Tutorial 2: Quantum Mechanics

1. Given that $\psi(x) = (\pi/\alpha)^{-\frac{1}{4}} e^{-\frac{\alpha x^2}{2}}$,

a) calculate $\langle x^n \rangle$ for n even. Why does this vanish for n odd?

b) calculate the positional spread $\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$.

2. Show that the operator relation $e^{iap/\hbar} x e^{-iap/\hbar} = x - a$ holds, using

a) the taylor expansion of the exponential and commutation relations,

b) using the momentum representation.

3. A particle is known to be localized in the left half of a box with sides at $x = \pm a/2$, with the wavefn.

$$\psi(x) = \sqrt{2/a}, \quad -a/2 < x < 0,$$
$$= 0, \; 0 < x < a/2.$$

a) Will the particle remain localized at later times?

b) Calculate the probabilities that an energy measurement yields the ground state energy and the energy of the first excited state.

4. A particle in free space is initially in a wave packet described by: $\psi(x) = (\alpha/\pi)^{\frac{1}{4}} e^{-\alpha x^2/2}$.

a) What is the probability that its momentum is in the range (p,p+dp)?

b) What is the expectation value of the energy? Can you justify the answer using the size of the wavefunction and uncertainty principle?

5. Consider the eigenfunctions for particle in a box with sides at $x = \pm a$. Without working out the integral, find the expectation value of the operator $x^2 p^3 + 3xp^3 x + p^3 x^2$ for all eigenfunctions.