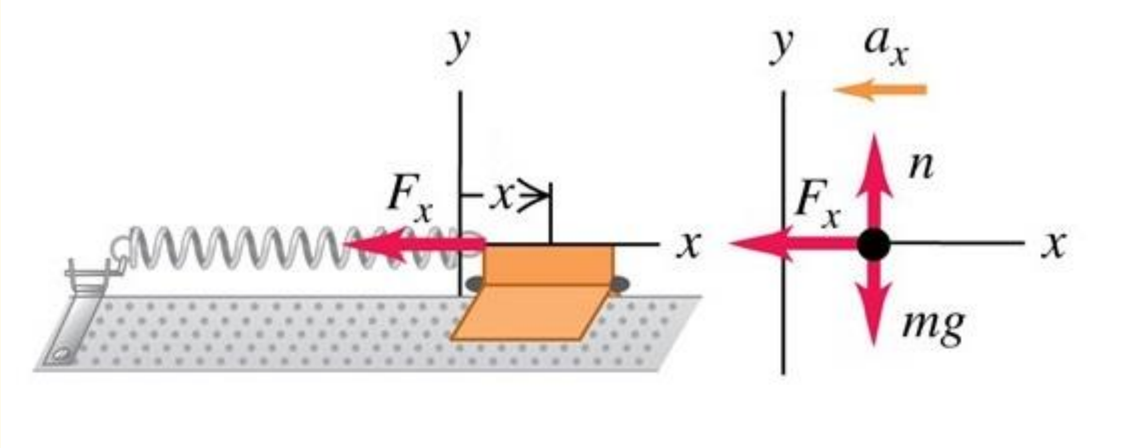
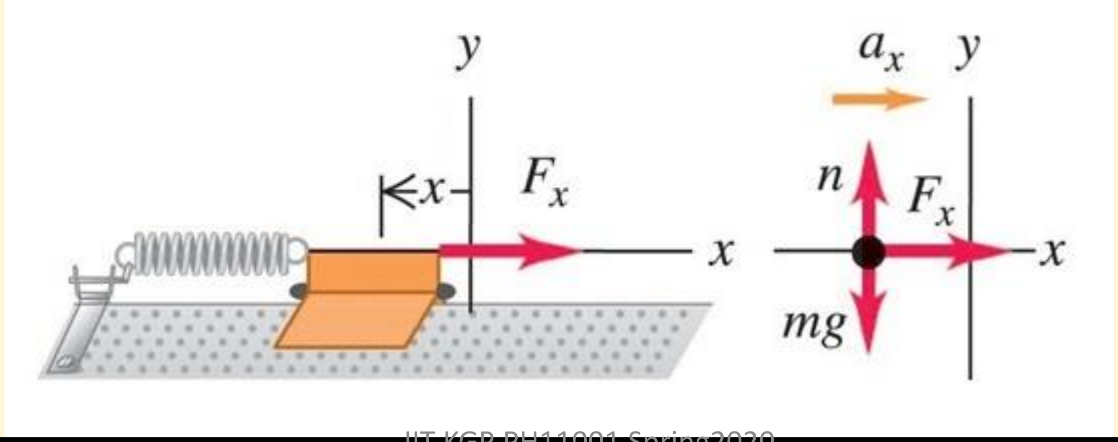
If a body attached to a spring is displaced from its equilibrium position, the spring exerts restoring force which causes oscillation or periodic motion

