\vec{p}}{r^3}\right)$$

   Find the flux of this electric field through the *hemisphere* given by $x^2+y^2+z^2 = 1,\ z \geq 0$.

   (b) Consider spherical polar coordinates $(r,\theta,\phi)$, which has the usual relation with cartesian coordinates $(x,y,z)$. Also $(\hat{\imath},\hat{\jmath},\hat{k})$ are unit vectors along $x, y$ and $z$ respectively. Now the magnetic field in a region is given by

   $$\vec{B}(r,\theta,\phi) = B_0\left(-\sin\phi\ \hat{\imath} + \cos\phi\ \hat{\jmath}\right),$$

   where $B_0$ is a constant. Calculate the flux of this magnetic field through the $x = 0$ plane (in the positive $x$-direction).

2. *Faraday's law*

   A current configuration creates a time-dependent electric and magnetic field given, in cylindrical polar coordinates, by (consider spherical polar coordinates $(r,\theta,\phi)$)

   $$\vec{B} = \frac{e^{-t}}{r}\ \hat{k}$$

   Now consider a moving loop of wire $C(t)$ in this electric and magnetic field. This loop is a circular loop and it lies in the $z = 0$ plane with its center at the origin, and it has a time varying radius $R(t) = 1+t$. Find the induced EMF in this loop. Clearly explain the origin(s) of this EMF in the loop.

3. *Electromagnetic waves*

   The Electric and magnetic field in a region is given by

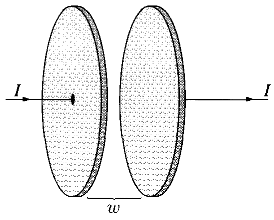
   $$\vec{E} = E_0\ \hat{\imath}\cos\left(\omega\left(t - \frac{z}{c}\right)\right),\ \vec{B} = B_0\ \hat{\jmath}\cos\left(\omega\left(t - \frac{z}{c}\right)\right),$$

   where $E_0, B_0$ and $\omega$ are constants and $c$ is the speed of light in vacuum.

   Calculate the divergence and curl of $\vec{E}$ and $\vec{B}$ in the given region. From your answer, can you predict the charge density and current density in the given region?

4. *Poynting Vector*

   Consider a steady current $I$ resulting in the charging of a circular parallel plate capacitor as shown below. The charge is zero at time $t = 0$, when the steady current starts flowing and a uniform surface charge density starts developing with the flow of the current. An external agency is responsible to maintain the steady current.

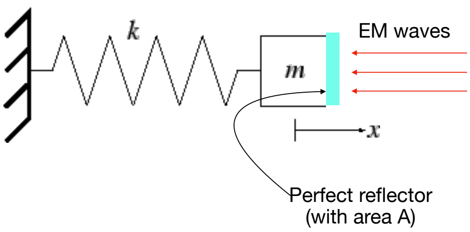



   (a) Find the electric and magnetic fields in the gap, as a function of time. Ignore any edge effects (you can thinking of finding the electric and magnetic fields deep inside the gap away from the edges).

   (b) Calculate the energy density and the Poynting vector inside the gap.

5. *Radiation pressure*

   Consider a situation as shown in the following figure. A perfect reflector is attached to the mass of a spring-mass system; a beam of plane electromagnetic waves is incident on the reflector and reflected away.




   The beam of plane electromagnetic wave is incident on the mass $m$ at time t=0 and is kept on upto time $t = \tau$. At $t = 0$, the mass $m$ is at rest and is located at the equilibrium position of the spring. The electric field in the beam of light is given by

$$\vec{E} = E_0 \ \hat{k} \cos\left(\omega\left(t - \frac{x}{c}\right)\right)$$

   Find the amplitude of oscillation after for time $t > \tau$.

   Note that we should assume $\omega \gg \sqrt{\frac{k}{m}}$ and the area $A$ is big enough so that it can be considered as a macroscopic object where radiation pressure due to the beam can act.