Solution of Tutorial 10- for PH11001 course
Spring 2020, IIT Kharapur
April 12, 2020

Question 1.
Unpolarized light with an intensity of $I_0 = 16W/m2$ is incident on a pair of polarizers. The first polarizer has its transmission axis aligned at 50° from the vertical. The second polarizer has its transmission axis aligned at 20° from the vertical. What is the intensity of the light when it emerges from the second polarizer?

Solution.
For the unpolarised light (intensity I_0) the first polariser, polarises it along its transmission axis and the intensity is

$$I_1 = \frac{I_0}{2}$$

Now, the second polariser acts as an analyser to this polarised light. The relative angle between the polariser and analyser is 30°, according to Malus's law the final intensity will be

$$I_2 = I_1 \cos^2 30^\circ = \frac{3I_0}{8} = 6W/m^2$$

Question 2.
What kind of polarization has an electromagnetic wave if the projections of the vector $\vec{E}$ on the x and y axes perpendicular to the direction of propagation are:

(a) $E_x = E\cos(\omega t - kz), E_y = E\sin(\omega t - kz)$

(b) $E_x = E\cos(\omega t - kz), E_y = E\cos(\omega t - kz + \pi/4)$

(c) $E_x = E\cos(\omega t - kz), E_y = E\cos(\omega t - kz + \pi)$

Solution.

(a) $E_x = E\cos(\omega t - kz), E_y = E\sin(\omega t - kz)$
We can see that on the plane perpendicular to the propagation, the $\vec{E}$ of the wave has a constant magnitude but its direction rotates at a constant rate

$$E_x^2 + E_y^2 = E^2$$

This is a (anti-clockwise) circular polarisation (refer to Fig. 1).

(b) $E_x = E\cos(\omega t - kz), E_y = E\cos(\omega t - kz + \pi/4)$

$$\begin{aligned} E_y &= E\cos(\omega t - kz)\frac{1}{\sqrt{2}} - E\sin(\omega t - kz)\frac{1}{\sqrt{2}} \\ E_y &= E_x\frac{1}{\sqrt{2}} - \sqrt{E^2 - E_x^2}\frac{1}{\sqrt{2}} \\ E^2 - E_x^2 &= \left(E_x - \sqrt{2}E_y\right)^2 \\ E^2 &= 2E_x^2 + 2E_y^2 - 2\sqrt{2}E_xE_y \end{aligned}$$

This is an (clockwise) ellipse equation, so this is an elliptical polarisation (refer to Fig. 2).

(c) $E_x = E\cos(\omega t - kz), E_y = E\cos(\omega t - kz + \pi)$

$$E_y = E\cos(\omega t - kz + \pi) = -E\cos(\omega t - kz) = -E_x$$

This is linear polarisation.

Question 3.
Unpolarized light of intensity I_0 is incident normally on three polarizers P1, P2 and P3 all arranged in series. The pass axis of each polarizer makes an angle of 45° with the earlier sheet (in the same diection).

(i) What is the intensity of the transmitted beam?

(ii) What is the intensity of the transmitted beam if the polarizer P2 is replaced with a quarter wave plate Q2 with its optic axis along the pass axis of P2?

Solution.
(i) The intensity of the unpolarized light after
P1 : $\frac{I_0}{2}$ (linearly polarized)
P2 : $\frac{I_0}{2}\cos^2 45° = \frac{I_0}{4}$ (linearly polarized)
P3 : $\frac{I_0}{4}\cos^2 45° = \frac{I_0}{8}$ (linearly polarized)
(ii) The intensity of the unpolarized light after
P1 : $\frac{I_0}{2}$ (linearly polarized)
Q2 : $\frac{I_0}{2}$ (circularly polarized : as quarter wave plate does not change the intensity but the polarization)
P3 : circularly polarized light is a superposition of two linearly polarized light which are out of phase by $\delta = \pi/2$. Let's suppose that one component of the electric field say E_x, makes an angle of θ with the pass axis of P3 , then the other perpendicular component E_y makes an angle of $\frac{\pi}{2} - \theta$ with the pass axis. The intensity due to E_x will be $I_x = \frac{I_0}{2}\cos^2\theta$ where as the intensity due to E_y will be $I_y = \frac{I_0}{2}\sin^2\theta$. So the resultant intensity of the transmitted light will be $I = I_x + I_y + 2\sqrt{I_x I_y}\cos\delta = \frac{I_0}{2}$ (linearly polarized).

Question 4.
Carvone molecule is a chiral molecule, which has two enantiomers, S and R. The specific rotation of $(S)-Carvone$ is $(+)61°$ (measured "neat", i.e. without any solvent). The optical rotation of a "neat" sample of a mixture of (S) and $(R)-Carvone$ is measured as $(-)23°$. What are the percentages of (S) and $(R)-Carvone$ in the sample?

Solution.
The observed rotation of the mixture is negative (counter-clockwise), and the specific rotation of the pure $(S)-Carvonne$ enantiomer is given as positive (clockwise), meaning that the specific rotation of pure $(R)-Carvonne$ enantiomer must be negative, and the mixture must contain more of the R enantiomer than of the S enantiomer.

$$Rotation(mixture sample) = [Fraction(S) \times Rotation(S)] + [Fraction(R) \times Rotation(R)]$$
$$\text{Let } Fraction(S) = x, \text{therefore } Fraction(R) = 1-x$$

$$\begin{aligned} -23° &= +61° \times x + (-61°)(1-x) \\ x &= \frac{38}{122} = 0.311475 \quad \text{and, } 1-x = 0.68852 \end{aligned}$$

There is 31.15% of $(S) - Carvone$ and 68.85% of $(R) - Carvonne$ in the mixture sample.

