Tutorial – 7

1. Consider a two-slit Young's interference experiment with λ= 500 nm where fringes are generated on a screen N placed at D = 1 meter apart from the slits.

(a) The fringe width decreases 1.2 times when the slit width is increased by 0.2 mm. Calculate the original fringe width.

(b) When a thin film of a transparent material is put behind one of the slits, the zero order fringe moves to the position previously occupied by the 4th order bright fringe. The index of refraction of the film is n = 1.2. Calculate the thickness (t) of the film.

Solution:

(a) We know that fringe width , $\Delta x = \frac{\lambda D}{d}$

(where d is the slit width and D distance between screen and the slits)

$$\Delta x \ \alpha \ \frac{1}{d}$$

so $\frac{\Delta x'}{\Delta x} = \frac{d}{\Delta d + 0.2\text{mm}} \quad (\Delta x' = \frac{\Delta x}{1.2})$

on solving the above equation we get slit width , d = 1 mm

original fringe width $\Delta x = \frac{\lambda D}{d} = \frac{500\times10^{-9}\times1}{1\times10^{-3}} = 0.5$ mm

(b) Thus the fringe pattern gets shifted by a distance Δ which is given by

$$\Delta = \frac{D(n-1)t}{d}$$

Here the zero order fringe moves to the position previously occupied by the 4th order bright fringe

We have $4\lambda = (n-1)t$

Thickness of the film, $t = \frac{4\lambda}{n-1} = \frac{4\times500\times10^{-9}}{0.2} = 10\ \mu m$

2. In a Lloyd's mirror experiment (see Figure 1), a bright wave emitted directly by the source S interferes with the wave reflected by the mirror M. As a result, an interference fringe pattern is formed on the screen N. The source and the screen is separated by a distance l = 1 m. At a certain position of the source the fringe width on the screen is equals to Δx = 0.25 mm. After the source is moved away from the plane of mirror by Δh= 0.60 mm, the fringe width decreases by a factor η= 1.5. Find the wavelength of the light.

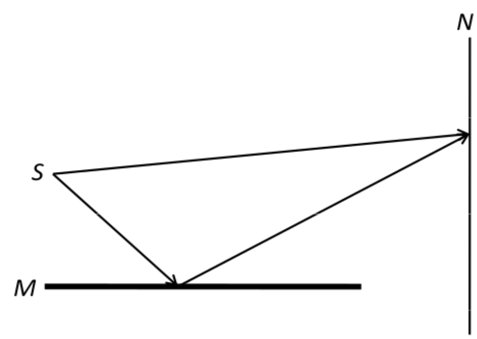


Figure 1: Lloyd's Mirror

<u>Solution:</u>

General formula is given by fringe width, $\Delta x = \frac{l\ \lambda}{d}$ (2a)

After the source moved away from the plane of the mirror,

we have $\frac{\Delta x}{\eta} = \frac{l\ \lambda}{d+2\Delta h}$ (2b)

Since d increased to d + 2Δh when source is moved away from the mirror by Δh.

Using equation (2a) and (2b) we get

$$\eta d = d + 2\Delta h$$

$$d = \frac{2\Delta h}{\eta - 1}$$

using eq. (2a), $\lambda = \frac{2\Delta h \times \Delta x}{l(\eta-1)} = \frac{2 \times 0.25 \times 0.6 \times 10^{-6}}{0.5 \times 1} = 0.6\ \mu m$

3. A plane light wave falls on a Fresnel mirrors with an angle $\alpha = 2.0'$ between them. Determine the wavelength of light if the fringe-width on the screen is

0.55 mm.

Solution:

In this case we must let $r \to \infty$ in the formula (A plane wave is like light emitted from a point source at ∞).

So fringe width, $\Delta x = \frac{(b+r)\lambda}{2\alpha r} \approx \frac{\lambda}{2\alpha}$

Then $\lambda = 2\alpha \cdot \Delta x = 2 \times \frac{2 \times \pi}{180 \times 60} \times 0.55 \times 10^{-3} = 0.64\ \mu m$

4. A lens of diameter 5 cm and focal length 25 cm is cut along its diameter into two identical halves. A layer of the lens a= 1 mm in thickness is removed and the two remaining halves of the lens are cemented together to form a composite lens. A slit is placed in the focal plane emitting monochromatic light of wavelength 0.60 µm. A screen is located behind the lens at a distance 50 cm from the lens.
(a) Find the fringe width on the screen and the number of possible maximum
(b) Find the maximum width of the slit at which the fringes on the screen will still be visible sufficiently sharp.