Glider displaced to the right w.r.t. to equilibrium position, $F_x < 0$



Glider displaced to the left w.r.t. to equilibrium position, $F_x > 0$



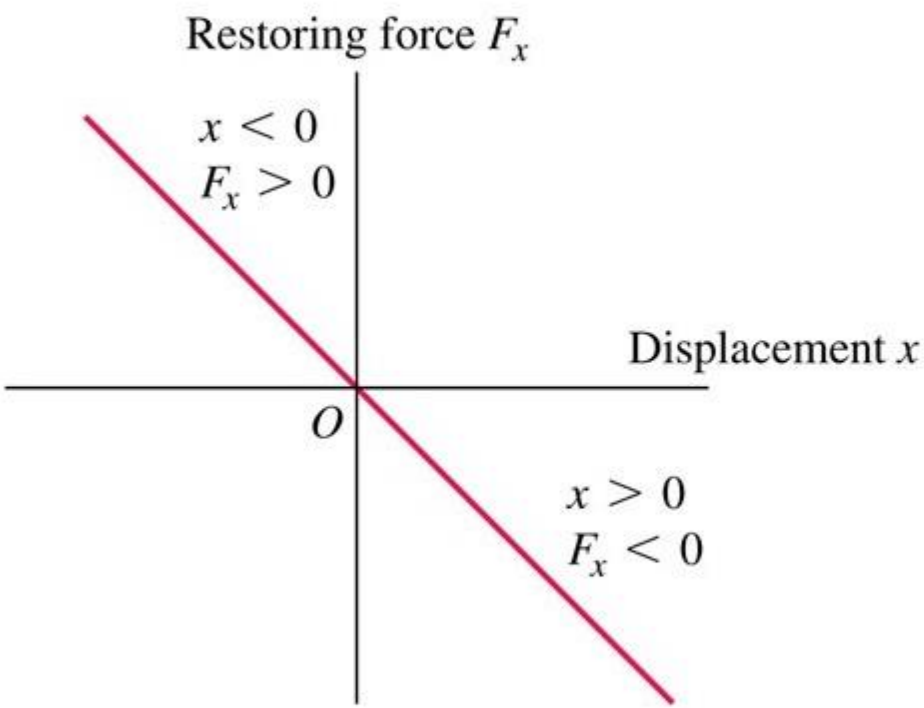
Characteristics of Periodic Motion

- The **amplitude**, A, is the maximum magnitude of displacement from equilibrium.
- The **period**, T, is the time for one cycle.
- The **frequency**, f, is the number of cycles per unit time.
- The **angular frequency**, is 2π times the frequency:
- The frequency and period are reciprocals of each other: $f = 1/T$ and $T = 1/f$.

Simple Harmonic Motion

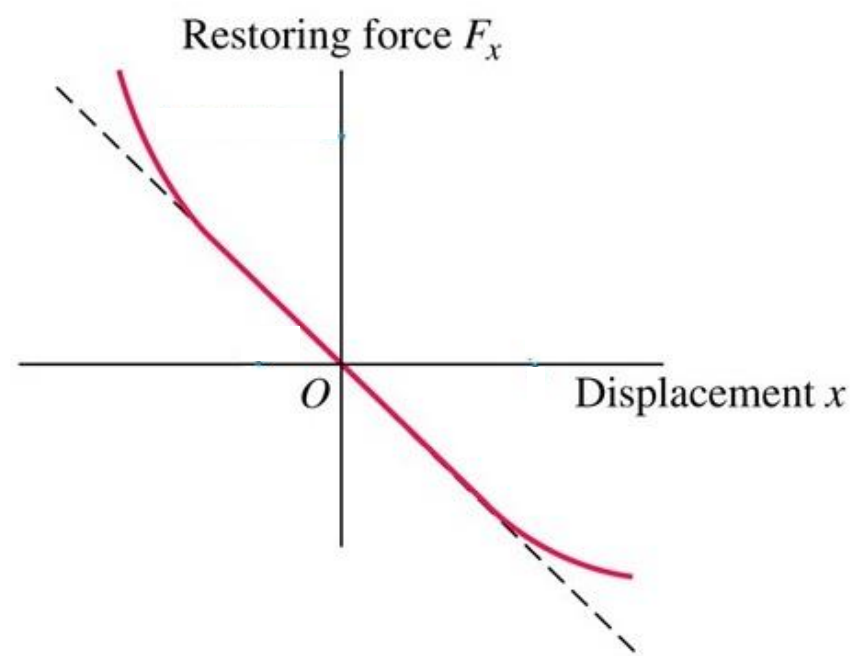
When the restoring force is *directly proportional* to the displacement from equilibrium, the resulting motion is called **simple harmonic motion** (SHM).

Restoring force is directly proportional to displacement (Hooke's Law: F = -kx)



In real cases, restoring force deviates from Hooke's law for larger displacements

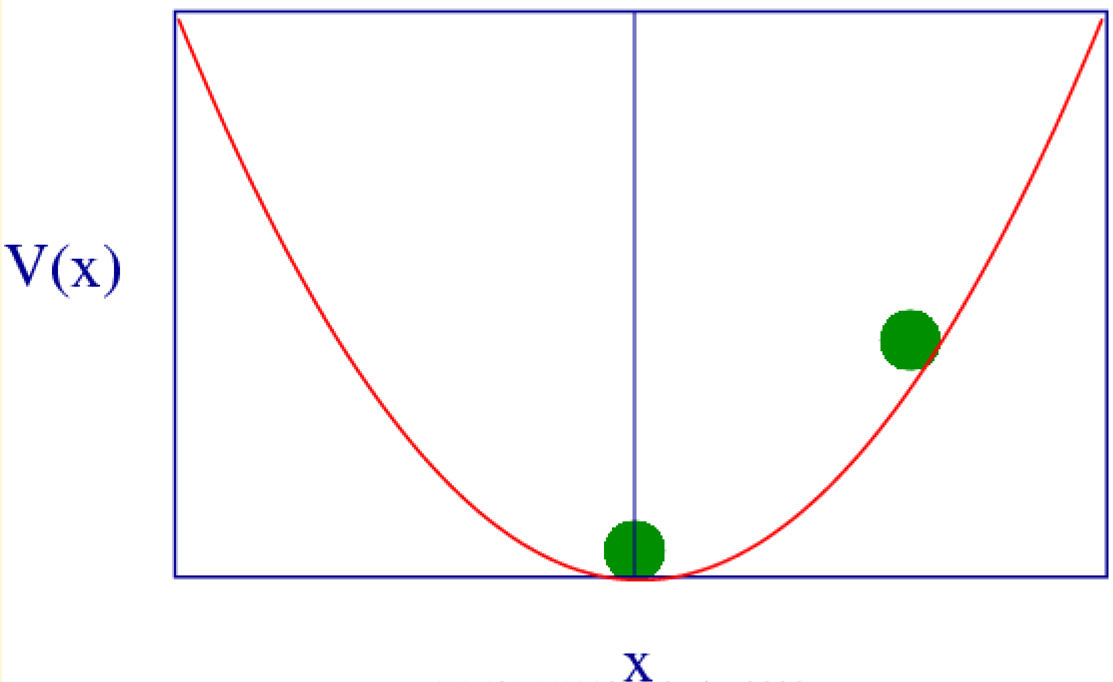
For small displacements, Hooke's law is a good approximation



