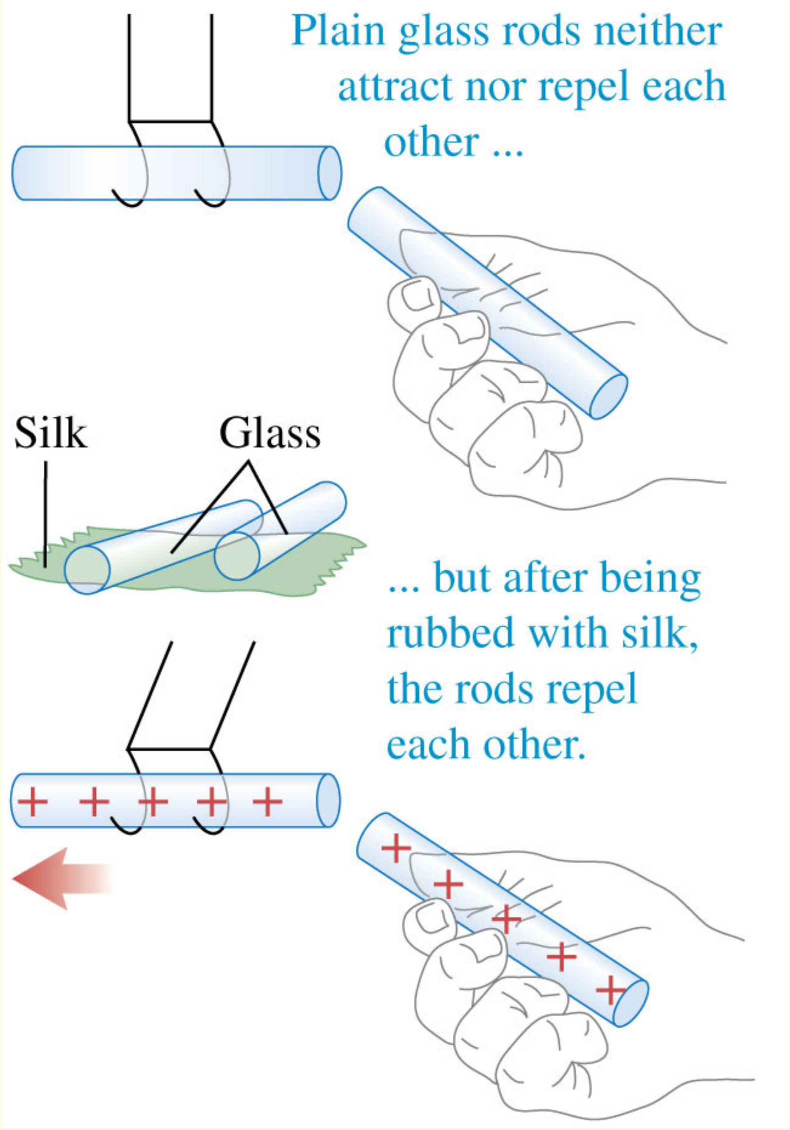
Electric Charge & Electrostatic Force

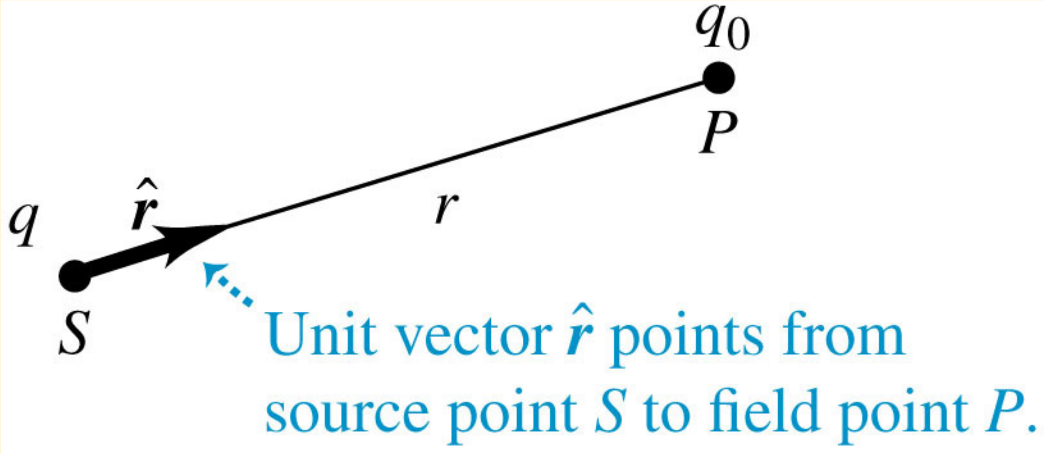
- Electric charges that lead electrostatic forces exists and simple experiments like the one demonstrated in the diagram shows there are two types of charges.
- The assignment of positive and negative charges are purely based on convention.
- Like charges repel while the opposite charges attract.
- The Forces are governed by the Coulomb's law



