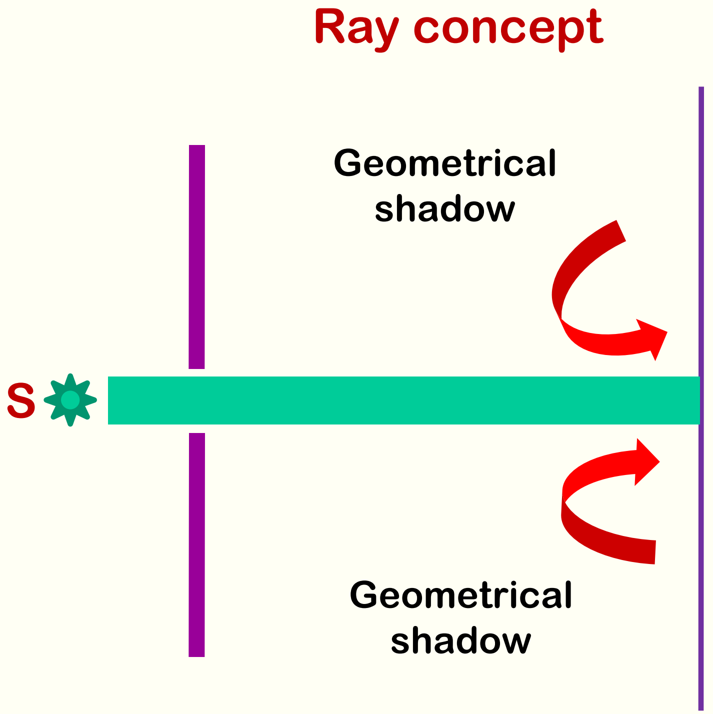
Diffraction

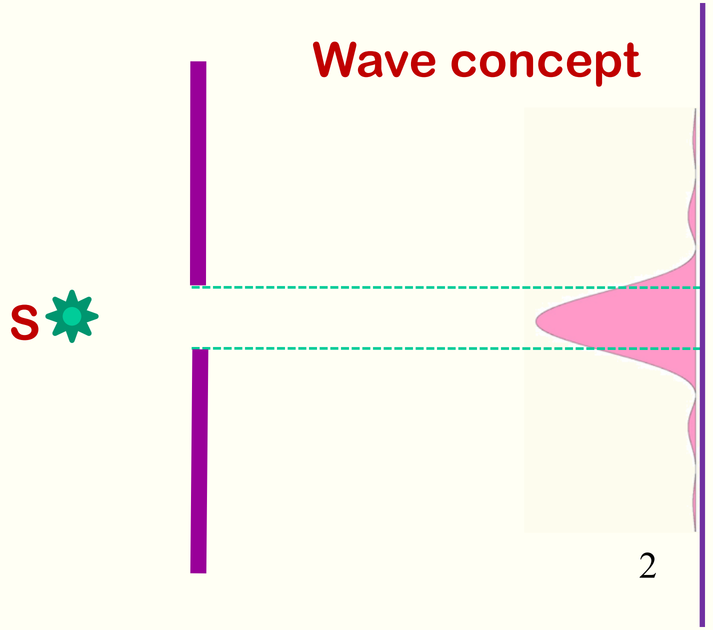
Diffraction

"Any deviation of light rays from rectilinear path which is neither reflection nor refraction is known as diffraction." (Sommerfeld)

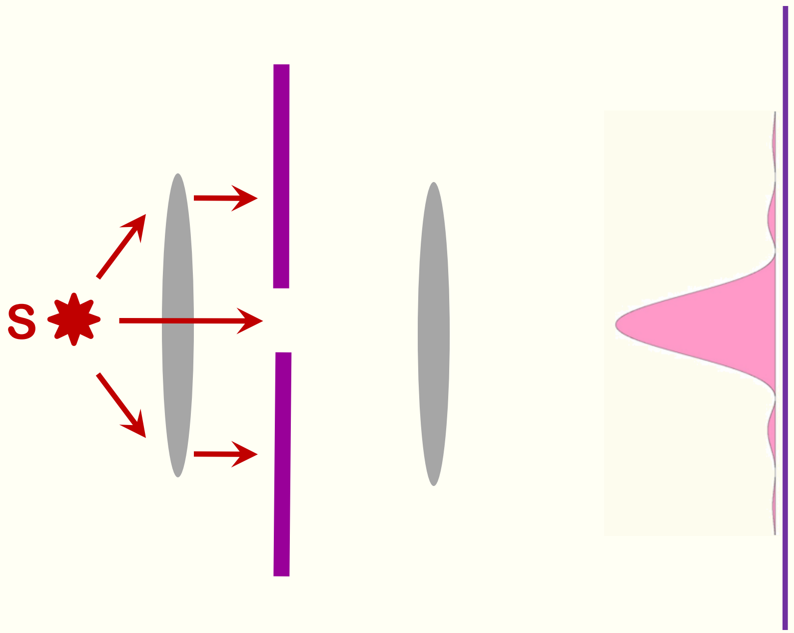


Types or kinds of diffraction:

1. Fraunhofer (1787-1826)
2. Fresnel (1788-1827)

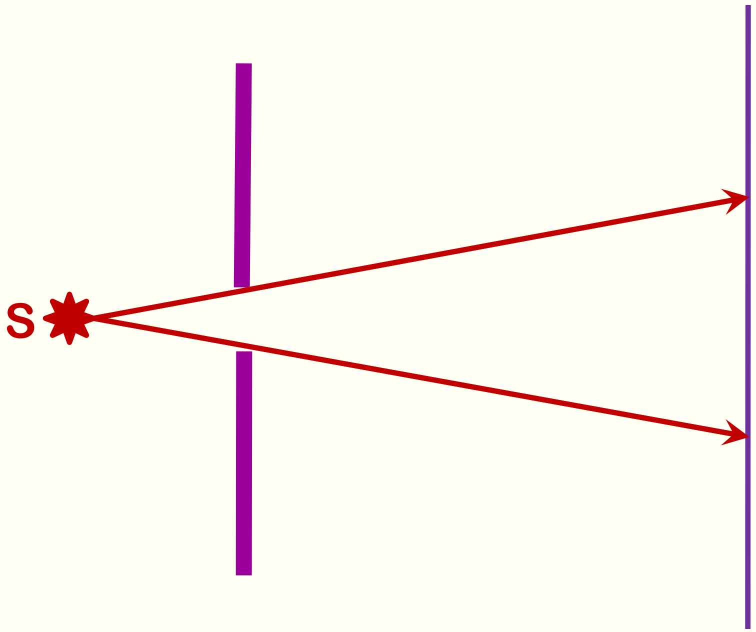


Fraunhofer Diffraction



Both source and screen are in infinity- Fraunhofer class

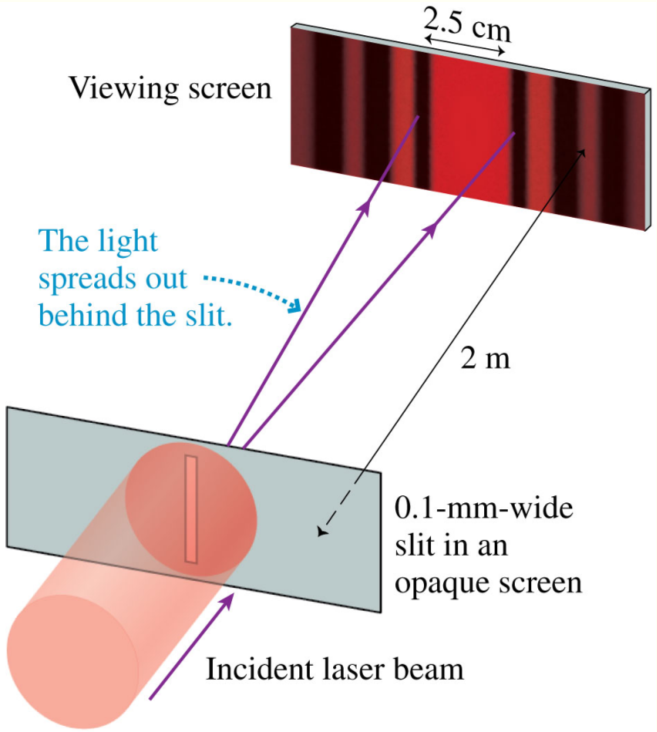
Fresnel Diffraction



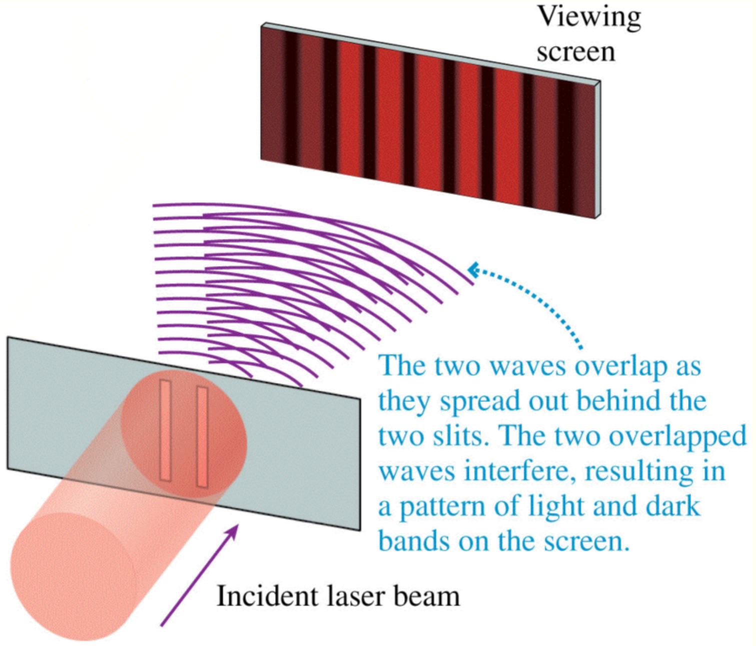
Either the source or the screen (or both) are at finite distance - Fresnel class