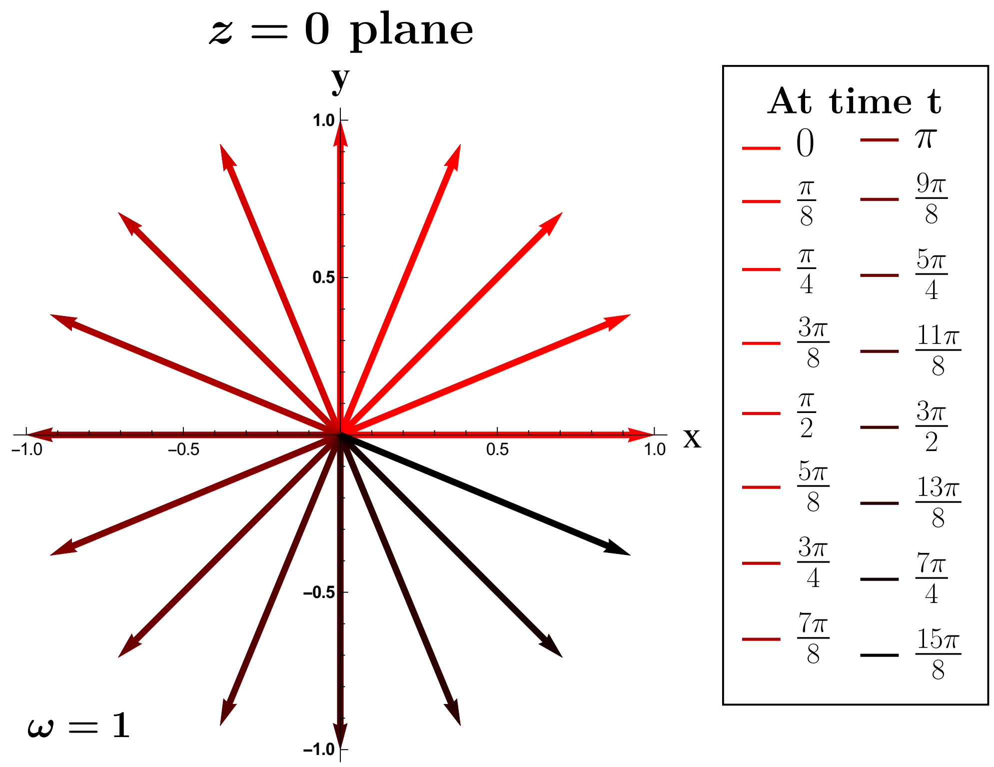

Figure 1: Prob 2a : Electric Field vectors for $\omega = 1$ (say)

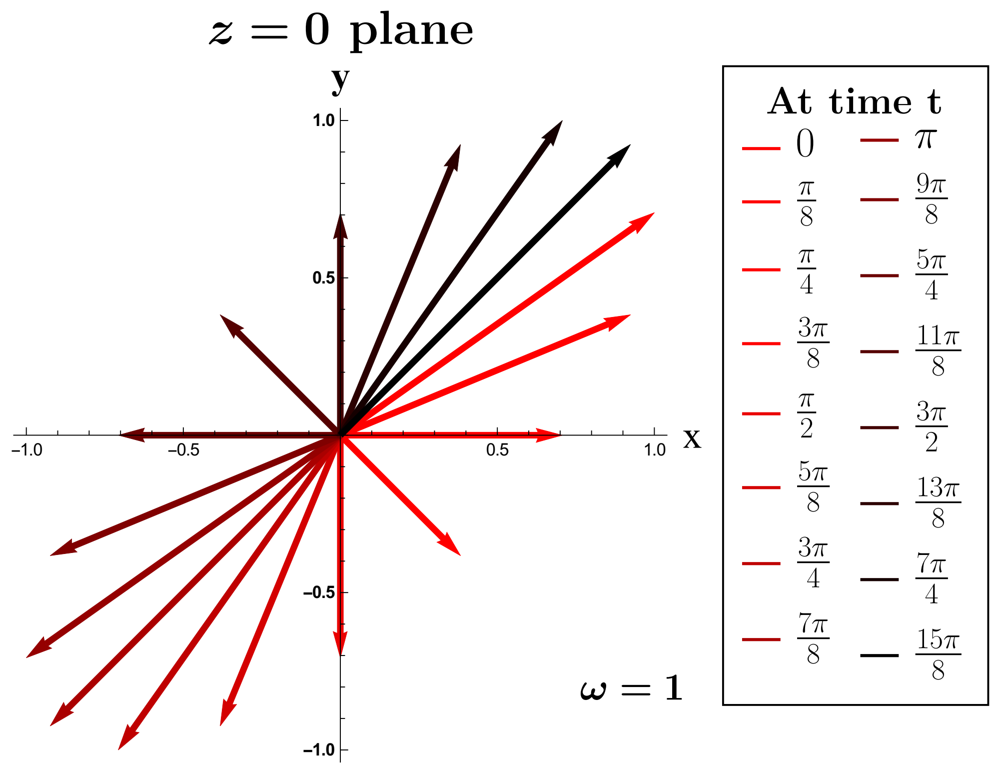

Figure 2: Prob 2b : Electric Field vectors for $\omega = 1$ (say)