Solution:

Concept:

Consider the Young's double slit arrangement as shown in figure 2. A point source placed far away from the biprism produce plane waves which would incident on biprism. This would be equivalent to the situation where two plane waves emerging from the same source and they will interfere to form fringes on the screen.

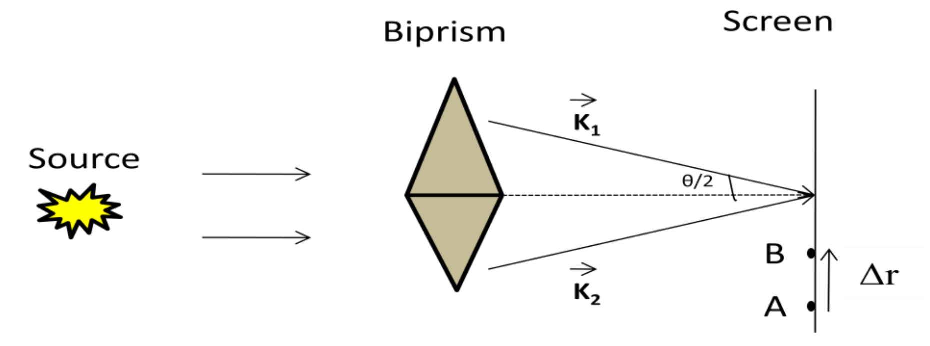


Figure 2: Young' double slit arrangement

Assume that at point A , both waves arrives at same phase, $E_1 = E_2 = E\, e^{i\phi(A)}$ and the intensity is maximum at point A.

As we move from A to B, both the waves shift with a phase difference

$$\phi_2(B) - \phi_1(B) = -(\mathbf{K_2} - \mathbf{K_1}).\,\Delta\mathbf{r}$$

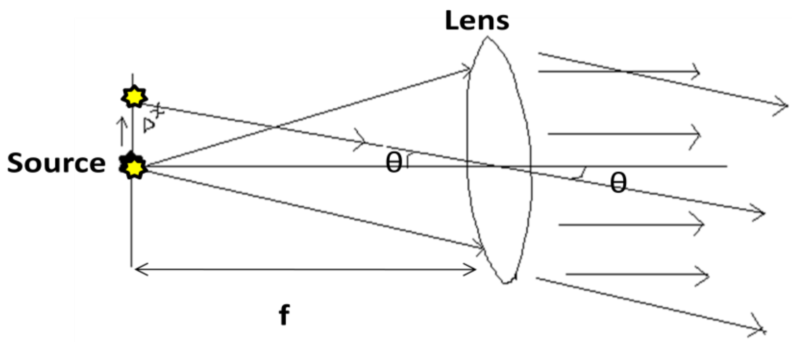
So the intensity is no longer maximum, it goes down as we reaches point B.

Similarly the condition for next maxima to occur at a point on the screen would be

$(\mathbf{K_2} - \mathbf{K_1}).\,\Delta\mathbf{r} = 2N\pi$ and for minima it is $(\mathbf{K_2} - \mathbf{K_1}).\,\Delta\mathbf{r} = 2(N+1)\pi$.

Let us try to solve this problem by using the above mentioned concept. Before that lets brush up some ideas tha we have already know.

