Quantum Mechanics

Lectrue 2

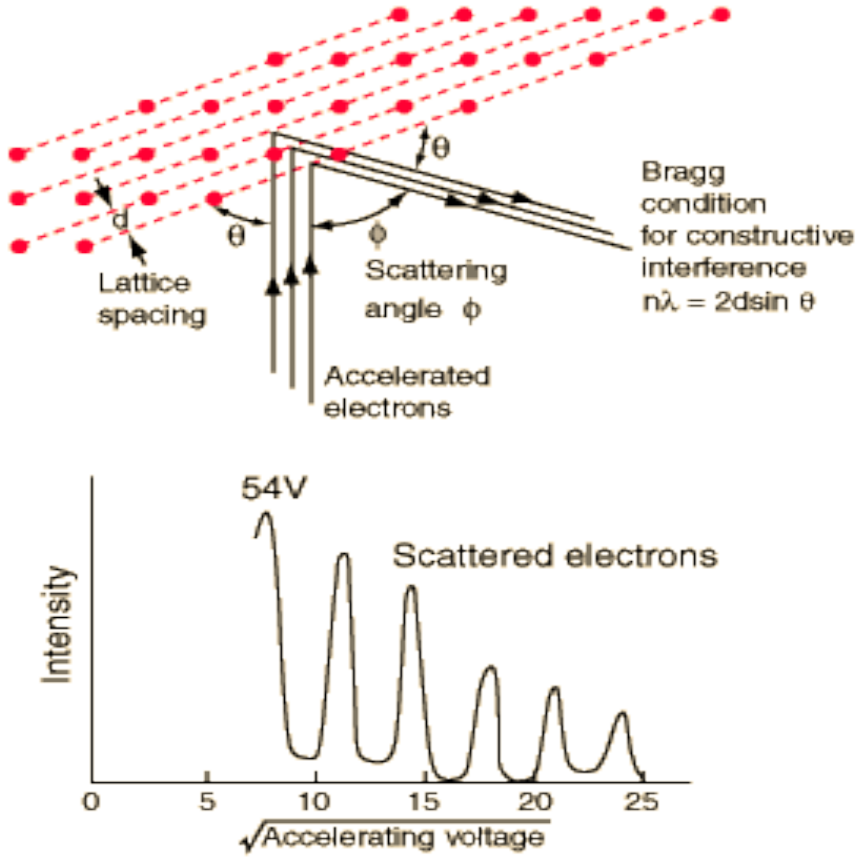
The story so far

- Electromagnetic waves must have a dual description, which simultaneously explains its wave nature and particle nature.

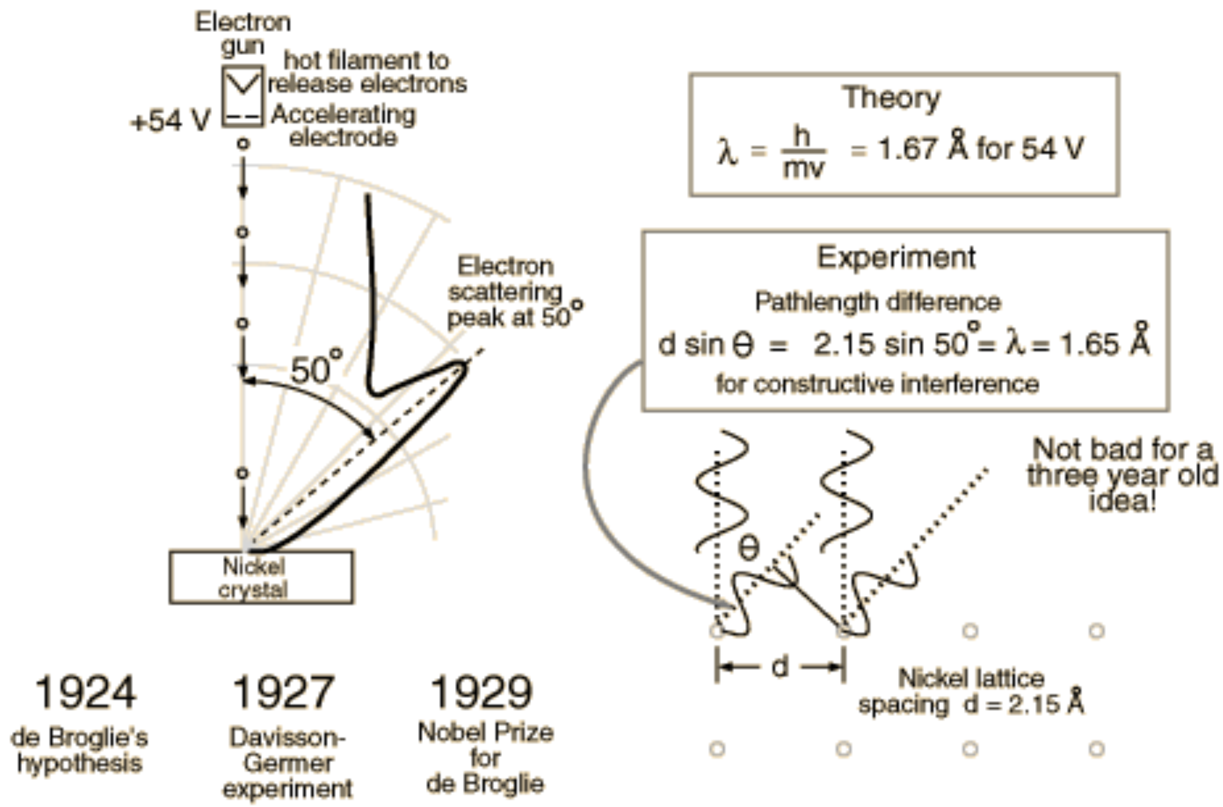
- If EM radiation has both characteristics, then matter particles must also demonstrate some wave-properties (de Broglie hypothesis).

