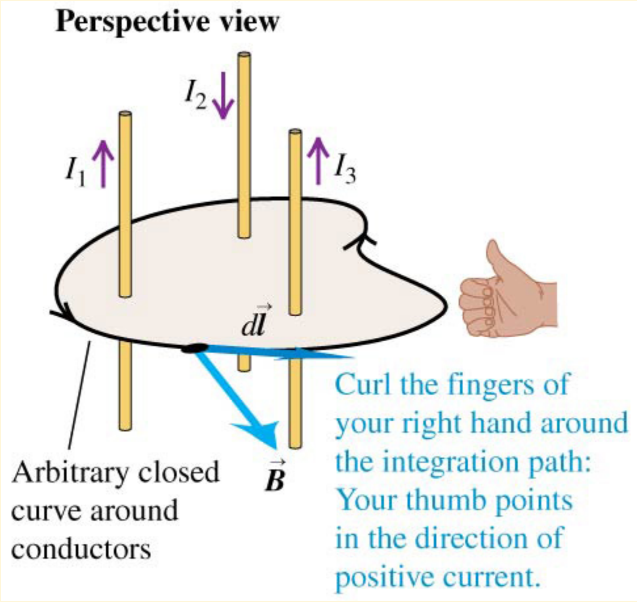
Amperes Law

One can reformulate the Biot-Savarts Law in the integral form as follows:



Line integral around a closed path

Ampere's law: $$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\mathrm{encl}}$$

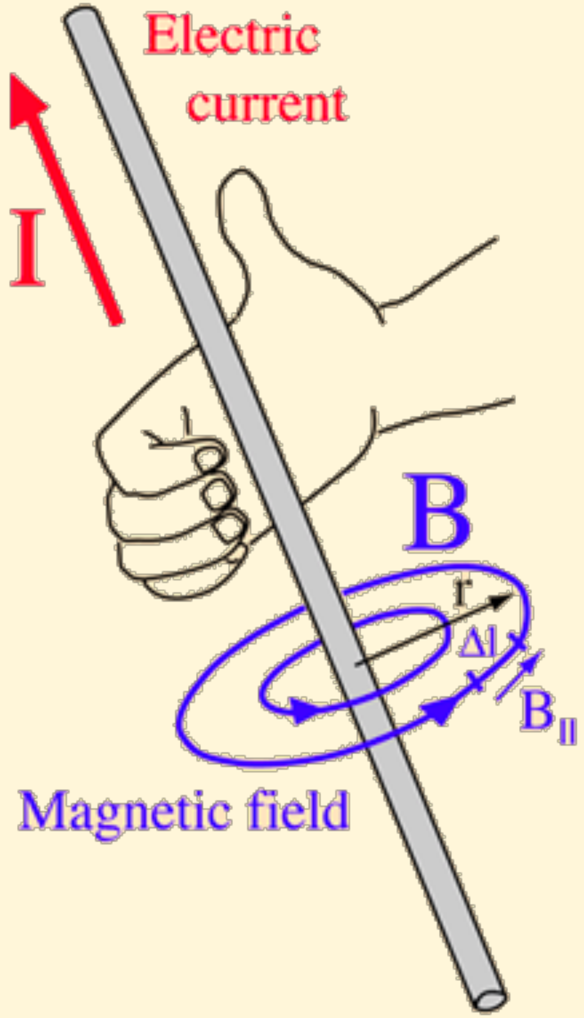
Magnetic constant

Net current enclosed by path

Scalar product of magnetic field and vector segment of path

Amperes Law application I: The infinite wire

Note the direction of the magnetic field is given by the right hand thumb rule as shown in the diagram here. Given this direction of the magnetic field we can now use the Amperes law to determine the magnetic field due to the infinite wire.



$$\oint \vec{B}.d\vec{l} = \mu_0 I$$

$$B \oint \hat{B}_{||}.\hat{l}dl = \mu_0 I$$

$$B \oint dl = B2\pi r = mu_0 I$$

$$\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{B}_{||}$$

Thick infinite wire

Inside the conductor:

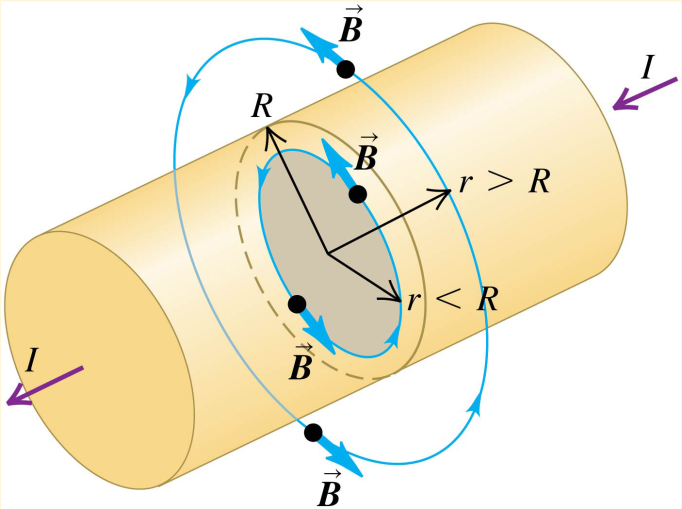
$$\oint \vec{B}.d\vec{l} = \mu_0 I_{enclosed}$$

