

# 致密星：白矮星，脉冲星（中子星）和黑洞

李广兴

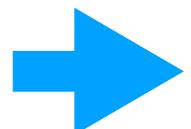
# Warm-up 1 : Order-of-magnitude physics

**Question:** Figure out the period of a pendulum

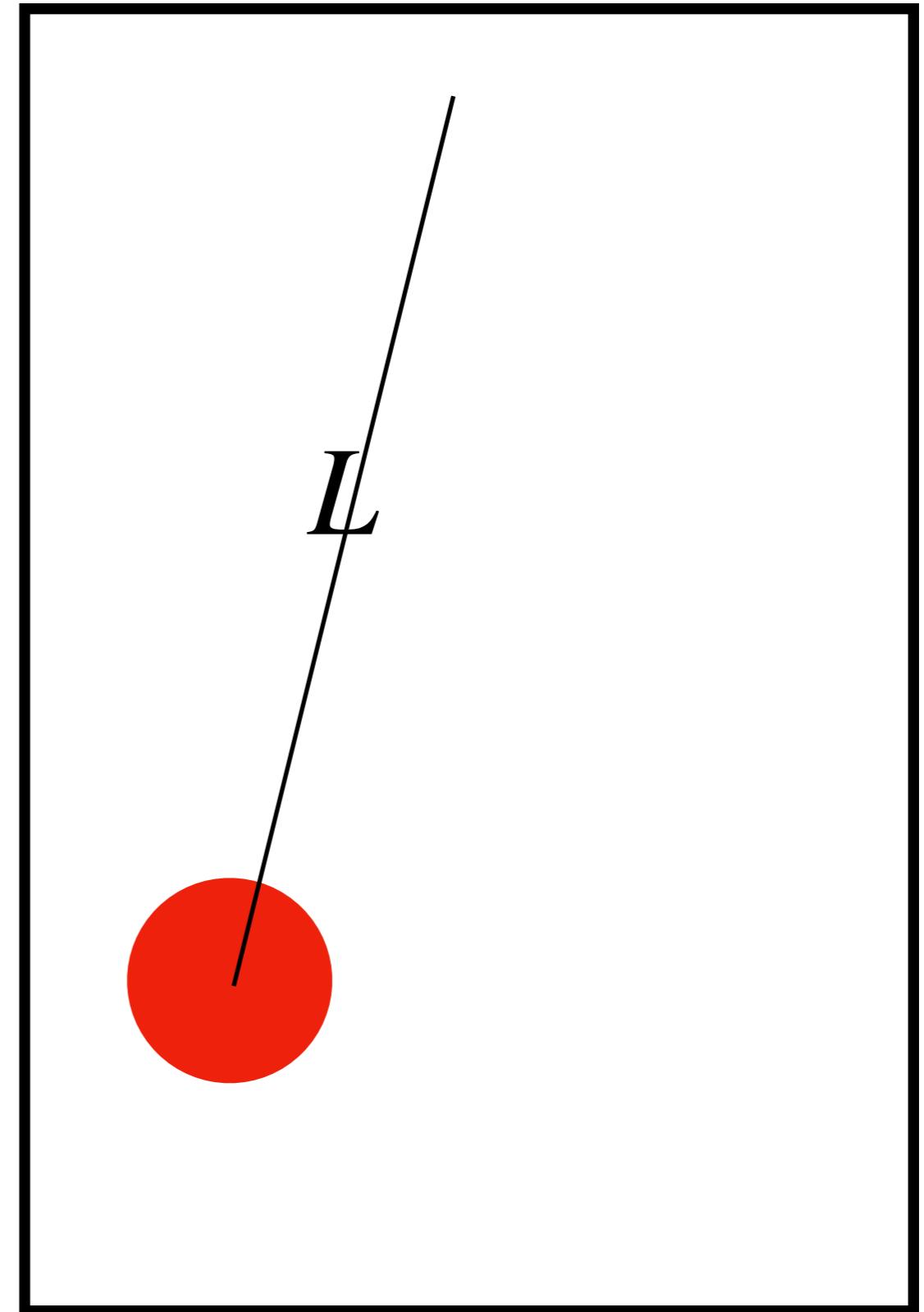
量纲分析:  $[]$  : Dimension

已知量:  $[g] = L/T^2$

$$[l] = L$$

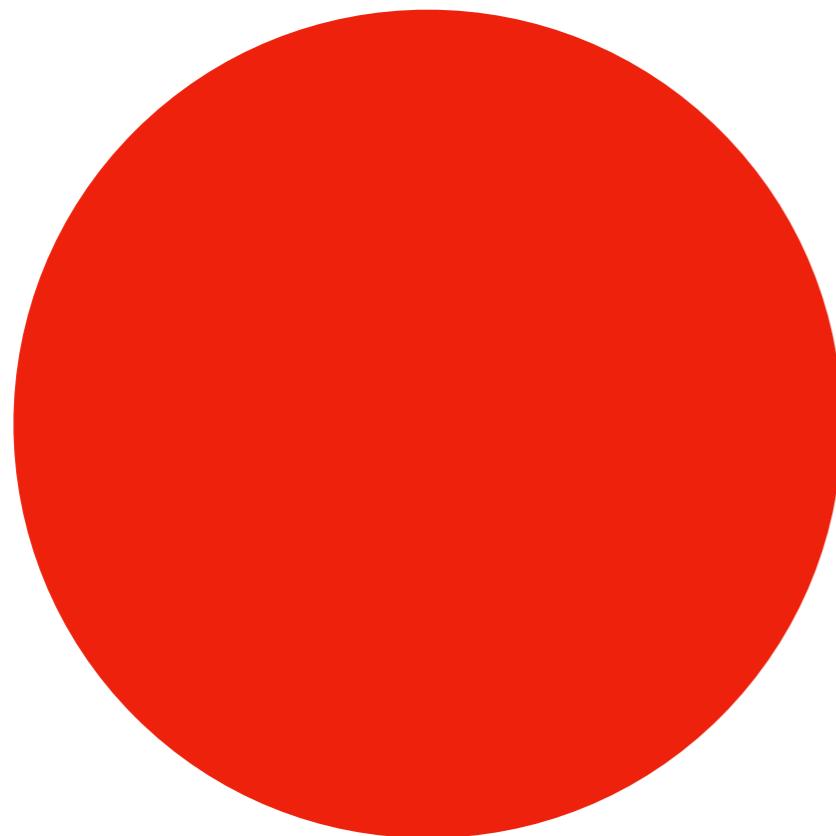


$$t \approx \sqrt{l/g}$$



# Warm-up 2 : Order-of-magnitude physics

引力塌缩



已知:  $m, L, G$

未知:  $t$

## Free-fall timescale

$$\frac{dr}{dt} \frac{d^2r}{dt^2} = -\left(\frac{4\pi}{3} G \rho_0 r_0^3\right) \frac{1}{r^2} \frac{dr}{dt},$$

$$\frac{1}{2} \left(\frac{dr}{dt}\right)^2 = \left(\frac{4\pi}{3} G \rho_0 r_0^3\right) \frac{1}{r} + C_1.$$

$dr/dt = 0$  when  $r = r_0$

$$C_1 = -\frac{4\pi}{3} G \rho_0 r_0^2.$$

$$\frac{d^2r}{dt^2} = -G \frac{M}{r^2} \rightarrow \frac{dr}{dt} = -\left[\frac{8\pi}{3} G \rho_0 r_0^2 \left(\frac{r_0}{r} - 1\right)\right]^{1/2}$$

$$\theta \equiv \frac{r}{r_0} \quad \left. \begin{array}{l} \\ \end{array} \right\} \rightarrow \frac{d\theta}{dt} = -\chi \left(\frac{1}{\theta} - 1\right)^{1/2}$$
$$\chi \equiv \left(\frac{8\pi}{3} G \rho_0\right)^{1/2}$$

$$\frac{\xi}{2} + \frac{1}{4} \sin 2\xi = \frac{\chi}{2} t + C_2$$

$r = r_0$  when  $t = 0$ ,  $\theta = 1$ , or  $\xi = 0$

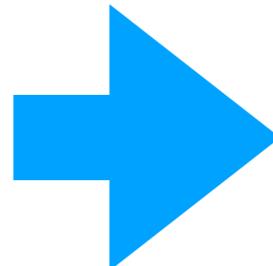
$$C_2 = 0.$$

$$(\theta = 0 \ \xi = \pi/2)$$

$$\xrightarrow{\theta = \cos^2 \xi} \cos^2 \xi \frac{d\xi}{dt} = \frac{\chi}{2} \rightarrow \xi + \frac{1}{2} \sin 2\xi = \chi t$$

$$t_{ff} = \left( \frac{3\pi}{32} \frac{1}{G \rho_0} \right)^{1/2}$$

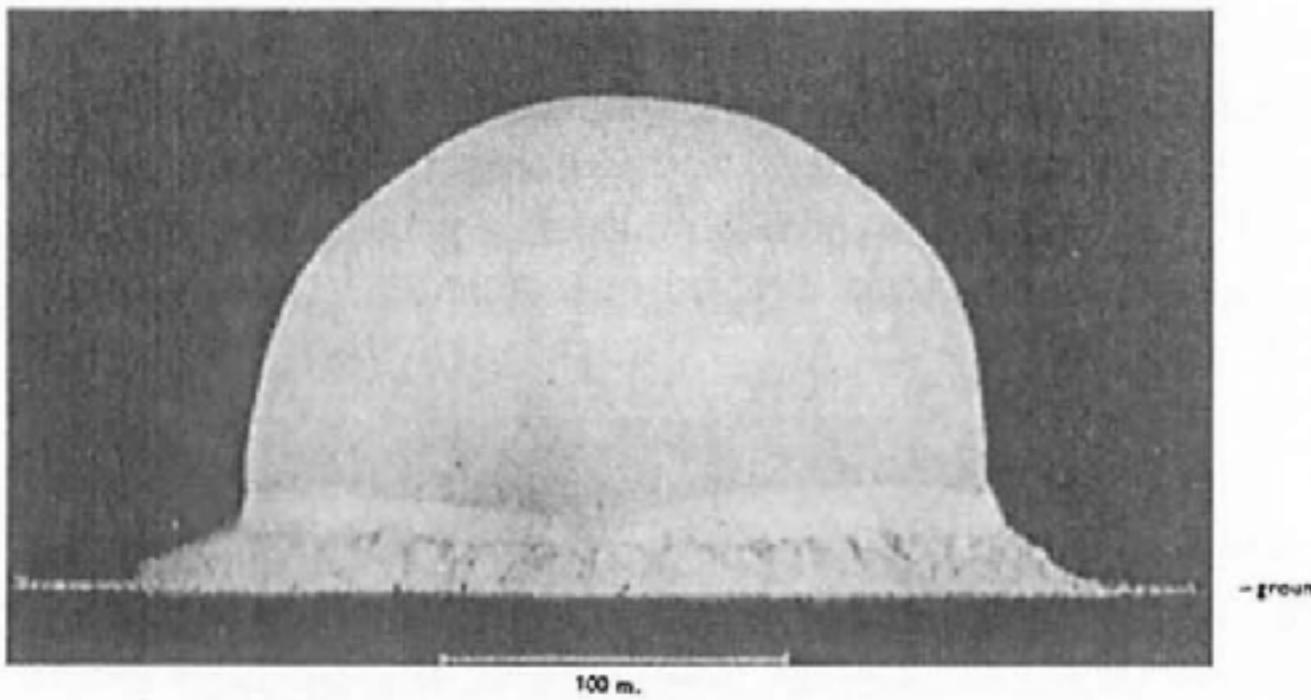
Free-fall time is independent of the initial radius of the sphere.



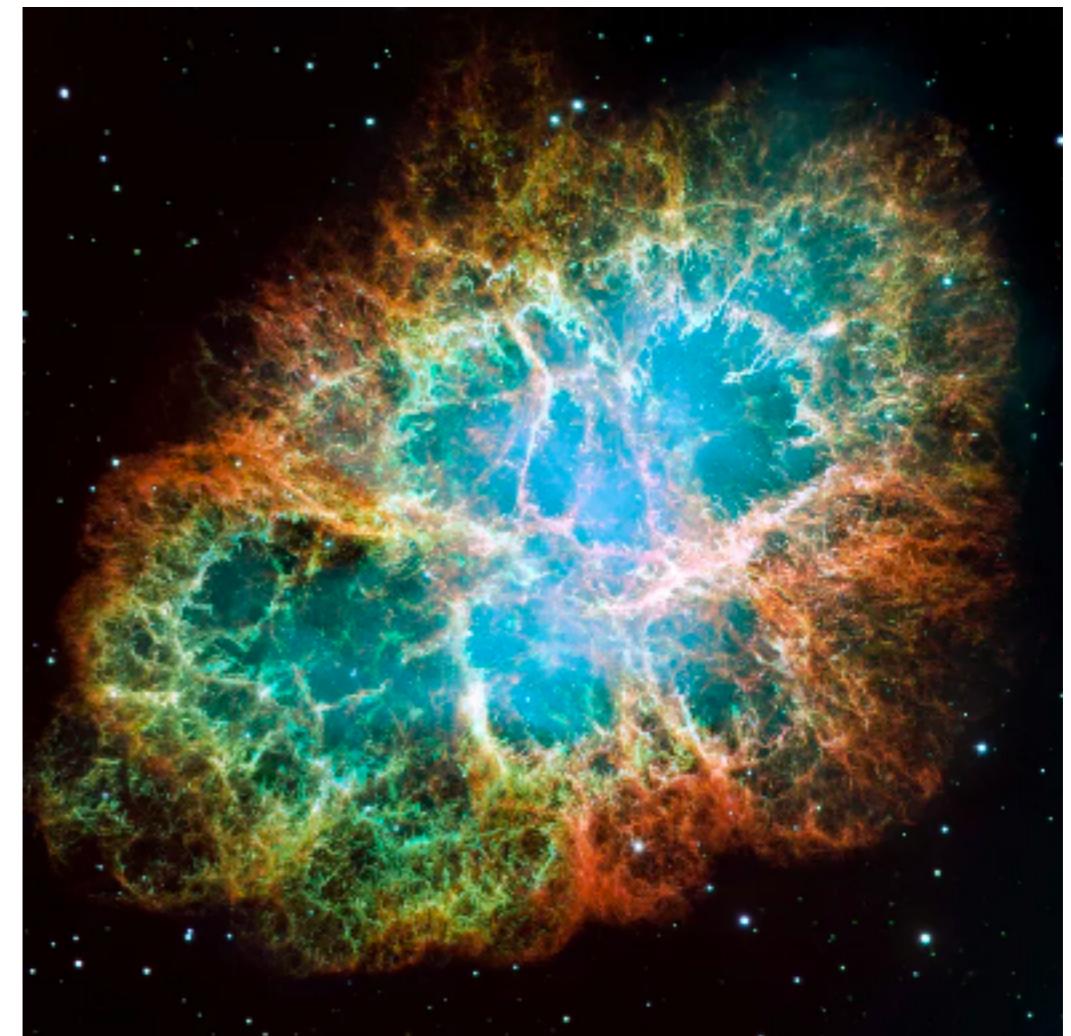
$$t = \sqrt{1/G\rho}$$

# Warm-up 2 : Order-of-magnitude physics

天文中的爆发现象



原子弹爆炸



超新星爆发

# Warm-up 2 : Order-of-magnitude physics

天文中的爆发现象

已知量：

能量  $E$

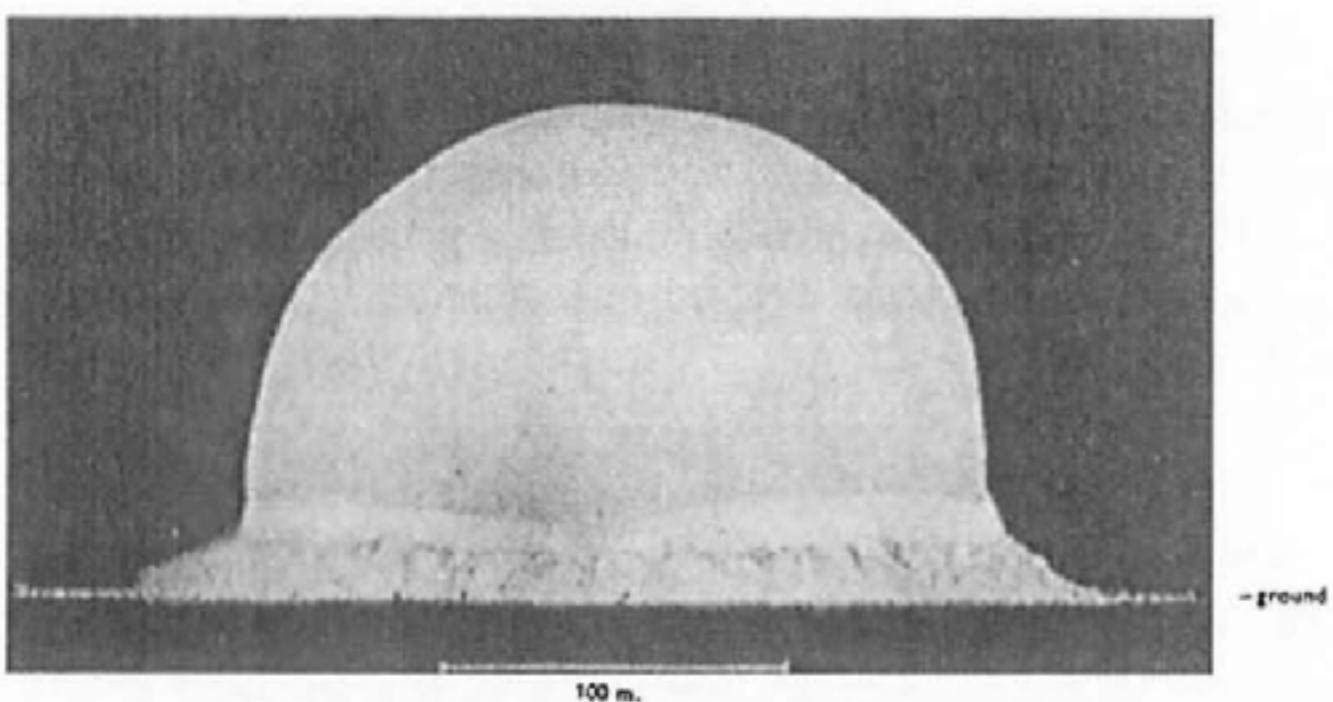
空气密度  $\rho$

时间  $t$

未知量：

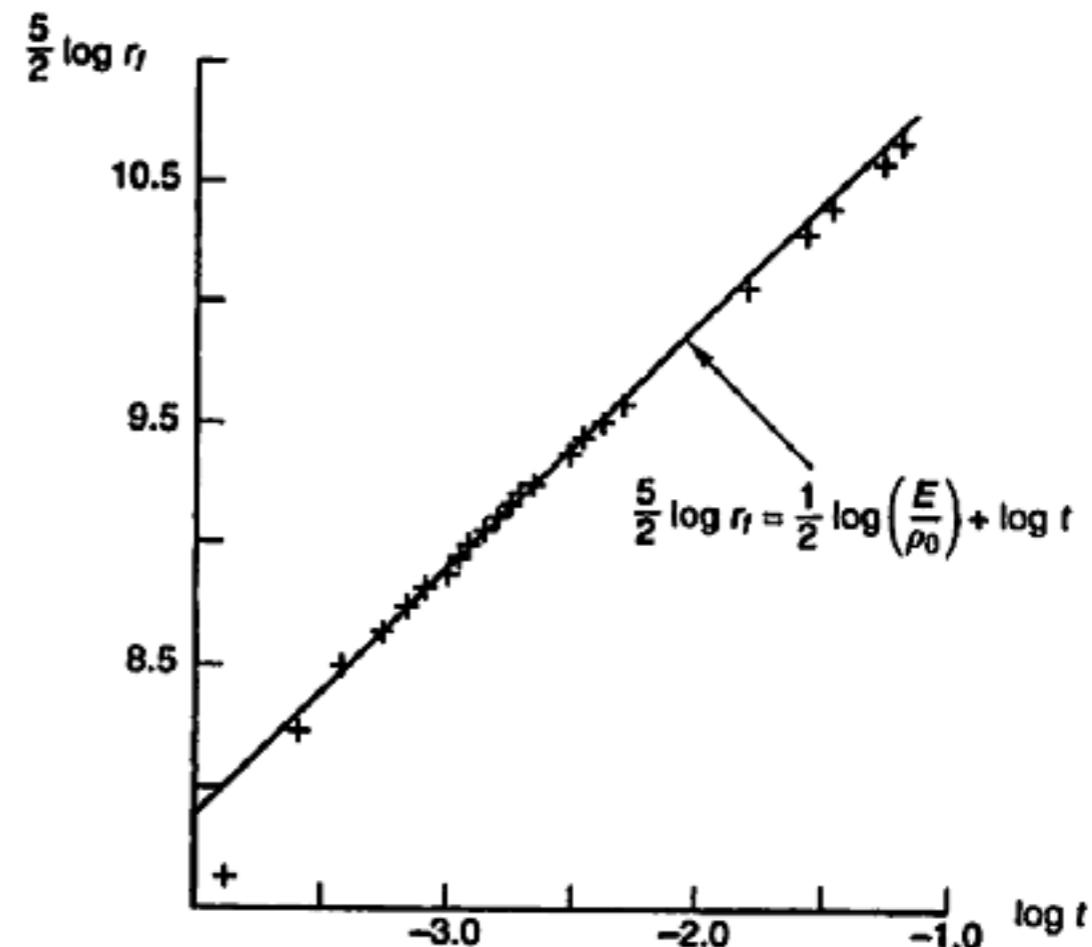
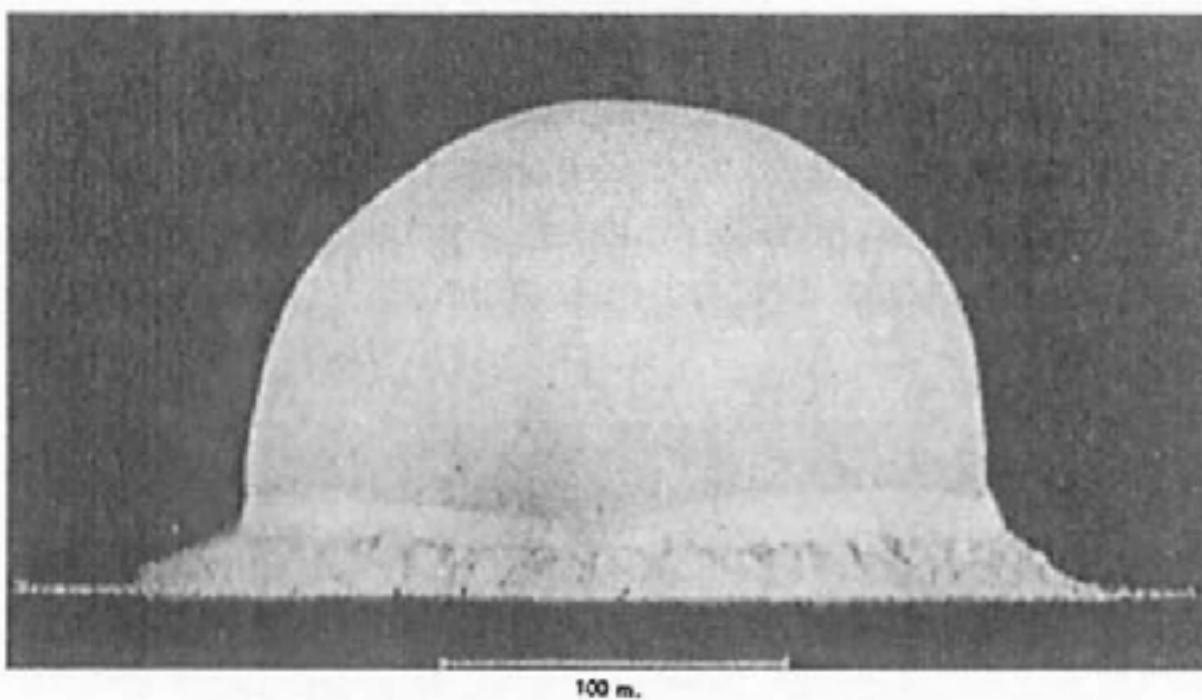
半径  $r$

