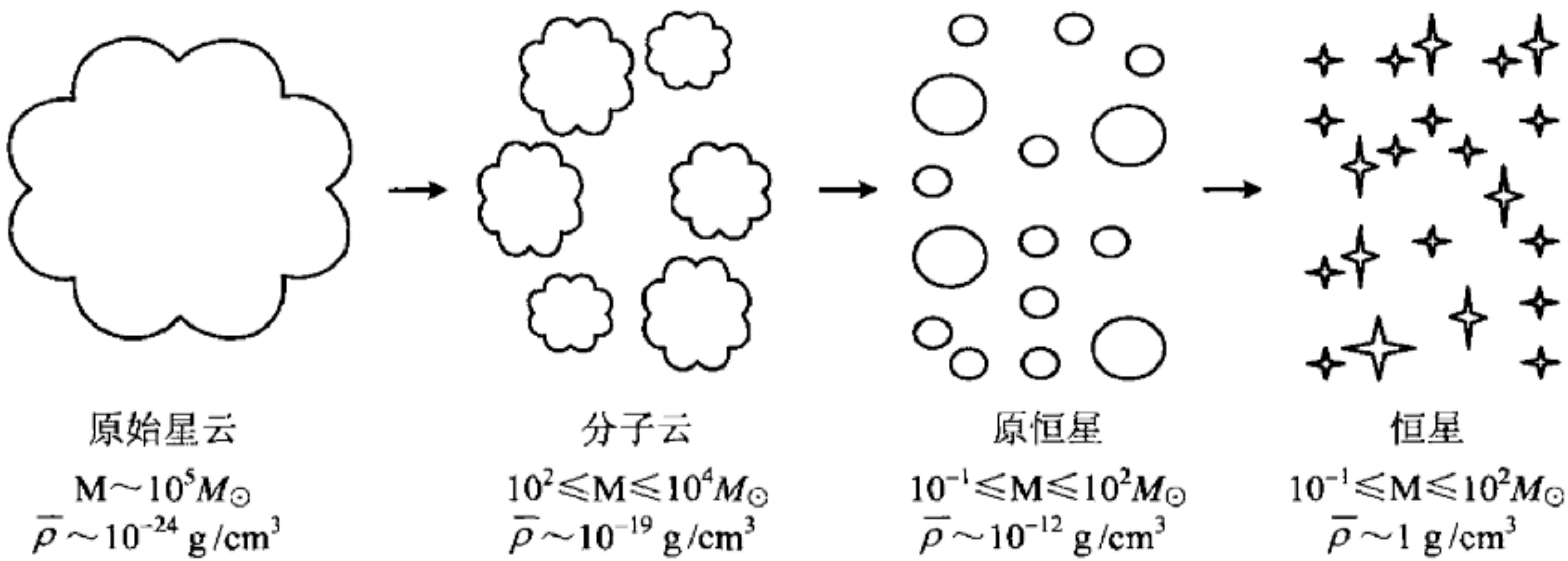
Relativistic Astrophysics
相对论天体物理

Guang-Xing Li (李广兴)

Course, textbook, etc…

星际介质与恒星形成



?

图 3.1 原始星云的碎裂、凝聚过程

宇宙膨胀得较快，这样核合成开始时剩下的中子就较多，从而产生较多的氦，与观测到的氦丰度不符。再有，G 变大时引力增强，行星就靠近太阳，行星的公转周期就会变短；反之，G 变小时，公转周期就会变长。行星的公转周期有没有变化，可以同原子钟来进行比较，因为原子钟的走时与万有引力无关。利用雷达信号从水星和金星表面的反射并用原子钟测时，可以精确测定它们的绕日轨道周期，这样得到的 G 的变化率是

$$\left|\frac{\dot{G}}{G}\right| \leqslant 4\times10^{-10}/\text{年} \tag{7.7.71}$$

以上这些分析都表明，在宇宙过去的时间内，万有引力常数 G 看来不会有明显的变化。

对电子的电荷 e 和质量 m_e 也可做类似的分析。如果 e 随时间变化，就会对原子结构和原子核的性质产生影响，例如放射性同位素的半衰期会发生改变。计算表明，如果 e 的减小速率为 5×10^{-11}/年，宇宙中的锇同位素（^{187}Os）就该几乎衰变光了，但至今地壳中仍有不少 ^{187}Os 存在。由 ^{187}Os 现在的丰度可以推断出，e 的相对年变化率应当小于 10^{-13}/年。同样，如果电子的质量 m_e 发生变化，也会导致 ^{187}Os 的衰变速率加快。由此推断出，m_e 的相对变化率不应大于 4×10^{-13}/年。而由恒星演化的分析得到的结果是，m_e 的时间变化率 $<4\times10^{-12}$/年。

PROBLEM SET

1 In 1672, an international effort was made to measure the parallax angle of Mars at the time of opposition, when it was closest to Earth; see the below figure.

(a) Consider two observers who are separated by a baseline equal to Earth's diameter. If the difference in their measurements of Mars's angular position is 33.6″, what is the distance between Earth and Mars at the time of opposition? Express your answer both in units of m and in AU.

(b) If the distance to Mars is to be measured to within 10%, how closely must the clocks used by the two observers be synchronized? *Hint:* Ignore the rotation of Earth. The average orbital velocities of Earth and Mars are 29.79 km s^{-1} and 24.13 km s^{-1}, respectively.

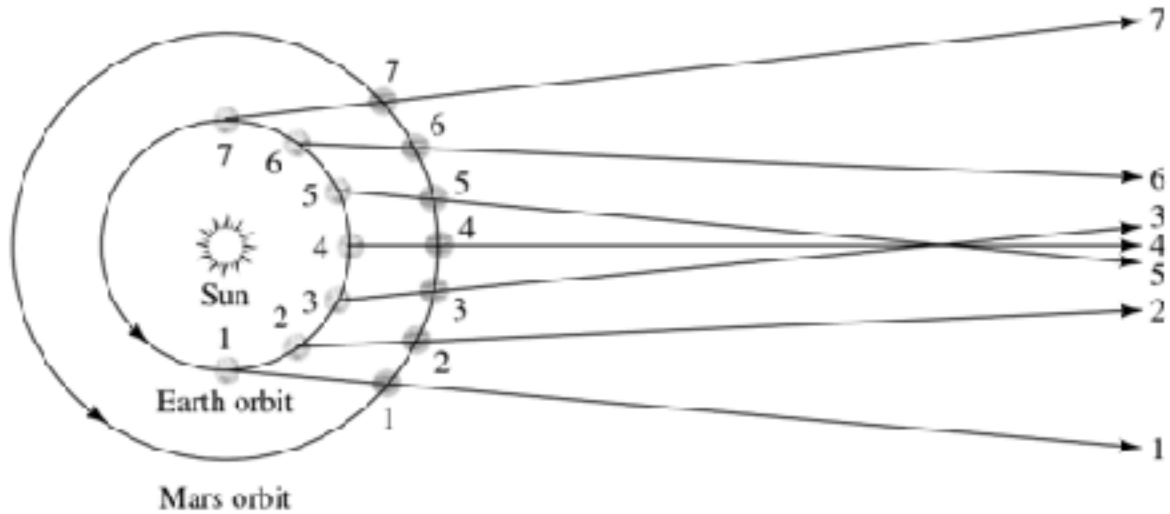
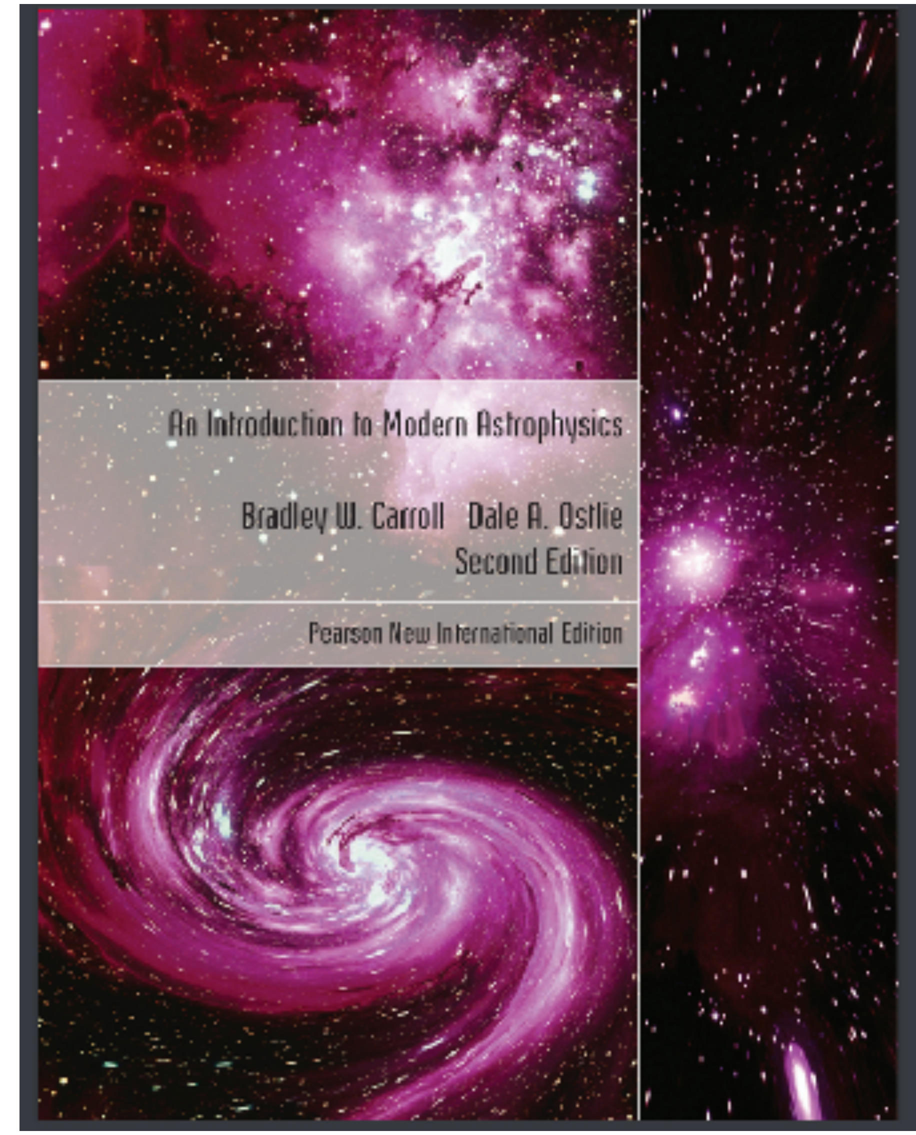


FIGURE The retrograde motion of Mars as described by the Copernican model. Note that the lines of sight from Earth to Mars cross for positions 3, 4, and 5. This effect, combined with the slightly differing planes of the two orbits result in retrograde paths near opposition.



2 At what distance from a 100-W light bulb is the radiant flux equal to the solar irradiance?

3 The parallax angle for Sirius is 0.379″.

(a) Find the distance to Sirius in units of (i) parsecs; (ii) light-years; (iii) AU; (iv) m.

(b) Determine the distance modulus for Sirius.

4 Using the information in Example 6.1 and Problem 3, determine the absolute bolometric magnitude of Sirius and compare it with that of the Sun. What is the ratio of Sirius's luminosity to that of the Sun?

5 **(a)** The Hipparcos Space Astrometry Mission was able to measure parallax angles down to nearly 0.001″. To get a sense of that level of resolution, how far from a dime would you need to be to observe it subtending an angle of 0.001″? (The diameter of a dime is approximately 1.9 cm.)

Part 1 — Planck Scale and Spacetime Bubble

Fundamental physics

Planck constant:

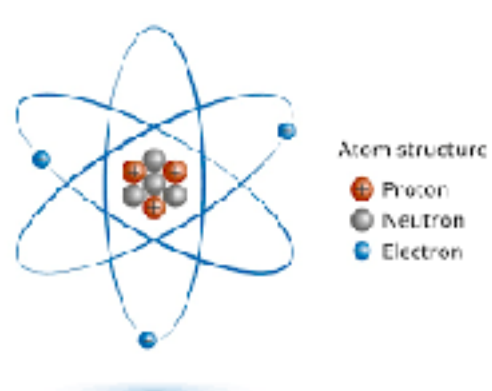
$$h = 6.626\,070\,15 \times 10^{-34}\ \mathrm{J{\cdot}s.}$$ [2][3]

$$E = hf.$$

Uncertainty Principle:

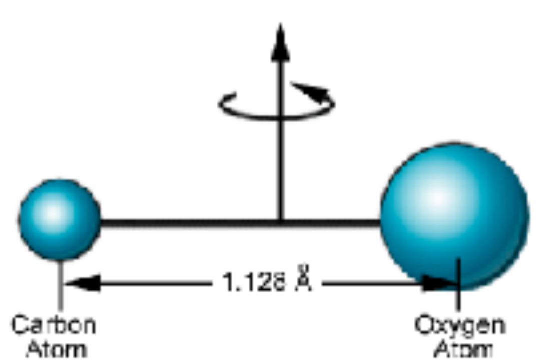
Position-Momentum

$$\Delta x\, \Delta p_x \geq \frac{\hbar}{2},$$



Angular momentum

$$I\omega^2 = h\omega$$



Energy-time

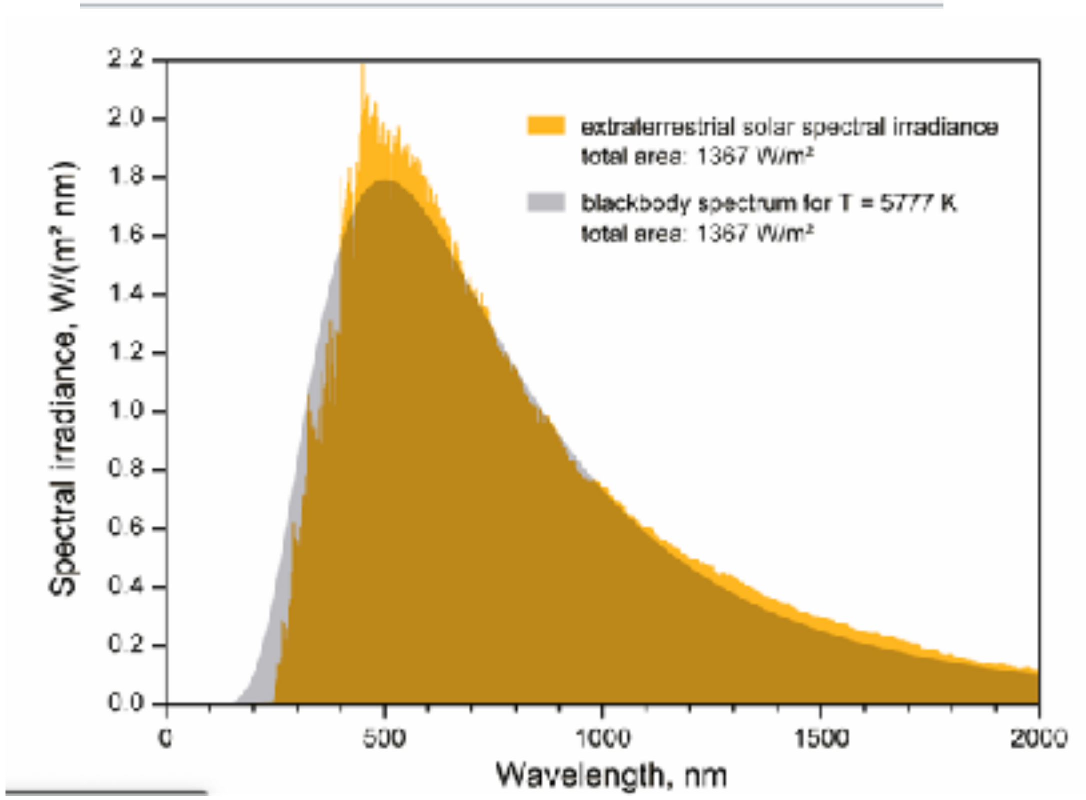
$$\delta E\, \delta t \approx h$$

?

Max-Planck

- Max Karl Ernst Ludwig Planck
- Age 38 (1900):
  - Discover the Planck Law
- Max-Planck Institute

$$B_\nu(\nu, T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/(k_\mathrm{B}T)} - 1}$$



Max Planck Society

Non-profit association

MAX-PLANCK-GESELLSCHAFT

mpg.de/en

The Max Planck Society for the Advancement of Science is a formally independent non-governmental and non-profit association of German research institutes founded in 1911 as the Kaiser Wilhelm Society and renamed the Max Planck Society in 1948 in honor of its former president, theoretical physicist Max Planck.
Wikipedia

About lectures….

12.1 Planck's early career

Max Planck was typically straightforward and honest about his early career, as can be learned from his short scientific autobiography.[1] He studied under Helmholtz and Kirchhoff in Berlin, but in his own words

I must confess that the lectures of these men netted me no perceptible gain. It was obvious that Helmholtz never prepared his lectures properly . . . Kirchhoff was the very opposite . . . but it would sound like a memorised text, dry and monotonous.

Helmholz: 亥姆霍兹

Kirchhoff: 基尔霍夫

Planck Scale

Size of a black hole:

$$r_{\rm BH} = Gm/c^2$$

Energy:

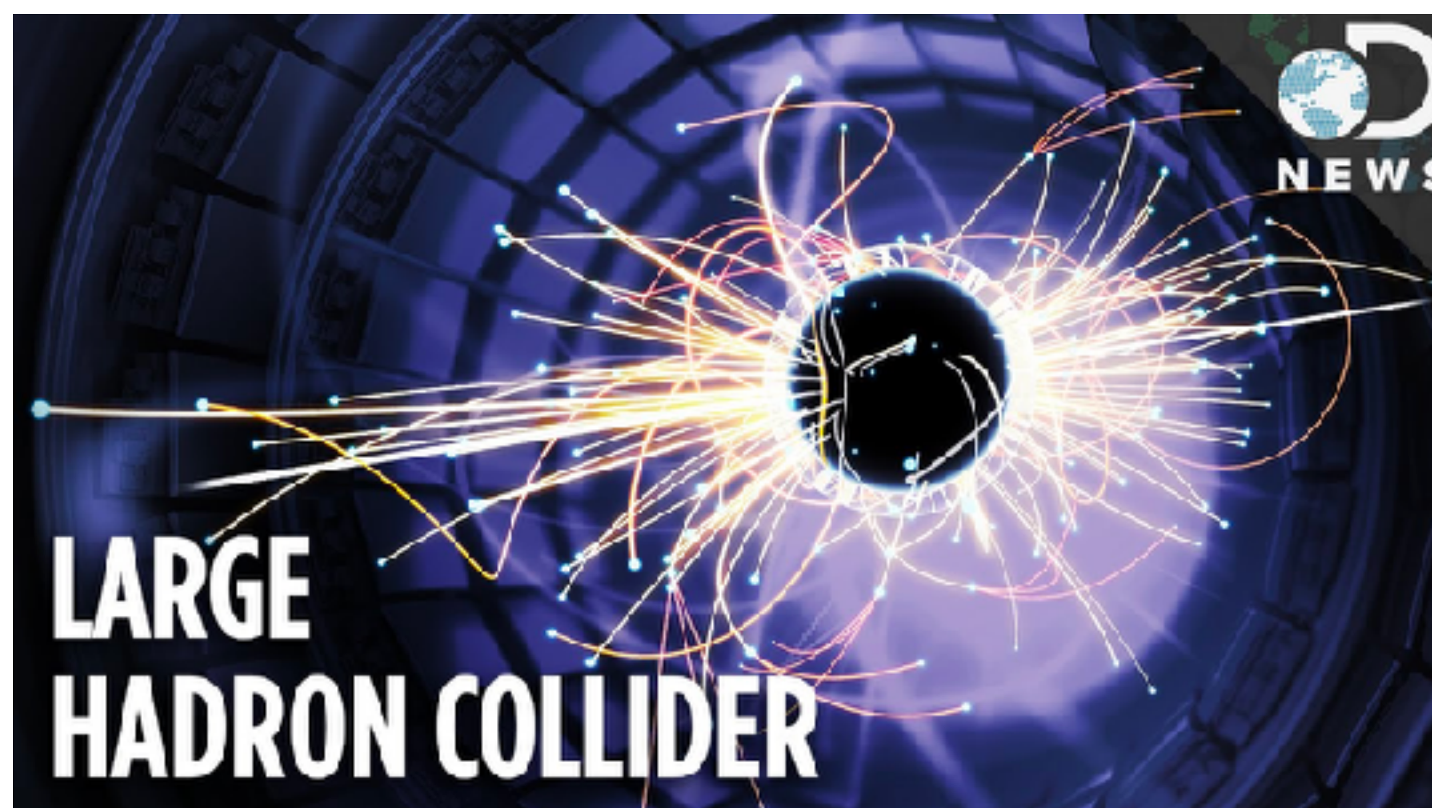
$$E = mc^2$$

Quantum Effect:

$$\lambda = \frac{h}{mc},$$

Table 2: Base Planck units

Name	Dimension	Expression	Value (SI unit)s[2]
Planck length	Length (L)	$l_{\rm P} = \sqrt{\frac{\hbar G}{c^3}}$	$1.616\,255(18) \times 10^{-35}$ m[6]
Planck mass	Mass (M)	$m_{\rm P} = \sqrt{\frac{\hbar c}{G}}$	$2.176\,435(24) \times 10^{-8}$ kg[7]
Planck time	Time (T)	$t_{\rm P} = \frac{l_{\rm P}}{c} = \frac{\hbar}{m_{\rm P} c^2} = \sqrt{\frac{\hbar G}{c^5}}$	$5.391\,245(60) \times 10^{-44}$ s[8]
Planck charge	Electric charge (Q)	$q_{\rm P} = \sqrt{4\pi\varepsilon_0 \hbar c} = \frac{e}{\sqrt{\alpha}}$	$1.875\,545\,956(41) \times 10^{-18}$ C[9][4][10]
Planck temperature	Temperature (Θ)	$T_{\rm P} = \frac{m_{\rm P} c^2}{k_{\rm B}} = \sqrt{\frac{\hbar c^5}{G k_{\rm B}^2}}$	$1.416\,785(16) \times 10^{32}$ K[11]



Planck Scale

Size of a black hole:

$$r_{\rm BH} = Gm/c^2$$

Energy:

$$E = mc^2$$

Quantum Effect:

$$\lambda = \frac{h}{mc},$$

Table 2: Base Planck units

Name	Dimension	Expression	Value (SI unit)s[2]
Planck length	Length (L)	$l_P = \sqrt{\frac{\hbar G}{c^3}}$	$1.616\,255(18) \times 10^{-35}$ m[6]
Planck mass	Mass (M)	$m_P = \sqrt{\frac{\hbar c}{G}}$	$2.176\,435(24) \times 10^{-8}$ kg[7]
Planck time	Time (T)	$t_P = \frac{l_P}{c} = \frac{\hbar}{m_P c^2} = \sqrt{\frac{\hbar G}{c^5}}$	$5.391\,245(60) \times 10^{-44}$ s[8]
Planck charge	Electric charge (Q)	$q_P = \sqrt{4\pi\varepsilon_0 \hbar c} = \frac{e}{\sqrt{\alpha}}$	$1.875\,545\,956(41) \times 10^{-18}$ C[9][4][10]
Planck temperature	Temperature (Θ)	$T_P = \frac{m_P c^2}{k_B} = \sqrt{\frac{\hbar c^5}{G k_B^2}}$	$1.416\,785(16) \times 10^{32}$ K[11]

Table 6: How Planck units simplify the key equations of physics

	SI form	Nondimensionalized form
Newton's law of universal gravitation	$F = G\frac{m_1 m_2}{r^2}$	$F = \frac{m_1 m_2}{r^2}$
Einstein field equations in general relativity	$G_{\mu\nu} = 8\pi \frac{G}{c^4} T_{\mu\nu}$	$G_{\mu\nu} = 8\pi T_{\mu\nu}$
Mass–energy equivalence in special relativity	$E = mc^2$	$E = m$
Energy–momentum relation	$E^2 = m^2 c^4 + p^2 c^2$	$E^2 = m^2 + p^2$
Thermal energy per particle per degree of freedom	$E = \frac{1}{2} k_B T$	$E = \frac{1}{2} T$
Boltzmann's entropy formula	$S = k_B \ln \Omega$	$S = \ln \Omega$