- This hypothesis was first experimentally verified in an experiment performed by Davisson and Germer (1927), where it was demonstrated that electrons can also exhibit the diffraction phenomenon.

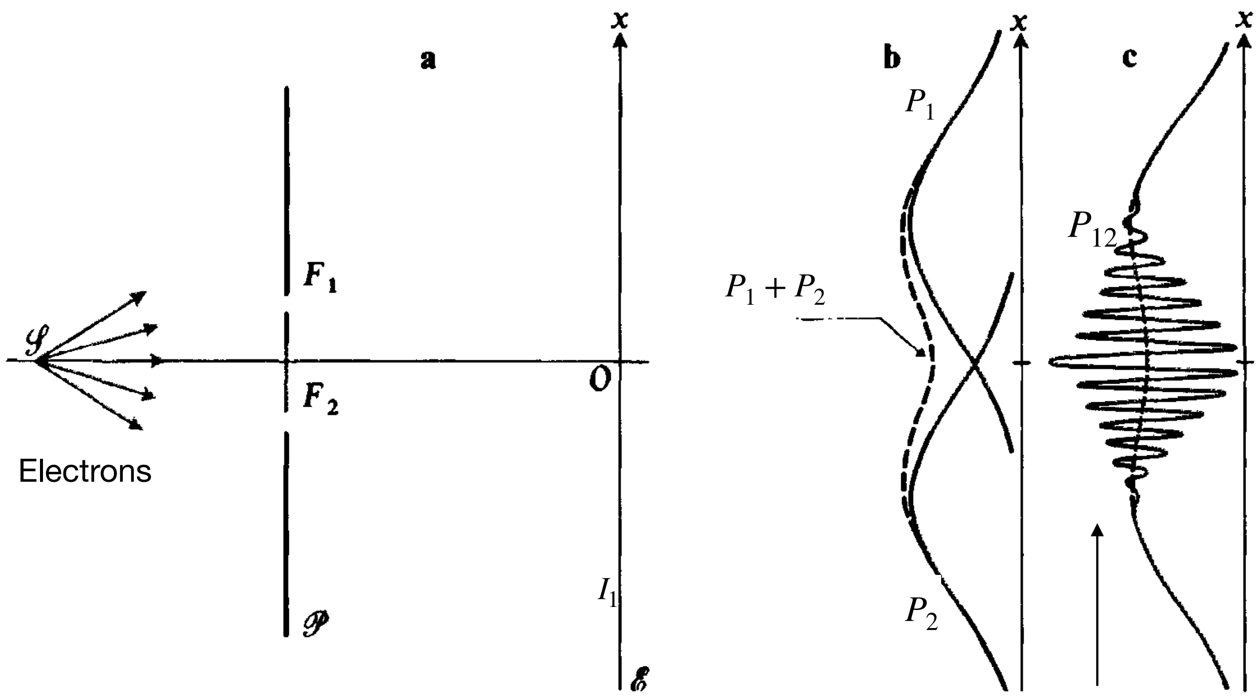
The Davisson-Germer experiment (1927)



Electron diffraction: Verification of de Broglie hypothesis



YDS experiment with electrons



Interference pattern observed when a large no of electrns are allowed to strike the screen.