原子弹爆炸



# Warm-up 2 : Order-of-magnitude physics

## 天文中的爆发现象



$$[E] = ML^2T^{-2}, \quad [t] = T, \quad [\rho_0] = ML^{-3},$$

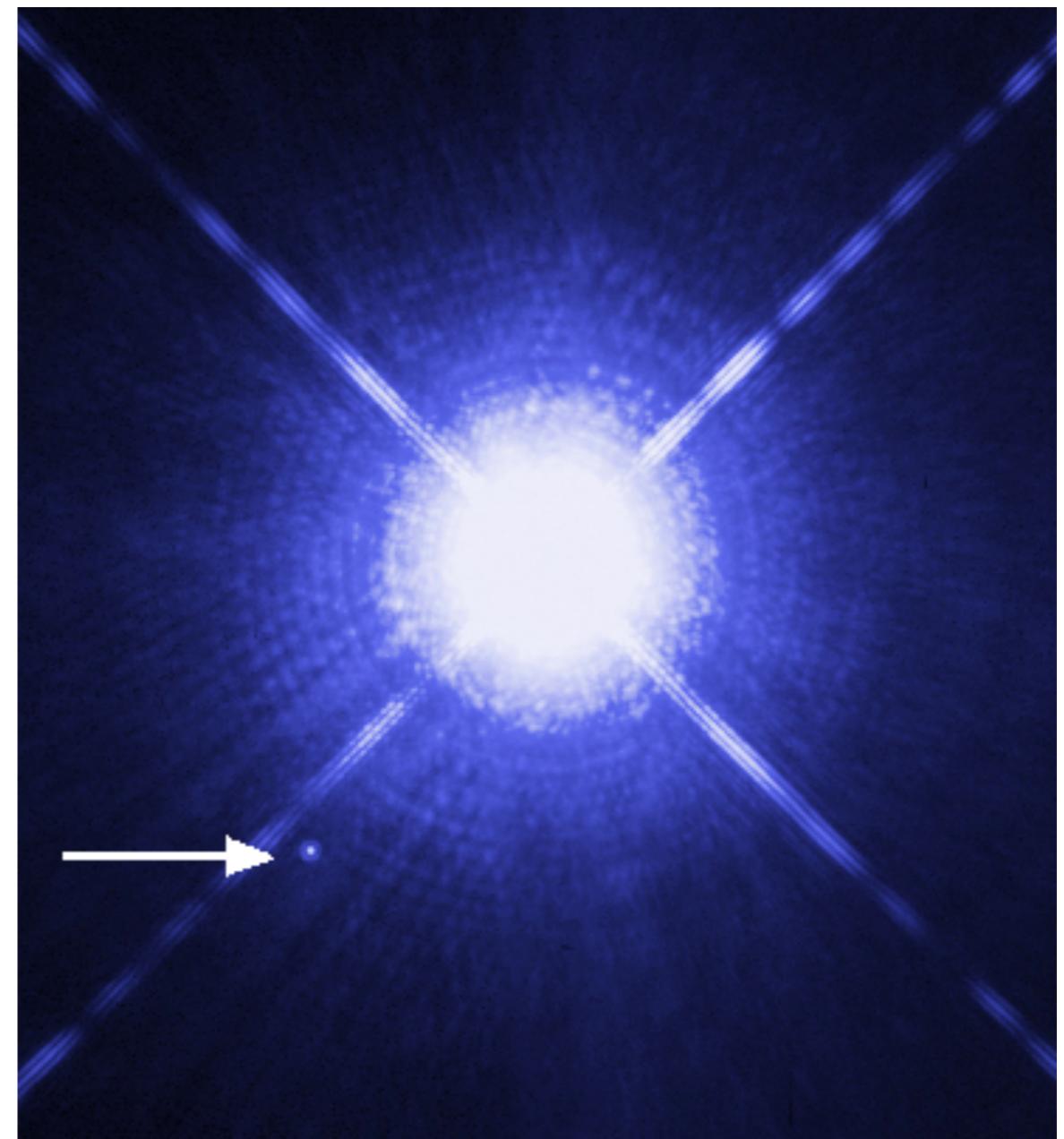
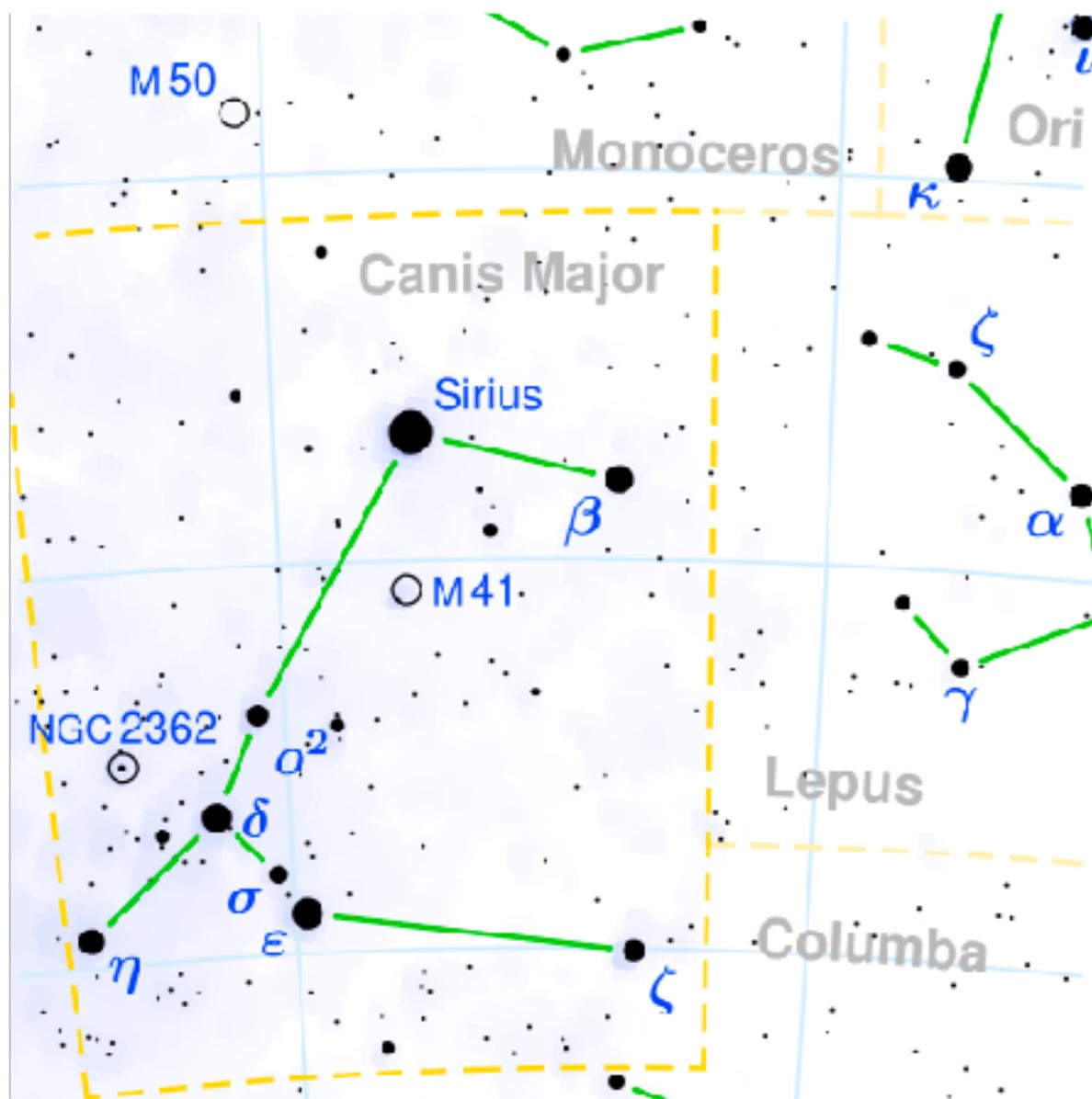
原子弹爆炸

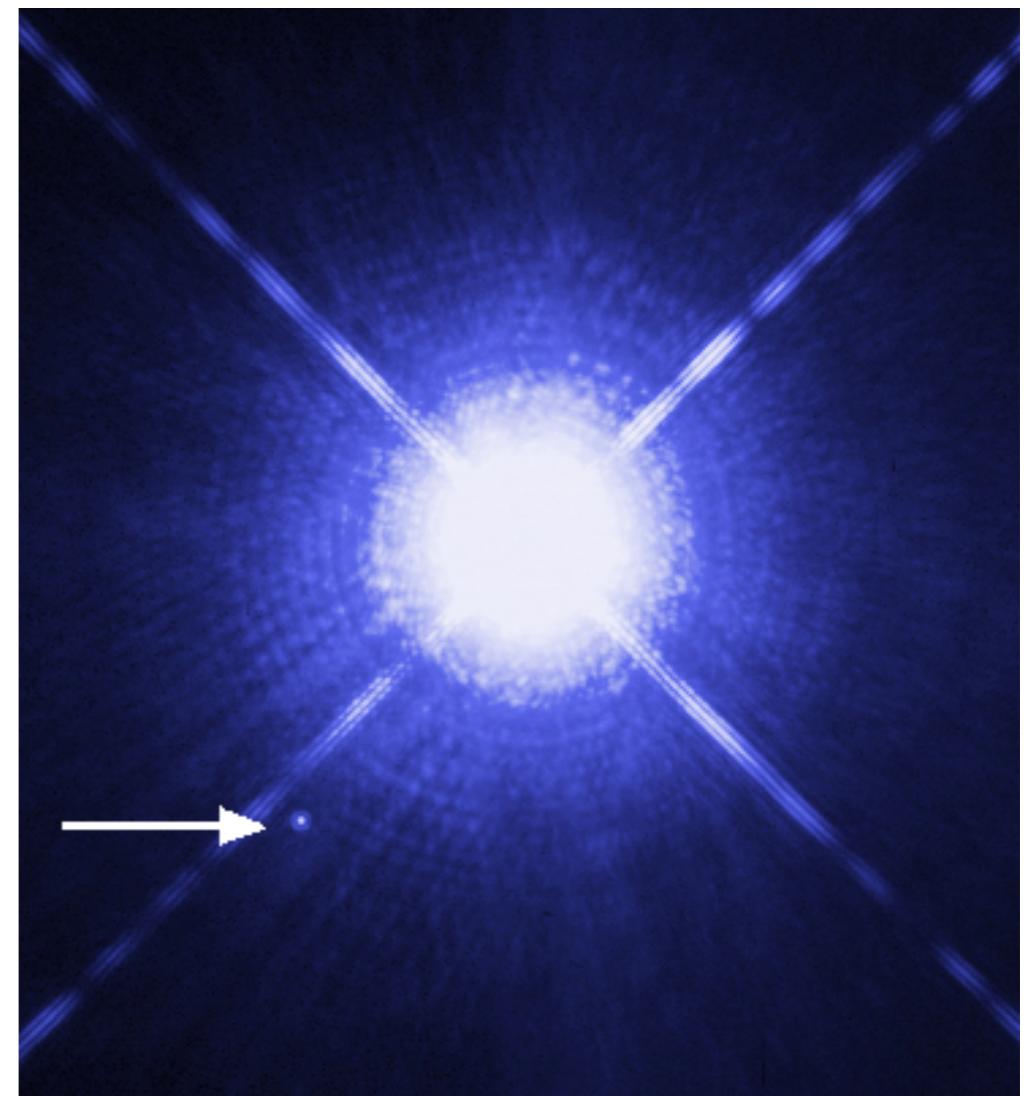
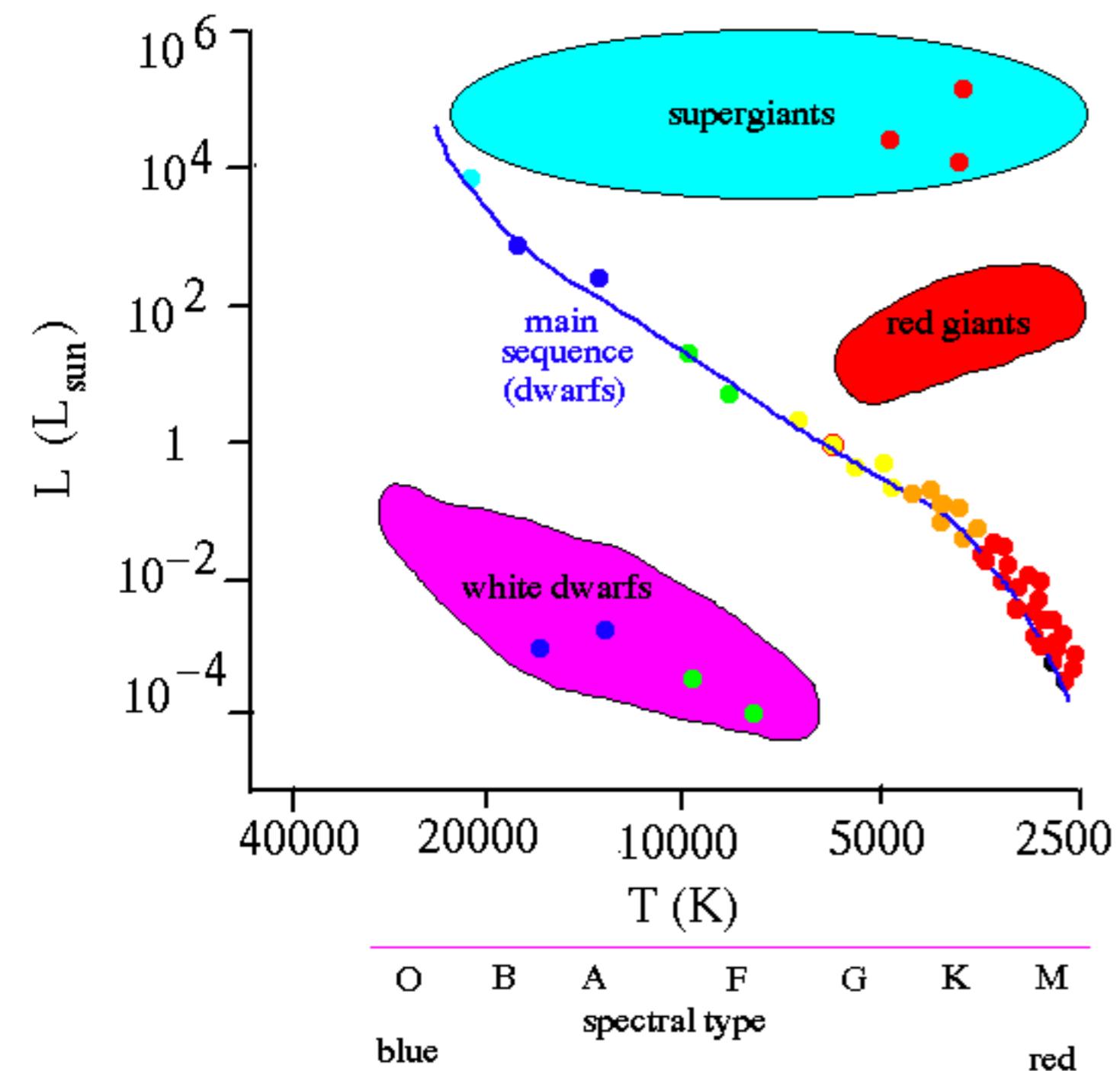
$$[r_f] = [E]^{1/5}[t]^{2/5}[\rho_0]^{-1/5}.$$

# Plan for today

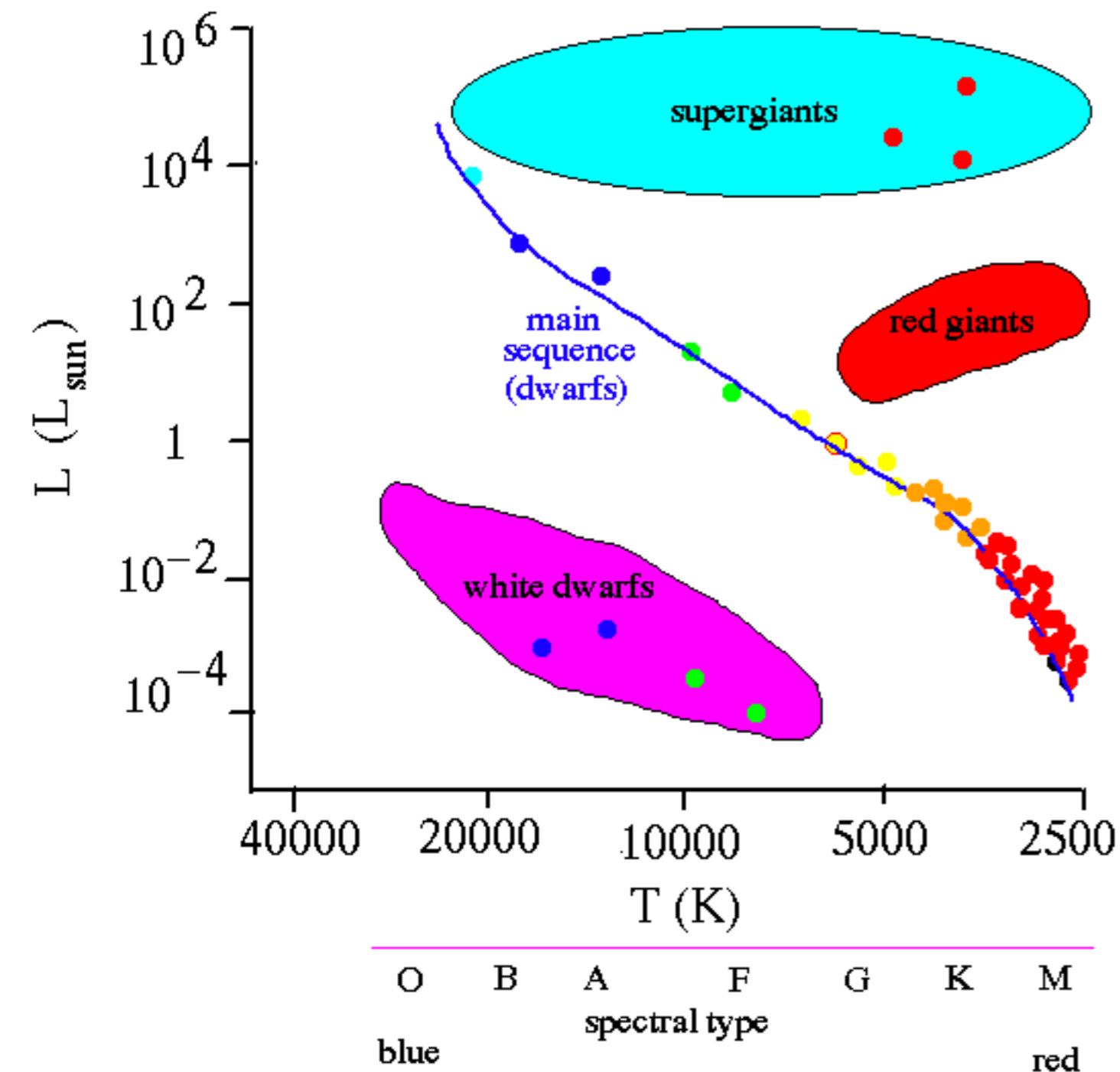
- White Dwarf 白矮星
- Neutron star 中子星 -> 夸克星
- 黑洞

# 白矮星





$$L_* \sim S_* T^4$$



$$L_* \sim S_* T^4$$

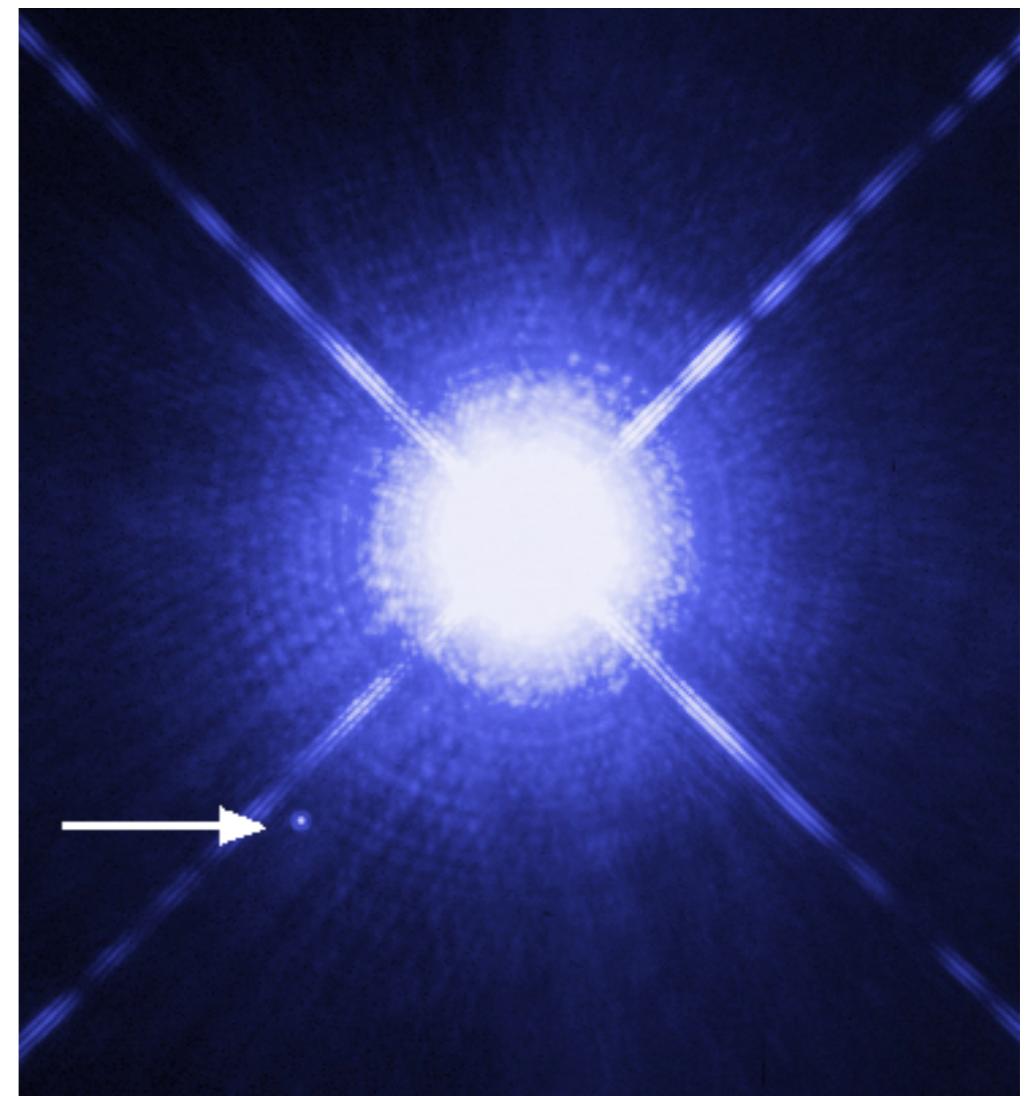
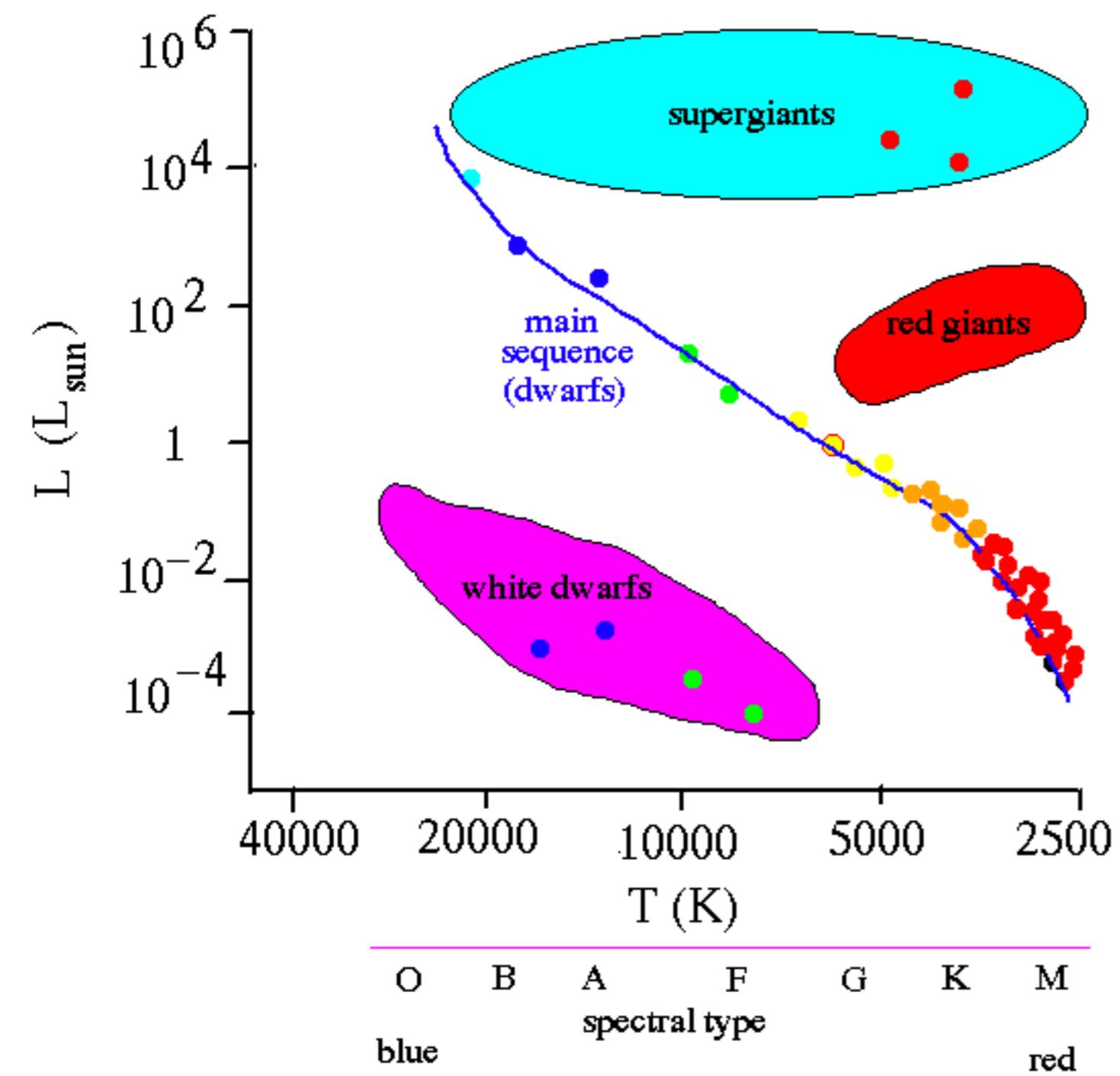
**L:** 总光度