Natural units

- Fundamental constants

Table 1: Dimensional universal physical constants

Constant	Symbol	Dimension
Speed of light in vacuum	c	$\mathsf{L}\,\mathsf{T}^{-1}$
Gravitational constant	G	$\mathsf{L}^3\,\mathsf{M}^{-1}\,\mathsf{T}^{-2}$
Reduced Planck constant	$\hbar = \frac{h}{2\pi}$ where h is the Planck constant	$\mathsf{L}^2\,\mathsf{M}\,\mathsf{T}^{-1}$
Coulomb constant	$k_e = \frac{1}{4\pi\varepsilon_0}$ where ε_0 is the permittivity of free space	$\mathsf{L}^3\,\mathsf{M}\,\mathsf{T}^{-2}\,\mathsf{Q}^{-2}$
Boltzmann constant	k_B	$\mathsf{L}^2\,\mathsf{M}\,\mathsf{T}^{-2}\,\Theta^{-1}$

Natural units

- Fundamental constants

Table 1: Dimensional universal physical constants

Constant	Symbol	Dimension
Speed of light in vacuum	c	$\mathrm{L\,T^{-1}}$
Gravitational constant	G	$\mathrm{L^3\,M^{-1}\,T^{-2}}$
Reduced Planck constant	$\hbar = \frac{h}{2\pi}$ where h is the Planck constant	$\mathrm{L^2\,M\,T^{-1}}$
Coulomb constant	$k_e = \frac{1}{4\pi\varepsilon_0}$ where ε_0 is the permittivity of free space	$\mathrm{L^3\,M\,T^{-2}\,Q^{-2}}$
Boltzmann constant	k_B	$\mathrm{L^2\,M\,T^{-2}\,\Theta^{-1}}$

$$l_P = c\, t_P$$

$$F_P = \frac{m_P\, l_P}{t_P^2} = G\,\frac{m_P^2}{l_P^2}$$

$$E_P = \frac{m_P\, l_P^2}{t_P^2} = \hbar\,\frac{1}{t_P}$$

$$F_P = \frac{m_P\, l_P}{t_P^2} = \frac{1}{4\pi\varepsilon_0}\,\frac{q_P^2}{l_P^2}$$

$$E_P = \frac{m_P\, l_P^2}{t_P^2} = k_B\, T_P\,.$$

α_G is typically defined[citation needed] in terms of the gravitational attraction between two electrons. More precisely,

$$\alpha_G = \frac{Gm_e^2}{\hbar c} = \left(\frac{m_e}{m_P}\right)^2 \approx 1.7518 \times 10^{-45}$$

宇宙膨胀得较快，这样核合成开始时剩下的中子就较多，从而产生较多的氦，与观测到的氦丰度不符。再有，G 变大时引力增强，行星就靠近太阳，行星的公转周期就会变短；反之，G 变小时，公转周期就会变长。行星的公转周期有没有变化，可以同原子钟来进行比较，因为原子钟的走时与万有引力无关。利用雷达信号从水星和金星表面的反射并用原子钟测时，可以精确测定它们的绕日轨道周期，这样得到的 G 的变化率是

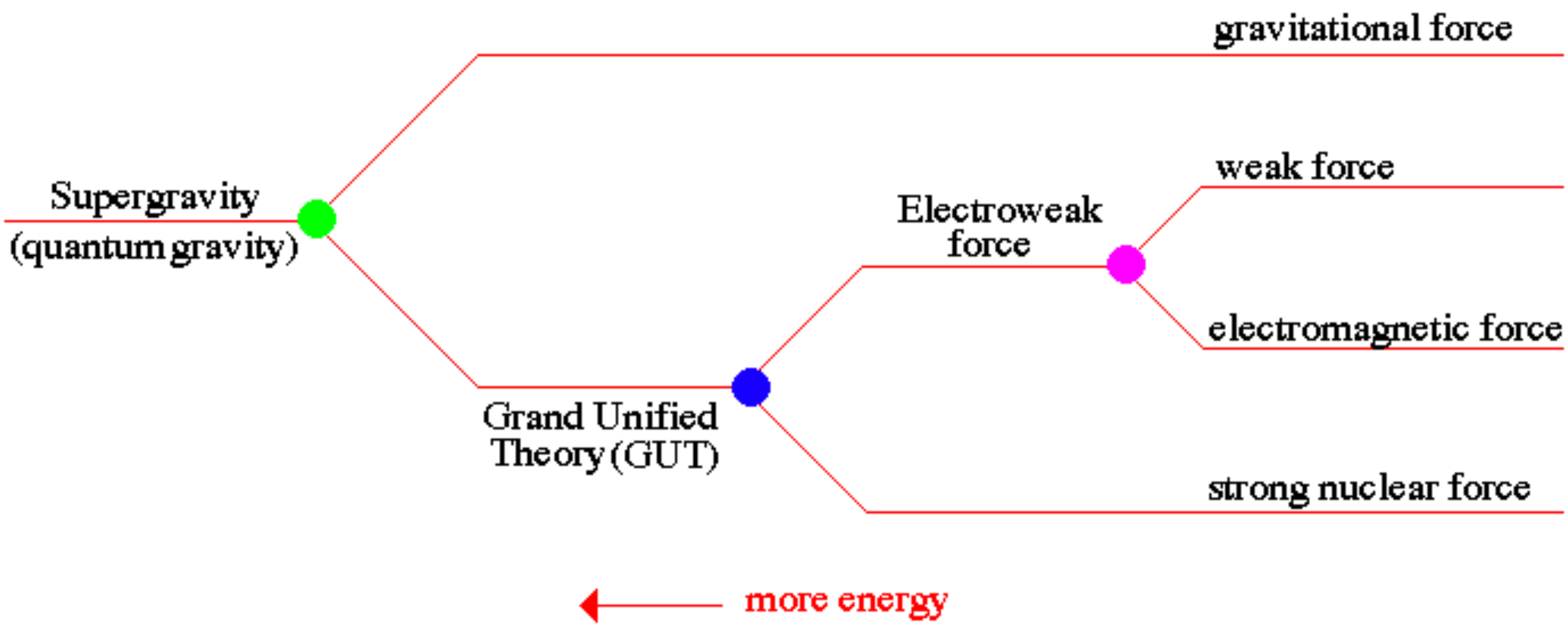
$$\left|\frac{\dot{G}}{G}\right| \leqslant 4\times10^{-10}/\text{年} \tag{7.7.71}$$

以上这些分析都表明，在宇宙过去的时间内，万有引力常数 G 看来不会有明显的变化。

对电子的电荷 e 和质量 m_e 也可做类似的分析。如果 e 随时间变化，就会对原子结构和原子核的性质产生影响，例如放射性同位素的半衰期会发生改变。计算表明，如果 e 的减小速率为 5×10^{-11}/年，宇宙中的锇同位素(^{187}Os)就该几乎衰变光了，但至今地壳中仍有不少 ^{187}Os 存在。由 ^{187}Os 现在的丰度可以推断出，e 的相对年变化率应当小于 10^{-13}/年。同样，如果电子的质量 m_e 发生变化，也会导致^{187}Os 的衰变速率加快。由此推断出，m_e 的相对变化率不应大于 4×10^{-13}/年。而由恒星演化的分析得到的结果是，m_e 的时间变化率$<4\times10^{-12}$/年。

Unification

all the forces of Nature should be capable of being described by a single theory. But only at high energies should the behavior of the forces combine, this is called unification

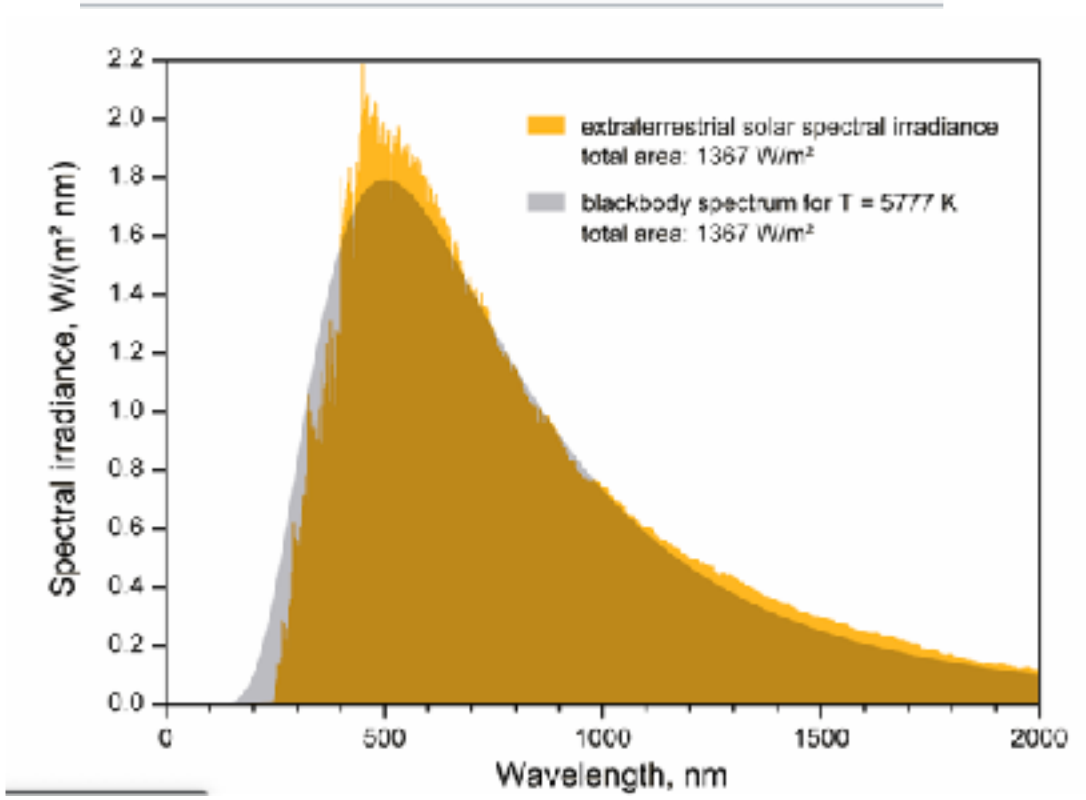


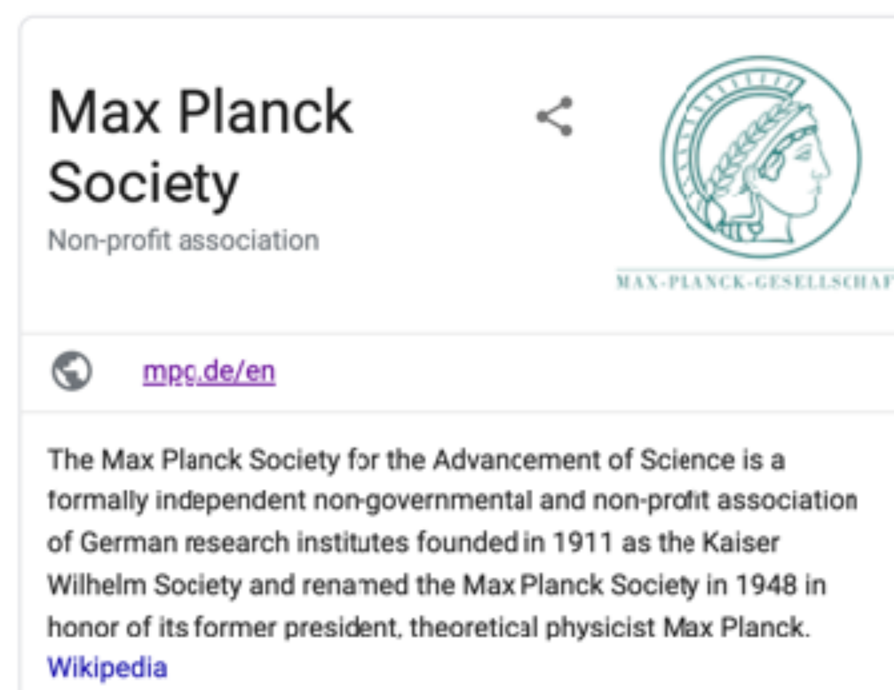
before the unification point, the forces are indistinguishable and have symmetry. After the unification point, the forces act differently and the symmetry is broken.

Max-Planck

- Max Karl Ernst Ludwig Planck
- Age 38 (1900):
    - Discover the Planck Law
- Max-Planck Institute

$$B_\nu(\nu, T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/(k_\mathrm{B}T)} - 1}$$





Summary

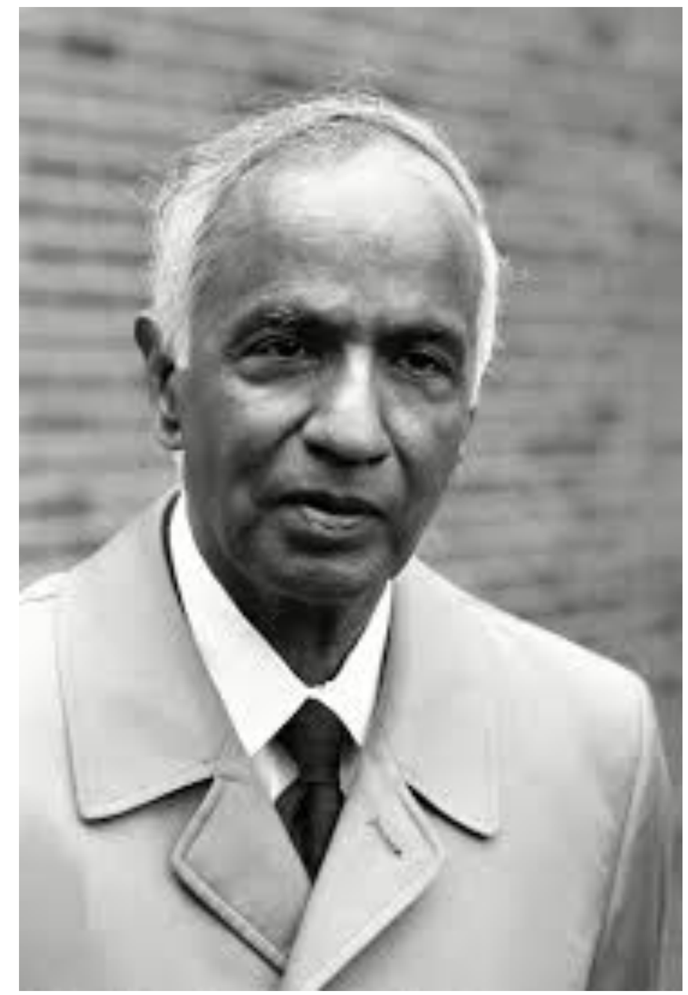
- Everything you can measure is dimensionless
- Planck Units: Key to Blackbody radiation
- Planck Units: Limit of physics

Part 2 — WD, NS and Chandrasekhar Mass

Chandrasekhar Mass

$$M_{\text{limit}} = \frac{\omega_3^0 \sqrt{3\pi}}{2} \left(\frac{\hbar c}{G} \right)^{\frac{3}{2}} \frac{1}{(\mu_e m_H)^2}$$

- $\hbar$ is the reduced Planck constant
- c is the speed of light
- G is the gravitational constant
- μ_e is the average molecular weight per e
- m_H is the mass of the hydrogen atom.



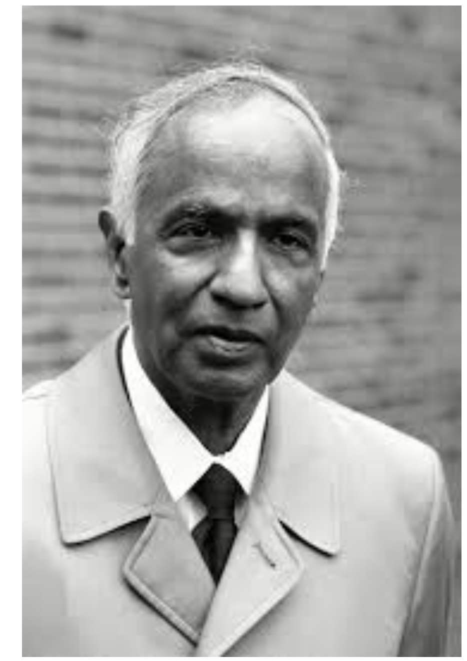
As $\sqrt{\hbar c/G}$ is the Planck mass,

$$\frac{M_{\text{Pl}}^3}{m_H^2}$$

Chandrasekhar(?) Mass

$$M_{\text{limit}} = \frac{\omega_3^0 \sqrt{3\pi}}{2} \left(\frac{\hbar c}{G} \right)^{\frac{3}{2}} \frac{1}{(\mu_e m_H)^2}$$

- $\hbar$ is the reduced Planck constant
- c is the speed of light
- G is the gravitational constant
- μ_e is the average molecular weight per e
- m_H is the mass of the hydrogen atom.



1930

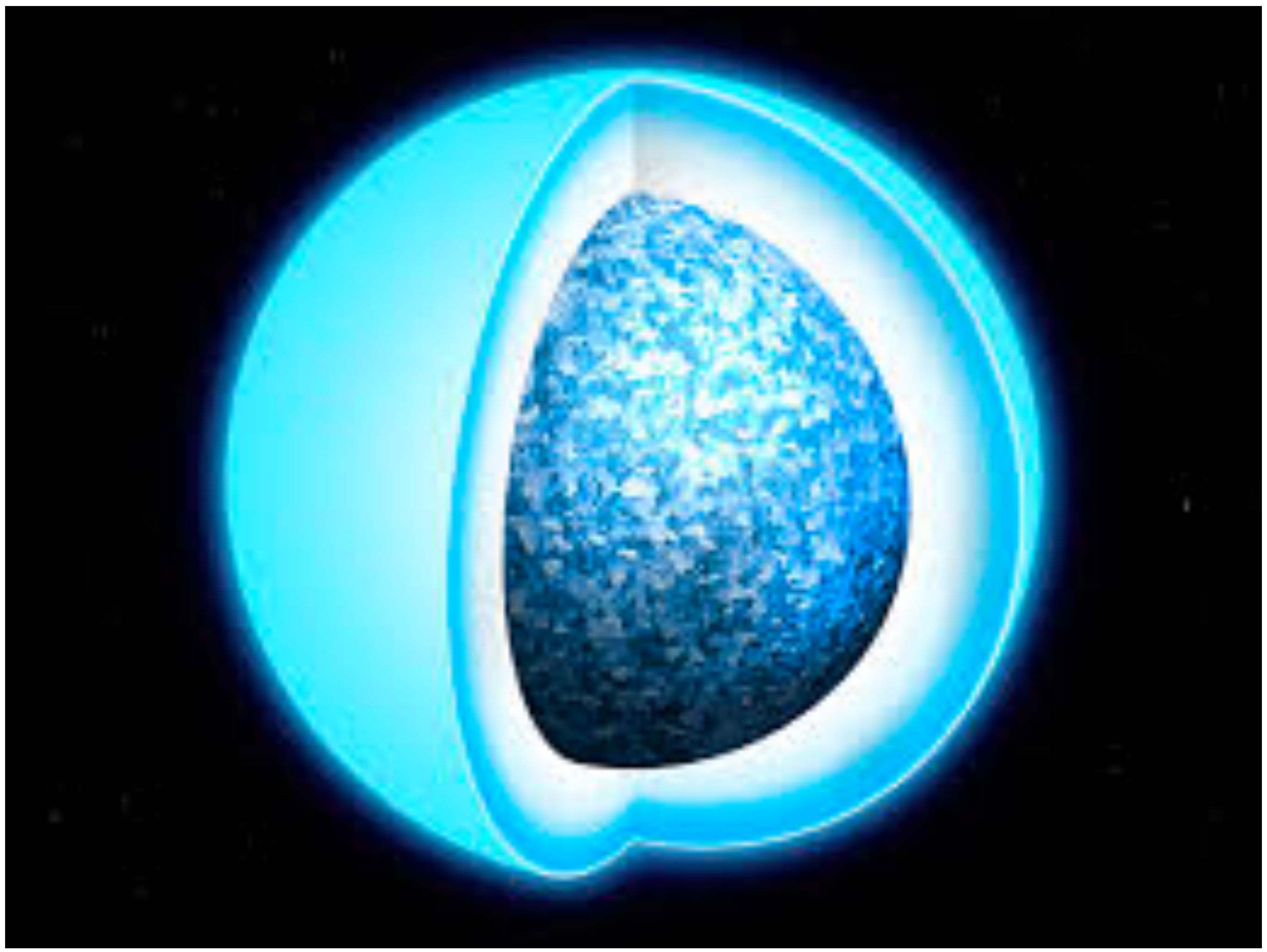
1929

discovered in separate papers published by Wilhelm Anderson and E. C. Stoner in **1929**. The limit was initially ignored by the community of scientists because such a limit would logically require the existence of black holes, which were considered a scientific impossibility at the time.

Eddington

Part 2.1 — WD, NS and Chandrasekhar Mass

Chandrasekhar Mass



Gravity:

$$E = Gm^2/r$$

Electron Energy Momentum:

$$\boxed{E^2 = (pc)^2 + \left(m_0 c^2\right)^2}$$

(1) **Stability of star**
(2) **NS vs. WD**
(3) **Limiting mass**

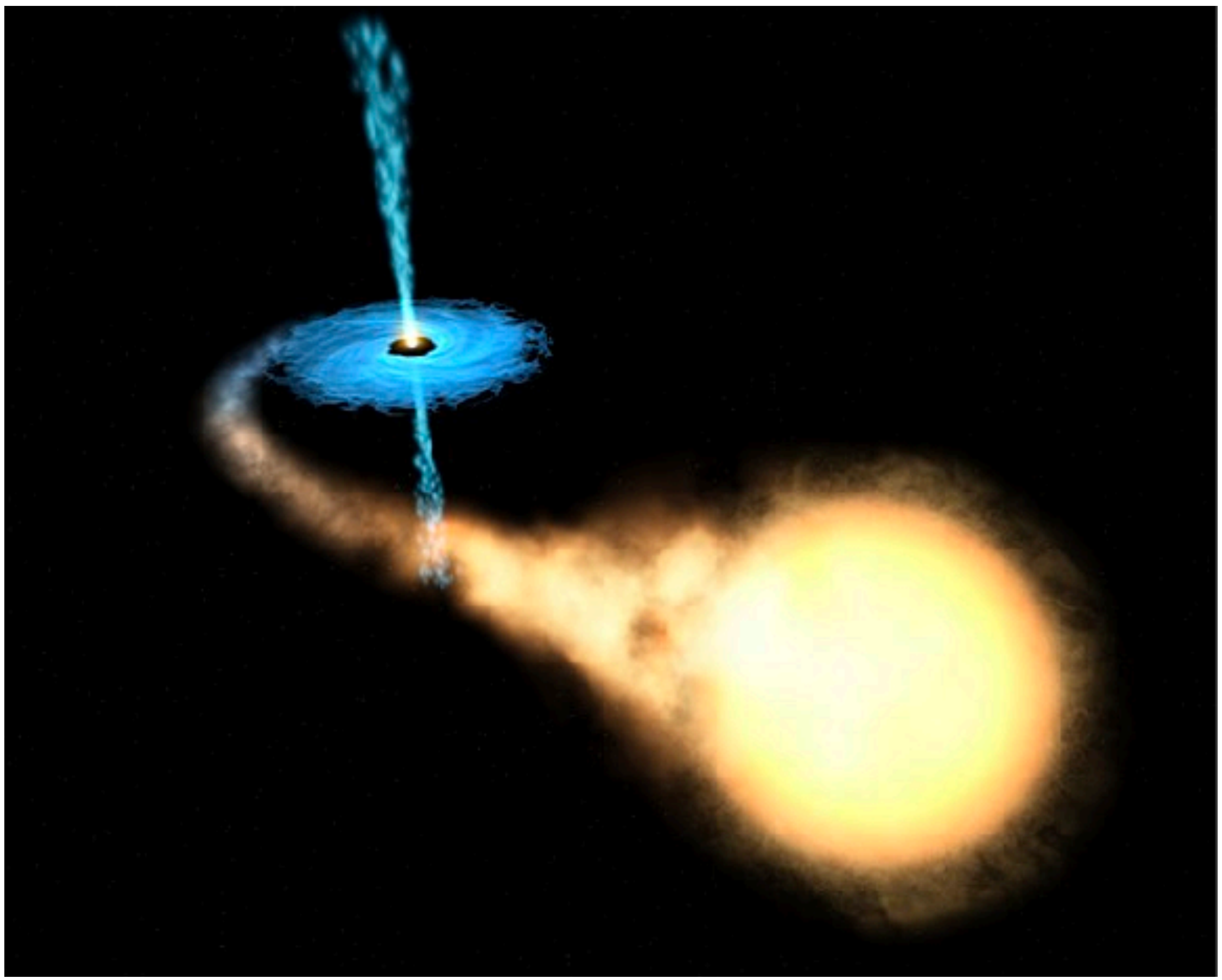
WD & NS

- Basic ingredients
    - Gravity, show that p~rho^{5/3}
- Degenerate Fermi gas
    - Assuming $n \sim (1/\delta x) * 3$
    - Show that
        - Relativistic $\boxed{E^2 = (pc)^2 + (m_0 c^2)^2}$
        - non-relativistic p~rho^{5/3}
- Explain why NS have a higher density compared to ND

