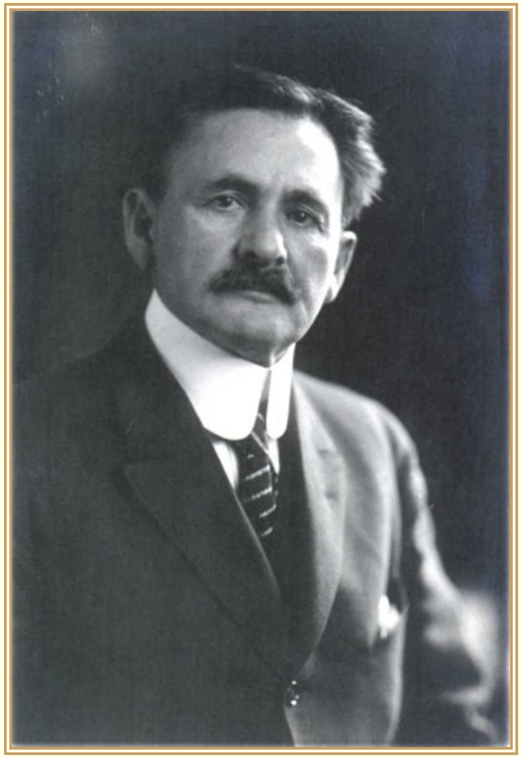
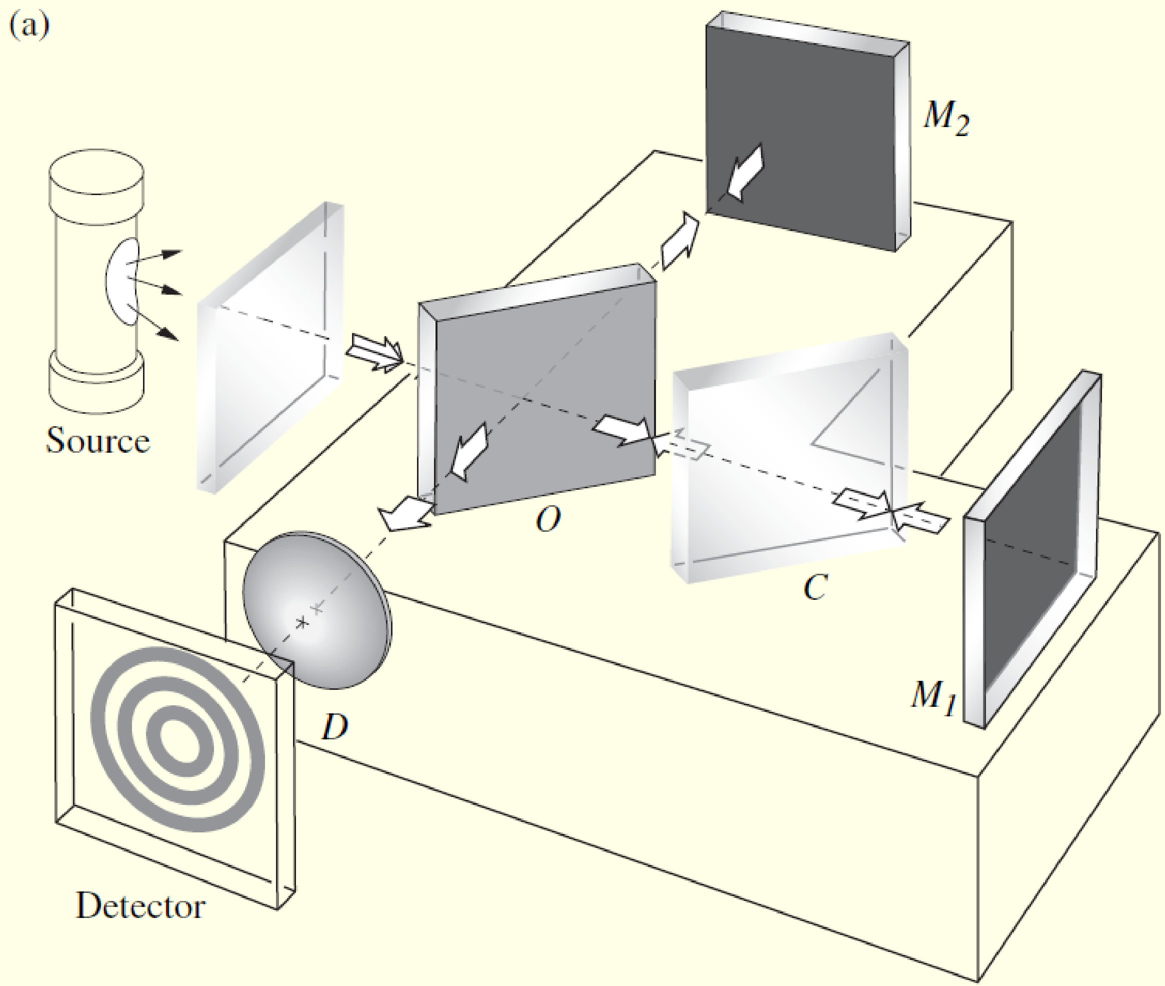
Michelson Interferometer

