# Solutions to Problem Set-9

*Optics: Interference, Diffraction*

**Problem 1: Fizeau fringes**

A plane monochromatic light wave with a wavelength $\lambda$ falls on the surface of a glass wedge of refractive index $n$. The angle of the wedge $\alpha << 1°$. The plane of incidence is normal to the edge and the angle of incidence is $\theta_1$. Find the distance between neighbouring fringe maxima on a screen placed at right angles to the reflected light.

**Solution:** We have usual equation for the $k$-th maxima,

$$2n\ d_k\ cos\theta_2 = \left(k + \frac{1}{2}\right)\lambda \qquad (1)$$

where, $d_k$ is the thickness of the film for $k$-th fringe and $\theta_2$ is the reflected angle, thus we can write,

$$\begin{aligned}\cos\theta_2 =& \sqrt{1 - sin^2\theta_2}\\ =& \sqrt{1 - \frac{sin^2\theta_1}{n^2}}\end{aligned}$$

and $d_k = h_k\alpha$ ( $\alpha$ being small angle ), where $h_k$ = distance of the fringe from top. Henceforth, we got from eqn.(1),

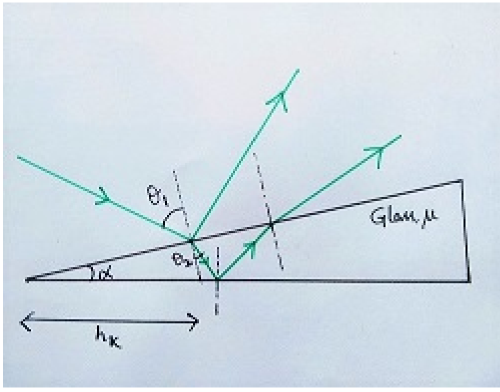


Figure 1: Interference from glass wedge

$$2h_k\alpha\sqrt{n^2 - sin^2\theta_1} = \left(k + \frac{1}{2}\right)\lambda$$

Now, since the screen is placed at right angles to the reflected light, the distance between neighbouring fringe maxima on a screen is,

$$\begin{aligned}\triangle\ x =& (h_k - h_{k-1})\ \ cos\theta_1\\ \triangle\ x =& \frac{\lambda cos\theta_1}{2\alpha\sqrt{n^2 - sin^2\theta_1}} \quad \text{(Ans)}\end{aligned}$$

**Problem 2: Newton's rings**

The spherical surface of a plano-convex lens comes into contact with a glass plate. The space between the lens and the plate is filled up with a transparent liquid. The refractive indices of the lens, liquid and plate are given by: $n_1 = 1.5; n_2 = 1,63; n_3 = 1.70$ respectively. The radius of curvature of the spherical lens is equal to R = 100 cm. Find the radius of the fifth dark Newton's ring in reflected light of wavelength $\lambda = 0.50\ \mu m$.

**Solution:** The condition for minima are,

$$\frac{r^2}{R}n_2 = \left(k + \frac{1}{2}\right)$$

Since the light is travelling from rarer to denser medium, there occur phase changes at both surfaces on reflection, hence minima will occur when path difference is half-multiple of $\lambda$.

Here $k = 4$ for fifth dark ring, thus we can write,

$$\begin{aligned}r =& \sqrt{(2k+1)\frac{\lambda\ R}{2n_2}}\\ r =& \sqrt{\frac{9 \times 0.50 \times 10^-6 \times 1}{2 \times 1.63}}\\ r =& 1.17 \times 10^{-3}\ m\\ r =& 1.17\ mm \quad \text{(Ans)}\end{aligned}$$

**Problem 3: Thin film interference**

A soap film of thickness $5.5 \times 10^{-5}$ cm is viewed at an angle of 45°. Its index of refraction is 1.33. Find the wavelength of light in the visible spectrum which will be absent from the reflected light.

