Collective Modes in Gapless Systems

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Classic Results of Parabolic Systems

- Long-wavelength Plasmon in metals (e-e interation, Jellium model)

Classical Picture
$$\begin{cases} \nabla \cdot E = \frac{1}{\epsilon}(n-n_0) \\ m\frac{dv}{dt} = -eE \\ n-n_0 \ll n_0 \end{cases} \Rightarrow \frac{\partial n}{\partial t} + \nabla(nv) \approx \frac{\partial n}{\partial t} + n\nabla v = 0, \frac{\partial^2}{\partial t^2}(n-n_0) + \frac{n_0 e^2}{\epsilon m}(n-n_0) = 0$$

$$\omega_p = \sqrt{\frac{ne^2}{\epsilon m}}$$

$$P(q,\omega) \approx \frac{n}{m}\frac{q^2}{\omega^2} + O\left(\frac{q^4}{\omega^4}\right) \ \& \ \epsilon(q,\omega_P) = 1 - v(q)P(q,\omega_P) = 0$$

$$\omega_P^{(1)} = \sqrt{2e^2/\kappa m} \cdot n_1^{1/2} q\sqrt{|\ln qa|} + O(q^3)$$
$$\omega_P^{(2)} = \sqrt{2\pi e^2/\kappa m} \cdot n_2^{1/2} q^{1/2} + O(q^{3/2})$$
$$\omega_P^{(3)} = \sqrt{4\pi e^2/\kappa m} \cdot n_3^{1/2} + O(q^2)$$

$$\omega_P^{(D)} \propto n_D^{1/2}$$
$$\omega_P^{(D)} \propto \begin{cases} q\sqrt{|\ln qa|} \\ q^{1/2} \\ 1 \end{cases}$$

- Electron Density (A. Sommerfeld, free electron gas, $T \to 0K$)

$$E(k) = \frac{\hbar^2 k^2}{2m}$$

$$n_D = \begin{cases} 1D: \frac{4}{h}\sqrt{2mE_F} \\ 2D: \frac{4\pi m}{h^2}E_F \\ 3D: \frac{8\pi}{3}\left(\frac{2mE_F}{h^2}\right)^{3/2} \end{cases} = \begin{cases} \frac{2}{\pi}k_F \\ \frac{1}{2\pi}k_F^2 \\ \frac{1}{3\pi^2}k_F^3 \end{cases} \equiv \frac{g_s \pi^{D/2}}{(2\pi)^D \Gamma(1+D/2)} k_F^D$$

Electrons around the Fermi surface contribute to the collective modes.

H. Haug et al, Quantum theory of the optical and electronic properties of semiconductors

Lindhard Theory (RPA+1st Pertu.)

To calculating the effects of electric field screening by electrons in a solid.

- The Lindhard formula of dielectric function

$$\epsilon(q,\omega) = 1 - v(q)P(q,\omega) = 1 - v(q)g\sum_k \frac{f(E_k) - f(E_{k+q})}{\hbar(\omega + i\delta) + E_k - E_{k+q}} F(q,\omega)$$

g: the degeneracy factor
$F(q,\omega)$: the overlap form factor due to chirality
f: the Fermi-Dirac distribution function

$$v(q) = \begin{cases} 3D: 4\pi/\kappa q^2 \\ 2D: 2\pi e^2/\kappa q \\ 1D: 2e^2 K_0(qa)/\kappa \end{cases}$$

- The Lindhard formula for two-band model

$$\epsilon(q,\omega) = 1 - v(q)g \sum_{\alpha,\alpha' \in\{\pm 1\}} \sum_k \frac{f_\alpha(E_k) - f_{\alpha'}(E_{k+q})}{\hbar(\omega + i\delta) + E_{k,\alpha} - E_{k+q,\alpha'}} F_{\alpha,\alpha'}(q,\omega)$$

- The longitudinal plasma eigen-modes: $Re[\epsilon(q,\omega)] = 0$ $\quad \phi(q,\omega) = \dfrac{\phi^e(q,\omega)}{\epsilon(q,\omega)}$

$$\lim_{\delta\to 0^+} \frac{1}{z - i\delta} = P\left(\frac{1}{z}\right) + i\pi\delta(z)$$

$$|z| \gg 0 \Rightarrow \delta(z) = 0 \Rightarrow \mathrm{Im}(\epsilon) = 0$$

Prasanta K. Misra, Physics of Condensed Matter, Ch-7.4
H. Haug et al, Quantum theory of the optical and electronic properties of semiconductors, Ch-8