(Division of Amplitude)

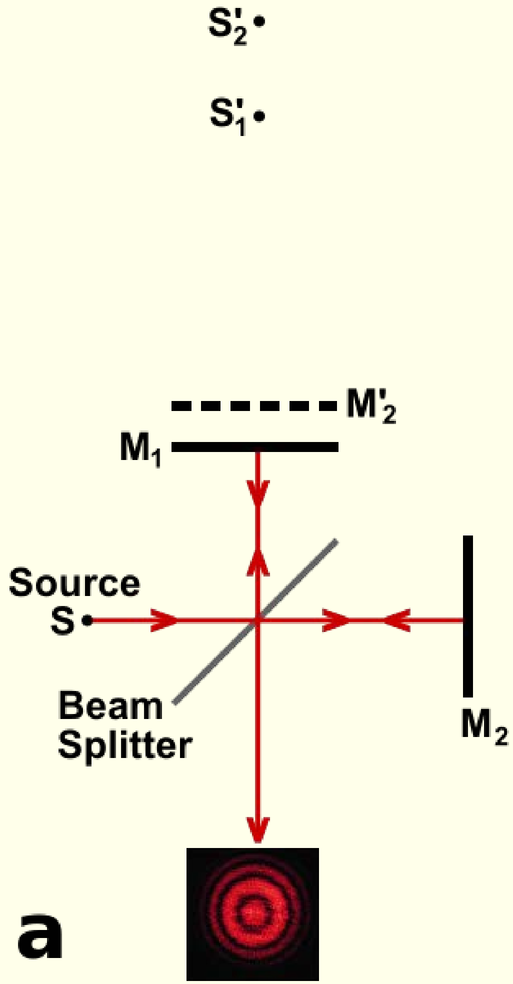
Experimental set up

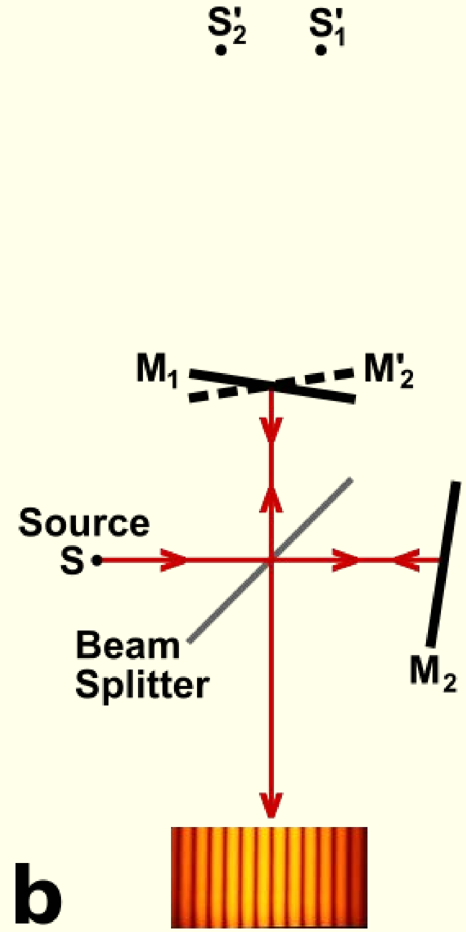


Albert Abraham Michelson
(1852-1931)



