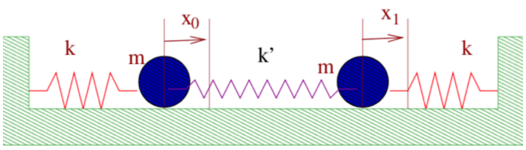
Tutorial 2 (Lecture 4-6): Forced Oscillations & Coupled Oscillators

1. A particle of mass m can perform undamped harmonic oscillations due to an elastic restoring force with a spring constant k. When the particle is in equilibrium, a constant force F is applied to it for τ seconds. Find the amplitude of oscillation of the particle after the force ceases.

2. Show that, $x(t) = \frac{f}{\omega_0^2-\omega^2}(\cos\omega t - \cos\omega_0 t)$, is a solution of the undamped forced system, $\ddot{x} + \omega_0^2 x = f\cos\omega t$, with initial conditions, $x(0) = \dot{x}(0) = 0$. Show that near resonance, $\omega \to \omega_0$, $x(t) \approx \frac{f}{2\omega_0} t \sin\omega_0 t$, that is the amplitude of the oscillations grow linearly with time. Plot the solution near resonance. (Hint: Take $\omega = \omega_0 - \Delta\omega$, and expand the solution taking $\Delta\omega \to 0$.)

3. A mildly damped oscillator driven by an external force is known to have a resonance at an angular frequency somewhere near $\omega = 1\ MHz$ with a quality factor of 1100. Further, for the force (in Newtons) $F(t) = 10\cos\omega t$, the amplitude of oscillations is $8.26\ mm$ at $\omega = 1.0\ KHz$ and $1.0\ \mu m$ at $100\ MHz$.
   a) What is the spring constant of the oscillator?
   b) What is the natural frequency ω_0 of the oscillator?
   c) What is the FWHM?
   d) What is the phase difference between the force and the oscillations at $\omega = \omega_0 + \frac{FWHM}{2}$?

4. For the coupled oscillator shown in figure below with $k = 10\ Nm^{-1}$, $k' = 30\ Nm^{-1}$ and $m = 1\ kg$, both particles are initially at rest. The system is set into oscillations by displacing x_0 by $40\ cm$ while $x_1 = 0$.
   a) What is the angular frequency of the faster normal mode?
   b) Calculate the average kinetic energy of the mass with coordinate x_1?
   c) How does the average kinetic energy of x_1 change if the mass of both the particles is doubled? (Answer: (a)$8.37\ s^{-1}, b) 8.00 \times 10^{-1} J, c)\ No\ Change$)



5. Two identical spring-mass systems A and B are coupled by a weak middle spring having a spring constant smaller by a factor of 100 (i.e. $100k_{middle} = k_A = k_B$). Mass A is pulled by a small distance and released from rest, while mass B is released from rest at its equilibrium position, at $t = 0$. Calculate the approximate number of oscillations completed by mass A before its oscillations die down first. (Answer: 50)