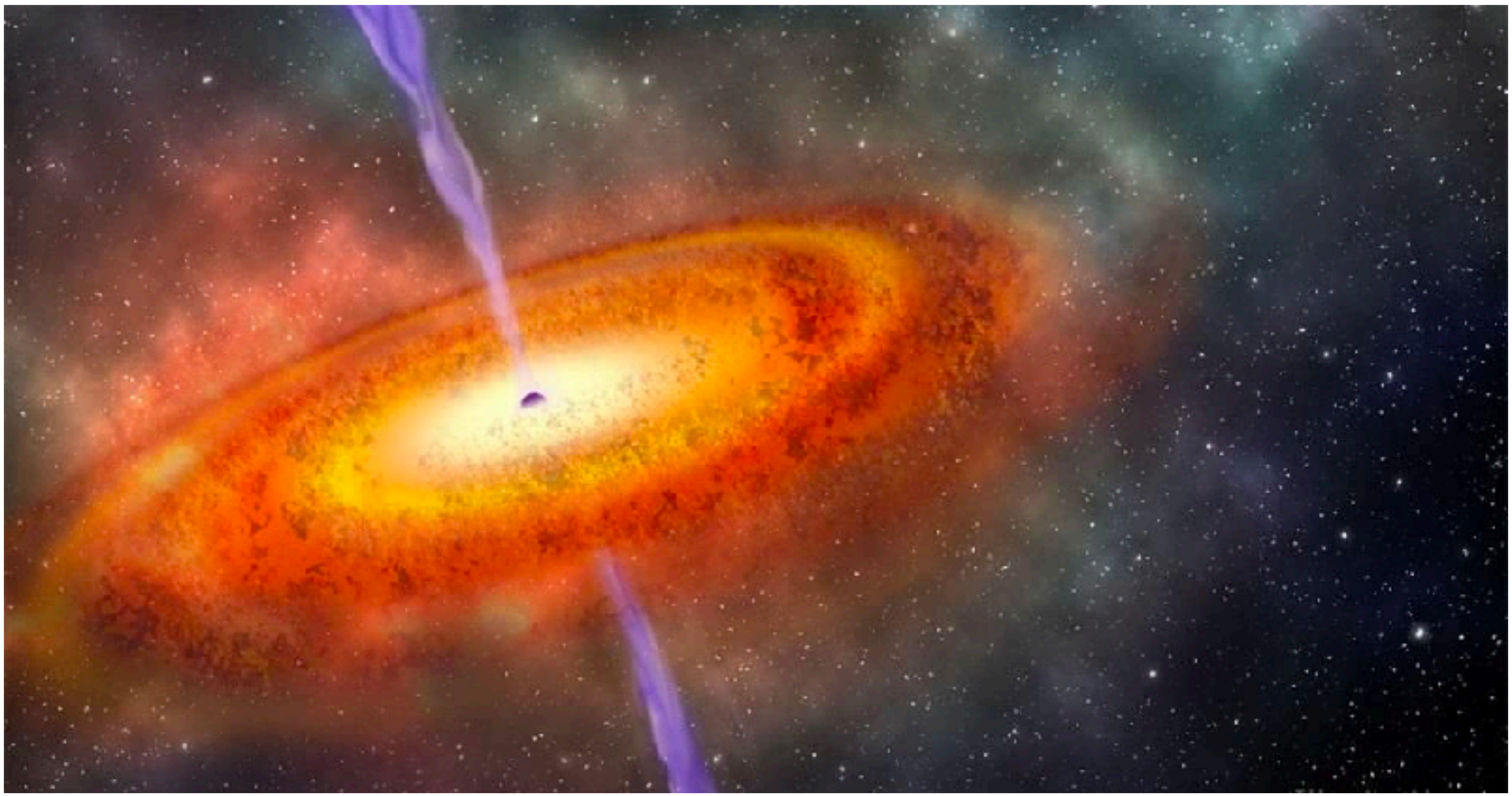
Part 3 — Black holes

Part 3.1 — Black hole mass

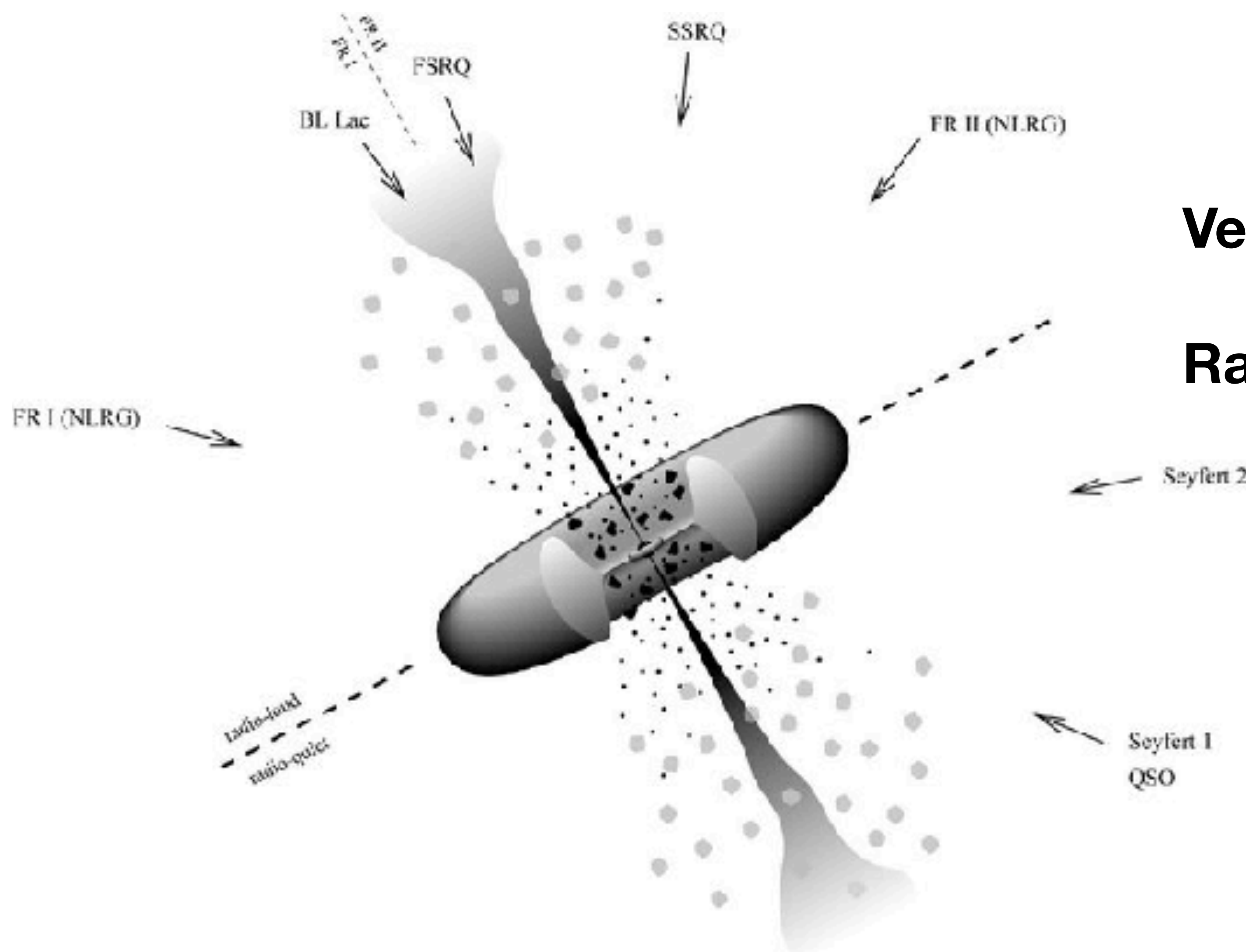
Stellar-Mass Black Hole



Supermassive Black Hole



Supermassive Black Hole



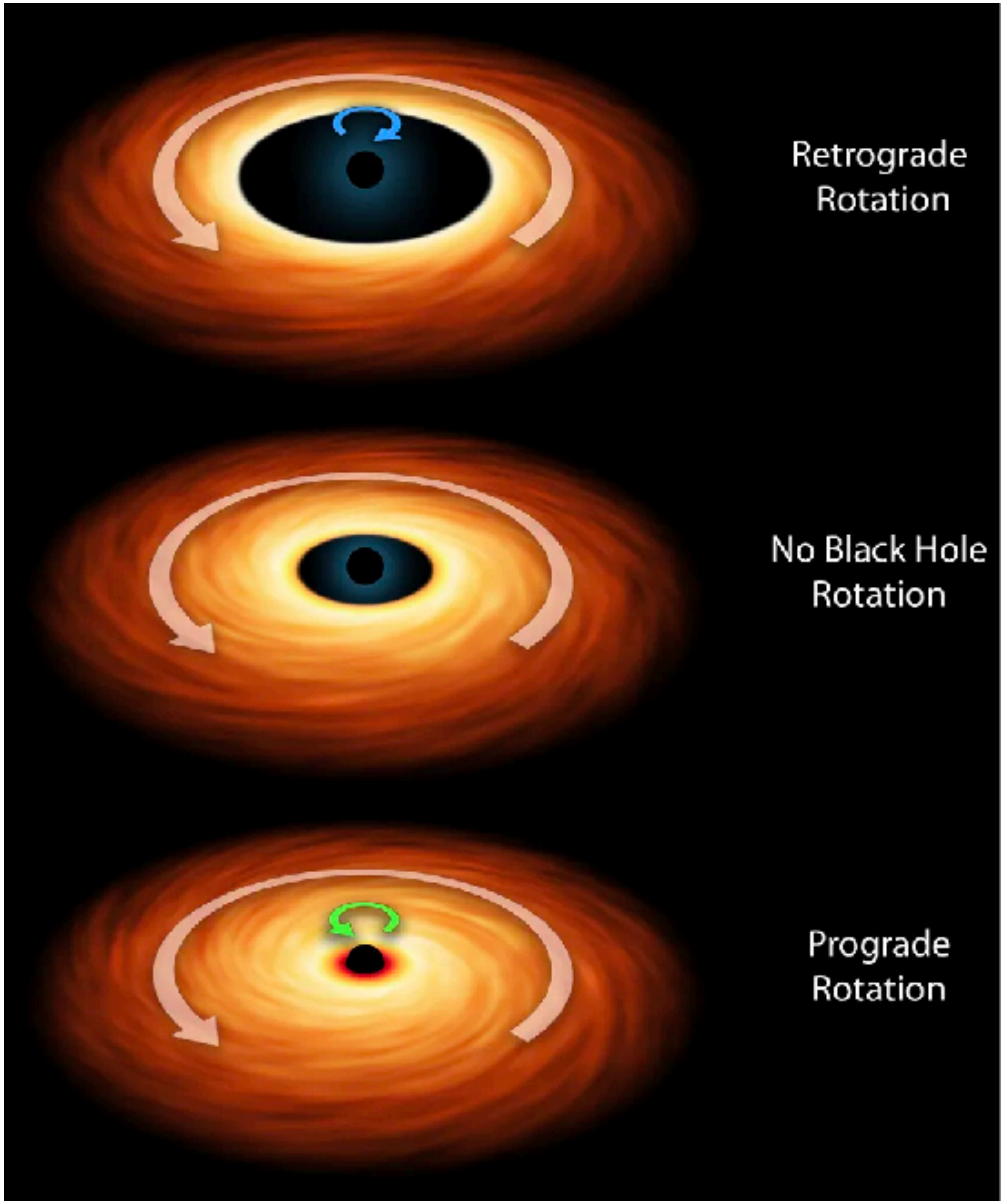
$$M = \sigma_v^2 R/G$$

Velocity dispersion: line width

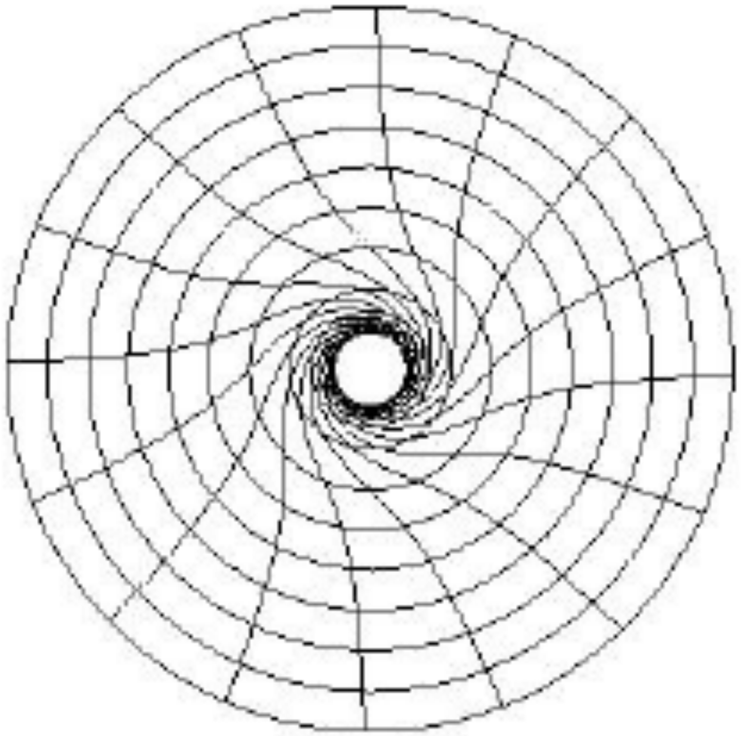
Radius:

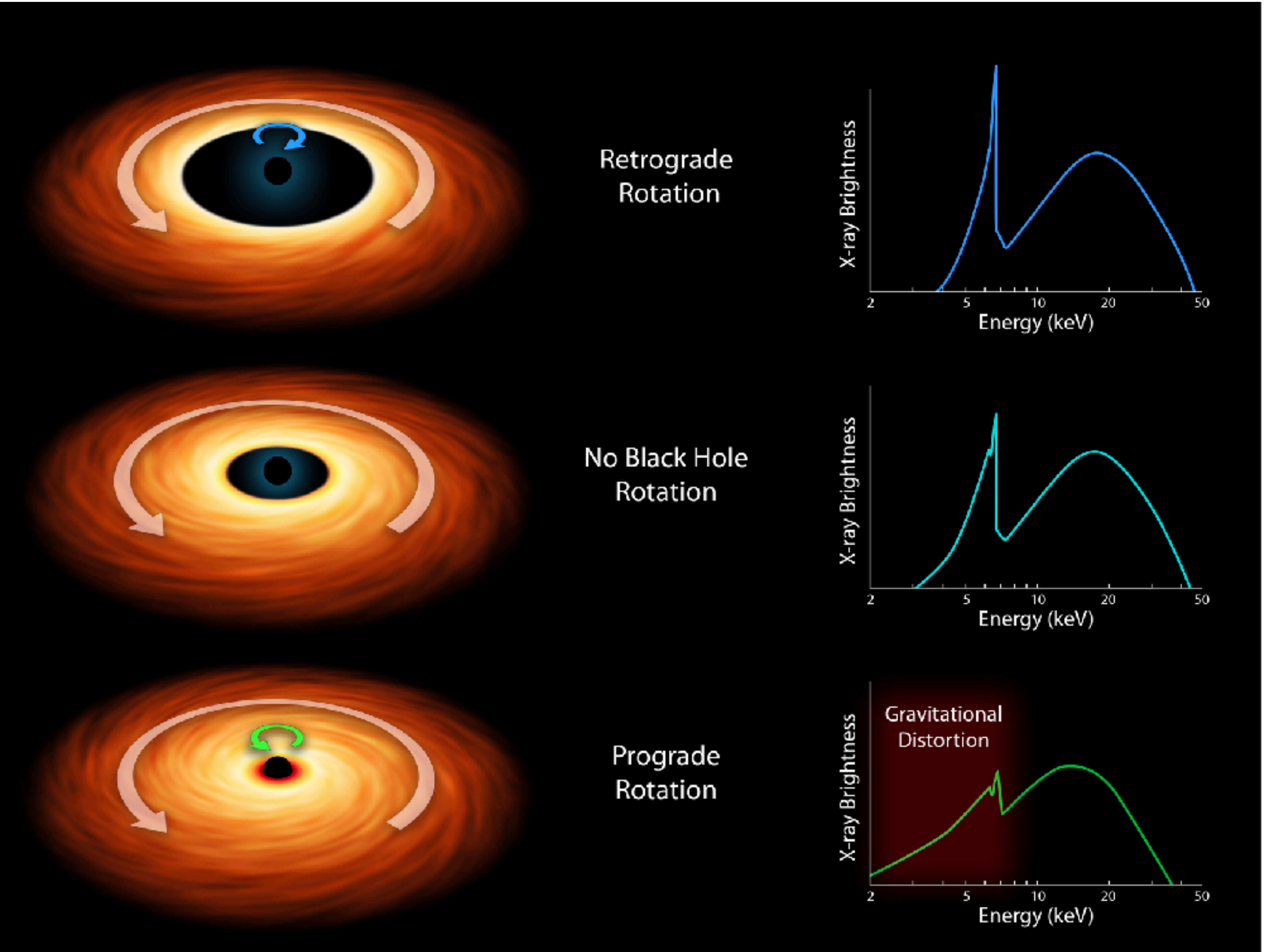
1. **Ionisation equilibrium**
2. **Reverberation mapping**

Part 3.2 — Black hole spin



Spinning black hole distorts space-time

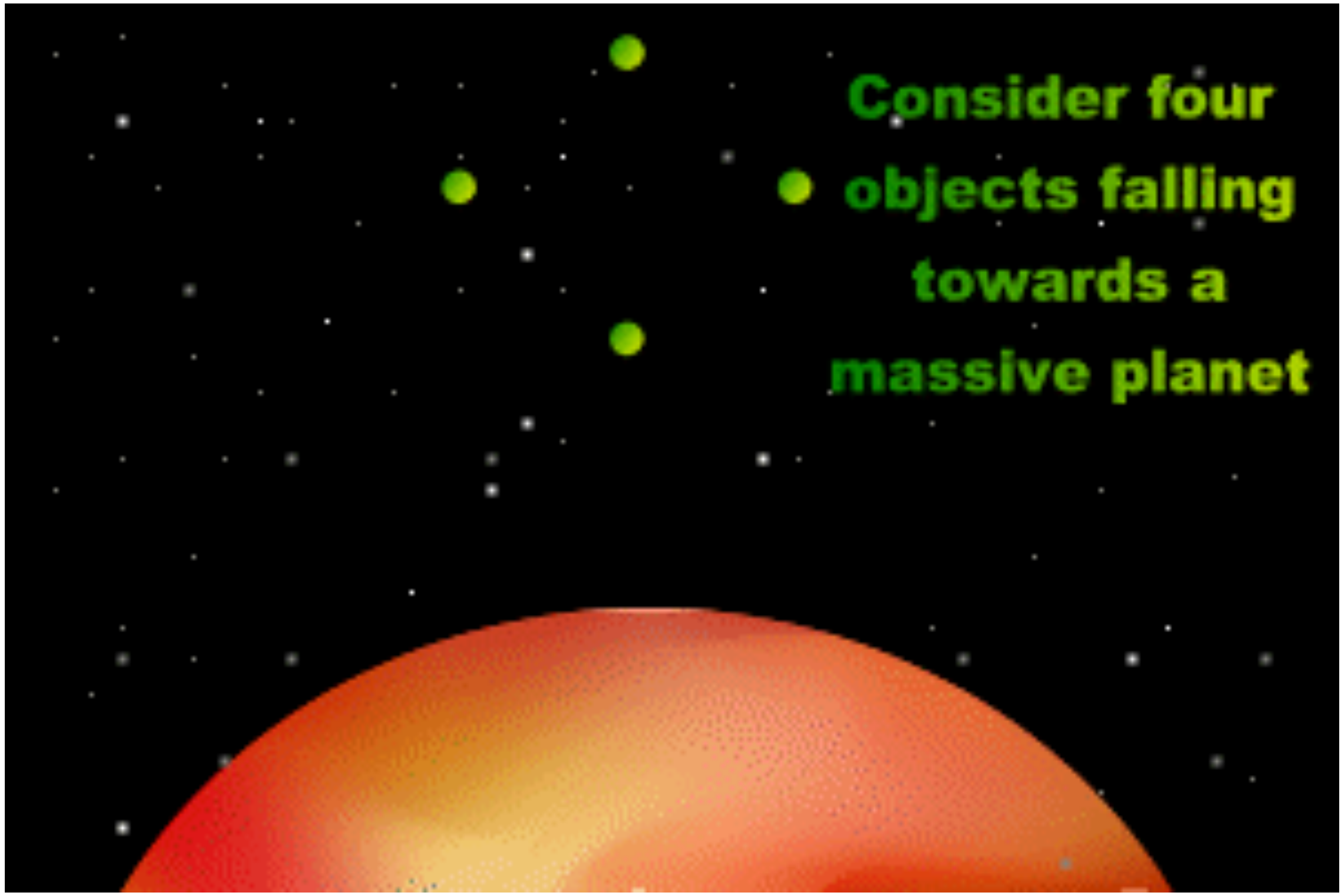



Retrograde Rotation
X-ray Brightness
2 5 10 20 50
Energy (keV)
No Black Hole Rotation
X-ray Brightness
2 5 10 20 50
Energy (keV)
Prograde Rotation
Gravitational Distortion
X-ray Brightness
2 5 10 20 50
Energy (keV)

