Technique of Phasor Addition

Phasor Addition

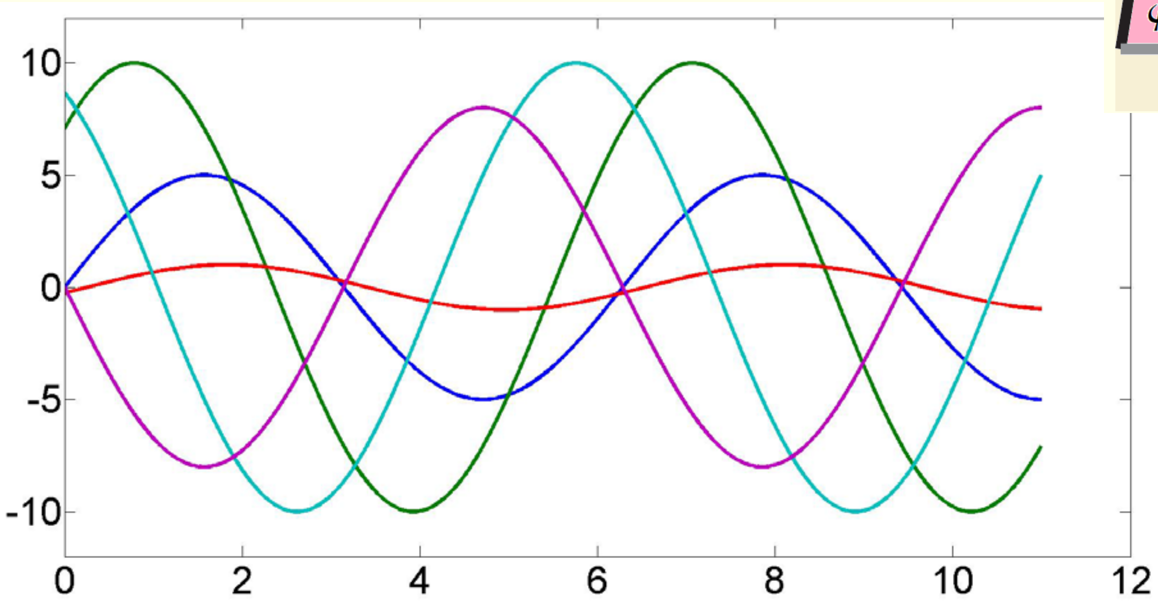
$$E_1 = 5 \sin \omega t$$

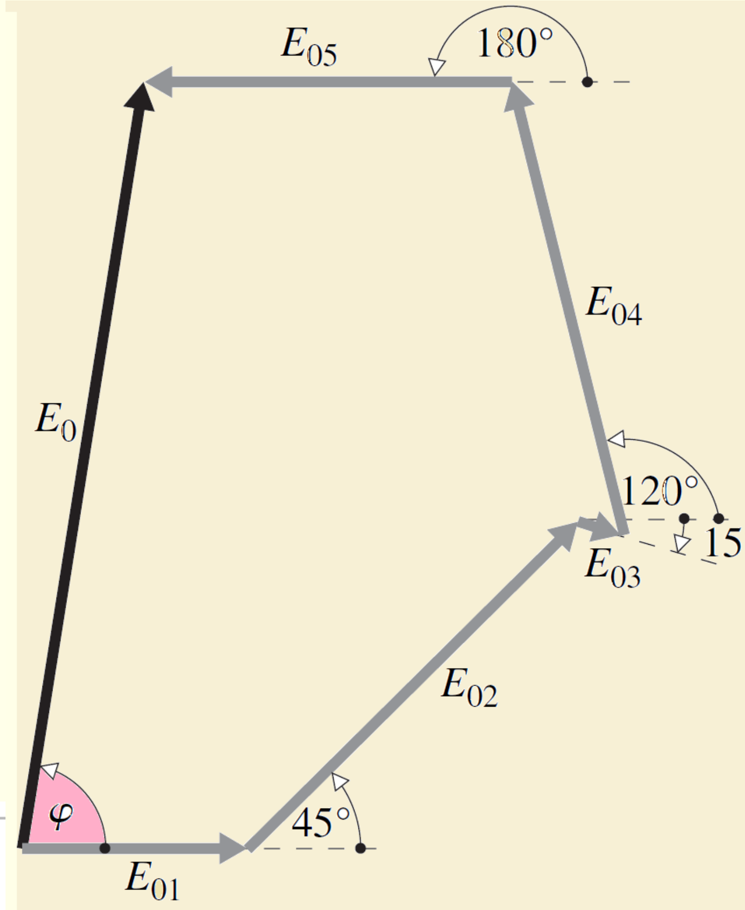
$$E_2 = 10 \sin(\omega t + 45°)$$

$$E_3 = \sin(\omega t - 15°)$$

$$E_4 = 10 \sin(\omega t + 120°)$$

$$E_5 = 8 \sin(\omega t + 180°)$$





Resultant wave

$$E = \sum_{i=1}^{5} E_i = E_0 \sin(\omega t + \varphi)$$

Problem

Superposition of a large number of phasors of equal amplitude and equal successive phase difference ϑ. Find the resultant phasor.

