Tutorial 1: Quantum Mechanics

1. a) Consider the following function in the momentum space:

$$g(k) = g_0 \ (-k_0 \leq k \leq k_0),$$
$$= 0 \text{ otherwise.}$$

Find the corresponding function in the position space, and also evaluate $\Delta x \Delta k$.

2. Consider the following thought experiment (proposed by Dicke-Wittke, see the figure at the end). Suppose there is a cylindrical bird cage with regular spacing $a = \frac{2\pi R}{N}$ between the bars, where N is the number of bars and R is the radius of cylinder. If radiation is emitted from the axis of the cylinder, the bars can act as a diffraction grating.

a) If the beam emerges from the cage at an angle θ with the original direction, find the condition for maximum intensity which relates θ and the wavelength λ.

b) The intensity peak can also be interpreted by assuming that the particles are scattered through θ off the bars of the cage. Considering the momentum transferred to the cage, find the angular momentum transferred to the cage?

c) Using the above and the De Broglie relation, show that angular momentum is quantized.

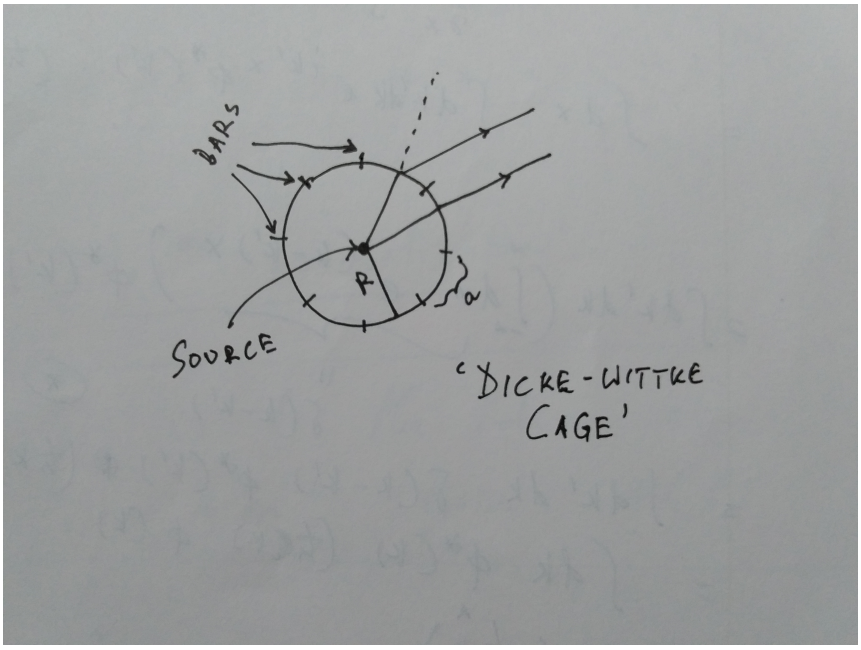
3. Use the uncertainty relation to esstimate the ground state energy of:

a) a harmonic oscillator, the energy being given by: $E = p^2/2m + 1/2m\omega^2 x^2$

b) a particle in the potentian $V = \alpha x^4$, $\alpha = const.$

4. Monochromatic light with $\lambda = 6000$ angstrom passes through a fast shutter that opens for 10^{-9} sec. What will be the spread in the wavelengths in the light that is no longer monochromatic after passing through the shutter?

5. The smallest separation resolvable by a microscope is of the order of magnitude of the wavelength used. What energy would one need for the electrons to have in order to resolve separations of 5 angstrom in an electron microscope?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