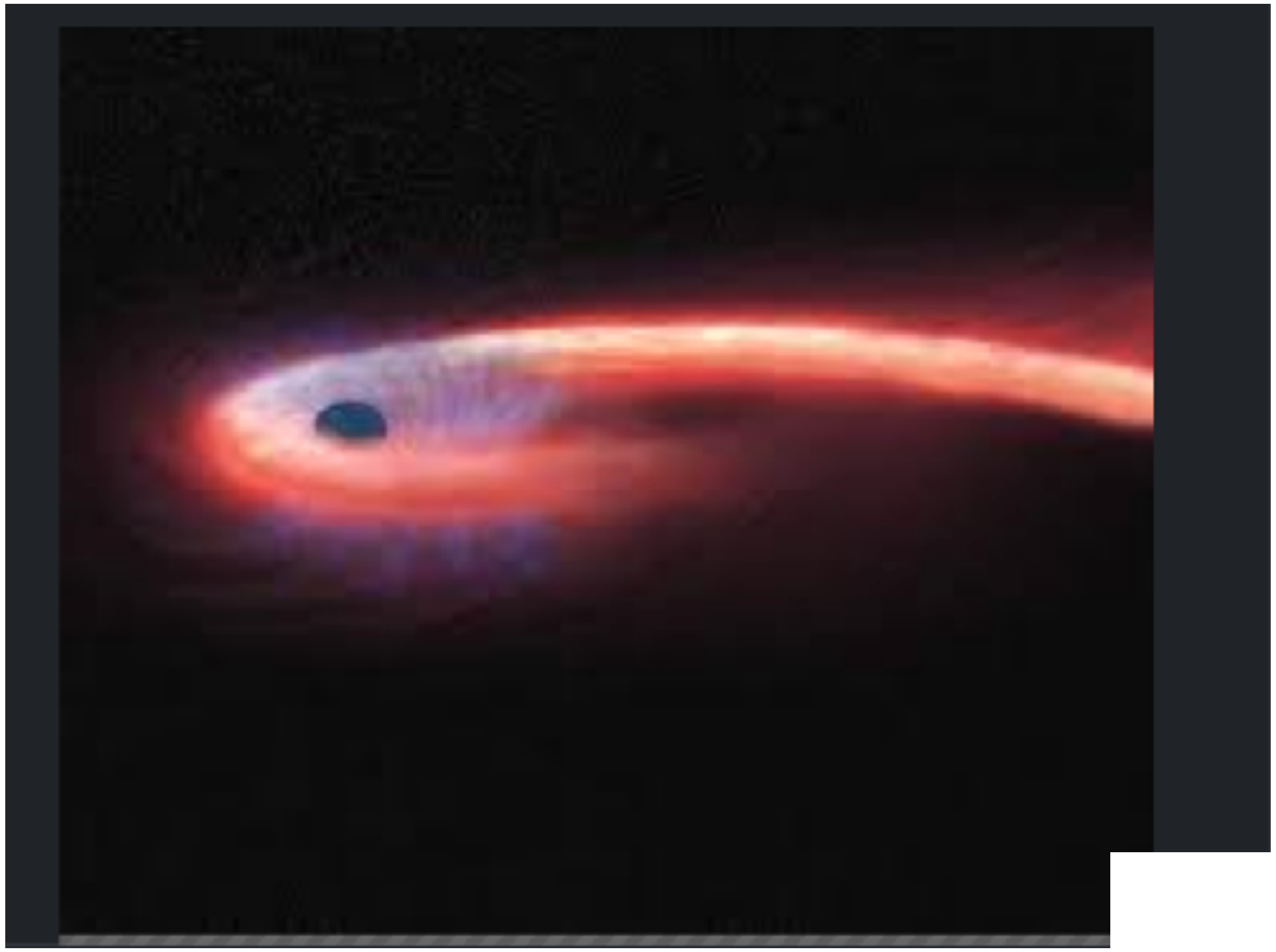
Part 3.2 — Tidal disruption events

Tidal force

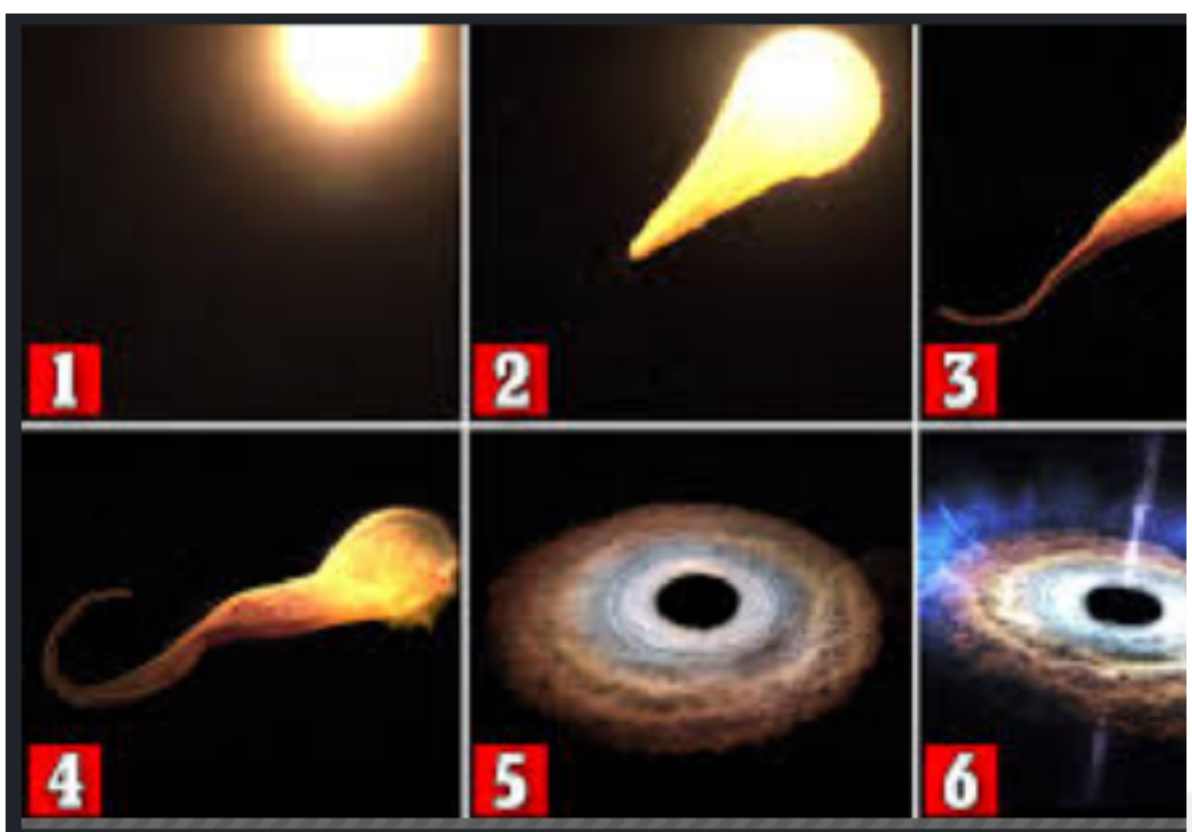
Measurement of tidal force?

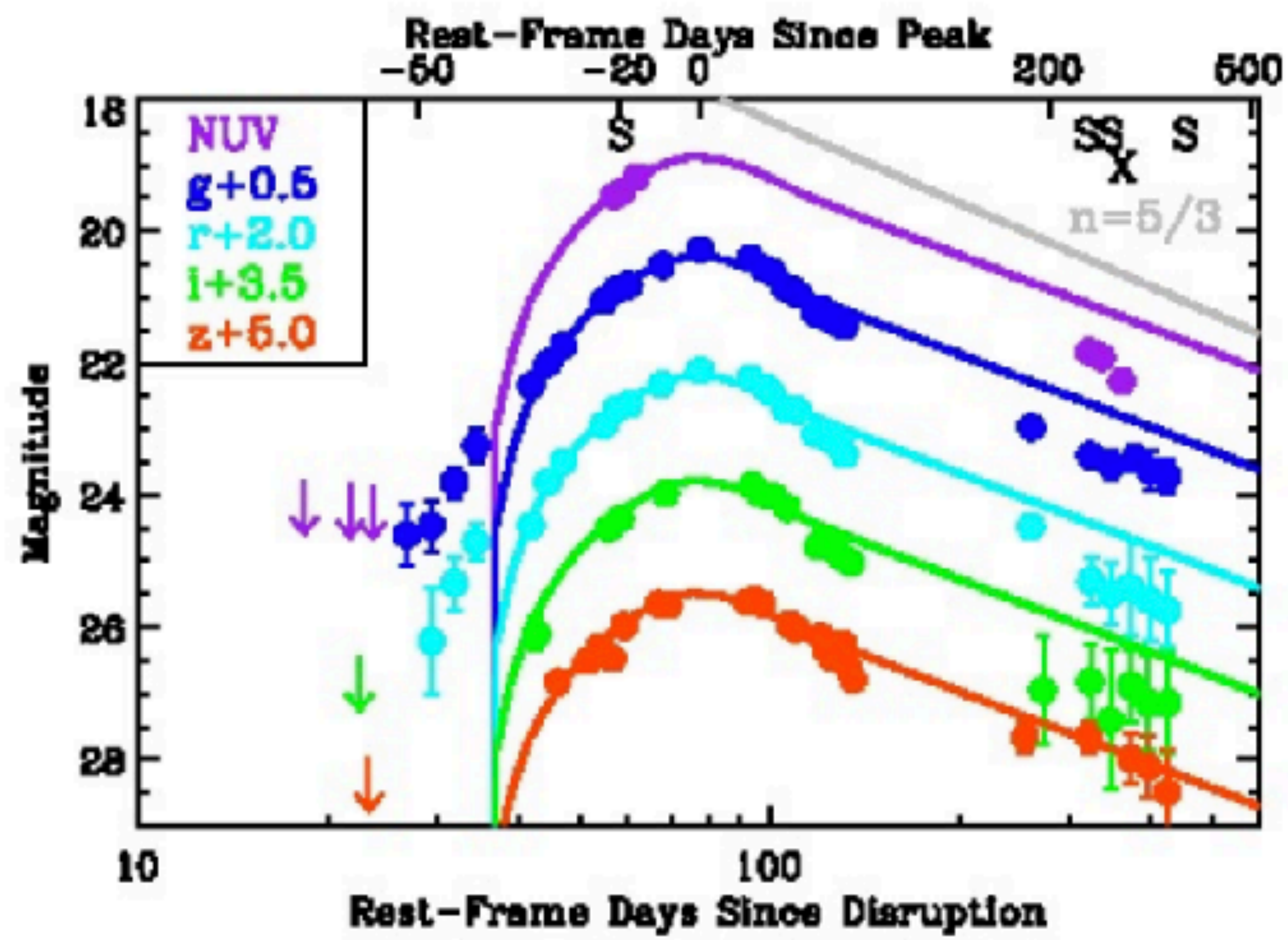


Tidal disruption events



An ultraviolet-optical flare from the tidal disruption of a helium rich stellar core





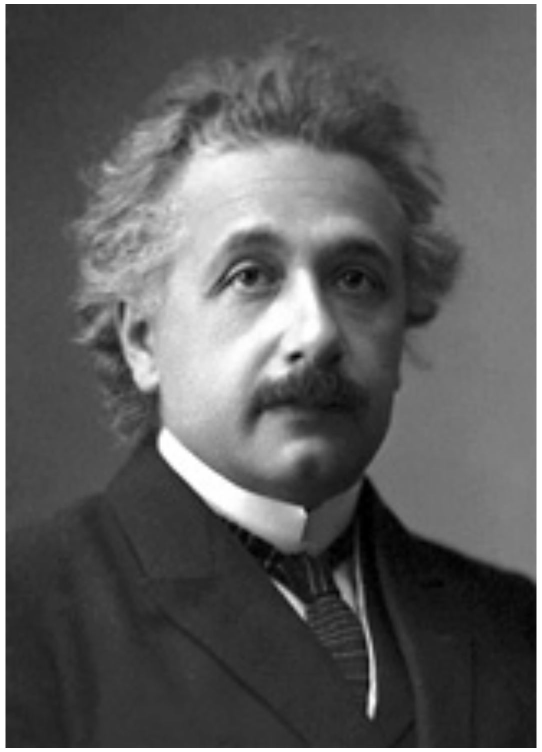
Part 4 — Relativity

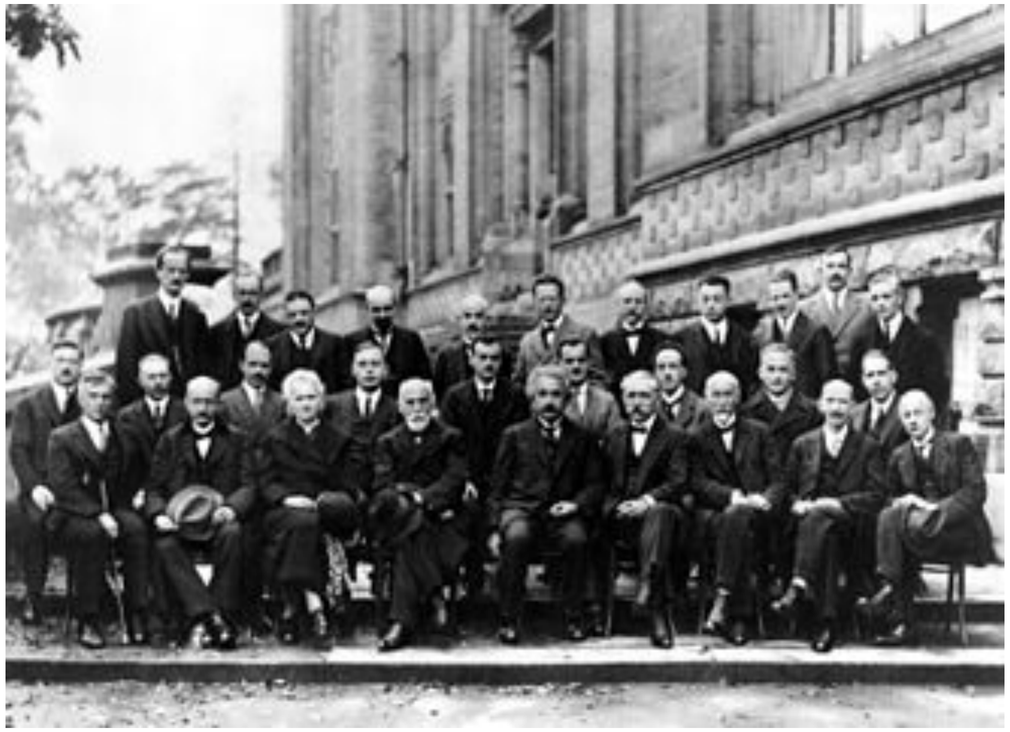
Part 4.1 — History

World Year of Physics 2005

- Microscopic
  - Photoelectric effect
  - Brownian motion
- General Relativity

Einstein's special theory of relativity heralded a new kind of physics, one that digressed from the classical mechanics that had been derived from Newton's calculus.
— from Wikipedia





- Physical review in 1902
  - On the Specific Heat of **Supercooled Water**
  - The Penetration of Totally Reflected Light into the **Rarer Medium**
  - On the Effect of Low Temperature on the Recovery of Overstrained **Iron and Steel**
  - Lecture Room Demonstrations of **Astigmamatism and of Distortion**
  - The Invisibility of **Transparent Objects**
- *On the Electrodynamics of **Moving Bodies***

Olympia Academy

What they actually do?

- Ernst Mach, Henri Poincaré, David Hume , Karl Pearson, John Stuart Mill ``A System of Logic'', Spinoza, ``Ethics'', and Don Quixote
- Karl Pearson, *The Grammar of Science*
- Henri Poincaré, ...

- *Karl Pearson.*
- Mathematical statistics, biometrics, meteorology, theories of social Darwinism and eugenics.
- 1857 – 1936
- Contributions:
    - PCA, chi-squared test
    - ~ 100 publication first authors
    - Writings about physical science

The grammar of science

- Relativity of motion to a **frame of reference** (fixed stars)
- **Equivalence of "matter" and energy**
- ***Physics as geometry***
- the non-existence of the ether
- *the importance of creative imagination rather than mere fact-gathering*
- fourth dimension

The grammar of science

- First published 1892
- Ideas comes from his 10 years teaching
- Author is an idealist (唯心主义)
  - ...science is in reality a classification and analysis of the contents of the mind...

Sense-impression

W

a

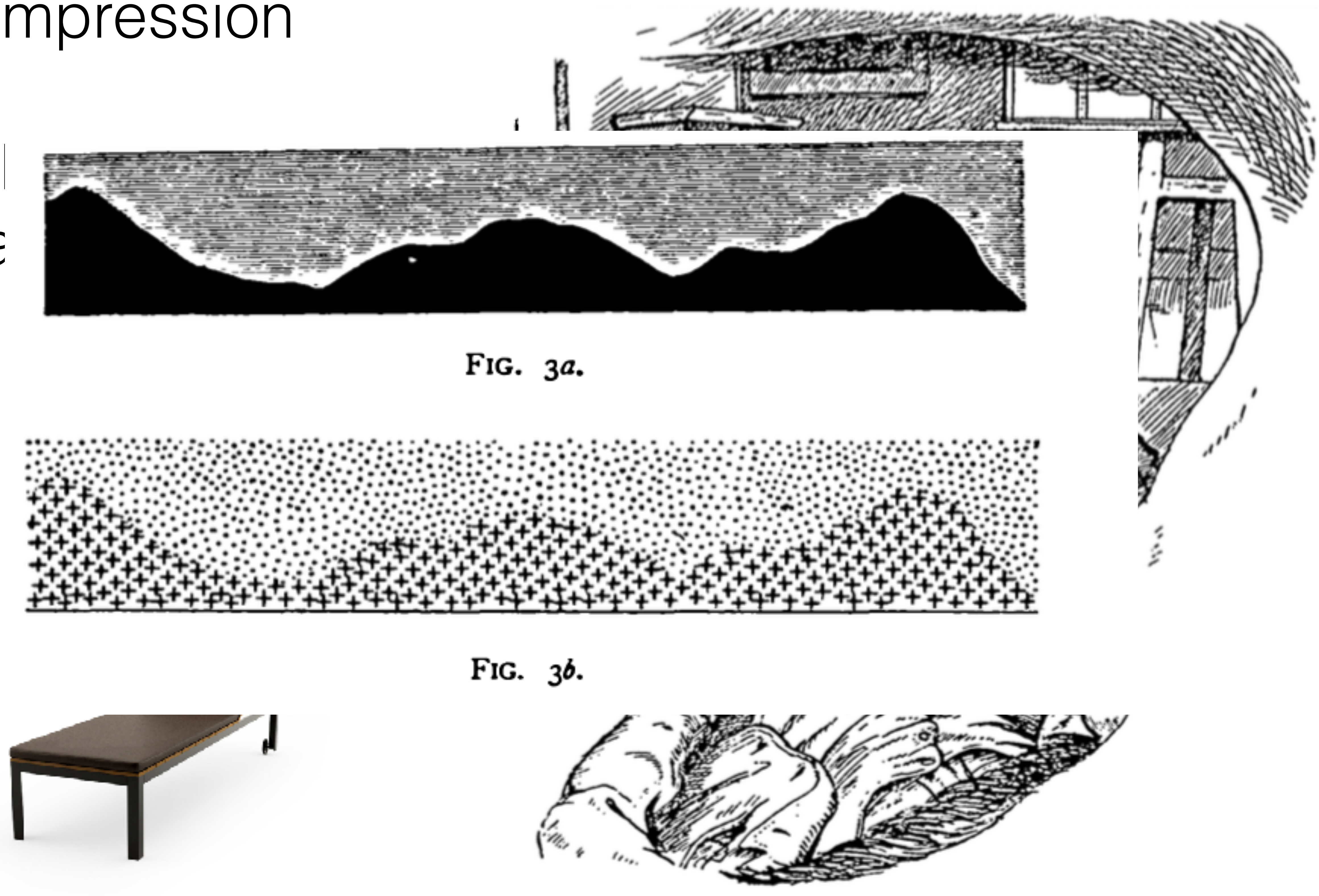


FIG. 3a.

FIG. 3b.

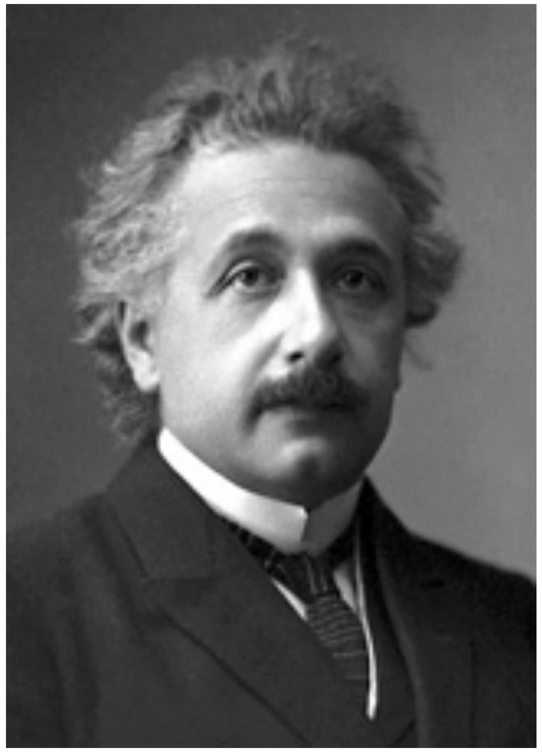
FIG. 1.

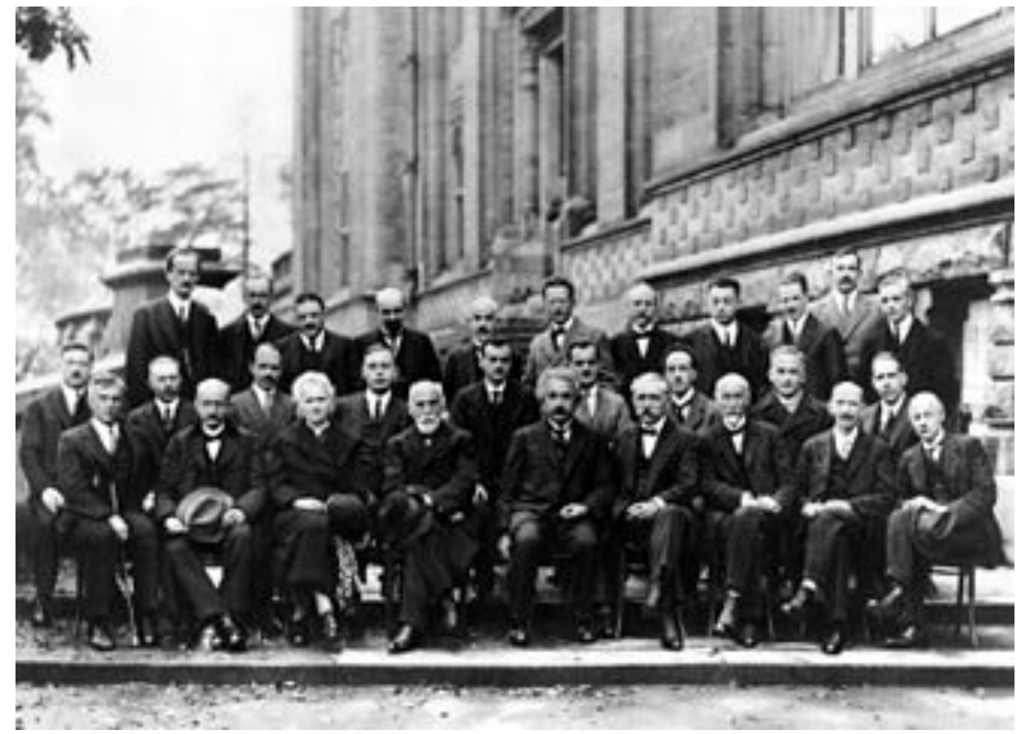
Matter is a primary conception of human mind (Pearson)

… Matter only as that which may have energy communicated to it from other matter ..