$$A = |A| \exp(i\phi) = a + a\exp(i\theta) + a\exp(i2\theta) \\ + a\exp(i3\theta) + \cdots a\exp(i(n-1)\theta)$$

Remember: The sum of the first *n* terms of a geometric series is:

$$a + ar + ar^2 + ar^3 + \ldots + ar^{(n-1)} = \sum_{k=0}^{n-1} ar^k = a\frac{1-r^n}{1-r}$$

Addition of Phasors

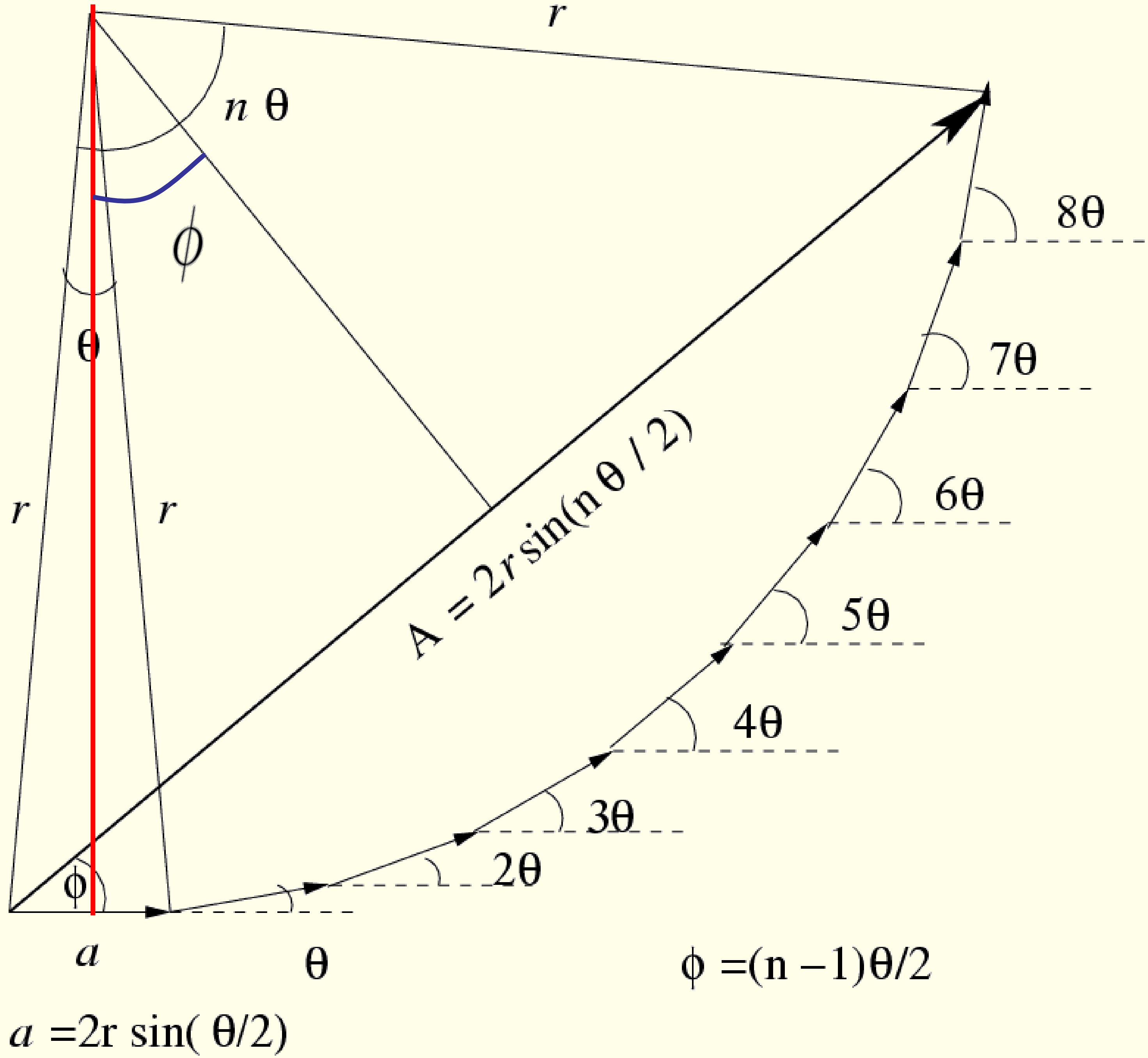
$$\boxed{\begin{aligned} A = |A|\exp(i\phi) &= a + a\exp(i\theta) + a\exp(i2\theta) \\ &+ a\exp(i3\theta) + \cdots a\exp(i(n-1)\theta) \end{aligned}}$$

