Collective Modes of the Massless Dirac Plasma

S. Das Sarma and E. H. Hwang

Condensed Matter Theory Center, Department of Physics, University of Maryland, College Park, Maryland 20742-4111, USA and Kavli Institute for Theoretical Physics, Santa Barbara, California 93106, USA

(Received 24 March 2009; published 22 May 2009)

We develop a theory for the long-wavelength plasma oscillation of a collection of charged massless Dirac particles in a solid, as occurring, for example, in doped graphene layers, interacting via the long-range Coulomb interaction. We find that the long-wavelength plasmon frequency in such a doped massless Dirac plasma is explicitly nonclassical in all dimensions with the plasma frequency being proportional to $1/\sqrt{\hbar}$. We also show that the long-wavelength plasma frequency of the D-dimensional superlattice made from such a plasma does not agree with the corresponding $D+1$-dimensional bulk plasmon frequency. We compare and contrast such Dirac plasmons with the well-studied regular palsmons in metals and doped semiconductors which manifest the usual classical long-wavelength plasma oscillation.

DOI: 10.1103/PhysRevLett.102.206412 PACS numbers: 71.45.Gm, 52.27.Ny, 81.05.Uw

A collection of charged particles (i.e., a plasma), electrons or holes or ions, is characterized by a *collective mode* associated with the self-sustaining in-phase density oscillations of all the particles due to the restoring force arising from the long-range $1/r$ Coulomb potential. The classical plasma frequency in three-dimensional (3D) plasmas [1] is well known to be $\omega_3 = (4\pi n_3 e^2/m)^{1/2}$, where e and m are, respectively, the charge and the mass of each particle, and n_3 is the 3D particle density. (In this Letter, we use ω_D and n_D as the D-dimensional long-wavelength plasma frequency and particle density, respectively.) A solid state degenerate plasma [2–4] exists in metals and doped semiconductors where free carriers can move around quantum mechanically in the ionic lattice background. Such a degenerate quantum plasma has the quantized version of exactly the same collective mode, the so-called plasmon [2–4], which dominates the spectral weight of the long-wavelength elementary excitation spectrum of an electron liquid. (We will use the world "electron" generally throughout this Letter to indicate either electron or hole.) The collective plasmon modes of solid state quantum plasmas have been extensively studied experimentally and theoretically over the last 60 years in both metals and doped semiconductors. In the present work, we study theoretically the collective plasmon mode in a solid state plasma of massless Dirac fermions, as occurring, for example, in 2D graphene layers. We define the Dirac plasma as a system of charged carriers whose energy-momentum dispersion is linear, obeying the Dirac equation.

Our main qualitative result is that the massless Dirac plasma is manifestly quantum, and does not have a classical limit in the form of an $\hbar$-independent long-wavelength plasma frequency, in a striking contrast to the corresponding parabolic dispersion electron liquids familiar from the extensive study of plasmons in metals and semiconductors [3–5]. The long-wavelength plasmon frequency of a Dirac plasma is necessarily quantum with '$\hbar$' appearing manifestly in the long-wavelength plasma frequency in $D=1$, 2, 3 dimension (and in between). By contrast the long-wavelength plasma frequency of ordinary electron liquids is classical, and quantum effects show up only as nonlocal corrections in higher order wave vector dispersion of the plasmon mode. This is quite unexpected in view of the popular belief that the long-wavelength quantum plasmon dispersion is necessarily a classical plasma frequency [2–4]. The popular belief seems to be true for the usual parabolic energy dispersion, but not for the linear Dirac spectrum.

We start from the fundamental many-body formula defining the collective plasmon mode in an electron system:

$$\epsilon(q,\omega) = 1 - v(q)\Pi(q,\omega) = 0, \tag{1}$$

where $\epsilon(q,\omega)$ is the wave vector (q) and frequency (ω) dependent dynamical dielectric function of the system, with $\Pi(q,\omega)$ the irreducible polarizability and $v(q)$ the Coulomb interaction between the electrons in the wave vector space. The zero of the dielectric function in Eq. (1) signifies a self-sustaining collective mode, with the solution of Eq. (1) giving the plasmon frequency as a function of wave vector. We first recapitulate the known results for the parabolic dispersion electron system before discussing the novel collective dispersion for massless Dirac plasma.