Energy, we know only as that which in all natural phenomena is continually passing from one portion of matter to another

… no hope, the only way to understand energy is through matter





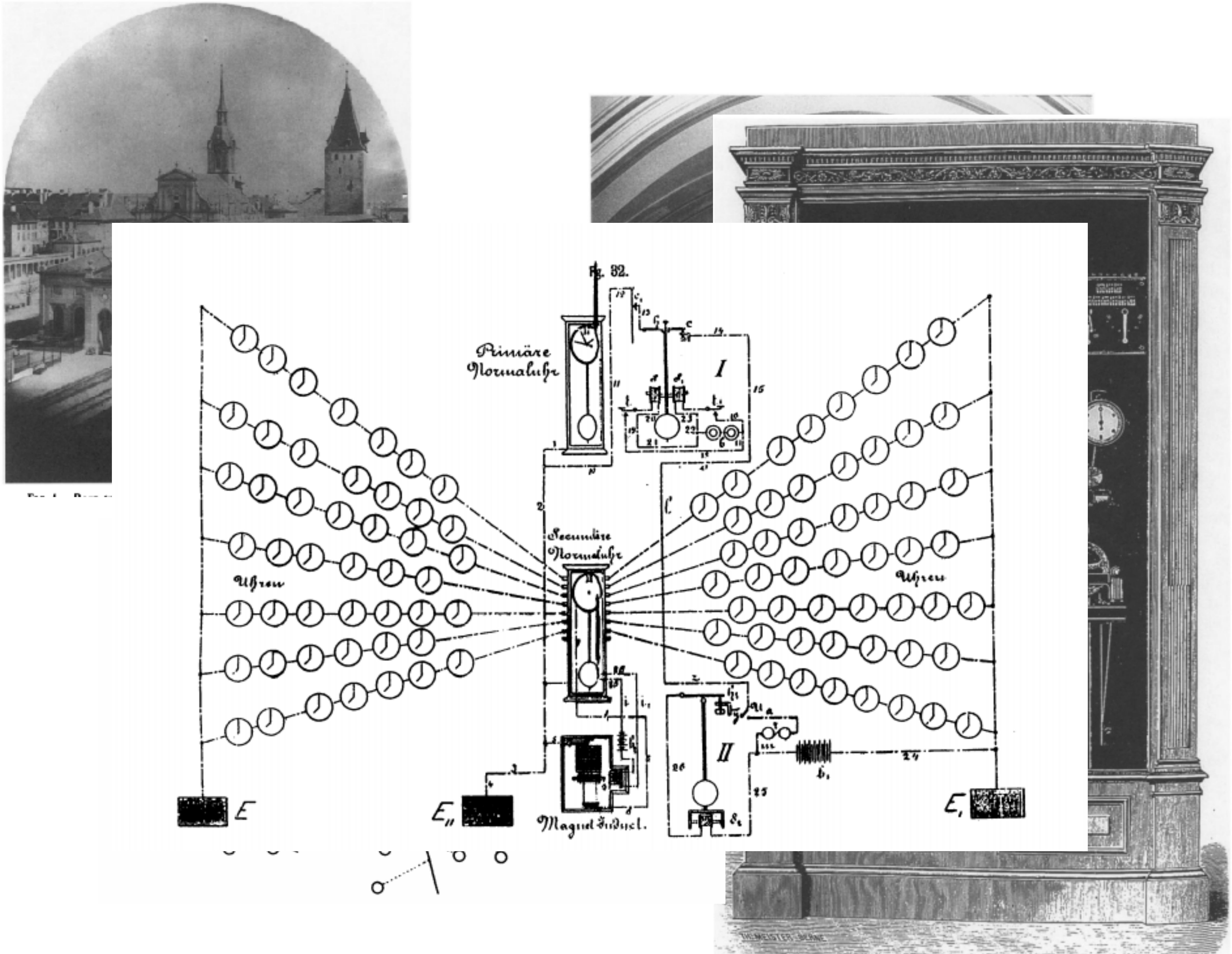


Fig. 32.
Primäre Normaluhr
I
Secundäre Normaluhr
Uhren
Uhren
II
E
E,,
Magnet Induct.
E,

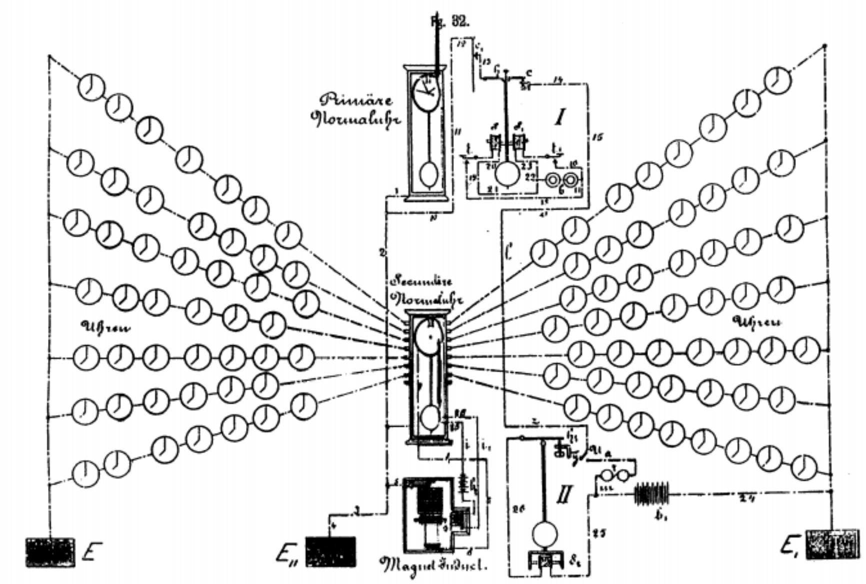
Conclusions

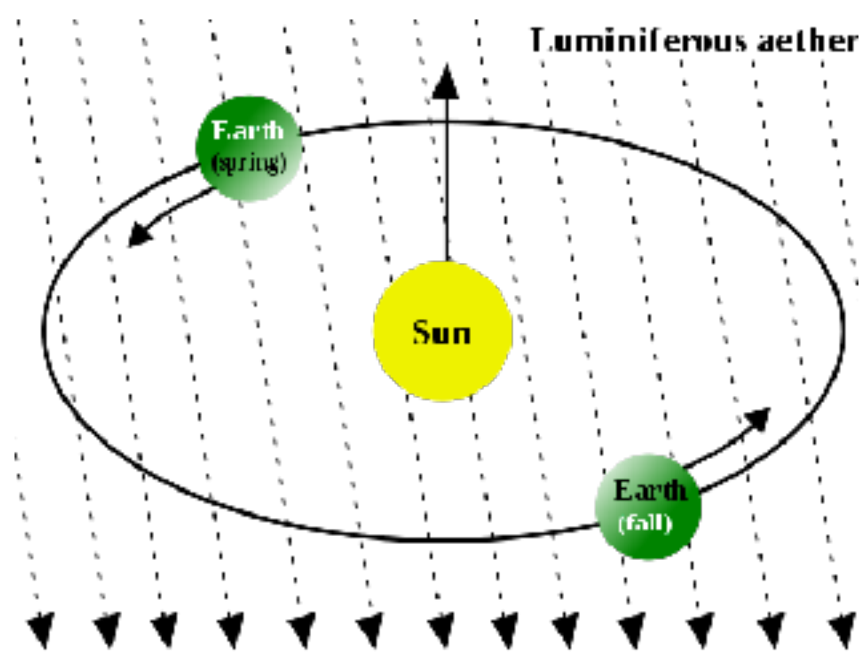
Fig. 1.

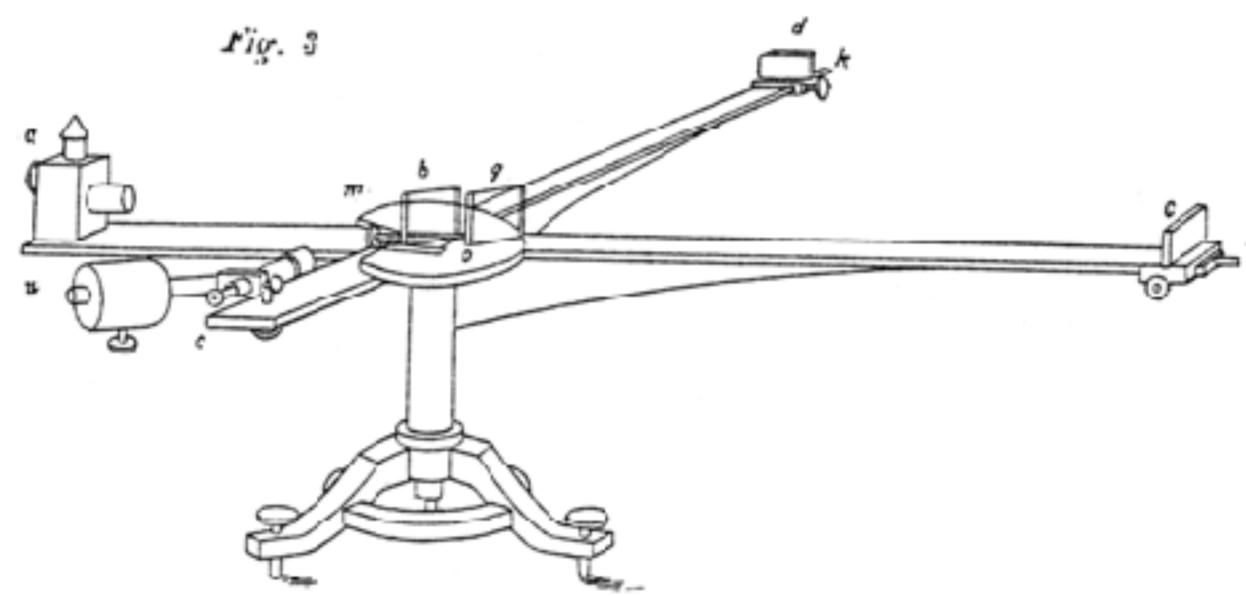
- Genera relativity
  - Philosophy of Karl Pearson
  - Synchronising clocks (patent office)
  - Friends & Competitors

Motivation for special relativity

- Speed of light is constant (from experiments)!
    - c as a fundamental parameter in nature!
- Difficulty in tuning a clock — What is time?

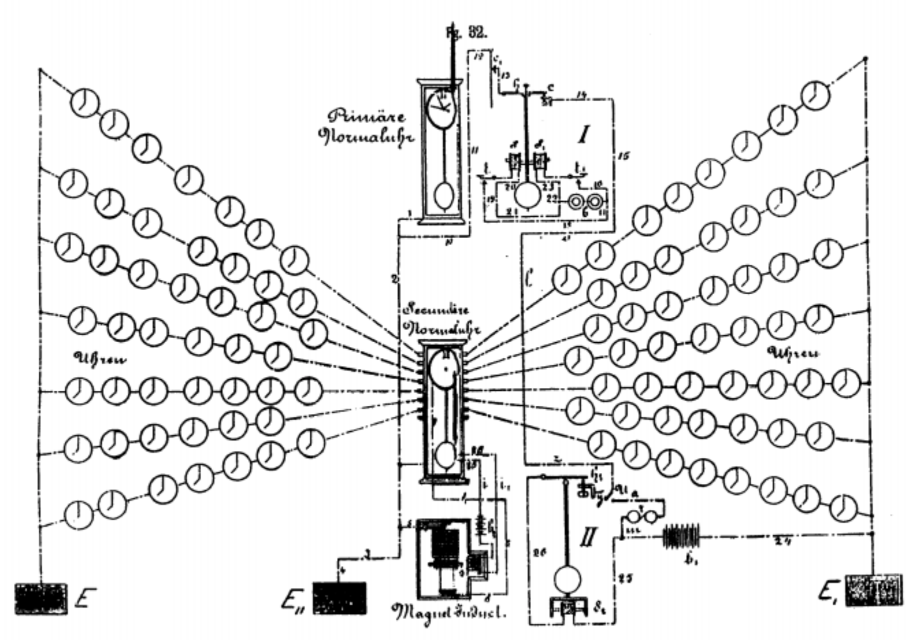


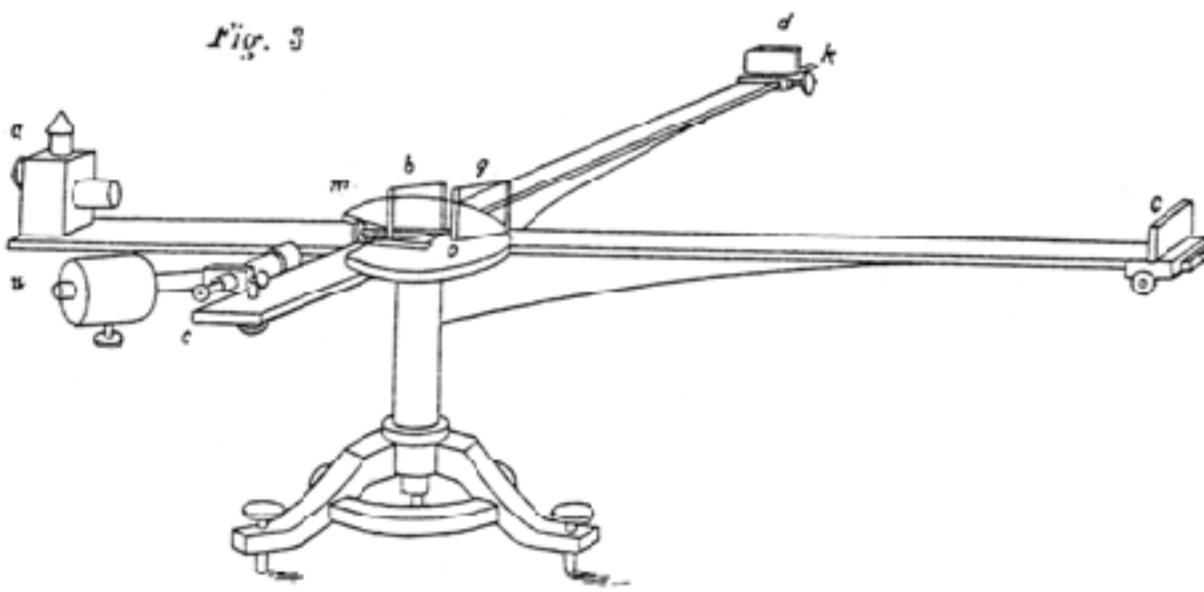




Motivation for special relativity

- Speed of light is constant (from experiments)!
    - c as a fundamental parameter in nature! -> Speed of light as a fundamental parameter
- Difficulty in tuning a clock — What is time? -> Space, time -> Spacetime

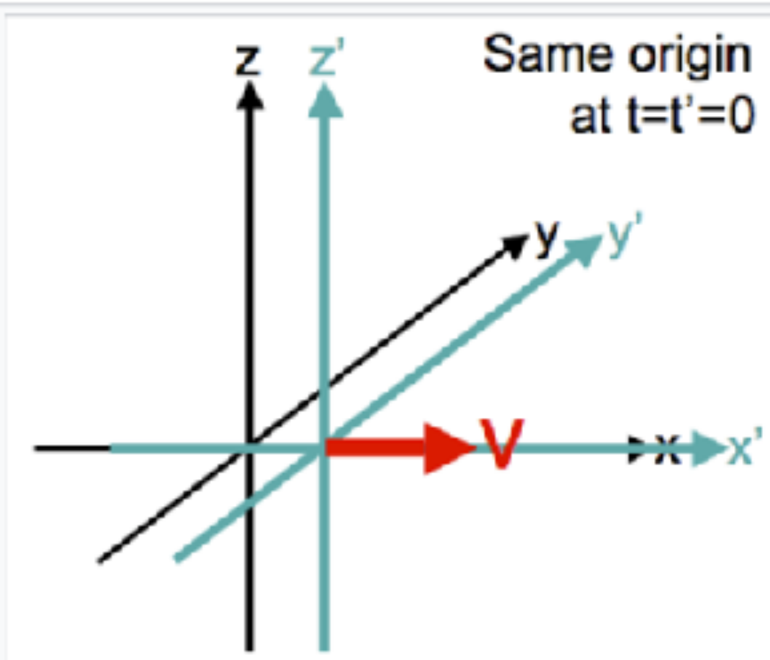




Newton -> Einstein

Galilean transformation (伽利略变换)

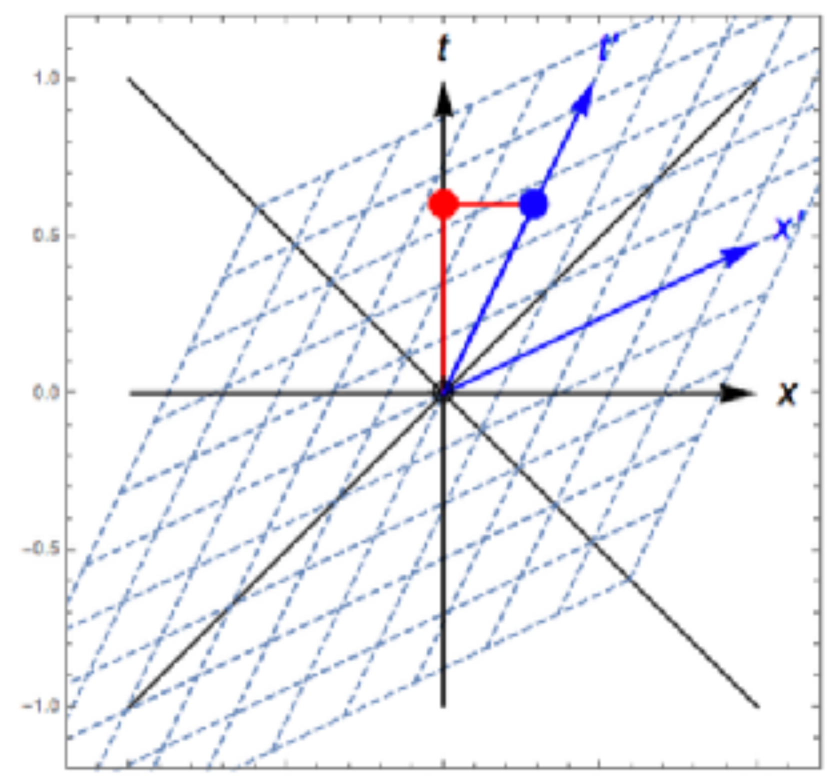
$$
\begin{aligned}
x' &= x - vt \\
y' &= y \\
z' &= z \\
t' &= t.
\end{aligned}
$$



Standard configuration of coordinate systems for Galilean transformations.

Lorentz transformation (洛伦兹变换)

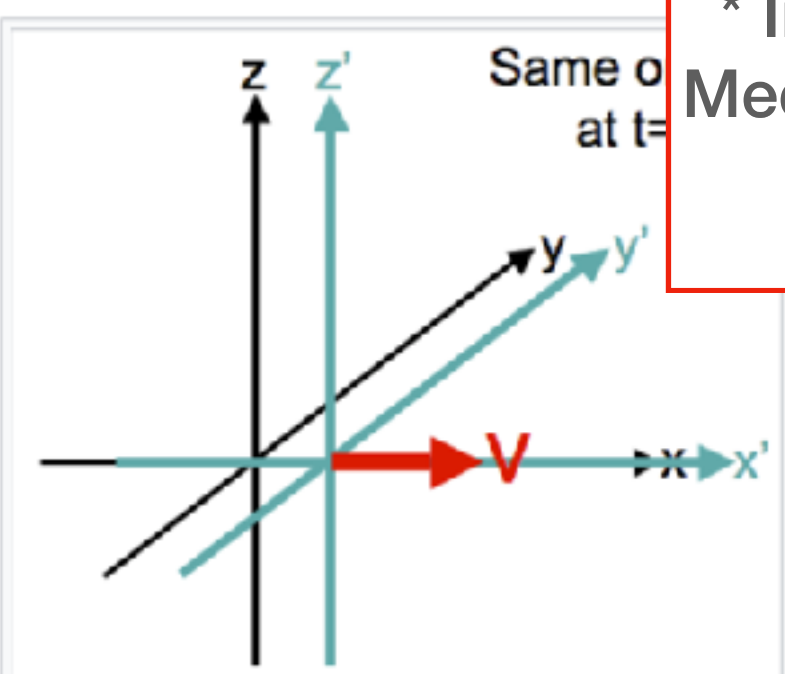
$$
\begin{aligned}
t' &= \gamma\left(t - \frac{vx}{c^2}\right) \\
x' &= \gamma\,(x - vt) \\
y' &= y \\
z' &= z
\end{aligned}
$$



Newton -> Einstein

Galilean transformation (伽利略变换)

$$x' = x - vt$$
$$y' = y$$
$$z' = z$$
$$t' = t.$$



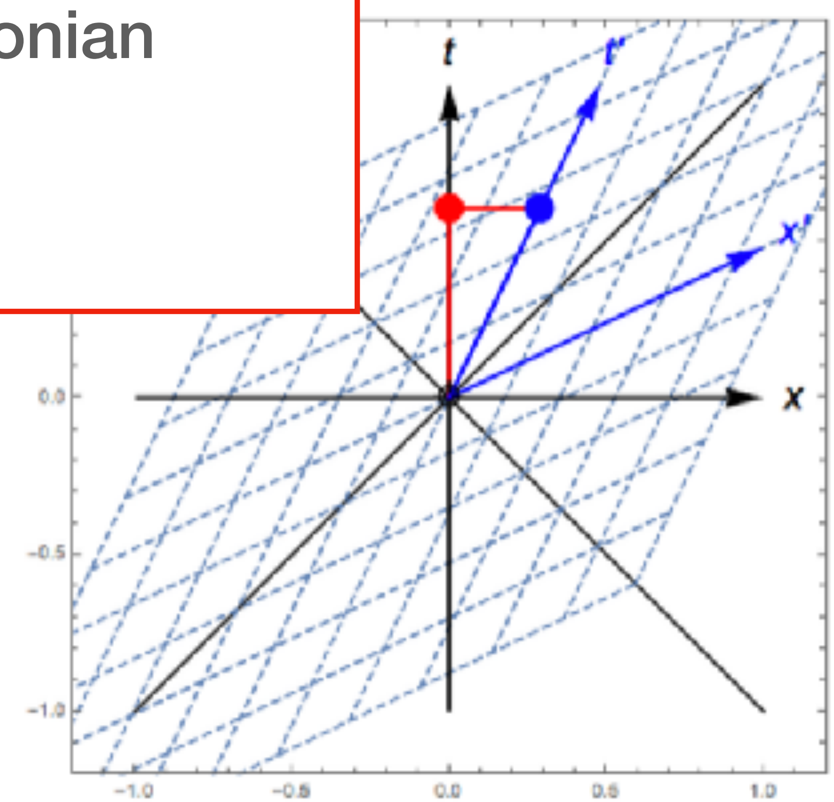
Standard configuration of coordinate systems for Galilean transformations.

Motivations:

Speed of light is constant
* Experiments
* Predicted by Maxwell Eqs.
* Impossible in Newtonian Mechanism

Lorentz transformation (洛伦兹变换)

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$
$$\gamma\left(x - vt\right)$$
$$y$$
$$z$$



Rotation

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \qquad R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \qquad R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Lorentz Transform —Rotation of Space-time

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \gamma ct - \gamma\beta x \\ \gamma x - \beta\gamma ct \\ y \\ z \end{pmatrix}.$$

Minkowski Space-time

Hermann Minkowski

Born	22 June 1864 Aleksotas, Kovno Governorate, Russian Empire (now in Kaunas, Lithuania)
Died	12 January 1909 (aged 44) Göttingen, German Empire
Nationality	German

Work on relativity [edit]

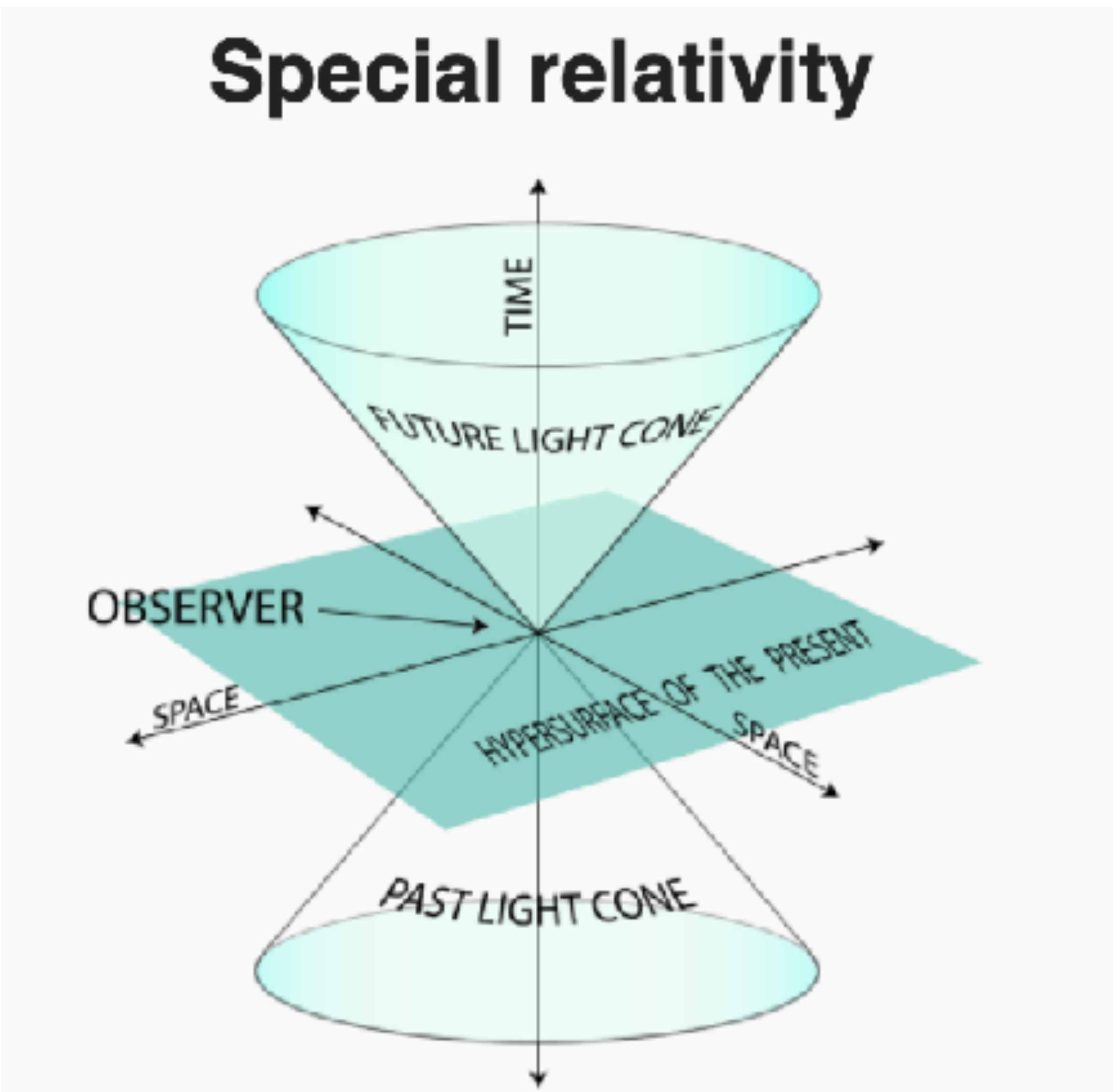
Further information: Minkowski space and Minkowski diagram

By 1908 Minkowski realized that the special theory of relativity, introduced by his former student Albert Einstein in 1905 and based on the previous work of Lorentz and Poincaré, could best be understood in a four-dimensional space, since known as the "Minkowski spacetime", in which time and space are not separated entities but intermingled in a four-dimensional space–time, and in which the Lorentz geometry of special relativity can be effectively represented using the invariant interval $x^2 + y^2 + z^2 - c^2t^2$ (see History of special relativity).

