Optics

Reference Books

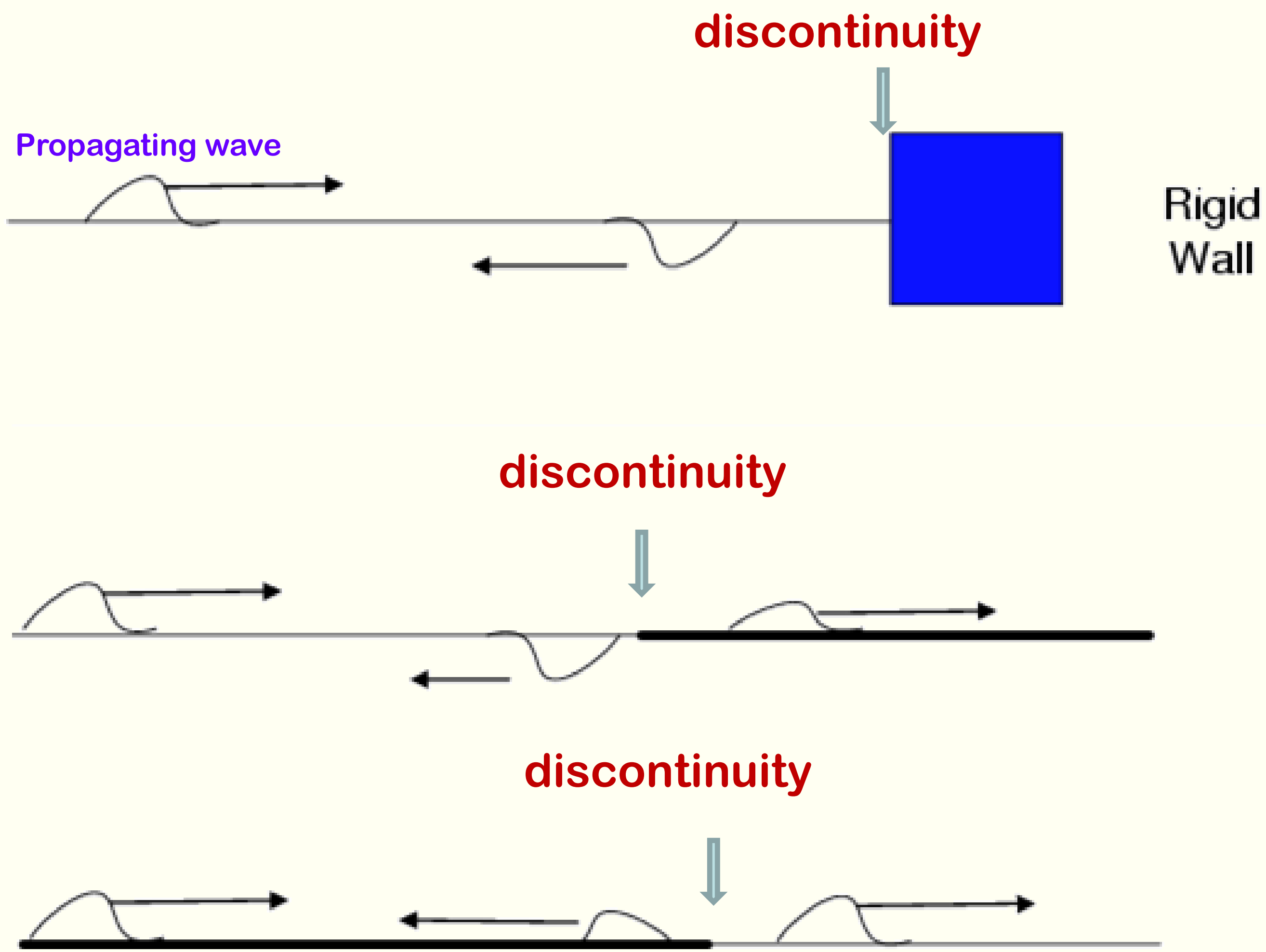
1. Optics by E. Hecht & A.R Ganesan
2. Optics by A. Ghatak