Interference Vs Diffraction

Diffraction



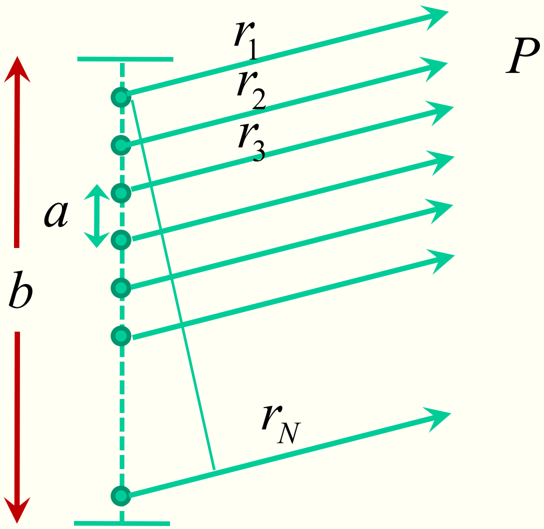
Interference



1. Two separate wave fronts originating from two coherent sources produce interference. Secondary wavelets originating from different parts of the same wave front constitute diffraction. Thus the two are entirely different in nature.
2. The region of minimum intensity is perfectly dark in interference. In diffraction they are not perfectly dark.
3. Width of the fringes is equal in interference. In diffraction they are never equal.
4. The intensity of all positions of maxima are of the same intensity in interference. In diffraction they do vary.

Single slit diffraction pattern

N coherent oscillator model



The sum of the interfering wavelet at P is

$$E = E_0 e^{i(\mathrm{kr}_1-\omega \mathrm{t})} + E_0 e^{i(\mathrm{kr}_2-\omega \mathrm{t})} + E_0 e^{i(\mathrm{kr}_3-\omega \mathrm{t})} + \ldots + E_0 e^{i(\mathrm{kr}_N-\omega \mathrm{t})}$$

$$= E_0 e^{i(\mathrm{kr}_1-\omega \mathrm{t})}\left[1 + e^{i\mathrm{k}(\mathrm{r}_2-r_1)} + e^{i\mathrm{k}(\mathrm{r}_3-r_1)} \ldots + e^{i\mathrm{k}(\mathrm{r}_N-r_1)}\right]$$

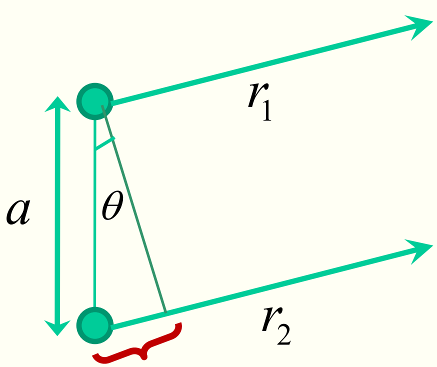
Phase difference between the adjacent source $\boxed{\phi = k\Lambda = ka\sin\theta}$

$$E = E_0 e^{i(\mathrm{kr}_1-\omega \mathrm{t})}\left[1 + e^{i\phi} + e^{2i\phi} \ldots + e^{i(\mathrm{N}-1)\phi}\right]$$

$$= E_0 e^{i(\mathrm{kr}_1-\omega \mathrm{t})}\left(\frac{e^{iN\phi}-1}{e^{i\phi}-1}\right)$$

$$= E_0 e^{-i\omega \mathrm{t}} e^{i[\mathrm{kr}_1+(\mathrm{N}-1)\frac{\phi}{2}]}\frac{\sin\left(\frac{N}{2}\phi\right)}{\sin\left(\frac{\phi}{2}\right)}$$

$$\boxed{I = A\frac{\sin^2\left(\frac{N}{2}\phi\right)}{\sin^2\left(\frac{\phi}{2}\right)}}$$



$\Lambda = a\sin\theta$

$b = \left(N-1\right)a =$ **Slit width**

We have........

$$I = A\frac{\sin^2\left(\frac{N}{2}\phi\right)}{\sin^2\left(\frac{\phi}{2}\right)}$$

$$\phi = k\Lambda = ka\sin\theta$$

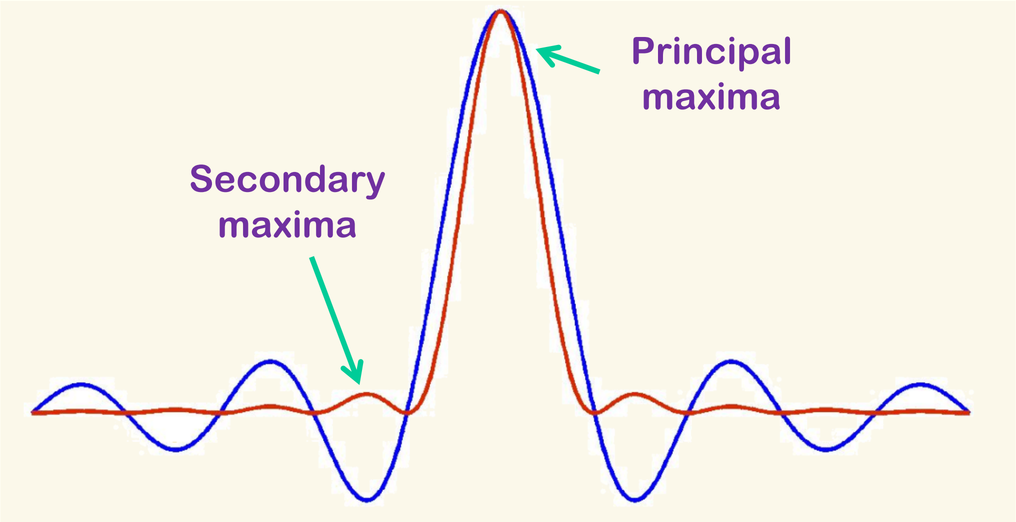
$b = \left(N-1\right)a =$ **Slit width**