$$A = \frac{a\left(1-e^{in\theta}\right)}{\left(1-e^{i\theta}\right)} = \frac{ae^{in\theta/2}\left[e^{in\theta/2}-e^{-in\theta/2}\right]}{e^{i\theta/2}\left[e^{i\theta/2}-e^{-i\theta/2}\right]}$$

GP series

$$= a\frac{\sin(n\theta/2)}{\sin(\theta/2)}e^{i(n-1)\theta/2}$$

$$|A| = a\frac{\sin(n\theta/2)}{\sin(\theta/2)} \qquad \phi = (n-1)\theta/2$$

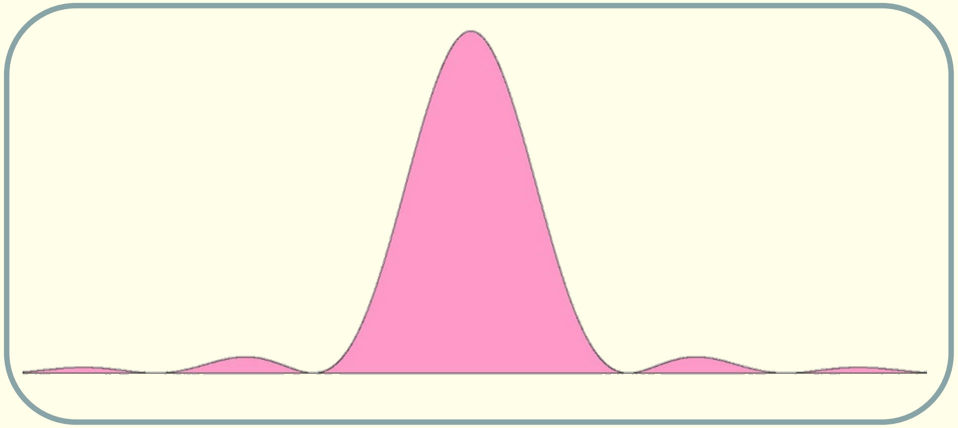

r
n θ
ϕ
θ
r
r
8θ
7θ
6θ
5θ
4θ
3θ
2θ
A = 2r sin(n θ / 2)
ϕ
a
θ
ϕ =(n −1)θ/2
a =2r sin(θ/2)