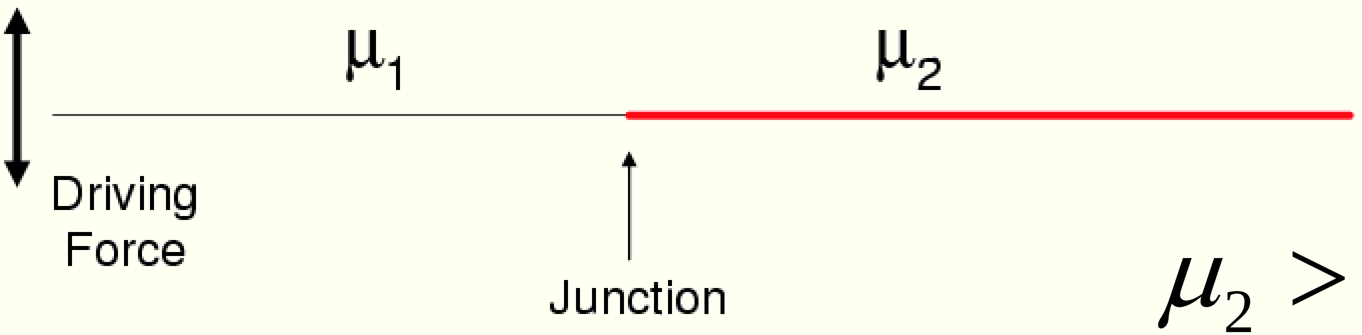
Acknowledgement: Prof. Shivakiran Bhaktha B N

Reflection and Transmission

(for mechanical wave)

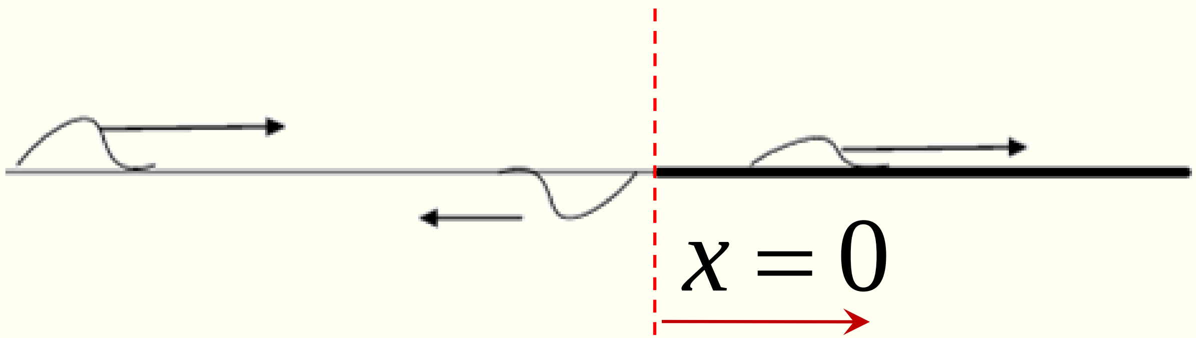
In an inhomogeneous medium (Reflection and Transmission at a discontinuity)





