Class (buys)

$$\text{Info}(D) = -\sum_{i=1}^{n} p_i \log_2(p_i)$$

$$= I(9,5)$$

$$= -\left(\frac{9}{14}\log_2\frac{9}{14}\right) + \left(-\frac{5}{14}\log_2\frac{5}{14}\right)$$

$$= -\frac{9}{14}\log_2\frac{9}{14} - \frac{5}{14}\log_2\frac{5}{14}$$

$$= -\frac{9}{14}(-0.637) - \frac{5}{14}(-1.485)$$

$$= 0.940 \quad \#$$

Feature

$$\text{Info}_{age}(D) = \sum_{j=1}^{v} \left|\frac{D_j}{D}\right| \times \text{Info}(D_j)$$

$$= \frac{5}{14} I(2,3) + \frac{4}{14} I(4,0) + \frac{5}{14} I(3,2)$$

$$= \frac{5}{14}\left[-\frac{2}{5}\log_2\left(\frac{2}{5}\right) - \frac{3}{5}\log_2\left(\frac{3}{5}\right)\right] + \frac{4}{14}\left[-\frac{4}{4}\cancelto{0}{\log_2\left(\frac{4}{4}\right)} - \frac{0}{4}\cancelto{\text{หาค่าไม่ได้}}{\log_2\left(\frac{0}{4}\right)}\right] + \frac{5}{14}\left[-\frac{3}{5}\log_2\left(\frac{3}{5}\right) - \frac{2}{5}\log_2\left(\frac{2}{5}\right)\right]$$

$$= \frac{5}{14}(0.529 + 0.442) + \frac{4}{14}(0 + \underline{\text{หาค่าไม่ได้}}) + \frac{5}{14}(0.442 + 0.529)$$

$$= \frac{5}{14}(0.971) + \frac{5}{14}(0.971)$$

$$= 0.347 + 0.347$$

$$= 0.694 \quad \#$$

$$\text{Info}_{income}(D) = \sum_{j=1}^{v} \left|\frac{D_j}{D}\right| \times \text{Info}(D_j)$$

$$= \frac{4}{14} I(2,2) + \frac{6}{14} I(4,2) + \frac{4}{14} I(3,1)$$

$$= \frac{4}{14}\left[-\frac{2}{4}\log_2\left(\frac{2}{4}\right) - \frac{2}{4}\log_2\left(\frac{2}{4}\right)\right] + \frac{6}{14}\left[-\frac{4}{6}\log_2\left(\frac{4}{6}\right) - \frac{2}{6}\log_2\left(\frac{2}{6}\right)\right] + \frac{4}{14}\left[-\frac{3}{4}\log_2\left(\frac{3}{4}\right) - \frac{1}{4}\log_2\left(\frac{1}{4}\right)\right]$$

$$= \frac{4}{14}(0.5 + 0.5) + \frac{6}{14}(0.390 + 0.528) + \frac{4}{14}(0.311 + 0.5)$$

$$= \frac{4}{14} + \frac{6}{7}(0.918) + \frac{4}{14}(0.811)$$

$$= 0.286 + 0.394 + 0.232$$

$$= 0.912 \quad \#$$

$$\text{Info}_{\text{student}}(D) = \sum_{j=1}^{v} \left|\frac{D_j}{D}\right| \times \text{Info}(D_j)$$

$$= \frac{7}{14} I(3,4) + \frac{7}{14} I(6,1)$$

$$= \frac{7}{14}\left[-\frac{3}{7}\log_2\left(\frac{3}{7}\right) - \frac{4}{7}\log_2\left(\frac{4}{7}\right)\right] + \frac{7}{14}\left[-\frac{6}{7}\log_2\left(\frac{6}{7}\right) - \frac{1}{7}\log_2\left(\frac{1}{7}\right)\right]$$

$$= \frac{7}{14}(0.524 + 0.461) + \frac{7}{14}(0.191 + 0.401)$$

$$= \frac{7}{14}(0.985) + \frac{7}{14}(0.592)$$

$$= 0.493 + 0.296$$

$$= 0.789$$ #

$$\text{Info}_{\text{credit}}(D) = \sum_{j=1}^{v} \left|\frac{D_j}{D}\right| \times \text{Info}(D_j)$$