YDS experiment with electrons

- Qualitative features of the result is same as that of photons.
- For small time of exposure we see a few electrons hitting here and there on the screen. While for large exposure time, all these hits accumulate to give a interferene pattern.
- When the number of electrons is large we can provide a probabilistic interpretation. Let us consider the ratio of number of electrons at a point x on the screen divided by the total number of electrons. This ratio can be interpreted as the probability of the electron arriving at the point x.

Reconciling the particle and wave characteristics

- The electrons arrive as particles on the screen.

- The probability of their arrival is distributed (along x) like the intensity distribution pattern seen in interference of waves. This is why we see fringes when we have a large number of electrons hitting the screen.

- It is in this sense that the electron sometimes behaves as a particle and sometimes as a wave.

- The particle aspect can also be seen emerging out of the wave aspect through the notion of a wave-packet (see next Lecture-3 for more details).

Which hole did the electron pass through

- If the electrons passes through either hole 1 or hole 2 then the combined probability distribution P_{12} should be $P_{12} = P_1 + P_2$ which is **not true**, since there are interference-like effects.

- The idea that the electron has to pass through either hole 1 or 2 is not viable in the new quantum paradigm.

- This suggests that the classical deterministic notion of a trajectory of a particle breaks down.

Probability amplitude

- In order to explain the probability distribution of the electrons resembling the interference pattern of waves, we need to associate with the electron a concept which is 'analogues' * to the electric field for the electromagnetic waves.

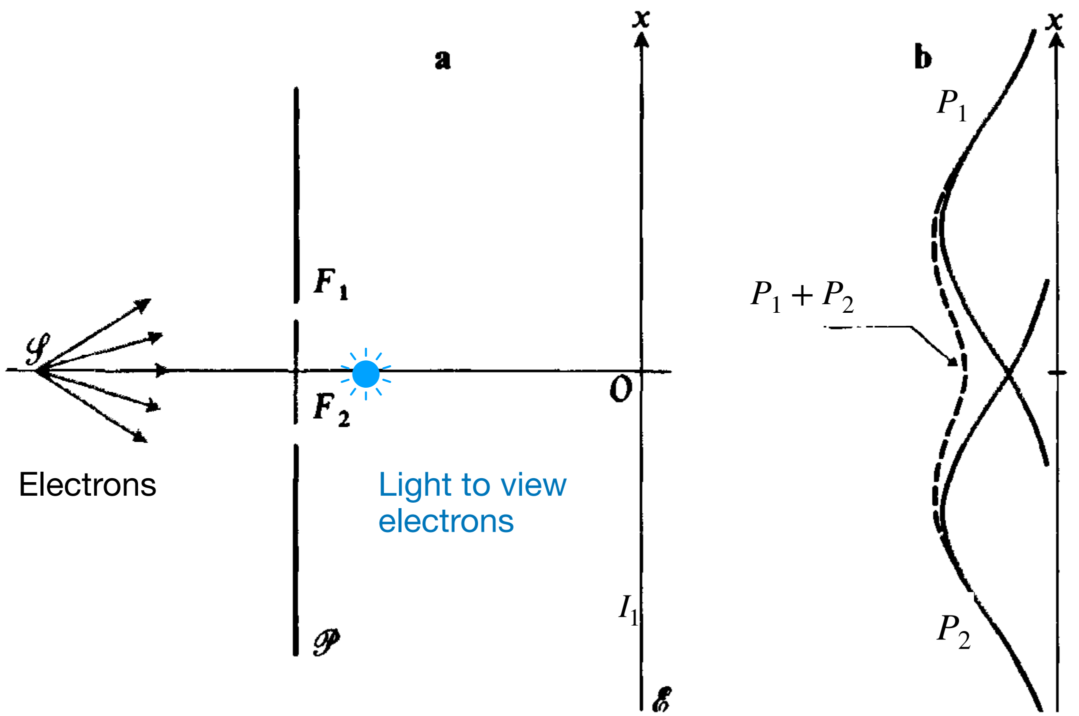
- We can associate two probability amplitudes (complex functions) ψ_1 and ψ_2 with the two holes, such that $P_1 = |\psi_1|^2$ and $P_2 = |\psi_2|^2$, but $P_{12} = |\psi_1 + \psi_2|^2$.

- Because we add probability amplitudes instead of adding probabilities, there would be an interference cross term in P_{12} (in the same way as an interference cross term exists in combined intensity distribution when we superpose the electric fields of two EM waves).

$\psi(x) \rightarrow$ **the probability amplitude or wavefunction of the electron.**

*This analogy should not be taken very seriously; we have to associate a probability amplitude even for the photon. The electric field in the classical description is not enough.