In the limit...... $N \to \infty \quad a \to 0$

$$I = A\frac{\sin^2\left(\frac{N}{2}ka\sin\theta\right)}{\sin^2\left(\frac{1}{2}ka\sin\theta\right)} = nA\frac{\sin^2\left(\frac{\pi b}{\lambda}\sin\theta\right)}{\left(\frac{\pi b}{\lambda}\sin\theta\right)^2}$$

Now if $I_0 = nA \qquad \beta = \frac{\pi b}{\lambda}\sin\theta$

$$\boxed{I = I_0\frac{\sin^2\beta}{\beta^2}}$$



Condition of maxima and minima

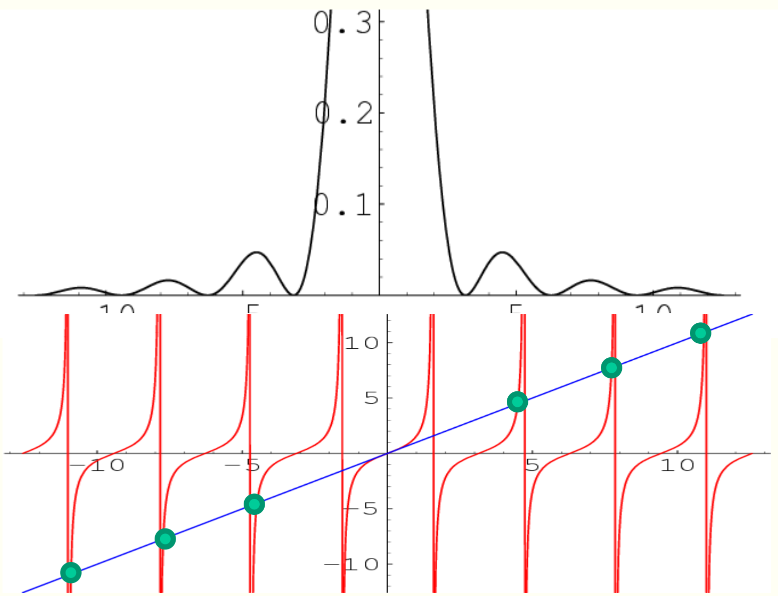
$$\boxed{I = I_0 \frac{\sin^2 \beta}{\beta^2}}$$

$$\beta = m\pi, \quad (\mathrm{m} \neq 0) \Longrightarrow$$ **Condition of minima**

$$\beta = \frac{\pi b}{\lambda} \sin\theta$$

For Principal maxima $$\theta = 0; \quad \frac{\sin\beta}{\beta} = 1, \;\; I(\theta) = I(0)$$

For secondary maxima $$\frac{dI}{d\beta} = I(0) \frac{2\sin\beta(\beta\cos\beta - \sin\beta)}{\beta^3} = 0 \Longrightarrow \boxed{\tan\beta = \beta}$$