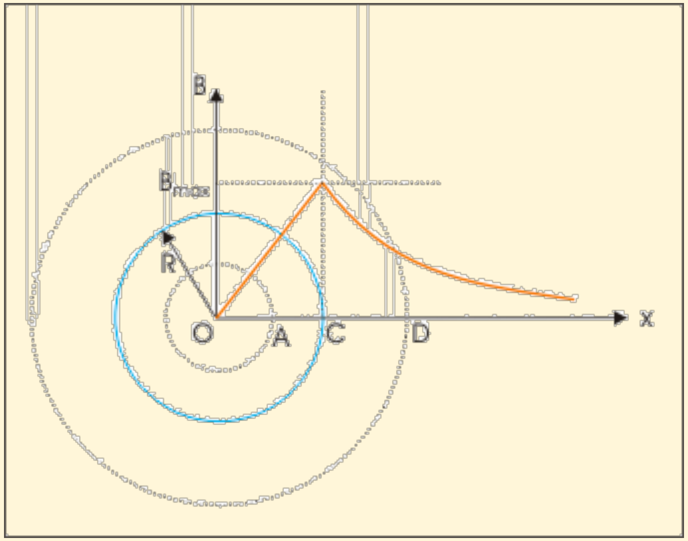
$$B(2\pi r) = \mu_0 \left(I \frac{r^2}{R} \right)$$

$$B = \frac{\mu_0 I}{2\pi} \frac{r}{R^2} \qquad \text{(inside the conductor, } r < R\text{)}$$

Outside the conductor:
Following what we did previously

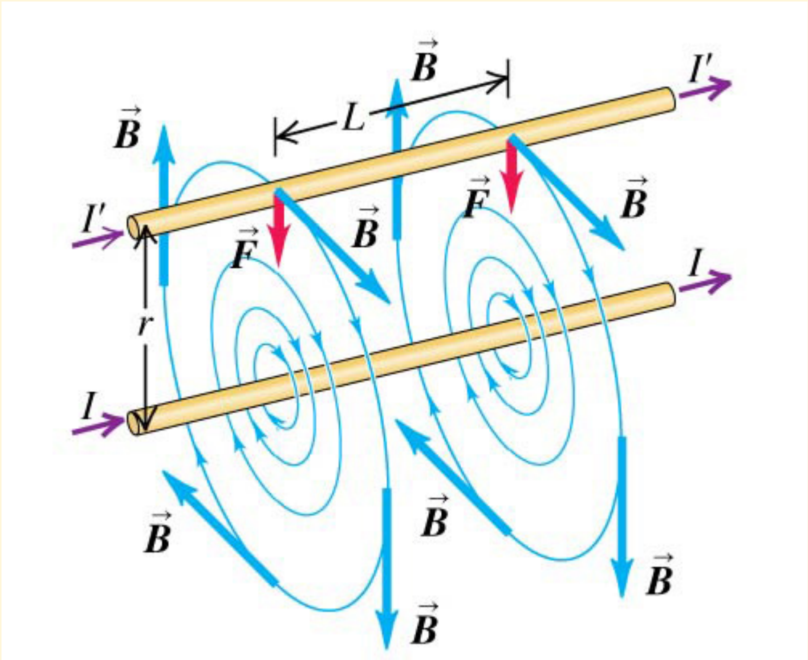
$$B = \frac{\mu_0 I}{2\pi r} \qquad \text{(outside the conductor, } r > R\text{)}$$





Force on one wire due to another

- The figure shows segments of two long, straight, parallel conductors separated by a distance r and carrying currents I and I' in the same direction.
- Each conductor lies in the magnetic field set up by the other, so each experiences a force.



Using the Force Law:

Magnetic constant

Current in first conductor

Magnetic force per unit length between two long, parallel, current-carrying conductors

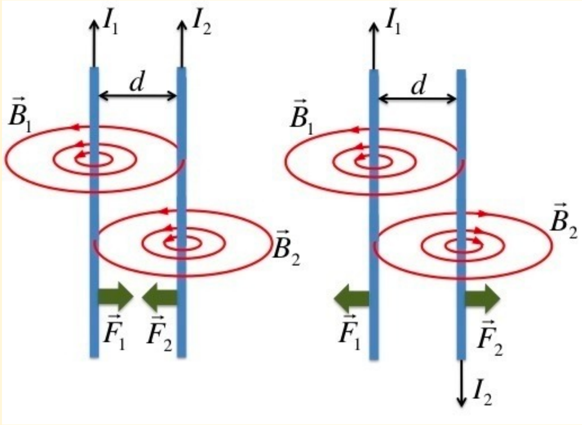
$$\frac{F}{L} = \frac{\mu_0 I I'}{2\pi r}$$