The Coulomb interaction in the wave vector space is given by the appropriate D-dimensional Fourier transform of the Coulomb interaction $v(r) = e^2/\kappa r$

$$v(q) = \frac{4\pi e^2}{\kappa q^2} \qquad D=3, \tag{2a}$$

$$= \frac{2\pi e^2}{\kappa q} \qquad D=2, \tag{2b}$$

$$= \frac{2e^2}{\kappa} K_0(qa) \qquad D=1, \tag{2c}$$

where we have introduced a background dielectric constant (κ) which, in general, differs from unity in semiconductor based electron systems, and K_0 is the zeroth-order modified Bessel function of the second kind. We note that $K_0(x) \sim |\ln(x)|$ for $x \to 0$, and the length "a" in the 1D Coulomb interaction in Eq. (2c) characterizes the typical lateral confinement size of the 1D electron system (ES) which is obviously necessary in defining a 1DES.

The irreducible polarizability function $\Pi(q, \omega)$ of an interacting ES is, in general, unknown since self-energy and vertex corrections cannot be calculated exactly. A great simplification, however, occurs in the long-wavelength limit ($q \to 0$) when the dielectric function, and, consequently, the plasmon frequency is determined entirely by the noninteracting irreducible polarizability, the electron-hole "bubble" diagram. The noninteracting irreducible polarizability is given by the expression

$$\Pi(q, \omega) = g \int \frac{d^D k}{(2\pi)^D} \frac{n_F(\xi_k) - n_F(\xi_{k+q})}{\hbar\omega + \xi_k - \xi_{k+q}} F(k, q), \quad (3)$$

where ξ_k is the single-particle energy dispersion, i.e., $\xi_k = \hbar^2 k^2/2m$ for parabolic systems (and $\xi_k = \hbar v_F k$ for the massless Dirac plasma), n_F is the Fermi distribution function, and $F(k, q)$ is the overlap form factor due to chirality. For nonchiral systems $F(q, k) = 1$. The factor "g" in Eq. (3) is the degeneracy factor: $g = g_s g_v$ where g_s ($=2$) is the spin degeneracy and g_v is the valley or pseudospin degeneracy.

Putting $\xi_k = \hbar^2 k^2/2m$, we can easily calculate Eq. (3) upto the leading order in wave vector (i.e., the long-wavelength limit) to obtain

$$\Pi(q, \omega) \approx \frac{n_D}{m} \frac{q^2}{\omega^2} + O(q^4/\omega^4). \quad (4)$$

Combining Eqs. (1)–(4) we immediately obtain the well-known long-wavelength plasma frequency in a D-dimensional ES

$$\omega_1^{(p)} = \sqrt{\frac{2e^2 n_1}{\kappa m}} q \sqrt{|\ln(qa)|} + O(q^3), \quad (5a)$$

$$\omega_2^{(p)} = \sqrt{\frac{2\pi n_2 e^2}{\kappa m}} q^{1/2} + O(q^{3/2}), \quad (5b)$$

$$\omega_3^{(p)} = \sqrt{\frac{4\pi n_3 e^2}{\kappa m}} + O(q^2), \quad (5c)$$

where $\omega_D^{(p)}$ denotes the long-wavelength ($q \to 0$) plasmon mode in the D-dimensional parabolic dispersion ES (with the carrier density n_D per unit D-dimensional volume) where the one particle energy is given by $\xi = \hbar^2 k^2/2m \to p^2/2m = mv^2/2$ classically (where $p = \hbar k$ is momentum). The long-wavelength plasmon frequencies for parabolic dispersion systems given in Eq. (5) are, of course, well known and have been verified experimentally extensively [4–6]. Our purpose of deriving Eq. (5) is the explicit demonstration, to be contrasted below with the corresponding massless Dirac plasma, that the long-wavelength plasmon frequency $\omega^{(p)}$ for parabolic systems is completely classical since '$\hbar$' does not appear in the leading term of Eq. (5) in any dimension. The second order dispersion correction term in Eq. (5), i.e., the $O(q^2, q^{3/2}, q^3)$ term in $D = 3, 2, 1$, respectively, is fully quantum mechanical (i.e., "$\hbar$" shows up explicitly in the nonlocal wave vector corrections), and is affected by interaction corrections (both self-energy and vertex corrections to the irreducible polarizability).

