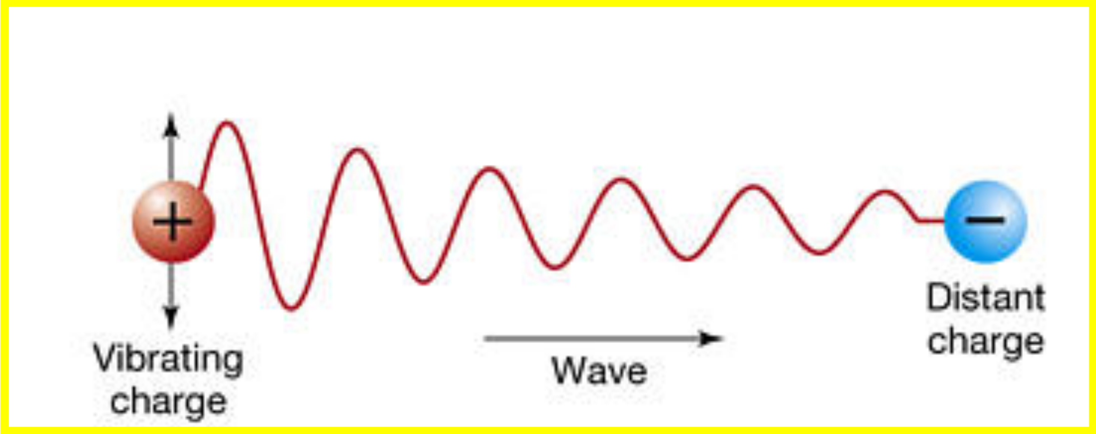
Electromagnetic Waves

Electromagnetic Radiation



If a charge moves non-uniformly, it radiates
The Radiation is known as electromagnetic wave.

Electromagnetic Waves: Spectrum

The frequencies and wavelengths of electromagnetic waves found in nature extend over such a wide range that we have to use a logarithmic scale to show all important bands