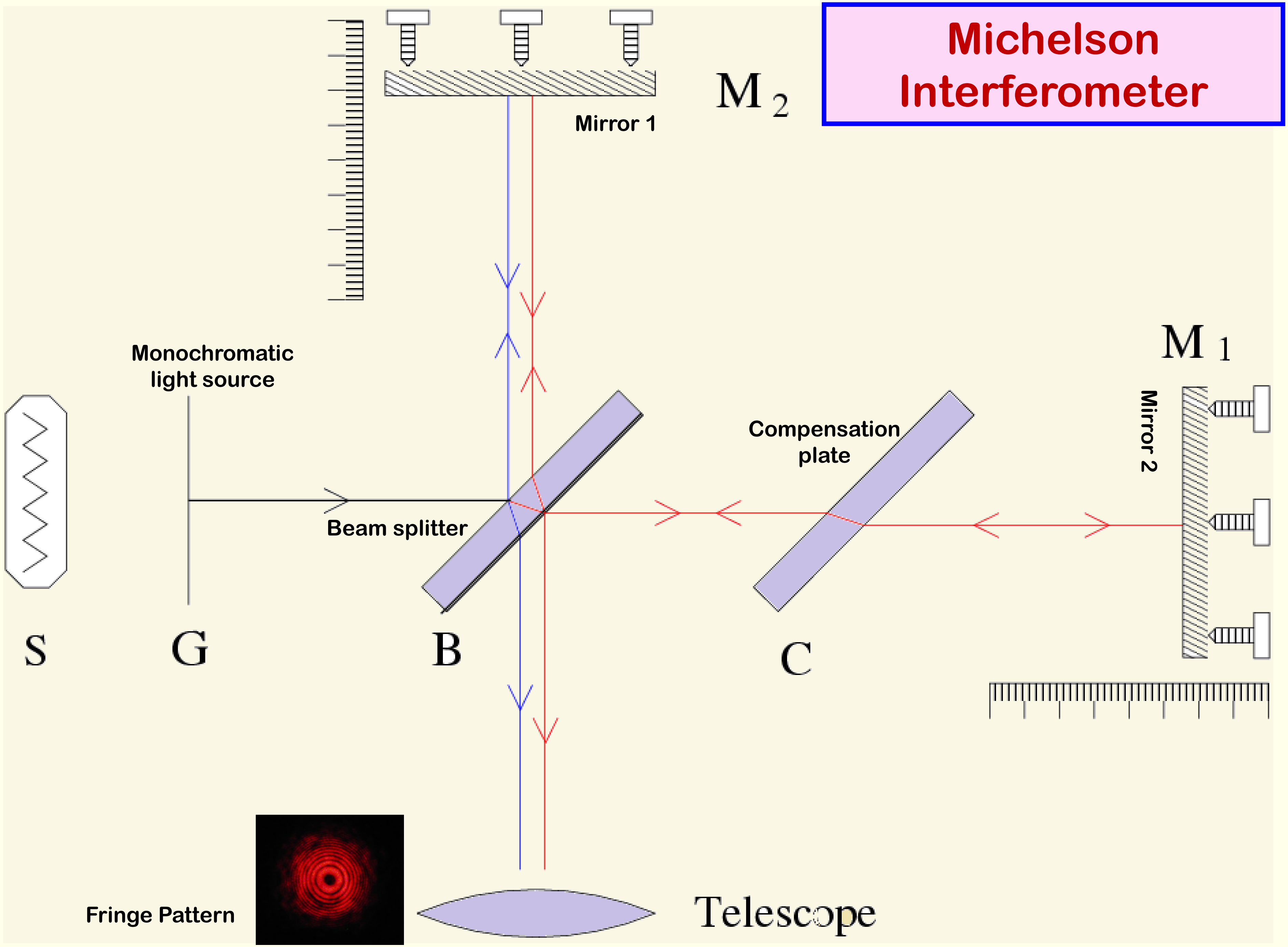
Michelson Interferometer





This instrument can produce both types of interference fringes i.e., *circular fringes of equal inclination at infinity* and *localized fringes of equal thickness*