$$= \frac{8}{14} I(6,2) + \frac{6}{14} I(3,3)$$

$$= \frac{8}{14}\left[-\frac{6}{8}\log_2\left(\frac{6}{8}\right) - \frac{2}{8}\log_2\left(\frac{2}{8}\right)\right] + \frac{6}{14}\left[-\frac{3}{6}\log_2\left(\frac{3}{6}\right) - \frac{3}{6}\log_2\left(\frac{3}{6}\right)\right]$$

$$= \frac{8}{14}(0.311 + 0.5) + \frac{6}{14}(0.5 + 0.5)$$

$$= \frac{8}{14}(0.811) + \frac{6}{14}$$

$$= 0.464 + 0.429$$

$$= 0.893$$ #

Gain

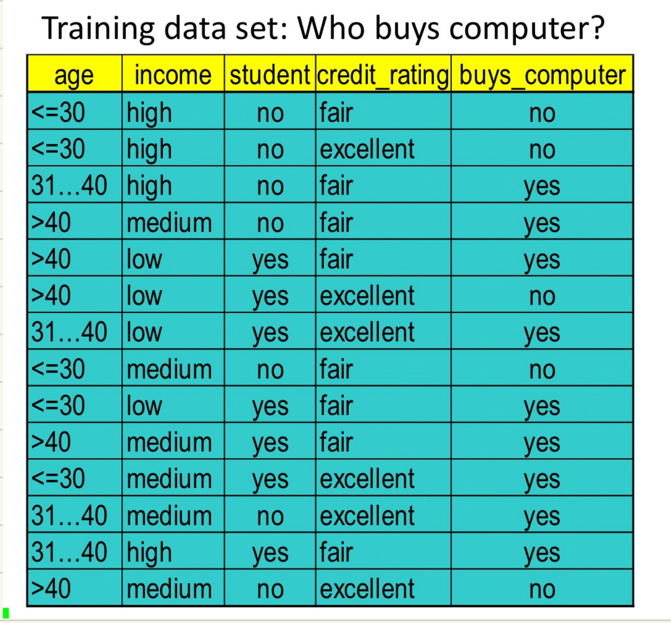
Gain (age) $= \text{Info}(D) - \text{Info}_{\text{age}}(D) = 0.940 - 0.649 = 0.291$

Gain (income) $= \text{Info}(D) - \text{Info}_{\text{income}}(D) = 0.940 - 0.912 = 0.028$

Gain (student) $= \text{Info}(D) - \text{Info}_{\text{student}}(D) = 0.940 - 0.789 = 0.151$

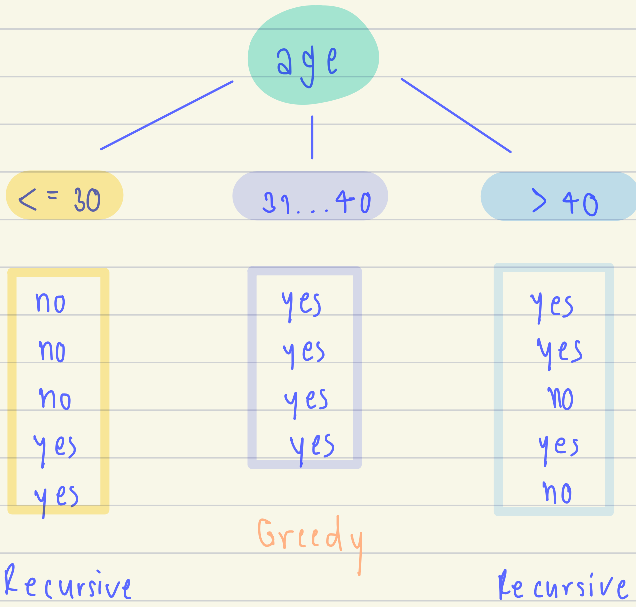
Gain (credit_rating) $= \text{Info}(D) - \text{Info}_{\text{credit}}(D) = 0.940 - 0.893 = 0.047$

∴ เลือก **Gain (age)** เพราะมีค่าเยอะที่สุด แปลว่าเป็นทางเลือกที่ดีที่สุด



Training data set: Who buys computer?