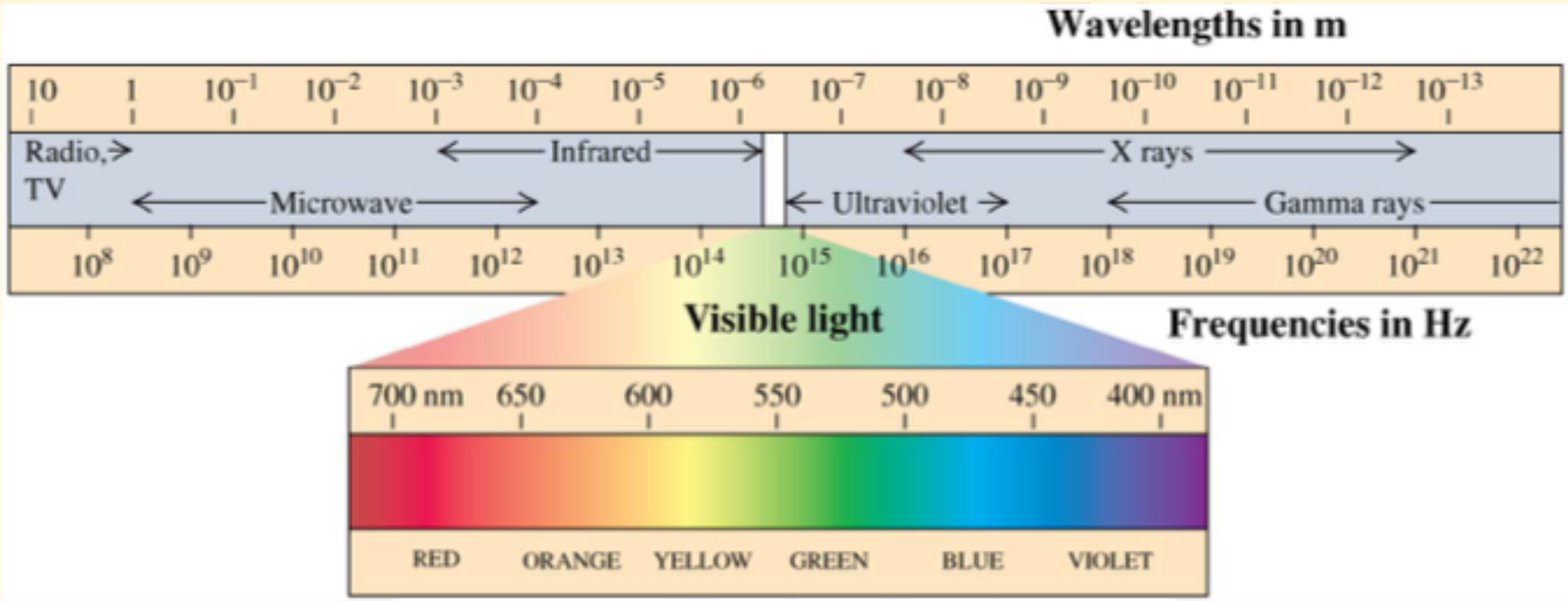
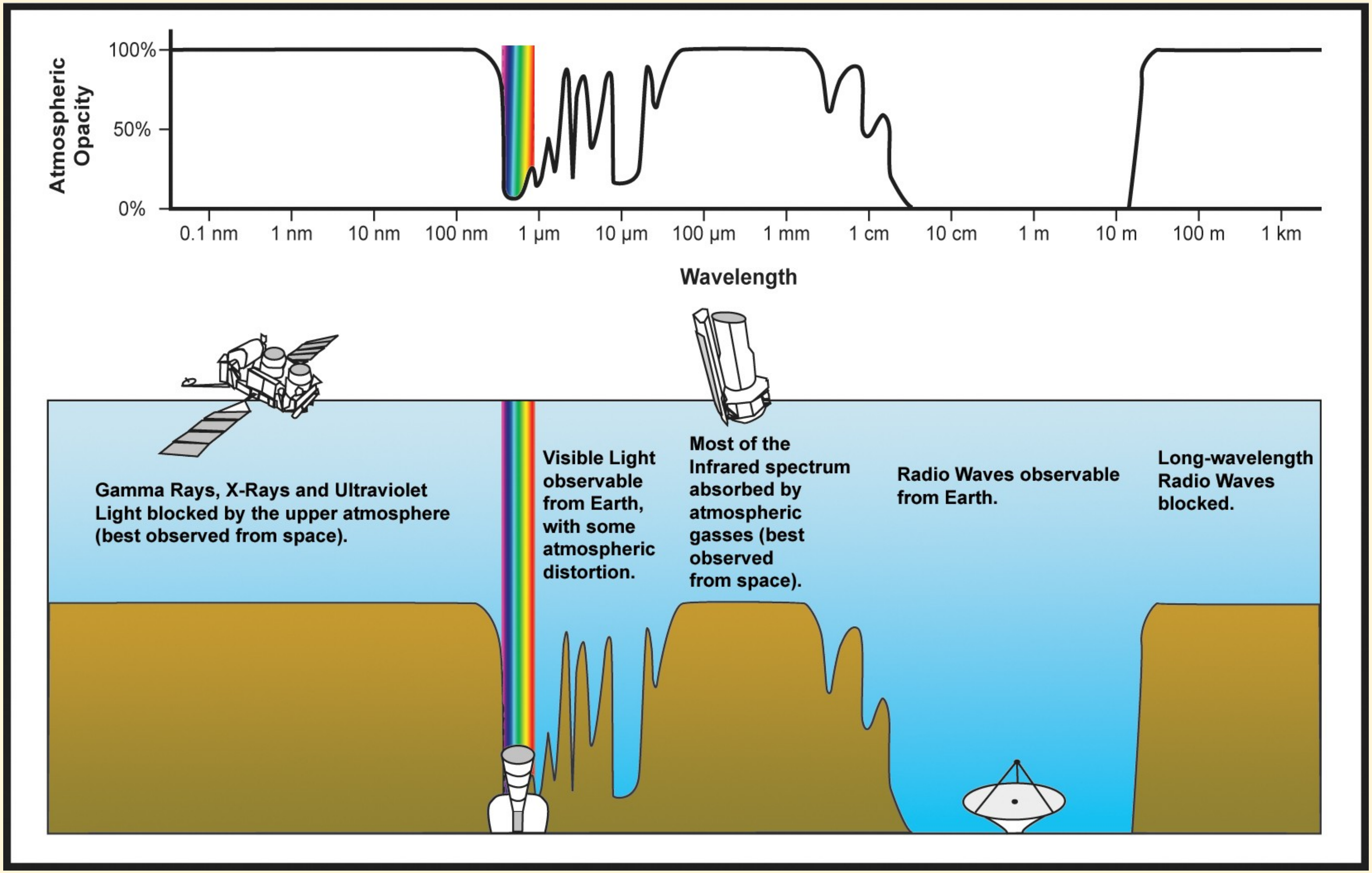