$$\mu = \frac{mass}{length}$$

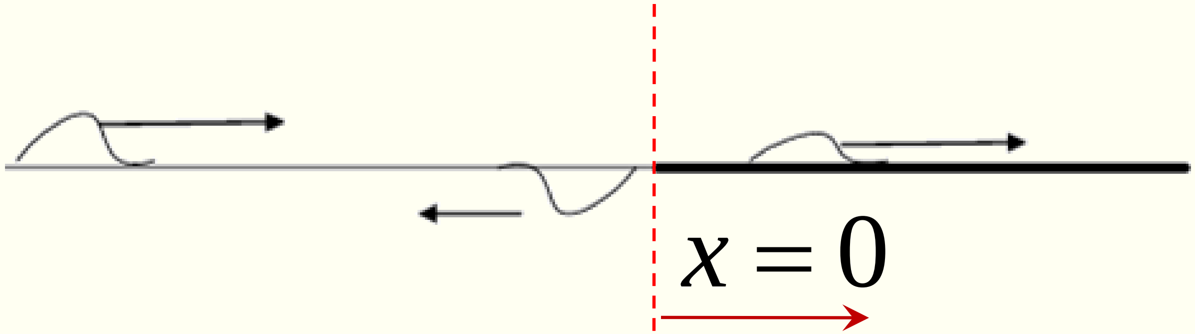
$$\mu_2 > \mu_1$$



$$y_{inc} = A\cos(k_1\,x - \omega\,t)$$

$$y_{ref} = B\cos(k_1\,x + \omega\,t)$$

$$y_{trans} = C\cos(k_2\,x - \omega\,t)$$



On the left side of
the junction we have

$$y_l = y_{inc} + y_{ref}$$
$$= A\cos(k_1\, x - \omega\, t) + B\cos(k_1\, x + \omega\, t)$$

and on the right side
of the junction we
have

$$y_r = y_{trans} = C\cos(k_2\, x - \omega\, t).$$

At the boundary $x = 0$ the wave must be continuous, (as there are no kinks in it).
Thus we must have

$$y_l(0,t) = y_r(0,t)$$

$$\left.\frac{\partial y_l(x,t)}{\partial x}\right|_{x=0} = \left.\frac{\partial y_r(x,t)}{\partial x}\right|_{x=0}$$

So from the first equation

$$A\cos(\omega t) + B\cos(\omega t) = C\cos(\omega t)$$

$$\boxed{A + B = C}$$

Second boundary condition.........

$$\left.\frac{\partial y_l(x,t)}{\partial x}\right|_{x=0} = \left.\frac{\partial y_r(x,t)}{\partial x}\right|_{x=0}$$

$$-A\,k_1\,\sin(-\omega\,t) - k_1\,B\,\sin(\omega\,t) = -k_2\,C\,\sin(-\omega\,t)$$

$$(A - B)\,k_1\,\sin\omega\,t = C\,k_2\,\sin\omega\,t$$

$$A - B = \frac{k_2}{k_1}\,C$$

$$A + B = C$$

$$A - B = \frac{k_2}{k_1}\, C$$

$$2\, A = \left(1 + \frac{k_2}{k_1}\right) C$$

We can define the transmission coefficient: $t_r = C/A$

$$t_r \equiv C/\,A = \frac{2\, k_1}{k_1 + k_2}$$

$$y_{inc} = A \cos(k_1\, x - \omega\, t)$$

$$y_{ref} = B \cos(k_1\, x + \omega\, t)$$

$$y_{trans} = C \cos(k_2\, x - \omega\, t)$$

We can define the Reflection coefficient: *r=B/A*

$$A + B = C$$

$$r \equiv B/A = \frac{C}{A} - 1 = \frac{k_1 - k_2}{k_1 + k_2}$$

$$t_r \equiv C/A = \frac{2\,k_1}{k_1 + k_2}$$

Rigid End: $\mu_2 \to \infty \; (\mu_2 >> \mu_1)$

$k_2 \to \infty$

$$r = \frac{k_1 - k_2}{k_1 + k_2} = \frac{\frac{k_1}{k_2} - 1}{\frac{k_1}{k_2} + 1}$$