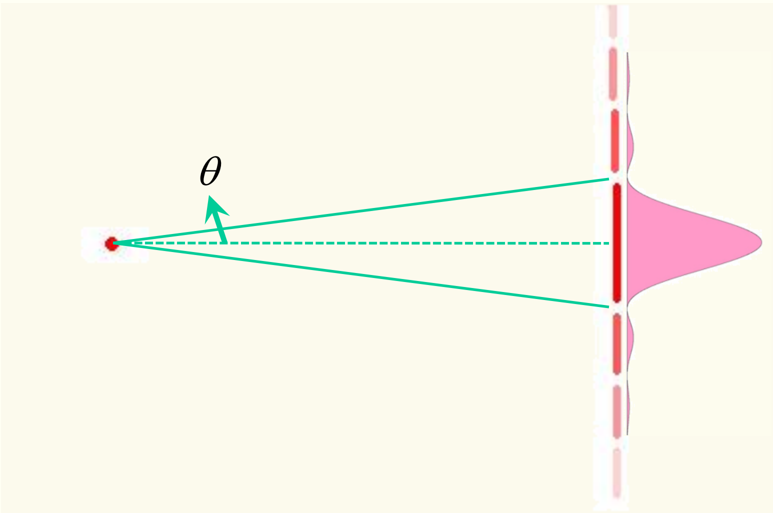


For secondary maxima

$\Downarrow$

$$\beta = \pm 1.4303\pi, \pm 2.4590\pi, \pm 3.470\pi, \ldots..$$

Angular width of central maximum



$$\beta = m\pi, \quad (\mathrm{m} \neq 0)$$

$$\beta = \frac{\pi b}{\lambda}\sin\theta$$

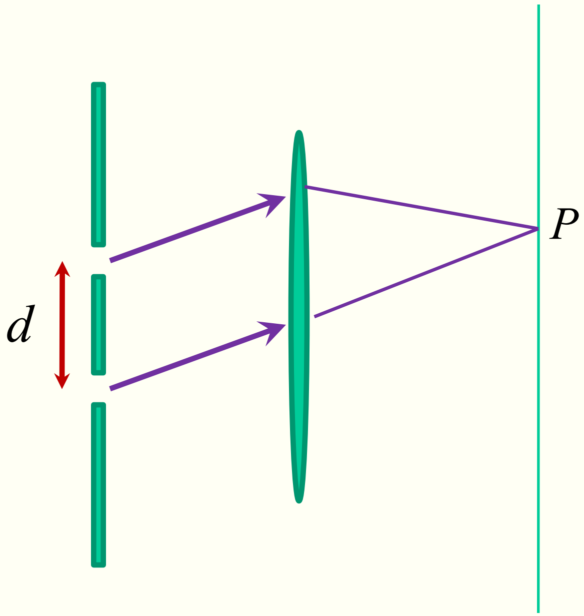
For first minima $m = 1 \Rightarrow \beta = \pi$

$$\beta = \frac{\pi b}{\lambda}\sin\theta = \pi \Rightarrow \sin\theta = \frac{\lambda}{b}$$

θ **is small** $\Rightarrow \sin\theta \approx \theta = \dfrac{\lambda}{b}$

Angular width $2\theta = \dfrac{2\lambda}{b}$

Two slit Fraunhofer diffraction pattern



We have so far…….

$$E_1 = E_0 e^{-i\omega t} e^{i[kr_1 + (N-1)\frac{\phi}{2}]} \frac{\sin\beta}{\beta}$$

$$\beta = \frac{\pi b}{\lambda}\sin\theta \qquad \phi = k\Lambda = ka\sin\theta$$

$$b = (N-1)a = \text{Slit width}$$

$$(N-1)\frac{\phi}{2} = (N-1)\frac{ka\sin\theta}{2} = \frac{\pi b\sin\theta}{\lambda} = \beta$$

$$E_1 = E_0 e^{-i\omega t} e^{i[kr_1 + \beta]} \frac{\sin\beta}{\beta}$$ **(Field due to 1st slit)**

$$E_2 = E_0 e^{-i\omega t} e^{i[kr_1 + \beta + \delta]} \frac{\sin\beta}{\beta}$$ **(Field due to 2nd slit)**

$$\delta = \frac{2\pi}{\lambda} d\sin\theta$$ ⇒ **Phase difference**

$$\frac{\delta}{2} = \frac{\pi}{\lambda} d\sin\theta = \gamma$$

Total field at P

$$E = E_1 + E_2$$

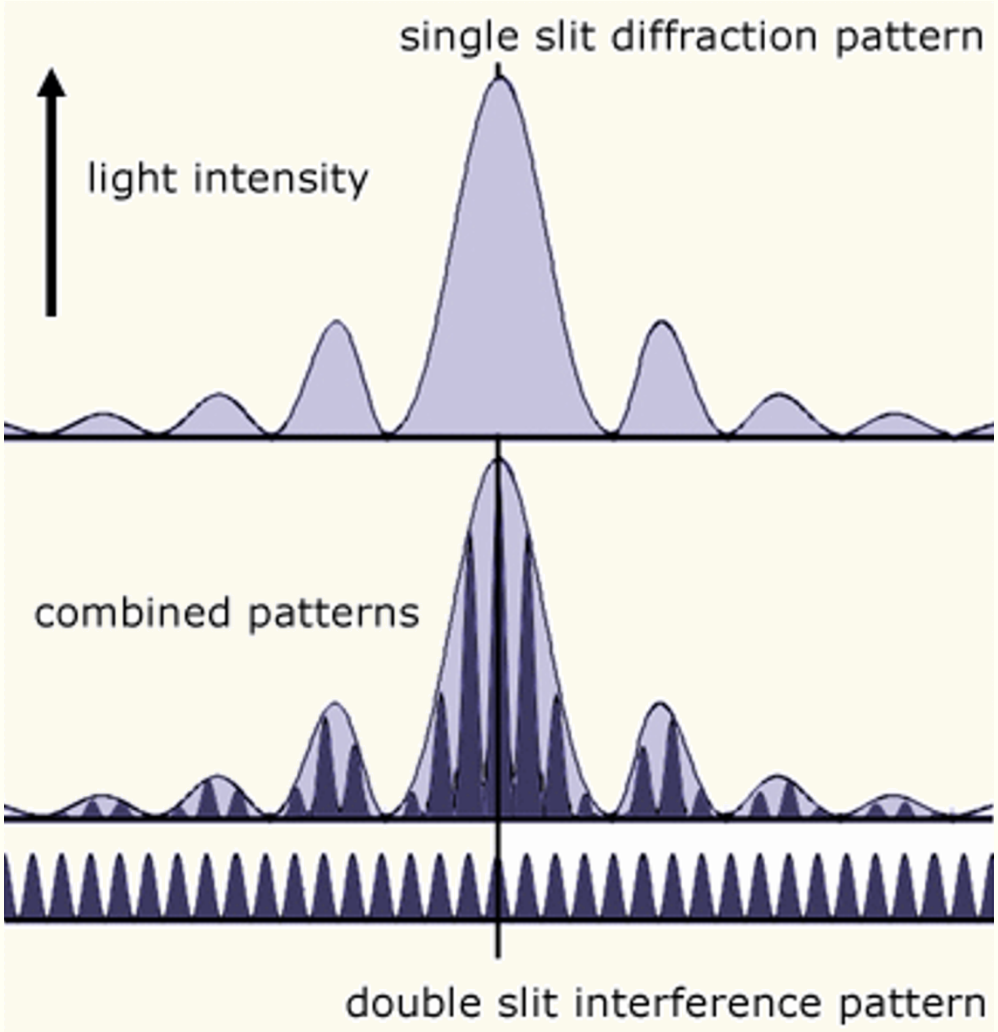
$$E = E_0 e^{i(kr_1 - \omega t + \beta + \delta/2)} \frac{\sin\beta}{\beta}\left(e^{i\frac{\delta}{2}} + e^{-i\frac{\delta}{2}}\right)$$