Electrostatics Force due to point charges

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2}\hat{r}$$

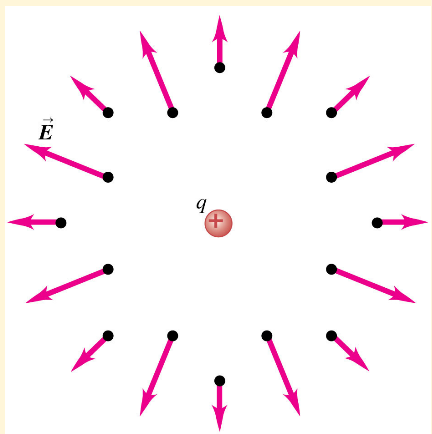
Constant of proportionality. In air it is approximately numerically equal to: $9.0 * 10^9$ N m^2/C^2



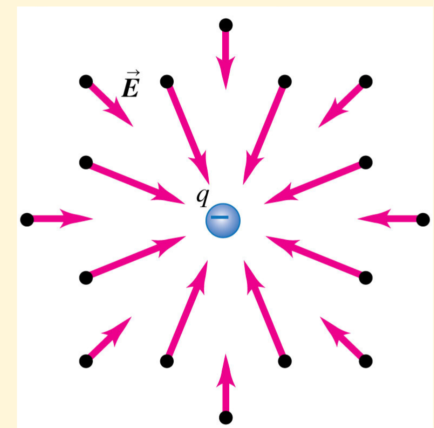
Electric Field

Interestingly one can construct an abstract concept of the electric field. The electro-static interaction of a charge in the presence of another can then be constructed as an interaction between of the charge with the electric field produced by the other charge:

Field Lines diverge away from positive charges



Field Lines diverge towards negative charges



Electric field due to a point charge

Value of point charge

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

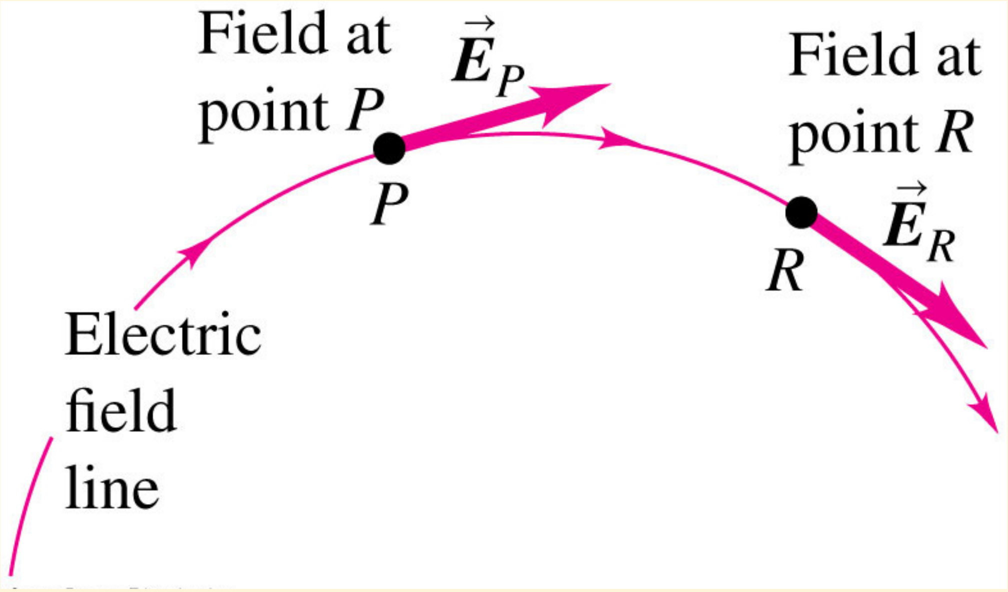
Unit vector from point charge toward where field is measured

Distance from point charge to where field is measured

Electric constant

Electric Field Lines

An **electric field line** is an imaginary line or curve whose tangent at any point is the direction of the electric field vector at that point.