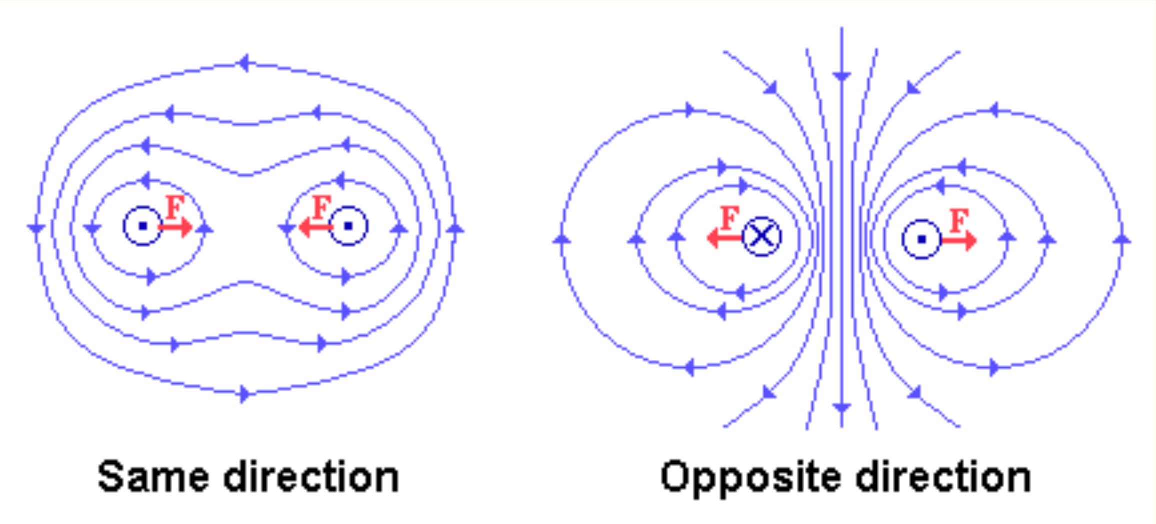
Current in second conductor

Distance between conductors

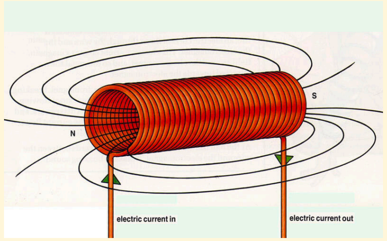
Practical Application

- Computer cables, or cables for audio-video equipment should create little or no magnetic field so as not to disturb the computer.
- This is one done by placing closely spaced wires carrying current in both directions along the length of the cable.
- The magnetic fields from these opposing currents cancel each other.





Amperes Law application II: Solenoid



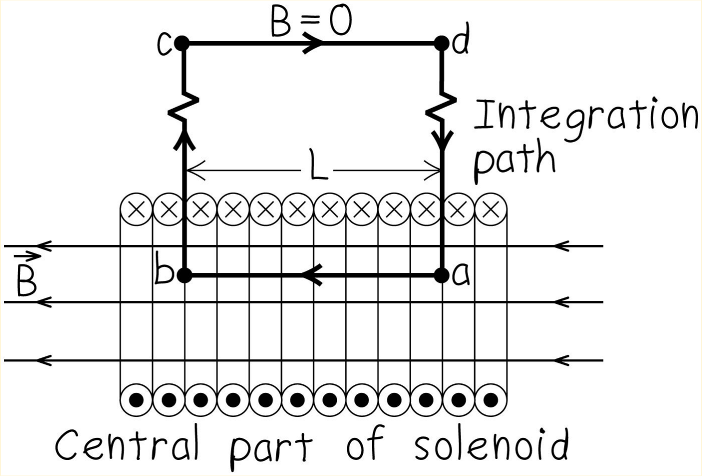
Ideal Solenoid of infinite length:

Where “n” is the number of windings per unit length

$$\oint \vec{B} d\vec{l} = BL = \mu_0 I_{encl}$$

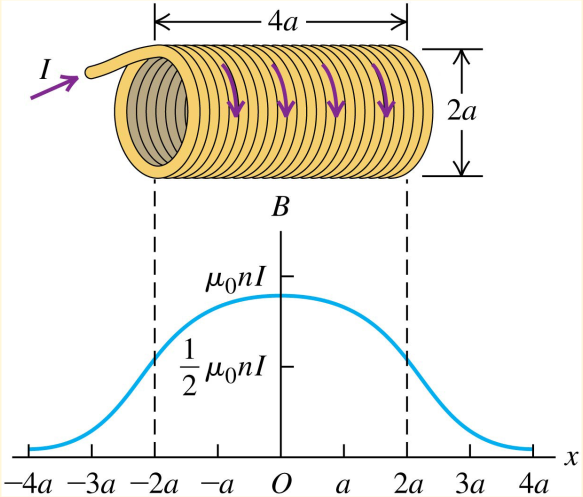
$$BL = \mu_0 I(nL)$$

$$B = \mu_0 n I$$



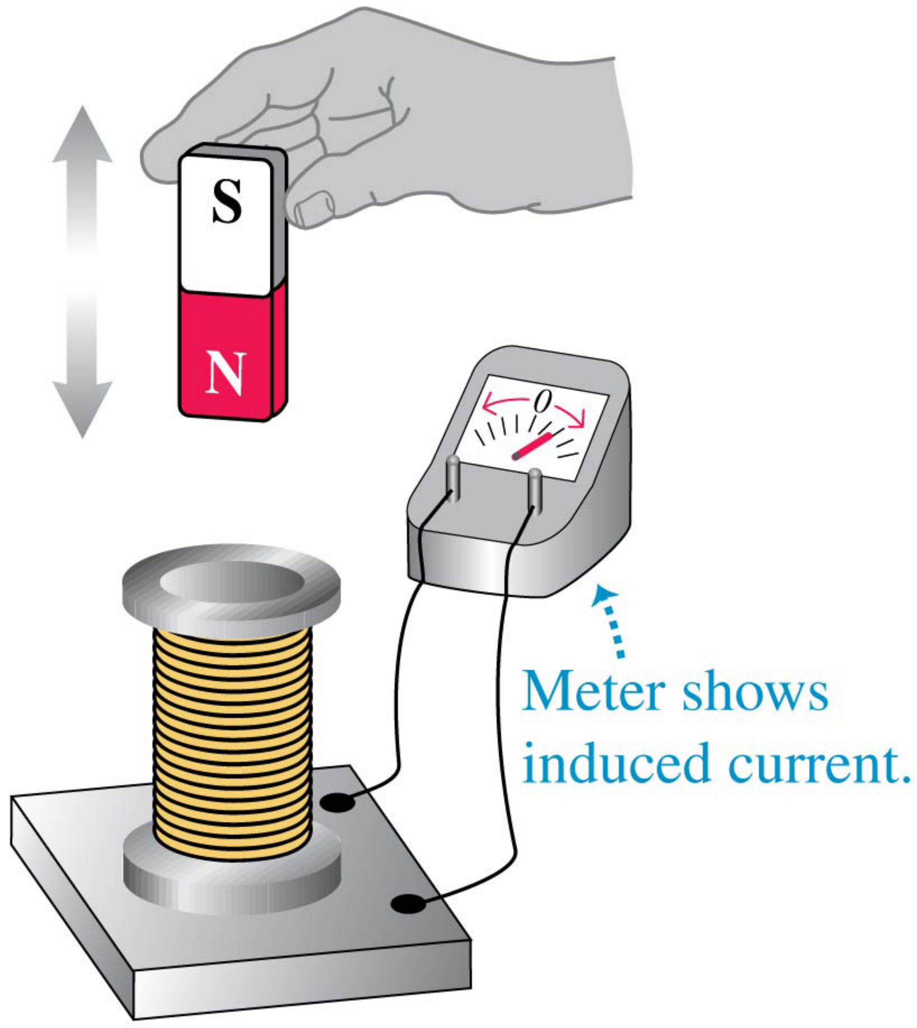
End Effect:

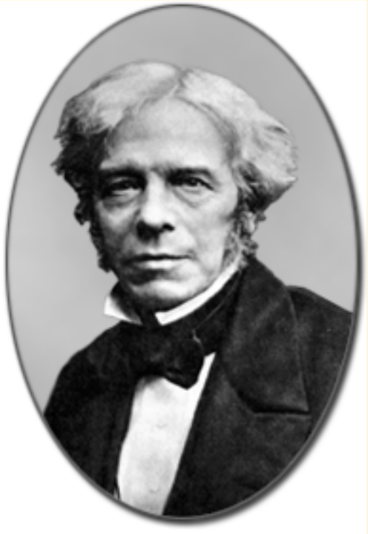
Real solenoid have finite edges!



Electromagnetic Induction

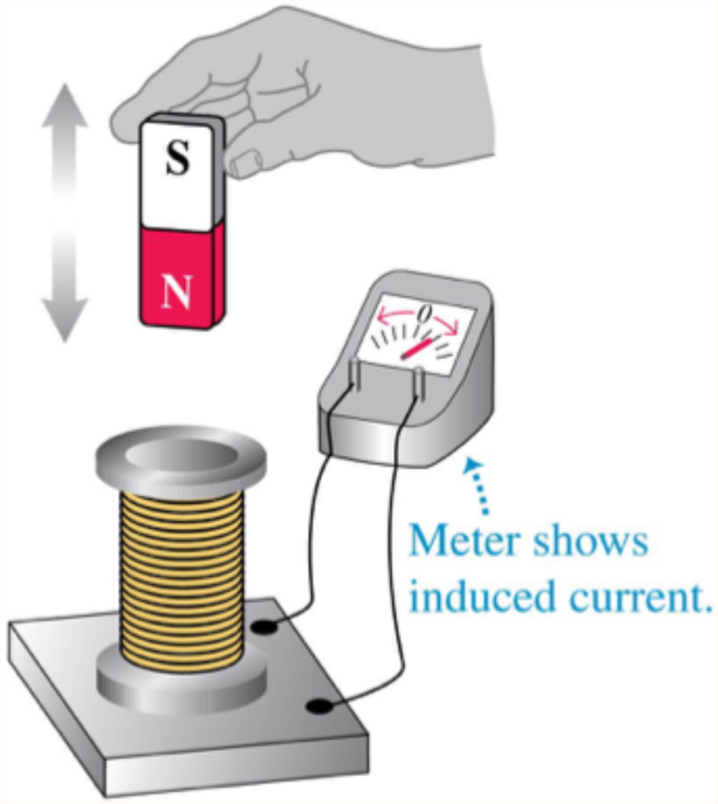
- When a closed circuit is placed in changing magnetic an emf is generated in the circuit.
- This is called electromagnetic induction
- The induced emf can drive a current in the circuit that may be measured in a galvanometer.
- The Laws of electromagnetic induction were discovered by the great British scientist Faraday