TABLE 32.1 Wavelengths of Visible Light

380–450 nm	Violet
450–495 nm	Blue
495–570 nm	Green
570–590 nm	Yellow
590–620 nm	Orange
620–750 nm	Red


Atmospheric Opacity
100%
50%
0%
0.1 nm
1 nm
10 nm
100 nm
1 µm
10 µm
100 µm
1 mm
1 cm
10 cm
1 m
10 m
100 m
1 km
Wavelength
Gamma Rays, X-Rays and Ultraviolet Light blocked by the upper atmosphere (best observed from space).
Visible Light observable from Earth, with some atmospheric distortion.
Most of the Infrared spectrum absorbed by atmospheric gasses (best observed from space).
Radio Waves observable from Earth.
Long-wavelength Radio Waves blocked.

Electromagnetic Wave Equation: Derivation I

Maxwell's Equations in free space → EM Wave Equation

$$\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt} \longrightarrow \vec{\nabla} \times \vec{\nabla} \times \vec{E} = -\vec{\nabla} \times \frac{d\vec{B}}{dt}$$

$$\vec{\nabla}.\vec{E} = 0 \longrightarrow LHS = \vec{\nabla}\left(\vec{\nabla}.\vec{E}\right) - \nabla^2\vec{E} = -\nabla^2\vec{E}$$

$$\vec{\nabla} \times \vec{B} = \mu_0\epsilon_0\frac{d\vec{E}}{dt} \longrightarrow RHS = -\mu_0\epsilon_0\frac{d^2\vec{E}}{dt^2}$$

$$\Longrightarrow \nabla^2\vec{E} = \mu_0\epsilon_0\frac{d^2\vec{E}}{dt^2}$$

$$\nabla^2\vec{E} = \frac{1}{c^2}\frac{d^2\vec{E}}{dt^2}$$

$$c = \frac{1}{\sqrt{\mu_o\epsilon_0}} \longleftarrow$$

Electromagnetic Wave Equation: Derivation I

Maxwell's Equations in free space $\rightarrow$ EM Wave Equation

$$\vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \frac{d\vec{E}}{dt} \rightarrow \vec{\nabla} \times \vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \vec{\nabla} \times \frac{d\vec{E}}{dt}$$

$$\vec{\nabla}.\vec{B} = 0 \rightarrow LHS = \vec{\nabla}\left(\vec{\nabla}.\vec{B}\right) - \nabla^2 \vec{B} = -\nabla^2 \vec{B}$$

$$\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt} \rightarrow RHS = -\mu_0 \epsilon_0 \frac{d^2 \vec{B}}{dt^2}$$

$$\implies \nabla^2 \vec{B} = \mu_0 \epsilon_0 \frac{d^2 \vec{B}}{dt^2}$$

$$c = \frac{1}{\sqrt{\mu_o \epsilon_0}} \leftarrow \nabla^2 \vec{B} = \frac{1}{c^2} \frac{d^2 \vec{B}}{dt^2}$$

Electromagnetic waves

$$\boxed{\nabla^2 \mathbf{E} = \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2}}$$