$$E = 2E_0 e^{i(kr_1 - \omega t + \beta + \delta/2)} \frac{\sin\beta}{\beta}\cos\left(\frac{\delta}{2}\right)$$

$$\Downarrow$$

$$\boxed{I = I_0 \frac{\sin^2\beta}{\beta^2}\cos^2\gamma}$$

Condition for Missing orders



$$I = I_0 \frac{\sin^2 \beta}{\beta^2} \cos^2 \gamma$$

Diffraction Minima at

$$b \sin \theta = m \lambda \qquad m \neq 0$$

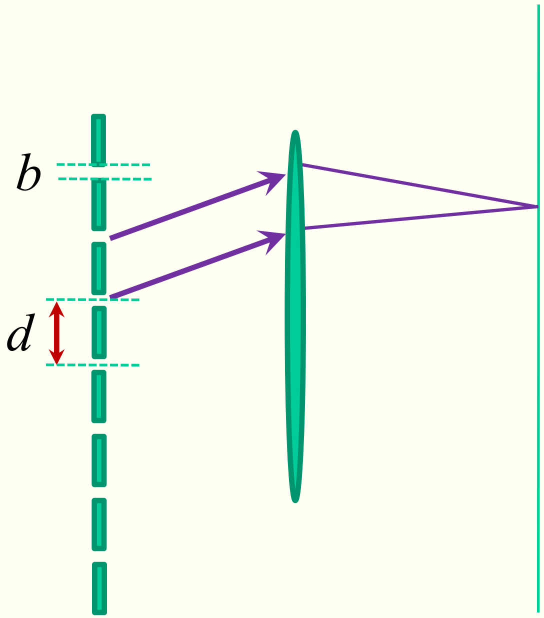
Interference Maxima at

$$d \sin \theta = n \lambda$$

When the above two equations are satisfied at the same point in the pattern (same θ), dividing one equation by the other gives the condition for *missing orders*.

$$\boxed{d = \left(\frac{n}{m}\right) b}$$

N slit Fraunhofer diffraction pattern (Grating)



$$E_1 = E_0 e^{-i\omega t} e^{i[\mathrm{kr}_1 + \beta]} \frac{\sin\beta}{\beta}$$ **(Field due to 1st slit)**

$$E_2 = E_0 e^{-i\omega t} e^{i[\mathrm{kr}_1 + \beta + \delta]} \frac{\sin\beta}{\beta}$$ **(Field due to 2nd slit)**

$$E_N = E_0 e^{-i\omega t} e^{i[\mathrm{kr}_1 + \beta + \delta(\mathrm{N}-1)]} \frac{\sin\beta}{\beta}$$ **(Field due to Nth slit)**

Total Field

$$E = \sum_{i=1}^{N} E_i$$

$$E = E_0 \frac{\sin\beta}{\beta} e^{i(\mathrm{kr}_1 + \beta - \omega t)} \left[1 + e^{i\delta} + e^{2i\delta} \ldots + e^{i(\mathrm{N}-1)\delta}\right]$$

$$= E_0 \frac{\sin\beta}{\beta} e^{i(\mathrm{kr}_1 + \beta - \omega t)} \left(\frac{e^{iN\delta} - 1}{e^{i\delta} - 1}\right)$$

$$= E_0 \frac{\sin\beta}{\beta} e^{-i\omega t} e^{i[\mathrm{kr}_1 + \beta + (\mathrm{N}-1)\frac{\delta}{2}]} \frac{\sin\left(\frac{N}{2}\delta\right)}{\sin\left(\frac{\delta}{2}\right)}$$

$$\beta = \frac{\pi b}{\lambda} \sin\theta$$

$$\delta = \frac{2\pi}{\lambda} d \sin\theta$$ ➡ **Phase difference**

$$\frac{\delta}{2} = \frac{\pi}{\lambda} d \sin\theta = \gamma$$

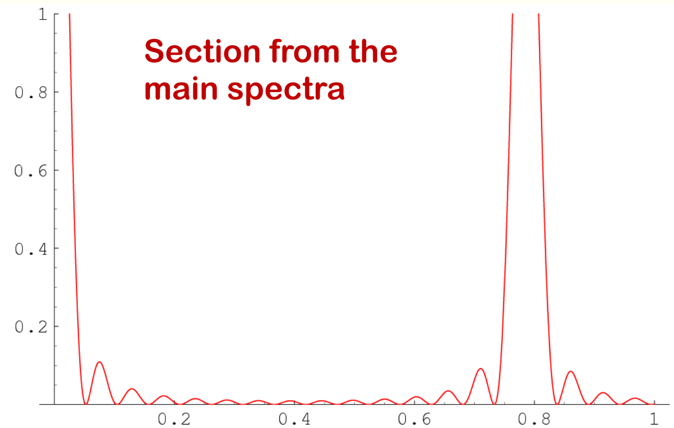
$$\boxed{I = I_0 \frac{\sin^2\beta}{\beta^2} \frac{\sin^2 N\gamma}{\sin^2\gamma}}$$