**S:** 面积

**T:** 温度

**The sun:**  $7 \times 10^{10} \text{ cm}^2$

**White dwarf:**  $10^8 \text{ cm}^2$   
大小大约是太阳的百分之一

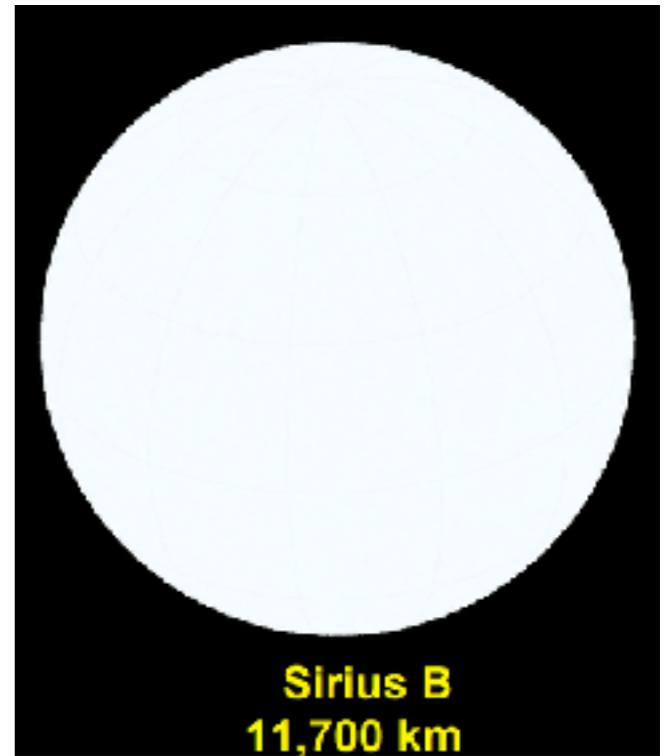


# 白矮星的密度

Let's work out the density

$$\rho = M/r^3 = 1M_{\odot}/(10^4 \text{ km})^3 = ?$$

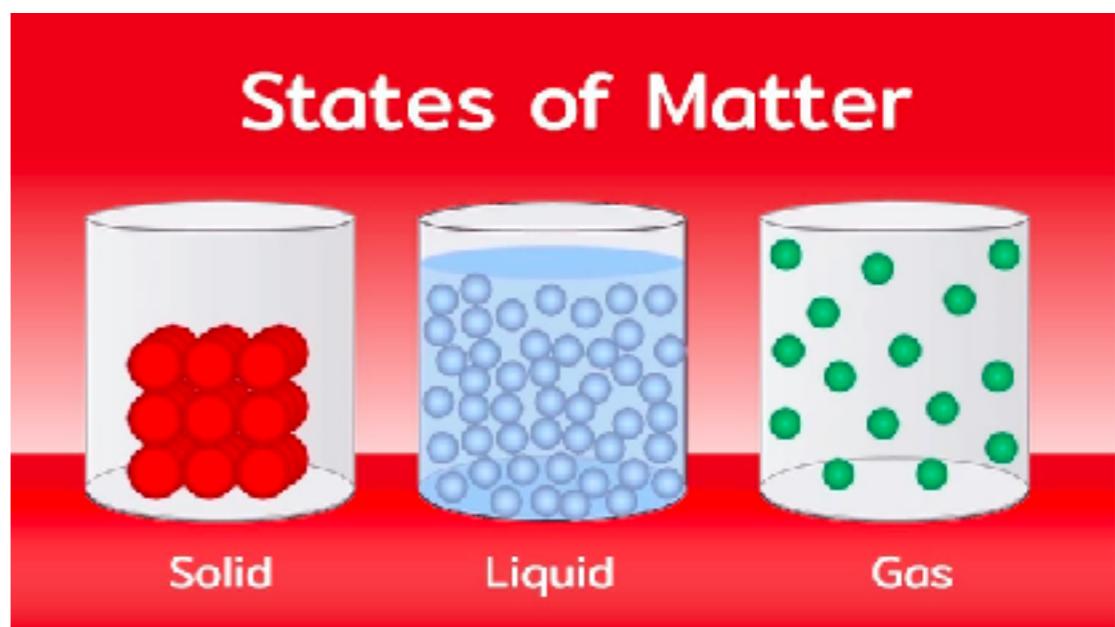
远大于普通物质的密度 (1-10 g cm<sup>-3</sup>) !



远大于原子的密度 -> 原子没法存在

气体可以压缩：原子间有间隙

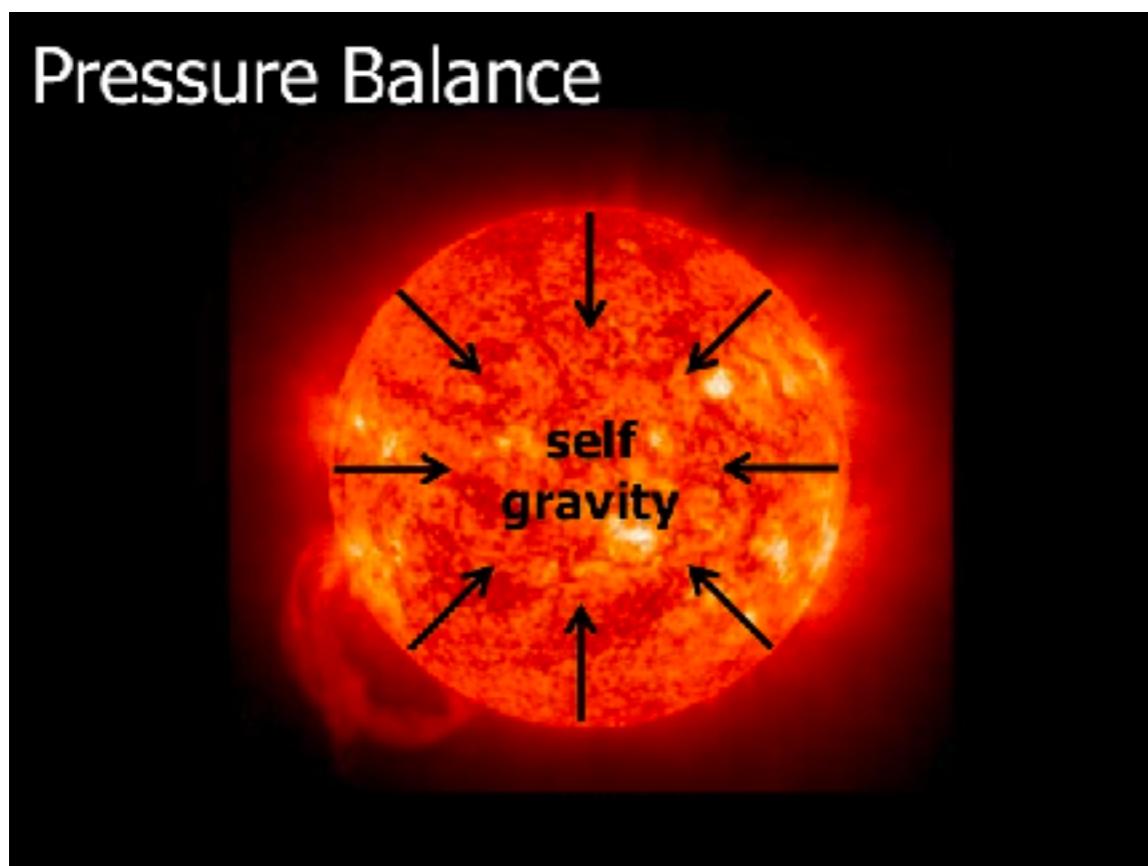
液体，固体：原子间没有间隙



# 白矮星是如何压碎原子的

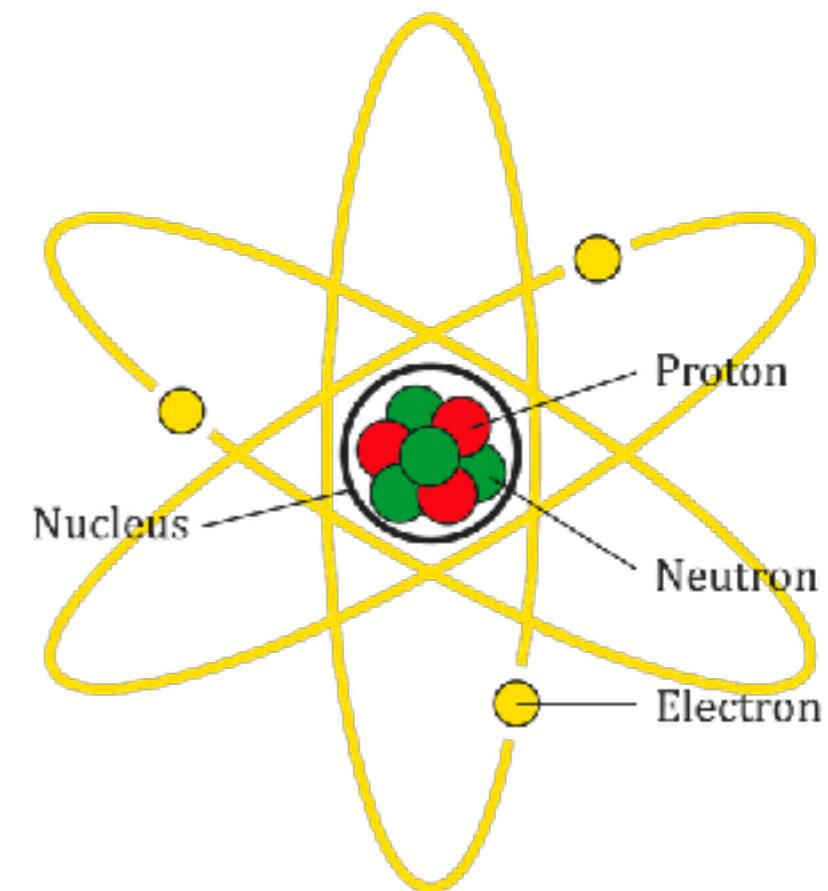
白矮星内部的压力

$$P = F/S$$



原子内部的压力

$$P = E/V$$



压碎原子：白矮星的压力 > 原子内部的压力

# 白矮星是如何压碎原子的

## Gravitational Pressure

$$P = F/S$$

**1 Msun = 2e33 gram**

**G = 6.67e18**

**r = 1e6 cm**

**p=?**

## Pressure inside an atom

$$P = E/V$$

**Binding Energy: ~10 eV**

1	=	1.60218e-12
Electronvolt	↔	Erg

**Size: 1e-8 cm**

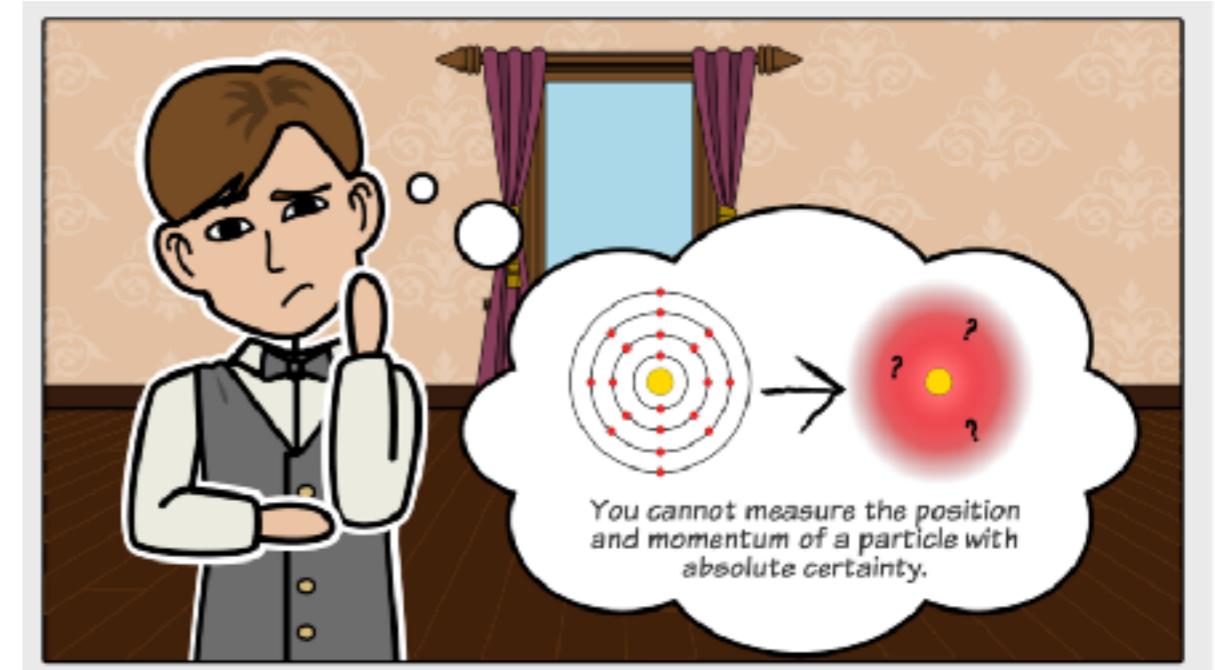
# 由电子支撑的白矮星

Fermi Energy (费米能, 简并能)

$$\delta_x = n^{-1/3} \quad (\text{n 数密度})$$

$$\delta x \delta p \approx h \quad (\text{测不准原理})$$

$$E_{\text{kin}} = p^2/2m \quad (\text{能量})$$



Compare to gas pressure

$$P_{\text{gas}} = nk_{\text{B}}T$$

$$E_k \approx \frac{N(\Delta p)^2}{2m} \approx \frac{N\hbar^2 n^{2/3}}{2m} \approx \frac{M^{2/3} N^{5/3} \hbar^2}{2mR^2}.$$

# 简并支撑作用

$$dx = (1/n)^{1/3}$$

密度 (数格子)

$$dp = h/dx$$

测不准原理

$$e = p^2/2m$$

能量

$$p = E/V = e/v$$

压力

# 质量半径关系

## Gravitational Energy

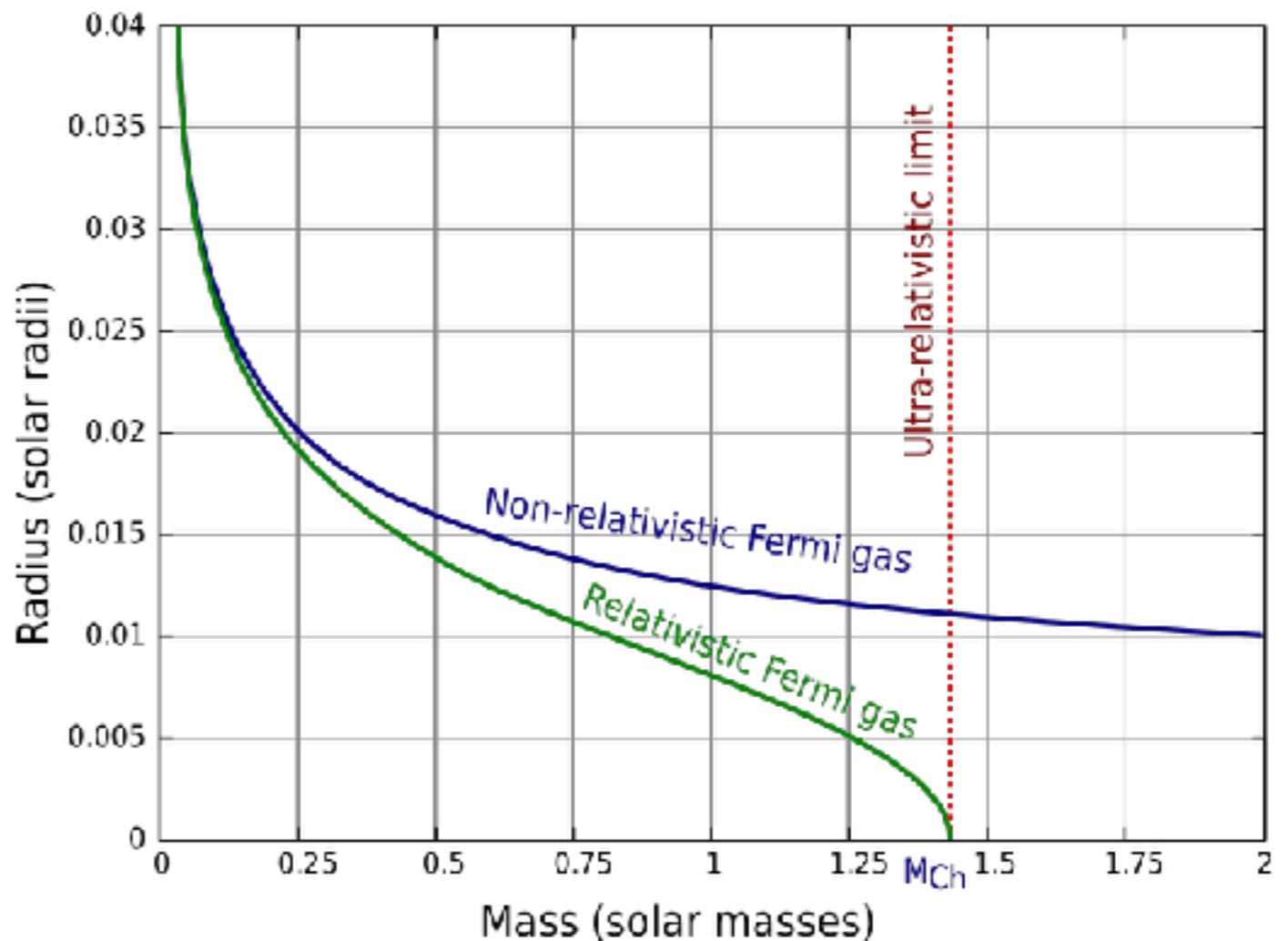
$$E_g \approx \frac{-GM}{R}.$$

## Kinetic Energy

$$E_k \approx \frac{N(\Delta p)^2}{2m} \approx \frac{N\hbar^2 n^{2/3}}{2m} \approx \frac{M^{2/3} N^{5/3} \hbar^2}{2m R^2}.$$

## Solve the radius

$$R \approx \frac{N^{5/3} \hbar^2}{2m G M^{1/3}}$$



# 白矮星极限质量

Chandrasekhar 1943:



$$M_{\text{limit}} = \frac{\omega_3^0 \sqrt{3\pi}}{2} \left( \frac{\hbar c}{G} \right)^{\frac{3}{2}} \frac{1}{(\mu_e m_H)^2} \approx 1.4 m_\odot$$

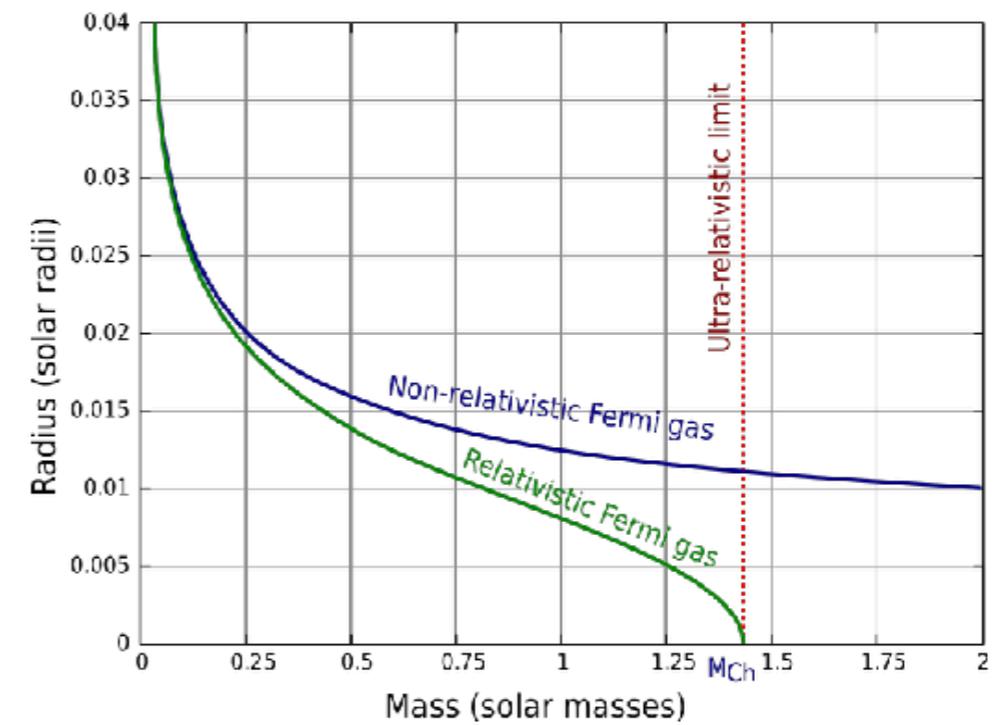
白矮星质量上限（钱德拉塞卡极限）

No-relativistic

$$E = p^2/2m$$

Relativistic

$$E = c * p$$



# 白矮星极限质量

Chandrasekhar 1943:

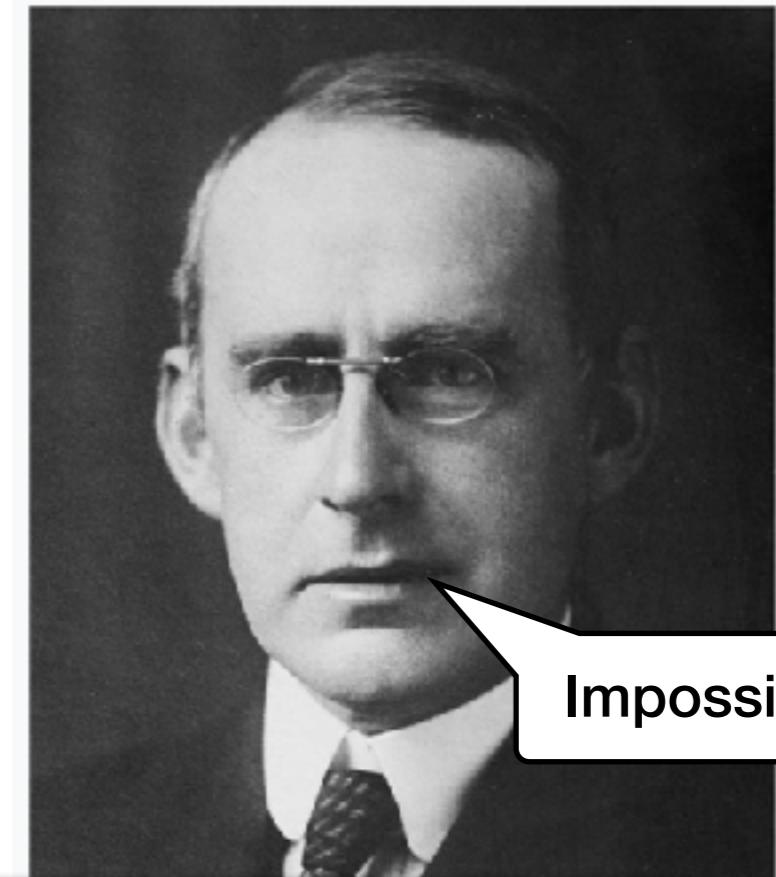


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白矮星质量上限 (钱德拉塞卡极限)

Uncle

Sir Arthur Eddington



Sir Chandrashekara Venkata Raman  
FRS Kt



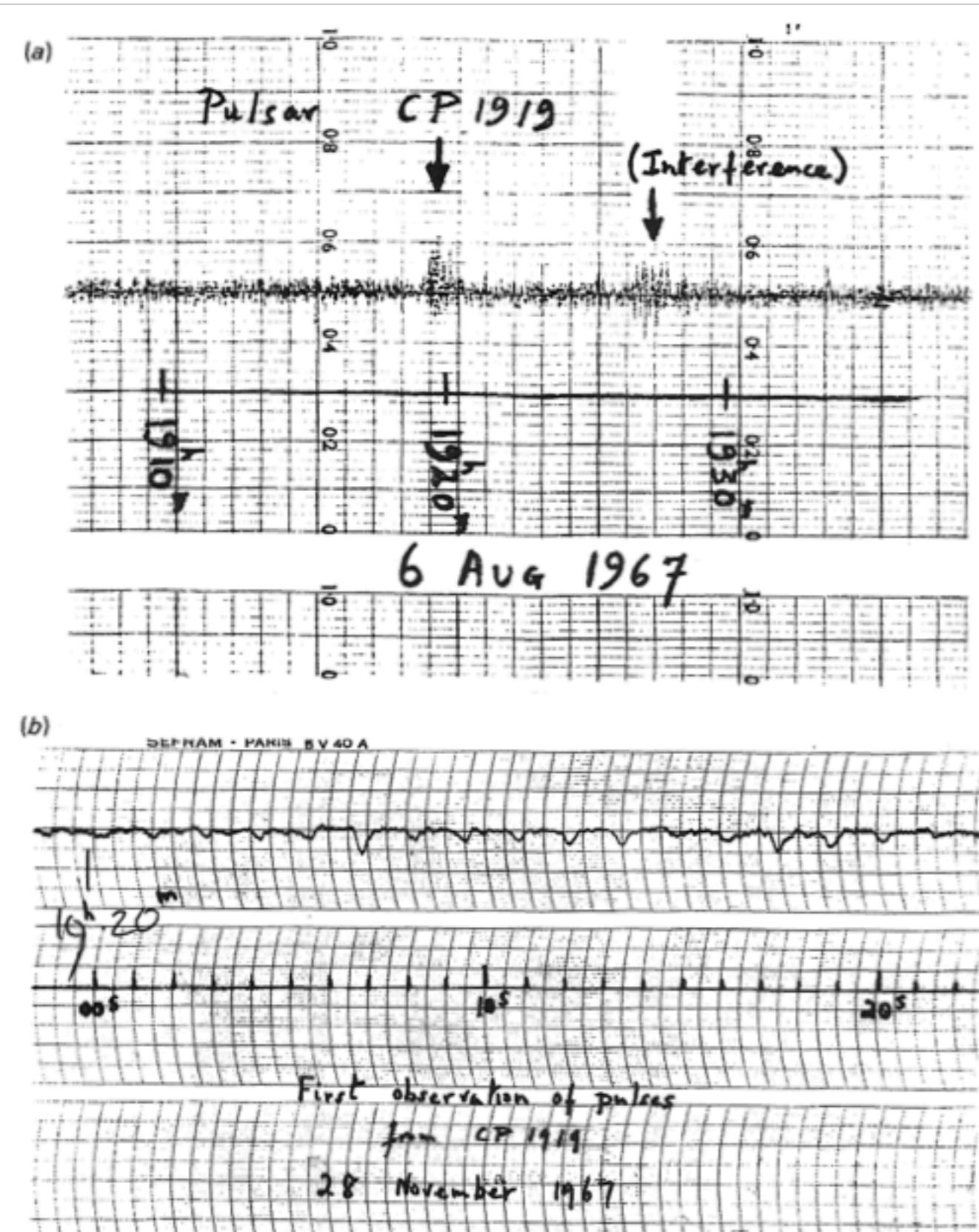
# 白矮星：总结

- 白矮星的观测：光学观测
- 形成：恒星塌缩
- 特点：光度小，温度高
- 物理：密度高，引力强，原子没法存在，靠电子简并压支撑（测不准原理）
- $1.4 m_{\text{sun}}$  质量上限（钱德拉塞卡极限） $\rightarrow$  超新星爆发