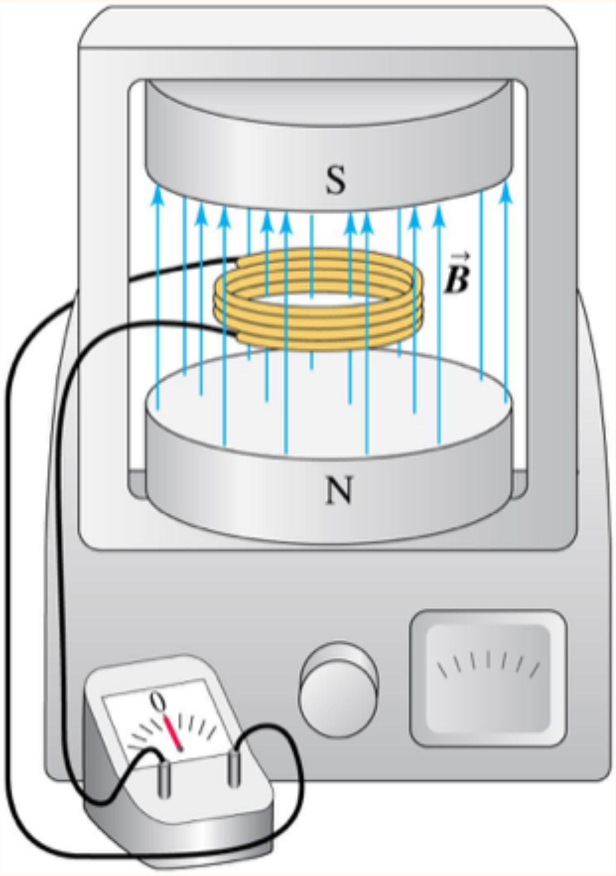


Faradays Findings

If you move a magnet near a coil an emf is generated



If you place a coil in a changing magnetic field an emf is generated

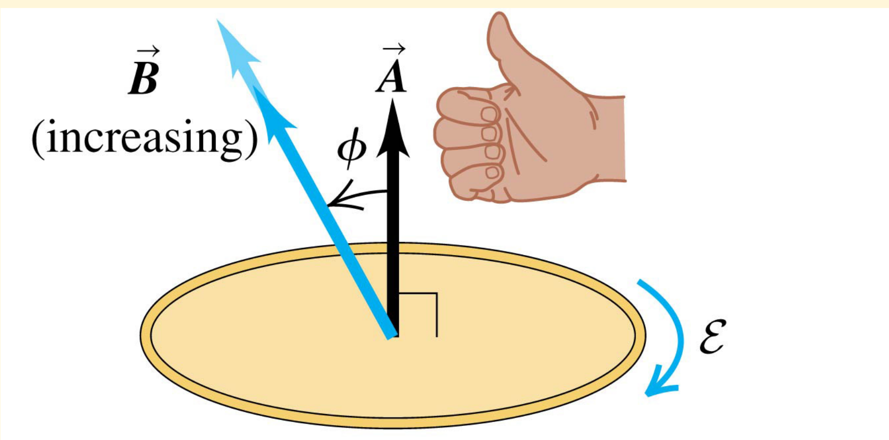


Faraday's Law

When the magnetic flux through a single closed loop changes with time, there is an induced emf that can drive a current around the loop

Faraday's law: The induced emf in a closed loop ... $\mathcal{E} = -\dfrac{d\Phi_B}{dt}$ ... equals the negative of the time rate of change of magnetic flux through the loop.