Now we consider plasmons in the D-dimensional massless Dirac plasma, where the single-particle energy dispersion is linear, i.e., $\xi_k = \hbar v_F |\mathbf{k}| \to vp$ classically, in $D = 1, 2, 3$. The long-wavelength quantum plasmon dispersion is still defined by the set of formulas given by Eqs. (1)–(3) with the explicit form of the noninteracting irreducible polarizability being calculated with $\xi_k = \hbar v_F |\mathbf{k}|$ in Eq. (3).

The long-wavelength ($q \to 0$) form for the noninteracting irreducible polarizability [Eq. (3)] can be calculated for linear energy dispersion relation (i.e., $\xi_k = \hbar v_F k$) in all dimensions, giving

$$\Pi(q, \omega) = \frac{g v_F k_F^{D-1}}{D(2\pi)^D} \frac{2\pi^{D/2}}{\Gamma(\frac{D}{2})} \frac{q^2}{\omega^2} + O(q^4/\omega^4), \quad (6)$$

where k_F is the Fermi momentum of the system and $\Gamma(x)$ is the Gamma function. (We note that the chirality factor $F(k, q)$ in Eq. (3) does not influence the long-wavelength limit.)

Combining Eqs. (1), (2), and (6) we get the following for the long-wavelength plasmon frequency, $\omega_D^{(l)}$, in $D = 1, 2, 3$ Dirac plasma:

$$\omega_1^{(l)} = \sqrt{r_s} \sqrt{\frac{g}{\pi}} v_F q \sqrt{|\ln(qa)|} + O(q^3), \quad (7a)$$

$$\omega_2^{(l)} = \sqrt{r_s} (g\pi n_2)^{1/4} v_F q^{1/2} + O(q^{3/2}), \quad (7b)$$

$$\omega_3^{(l)} = \sqrt{r_s} \left(\frac{32\pi g}{3}\right)^{1/6} n_3^{1/3} v_F + O(q^2), \quad (7c)$$

where we have introduced the dimensionless fine structure constant $r_s (\equiv e^2/(\kappa\hbar v_F))$ for notational simplicity.

Comparing Eqs. (5) and (7) we see that $\omega^{(p)}$ and $\omega^{(l)}$ have one important similarity and several striking differences. The similarity is that the plasmon dispersion is the same in the parabolic system and the massless Dirac plasma for all D. This is indeed required under very general principles, since for any Coulomb system the long-wavelength plasmon dispersion is set by the continuity equation (or equivalently, by particle conservation) to be $\omega_D(q \to 0) \sim q^{(3-D)/2}$ as is obeyed by both $\omega_D^{(p)}$ and $\omega_D^{(l)}$.

The most striking qualitative feature of $\omega_D^{(l)}$ in Eq. (7), in sharp contrast with the usual $\omega_D^{(p)}$ in Eq. (5), is that $\hbar$ appears explicitly in the *leading term*, not just the subleading nonlocal corrections. There is no classical plasma frequency in the massless Dirac plasma, i.e., the long-wavelength plasma frequency for ES with linear dispersion explicitly depends on $\hbar$, and is therefore, by definition, nonclassical. This absence of a classical long-wavelength plasma frequency in the Dirac plasma is a direct manifestation of the relativistic Dirac nature of the underlying quantum description, and such a Dirac plasma does not have a classical plasma frequency. Note that the nonclassical nature of the long-wavelength plasma oscillation of the Dirac plasma is independent of the chirality or gaplessness of graphene, and arises primarily from the linear Dirac spectrum.