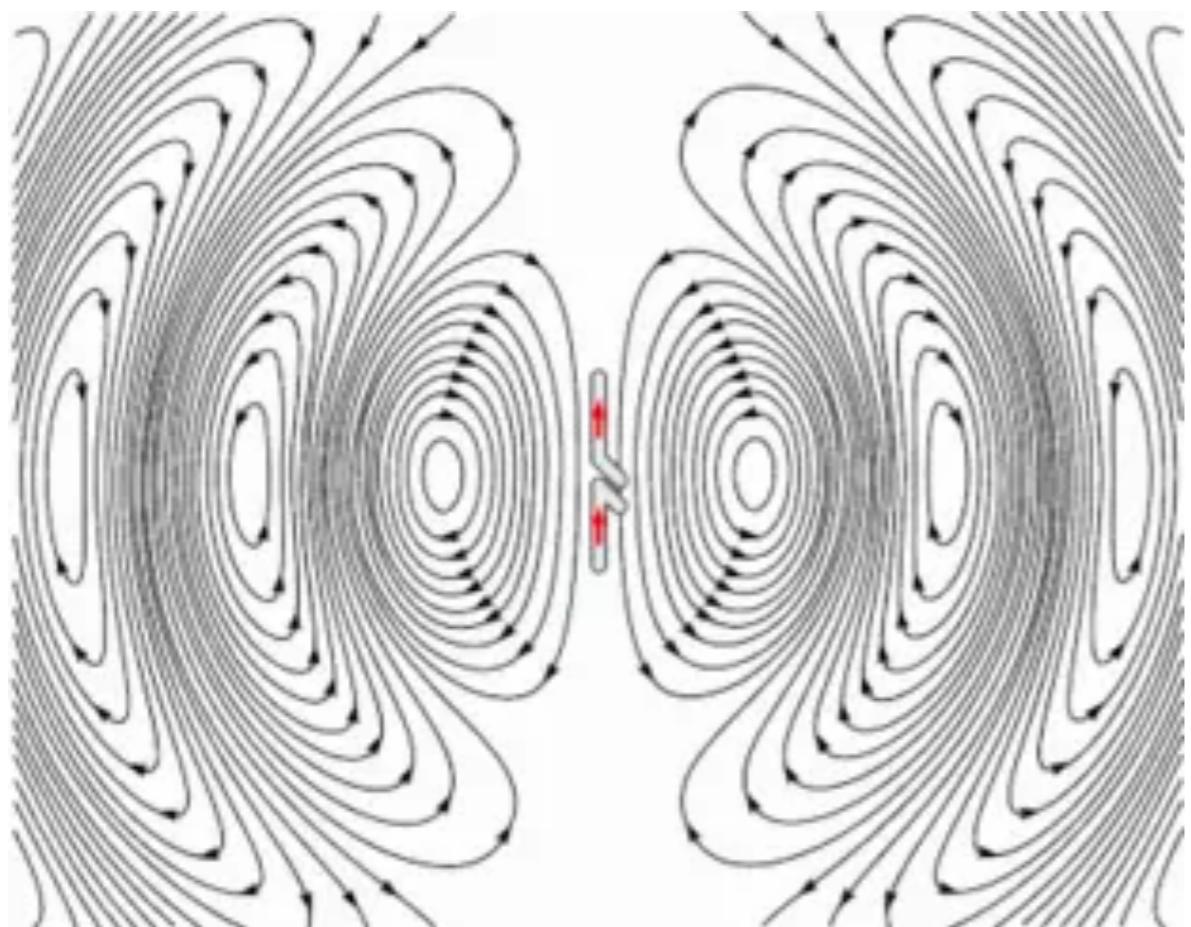
**射电脉冲星，中子星，夸克星**

# 脉冲星的发现

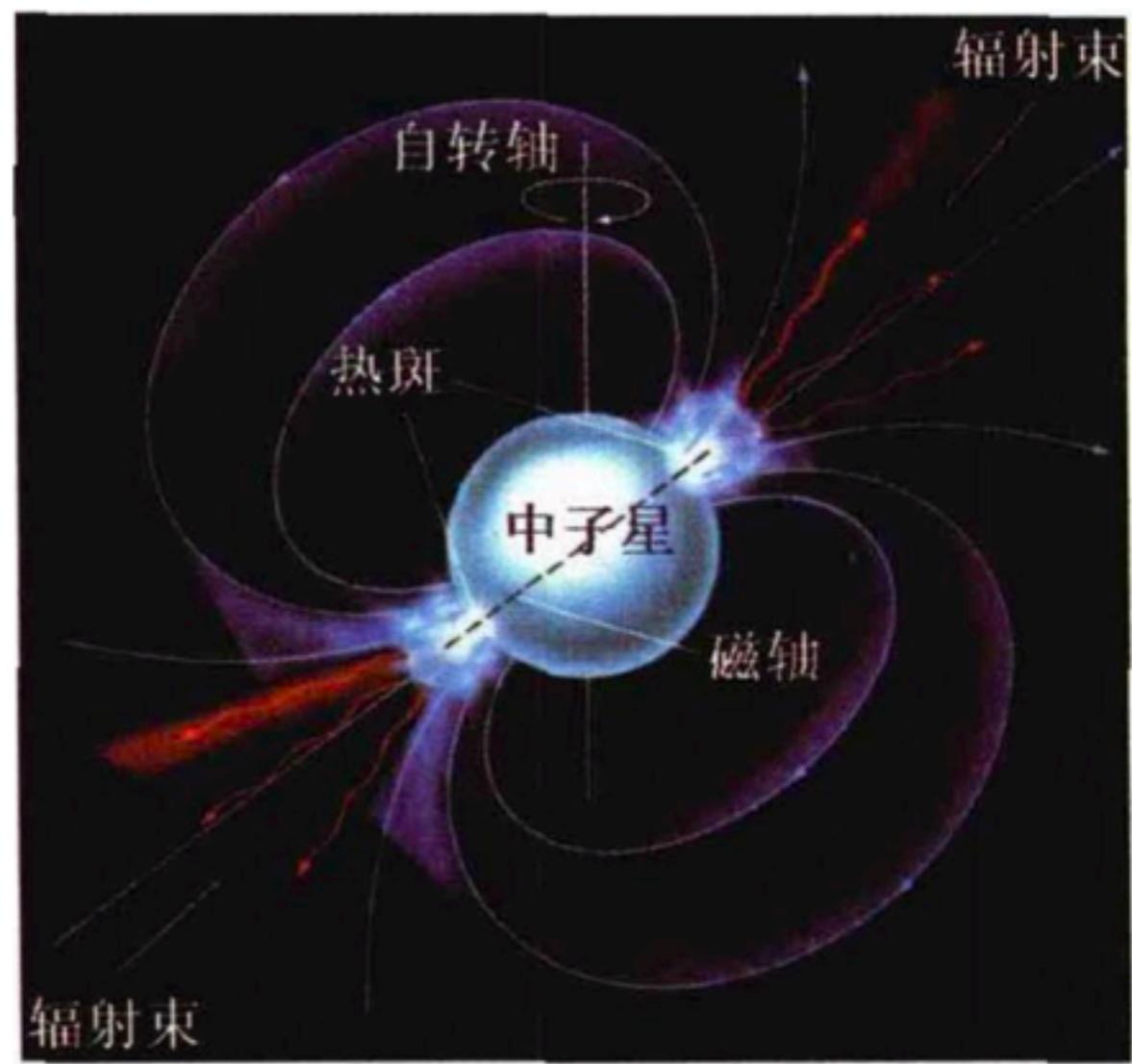
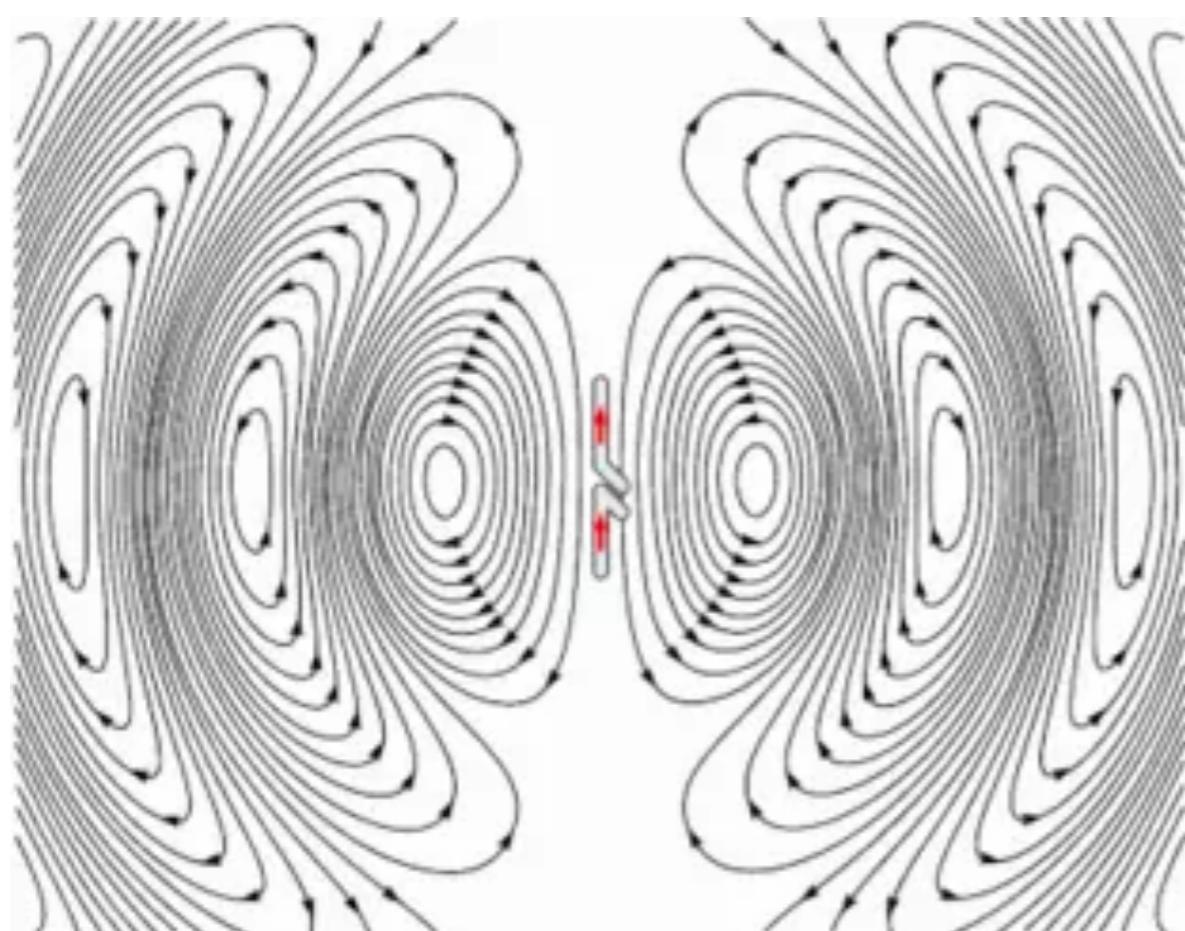
周期：毫秒到秒



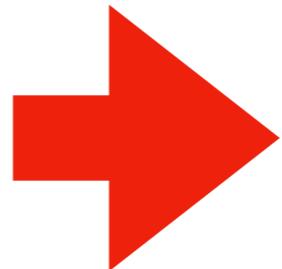
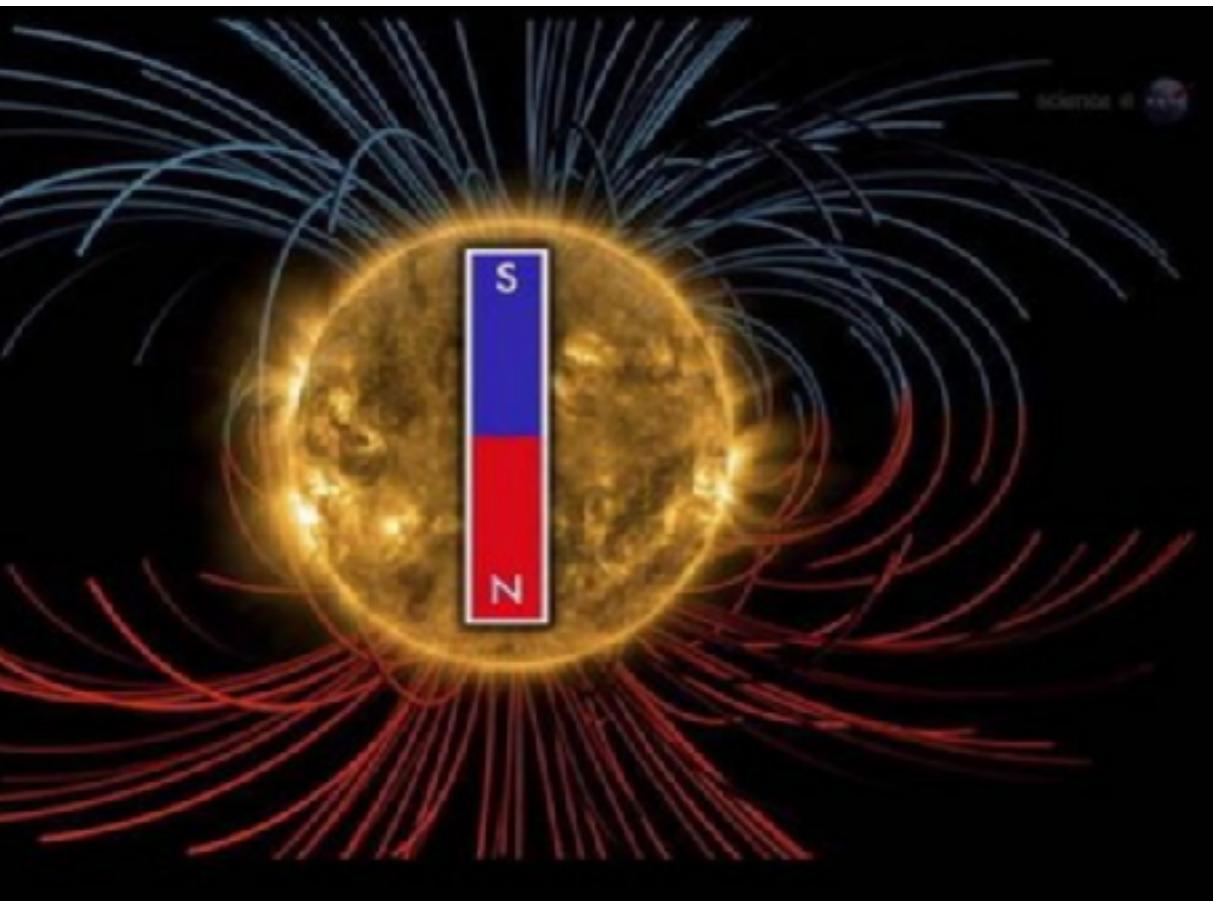
# 脉冲星的射电辐射



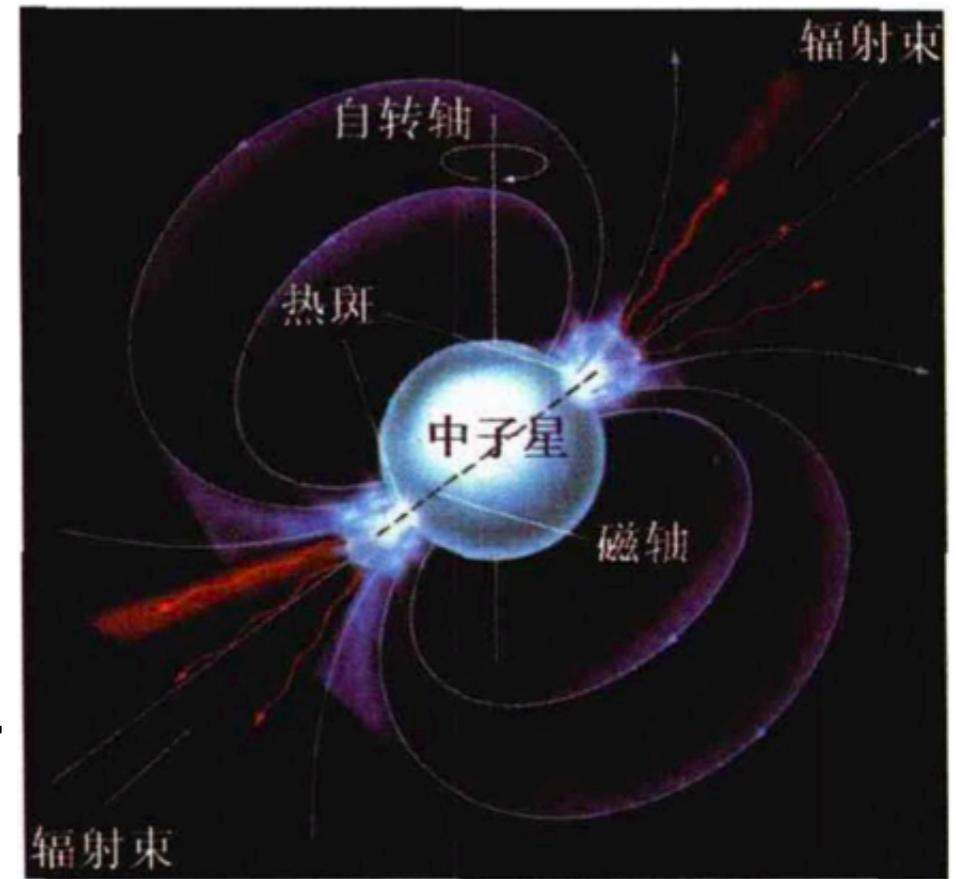
# 脉冲星的射电辐射：磁场+转动



# 脉冲星磁场的来源

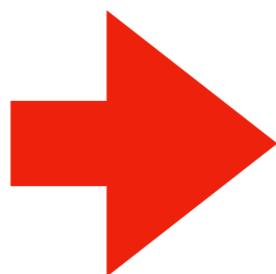


半径缩小  $10^4$



磁通量守恒:

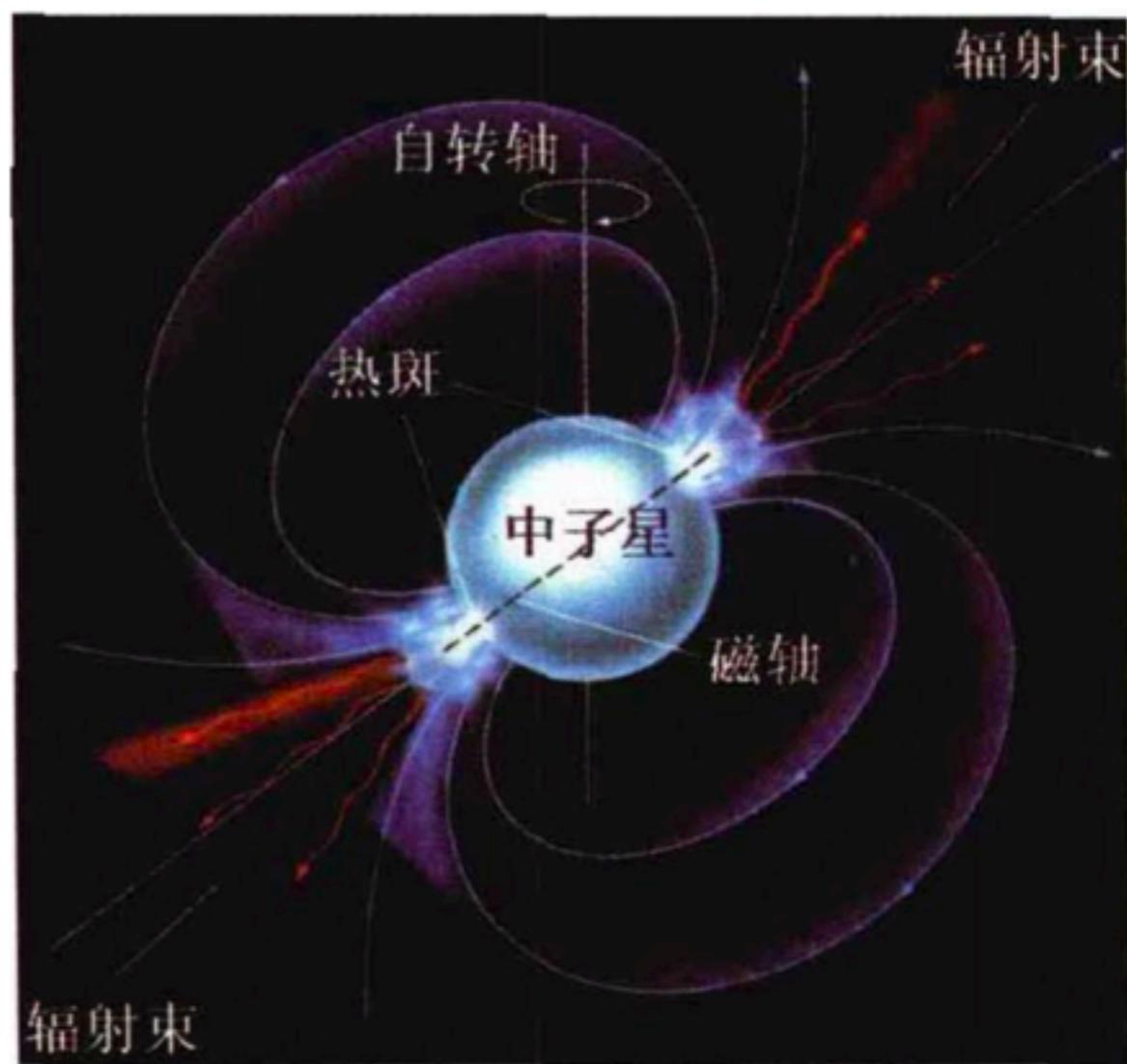
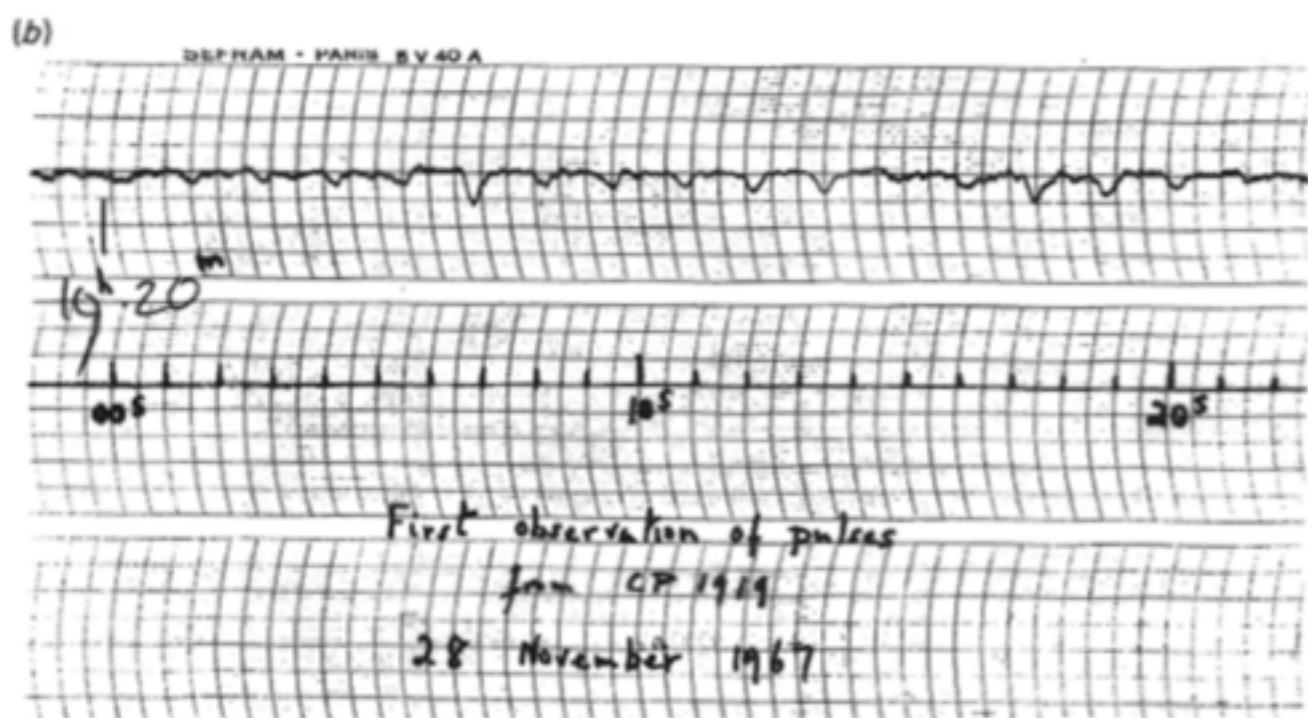
$$\phi = Br^2 = \text{constant}$$



$$B \sim r^{-2}$$

# 脉冲星的密度

中子星的周期在秒到毫秒之间，  
问密度大约有多少？



# 脉冲星的密度下限

中子星不因转动而破碎：

$$\Omega^2 r = Gm/r^2 \rightarrow t \approx 1/\Omega \approx \sqrt{1/G\rho}$$

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$$t = 1 \text{ ms} \rightarrow \rho_0 = 10^{13} \text{ g cm}^{-3}$$

$$t = 1 \text{ s} \rightarrow \rho_0 = 10^7 \text{ g cm}^{-3}$$

$\rho > \rho_0$       远大于白矮星的平均密度 ( $10^7$ )  
->只可能是新的星体-> 中子星?