Harmonic oscillator potential

$$F = -kx$$

$$V(x) = \frac{1}{2}kx^2 \qquad k = +\text{ve}$$

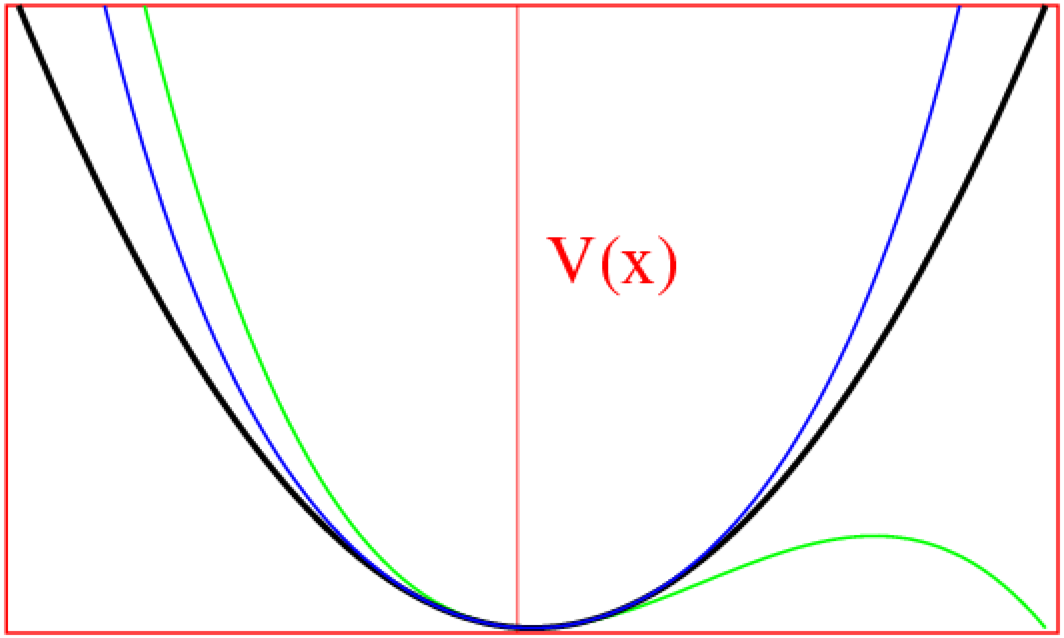


Consider a potential *V(x)*. Let us assume a minimum at $x=x_0$, Taylor expand around $x=x_0$,

$$V(x) = V(x_0) + (x - x_0)V'(x_0) + \frac{1}{2!}(x - x_0)^2 V''(x_0) + \cdots,$$

$V'(x_0)$ is zero and $V''(x_0)$ is positive since x_0 is a minimum of $V(x)$.

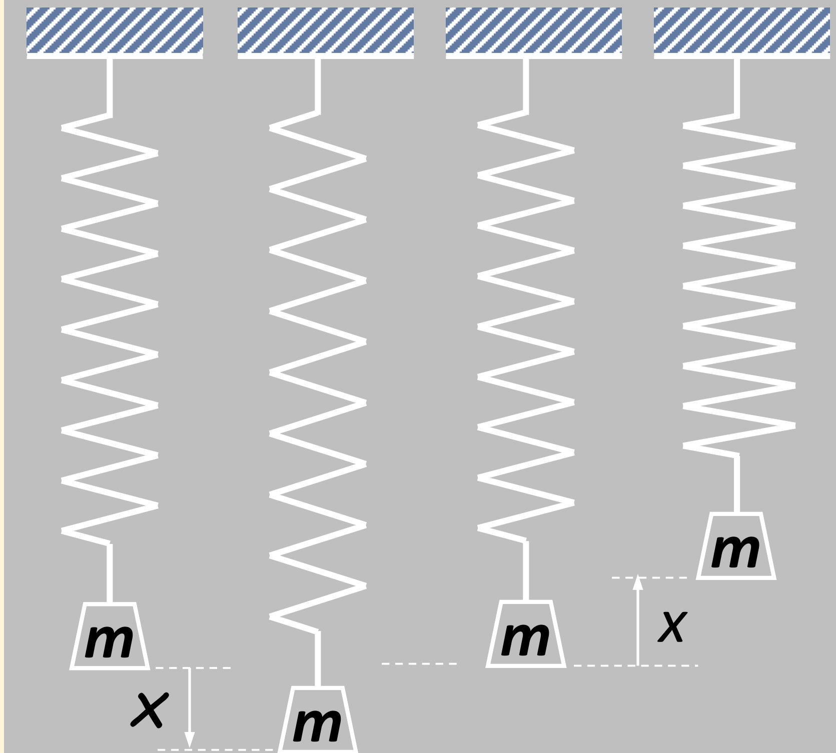
Close to minimum Harmonic oscillator potential is a good approximation