Watching the electrons



No interference pattern observed

Watching the electrons

- If we try to watch the electrons, which hole they pass through, with the help of a light source, we will get a definite answer for each electron.
- But if we choose to make this observation we will **not get any interference pattern**.
- We will simply get the new probability distribution to be $P_{12} = P_1 + P_2$.
- The photons from the source will affect the electrons so much that there will be this observable effect (interference pattern gone).

Summary: Key observations

The principles of the new paradigm of QM must have the following properties

- Dynamics is intrinsically probabilistic (in contrast to deterministic classical dynamics).
- We must introduce the notion of a probability amplitude ψ to describe the 'state' of a quantum particle.
- When an event can occur in two ways we should add probability amplitudes (rather than probabilities).
- Observation (or measurement) can influence the system drastically (interference effects due to inherent quantumness may be lost due to such observations).

These ideas will go into formulating a set of mathematically precise set of postulates for quantum mechanics, which must include the equation determining the dynamics of ψ.

The uncertainty principle

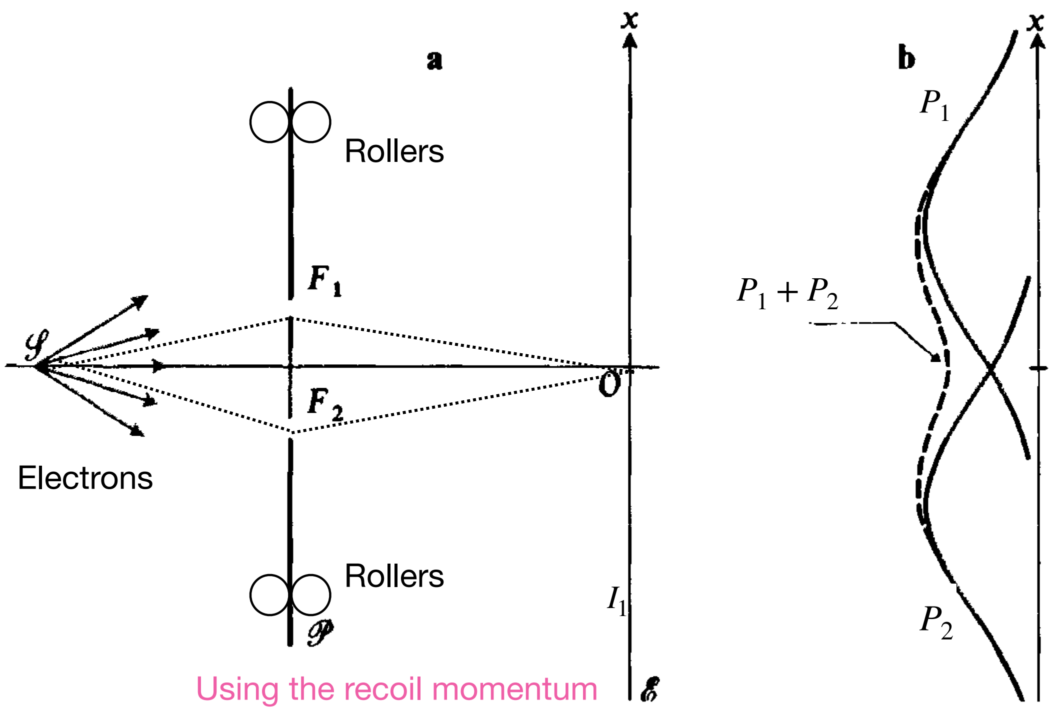
Heisenburg (1927)

If we make a measurement on a particle and determine the x-component of its momentum with uncertainty Δp, then we cannot at the same time know its x-coordinate position more accurately than Δx, so that $\Delta x \; \Delta p \geq \frac{\hbar}{2}$.

- Intrinsic to Quantum mechanics with no classical analogue.

- Applicable to all 'conjugate' variables like $\{x, p\}$ and $\{t, E\}$.

Double Slit on rollers



Using the recoil momentum determine which slit the electron passes through.

No Interference pattern!

Why?

The uncertainty principle!

Double Slit on rollers

- We may try to find which slit the individual electrons passes through by measuring the recoil momentum of the slit-wall as the electrons gets deflected at the slits.
- In order to find the momentum exactly we need to put the slit-wall on rollers. This would make the position of the slits "uncertain".
- Uncertainty in the position of the slits would cause uncertainty in the position of the maxima and minima of the fringe pattern. The averaged effect over a long time would blur out the fringes.
- In this case, uncertainty principle is making sure that fringe pattern disappears once we know which slit the individual electrons are passing through.

The uncertainty principle protects quantum mechanics from becoming inconsistent!