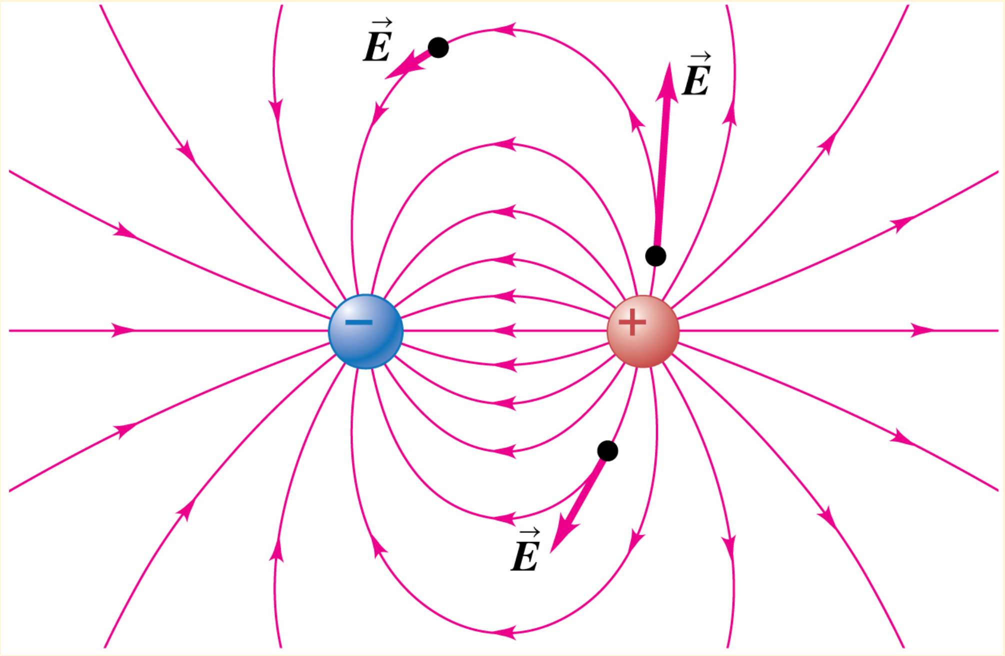
Superposition of Electric Field

For a system of charges the net electric field at a point is the vector sum of the electric fields produced by individual charges.

If E_i is the electric field produced by the i^{th} charge then the total field is given by:

$$E_{tot} = E_1 + E_2 + E_3 \cdots = \sum_i E_i$$

Example: Electric Dipole



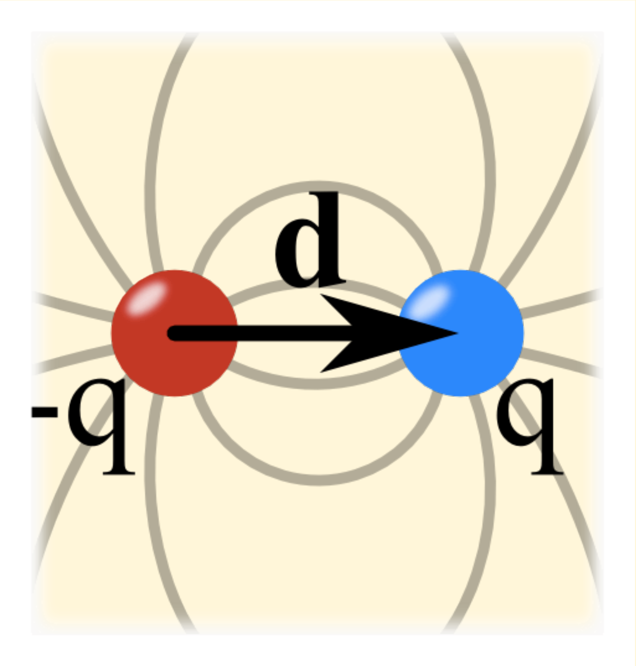
Electrical Dipole

- The electric dipole is a combination of two equal and opposite charges (+q & -q) placed at a certain distance from each other.

- The electric field lines can be obtained by obtaining the vector sum of the fields from the individual charges.

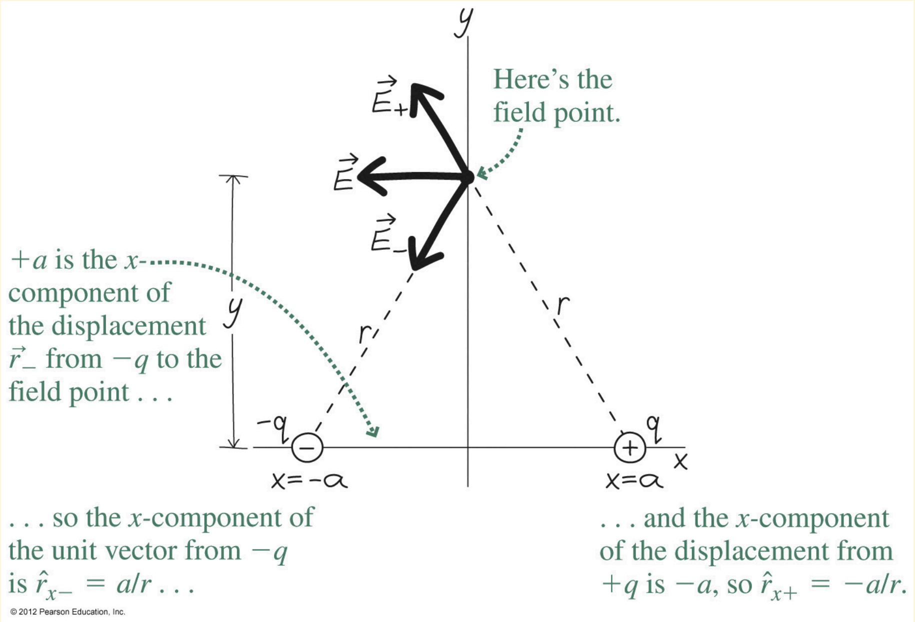
- The electric field can be expressed in terms of a useful parameter called the electric dipole moment