age	income	student	credit_rating	buys_computer
<=30	high	no	fair	no
<=30	high	no	excellent	no
31…40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
31…40	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
31…40	medium	no	excellent	yes
31…40	high	yes	fair	yes
>40	medium	no	excellent	no



F1 age <= 30

age	income	student	credit	buys
<= 30	high	no	fair	no
<= 30	high	no	excellent	no
<= 30	medium	no	fair	no
<= 30	low	yes	fair	yes
<= 30	medium	yes	excellent	yes

$$\text{Info}(D) = \sum_{i=1}^{m} p_i \log_2(p_i)$$

$$= I(2,3)$$

$$= -\frac{2}{5}\log_2\left(\frac{2}{5}\right) - \frac{3}{5}\log_2\left(\frac{3}{5}\right)$$

$$= 0.971$$

$$\text{Info}_{\text{income}}(D) = \frac{2}{5} I(0,2) + \frac{2}{5} I(1,1) + \frac{1}{5} I(1,0)$$

$$= \frac{2}{5}\left[\cancelto{\text{หาค่าไม่ได้}}{-\frac{0}{2}\log_2\left(\frac{0}{2}\right)} - \frac{2}{2}\log_2\left(\frac{2}{2}\right)\right] + \frac{2}{5}\left[-\frac{1}{2}\log_2\left(\frac{1}{2}\right) - \frac{1}{2}\log_2\frac{1}{2}\right] + \frac{1}{5}\left[-1\log_2\left(\frac{1}{5}\right) - \cancelto{\text{หาค่าไม่ได้}}{0\log_2(0)}\right]$$

$$= 0.4 \quad \#$$

$$\text{Info}_{\text{student}}(D) = \frac{3}{5} I(0,3) + \frac{2}{5} I(2,0)$$

$$= \frac{3}{5}\left[-\frac{0}{3}\log_2\left(\frac{0}{3}\right) - \frac{3}{3}\log_2\left(\frac{3}{3}\right)\right] + \frac{2}{5}\left[-\frac{2}{2}\log_2\left(\frac{2}{2}\right) - \frac{0}{2}\log_2\left(\frac{0}{2}\right)\right]$$

$$= 0 \quad \#$$

$$\text{Info}_{\text{credit}}(D) = \frac{3}{5} I(1,2) + \frac{2}{5} I(1,1)$$

$$= \frac{3}{5}\left[-\frac{1}{3}\log_2\left(\frac{1}{3}\right) - \frac{2}{3}\log_2\left(\frac{2}{3}\right)\right] + \frac{2}{5}\left[-\frac{1}{2}\log_2\left(\frac{1}{2}\right) - \frac{1}{2}\log_2\left(\frac{1}{2}\right)\right]$$