Associated with the appearance of $1/\sqrt{\hbar}$ in the long-wavelength plasma frequency of the Dirac plasma are several other interesting properties distinguishing it from the standard parabolic dispersion Schrödinger plasma: (i) The density dependence of the Dirac plasmon is different from the regular plasmon [7,8]—in particular, the density dependence is weaker in the sense that $\omega_D^{(l)} \propto n^{1/3}, n^{1/4}, n^0$ in $D = 3, 2, 1$, respectively, in contrast with $\omega_D^{(p)} \propto \sqrt{n}$ in all dimensions. In general, the plasmon frequency in the Dirac plasma is given by $\omega_D^{(l)} \propto n^{(D-1)/2D}$. (ii) The 1D Dirac plasmon frequency is curiously density independent. (iii) The quantum coupling parameter (i.e. the effective fine structure constant) shows up explicitly in the long-wavelength Dirac plasmon frequency, $\omega_D^{(l)} \propto \sqrt{r_s}$. (iv) The long-wavelength Dirac plasmon $\omega_D^{(l)}$ goes as $\hbar^{-1/2}$ for all dimensions whereas the long-wavelength regular plasmon $\omega_D^{(p)}$ goes as $n_D^{1/2}$ in all dimensions.

Before concluding, we consider another interesting and peculiar feature of the Dirac plasmon distinguishing it from the regular plasmon. We consider collective modes of periodic arrays of 2D Dirac plasma layers (for example, a graphene superlattice made of parallel 2D graphene sheets in the direction transverse to the 2D graphene plane) and of 1D Dirac plasma nanoribbons (i.e., a graphene superlattice made of identical 1D graphene nanoribbons placed parallel to each other in the 2D plane). Collective plasmon modes of such 2D [9] and 1D [10] superlattices have been theoretically studied in the context of regular parabolic systems, and have been experimentally observed in doped GaAs multiquantum well and multiquantum wire structures.

The main physics to be considered in describing the collective plasmon modes of such superlattices is the inclusion of the interlayer or inter-ribbon Coulomb interaction, which will necessarily couple all the layers (or the ribbons) due to the long-range nature of the Coulomb potential. This changes the fundamental collective mode equation [Eq. (1)] to an infinite matrix equation:

$$|\delta_{ll'} - v_{ll'}(q, \omega)\Pi_{l'}(q, \omega)| = 0, \tag{8}$$

where $\Pi_l = \Pi$ is the irreducible polarizability of each 2D layer (or 1D ribbon), which is exactly the same polarizability considered in Eq. (3). In Eq. (8), $v_{ll'}$ is the Coulomb interaction between the l and the l' layer or ribbon in the periodic array, which is given by

$$v_{ll'} = \frac{2\pi e^2}{\kappa q} e^{-qd|l-l'|} \qquad D = 2, \tag{9a}$$

$$v_{ll'} = \frac{2e^2}{\kappa}[K_0(qa) + K_0(qd|l - l'|)] \qquad D = 1, \tag{9b}$$

where d is the superlattice period (to be distinguished from the length a in $D = 1$ which defines the lateral width of each ribbon).

The periodic invariance of the superlattice and the associated Bloch's theorem allow an immediate solution of the infinite-dimensional determinantal equation defined by Eq. (8), leading to the following collective plasmon bands for the superlattice structure:

$$\tilde{\omega}_{2s}(q; k) = \omega_2(q)S_2(q, k) \qquad D = 2, \tag{10a}$$

$$\tilde{\omega}_{1s}(q; k) = \omega_1(q)S_1(q, k) \qquad D = 1, \tag{10b}$$

where $\tilde{\omega}_{Ds}$ is the plasmon band frequency for the superlattice ($D = 2$ for the multilayer and $D = 1$ for the multiribbon periodic arrays) and ω_D is the corresponding 2D ($D = 2$) and 1D ($D = 1$) plasmon modes discussed in Eqs. (5) and (7). The wave vector q in Eq. (10) is the same conserved 2D or 1D wave vector in each individual 2D layer or 1D nanoribbon defining the plasmon dispersion relation $\omega_D(q)$ whereas the additional wave vector k is a new continuous parameter defining the superlattice plasmon band (arising from the periodicity in the array structure). The band wave vector k is restricted to the first superlattice Brillouin zone, $k \leq \pi/d$, in the reduced zone scheme. For the 2D layer superlattice, if each layer is assumed to lie in the x-y plane, then $k = q_z$ is along the superlattice direction of the z axis. For the 1D ribbon superlattice, if each ribbon is assumed to be along the x axis (i.e., $q = q_x$) with a width of a defining the ribbon in the y direction, then $k = q_y$ is along the superlattice direction of the y axis.