$$\vec{P} = q\vec{d}$$



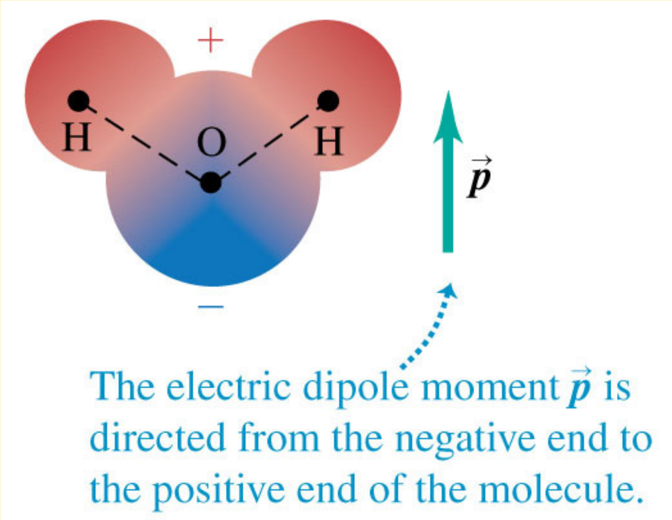
Field due to Electrical Dipole

To compute the total field due to the two charges one has vectorially add the fields due to the individual charges as depicted below:



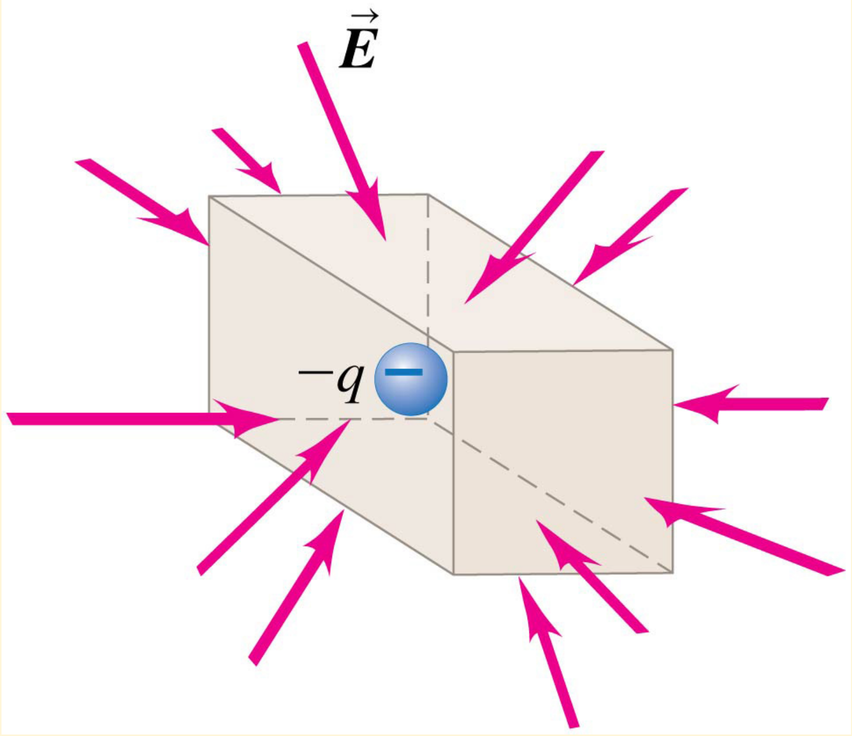
Example of an electric dipole in nature: H_20