for E field

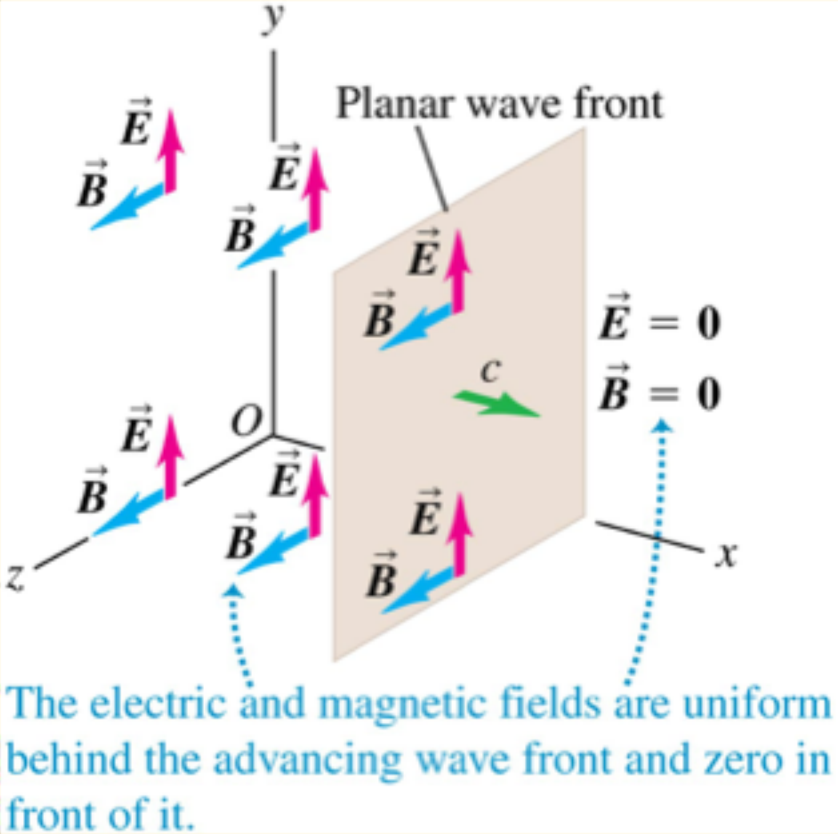
$$\boxed{\nabla^2 \mathbf{B} = \frac{1}{c^2}\frac{\partial^2 \mathbf{B}}{\partial t^2}}$$

for B field

Electromagnetic Wave: Properties

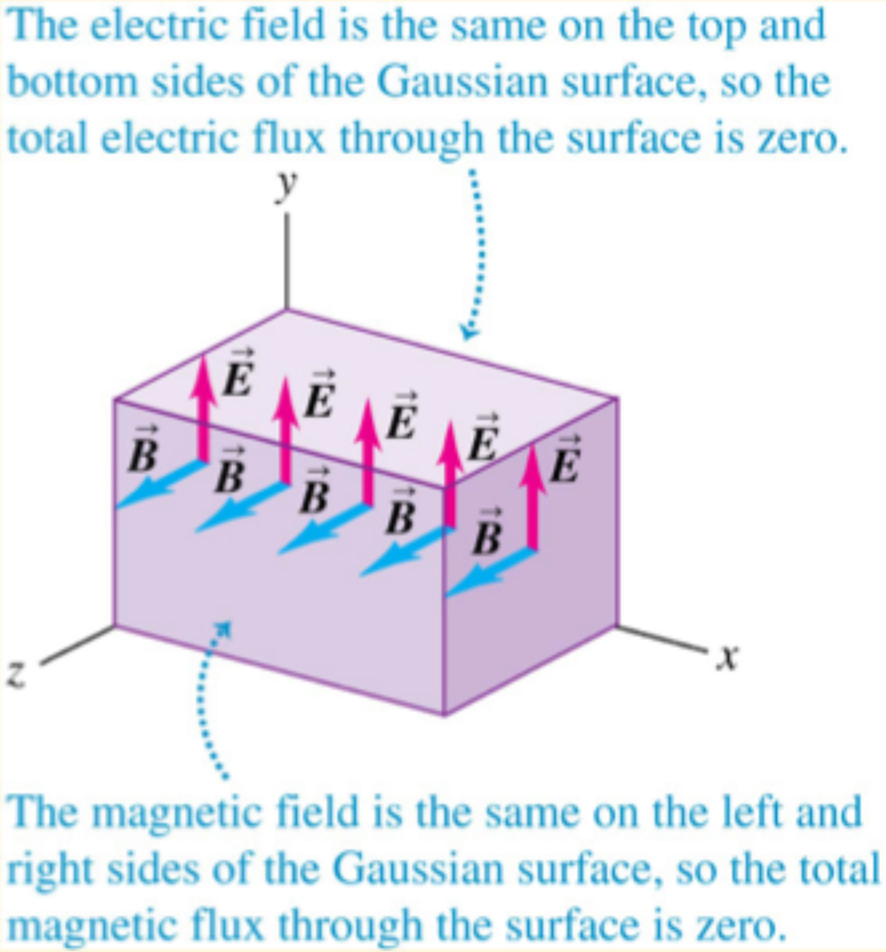
Plane Electromagnetic Wave

- Imagine that all space is divided into two regions separated by a plane perpendicular to the x-axis.
- At every point to the left of this plane there are uniform electric field, magnetic fields as shown.
- The boundary plane, which we call the wave front, moves in the +x-direction with a constant speed c.