The mathematical basis of Minkowski space can also be found in the hyperboloid model of hyperbolic space already known in the 19th century, because isometries (or motions) in hyperbolic space can be related to Lorentz transformations, which included contributions of Wilhelm Killing (1880, 1885), Henri Poincaré (1881), Homersham Cox (1881), Alexander Macfarlane (1894) and others (see History of Lorentz transformations).

Minkowski Space-time

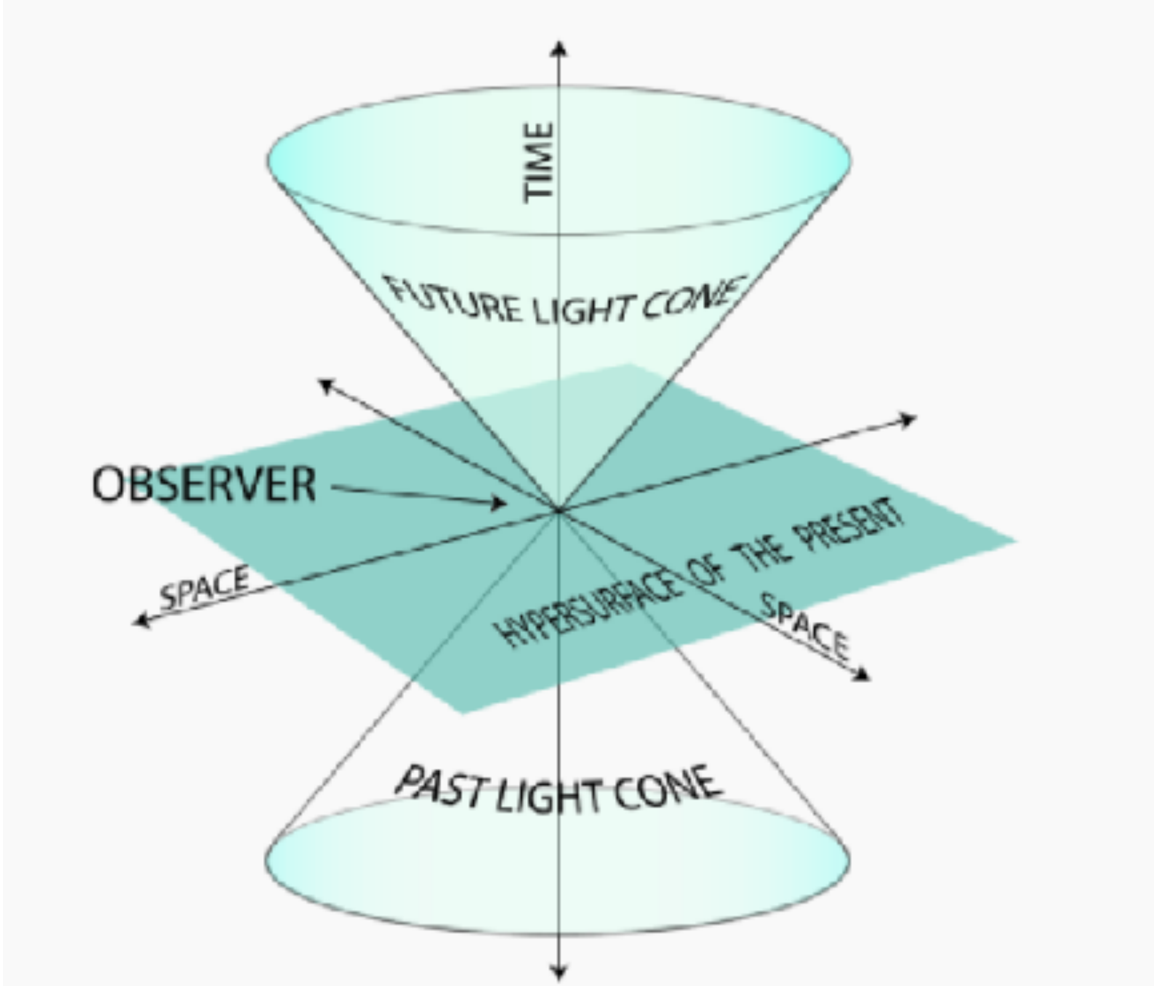
Proper time (原时：测试粒子的时钟)

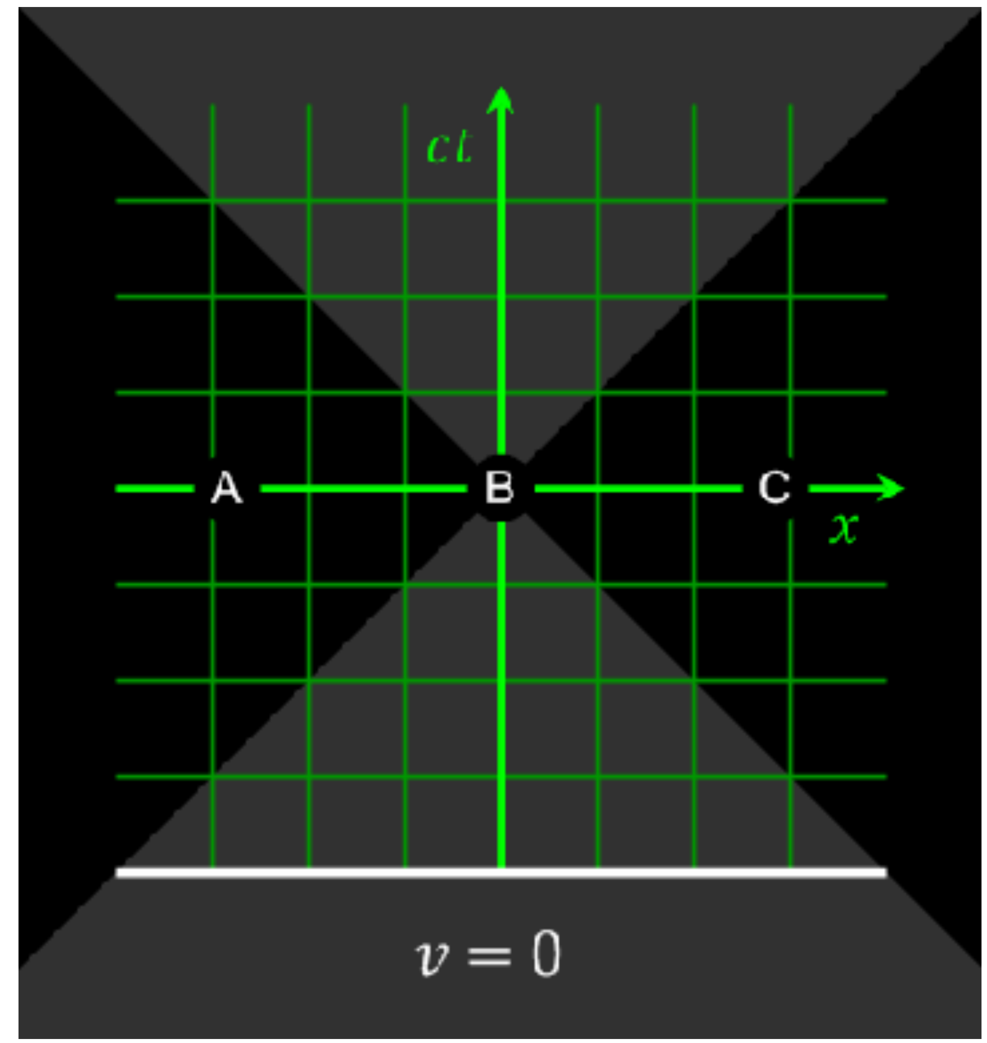


Hermann Minkowski

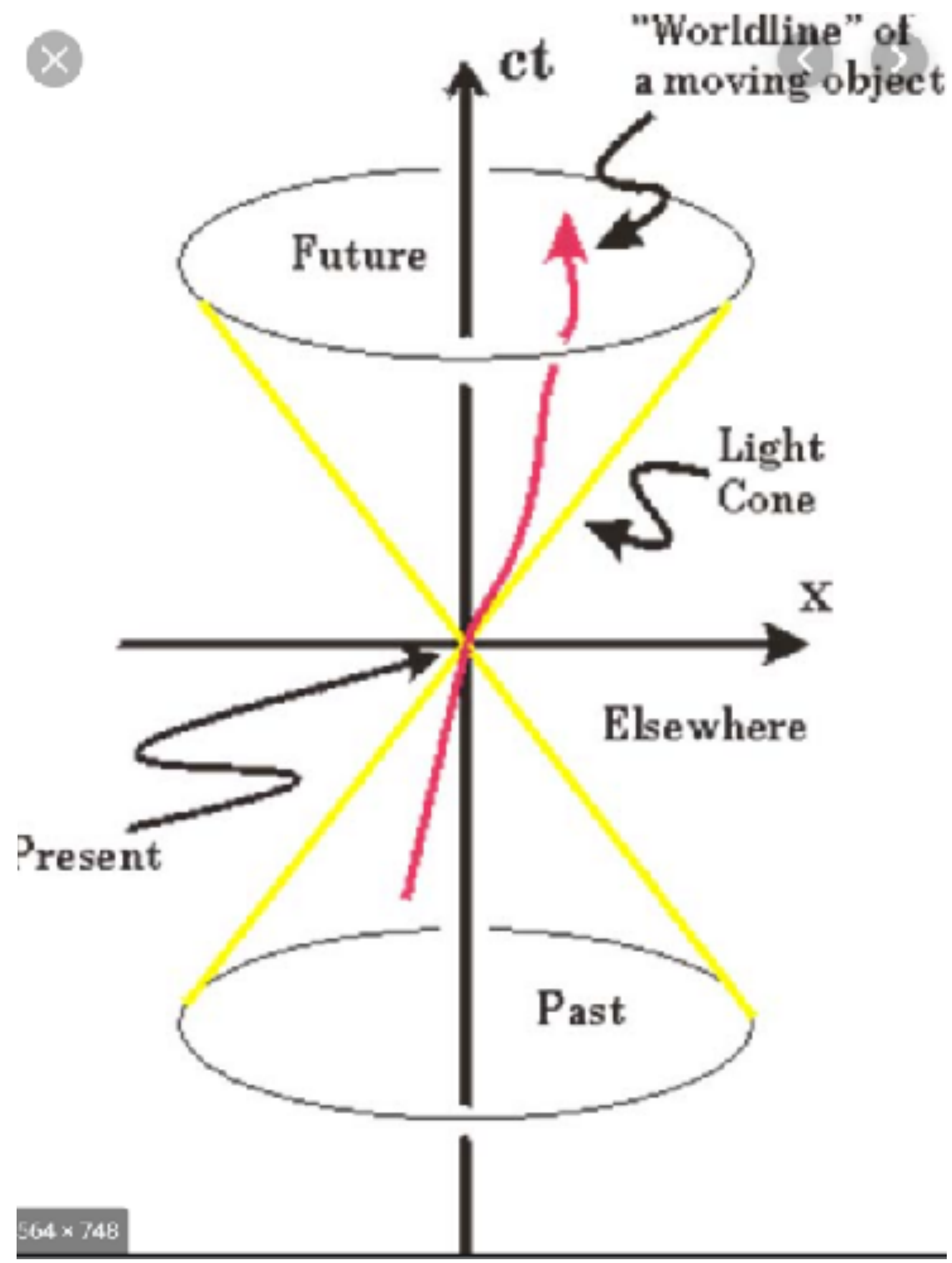
Born	22 June 1864 Aleksotas, Kovno Governorate, Russian Empire (now in Kaunas, Lithuania)
Died	12 January 1909 (aged 44) Göttingen, German Empire
Nationality	German

Special relativity





$$
\begin{aligned}
\Delta\tau &= \int_P \frac{1}{c}\sqrt{\eta_{\mu\nu}\,dx^\mu\,dx^\nu} \\
&= \int_P \sqrt{dt^2 - \frac{dx^2}{c^2} - \frac{dy^2}{c^2} - \frac{dz^2}{c^2}} \\
&= \int \sqrt{1 - \frac{1}{c^2}\left[\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2\right]}\,dt \qquad (3) \\
&= \int \sqrt{1 - \frac{v(t)^2}{c^2}}\,dt = \int \frac{dt}{\gamma(t)},
\end{aligned}
$$



Special Relativity

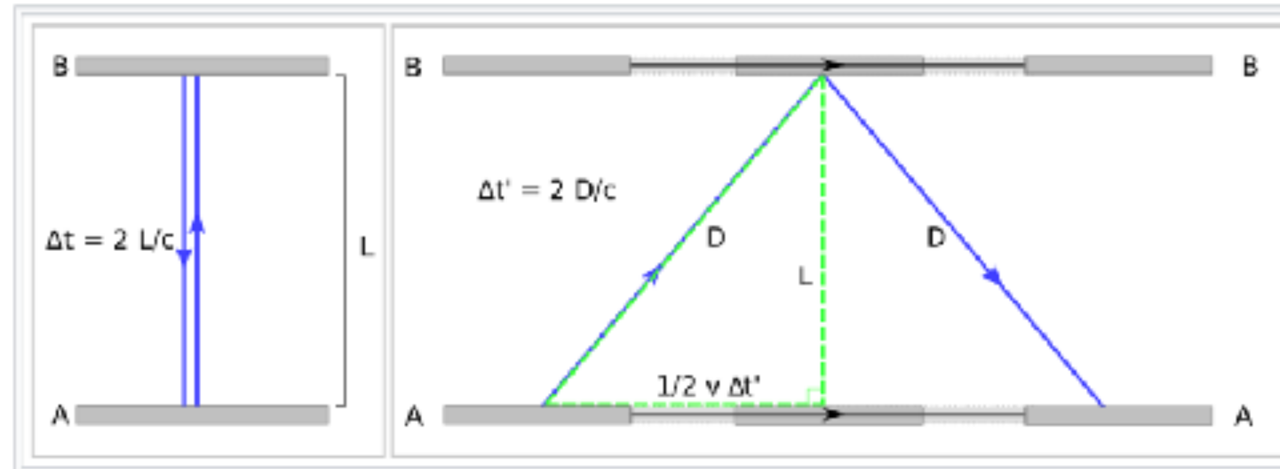
- Effect of special relativity
  - **Time dilation**

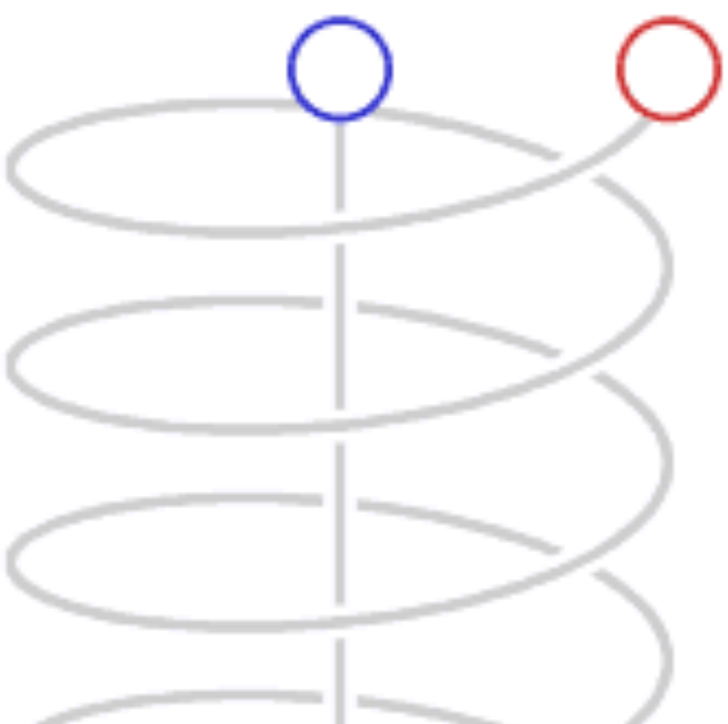
$$\Delta t' = \gamma\, \Delta t = \frac{\Delta t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

  - Relativistic Beaming

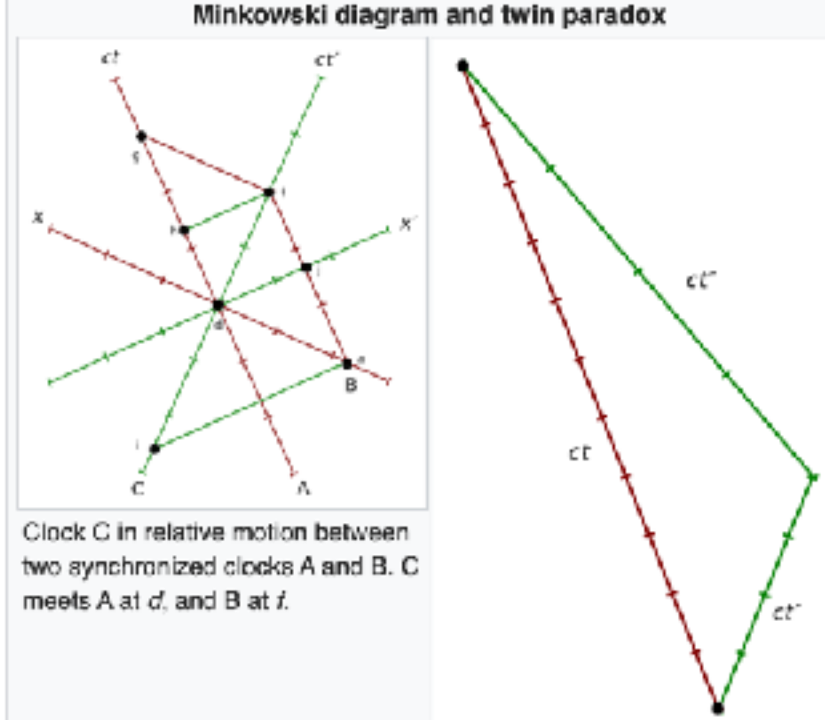
Time dilation (时钟变慢)

$$\Delta t' = \frac{\Delta t}{\sqrt{1 - \frac{v^2}{c^2}}},$$





Minkowski diagram and twin paradox

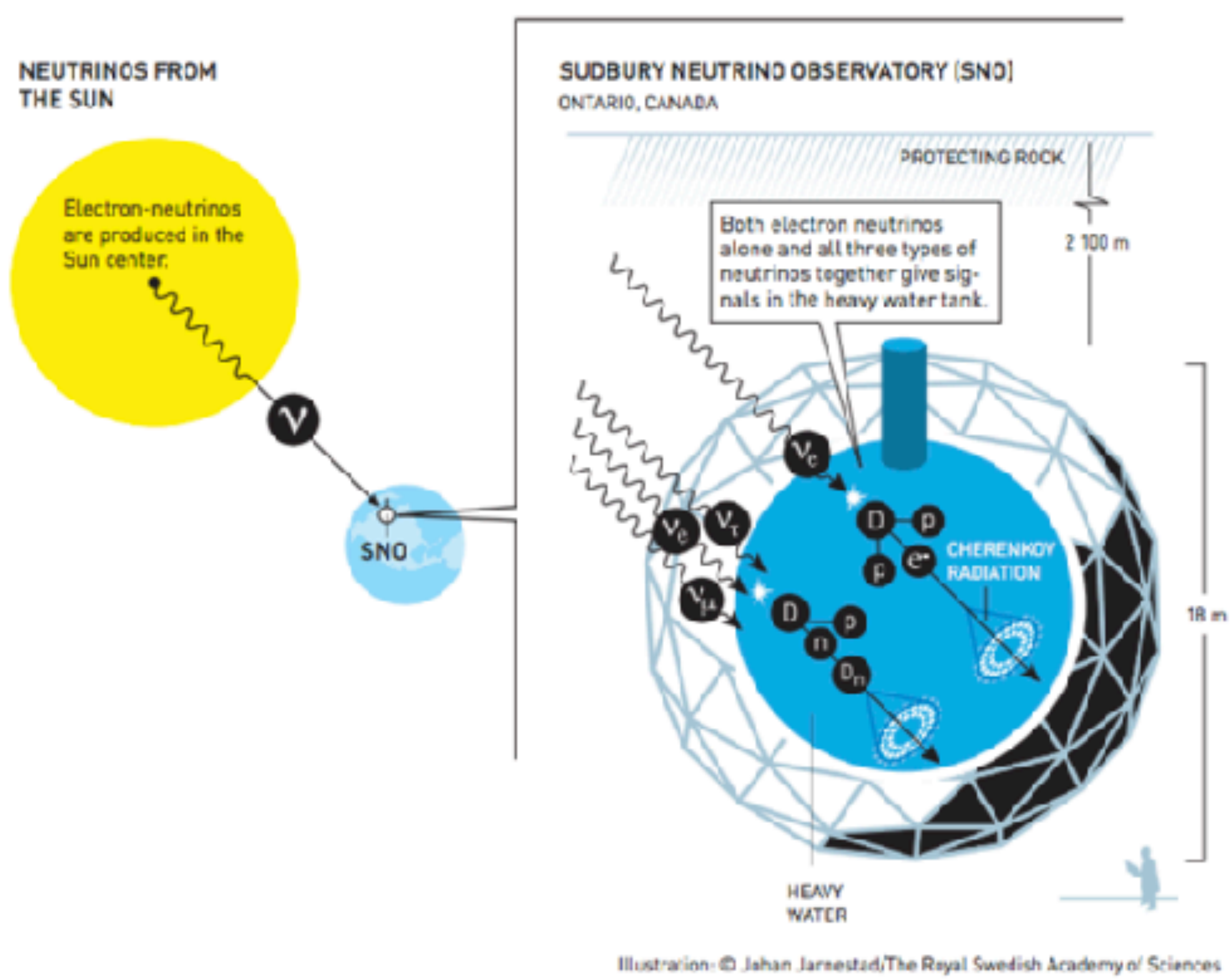


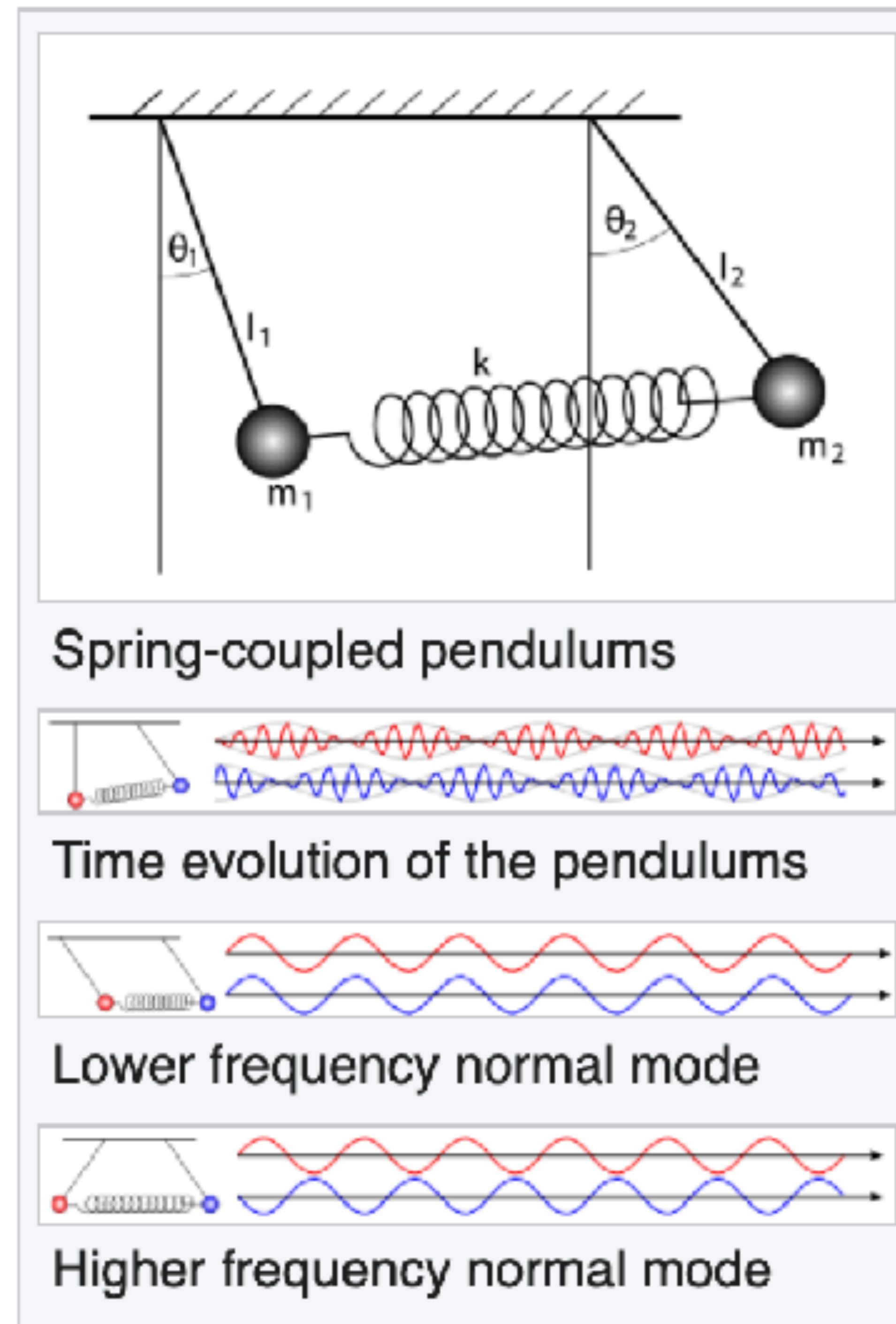
Clock C in relative motion between two synchronized clocks A and B. C meets A at *d*, and B at *f*.