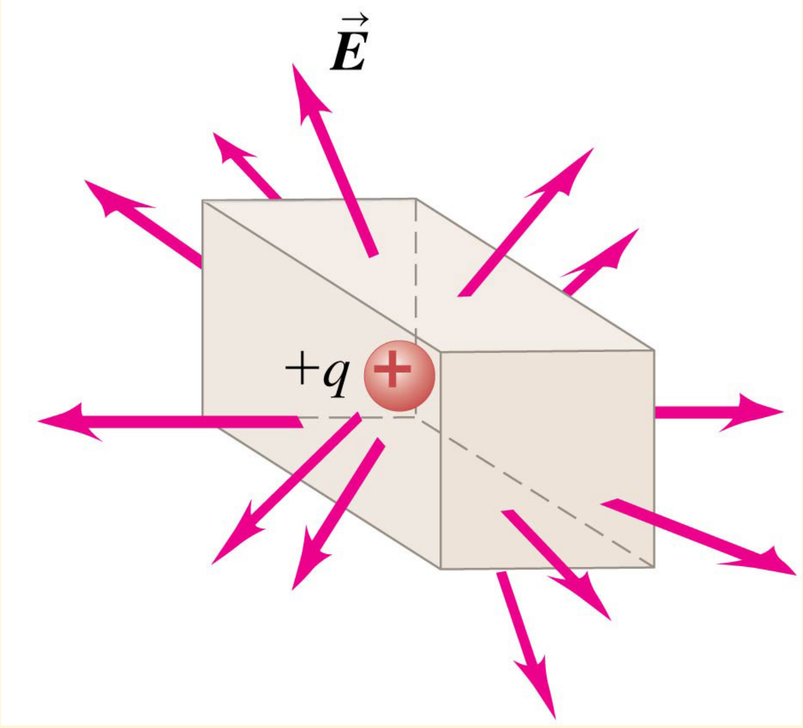
Water is a polar liquid resulting in its utility as a good solvent!



Gauss's Theorem for Electrostatics

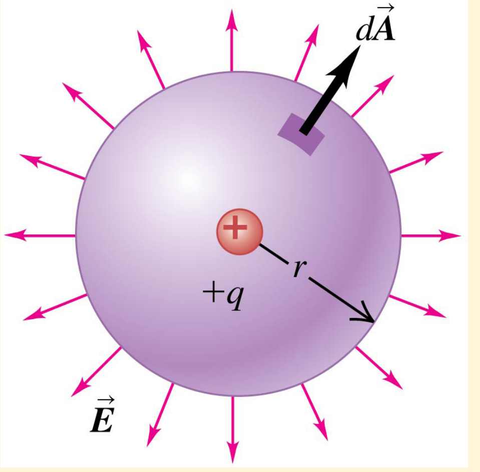
If one imagines a closed volume around a charge then the field files will either enter or leave the volume depending on the total charge enclosed by the volume as depicted in the diagrams below:





Electric Flux

Electric flux is the total amount of such field lines that is coming out or going in through the surface of such a closed volume enclosing a charge



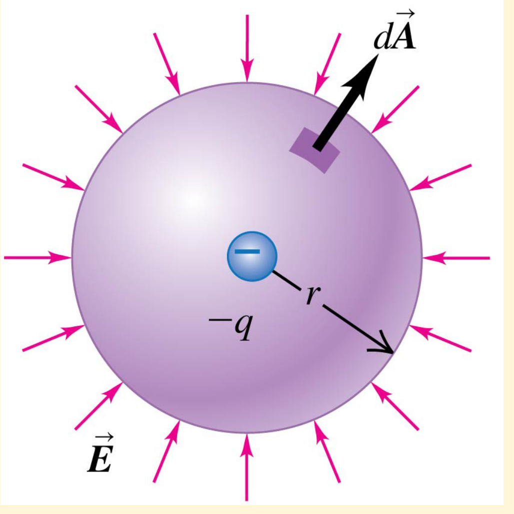
Magnitude of electric field $\vec{E}$

Component of $\vec{E}$ perpendicular to surface

Electric flux through a surface

$$\Phi_E = \int E\cos\phi \, \underbrace{dA}_{\text{Element of surface area}} = \int E_{\perp} \, \underbrace{dA}_{\text{Element of surface area}} = \int \vec{E} \cdot \underbrace{d\vec{A}}_{\text{Vector element of surface area}}$$

Angle between $\vec{E}$ and normal to surface



Statement of Gauss's Theorem

The net electric flux through any hypothetical closed surface is equal to $1/\varepsilon_0$ times the net electric charge within that closed surface.