$$= 0.551 + 0.4$$

$$= 0.951 \quad \#$$

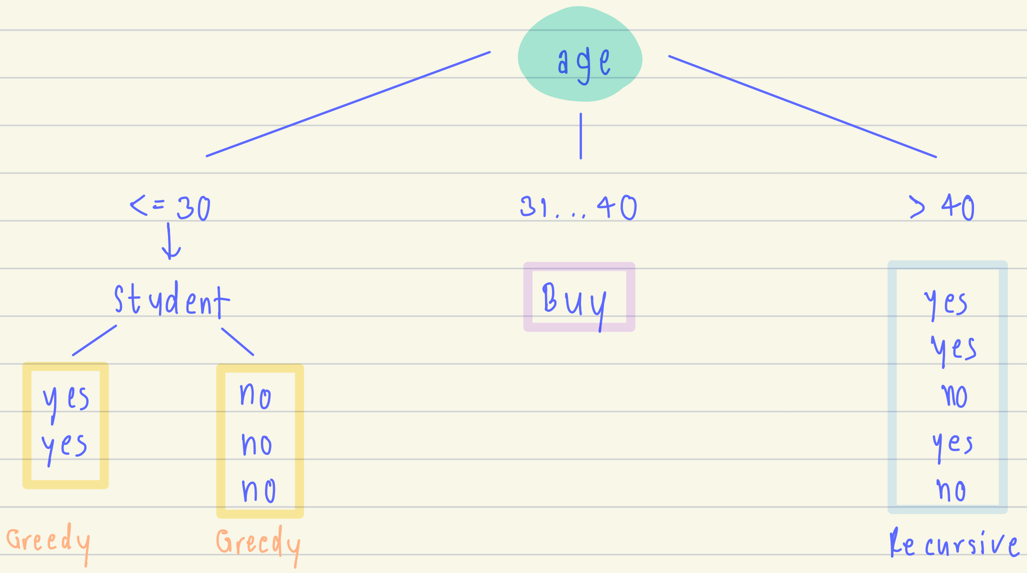
Gain

$\text{Gain}(\text{income}) = \text{Info}(D) - \text{Info}_{\text{income}}(D) = 0.971 - 0.4 = 0.571$

$\text{Gain}(\text{student}) = \text{Info}(D) - \text{Info}_{\text{student}}(D) = 0\ 971 - 0 = 0.971$

$\text{Gain}(\text{credit}) = \text{Info}(D) - \text{Info}_{\text{credit}}(D) = 0.971 - 0.951 = 0.020$

∴ Gain ที่มากที่สุด คือ Gain (student)



F2 age > 40

age	income	student	credit	buys
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
>40	medium	yes	fair	yes
>40	medium	no	excellent	no

$$\text{Info}(D) = I(3,2)$$
$$= -\frac{3}{5}\log_2\left(\frac{3}{5}\right) - \frac{2}{5}\log_2\left(\frac{2}{5}\right)$$
$$= 0.971$$

$$\text{Info}_{\text{income}}(D) = \frac{3}{5} I(2,1) + \frac{2}{5} I(1,1)$$
$$> \frac{3}{5}\left[-\frac{2}{3}\log_2\left(\frac{2}{3}\right) - \frac{1}{3}\log_2\left(\frac{1}{3}\right)\right] + \frac{2}{5}\left[-\frac{1}{2}\log_2\left(\frac{1}{2}\right) - \frac{1}{2}\log_2\left(\frac{1}{2}\right)\right]$$
$$= 0.551 + 0.4$$
$$= 0.951 \quad \#$$

$$\text{Info}_{\text{student}}(D) = \frac{2}{5} I(1,1) + \frac{3}{5} I(2,1)$$
$$= \frac{2}{5}\left[-\frac{1}{2}\log_2\left(\frac{1}{2}\right) - \frac{1}{2}\log_2\left(\frac{1}{2}\right)\right] + \frac{3}{5}\left[-\frac{2}{3}\log_2\left(\frac{2}{3}\right) - \frac{1}{3}\log_2\left(\frac{1}{3}\right)\right]$$
$$= 0.4 + 0.551$$
$$= 0.951 \quad \#$$

$$\text{Info}_{\text{credit}}(D) = \frac{3}{5} I(3,0) + \frac{2}{5} I(1,1)$$
$$= \frac{3}{5}\left[-\frac{3}{3}\log_2\left(\frac{3}{3}\right) - \overset{\text{หาค่าไม่ได้}}{\cancel{\frac{0}{3}\log_2\left(\frac{0}{3}\right)}}\right] + \frac{2}{5}\left[-\frac{1}{2}\log_2\left(\frac{1}{2}\right) - \frac{1}{2}\log_2\left(\frac{1}{2}\right)\right]$$
$$= 0.4 \quad \#$$