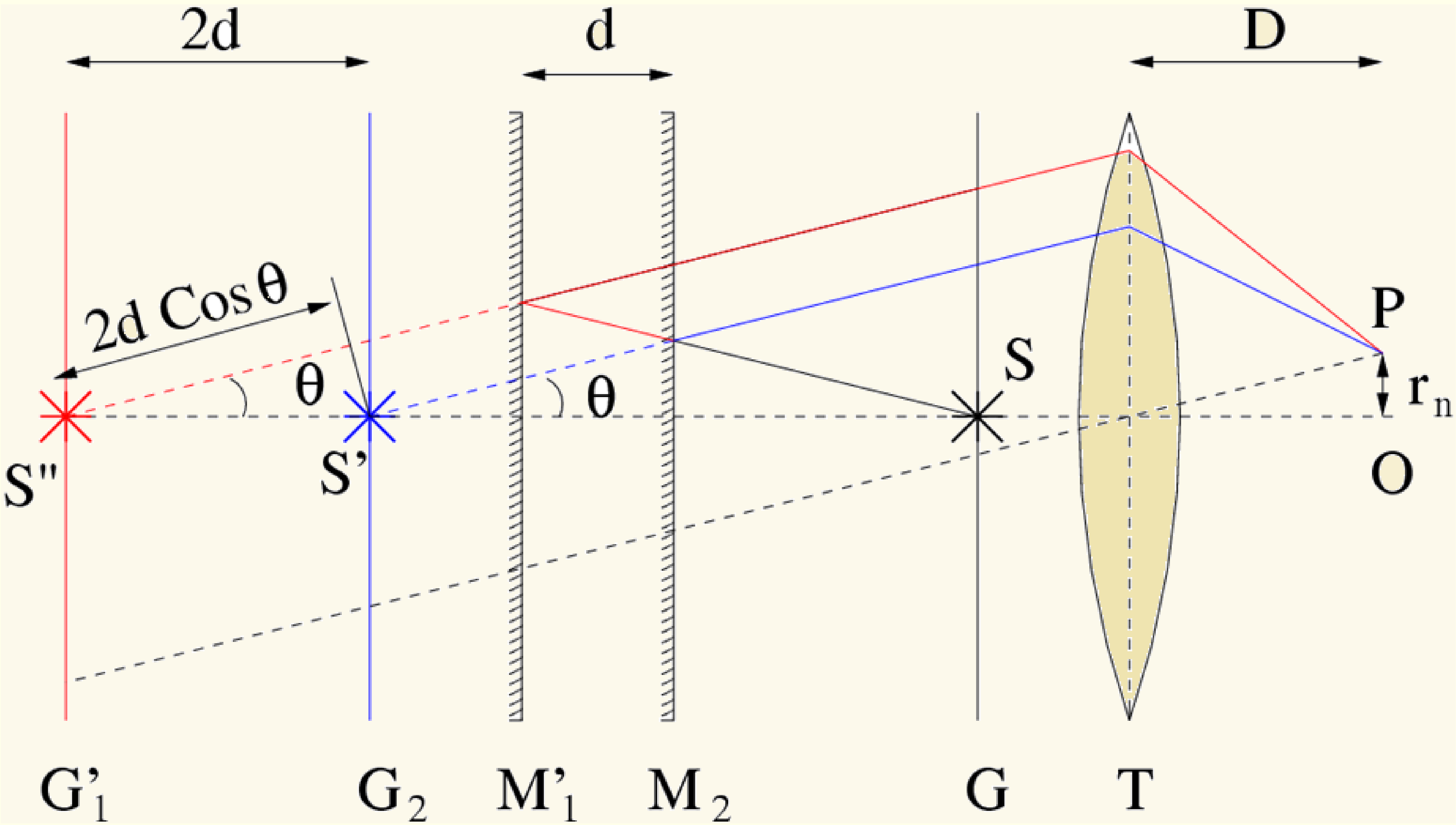
Michelson Interferometer



Effective arrangement of the interferometer

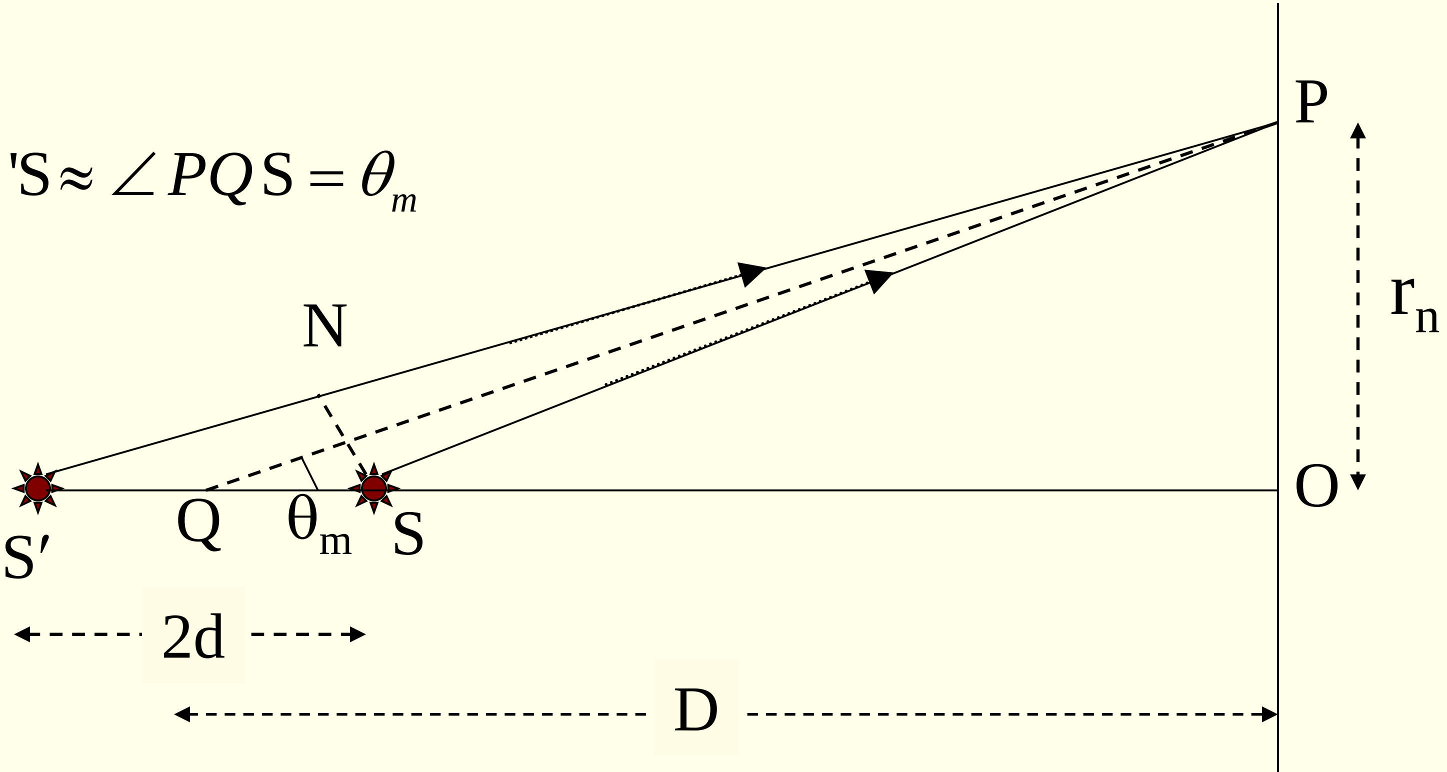
Circular fringes

An observer at the detector will see M_1, a reflected image of M_2($M_2^{//}$) and the images S' and S" of the source provided by M_1 and M_2. This may be represented by a linear configuration.



Longitudinal section –Circular fringes (general treatement)

$$\angle PS'S \approx \angle PQS = \theta_m$$