Gauss's law:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0}$$

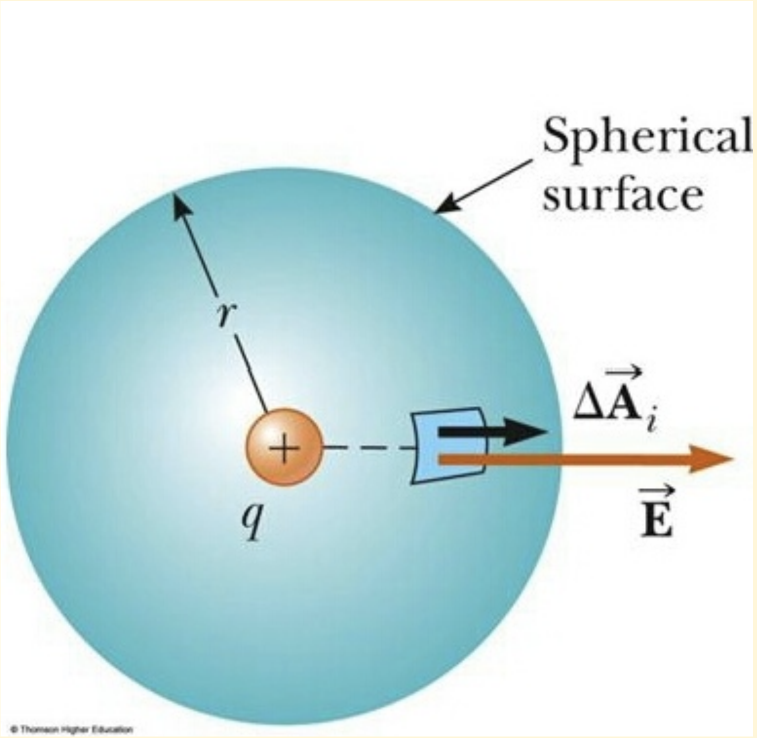
Total charge enclosed by surface

Electric flux through a closed surface of area A = surface integral of $\vec{E}$

Electric constant

Derivation Gauss's Theorem

$$
\begin{aligned}
\Phi_E &= \oint \vec{E} \cdot \vec{dA} \\
&= \oint \frac{q}{4\pi\epsilon_0 r^2} \hat{r} \cdot \vec{dA} \\
&= \frac{q}{4\pi\epsilon_0 r^2} \oint \hat{r} \cdot (dA\hat{r}) \\
&= \frac{q}{4\pi\epsilon_0 r^2} \oint dA \\
&= \frac{q}{4\pi\epsilon_0 r^2} (4\pi r^2) \\
&= \frac{q}{\epsilon_0} .
\end{aligned}
$$

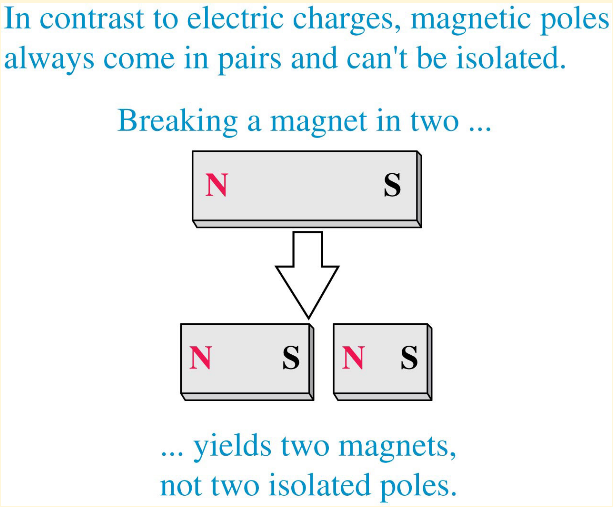


Local Equation

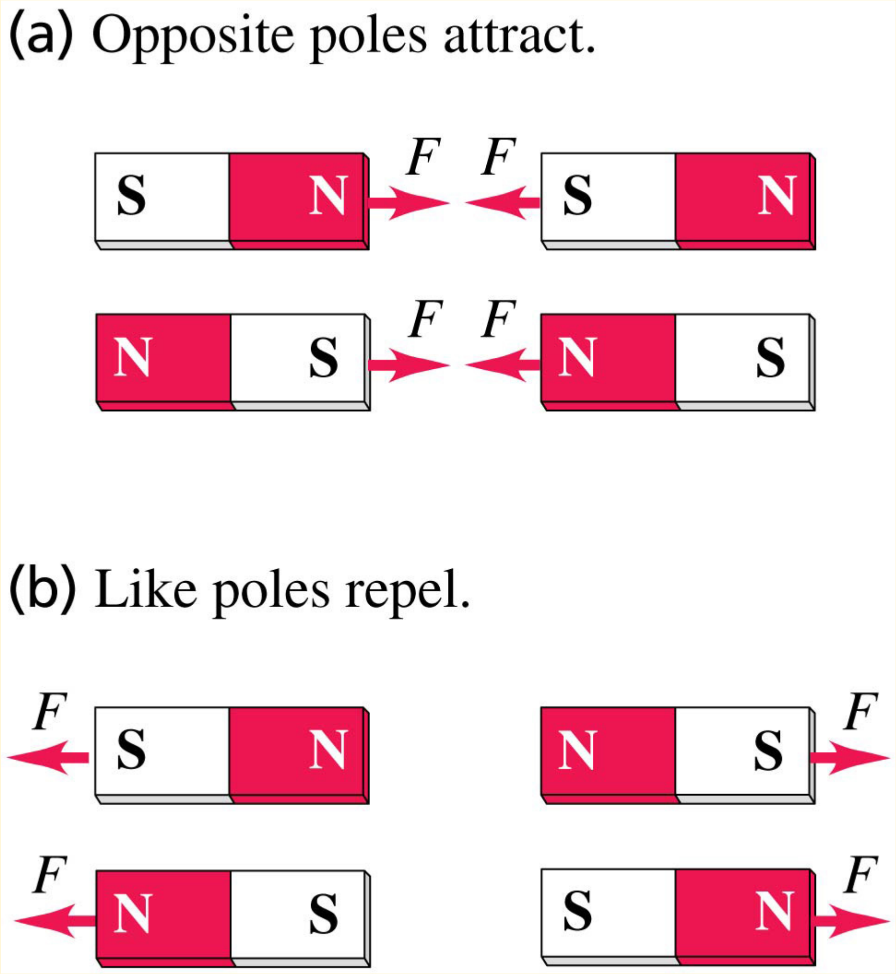
$$
\int \vec{\nabla} \cdot \vec{E} dV = \int \left(\frac{\rho}{\epsilon_0} \right) dV \rightarrow \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}
$$