$$V(x) = x^2$$

$$V(x) = \exp(x^2/2) - 1$$

$$V(x) = x^2 - x^3$$



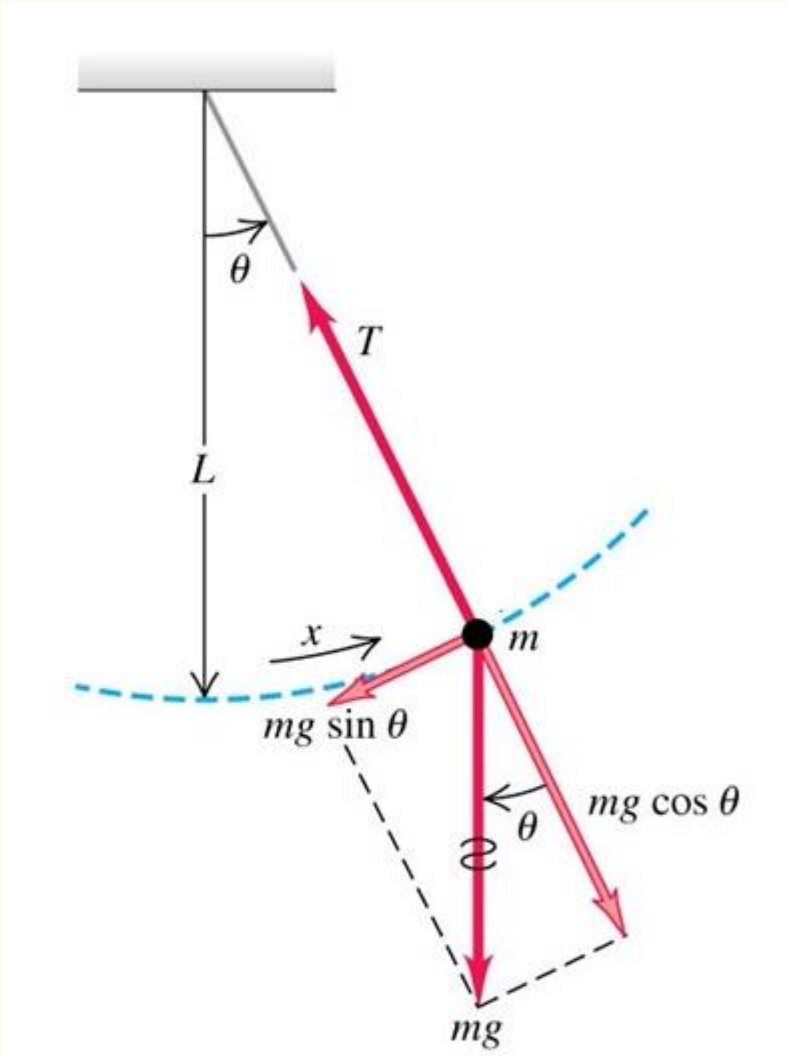
Equation of Motion

$$m\frac{d^2x}{dt^2} = -kx$$

k : stiffness constant

Above equation is second-order ordinary homogenous linear differential equation

Second order: because the highest derivative is second order.
Ordinary: because the derivatives are only with respect to one variable (t).
Homogeneous: because x or its derivatives appear in every term, and
Linear: because x and its derivatives appear separately and linearly in each term