Hamiltonian and Fermi Surface of semimetals

Weyl semimetal

$$H = k_x\sigma_x + k_y\sigma_y + (m - Bk_z^2)\sigma_z$$

$$E_F^2 = (m - Bk_z^2)^2 + k_\perp^2$$

Nodal line semimetal

$$H = (m - Bk_\perp^2)\sigma_x + k_z\sigma_z$$

$$E_F^2 = (m - Bk_\perp^2)^2 + k_z^2$$

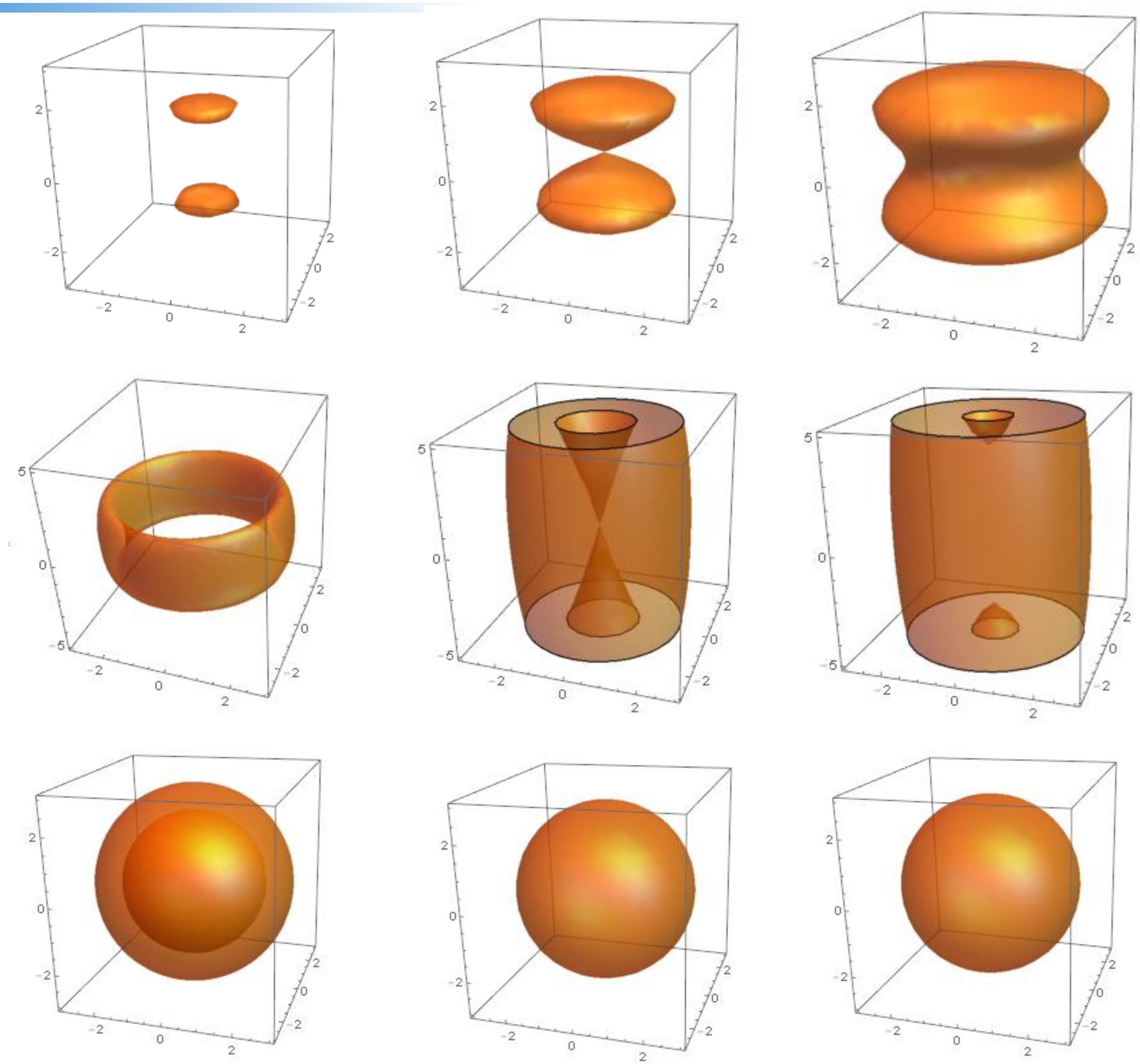
$$k_\perp^2 = \frac{m \pm \sqrt{E_F^2 - k_z^2}}{B}$$