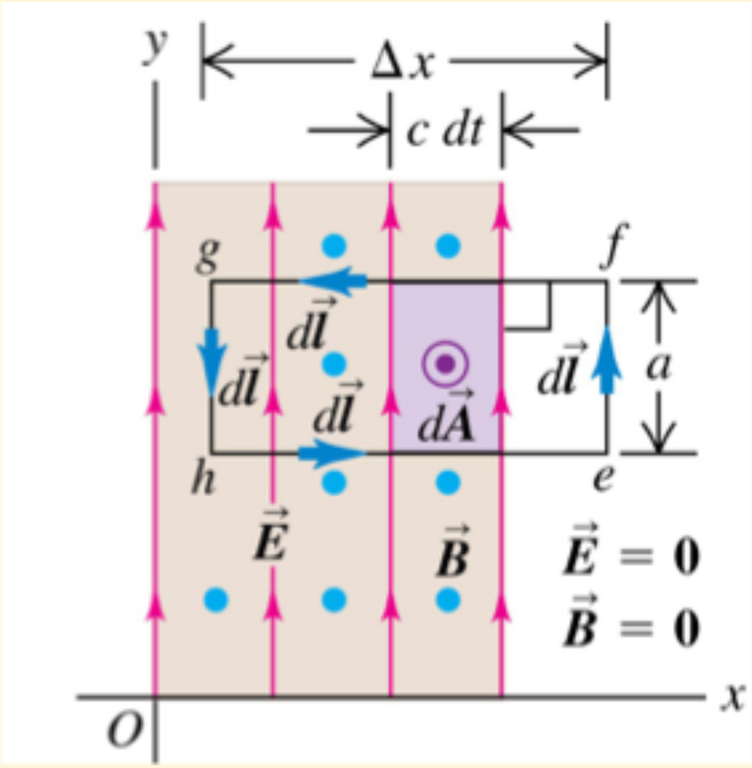
Gauss's laws and the simple plane wave

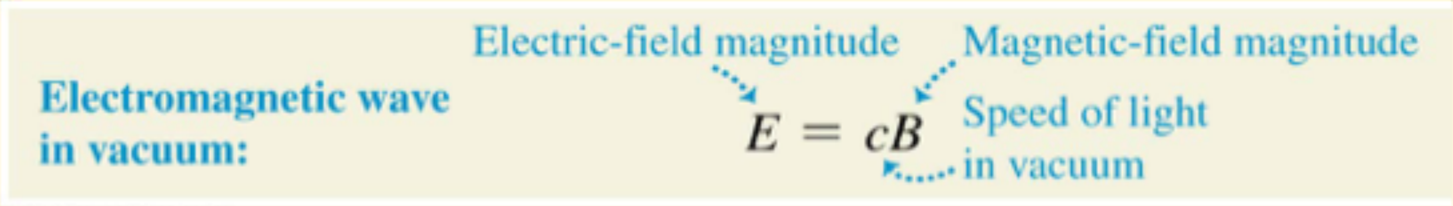
- Consider a Gaussian surface of a rectangular box, through which the simple plane wave is traveling.
- The box encloses no electric charge.
- In order to satisfy Maxwell's first and second equations, the electric and magnetic fields must be perpendicular to the direction of propagation; that is, the wave must be **transverse**.