The function S_D in Eq. (10) is a form factor arising from the Coulomb coupling between all the layers and the ribbons forming the periodic array, and is given by

$$S_2 = \sum_{l'} e^{-q|l-l'|d - iq_z|l-l'|d}, \tag{11a}$$

$$S_1 = \sum_{l'}[K_0(q|l - l'|d)\cos(lq_yd) + K_0(qa)]. \tag{11b}$$

Combining the above equations for superlattice plasmons, we get the following long-wavelength ($q \rightarrow 0$) plasmon bands for 2D and 1D arrays in parabolic and Dirac plasma systems, respectively:

$$\tilde{\omega}_{2s}^{(p,l)}(\mathbf{q}) = \omega_2^{(p,l)}(q)\left[\frac{\sinh(qd)}{\cosh(qd) - \cos(q_zd)}\right]^{1/2}, \tag{12a}$$

$$\tilde{\omega}_{1s}^{(p,l)}(\mathbf{q}) = \omega_1^{(p,l)}(q)\left[K_0(qa) + 2\sum_{n=1}^{\infty} K_0(nqd)\cos(q_ynd)\right]^{1/2}. \tag{12b}$$

Equation (12) above defines plasmon bands for superlattice arrays made out of periodic 2D layers and 1D ribbons in parabolic and linear plasma systems.

An interesting quantum feature of $\tilde{\omega}^{(l)}_{Ds}(q, k)$ is apparent when one looks at the long-wavelength plasmon ($q \rightarrow 0$) at the band-edge $k = 0$, and compares $\tilde{\omega}^{(l)}_{Ds}(q, k = 0)$ with $\tilde{\omega}^{(p)}_{Ds}(q, k = 0)$. We get

$$\tilde{\omega}^{(p)}_{2s}(q; q_z = 0) = \left(\frac{4\pi\tilde{n}_3 e^2}{\kappa m}\right)^{1/2} \quad \text{with } \tilde{n}_3 = \frac{n_2}{d}, \tag{13a}$$

$$\tilde{\omega}^{(p)}_{1s}(q; q_y = 0) = \left(\frac{2\pi\tilde{n}_2 e^2 q}{\kappa m}\right)^{1/2} \quad \text{with } \tilde{n}_2 = \frac{n_1}{d}, \tag{13b}$$

and

$$\tilde{\omega}^{(l)}_{2s}(q; q_z = 0) = \sqrt{r_s}(4\pi g)^{1/4}\left(\frac{\tilde{n}_3}{d}\right)^{1/4} v_F, \tag{14a}$$

$$\tilde{\omega}^{(l)}_{1s}(q; q_y = 0) = \sqrt{r_s}\sqrt{\frac{g}{d}} v_F \sqrt{q}. \tag{14b}$$

We note that Eq. (13) for the usual parabolic electron plasma has the appropriate physical limit at the band-edge $k = 0$, where the D-dimensional superlattice plasmon should have the precise character of the corresponding $(D + 1)$-dimensional bulk plasmon in the long-wavelength limit, and indeed $\tilde{\omega}^{(p)}_{2s}(q, k = 0)$ and $\tilde{\omega}^{(p)}_{1s}(q, k = 0)$ are identical to the corresponding 3D and 2D plasmons [in Eq. (5)], respectively, with $\tilde{n}_3 = n_2/a$ and $\tilde{n}_2 = n_1/a$. This is exactly what one expects since the D-dimensional superlattice "loses" its discrete periodic structure for $k = 0$ and simply becomes the $(D + 1)$-dimensional regular plasmon at long wavelength.