Nodal sphere semimetal

$$H = (m - Bk^2)\sigma_z$$

$$E_F^2 = (m - Bk^2)^2$$

$$k^2 = \frac{m \pm |E_F|}{B}$$



$E_F < m$ $\quad$ $E_F = m$ $\quad$ $E_F > m$

Roughly Thinking about ω_P^D

$$\epsilon(q,\omega) = 1 - v(q)P(q,\omega)$$

$$= 1 - v(q)g\sum_k \frac{f(E_k) - f(E_{k+q})}{\hbar(\omega + i\delta) + E_k - E_{k+q}} F(q,\omega)$$

$$\approx 1 - v(q)g\sum_k \frac{\delta(E_F - E_k)\frac{\partial E_k}{\partial k_\alpha} q_\alpha}{\hbar\omega - \frac{\partial E_k}{\partial k_\alpha} q_\alpha}$$

$T \ll T_F, \frac{\partial f}{\partial E} \approx -\delta(E_F - E)$

$\hbar\omega > E_{k+q} - E_k$

Up to 1st order

$$\approx 1 - v(q)g\sum_k \left(1 + \frac{\partial E_k}{\partial k_\alpha}\frac{q_\alpha}{\hbar\omega}\right)\frac{\partial E_k}{\partial k_\alpha}\frac{q_\alpha}{\hbar\omega}\delta(E_F - E_k)$$

$(1-x)^{-1} \approx 1 + x$

$$\approx 1 - \frac{v(q)g}{(\hbar\omega)^2}\sum_k \left(\frac{\partial E_k}{\partial k_\alpha}\right)^2 q_\alpha^2\,\delta(E_F - E_k)$$

$\sum_k \frac{\partial f(E_k)}{\partial E_k} = \int \frac{d^D k}{(2\pi)^D}\frac{\partial f(E_k)}{\partial E_k} = f(E_\infty) = 0$

$P(q,\omega) \propto \frac{q^2}{\omega^2}$ Plasma dispersion $\omega_P(q)$ seems to be the same up to prefactors.

$\frac{\partial E_k}{\partial k}$ *Different Energy dispersion may bring up something new.*

Collective modes of Dirac Plasma

$$E_k = \hbar v_F k$$

- Plasmon frequency

$$P(q,\omega) = \frac{v_F(\hbar k_F)^{D-1}}{\hbar D(2\pi)^D} \cdot \frac{2\pi^{D/2}}{\Gamma(D/2)} \frac{q^2}{\omega^2}$$

$$1 - v(q)P(q,\omega_P) = 0 \Rightarrow \omega_P$$

$$\omega_P^{(1)} = \sqrt{r_s}\sqrt{\frac{g}{\pi}} v_F q\sqrt{|\ln qa|}$$

$$\omega_P^{(2)} = \sqrt{r_s}(g\pi)^{1/4} v_F n_2^{1/4} q^{1/2}$$

$$\omega_P^{(3)} = \sqrt{r_s}\left(\frac{32\pi g}{3}\right)^{1/6} v_F n_3^{1/3}$$

$$r_s = \frac{e^2}{\kappa\hbar v_F}$$

- Dependent of electron density

$$\omega_P^{(1)} \propto 1$$

$$\omega_P^{(2)} \propto n_2^{1/4}$$

$$\omega_P^{(3)} \propto n_3^{1/3}$$

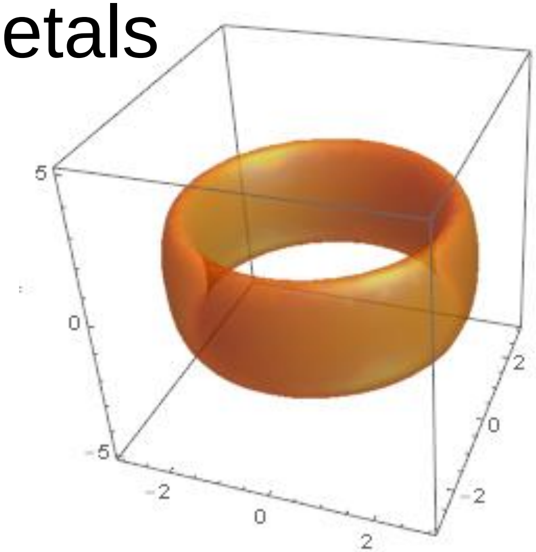
- Dependent of Planck constant

$$\omega_P^{(D)} \propto \hbar^{-1/2}$$

No classical correspondence.