$$r \to -1$$

When $\mu_2 > \mu_1$,

$r < 0$

Change in sign of the reflected pulse

External Reflection

Free End: $\mu_2 \rightarrow 0$ $(\mu_2 << \mu_1)$

$k_2 \rightarrow 0$

$$r = \frac{k_1 - k_2}{k_1 + k_2}$$

$$= \frac{k_1}{k_1}$$

$$r \rightarrow +1$$

When $\mu_2 < \mu_1$,

$r > 0$

No Change in sign of the reflected pulse

Internal Reflection

$$r = B/A = \frac{k_1 - k_2}{k_1 + k_2} = \frac{v_2 - v_1}{v_2 + v_1}$$

$$t_r = C/A = \frac{2k_1}{k_1 + k_2} = \frac{2v_2}{v_2 + v_1}$$

$$k_i = \frac{\omega}{v_i}$$

v_i is the velocity of the wave in the i^{th} medium

In either case: $\boldsymbol{t_r > 0}$

No Change in phase of the transmitted pulse

It can be seen that

$$r_{12} = -r_{21}$$

$$1 - r_{12}^2 = t_{12} t_{21}$$

Stokes relations

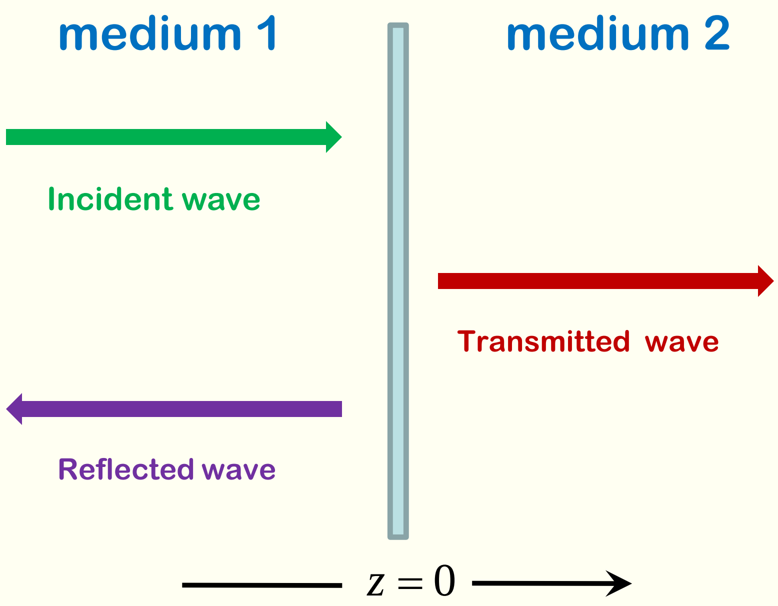
The reflectance and transmittance of Intensity is proportional to square of Amplitude

$$R_{12} = r_{12}^2$$

$$T_{12} = 1 - R_{12}$$

Reflection and Transmission
(of electromagnetic wave for normal incidence)

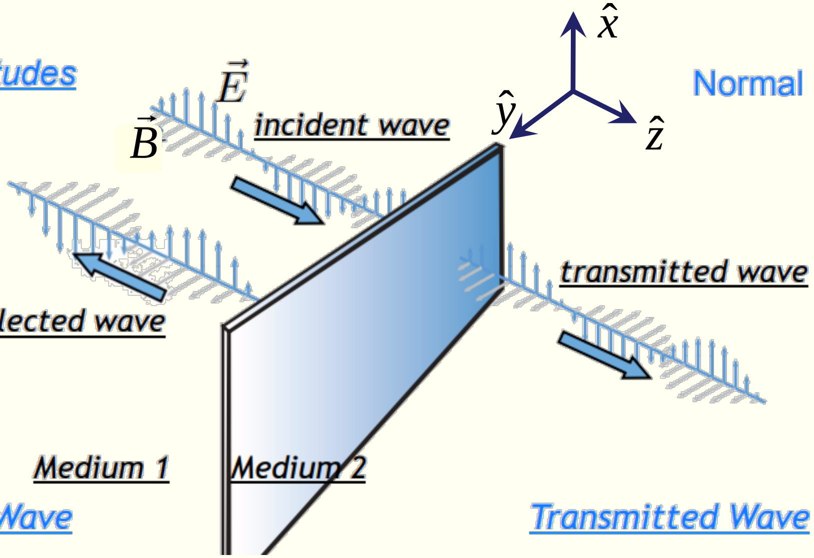
Basic Diagram



Reflection and Transmission (of EM wave for normal incidence)

same amplitudes

Normal Incidence



$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\Downarrow$$

$$\vec{k}\times\vec{E} = \omega\vec{B}$$

$$\Downarrow$$

$$\vec{B} = \frac{n}{c}\hat{k}\times\vec{E} \qquad \left(k = \frac{\omega}{c}n\right)$$