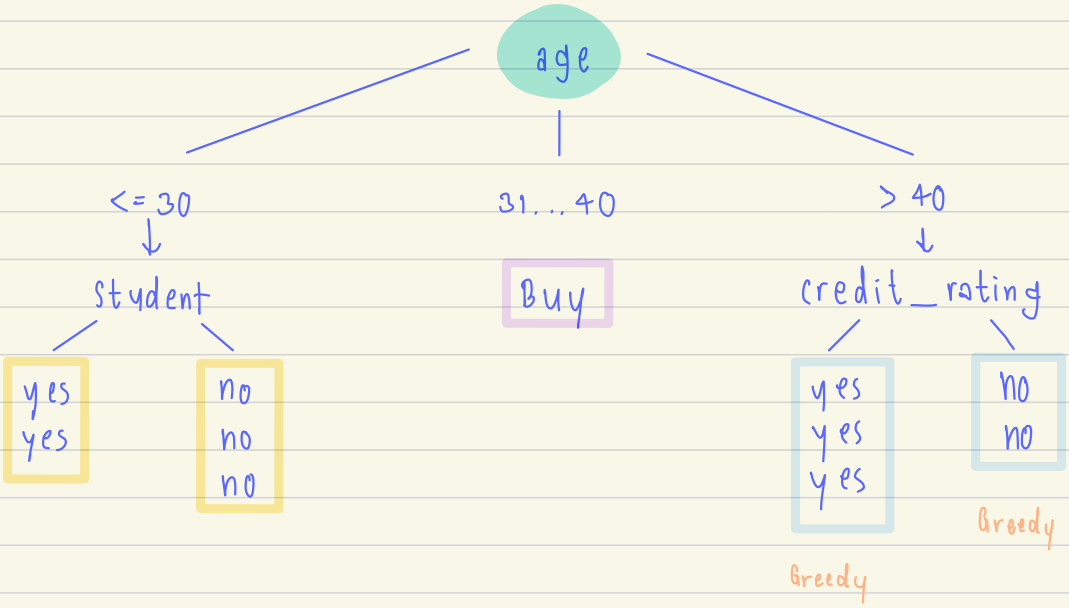
Gain

$\text{Gain(income)} = \text{Info}(D) - \text{Info}_{\text{income}}(D) = 0.971 - 0.951 = 0.2$

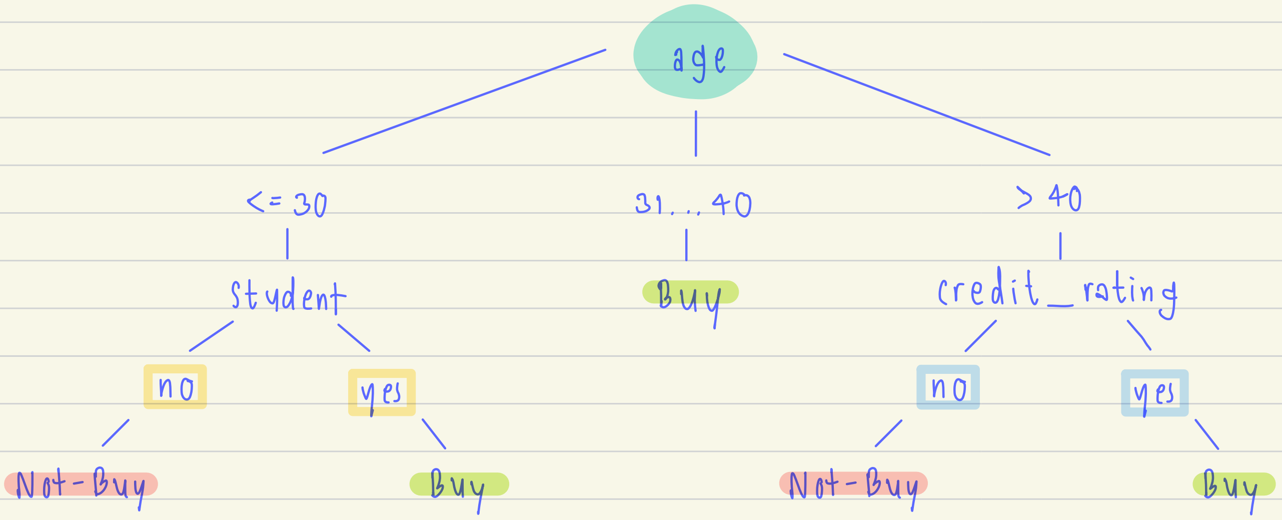
$\text{Gain(student)} = \text{Info}(D) - \text{Info}_{\text{student}}(D) = 0.971 - 0.951 = 0.2$

$\text{Gain(credit)} = \text{Info}(D) - \text{Info}_{\text{credit}}(D) = 0.971 - 0.4 = 0.571$

∴ Gain ที่มากที่สุด คือ Gain(credit)



Decision Tree Induction


age
<= 30
31...40
> 40
Student
Buy
credit_rating
no
yes
no
yes
Not-Buy
Buy
Not-Buy
Buy