Magneto-statics

- Unlike Electrostatic we have not discovered any evidence for a magnetic monopoles

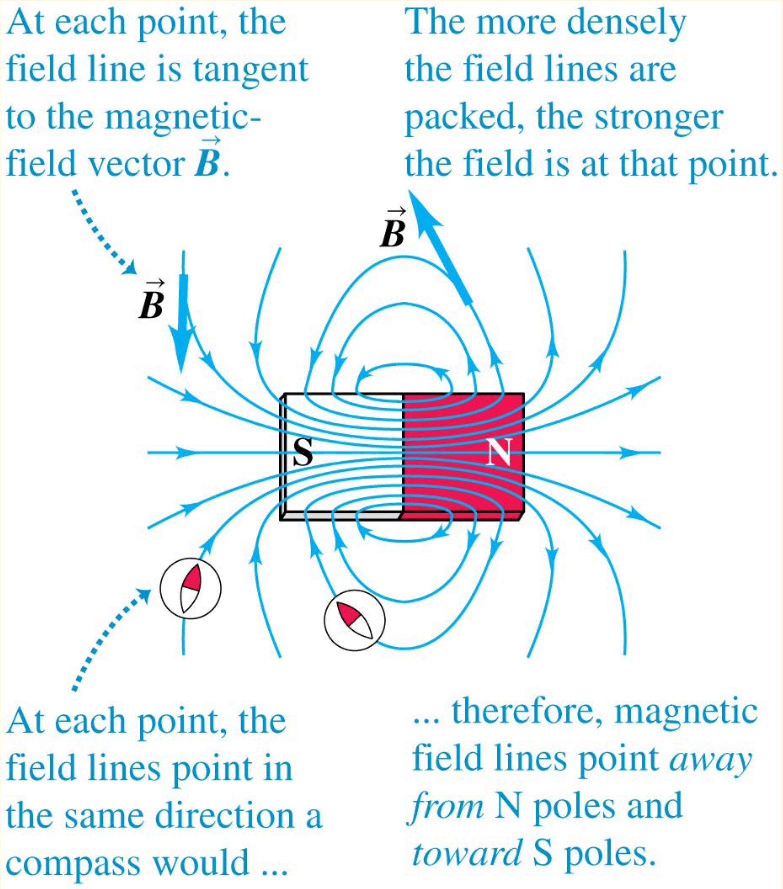


- So the minimal setup for a magnetic charge is a dipole. The popular notion of a piece of magnet is actually a magnetic dipole.
- Opposite poles attract each other, and like poles repel each other, as shown.



Magnetic Field Lines

- We can represent any magnetic field by **magnetic field lines**.
- We draw the lines so that the line through any point is tangent to the magnetic field vector at that point.
- Field lines never intersect.



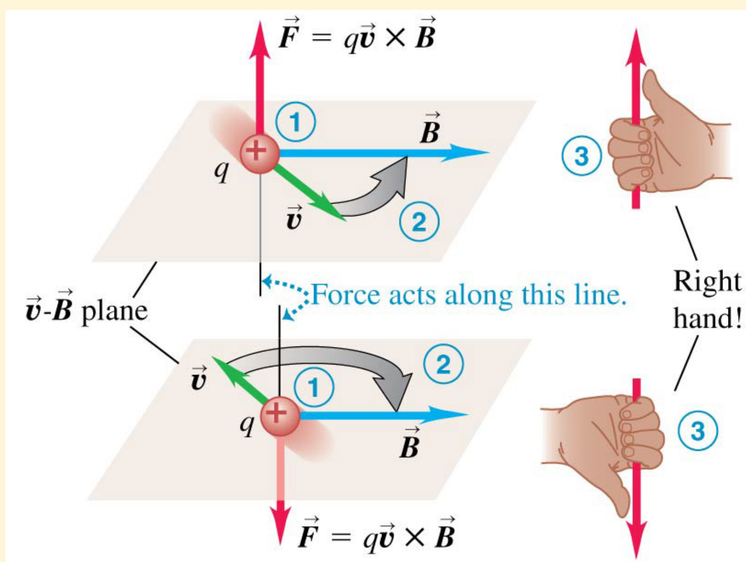
Force Law in Magnetic Field

Similar to electric field on can define a magnetic field B. Under this field a charge moving with some velocity is given by