Twin paradox. One twin has to change frames, leading to different proper times in the twin's world lines.

Experiment evidences

- Moving clocks
- Particle Decay
- Neutrino(中微子) oscillation and mass





Spring-coupled pendulums

Time evolution of the pendulums

Lower frequency normal mode

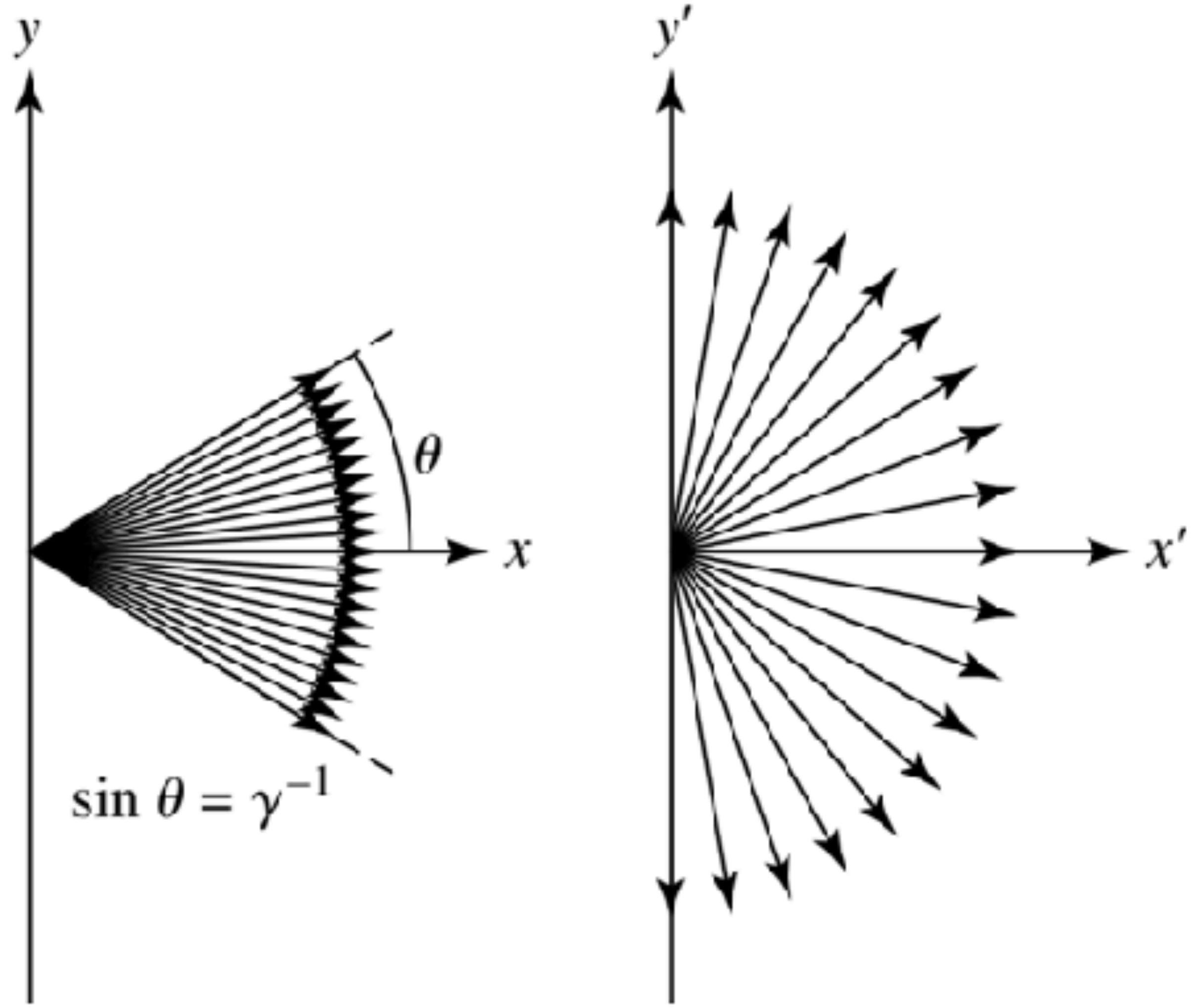
Higher frequency normal mode

Special Relativity

- Effect of special relativity
  - Time dilation
  - **Relativistic Beaming**

Relativistic Beaming

$$\boxed{\sin\theta = \frac{v_y}{v} = \sqrt{1 - u^2/c^2} = \gamma^{-1},}$$



(a) S (b) S'

relativistic beta

$$\beta = \frac{v_j}{c}$$

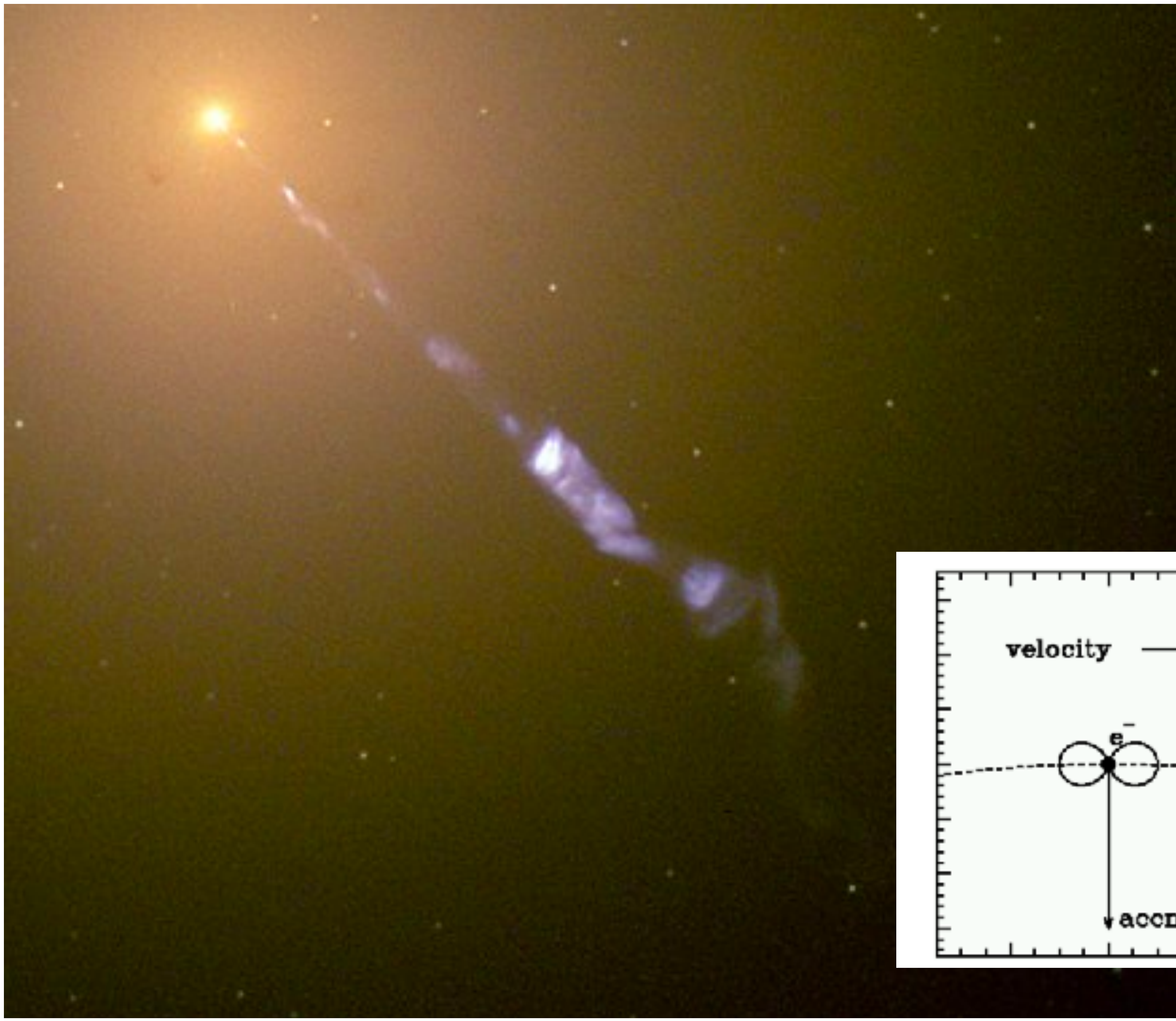
Lorentz factor

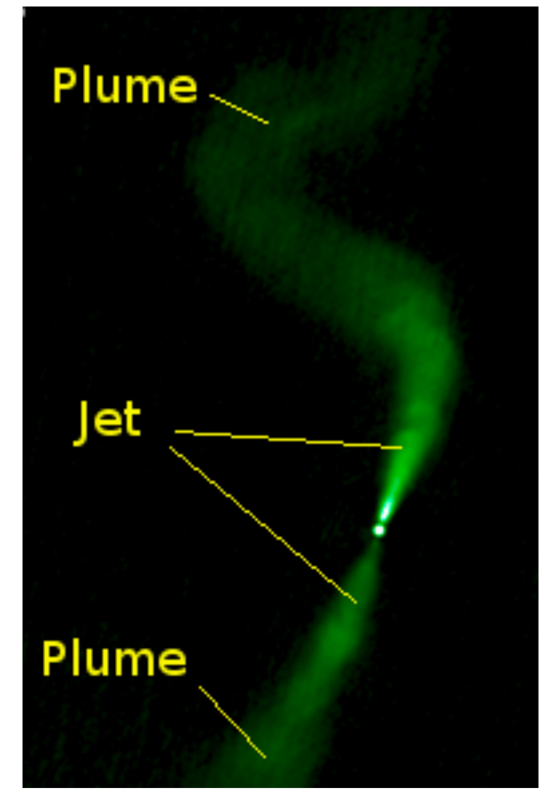
$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

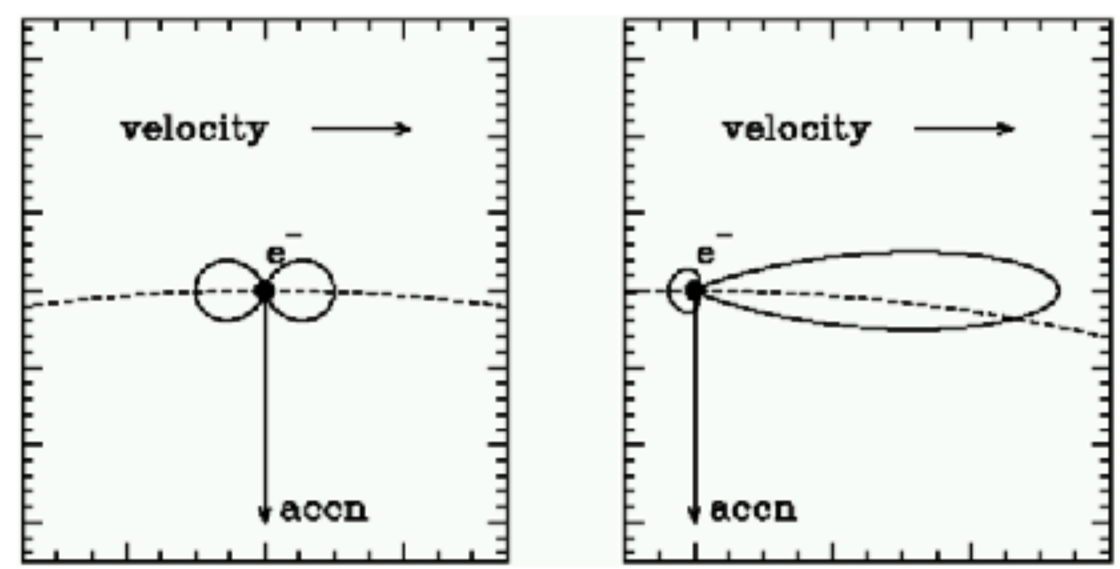
Doppler factor

$$D = \frac{1}{\gamma(1-\beta\cos\theta)}$$

Relativistic Beaming

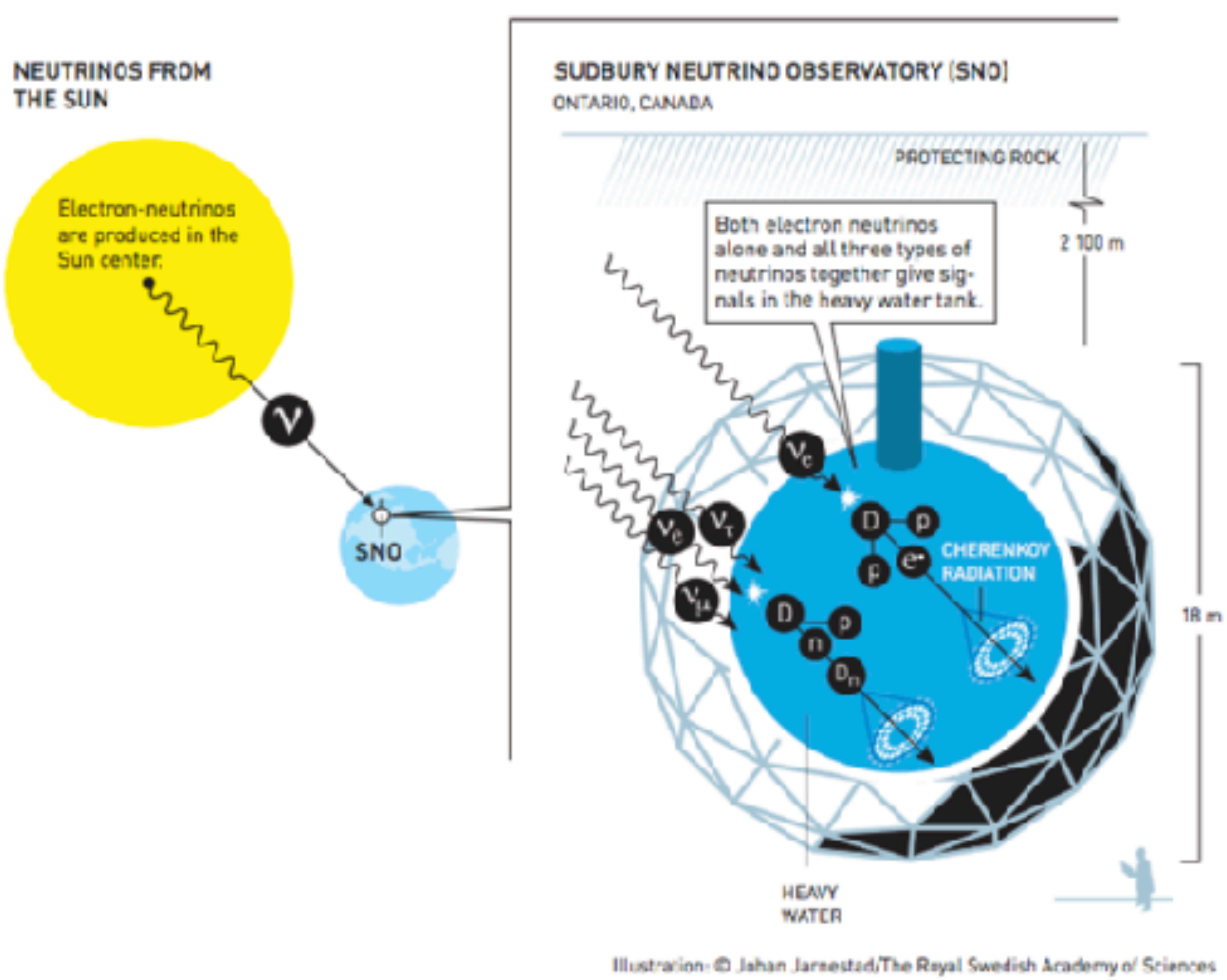


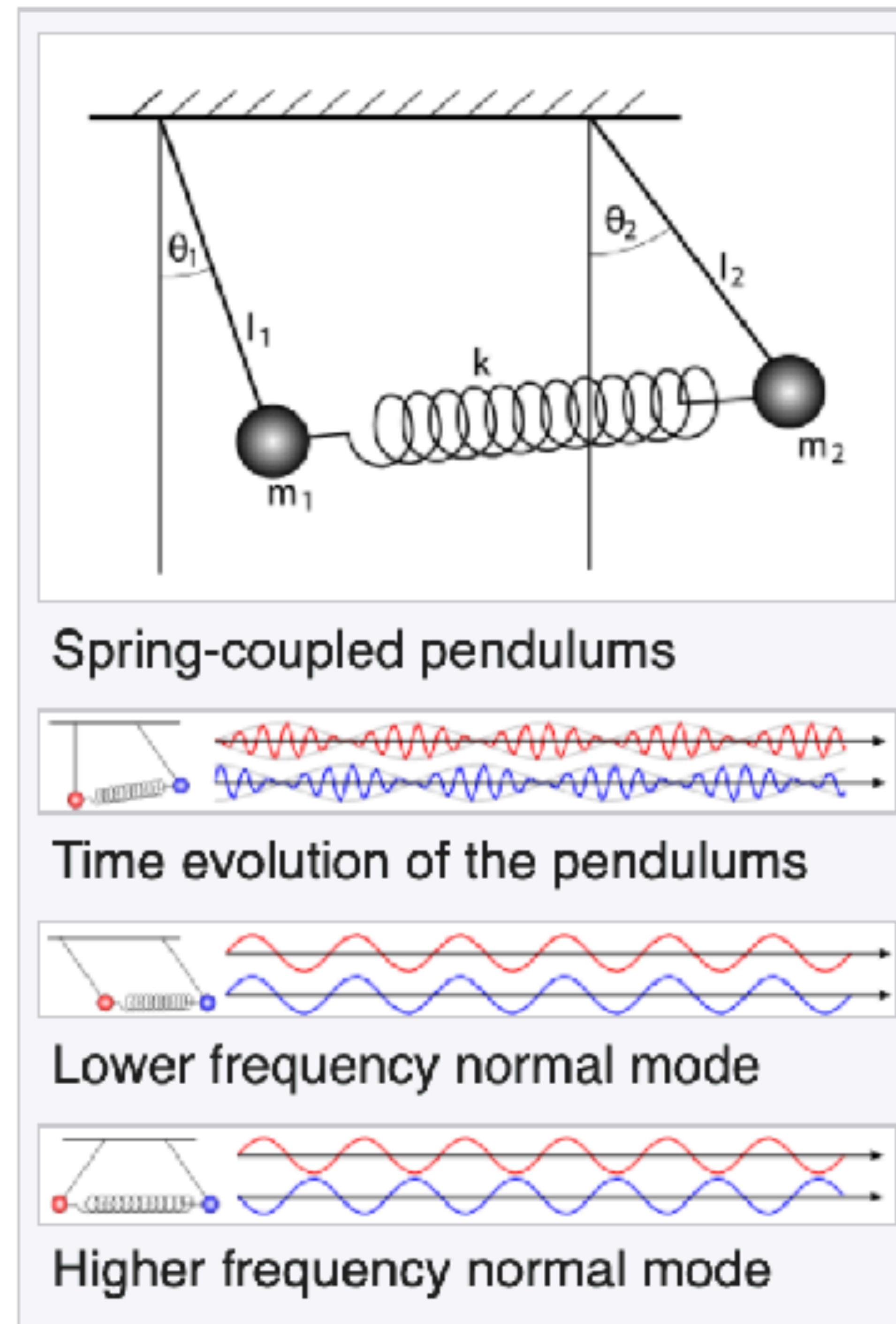




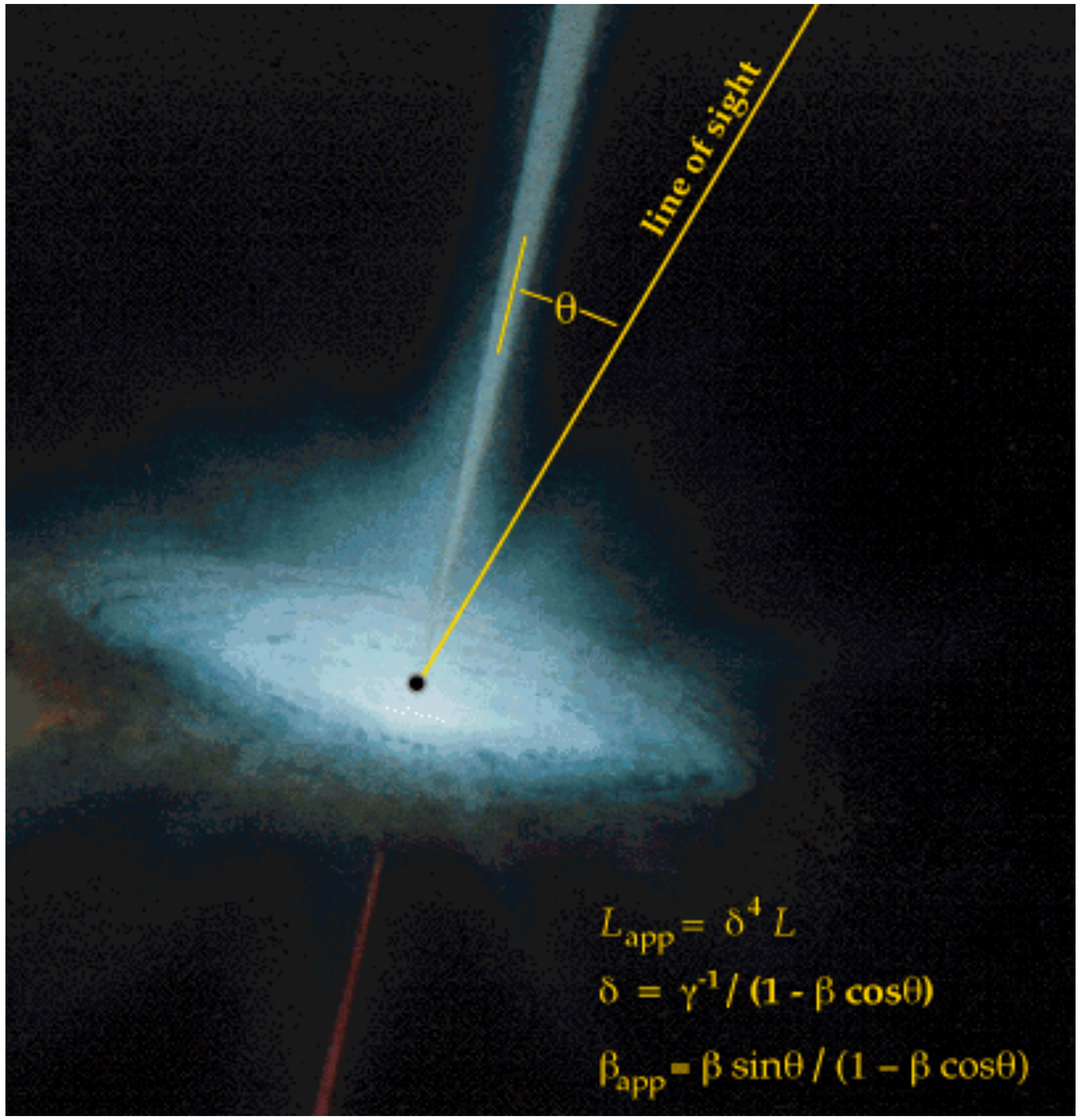
Experiment evidences

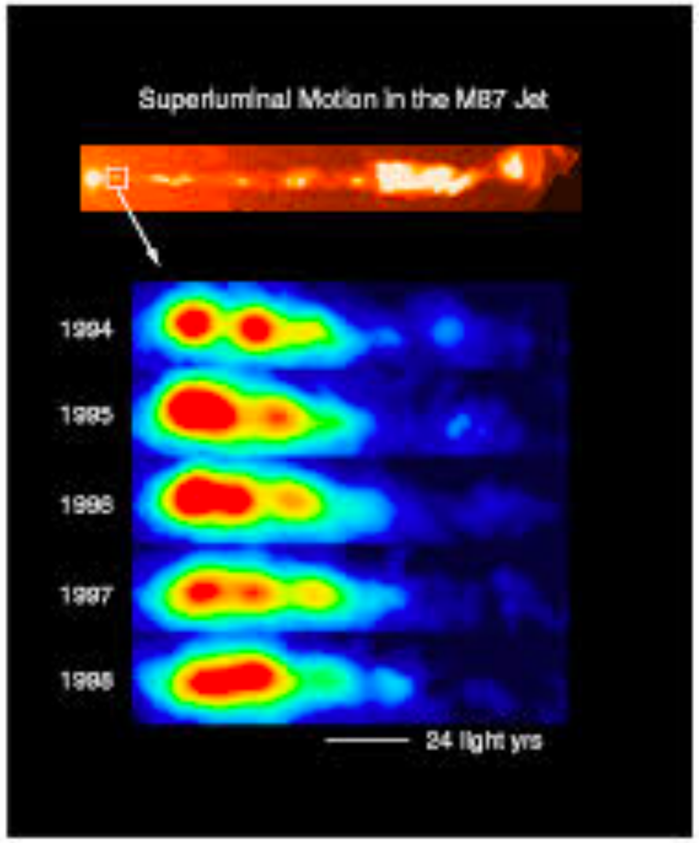
- Moving clocks
- Particle Decay
- Neutrino(中微子) oscillation and mass