$$S'P - SP \approx S'N \approx 2d\cos\theta_m = m\lambda$$

Condition of maxima (without any reflection)

$$\cos\theta_m \approx 1 - \frac{\theta_m^{\ 2}}{2} \qquad \text{(for small } \theta_m\text{)}$$

For small θ_m $$1 - \frac{\theta_m^{\ 2}}{2} = \frac{m\lambda}{2d}$$

Central bright fringe $2d = m_0\,\lambda$ **(Note: There is no reflection here)**

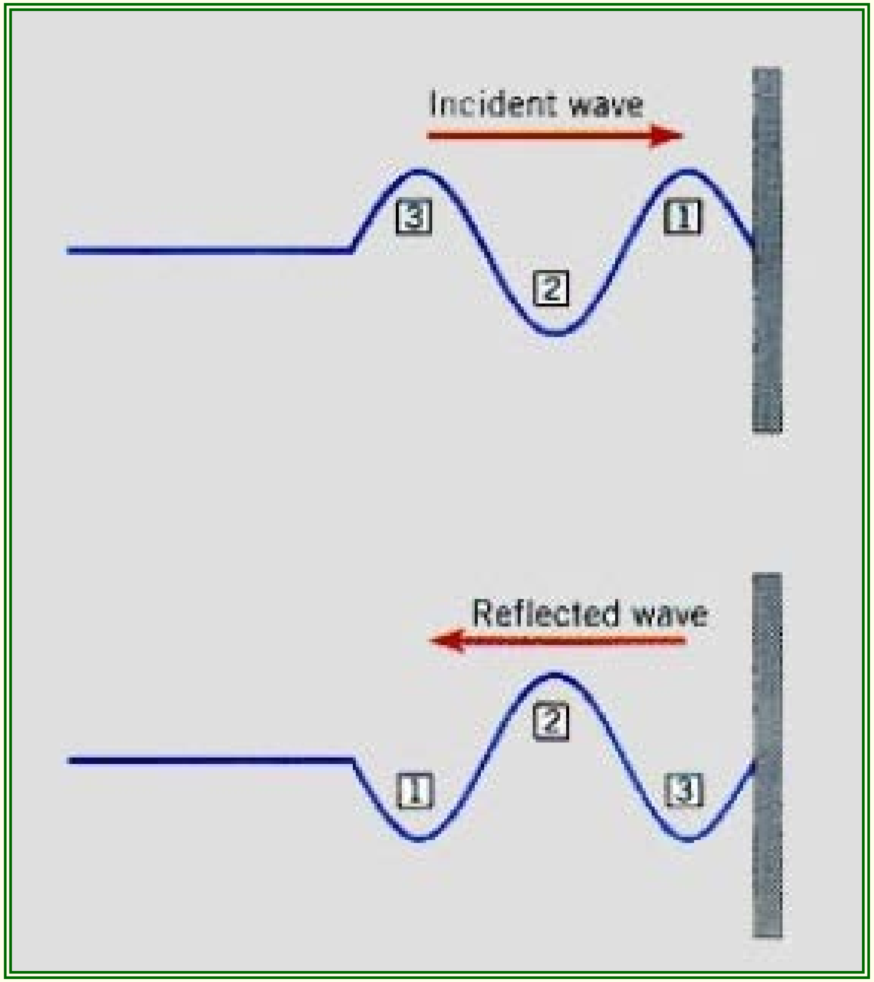
$$\theta_m^2 \approx \frac{(m_0 - m)\lambda}{d} = \frac{n\lambda}{d} \qquad (n = m_0 - m)$$