Particle's charge

Magnetic force on a moving charged particle $$\vec{F} = q\vec{v} \times \vec{B}$$ Magnetic field

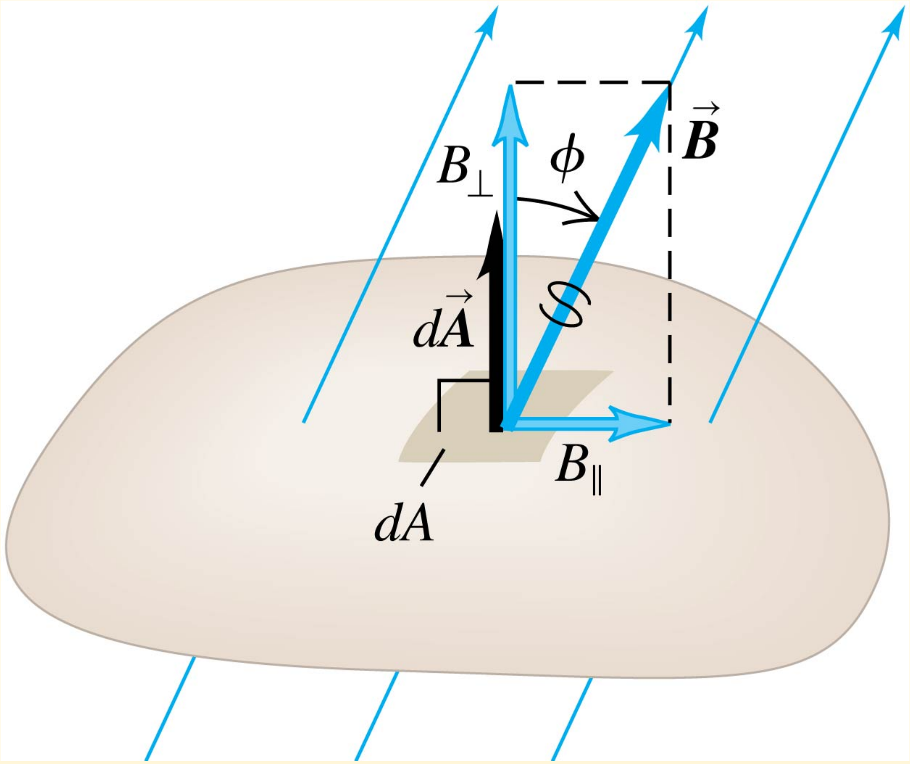
Particle's velocity



Magnetic Flux

The magnetic flux escaping from a small surface element dA is given by

$$d\Phi_B = \vec{B}.d\vec{A}$$



Due to absence of any monopole the field lines never diverges from point. So the total flux entering a closed volume is equal to the total flux exiting it.

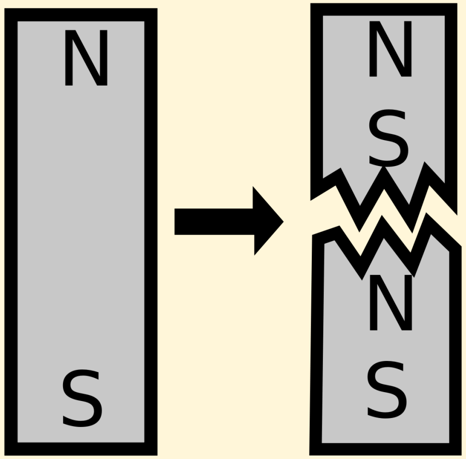
Gauss's Law for Magnetic Field

Gauss's law for magnetism:

The total magnetic flux through *any* **closed surface** ...

$$\overbrace{\oint \vec{B} \cdot d\vec{A}}^{\text{The total magnetic flux through any closed surface ...}} = 0 \quad \text{... equals zero.}$$

- The Magnetic field lines do not diverge from any point.
- This imply that there are no magnetic monopoles
- The smallest unit for a magnetic field is the dipole.



By Divergence theorem:

$$\int \vec{\nabla} \cdot \vec{B}\, dV = \oint \vec{B} \cdot \overrightarrow{dA} = 0 \;\rightarrow\; \vec{\nabla} \cdot \vec{B} = 0$$

Source of Magnetic Fields

- Like electric charges are the cause of electric fields. Currents can be considered as sources of magnetic field.
- Consider a small line element ***dl*** carrying current ***I***. The field at a distance ***r*** from the centre of tis line element is given by the *Biot-Savarts* law