When n is large and θ and a are small such that

$$n\theta/2 = \alpha$$
$$na = A_0$$

$$A = a\frac{\sin(n\theta/2)}{\sin(\theta/2)}e^{i(n-1)\theta/2}$$

$$A = (A_0 \sin\alpha/\alpha)\exp(i\alpha)$$
$$I = AA^* = A_0^2 \sin^2\alpha/\alpha^2$$



Interference of Waves

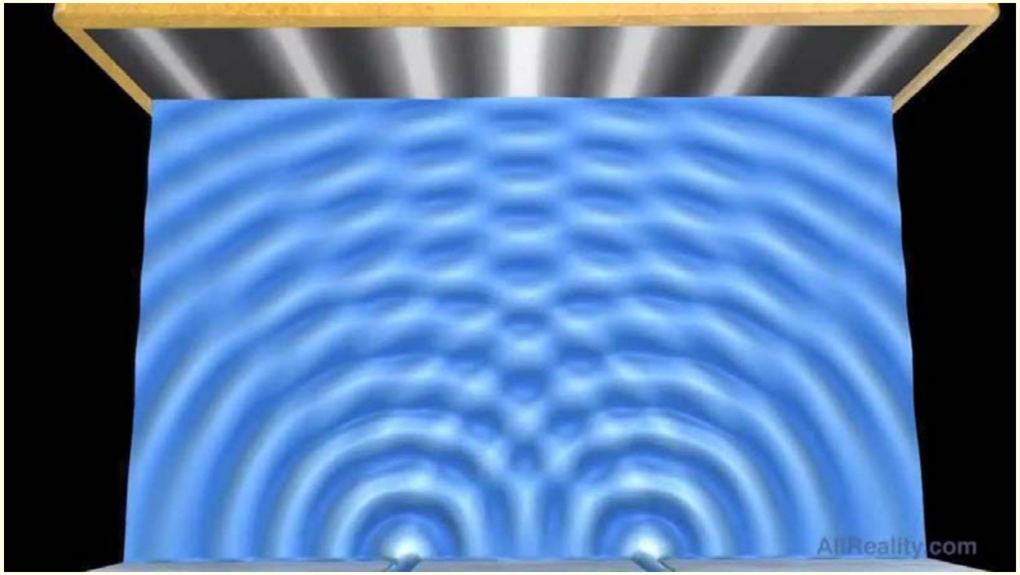
Interference of water waves

Two wave sources are said to be coherent:

- if the phase difference between the sources is constant,

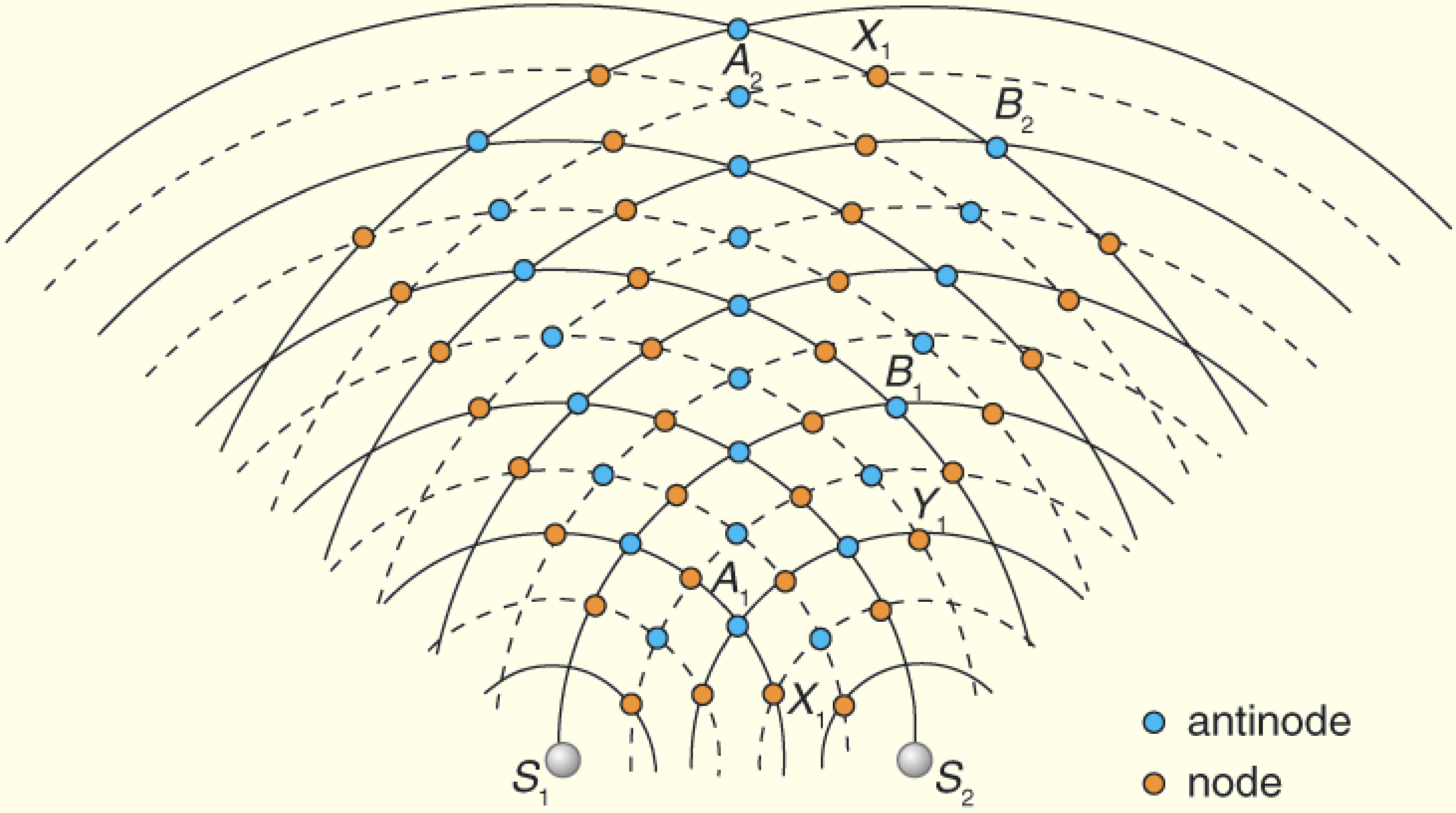
- if they have same frequency,

- if the two waves have comparable amplitudes.



The interference pattern produced in a ripple tank using two sources of circular waves which are in phase with each other.

Constructive & destructive interference

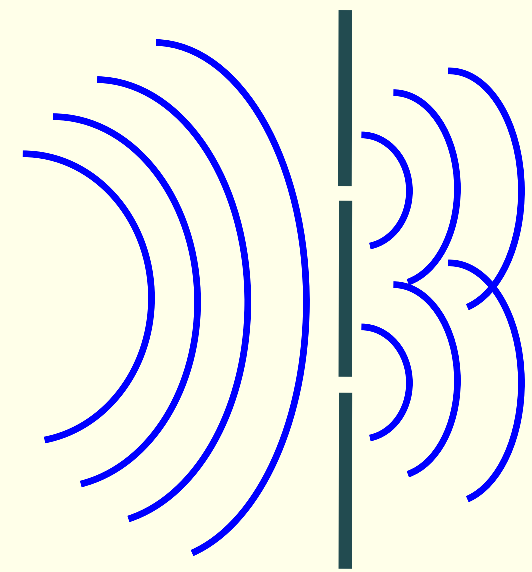


The two sources S_1 and S_2 are in phase and coherent. Therefore, the wavelengths of waves from S_1 and S_2 are the same, say λ.