**Solution:** For a wavelength to be absent in light reflected from a thin film, it must satisfy the condition for destructive interference (i.e., for an intensity minima). Referring to the formula derived in Lecture 20,

$$d\cos\theta_t = 2m(\frac{\lambda_f}{4}), \text{ where } \lambda_f = \lambda_0/n, \quad m = 0, 1, 2...$$
$$\implies \lambda_0 = \frac{2nd\cos\theta_t}{m}, \quad m = 1, 2, 3...$$

Using Snell's law, we can find refracted angle $\theta_t$ :

$$n_{air}\sin\theta_i = n_{soap}\sin\theta_t \implies \cos\theta_t = \sqrt{1 - \frac{sin^2\theta_i}{n_{soap}^2}} \approx 0.847. \tag{2}$$

$$\therefore \lambda_0 = \frac{2 \times 1.33 \times 5.5 \times 10^{-5} \times 0.847}{m}\text{cm}$$
$$= \frac{1.239 \times 10^{-4}}{m}\text{ cm} = \frac{1239}{m}\text{ nm}$$

Since light visible to the human eye has a wavelength range of around 380 nm to 740 nm, the wavelength in the visible spectrum that is absent from the reflected light corresponds to $m = 2$ here.

$\therefore \lambda_0 = 619.5$ nm. (Ans)

**Problem 4: Single slit diffraction**

The distance between the first and fifth minima of a single-slit diffraction pattern is 0.35 mm, with the screen 40 cm away from the slit, and light of wavelength 550 nm. Find the slit width.

**Solution:** For a minima in single slit diffraction, the following condition must be satisfied:

$$\beta = \frac{\pi b\sin\theta}{\lambda} = m\pi \quad (m = 1, 2, 3...)$$
$$\implies b\sin\theta = m\lambda$$

Since the maxima widths (less than 0.35 mm) are much smaller than the distance between the slit and screen ($D = 40$ cm), the angle $\theta$ is very small. Thus, the approximation $\sin\theta \approx \tan\theta = \frac{x}{D}$ can be used, where $x$ is the position on the screen.

For first minima,

$$\frac{bx_1}{D} = \lambda$$

For fifth minima,

$$\frac{bx_5}{D} = 5\lambda$$

Subtracting,

$$\frac{b\Delta x}{D} = 4\lambda$$

$$\implies b = \frac{4\lambda D}{\Delta x} = \frac{4 \times 550\text{nm} \times 40\text{cm}}{0.035\text{cm}} = 2.51 \times 10^6 \text{ nm} = 2.51 \text{ mm. (Ans)}$$

**Problem 5: Diffraction grating**

Find the wavelength of monochromatic light falling normally on a diffraction grating with period $d = 2.2\ \mu m$, if the angle between the directions to the Fraunhofer maxima of the first and the second order is equal to $\Delta\theta = 15°$.

**Solution:** For a maxima in the case of a diffraction grating, the following condition must be satisfied:

$$d \sin\theta = m\lambda, \quad m = 0, \pm 1, \pm 2...$$

For first order maxima,

$$d \sin\theta_1 = \lambda \implies \sin\theta_1 = \lambda/d \tag{3}$$

For second order maxima,

$$d \sin\theta_2 = d \sin(\theta_1 + 15°) = 2\lambda \tag{4}$$

Subtracting (1) from (2),

$$d[\sin(\theta_1 + 15°) - \sin\theta_1] = \lambda$$

$$\implies d[\sin\theta_1 \cos 15° + \cos\theta_1 \sin 15° - \sin\theta_1] = \lambda$$

Using eqn. (1),

$$\implies \lambda \cos 15° + \sqrt{d^2 - \lambda^2}\sin 15° - \lambda = \lambda$$

Inserting the given value of $d$ and solving for $\lambda$, we get

$$\lambda = \frac{d}{\sqrt{16.96}} = 0.53\mu\text{m. (Ans)}$$

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