Grating Spectrum

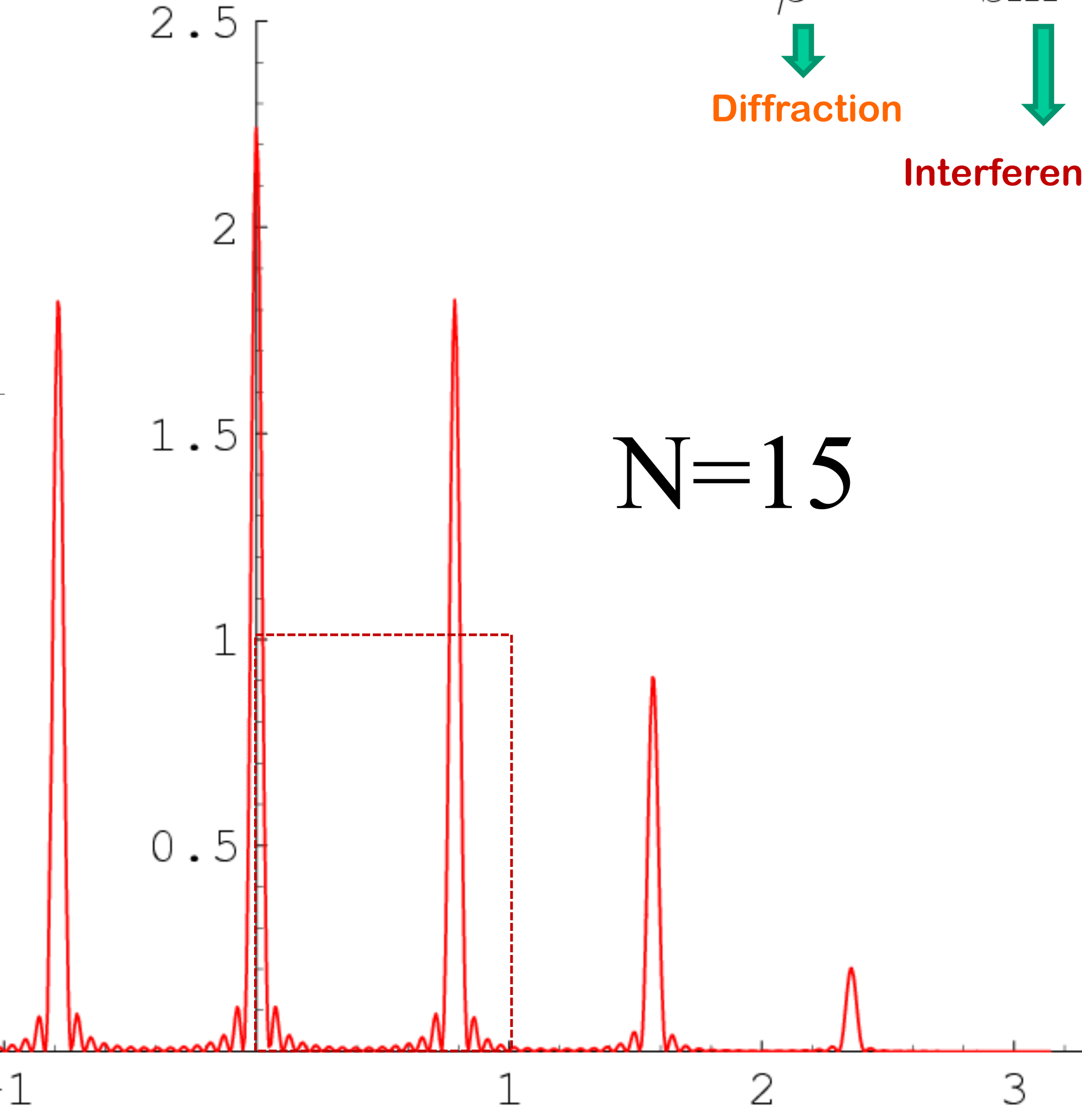
$$I(\theta) = I_0 \frac{\sin^2 \beta}{\beta^2} \cdot \frac{\sin^2 N\gamma}{\sin^2 \gamma}$$

Diffraction

Interference



The intensity becomes zero *(N-1)* times between two successive primary maxima. There are (*N-2*) secondary maxima in between.



When $\gamma = 0$, or $m\pi$ (m = 0, ±1, ±2,…)

$$\frac{\sin N\gamma}{\sin\gamma} = \pm N \qquad \left(\gamma = \frac{\pi}{\lambda} d \sin\theta\right)$$

$$d\sin\theta = m\lambda \qquad (\mathrm{m} = 0, \pm 1, \pm 2, \ldots)$$

The intensity is maximum when this condition is satisfied.
These are called the Primary maxima.
***m* gives the order of the maximum.**
The intensity drops away from the primary maxima.
The intensity becomes zero *(N-1)* times between two successive primary maxima.
There are (*N-2*) secondary maxima in between.
As *N* increases the number of secondary maxima increases and the primary maxima becomes sharper.