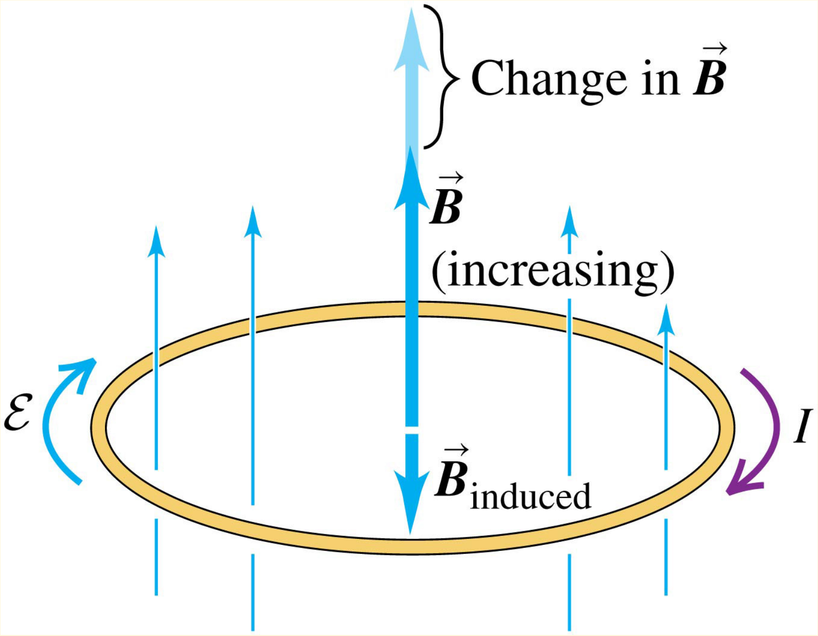
$$d\Phi_B = \vec{B}.d\vec{A}$$



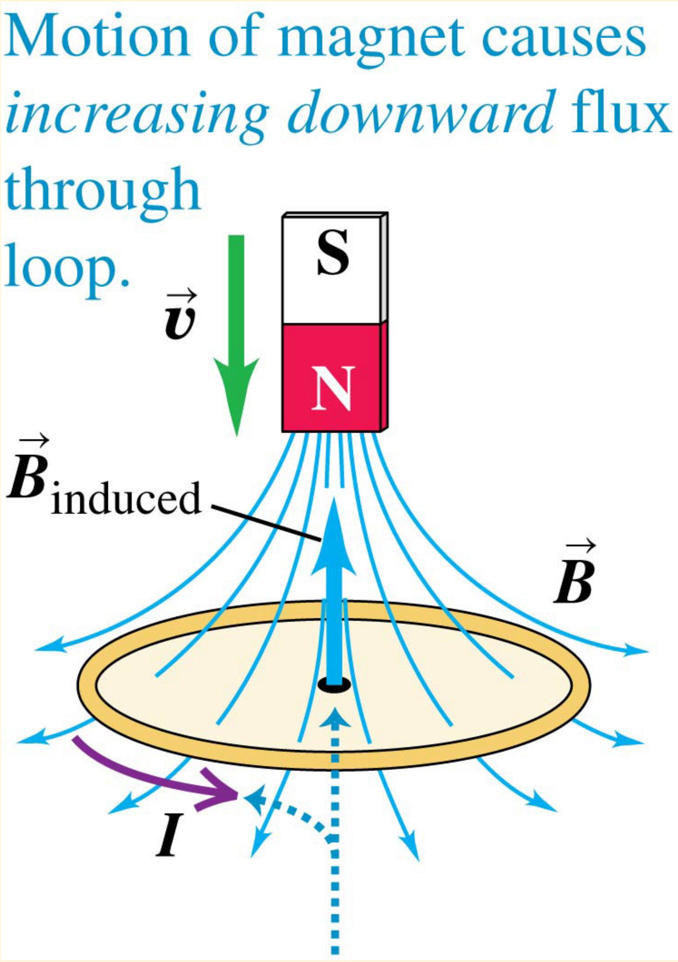
The unit of magnetic flux is the weber (Wb).
1 V = 1 Wb/s.

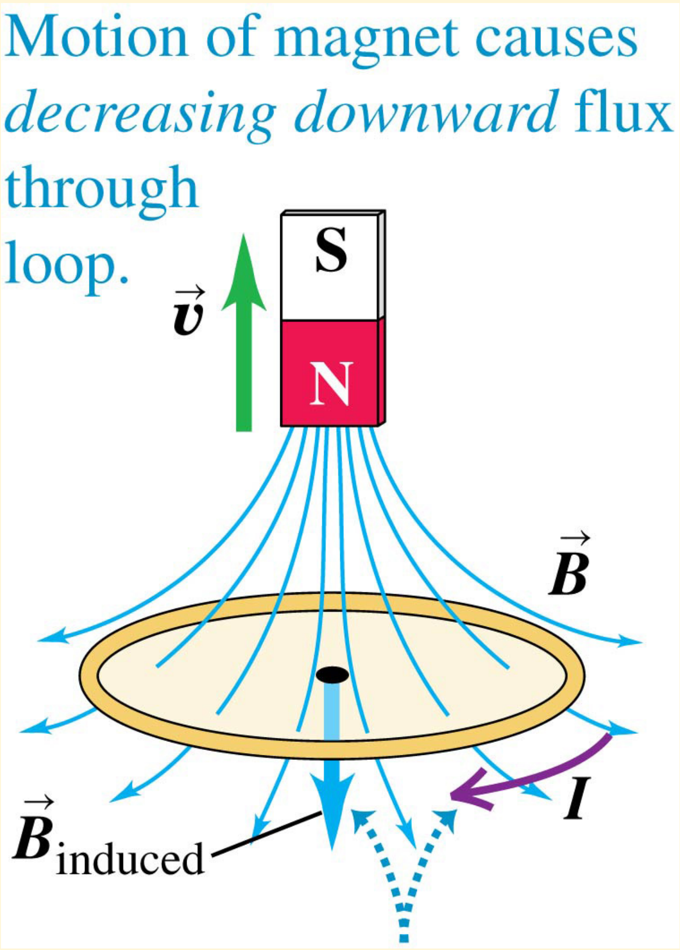
Lenz’s Law: The sign in the Faradays Law

- Lenz’s law states:
  - The direction of any electro-magnetic induction effect is such as to oppose the cause of the effect.
- For example, in the figure there is a uniform magnetic field through the coil.
- The magnitude of the field is increasing,
 so there is an induced emf
 driving a current, as shown.
- The direction of the current is such that by Amperes Law it will produce a field in the opposite direction to reduce the increase in magnetic field