Faraday's laws and the simple plane wave

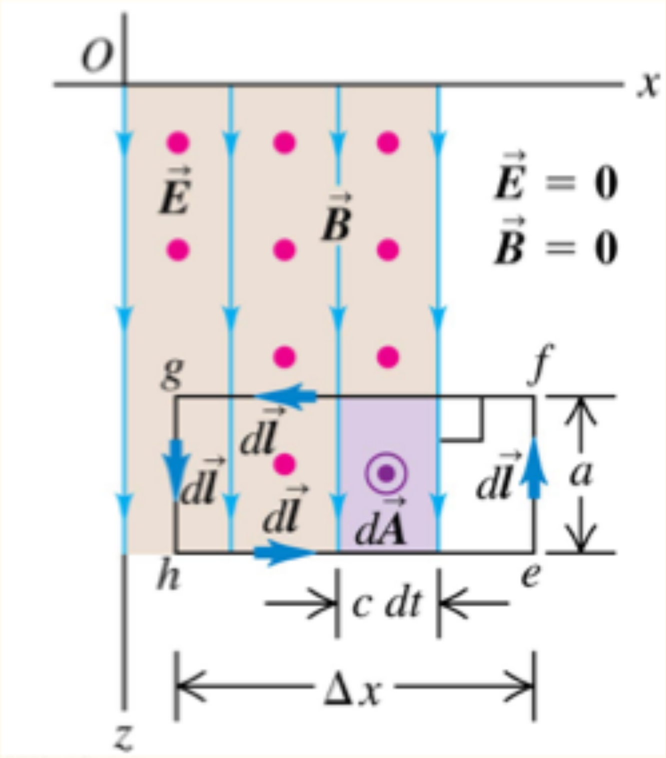
- The simple plane wave must satisfy Faraday's law in a vacuum.
- In a time dt, the magnetic flux through the rectangle in the xy-plane increases by an amount $d\Phi_B$.
- This increase equals the flux through the shaded rectangle with area $ac\,dt$; that is, $d\Phi_B = Bac\,dt$.
- Thus $d\Phi_B/dt = Bac$.

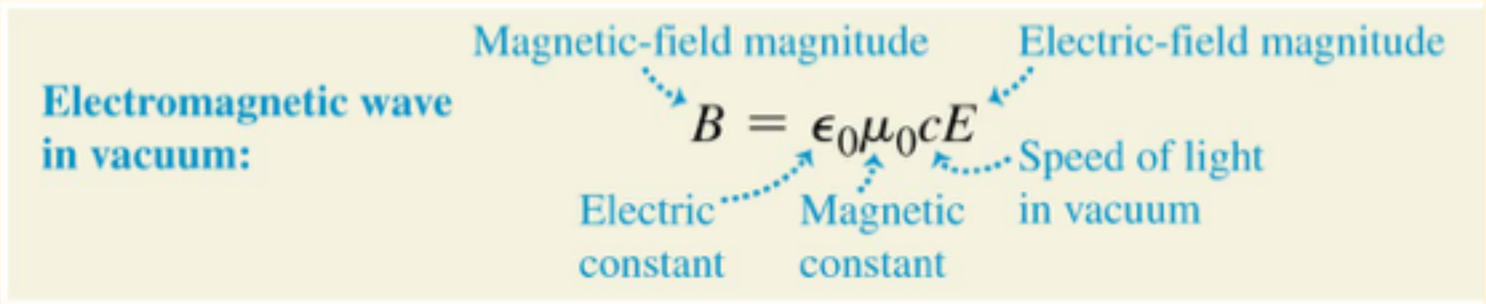




Ampere's laws and the simple plane wave

- The simple plane wave must satisfy Ampere's law in a vacuum.
- In a time dt, the electric flux through the rectangle in the xz-plane increases by an amount $d\Phi_E$.
- This increase equals the flux through the shaded rectangle with area $ac\,dt$; that is, $d\Phi_E = Eac\,dt$.
- Thus $d\Phi_E/dt = Eac$. This implies:





Properties of electromagnetic waves

Maxwell's equations imply that in an electromagnetic wave, both the electric and magnetic fields are perpendicular to the direction of propagation of the wave, and to each other.

In an electromagnetic wave, there is a definite ratio between the magnitudes of the electric and magnetic fields: $E = cB$.

Unlike mechanical waves, electromagnetic waves require no medium.

In fact, they travel in vacuum with a definite and unchanging speed:

Speed of electromagnetic waves in vacuum

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

Electric constant

Magnetic constant

Inserting the numerical values of these constants, we obtain
$c = 3.00 \times 108$ m/s.