However, this correspondence does not happen for the Dirac plasma, i.e., the D-dimensional superlattice plasmon for $k = 0$ does not become the corresponding $(D + 1)$-dimensional bulk plasma frequency as one would have expected intuitively. In particular, $\tilde{\omega}^{(l)}_{2s}(q, k = 0)$ would agree with the corresponding 3D Dirac plasmon $\tilde{\omega}^{(l)}_3(q)$ [in Eq. (7)] only if we define the corresponding effective 3D density to be $\tilde{n}_3 = (9\pi g/16)^{1/4}(n_2/d^2)^{3/4}$, rather than the intuitive definition $\tilde{n}_3 = n_2/d$. For the 1D superlattice Dirac plasmon, the situation is qualitatively different since $\tilde{\omega}^{(l)}_{1s}(q, k = 0)$ does not depend at all on the carrier density, and only the following substitution provides a correspondence between $\tilde{\omega}^{(l)}_{1s}(q, k = 0)$ and $\omega^{(l)}_2(q \rightarrow 0)$: $\tilde{n}_2 = g/(\pi d^2)$, which is a constant for all carrier density. This absence of correspondence between $\tilde{\omega}^{(l)}_{Ds}(k = 0)$ and $\omega^{(l)}_{D+1}$ is the direct consequence of the density dependence of the irreducible polarizability, Eq. (6), (i.e., the density response function). In linear response the density response function should depend linearly on the total density of the ES as it does for the ordinary parabolic ES. However, for the Dirac plasma the density response function is given by $\Pi \propto n^{(D-1)/D}$. This peculiar density dependence of the polarizability is a manifestation of the quantum nature of the Dirac plasma and gives rise to the lack of correspondence between the band-edge plasmon at $k = 0$ in the D-dimensional superlattice as obtained by solving Eq. (8) and the corresponding bulk plasmon in $(D + 1)$-dimension as given by Eq. (1).

In summary, we have found that the long-wavelength plasma frequency of a massless Dirac plasma with linear carrier energy dispersion is nonclassical with an explicit $1/\sqrt{\hbar}$ appearing in the plasma frequency. This is in sharp contrast with the widespread expectation that the long-wavelength plasmon is a classical plasma oscillation—in fact, a massless Dirac plasma has no classical analogy. We have also shown that the long-wavelength plasma mode of a D-dimensional superlattice of massless Dirac plasma does not reduce to the corresponding $(D + 1)$-dimensional bulk plasmon, as one would have expected intuitively. All of these peculiar results follow from the fact that a massless Dirac plasma is fundamentally nonclassical since the energy dispersion $E = vp$ characterizing a system with constant velocity (but variable momentum) simply cannot happen in classical physics. We believe that our predictions can be tested in doped graphene layers and multilayers, and in doped graphene ribbons and multiribbons arrays using electron scattering [11], light scattering [12], or infrared [13] spectroscopies. But the real importance of our results is conceptual as we establish a strange quantum behavior in the graphene world of a Dirac plasma where the long-wavelength plasmon is explicitly nonclassical.

This work is supported by US-ONR, NSF-NRI, SWAN, and DOE-Sandia.

[1] J. D. Jackson, *Classical Electrodynamics* (John Wiley & Sons, New York, 1999).
[2] P. M. Platzmann and P. A. Wolff, *Waves and Interactions in Solid State Plasmas*, Solid State Physics (Academic Press, New York, 1973).
[3] D. Pines and P. Nozieres, *The Theory of Quantum Liquids* (W. A. Benjamin, New York, 1966).
[4] G. D. Mahan, *Many Particle Physics* (Plenum Publisher, New York, 2000).
[5] T. Ando, A. B. Fowler, and F. Stern, Rev. Mod. Phys. **54**, 437 (1982).
[6] Q. P. Li and S. Das Sarma, Phys. Rev. B **43**, 11768 (1991).
[7] E. H. Hwang and S. Das Sarma, Phys. Rev. B **75**, 205418 (2007).
[8] L. Brey and H. Fertig, Phys. Rev. B **75**, 125434 (2007).
[9] S. Das Sarma and J. J. Quinn, Phys. Rev. B **25**, 7603 (1982).
[10] S. Das Sarma and Wu-yan Lai, Phys. Rev. B **32**, 1401 (1985).
[11] T. Eberlein *et al.*, Phys. Rev. B **77**, 233406 (2008); A. Nagashim *et al.*, Solid State Commun. **83**, 581 (1992).
[12] A. Pinczuk *et al.*, Phys. Rev. Lett. **56**, 2092 (1986); D. Olego *et al.*, Phys. Rev. B **25**, 7867 (1982); A. Pinczuk *et al.*, Phys. Rev. Lett. **47**, 1487 (1981); M. A. Eriksson *et al.*, Phys. Rev. Lett. **82**, 2163 (1999); A. R. Goni *et al.*, Phys. Rev. Lett. **67**, 3298 (1991).
[13] S. J. Allen *et al.*, Phys. Rev. Lett. **38**, 980 (1977).