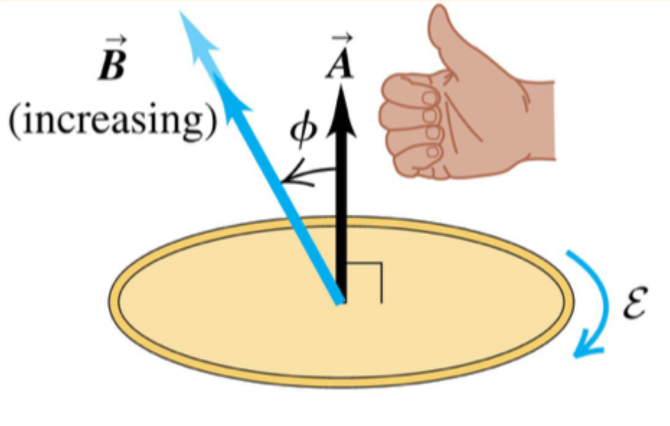


Lenz’s Law: direction of generated emf

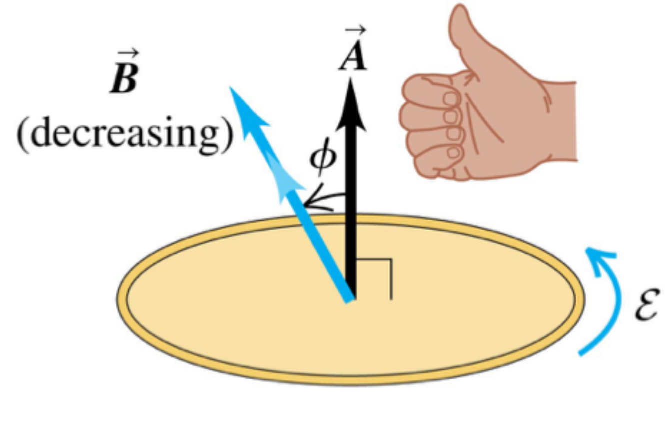




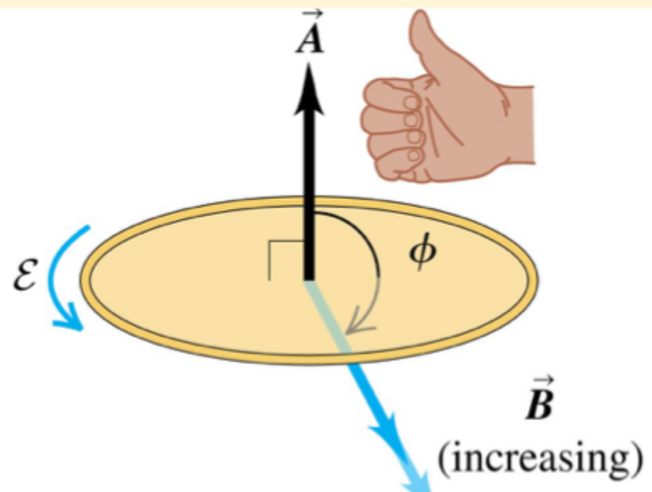
Determining the direction of the induced emf



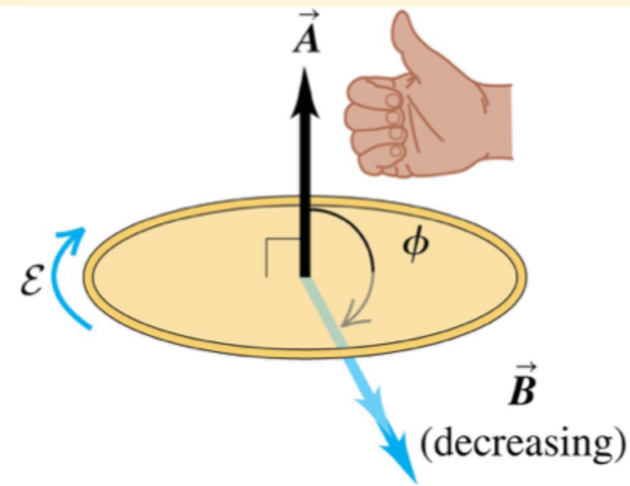
- Flux is positive ($\Phi_B > 0$) ...
- ... and becoming more positive ($d\Phi_B/dt > 0$).
- Induced emf is negative ($\mathcal{E} < 0$).



- Flux is positive ($\Phi_B > 0$) ...
- ... and becoming less positive ($d\Phi_B/dt < 0$).
- Induced emf is positive ($\mathcal{E} > 0$).



- Flux is negative ($\Phi_B < 0$) ...
- ... and becoming more negative ($d\Phi_B/dt < 0$).
- Induced emf is positive ($\mathcal{E} > 0$).



- Flux is negative ($\Phi_B < 0$) ...
- ... and becoming less negative ($d\Phi_B/dt > 0$).
- Induced emf is negative ($\mathcal{E} < 0$).