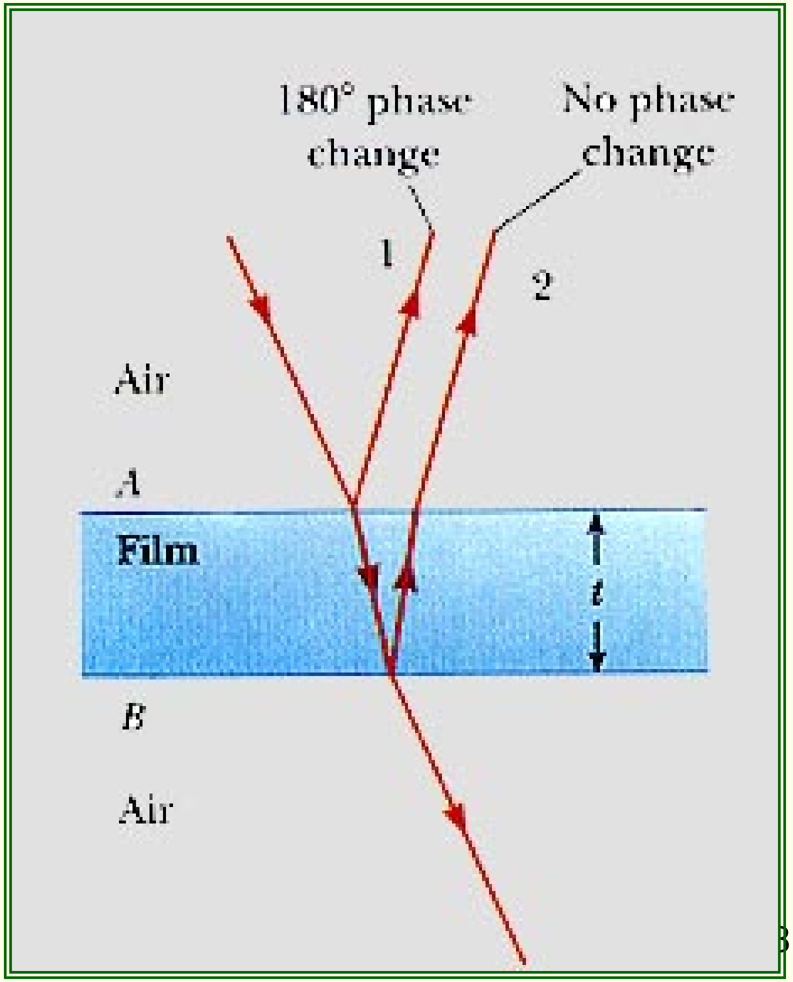
Radius of m^th^ bright ring

$$r_m^2 \approx D^2\theta_m^2 = \frac{D^2 n\lambda}{d}$$

Internal reflection **implies that the reflection is from an interface to a medium of lesser index of refraction.**

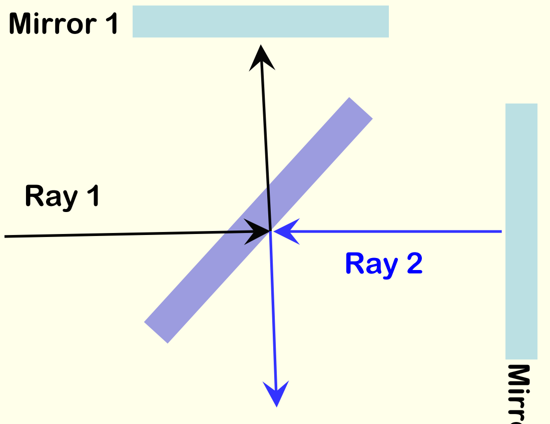
External reflection **implies that the reflection is from an interface to a medium of higher index of refraction.**





In Michelson interferometer

(when the phase change of ray 2 is considered)



$$2d\cos\theta_m = m\lambda \;(\mathrm{m}=0,1,2,\ldots): \mathrm{Minima}$$

$$2d\cos\theta_m = \left(m+\frac{1}{2}\right)\lambda \;(\mathrm{m}=0,1,2,\ldots): \mathrm{Maxima}$$

Note: Ray 2 experiences an additional π phase chance due to external reflection and as a result the conditions of maxima and minima are exchanged

Order of the fringe:

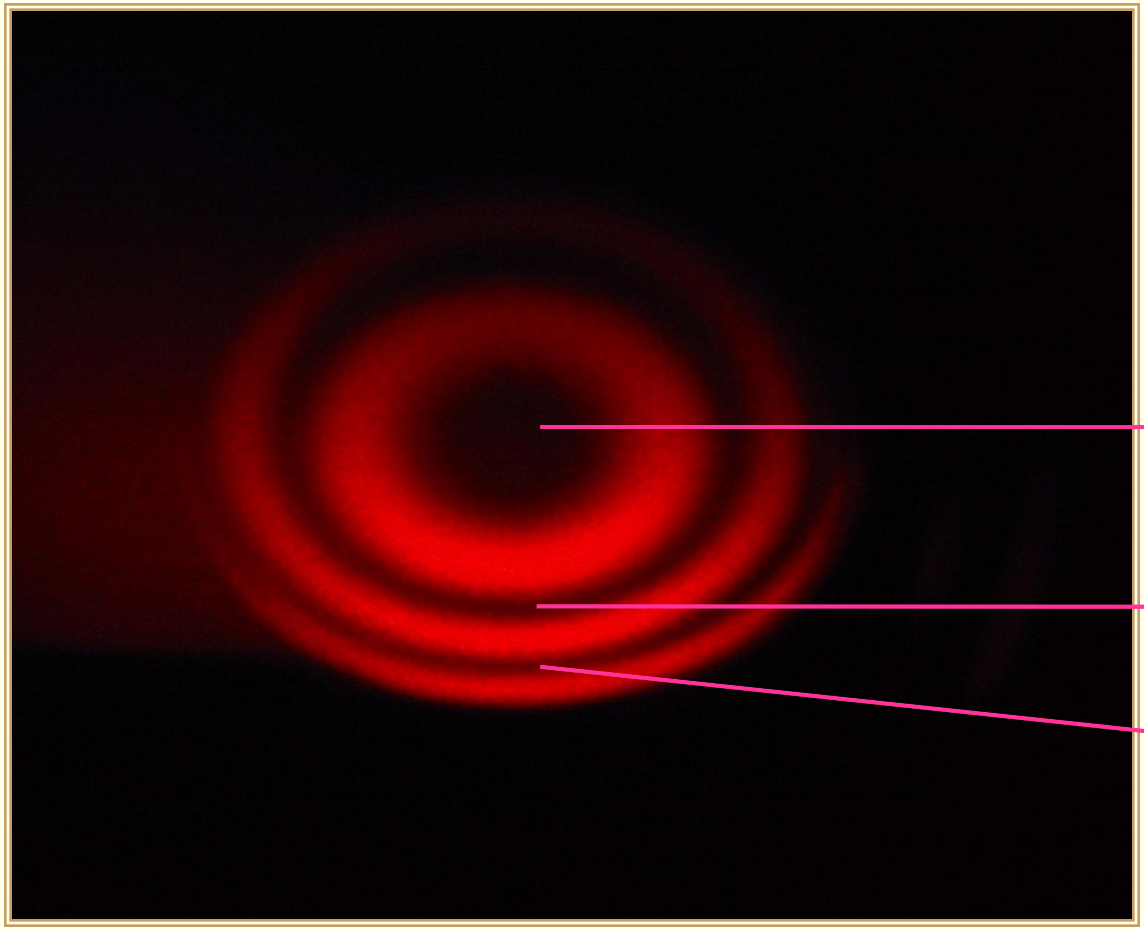
When the central fringe is dark the order of the fringe is

$$m = \frac{2d}{\lambda}$$

As d is increased new fringes appear at the centre and the existing fringes move outwards, and finally move out of the field of view.

For any value of d, the central fringe has the largest value of m.

Fringe shape



Central dark fringe
$$2d = m_o\lambda$$

1st dark ring
$$2d\cos\theta_1 = (m_0 - 1)\lambda$$

2nd dark ring
$$2d\cos\theta_2 = (m_0 - 2)\lambda$$

And so on............