Optical Interference

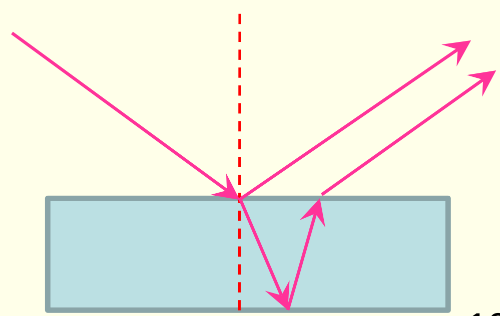
Optical interference corresponds to the interaction of two or more light waves yielding a resultant irradiance that deviates from the sum of component irradiance.

Optical Interference → Division of wavefront
Optical Interference → Division of amplitude



Division of wavefront

Wavefront Division: Involves taking one wavefront and dividing it up into more than one wave.
Eg: Young's double slit interference; Diffraction grating



Division of amplitude

Amplitude Division: Involves splitting a light beam into two beams at a surface of two media of different refractive index.
Eg: Michelson interferometer

Superposition of waves

$$\bar{E} = \bar{E}_1 + \bar{E}_2 + \bar{E}_3 + \bar{E}_4 + \ldots..$$

$$\bar{E}^2 = (\bar{E}_1 + \bar{E}_2).(\bar{E}_1 + \bar{E}_2)$$ **(for two waves)**

$$\vec{E}^2 = \vec{E}_1^2 + \vec{E}_2^2 + 2(\vec{E}_1 \cdot \vec{E}_2)$$

$$\boxed{\bar{E}_j = E_{0j} \cos(\vec{\mathrm{k}}_j . \vec{\mathrm{r}} - \omega \mathrm{t} + \varepsilon_j)}$$

Taking time average on both sides

$$I = I_1 + I_2 + I_{12}$$

Time Average

$$I_1 = \left\langle \overline{E}_1^2 \right\rangle_T$$

$$I_2 = \left\langle \overline{E}_2^2 \right\rangle_T$$

$$\mathrm{I}_{12} = 2\left\langle \overline{\mathrm{E}}_1 . \overline{\mathrm{E}}_2 \right\rangle_T$$

$$\left\langle \mathrm{f(t)} \right\rangle_T = \frac{1}{T} \int_t^{t+T} f\,(\mathrm{t'})\, dt'$$

Interference term

$$\left\langle \overline{E}_1 . \overline{E}_2 \right\rangle = \frac{1}{2} \overline{E}_{01} . \overline{E}_{02} \cos(\overline{k}_1 . \overline{r} + \varepsilon_1 - \overline{k}_2 . \overline{r} - \varepsilon_2)$$

$$I_{12} = \overline{E}_{01} . \overline{E}_{02} \cos\delta \qquad \left(\delta = \vec{k}_1 . \vec{r} + \varepsilon_1 - \vec{k}_2 . \vec{r} - \varepsilon_2\right)$$

The phase difference arising from a combined path length and initial phase difference.

Total irradiance $\Rightarrow$ $$\boxed{I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta}$$

$$\left(\delta = \vec{k}_1.\vec{r} + \varepsilon_1 - \vec{k}_2.\vec{r} - \varepsilon_2\right)$$

For maximum irradiance

$$\cos\delta = 1$$

$$I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

Total constructive interference

$$\delta = 0, \pm 2\pi, \pm 4\pi, \ldots\ldots\ldots$$

Total irradiance $\Rightarrow$ $$\boxed{I = I_1 + I_2 + 2\sqrt{I_1 I_2}\,cos\delta}$$

$$\left(\delta = \vec{k}_1.\vec{r} + \varepsilon_1 - \vec{k}_2.\vec{r} - \varepsilon_2\right)$$

For minimum irradiance

$$\cos\delta = -1$$

$$I_{\max} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

Total destructive interference

$$\delta = \pi, \pm 3\pi, \pm 5\pi, \ldots\ldots\ldots$$

Twin Source Interference Pattern

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$$

$$\delta = (\vec{k}_1.\vec{r} + \varepsilon_1 - \vec{k}_2.\vec{r} - \varepsilon_2)$$

For $I_1 = I_2 = I_0$

$$I = 2I_0(1+\cos\delta) \quad = 4I_0\cos^2\frac{\delta}{2}$$