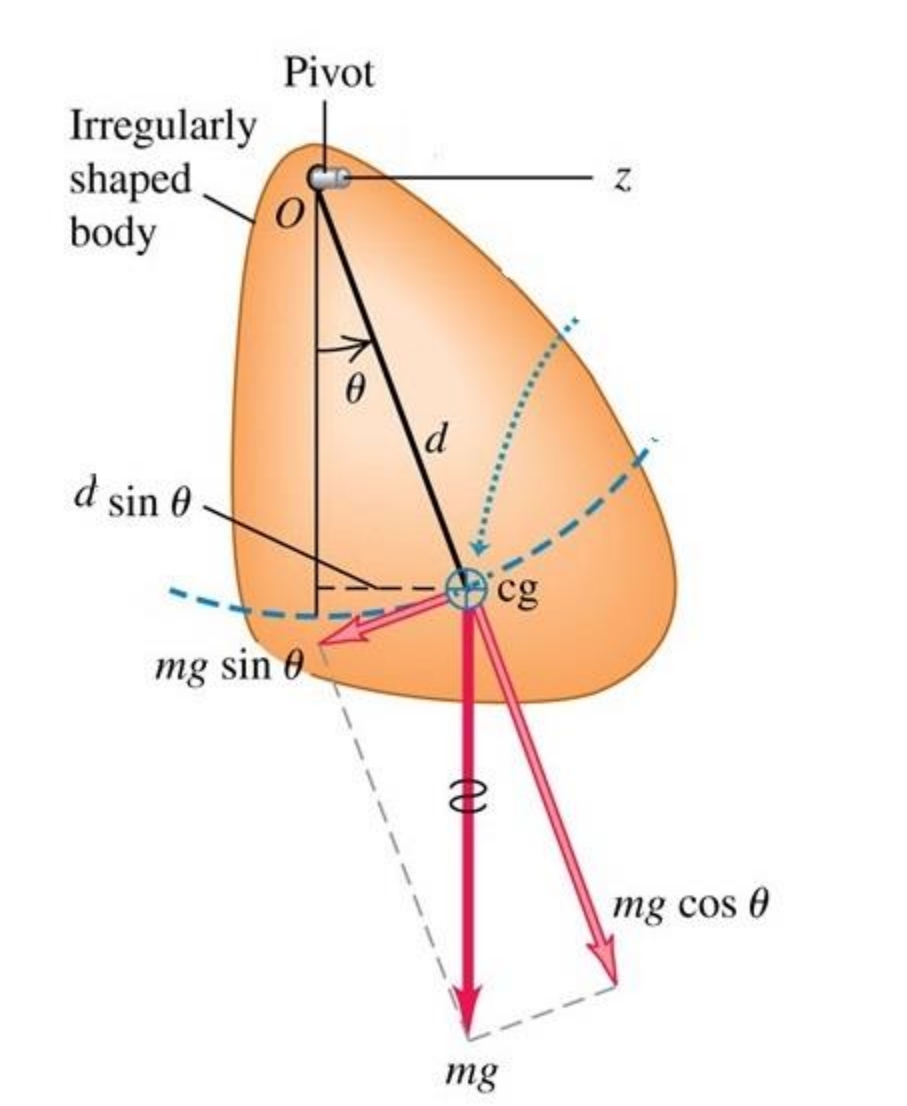


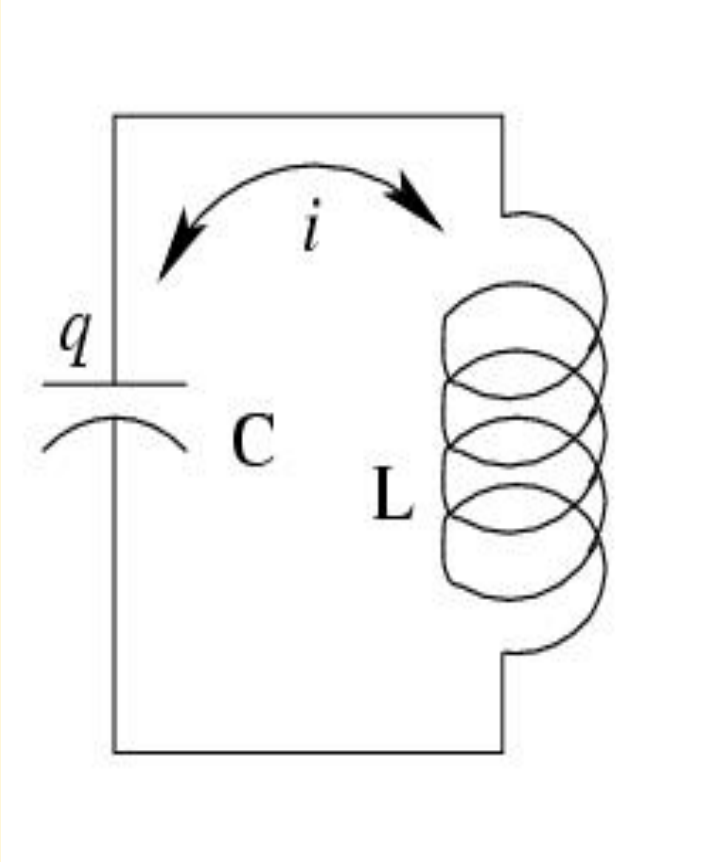
$$ml\ddot{\theta} = -mg\sin\theta$$

$$\theta \leq 4^\circ, \qquad \sin\theta \approx \theta$$

$$\ddot{\theta} + \frac{g}{l}\theta = 0$$

$$\sin 4^\circ = 0.0698, \qquad 4^\circ = 0.0698 \text{ rad}$$





$$I\ddot{\theta} = -Mgd\sin\theta$$

$$L\ddot{q} + q/C = 0$$

Hooke's Law:

$$F = -kx$$

$$m\ddot{x} + kx = 0$$

$$\ddot{x} + \omega_0^2 x = 0$$

Equation of SHM

$$\omega_0 = \sqrt{k/m}$$

Angular frequency

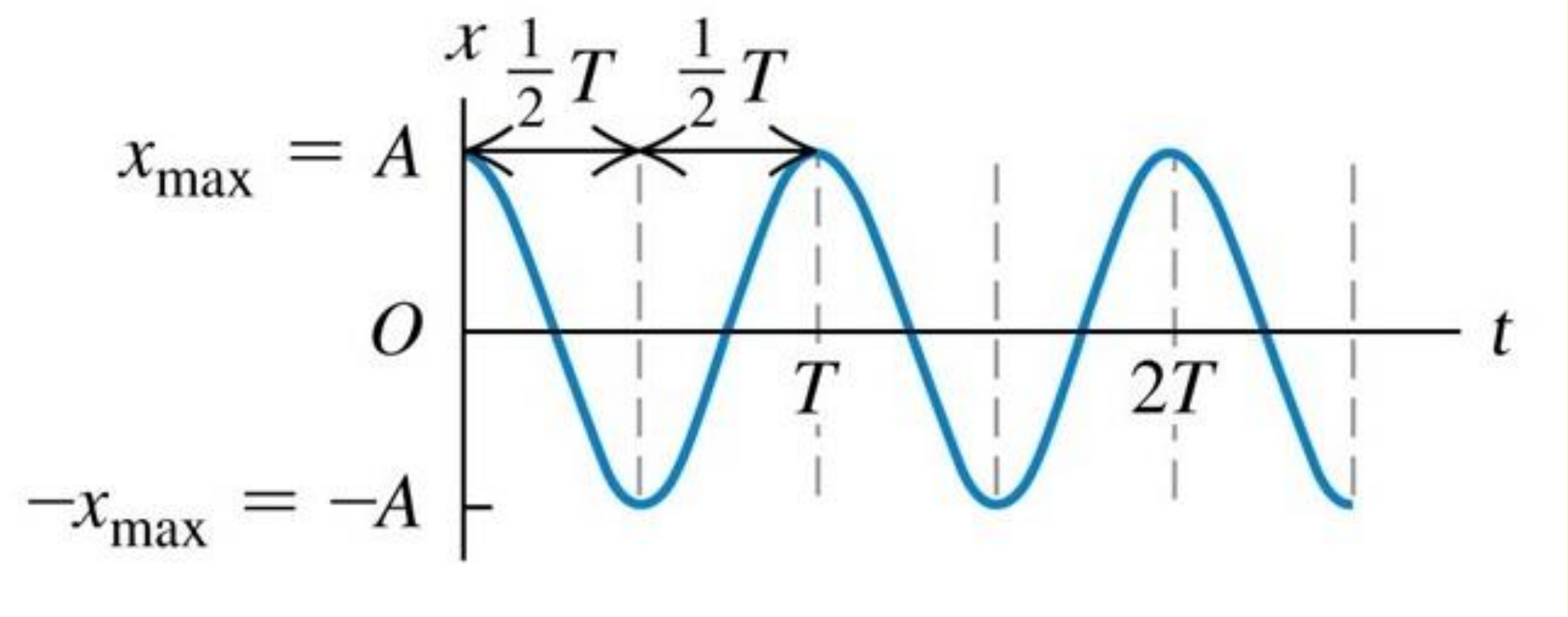
$$T = 2\pi/\omega_0$$

Time period

Solution:

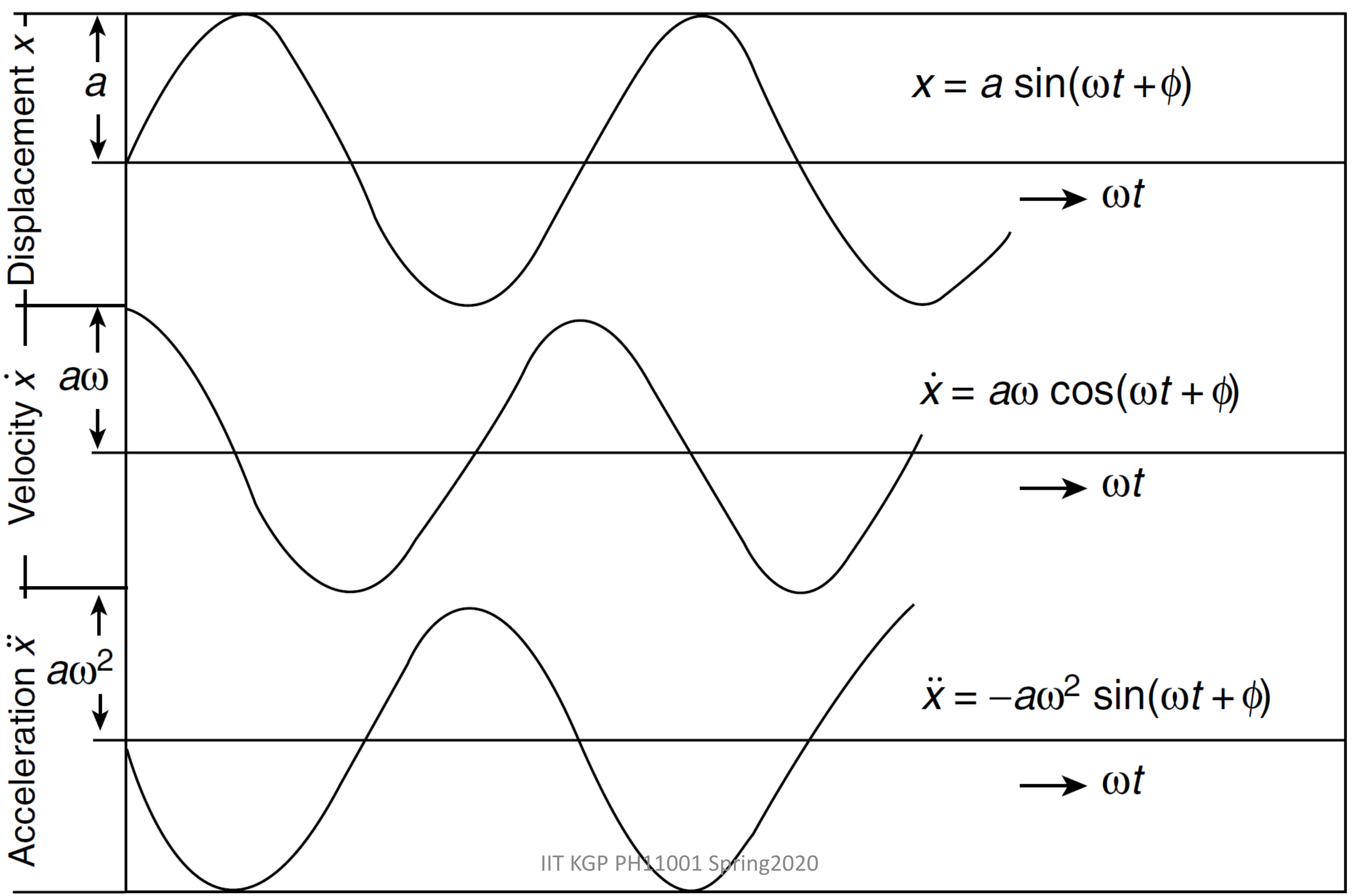
$$x(t) = A\cos(\omega_0 t + \phi)$$

A=Amplitude, ϕ =Phase, $T = \frac{2\pi}{\omega_0}$



Another Solution:

$$x = a\sin(\omega t + \phi)$$



Energy of simple harmonic oscillator:

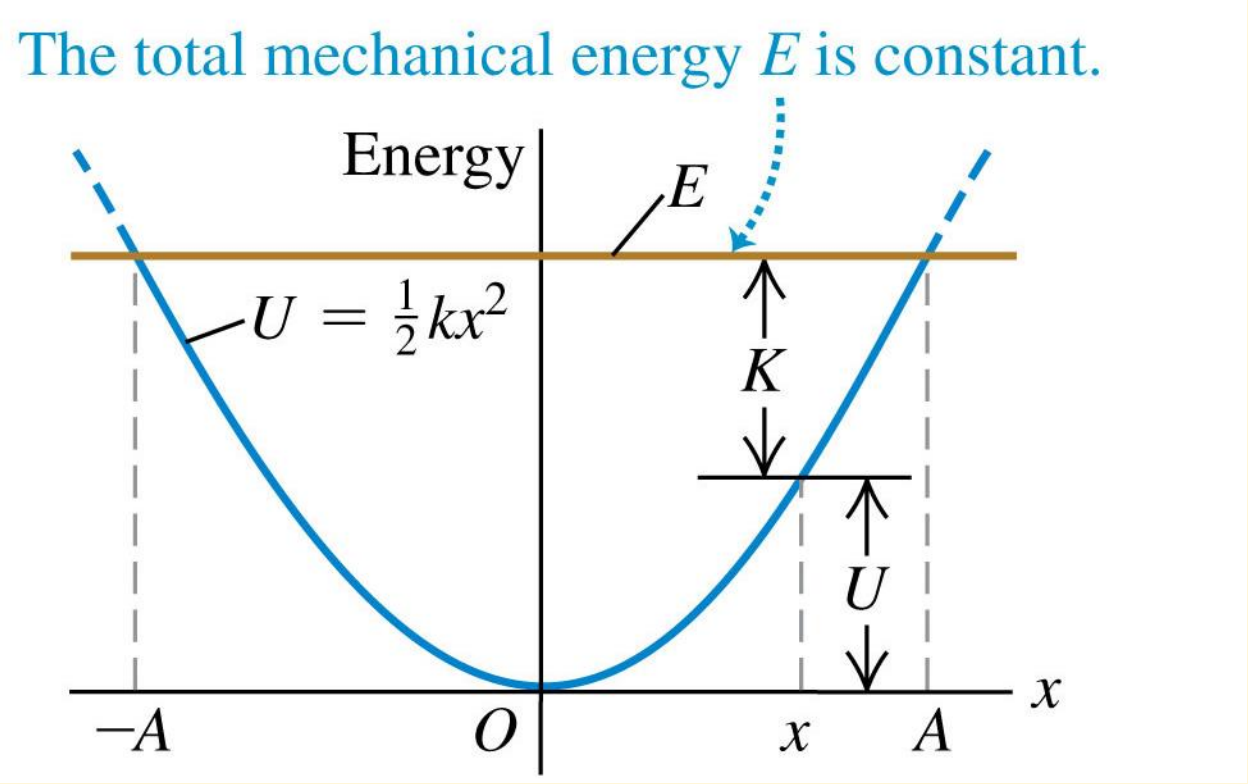
$$k.e. = \frac{1}{2}m\dot{x}^2 = \frac{1}{2}m\omega_0^2 A^2 \sin^2(\omega_0 t + \phi) = \frac{1}{2}kA^2 \sin^2(\omega_0 t + \phi)$$