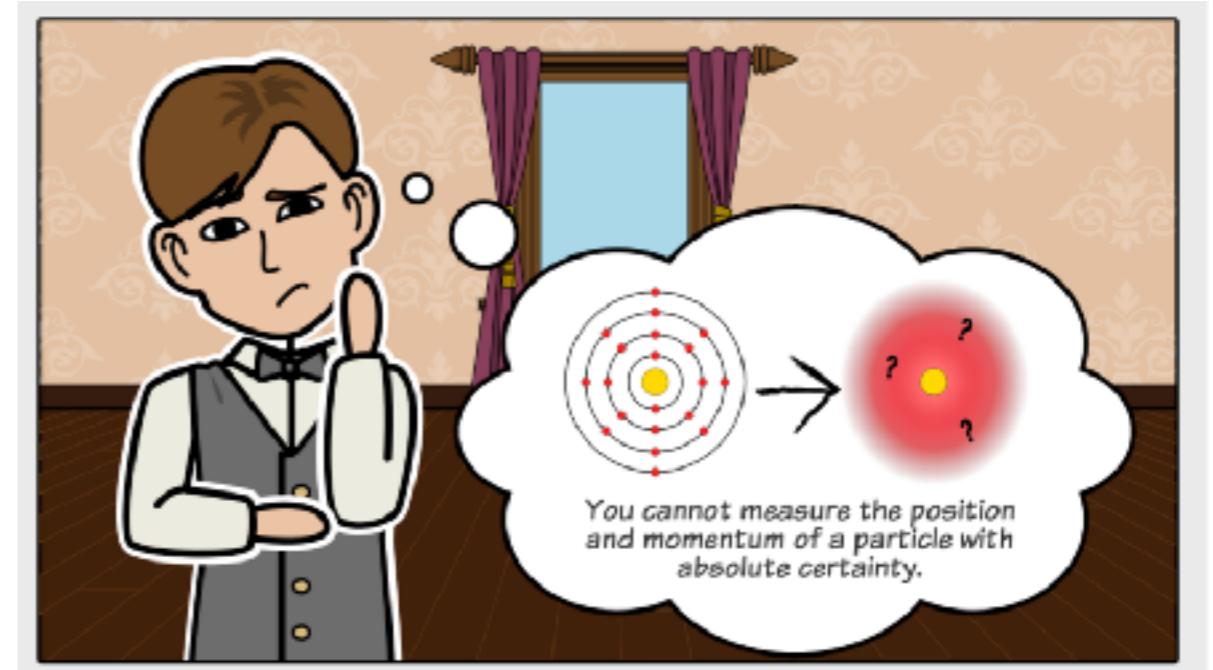
# 由电子支撑的白矮星

Fermi Energy (费米能, 简并能)

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Compare to gas pressure

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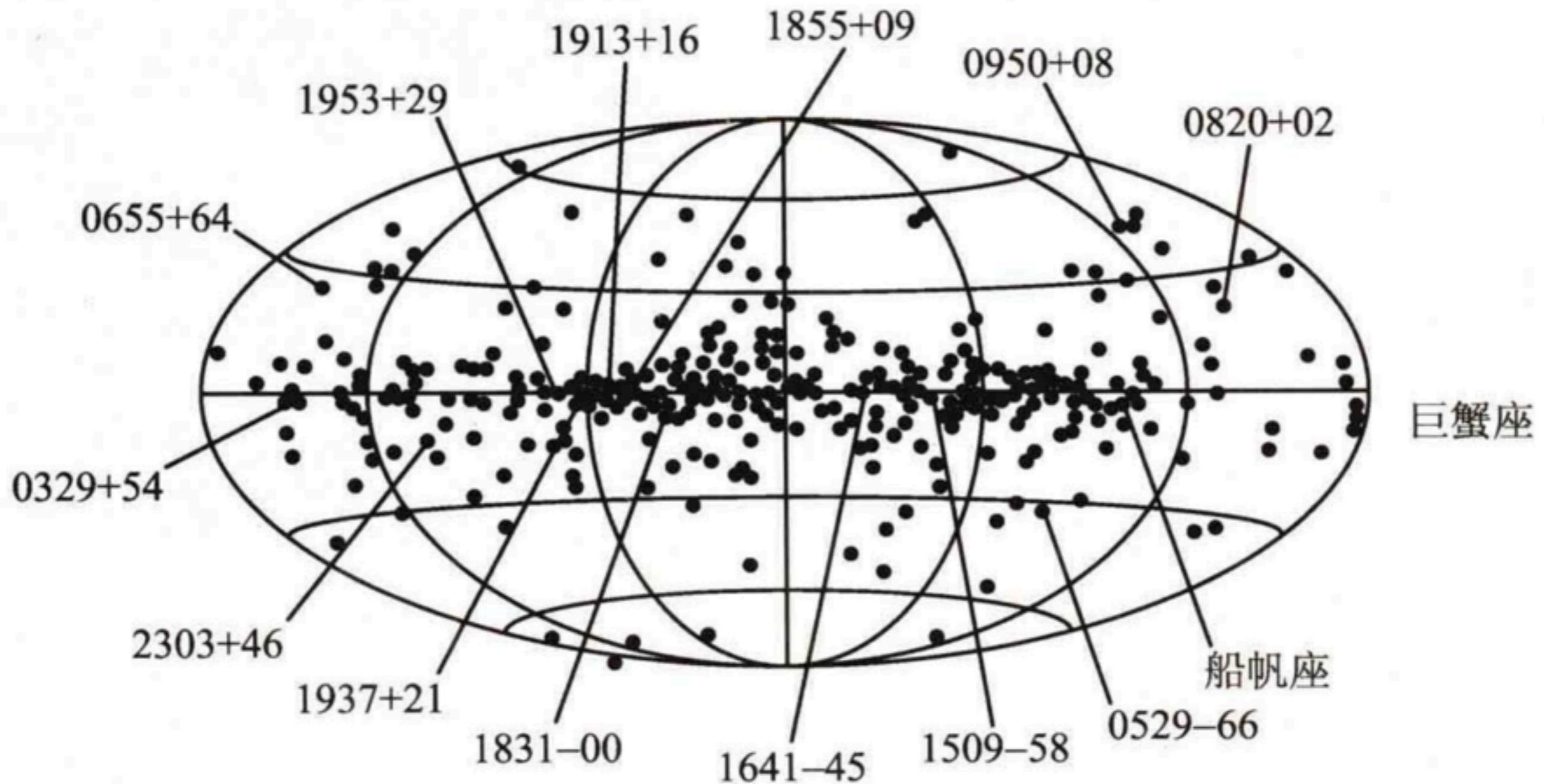


图 4.5 脉冲星在银道坐标系内的空间分布。它们主要集中在银道面附近,说明它们是银河系内的天体

# Observation of a Rapidly Pulsating Radio Source

by

A. HEWISH

S. J. BELL

J. D. H. PILKINGTON

P. F. SCOTT

R. A. COLLINS

Unusual signals from pulsating radio sources have been recorded at the Mullard Radio Astronomy Observatory. The radiation seems to come from local objects within the galaxy, and may be associated with oscillations of white dwarf or neutron stars.

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## The Nobel Prize in Physics 1974

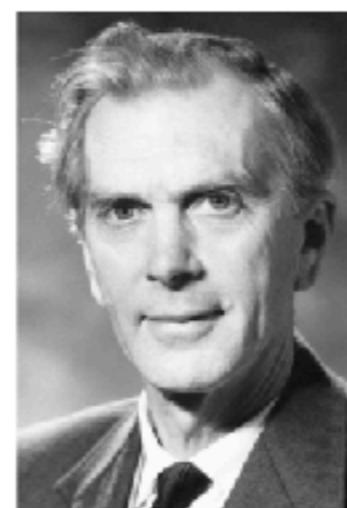


Photo from the Nobel Foundation archive.

Sir Martin Ryle

Prize share: 1/2



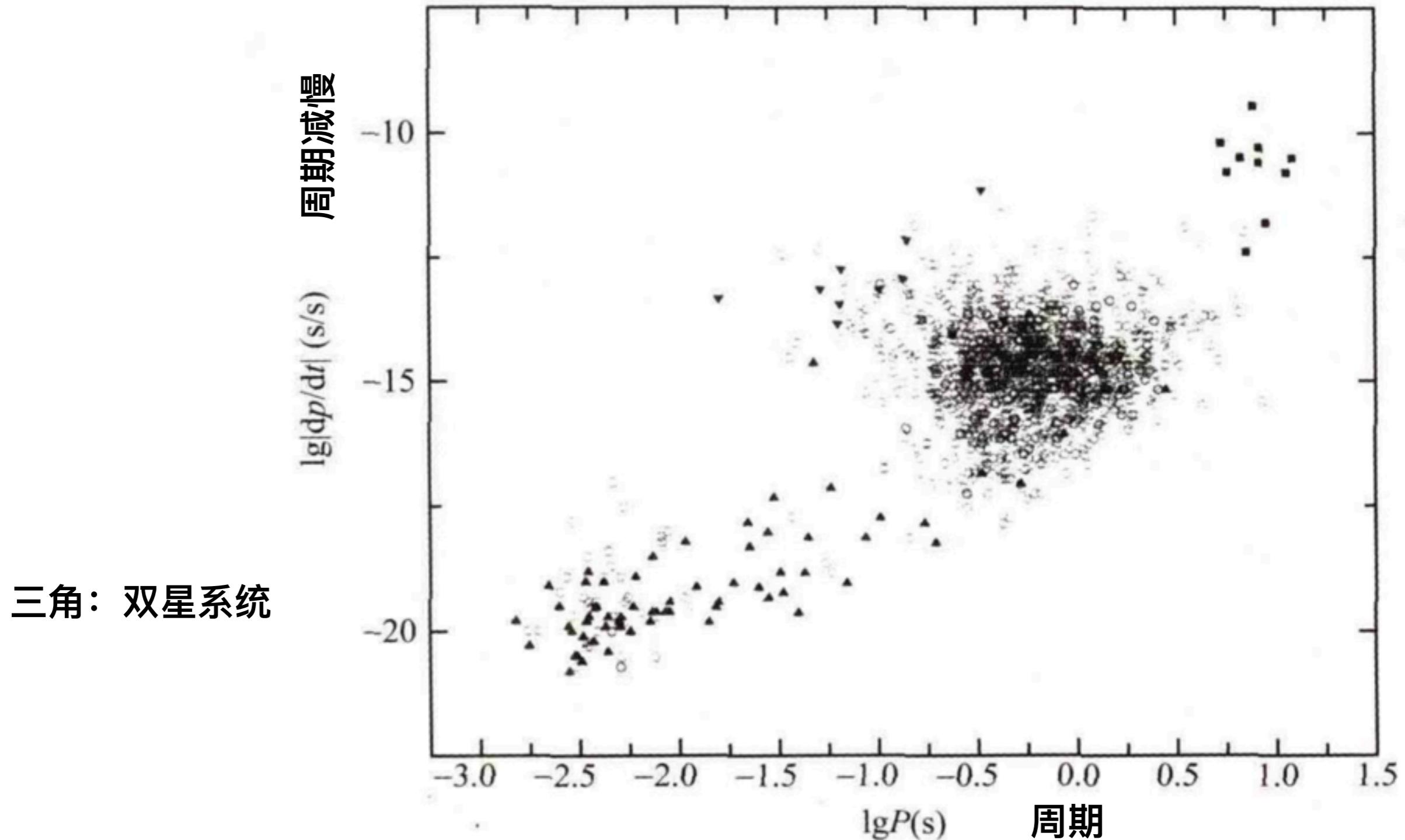
Photo from the Nobel Foundation archive.

Antony Hewish

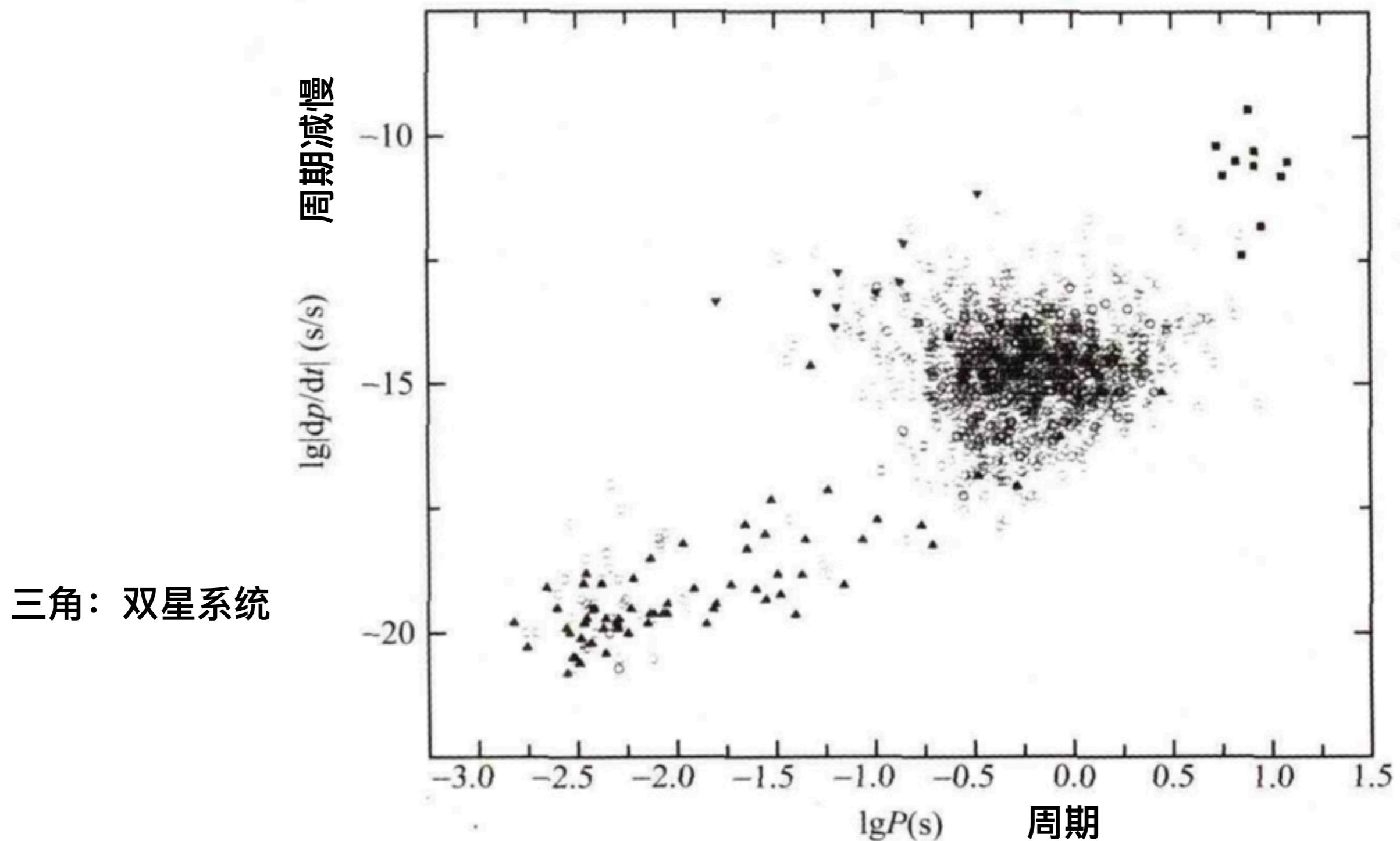
Prize share: 1/2

The Nobel Prize in Physics 1974 was awarded jointly to Sir Martin Ryle and Antony Hewish "for their pioneering research in radio astrophysics: Ryle for his observations and inventions, in particular of the aperture synthesis technique, and Hewish for his decisive role in the discovery of pulsars."