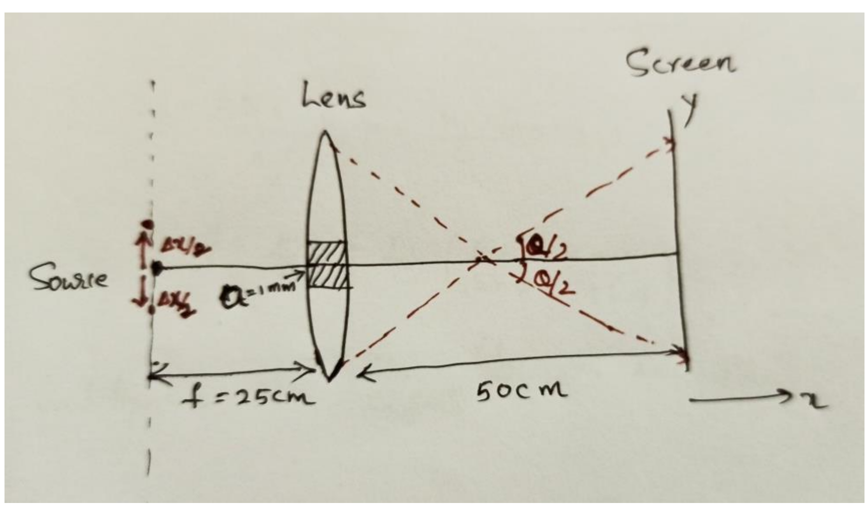
Consider the diagram drawn in the following figure,



As we can see when the source is placed on the axis of the lens at a distance equal to focal length of the lens , the emergent rays will be parallel rays. But, if we move the source by distance Δx **(**up or down) , then the emerging waves will make angle

$\theta = \frac{\Delta x}{f}$ with the symmetry axis of the lens.

(a) We have a lens with f = 25 cm, is cut in to two halves by removing a portion $\Delta x = a = 1$ mm. The two remaining halves of the lens are cemented together to form a composite lens(That means, the upper part of the lens is shifted down by an amount $a/2$ and lower part of the lens is shifted up by amount $a/2$). When we shift down upper part of the lens by a/2, this is equivalent to shifting the source up by an amount a/2.Hence, the upper part of the lens produce a light beam (dotted line)which makes an angle $\theta/2$ with symmetric axes of the lens. Similarly, for the lower part of the lens.



From the above figure the emergent light is at an angle $\theta/2 = \frac{a}{2f}$ with the symmetric axis of the lens .

Thus the divergence angle of the two incident light beams is

$\phi = 2.(\frac{\theta}{2}) = \frac{a}{f}$ (since θ is small)

When this two light beams interfere and the fringes produced on the screen would have a fringe width, $\Delta x = \frac{\lambda}{\phi} = \frac{\lambda f}{a} = \frac{0.6\times10^{-6}\times25\times10^{-2}}{1\times10^{-3}} = 0.15$ mm.

Number of possible maxima is equal to $\frac{b\phi}{\Delta x} = 13$ fringes
(where b is the distance between screen and the lens).

(b) If the slit width changes by a magnitude Δ (a/2 to a/2 +Δ)

The angle θ changes by Δ. θ (ie. Δ/2f)

So the fringe pattern will also shift by $\pm\frac{b.\Delta}{f}$

Equating this to $\frac{\Delta x}{2} = \frac{\lambda.f}{2a}$

We obtain $\Delta_{max} = \frac{\lambda.f^2}{2ab} = 37.5$ µm

5. Calculate frequency bandwidth for white light (frequency range 4×10^{14} Hz to $7.5 \times 10^{14}\ Hz$). Find coherence time and coherence length of white light.

<u>Solution:</u>

Coherence time , $T = \frac{1}{\Delta f} = \frac{1}{7.5\times10^{14} - 4\times10^{14}} = 2.85 \times 10^{-15}$ s

Coherence length $= cT = 3\times 10^{8} \times 2.85 \times 10^{-15} = 0.855$ µm

6. A quasi-monochromatic source emits radiation of mean wavelength 532 nm and has a frequency bandwidth of 10^{9} Hz. Calculate the coherence time, coherence length and degree of monochromaticity.

<u>Solution:</u>

Coherence time , $T = \frac{1}{\Delta f} = 10^{-9}$s

Coherence length $= cT = 3\times 10^{8} \times 10^{-9} = 0.3$ m

Frequency corresponding to mean wavelength,

$f_0 = \frac{c}{\lambda} = \frac{3\times10^{8}}{532\times10^{-9}} = 5.64 \times 10^{14}$ Hz

Degree of Monochromaticity $= \frac{\Delta f}{f_0} = 1.77 \times 10^{-6}$