Principal maxima

When $\gamma = 0$, or $m\pi$ (m = 0, ±1, ±2,…)

$$d \sin\theta = m\lambda \quad (m = 0, \pm 1, \pm 2, \ldots)$$

$$N\gamma = mN\pi \Leftrightarrow d\sin\theta_m = m\lambda$$

$$m = 0, \pm 1, \pm 2, \cdots$$

$$I(\theta) = I_0 \frac{\sin^2 N\gamma}{\sin^2 \gamma}$$

Minima

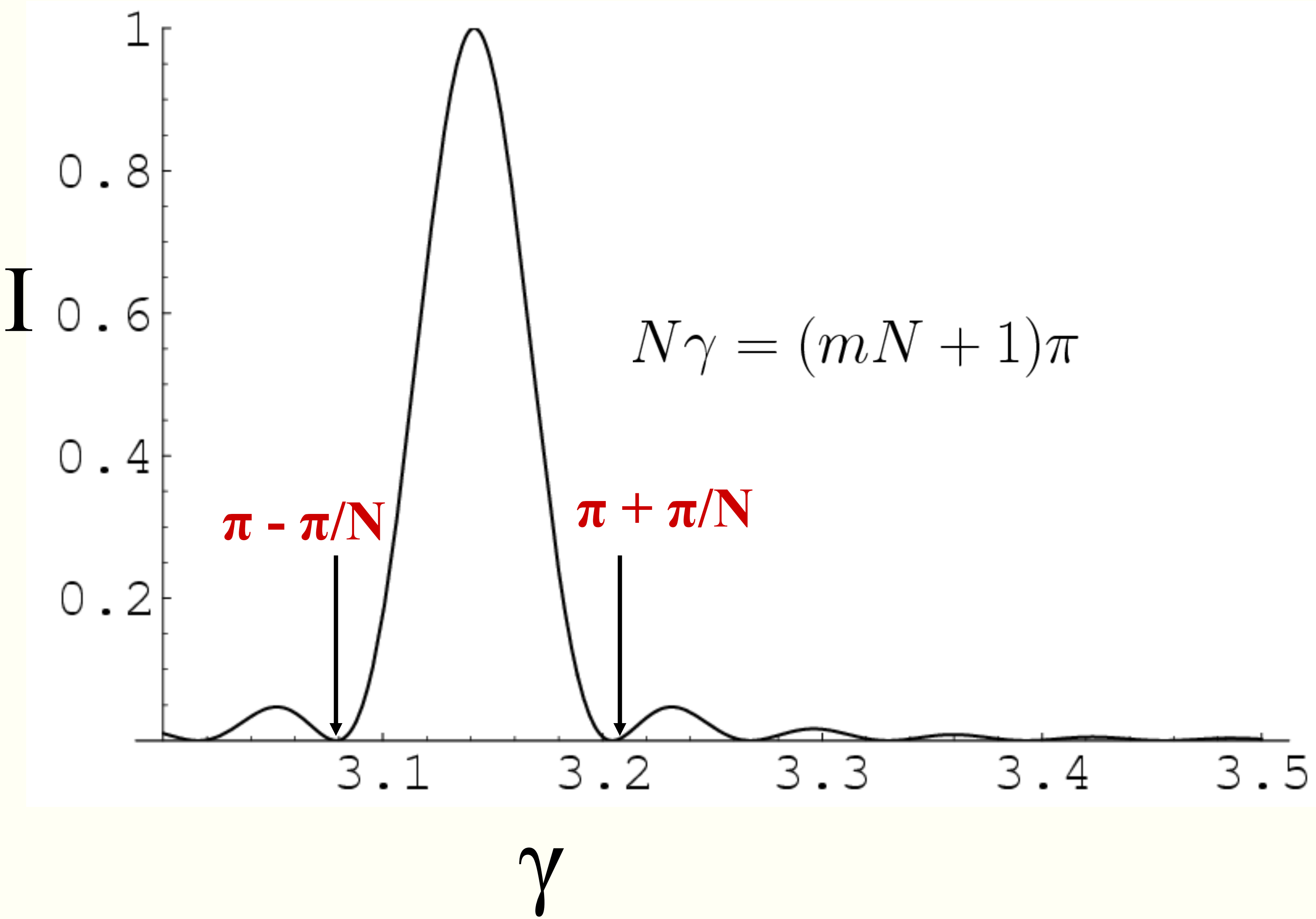
$$\sin N\gamma = 0 \Leftrightarrow N\gamma = \pm n\pi, n \neq 0, N, 2N, \cdots$$

Because at minima we should have, $\sin\gamma \neq 0$

$$N\gamma = (mN + 1)\pi$$

$$d \sin(\theta_m + (\Delta\theta)_w) = m\lambda + \frac{\lambda}{N}$$

If maximum is at θ_m and $\Delta\theta_w$ is the width, then the intensity should be zero at $(\theta_m + \Delta\theta_w)$



I
γ
0.2
0.4
0.6
0.8
1
3.1
3.2
3.3
3.4
3.5
$N\gamma = (mN+1)\pi$
π - π/N
π + π/N