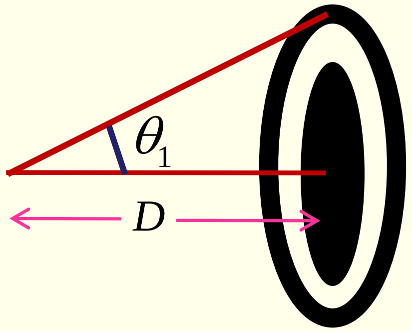
In Michelson interferometer

$$2d\cos\theta_m = m\lambda$$

For central dark fringe: $2d = m_o\lambda$

The **first dark fringe** satisfies: $2d\cos\theta_1 = (m_0 - 1)\lambda$

For small θ $\cos\theta_1 \approx 1 - \frac{\theta_1^{\,2}}{2}$



$$2d\left(1 - \frac{\theta_1^2}{2}\right) = (m_0 - 1)\lambda$$

$$d\theta_1^2 = \lambda$$

$$\boxed{r_1^{\,2} \approx D^2\theta_1^2 = \frac{D^2\lambda}{d}}$$

Radius of first dark fringe

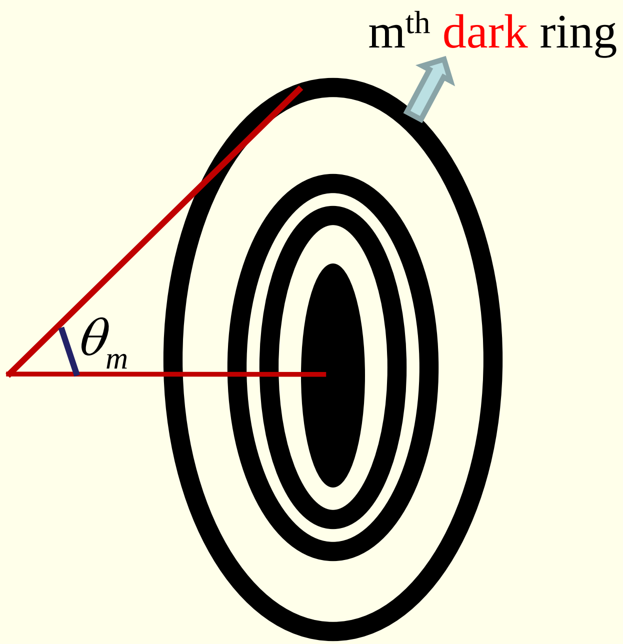
The **m^th dark fringe** satisfies: $2d\cos\theta_m = (m_0 - m)\lambda$

$$2d\left(1-\frac{\theta_m^2}{2}\right) = (m_0 - m)\lambda \quad \Rightarrow \quad d\theta_m^2 = m\lambda \qquad (2d = m_o\lambda)$$

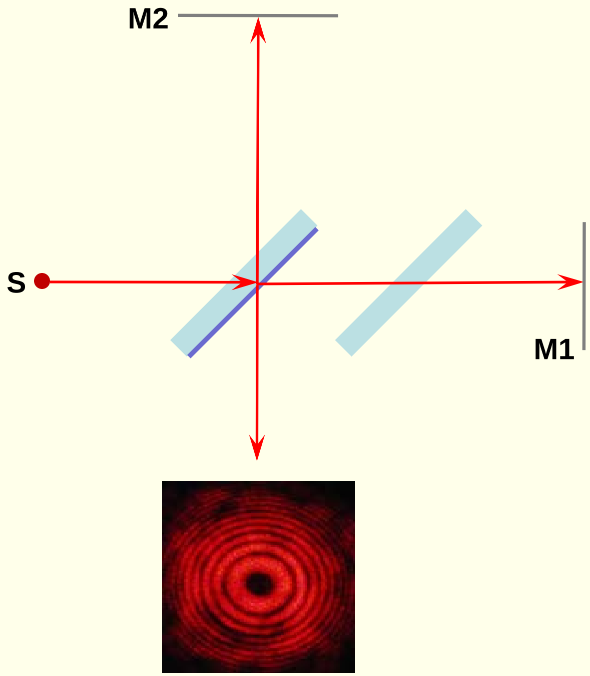
Radius of m^th dark ring:

$$d\theta_m^2 \approx m\lambda$$

$$r_m^2 \approx D^2\theta_m^2 = \frac{D^2 m\lambda}{d}$$



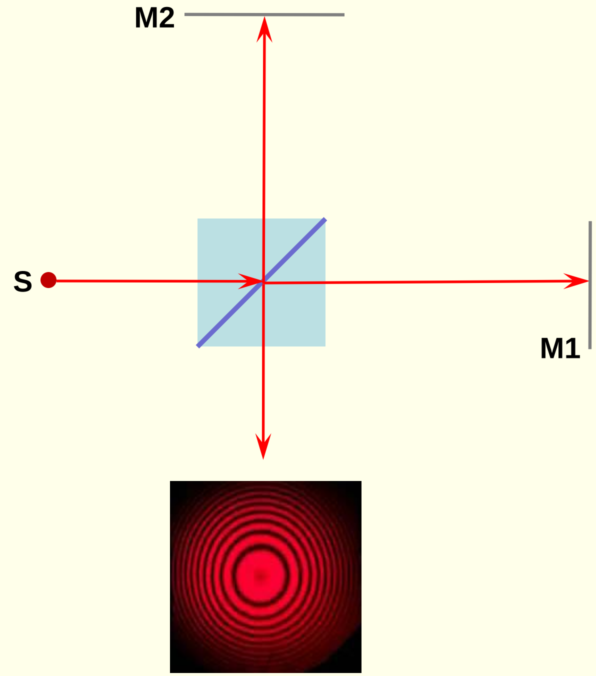
Michelson interferometer with compensator



Condition for central dark spot

$$2d = m_0\lambda \quad (\theta = 0)$$

Michelson interferometer without compensator



Condition for central bright spot

$$2d = m_0\lambda \quad (\theta = 0)$$

Measurement of wavelength of light

$$2d\cos\theta_m = m\lambda$$

$$2d = m_0\lambda \qquad (\theta = 0)$$

Move one of the mirrors to a new position *d'* so that the order of the fringe at the centre is changed from m_o to m.

$$2d' = m\lambda$$

$$2\left|d' - d\right| = \left|m - m_0\right|\lambda = \Delta m\lambda$$

$$\boxed{\lambda = 2\frac{\Delta d}{\Delta m}}$$