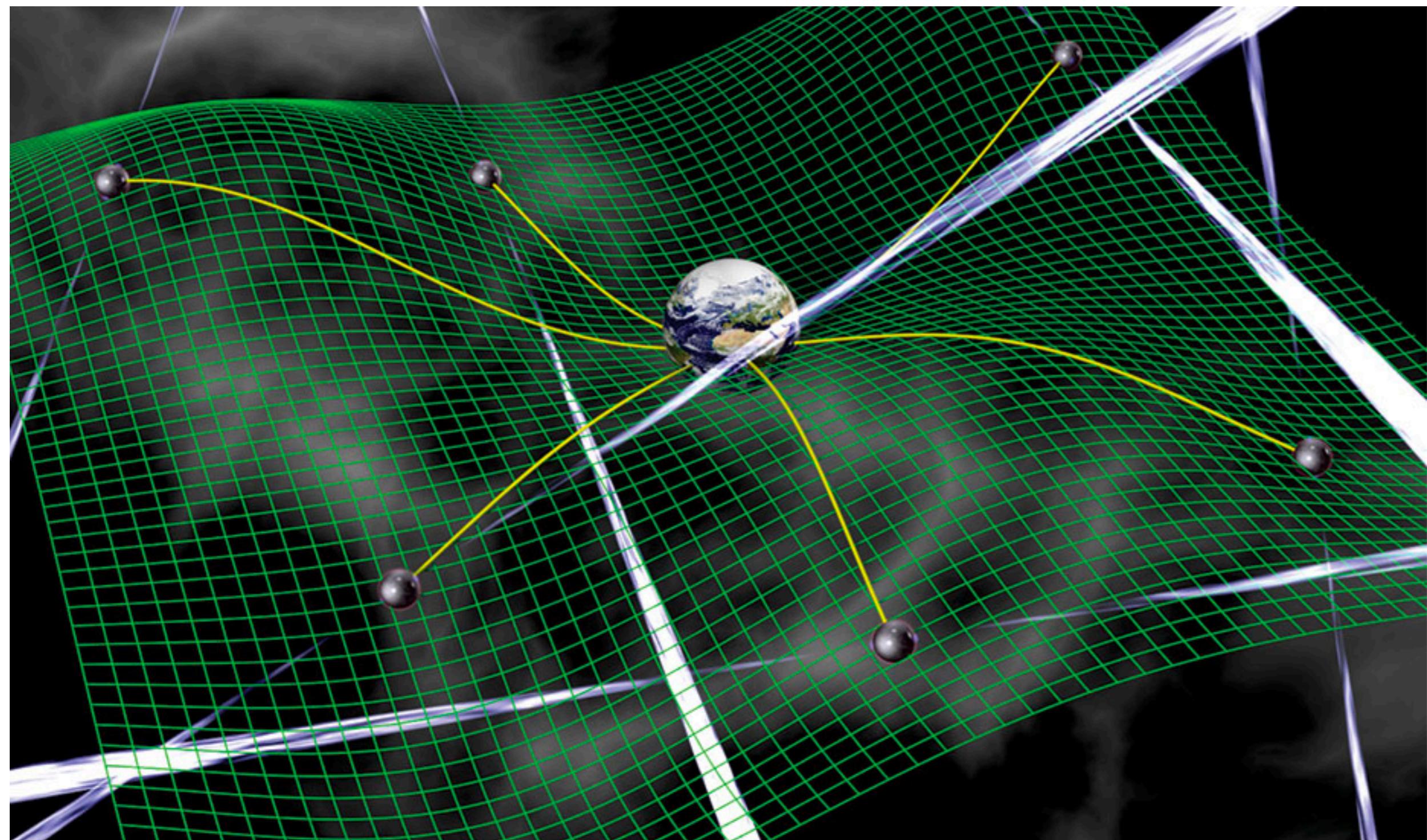
# 脉冲星的周期分布



# 脉冲星的周期分布

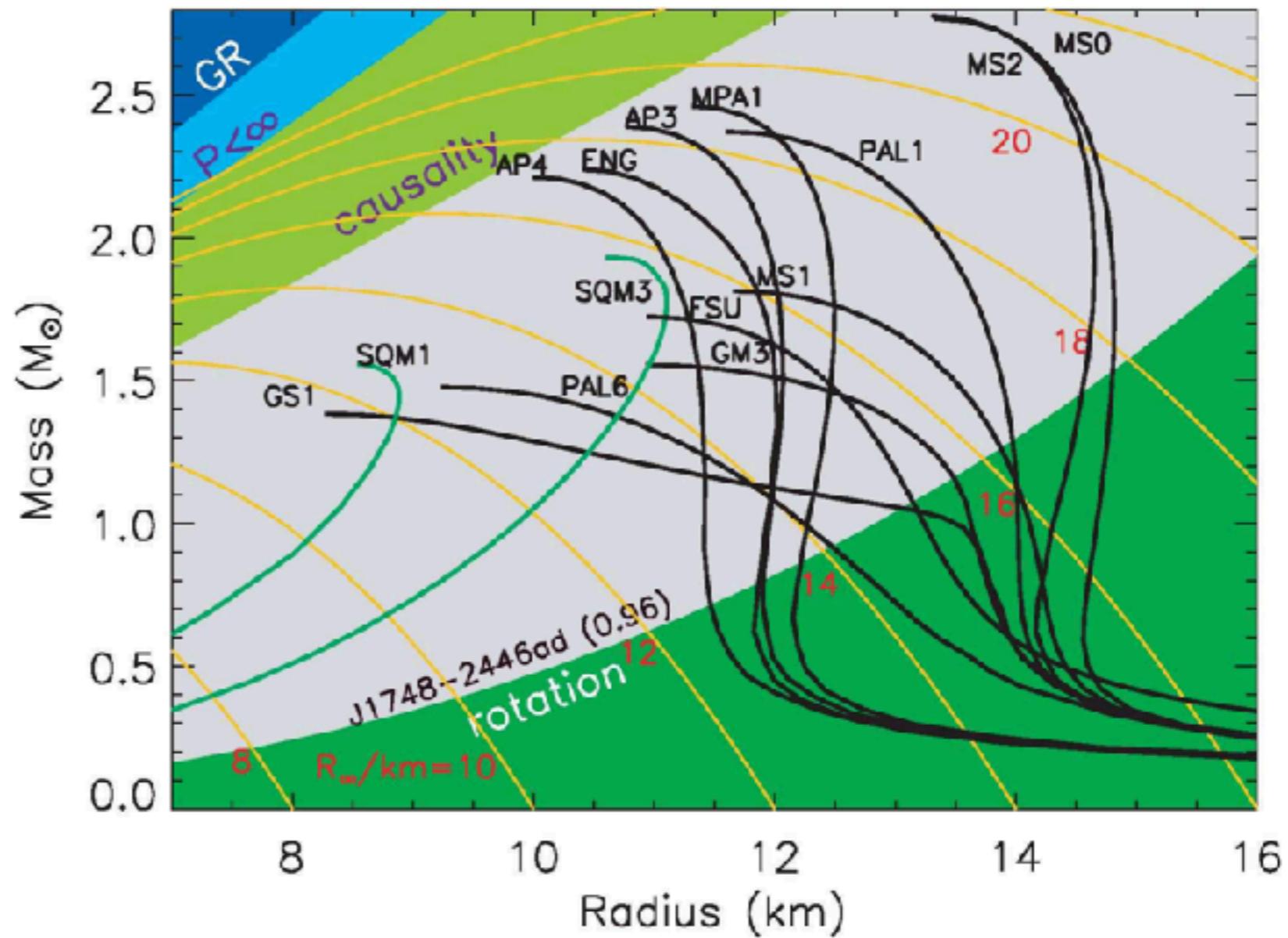


# 脉冲星计与引力波探测



# 脉冲星的构成 (不清楚)

- 中子：中子星
- 中子，质子是由夸克组成的。如果中子被破坏，脉冲星有可能是由夸克组成的：夸克星

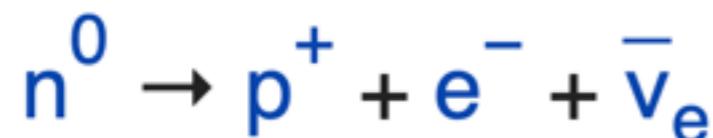


黑线：中子星 绿线：夸克星

# 中子星 — 形成

大质量白矮星的塌缩和超新星爆发

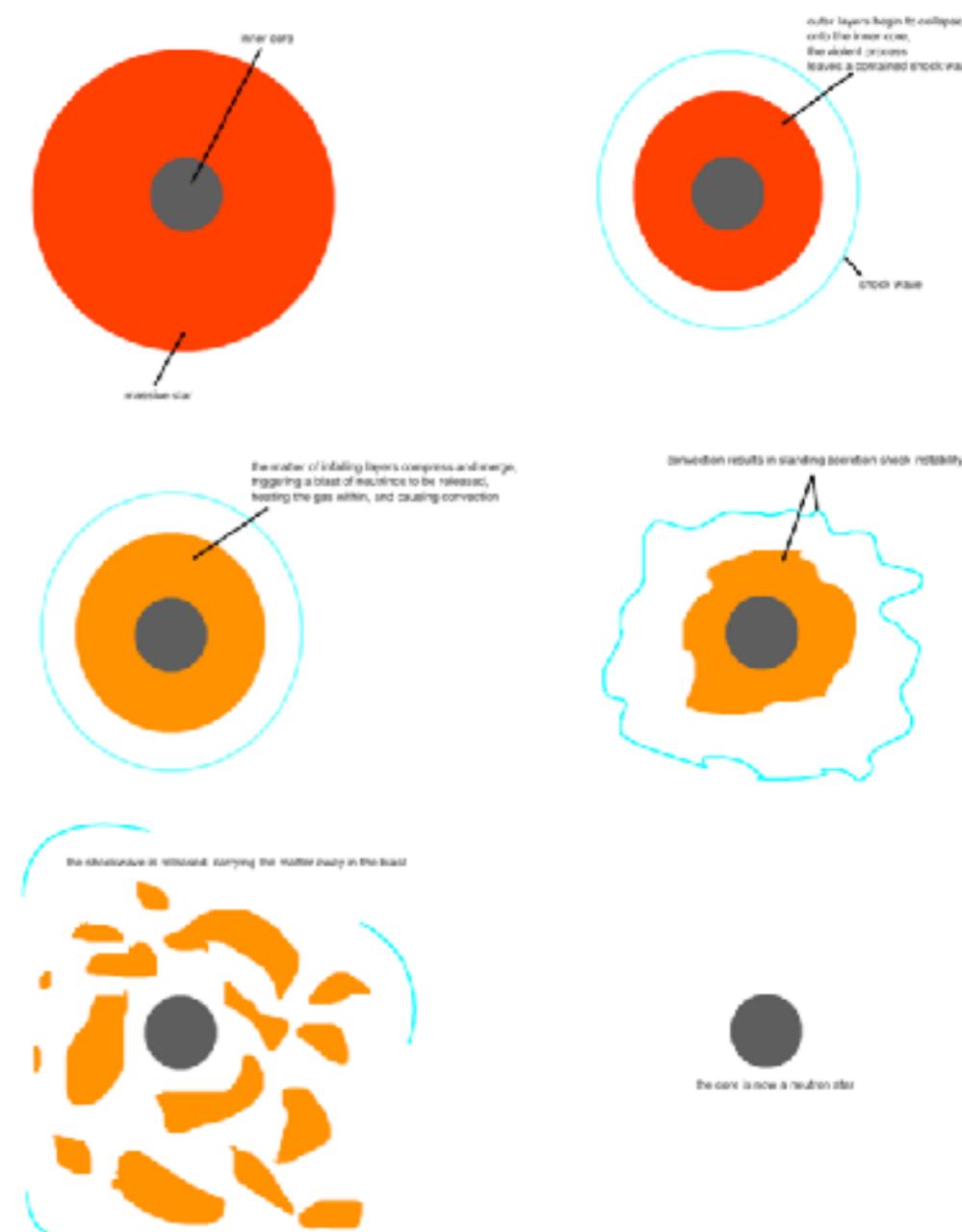
普通条件下：



中子-> 质子 + 电子 + 中微子

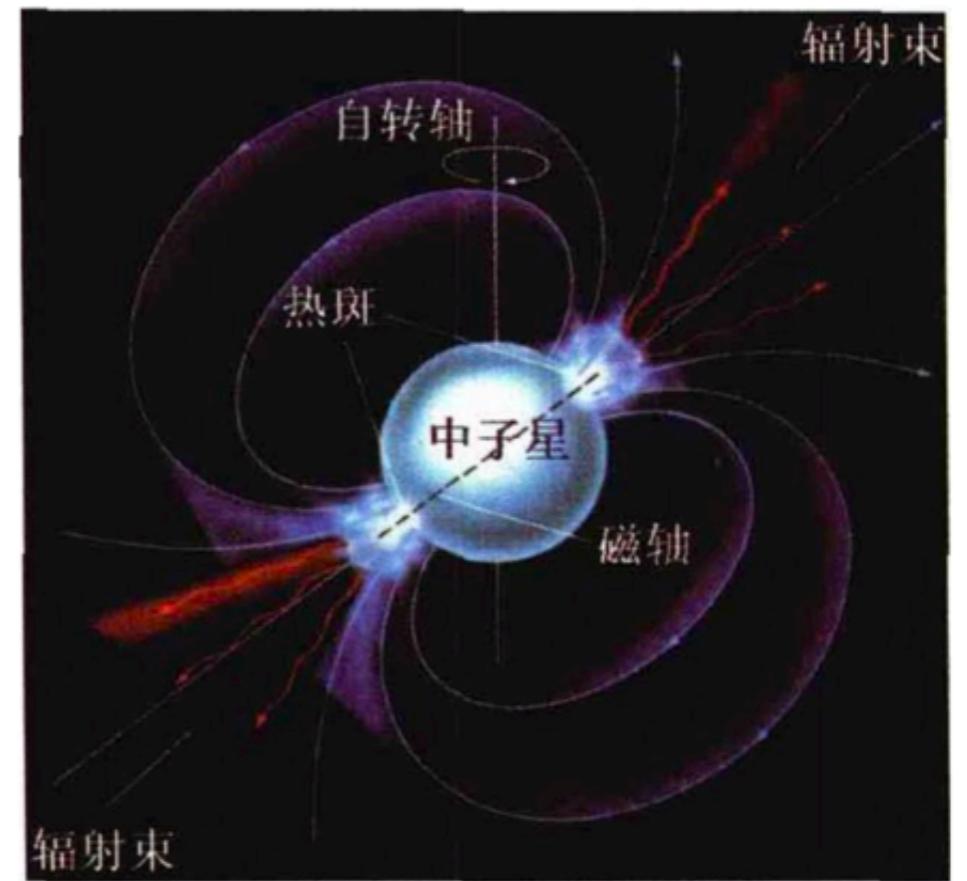
高温条件下：

质子俘获过程



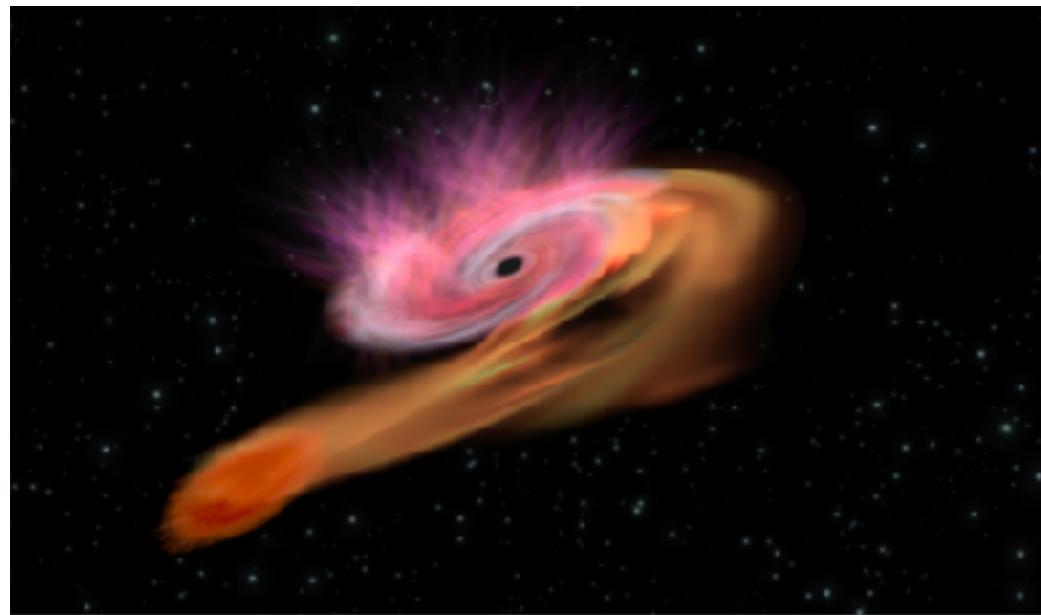
# 脉冲星总结

- 发现：射电观测
- 形成：白矮星塌缩
- 已知：强磁场，快速自转
- 可用于：计时，探测引力波
- 磁场：来源于塌缩
- 自转：角动量守恒，伴星吸积
- 内部结构，不清楚，可能中子，可能夸克



黑洞

# 回转半径：吸积盘的形成



Angular Momentum -> Kinetic Energy

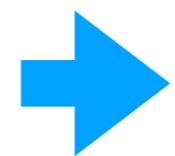
$$L = \nu \times r$$

$$e_{\text{kin}} \approx \nu^2 \approx L^2/r^2$$

Gravitational Energy

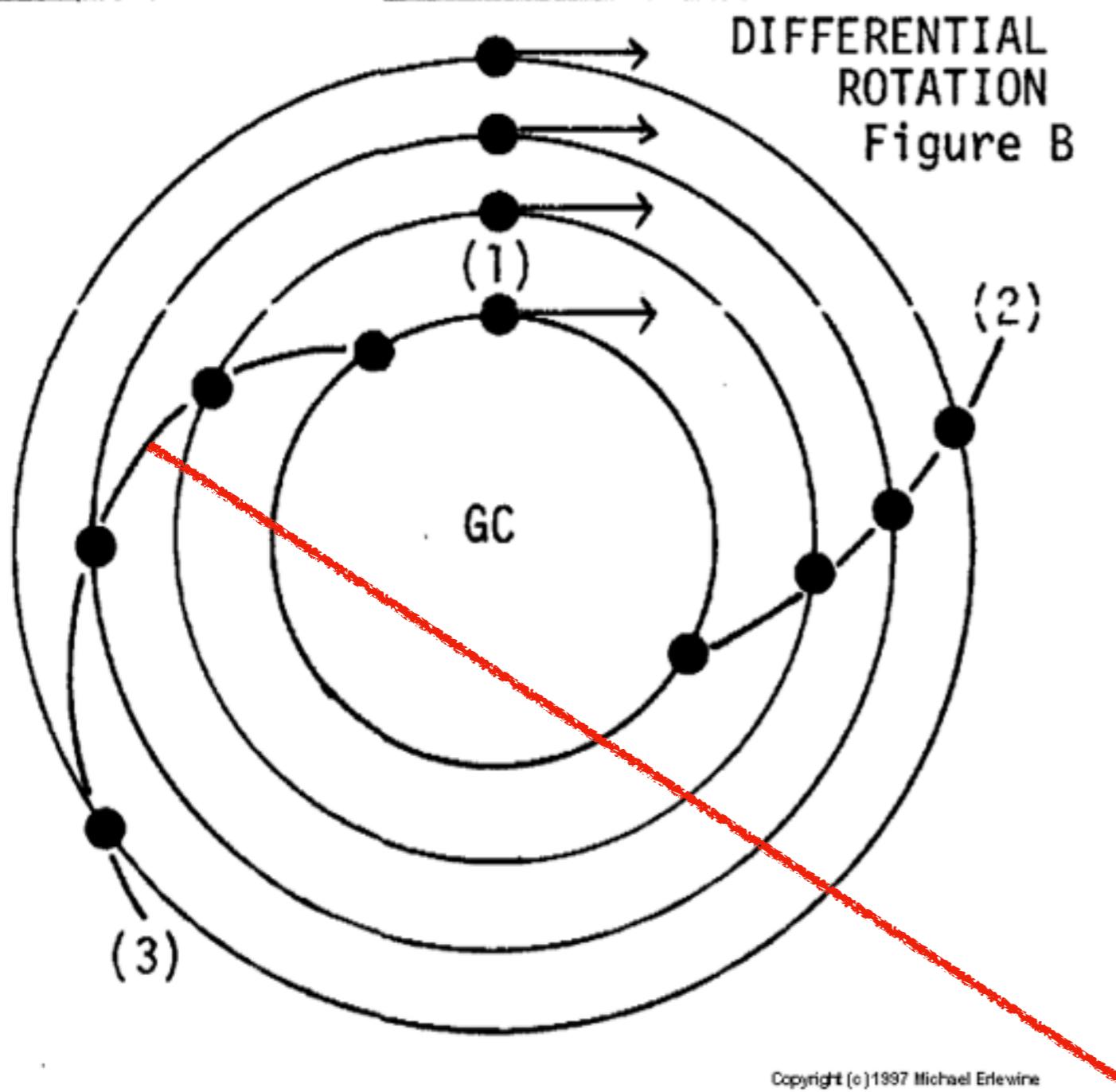
$$e_{\text{gravity}} \approx Gm_{\text{BH}}/r$$

Centrifugal Radius       $e_{\text{kin}} = e_{\text{gravity}}$



$$r_{\text{cf}} = \frac{L^2}{Gm_{\text{BH}}} >> r_{\text{BH}}$$

# 吸积盘：较差自转



刚体转动：

$$\Omega = \text{constant}$$

较差转动：

$$\Omega \neq \text{constant}$$

吸积盘：

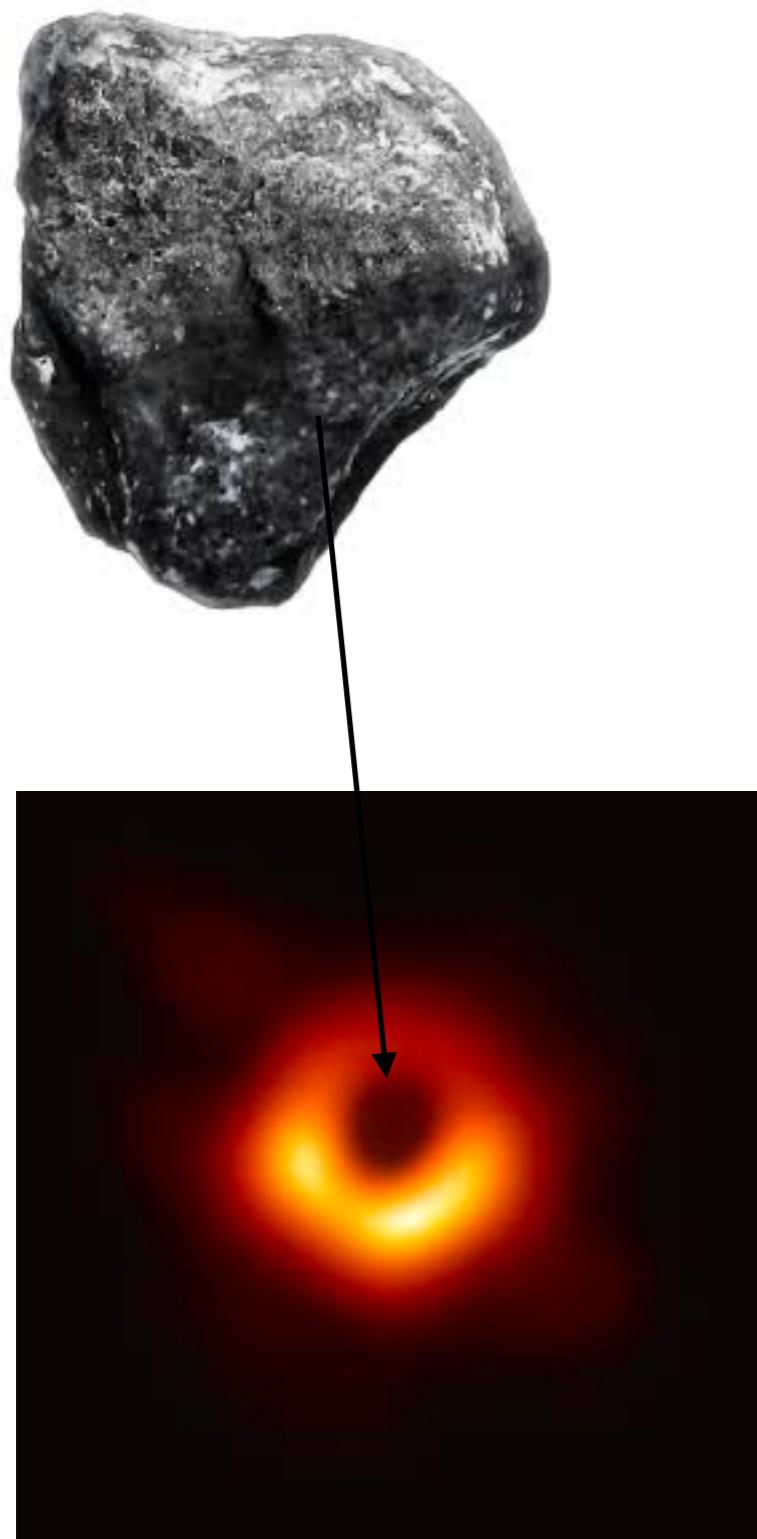
$$\Omega \sim v/r \sim r^{-1.5}$$

内层转得快，外层转的慢

粘滞（摩擦）加热 -> 辐射

# Observing black holes

50% Energy



Energy production:

$$E = \frac{Gm}{r}$$

Black hole radius:

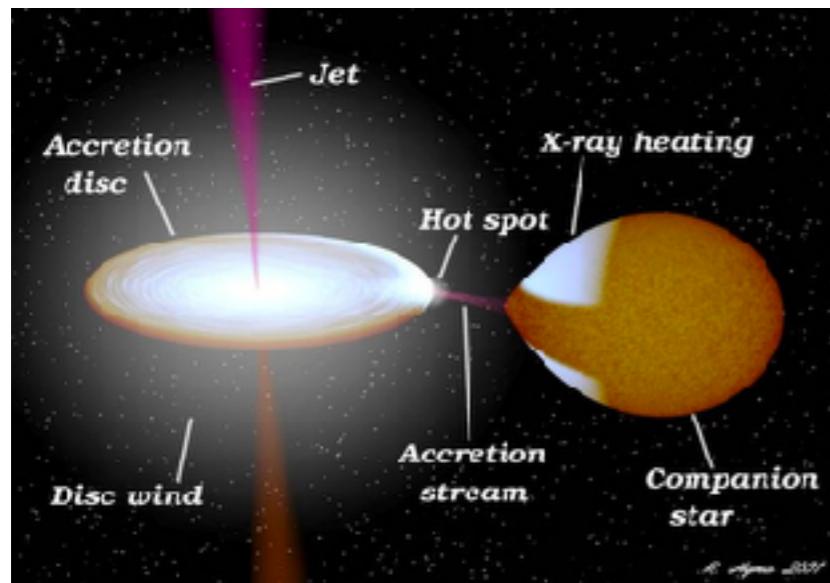
$$r = Gm/c^2$$

Thus:  $E = mc^2$

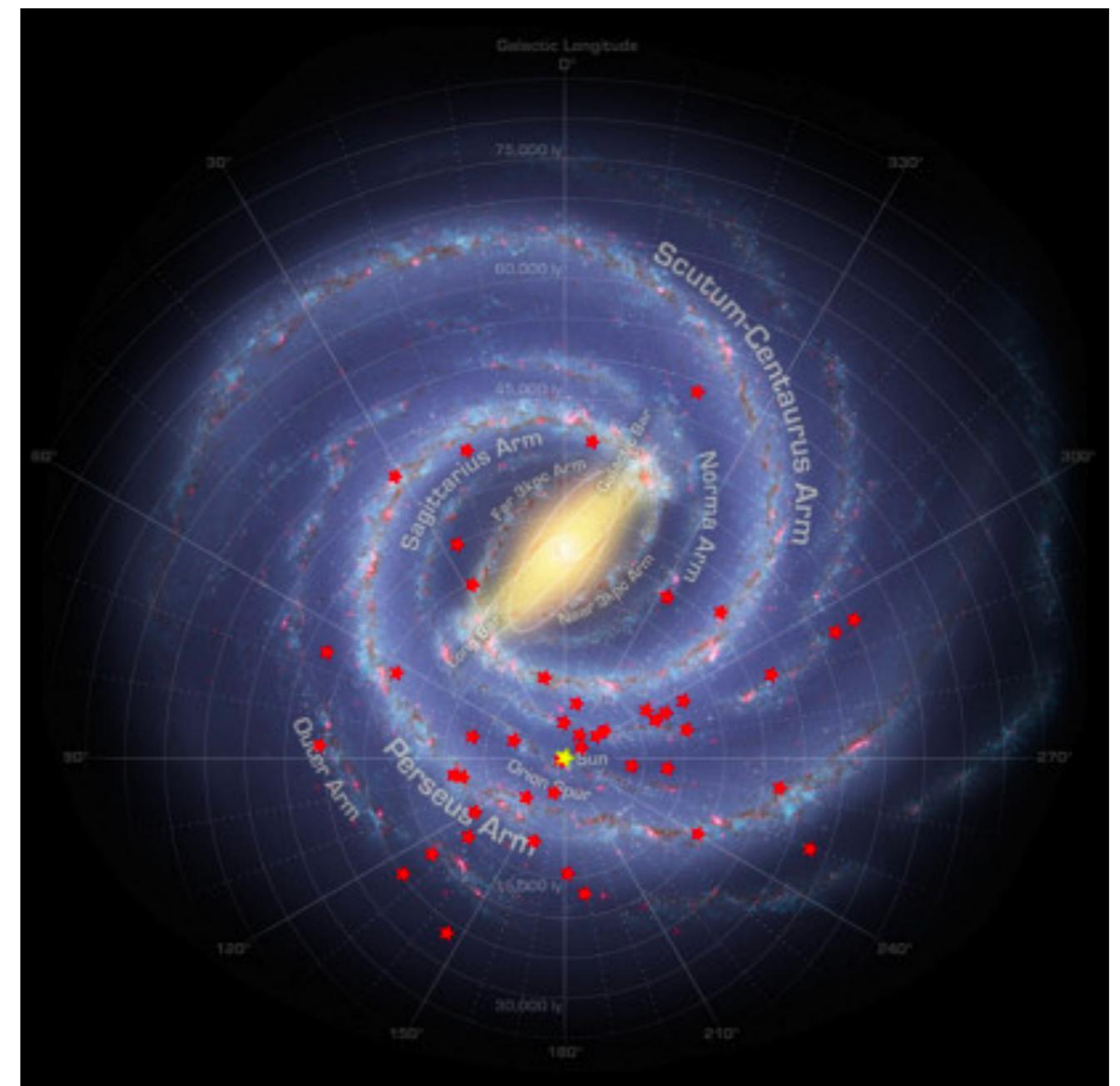
- (1) 黑洞可以放出很多能量
- (2) 前提：有物质注入

# 黑洞的分类

恒星级黑洞

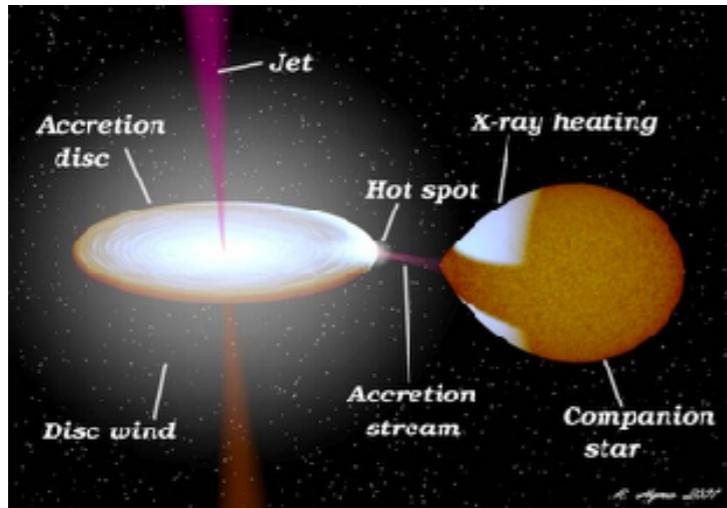


超大质量黑洞

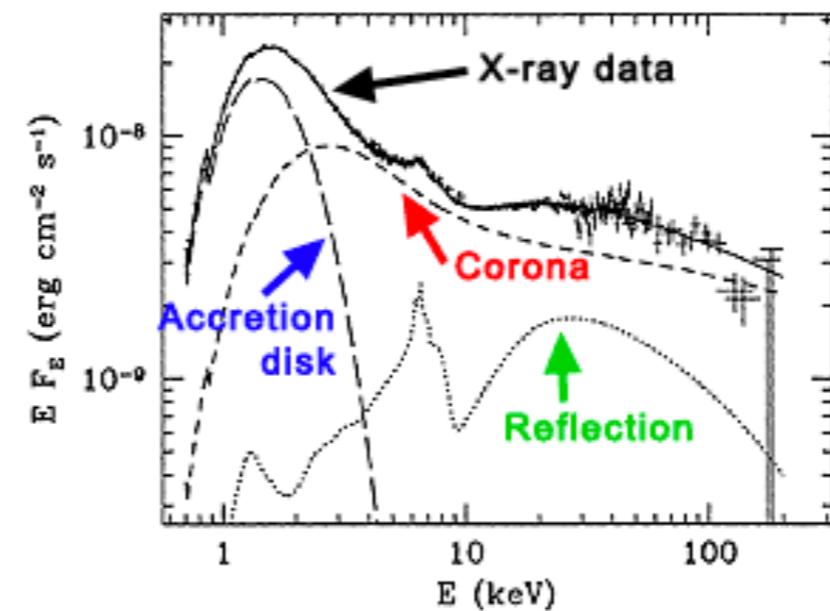


# 黑洞的分类

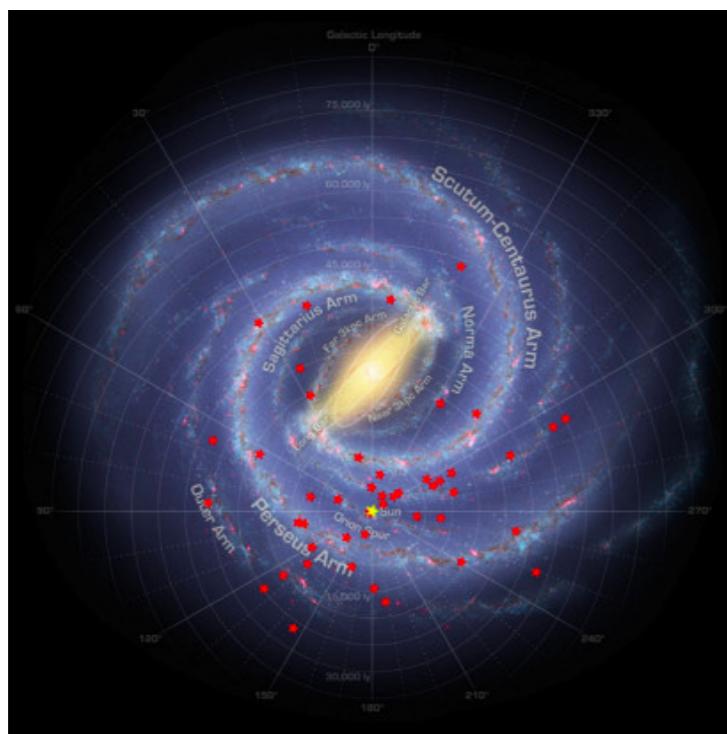
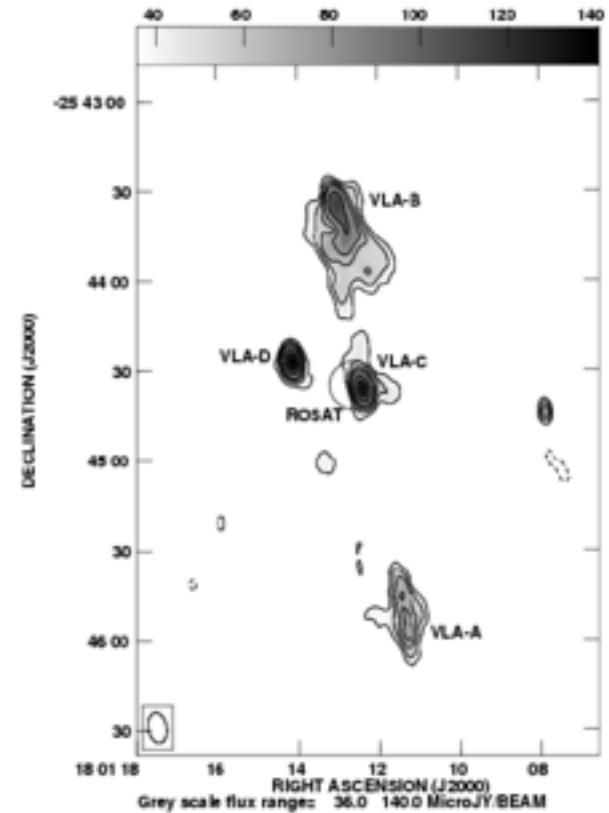
恒星级黑洞



X射线发射

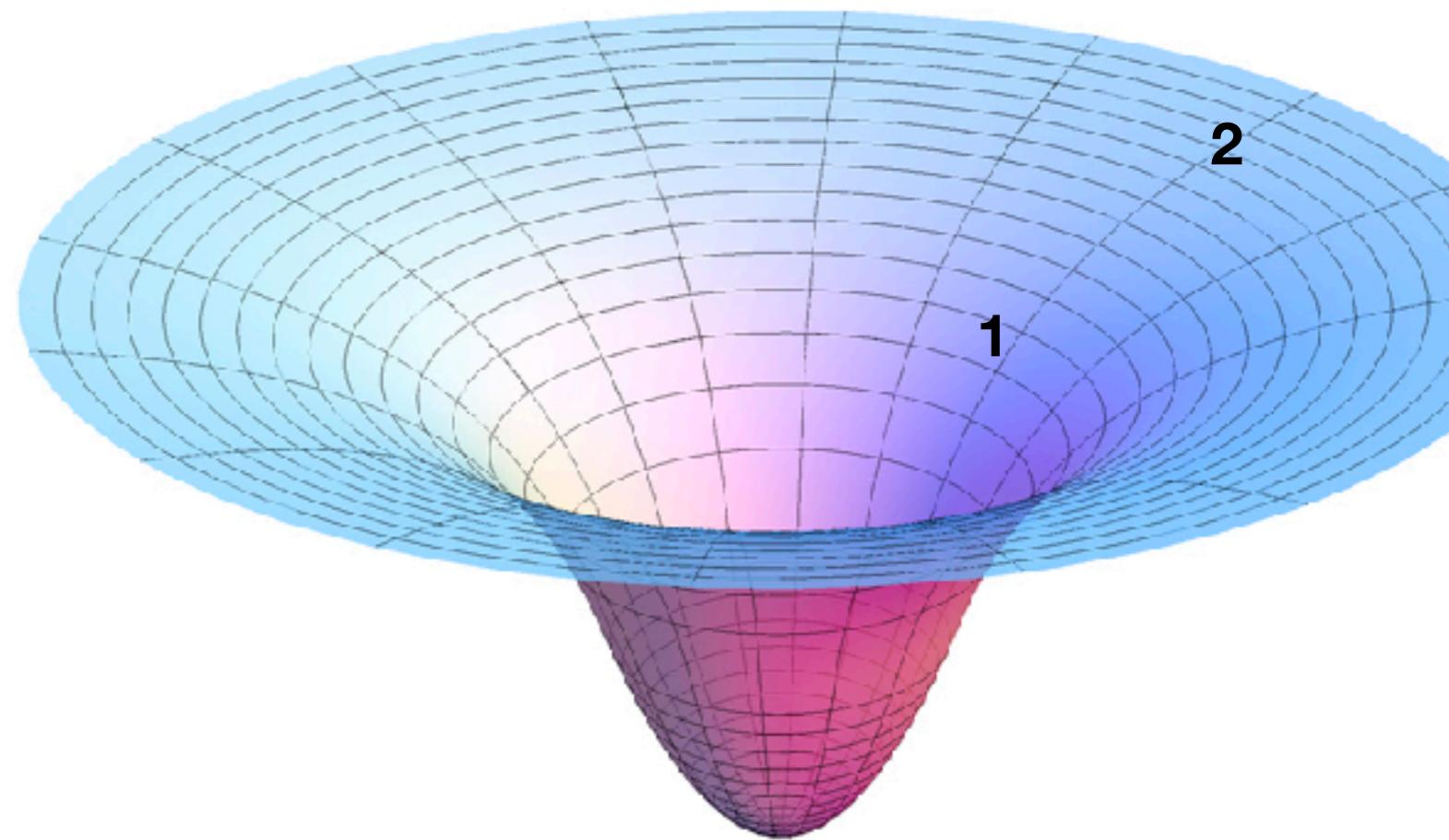
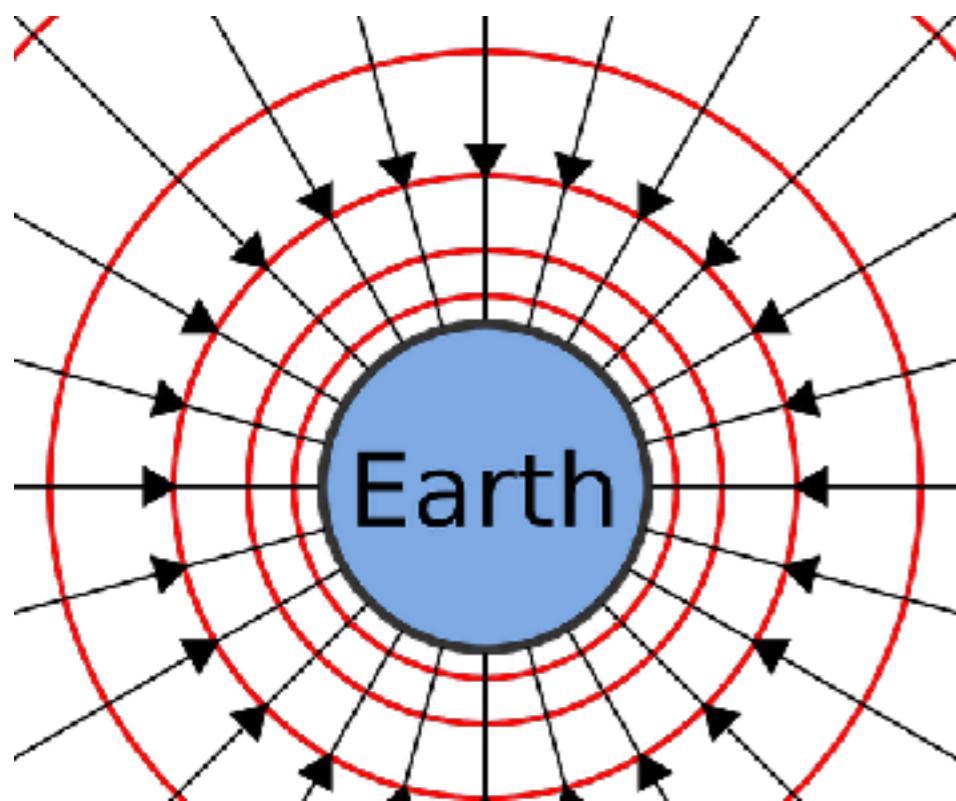


喷流



质量：恒星质量  
物质来源：伴星  
分布：星系中  
辐射：多波段，X-ray明显  
形成机制：恒星的演化

# 引力红移



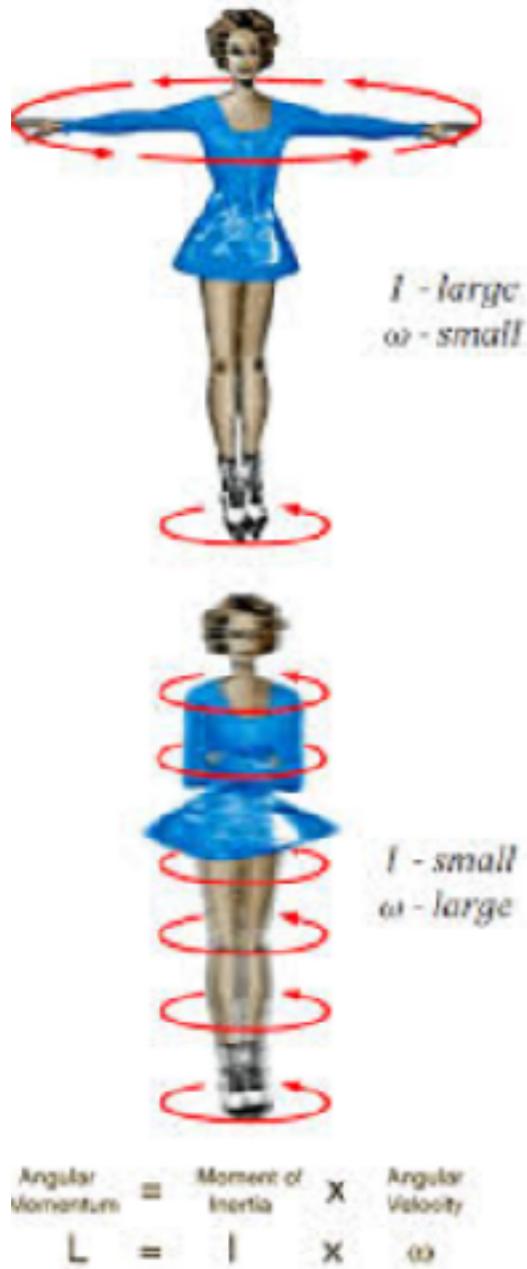
$$E_1 = h\nu_1 + \phi_1$$

$$E_2 = h\nu_2 + \phi_2$$

$$\frac{v_\infty}{v_0} = \left(1 - \frac{2GM}{r_0c^2}\right)^{1/2}.$$

# 黑洞自旋

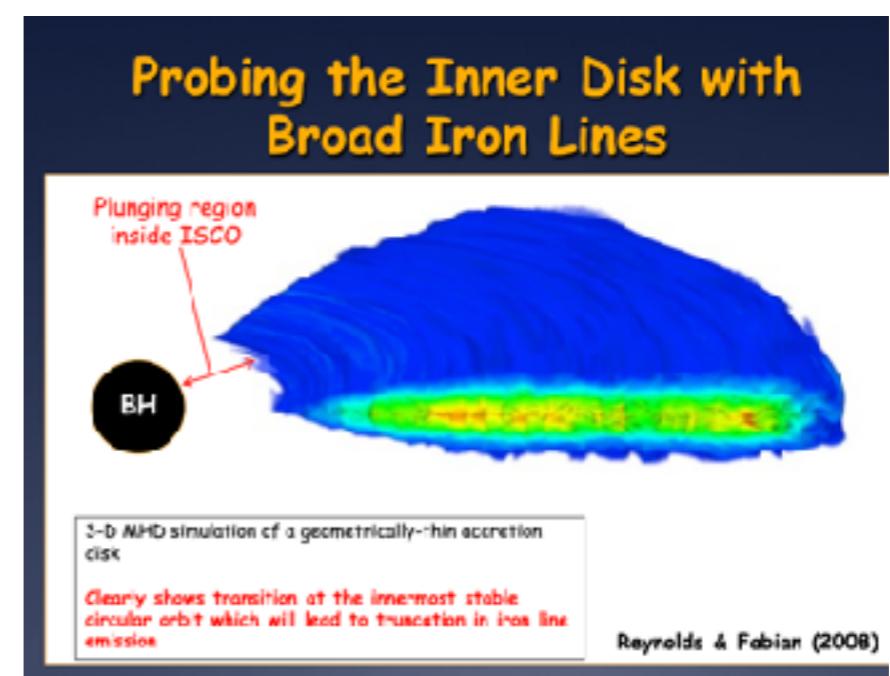
## 1、塌缩，角动量守恒



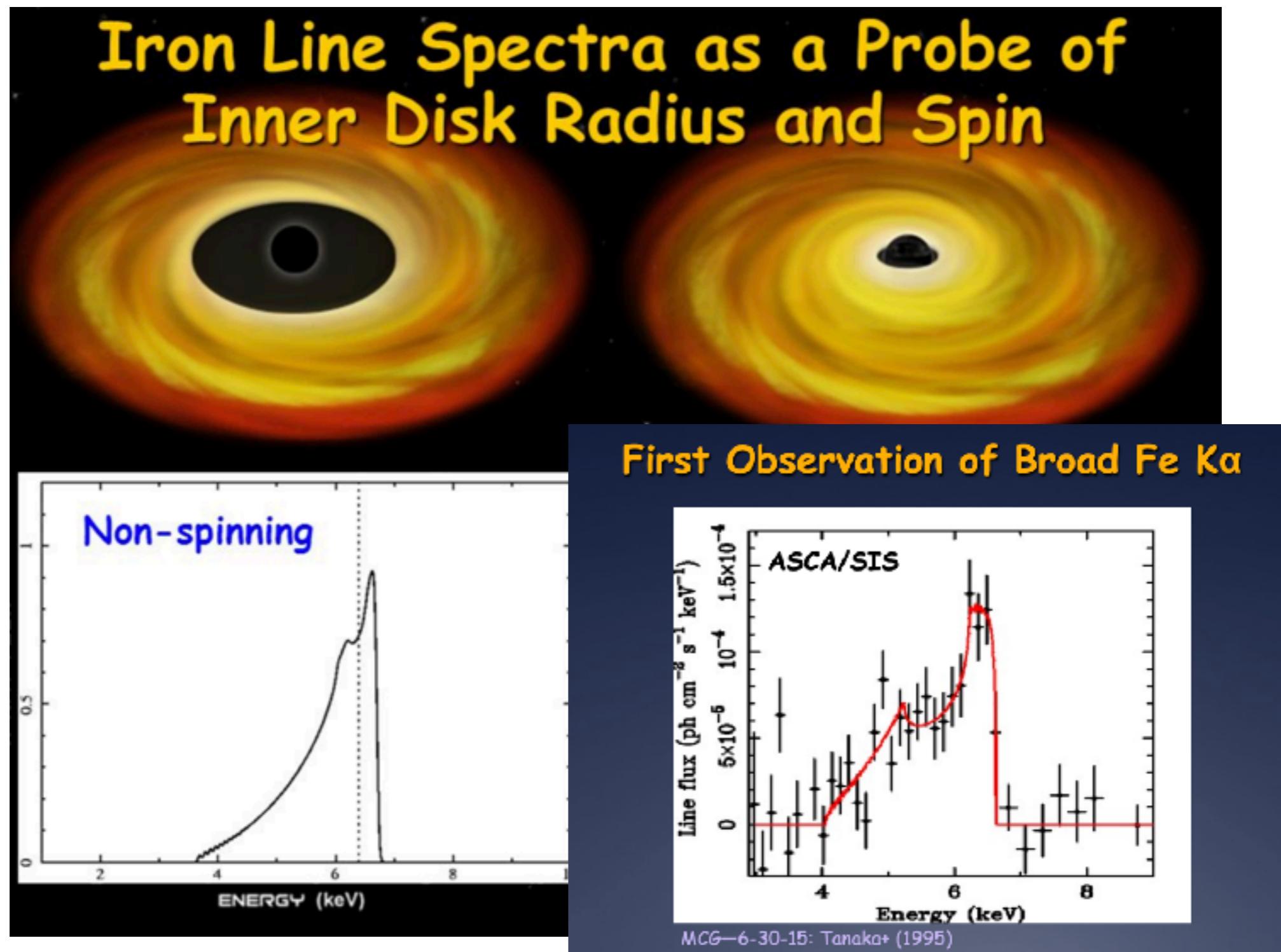
## 2、黑洞并合（引力波探测）



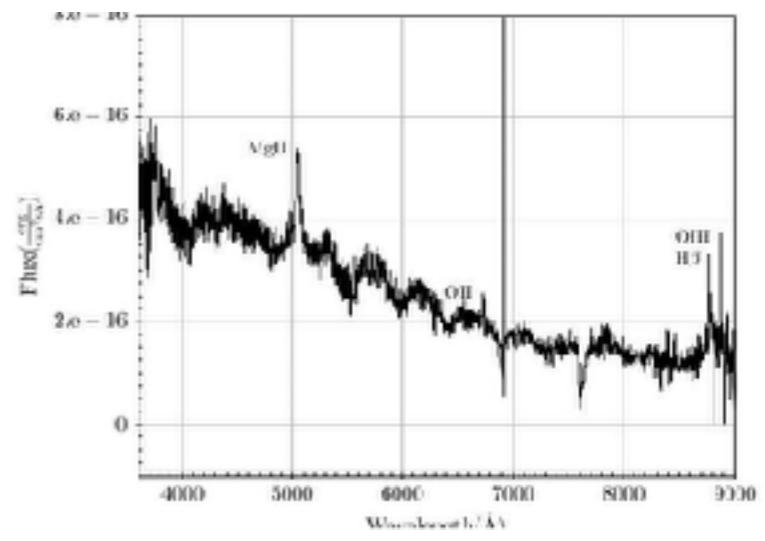
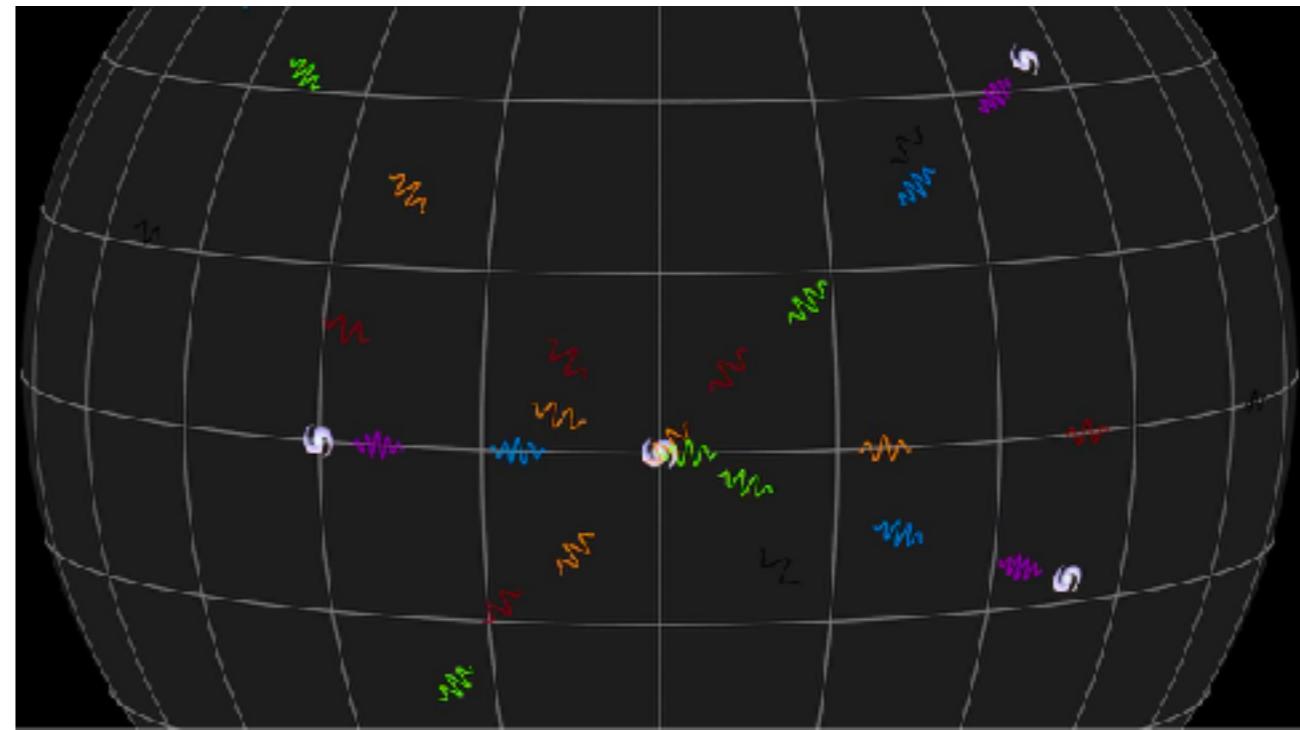
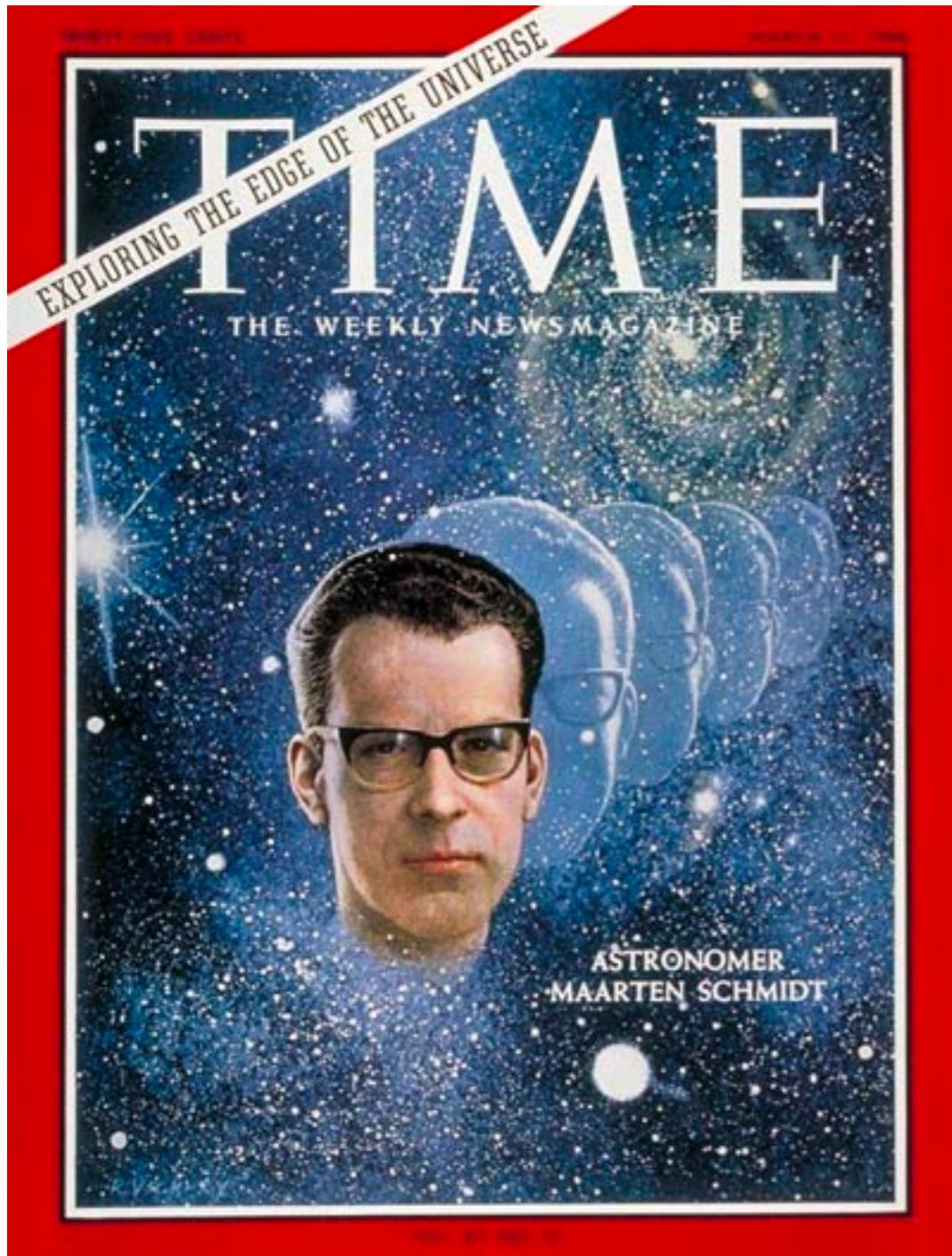
## 3、吸积物质