Faradays Law for coils

If the circuit has more than one loop then the resulting induced emf is far larger than would be possible with a single loop of wire

If a coil has N identical turns and if the flux varies at the same rate through each turn, total emf is:

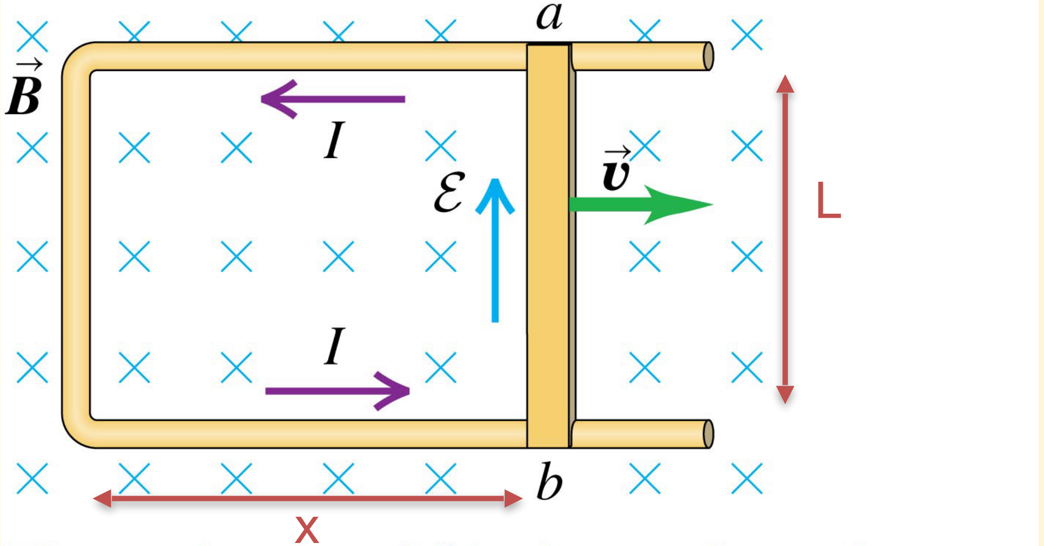
$$\mathcal{E} = -N\frac{d\Phi_B}{dt}$$

EMF on moving conductor in uniform magnetic field

$$\Phi_B = BLx(t)$$

$$\frac{d\Phi_B}{dt} = BL\frac{dx(t)}{dt}$$

$$= BLv$$



The motional emf $\mathcal{E}$ in the moving rod creates an electric field in the stationary conductor.

Motional emf, conductor length and velocity perpendicular to uniform $\vec{B}$ $\rightarrow$ $\mathcal{E} = vBL$

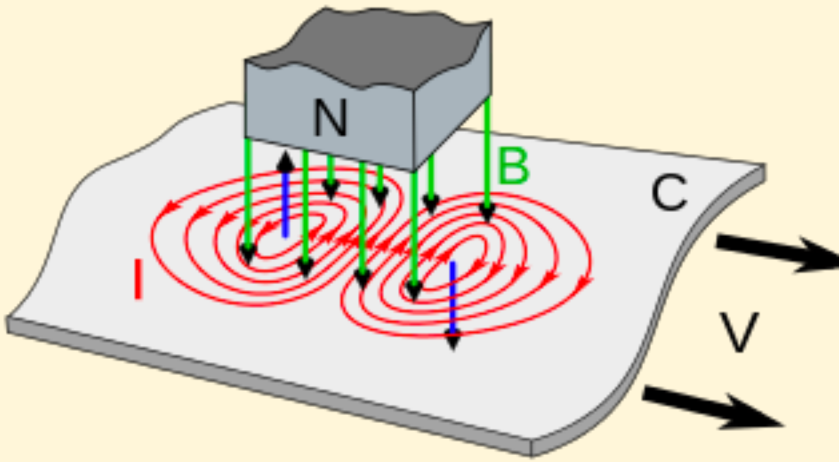
Conductor speed

Conductor length

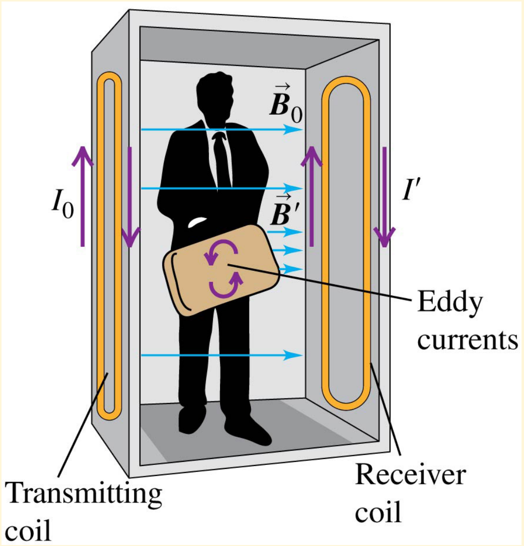
Magnitude of uniform magnetic field

Eddy Currents

When a conductor moves through a magnetic field or is located in a changing magnetic field, **eddy currents** are degenerated in virtual circuits due to electromagnetic induction.



The metal detectors operate by detecting eddy currents induced in metallic objects when an changing electric currents is used to generate a changing magnetic field in the operation zone!



Faradays Laws in the integral form

Definition of emf in terms of the electric field:

$$\mathcal{E} = \oint \vec{E} d\vec{l}$$

Using the Faradays Laws of electromagnetic induction we obtain,

Faraday's law for a stationary integration path:

$$\underbrace{\oint \vec{E} \cdot d\vec{l}}_{\text{Line integral of electric field around path}} = \underbrace{-\frac{d\Phi_B}{dt}}_{\text{Negative of the time rate of change of magnetic flux through path}}$$