S. Das Sarma et al, PRB 75, 205418 (2007)
S. Das Sarma et al, PRL 102, 206412 (2009)

Collective modes in nodal line semimetals



$$E_F^2 = \left(m - Bk_\perp^2\right)^2 + k_z^2$$

- Polarizability

$$P(q,\omega) = g\sum_k \frac{f(E_k) - f\left(E_{k+q}\right)}{\hbar(\omega + i\delta) + E_k - E_{k+q}} F(q,\omega)$$
$$= \frac{g}{(\hbar\omega)^2}\sum_k \left(\frac{\partial E_k}{\partial k_\alpha}\right)^2 q_\alpha^2\, \delta(E_F - E_k)$$
$$= \frac{mE_F}{4\pi}\frac{q_\perp^2}{\omega^2} + \frac{E_F}{8\pi B}\frac{q_z^2}{\omega^2}$$

- Electron density

$$n = \sum_k \theta(E_F - E_k) = \frac{1}{(2\pi)^2}\int_0^{+\infty} k_\perp \theta\left(E_F - \left|m - Bk_\perp^2\right|\right) dk_\perp \int_{-\sqrt{E_F - \left(m - Bk_\perp^2\right)^2}}^{\sqrt{E_F - \left(m - Bk_\perp^2\right)^2}} dk_z = \frac{E_F^2}{8\pi B}$$
$$\Rightarrow E_F \propto n^{1/2}$$

- The Lindhard formula

$$1 - v(q)P(q,\omega) = 1 - \frac{4\pi}{\kappa q^2}P(q,\omega) = 0 \Rightarrow \omega_P^{(3)} \propto n^{1/4}$$

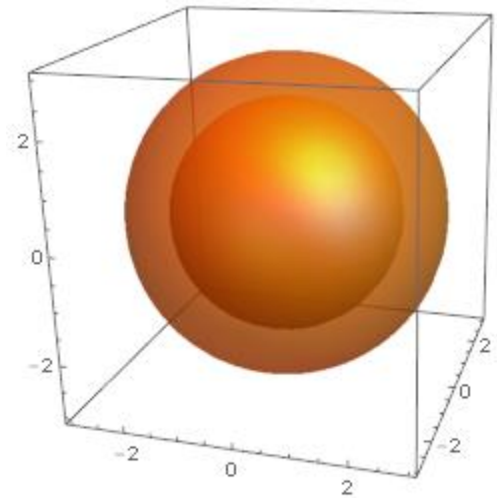
Ordinary parabolic plasma: $\omega_P^{(3)} \propto n^{1/2}$

Massless Dirac plasma: $\omega_P^{(3)} \propto n^{1/3}$

Zhongbo Yan et al, PRB 93,085138 (2016)

Collective modes in nodal sphere semimetals

- Polarizability $\quad m \gg E_F > 0$



$$P(q,\omega) = g\sum_k \frac{f(E_k) - f(E_{k+q})}{\hbar(\omega + i\delta) + E_k - E_{k+q}} F(q,\omega)$$

$$= \frac{g}{(\hbar\omega)^2}\sum_k \left(\frac{\partial E_k}{\partial k_\alpha}\right)^2 q_\alpha^2\, \delta(E_F - E_k)$$

$$= \frac{g}{(\hbar\omega)^2}\int \frac{d^3k}{(2\pi)^3}\left(4Bk_\alpha(m - Bk^2)\right)^2 q_\alpha^2 \delta(E_F - E_k) \qquad k_\alpha^2 q_\alpha^2 \approx k^2 q^2$$

$$\approx \frac{16B^2}{(2\pi)^3(\hbar\omega)^2} g \int E_F^2 k^2 q^2 \delta(E_F - E_k) k^2 dk \int sin\theta d\theta d\varphi$$

$$= \frac{8gB^2E_F^2q^2}{(\pi\hbar\omega)^2}\int_{\sqrt{(m-E_F)/B}}^{\sqrt{(m+E_F)/B}} k^4 dk = \frac{32gm^{3/2}}{(\pi\hbar)^2 B^{\frac{1}{2}}} E_F^3 \frac{q^2}{\omega^2}$$

$$E_F^2 = (m - Bk^2)^2$$

$$k^2 = \frac{m \pm |E_F|}{B}$$

- Electron density

$$n = \sum_k \theta(E_F - E_k) = \frac{1}{(2\pi)^3}\int_{\sqrt{(m-E_F)/B}}^{\sqrt{(m+E_F)/B}} 4\pi k^2 dk = \frac{(m+E_F)^{3/2} - (m-E_F)^{3/2}}{3\pi^2 B^{3/2}} \approx \frac{m^{\frac{1}{2}}}{\pi^2 B^{\frac{3}{2}}} E_F \propto E_F$$

- The Lindhard formula

$$1 - v(q)P(q,\omega) = 1 - \frac{4\pi}{\kappa q^2} P(q,\omega) = 0 \Rightarrow \omega_P^{(3)} \propto n^{3/2}$$