# 来自吸积盘的铁发射线

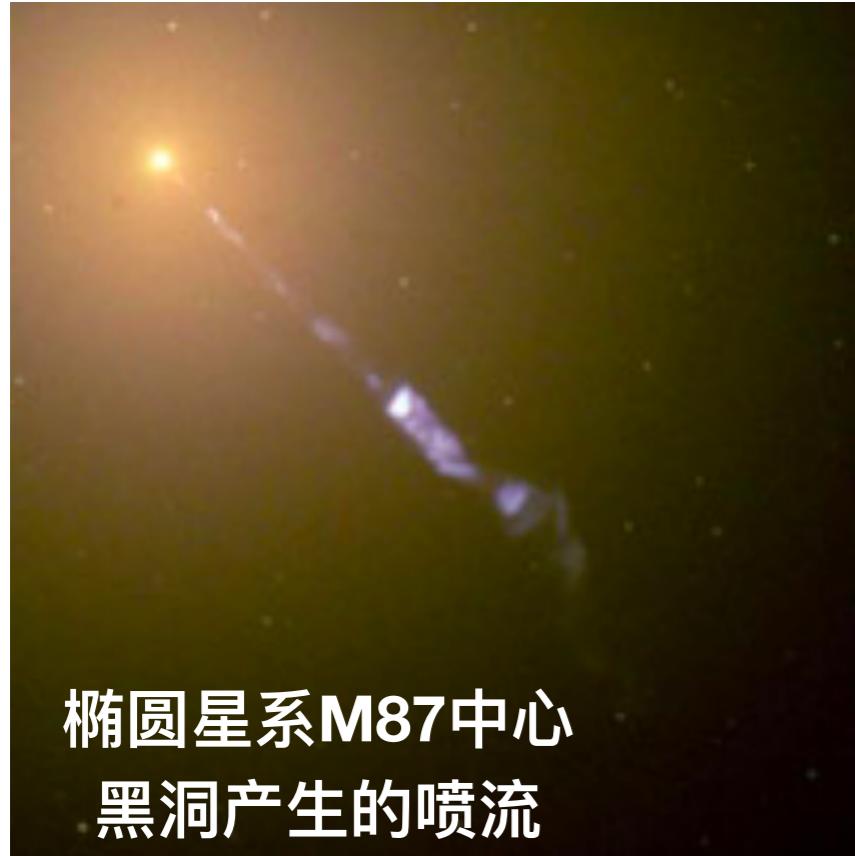


# 类星体：超大质量黑洞

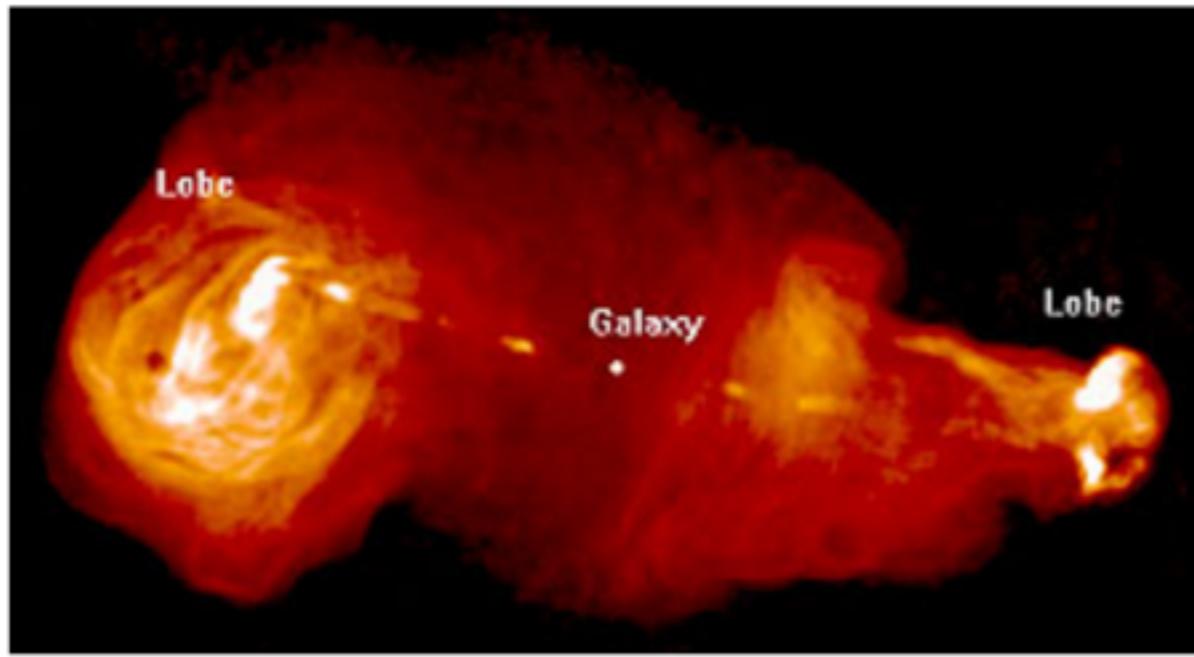
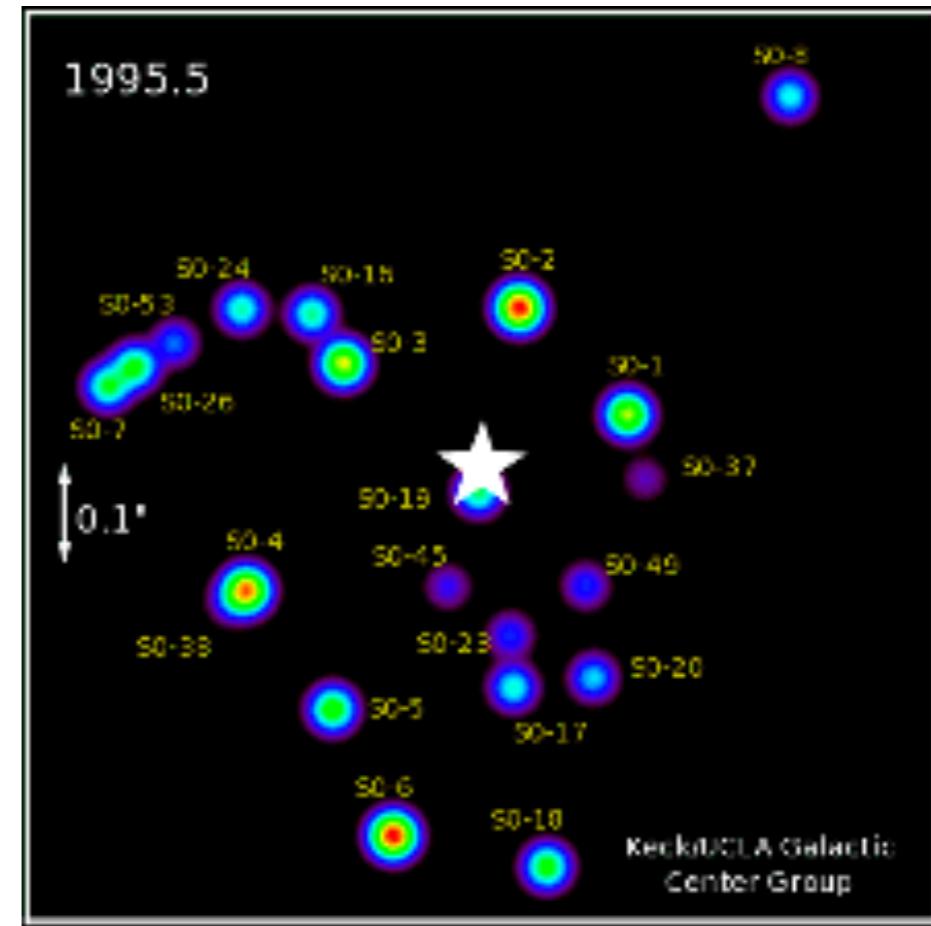


# 超大质量黑洞

超大质量黑洞



银河系中心黑洞  $10^6$

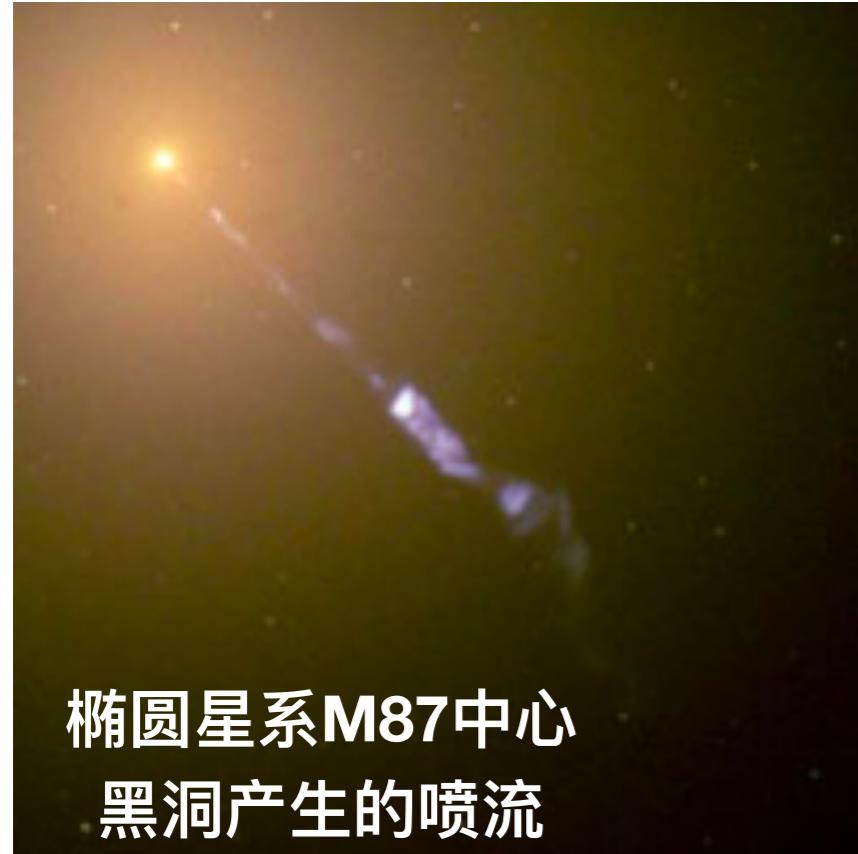


A false-color image of a radio galaxy, 3C 353.  
Image credit [NRDAO/Swain, Bridle, Baum](#).

质量:  $10^6\text{-}10^9$  太阳质量  
物质来源: 星系中的气体  
分布: 星系中心  
辐射: 多波段, X-ray明显  
形成机制: 和星系一起形成, 古老的天体!

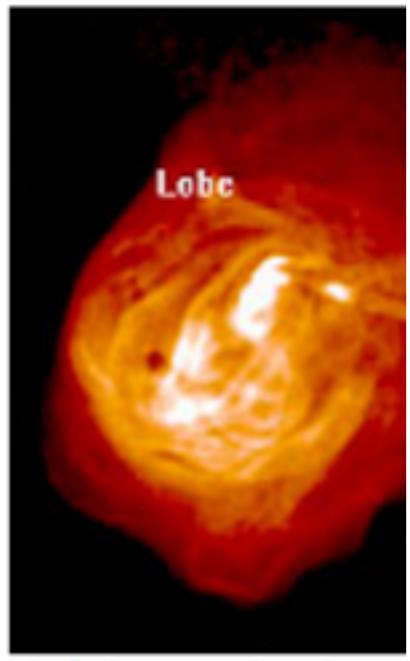
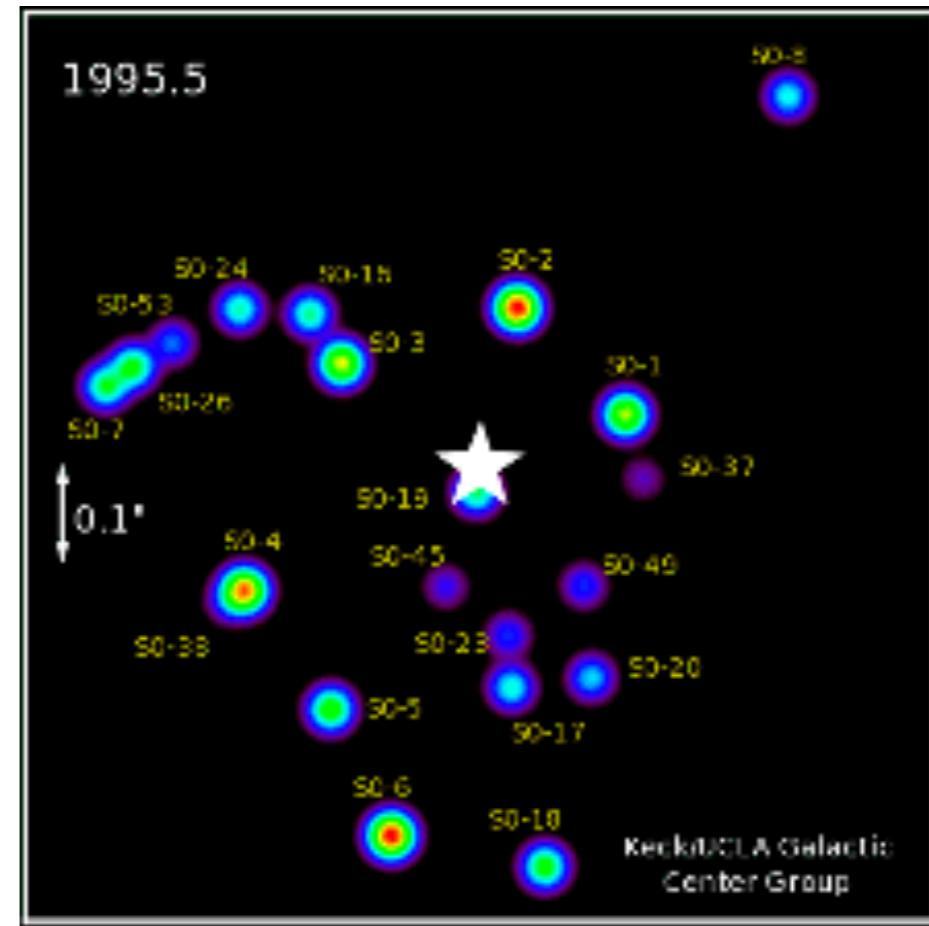
# 超大质量黑洞

超大质量黑洞



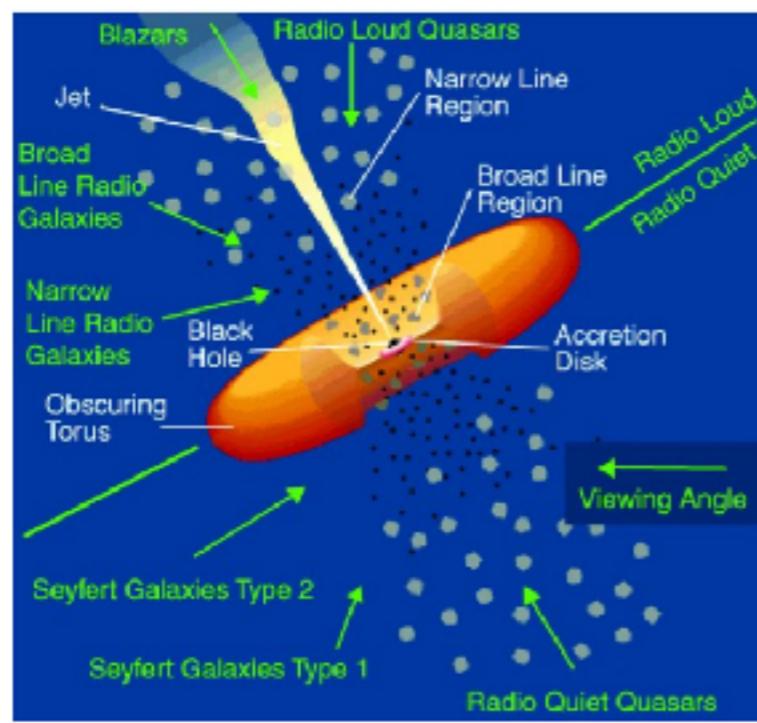
椭圆星系M87中心  
黑洞产生的喷流

银河系中心黑洞  $10^6$

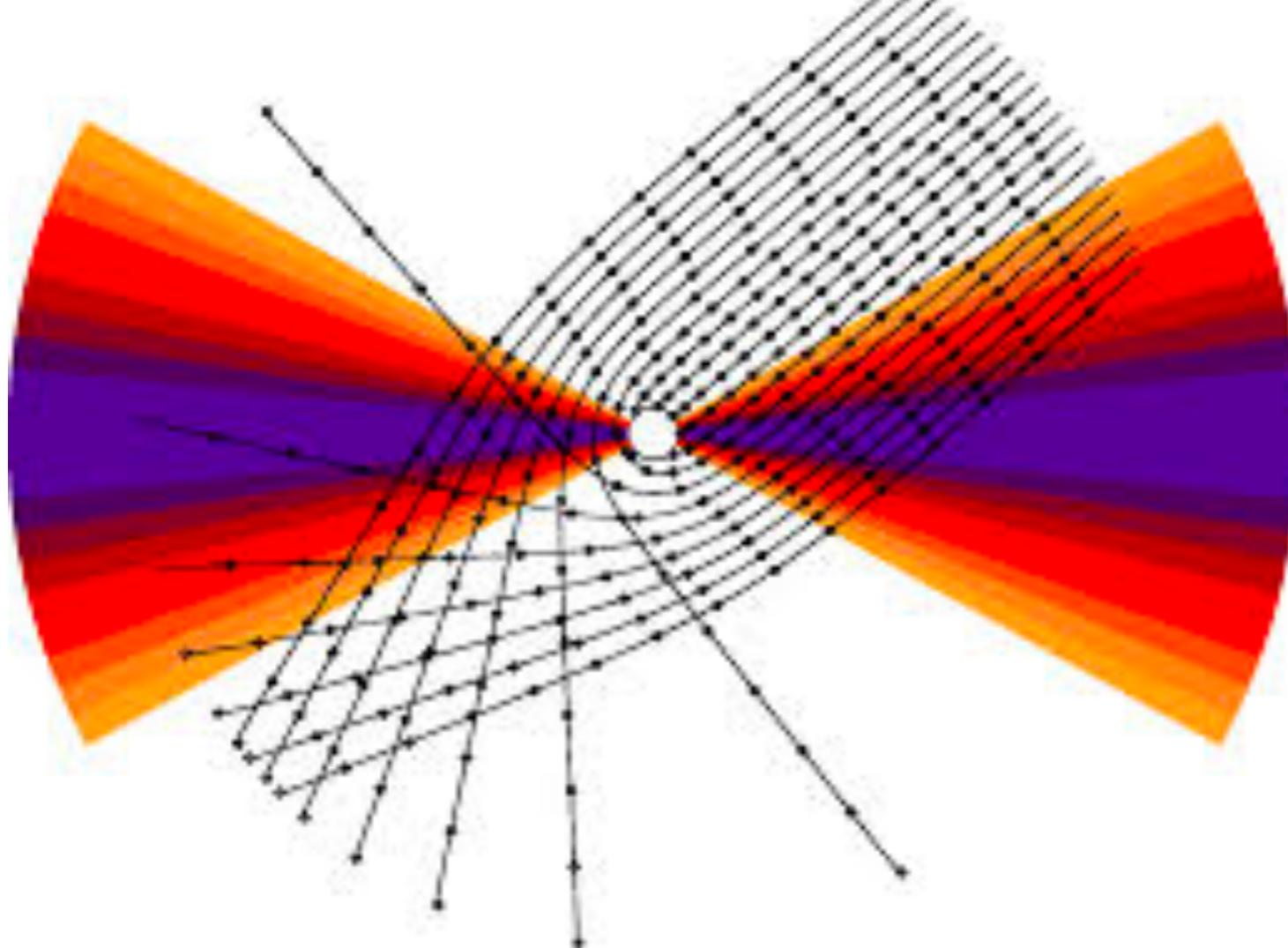


A false-color image

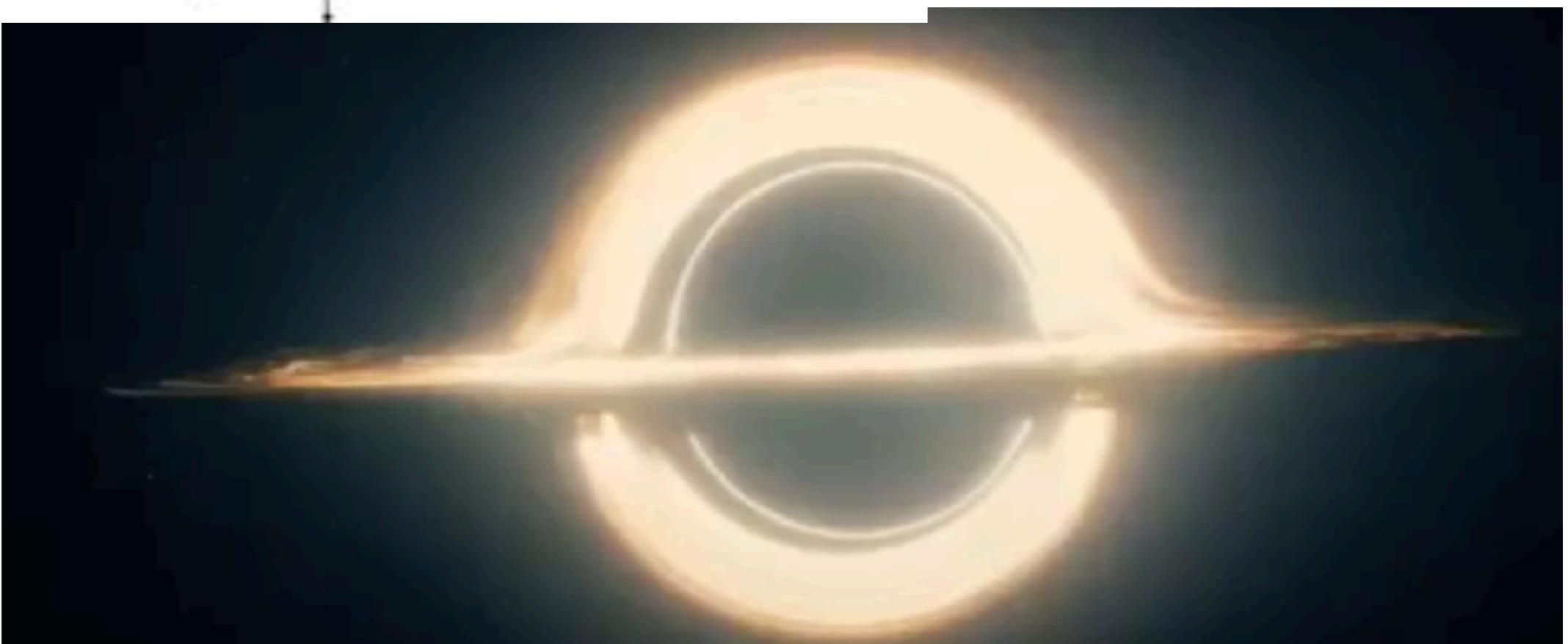
Image credit [NRAO/Swain, Bridle, Baum](#).



质量:  $10^6\text{-}10^9$  太阳质量  
物质来源: 星系中的气体  
分布: 星系中心  
辐射: 多波段, X-ray明显  
形成机制: 和星系一起形成, 古老的天体!



# 黑洞吸积盘： 图像



# 总结：黑洞

- 黑洞：致密天体
- 角动量守恒-> 黑洞吸积盘
- 吸积盘的较差自转与加热
- 黑洞的分类(恒星级黑洞与超大质量黑洞)
- 形成：恒星级黑洞起源于恒星演化，超大质量黑洞和黑洞一起形成
- 黑洞对光线的弯曲
- 思考，黑洞的光变