Because, $\omega_0 = \sqrt{k/m}$

$$p.e. = \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \cos^2(\omega_0 t + \phi) = \frac{1}{2}m\omega_0^2 A^2 \cos^2(\omega_0 t + \phi)$$

$$E = k.e. + p.e. = \frac{1}{2}kA^2 = \frac{1}{2}m\omega_0^2 A^2$$

Energy in vibration: KE and PE



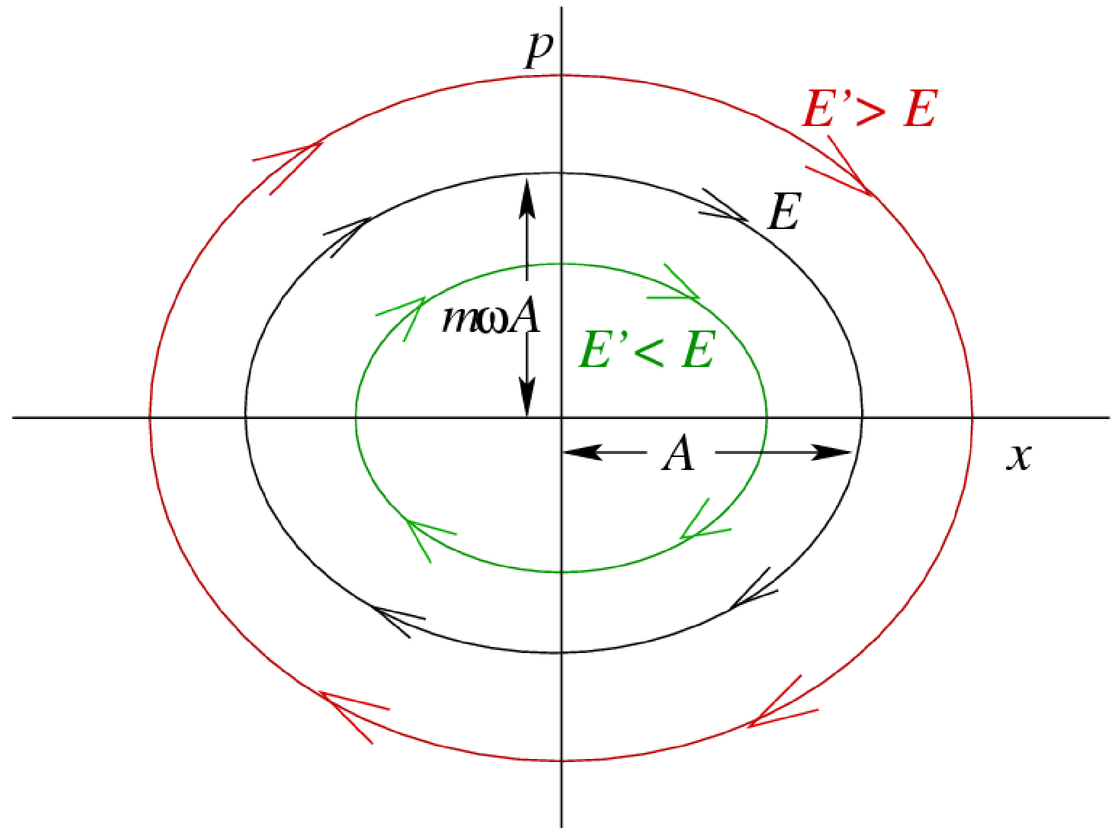
- Energy of SHO = Sum of potential (U) and kinetic (K) energy remains a constant.
- **Assumption**: Ideal case, total energy remains constant.
- *All P.E. becomes K.E. and vice versa.*

Phase Space

A **phase space** is a space in which all possible states of a system are represented, with each possible state of the system corresponding to one unique point in the phase space.

For mechanical systems, the phase space usually consists of all possible values of position and momentum variables.

It is often useful to picture the time-development of a system in ***phase space***



$$E = \frac{1}{2}kA^2 \;=\; p^2/2m + \frac{1}{2}kx^2$$

$$\text{or, } 1 \;=\; \frac{p^2}{m^2\omega_0^2 A^2} + \frac{x^2}{A^2}$$

The greater the total energy of the system, the larger will be the size of the ellipse.