Incident Wave

$$\vec{E}_i = E_{0i}e^{i(\mathrm{k_1 z}-\omega t)}\hat{x}$$

Transmitted Wave

$$\vec{E}_t = E_{0t}e^{i(\mathrm{k_2 z}-\omega t)}\hat{x}$$

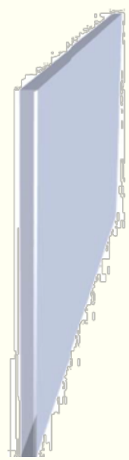
$$\vec{B}_i = \frac{n_1}{c}\hat{k}_1\times\vec{E}_i = \frac{n_1}{c}E_{0i}e^{i(\mathrm{k_1 z}-\omega \mathrm{t})}\hat{y}$$

$$\vec{B}_r = -\frac{n_1}{c}\hat{k}_1\times\vec{E}_r = -\frac{n_1}{c}E_{0r}e^{i(-\mathrm{k_1 z}-\omega \mathrm{t})}\hat{y}$$

$$\vec{B}_t = \frac{n_2}{c}\hat{k}_2\times\vec{E}_i = \frac{n_2}{c}E_{0t}e^{i(\mathrm{k_2 z}-\omega \mathrm{t})}\hat{y}$$

Reflected Wave

$$\vec{E}_r = E_{0r}e^{i(-\mathrm{k_1 z}-\omega t)}\hat{x}$$



$$\vec{E}_1 = \vec{E}_i + \vec{E}_r \qquad \vec{E}_2 = \vec{E}_t$$

---------------------------------- Medium 1   Medium 2 ----------------------------------

n_1

Refractive Index of medium 1

$$\vec{B}_1 = \vec{B}_i + \vec{B}_r \qquad \vec{B}_2 = \vec{B}_t$$

n_2

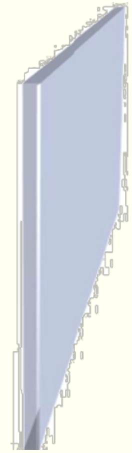
Refractive Index of medium 2

$$\vec{E}_1(\mathrm{z}=0) = \vec{E}_2(\mathrm{z}=0)$$

$$\vec{B}_1(\mathrm{z}=0) = \vec{B}_2(\mathrm{z}=0)$$

Boundary condition

$$\vec{E}_1 = \vec{E}_i + \vec{E}_r \qquad \vec{E}_2 = \vec{E}_t$$

Medium 1  *Medium 2*

$$\vec{B}_1 = \vec{B}_i + \vec{B}_r \qquad \vec{B}_2 = \vec{B}_t$$

$$\vec{E}_1(\mathrm{z}=0) = \vec{E}_2(\mathrm{z}=0)$$
$$\vec{B}_1(\mathrm{z}=0) = \vec{B}_2(\mathrm{z}=0)$$

$$\vec{E}_1(\mathrm{z}=0) = \vec{E}_2(\mathrm{z}=0)$$

$$\Rightarrow \quad E_{0i} + E_{0r} = E_{0t}$$

$$\vec{H}_1(\mathrm{z}=0) = \vec{H}_2(\mathrm{z}=0)$$

$$\Rightarrow \quad n_1 E_{0i} - n_1 E_{0r} = n_2 E_{0t}$$

$$\boxed{r = \frac{E_{0r}}{E_{0i}} = \frac{n_1 - n_2}{n_1 + n_2}}$$ **Reflection coefficient**

$$\boxed{t = \frac{E_{0t}}{E_{0i}} = \frac{2n_1}{n_1 + n_2}}$$ **Transmission coefficient**

Reflectance $$R = \frac{I_r}{I_i} = \left|\frac{n_1 - n_2}{n_1 + n_2}\right|^2$$

Transmittance $$T = \frac{I_t}{I_r} = \frac{n_2}{n_1}\left|\frac{2n_1}{n_1 + n_2}\right|^2$$

$$\Rightarrow \quad R + T = 1$$

$$\boxed{I = \frac{1}{2}\varepsilon_0 c n |E_0|^2}$$ **Intensity and amplitude relation**