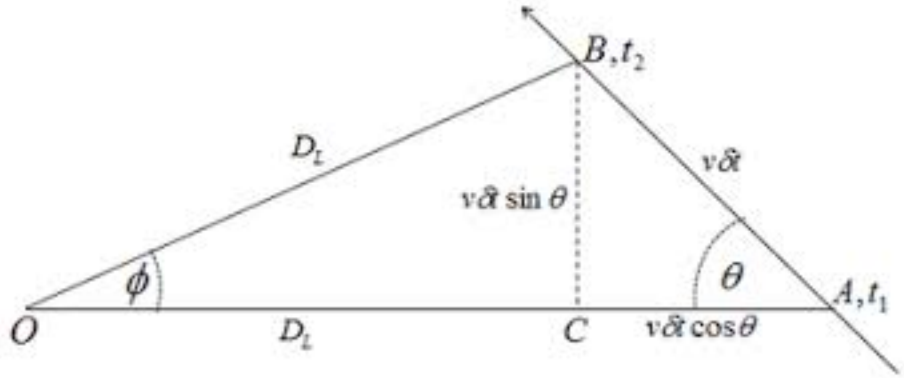




Superluminal Motion







Part 4.2 Differential Geometry & General relativity

注意：广义相对论不完整

- 没有讲的
  - 坐标系的建立和规范不变性。
  - **参考系的建立 (Definition of reference frames in GR)**
  - 仿射联络 （Christoffel symbols and the metric ）

General Relativity

What is gravity?

Before GR

From this postulate, he derived the time-dilation formula in a gravitational field

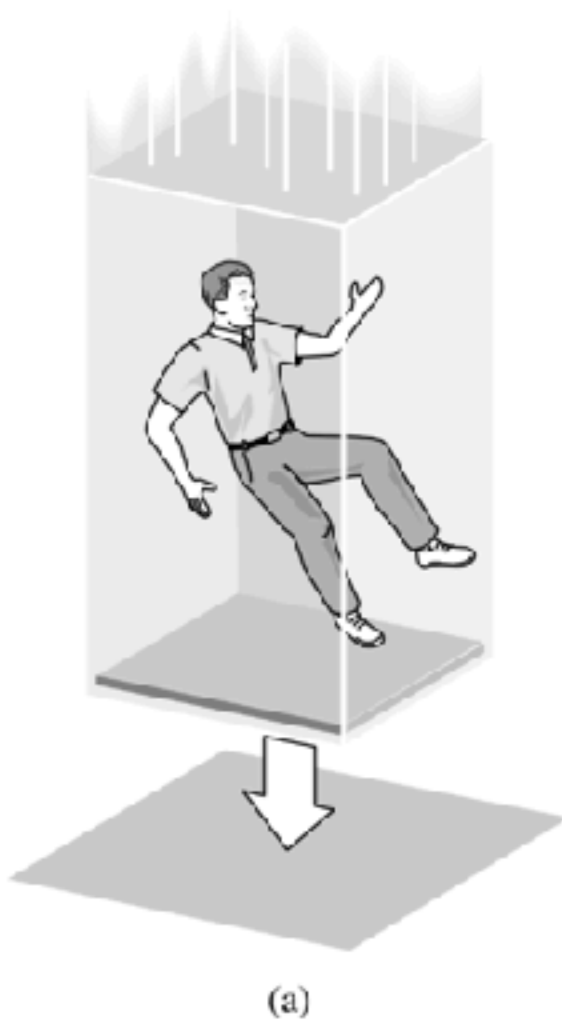
$$dt = d\tau\left(1 + \frac{\Phi}{c^2}\right), \tag{17.1}$$

where Φ is the gravitational potential, τ is the proper time and t is the time measured at zero potential. Then, applying Maxwell's equations to the propagation of light in a gravitational potential, he found that the equations are form-invariant provided the speed of light varies in the radial direction as

$$c(r) = c\left[1 + \frac{\Phi(r)}{c^2}\right], \tag{17.2}$$

Gravity changes light speed…

General relativity

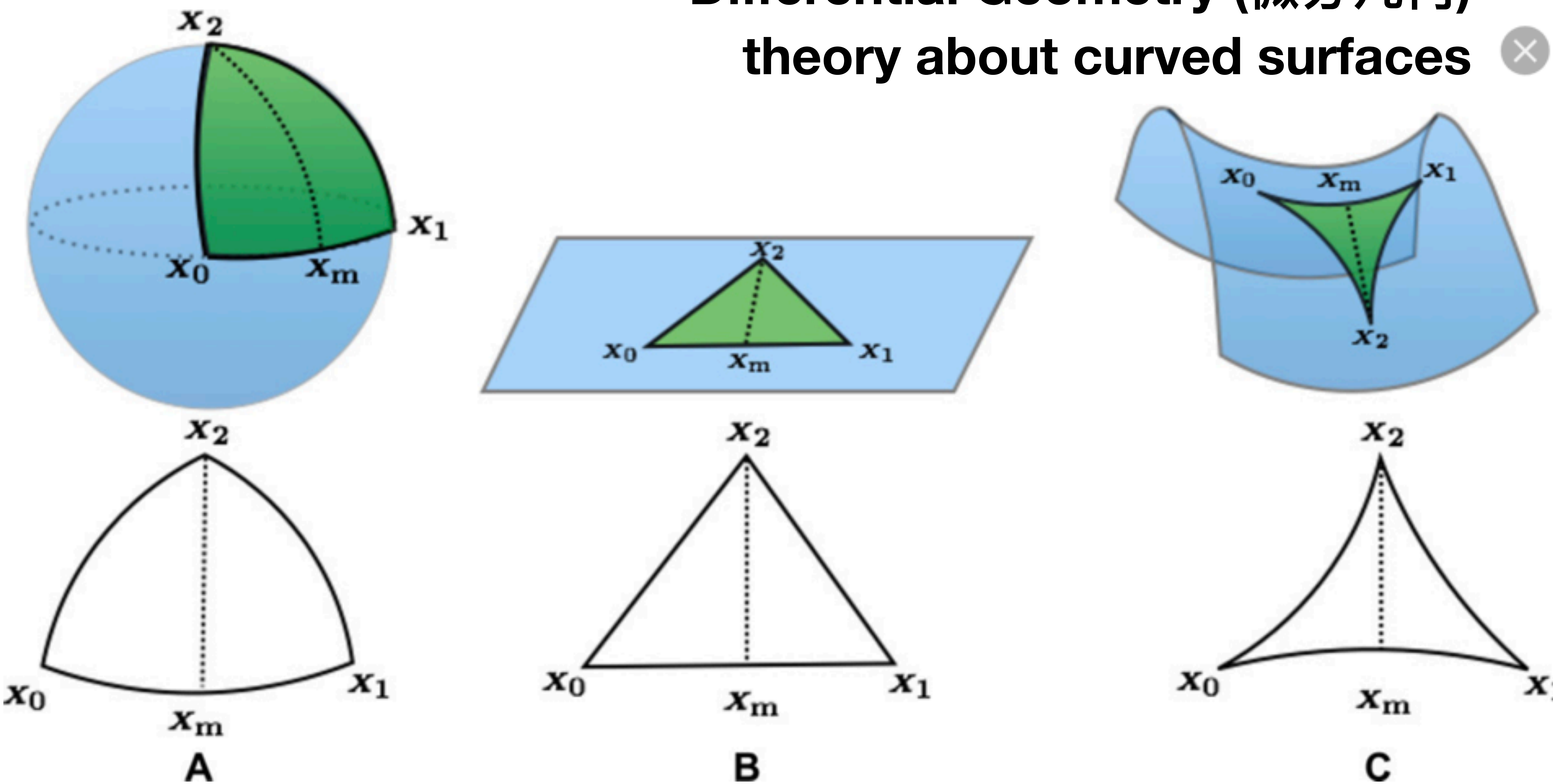


Albert Einstein at his lectern at the Federal Office for Intellectual Property in 1904.

In the same lecture, he remarks

I was sitting in a chair in the patent office in Bern when all of a sudden a thought occurred to me: 'If a person falls freely he will not feel his own weight.' I was startled. This simple thought made a deep impression upon me. It impelled me towards a theory of gravitation.

Differential Geometry (微分几何):
theory about curved surfaces



<u>Curvature (曲率)</u>

Positive (+) **Flat** **Negative (+-)**

Minkowski Space-time

Hermann Minkowski

Born	22 June 1864 Aleksotas, Kovno Governorate, Russian Empire (now in Kaunas, Lithuania)
Died	12 January 1909 (aged 44) Göttingen, German Empire
Nationality	German

Proper time (原时：测试粒子的时钟)

$$\begin{aligned}\Delta\tau &= \int_P \frac{1}{c}\sqrt{\eta_{\mu\nu}dx^\mu dx^\nu} \\ &= \int_P \sqrt{dt^2 - \frac{dx^2}{c^2} - \frac{dy^2}{c^2} - \frac{dz^2}{c^2}} \\ &= \int \sqrt{1 - \frac{1}{c^2}\left[\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2\right]}dt \\ &= \int \sqrt{1 - \frac{v(t)^2}{c^2}}dt = \int \frac{dt}{\gamma(t)},\end{aligned} \tag{3}$$

Four velocity

$$\mathrm{d}x^\mu = (dt, dx, dy, dx)$$

$$\eta = \pm\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

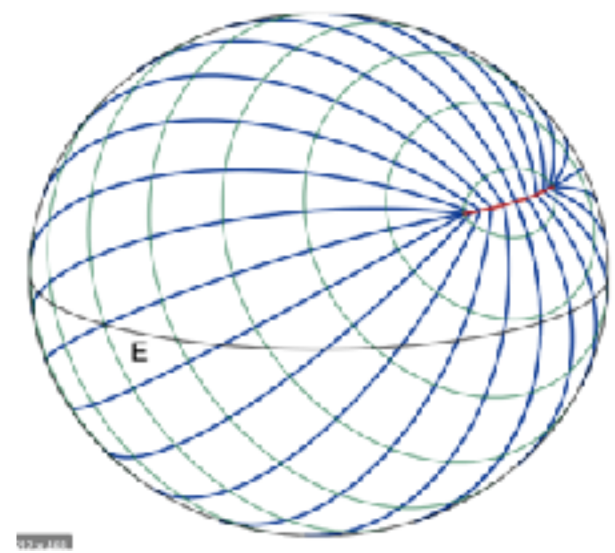
General relativity: statements

- Spacetime is curved

- Particle movements follows geodesics (测地线，短程线)

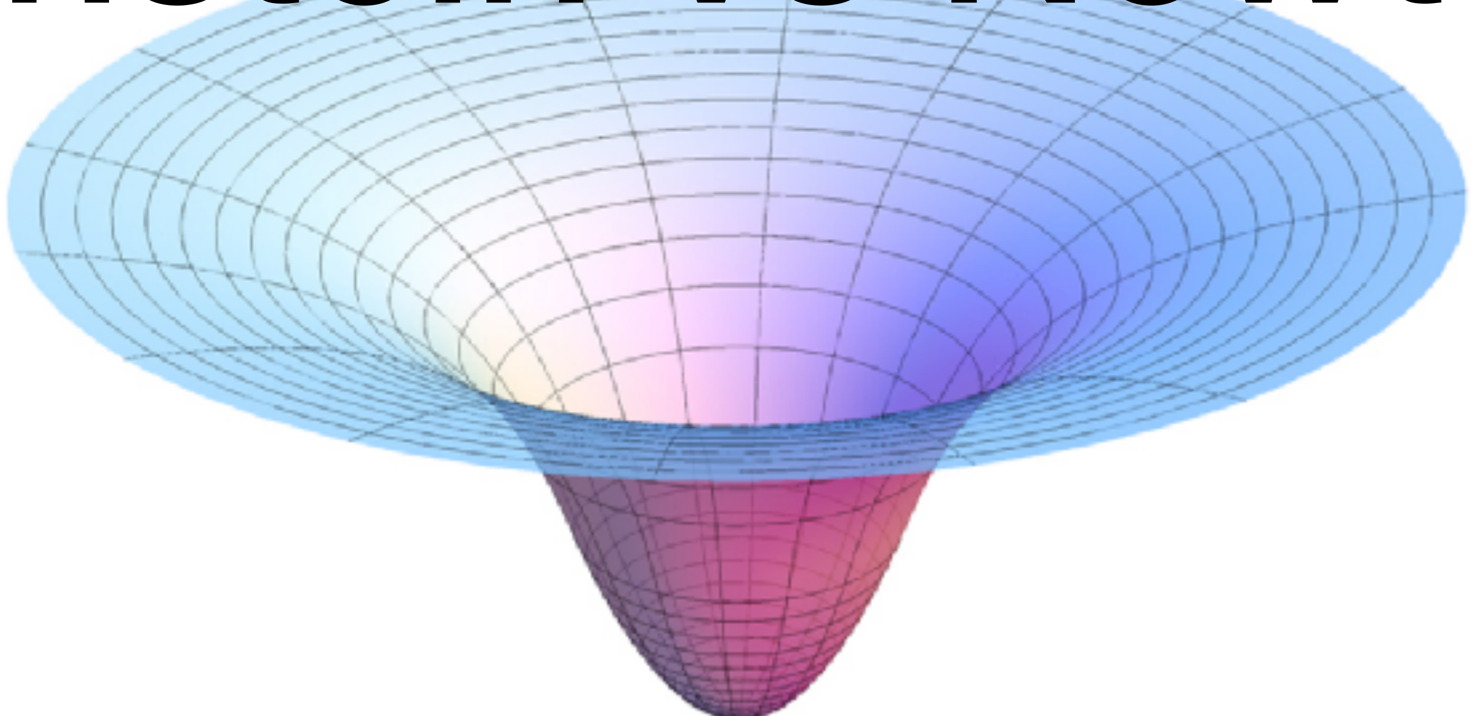
$$\frac{d^2 x^\mu}{ds^2} + \Gamma^\mu{}_{\alpha\beta} \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds} = 0$$

Photons: $ds^2 = 0$



- The geometry of spacetime is determined by the Field Equation

Einstein vs Newton



Einstein	**Newton**
Field Equation	**Poisson Equation**
$\boxed{R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}}$	$\nabla^2\phi = 4\pi G\rho.$
Metric	**Gravitational potential**
$g_{mn} = \mathrm{diag}(1 + 2U, -1 + 2U, -1 + 2U, -1 + 2U).$	$U = -GM / c^2r.$
Geodesics equation	**Newton's law**
$\frac{d^2x^\mu}{ds^2} + \Gamma^\mu{}_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds} = 0$	$\mathbf{F} = m\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = m\mathbf{a},$

3.1 The metric tensor

Consider the representation of two vectors $\vec{A}$ and $\vec{B}$ on the basis $\{\vec{e}_\alpha\}$ of some frame $\mathcal{O}$:

$$\vec{A} = A^\alpha \vec{e}_\alpha, \qquad \vec{B} = B^\beta \vec{e}_\beta.$$

Their scalar product is

$$\vec{A} \cdot \vec{B} = (A^\alpha \vec{e}_\alpha) \cdot (B^\beta \vec{e}_\beta).$$

(Note the importance of using *different* indices α and β to distinguish the first summation from the second.) Following Exer. 34, § 2.9, we can rewrite this as

$$\vec{A} \cdot \vec{B} = A^\alpha B^\beta (\vec{e}_\alpha \cdot \vec{e}_\beta),$$

which, by Eq. (2.27), is

$$\vec{A} \cdot \vec{B} = A^\alpha B^\beta \eta_{\alpha\beta}. \tag{3.1}$$

Metric of curved space-time

- Cartesian
  - $ds^2 = dx^2 + dy^2$
  - $ds^2 = dr^2 + r^2 d\theta^2$
- Curved
  - $ds^2 = dr^2 + r^2 f(r) d\theta^2$

- Special Relativity
  - $ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$
- Curved e.g. around a black hole

$$g = -c^2\, d\tau^2 = -\left(1 - \frac{r_s}{r}\right) c^2\, dt^2 + \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 + r^2 g_\Omega,$$

where g_Ω is the metric on the two sphere, i.e. $g_\Omega = \left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$.

Einstein Field Equation

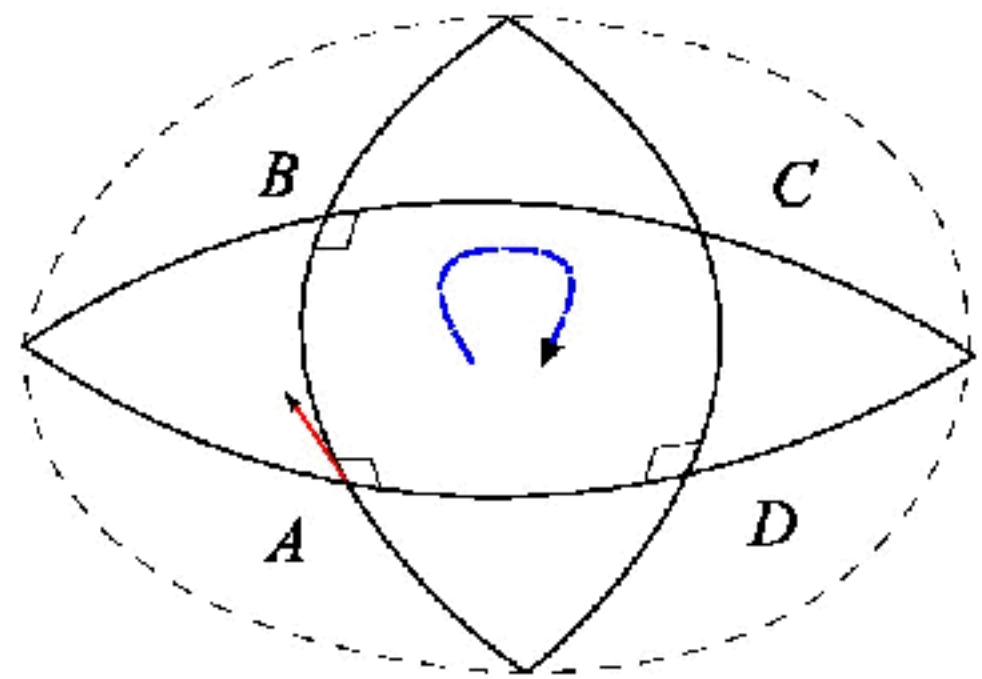
Rμν

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \cancel{\Lambda g_{\mu\nu}} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Rμν is the Ricci curvature tensor
R is the scalar curvature
gμν is the metric tensor
~~*Λ is the cosmological constant*~~
Tμν is the stress–energy tensor

Curvature of Spacetime ~ Matter

Rμν (gμν)

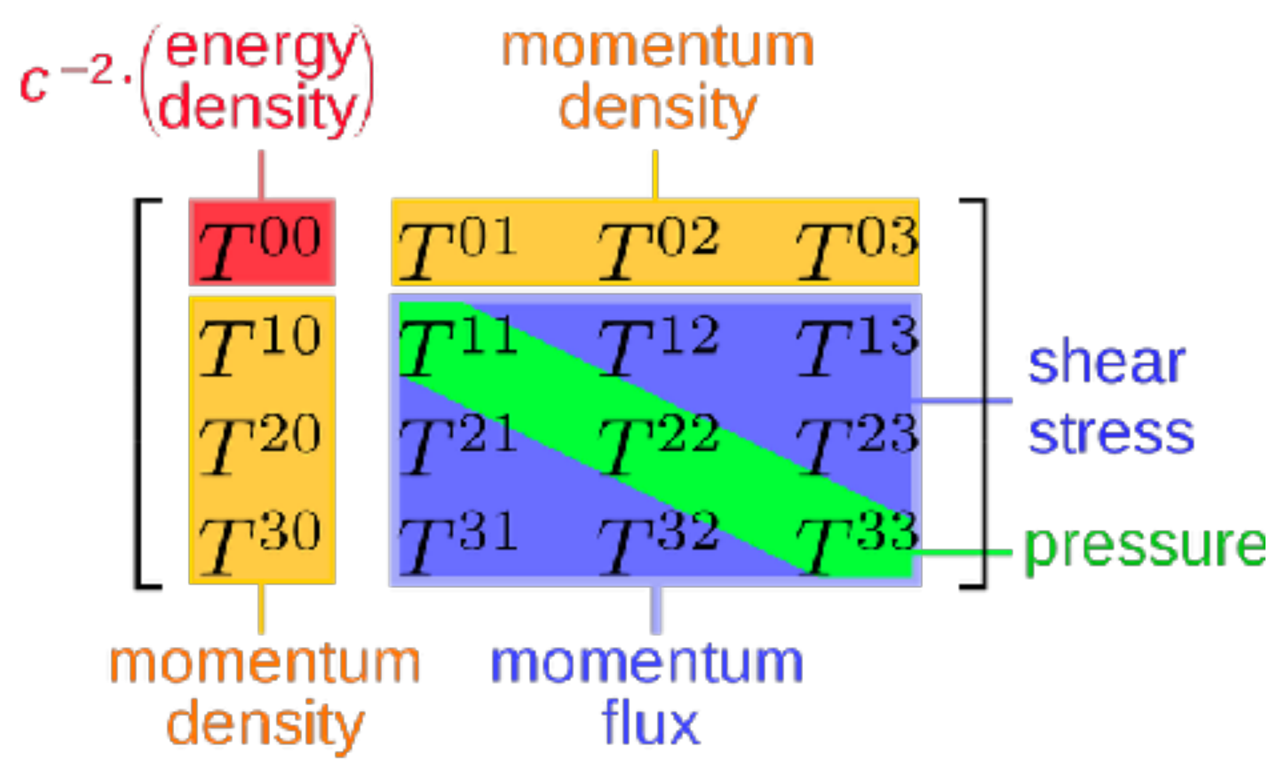


Einstein Field Equation

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \cancel{\Lambda g_{\mu\nu}} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

Rμν is the Ricci curvature tensor
R is the scalar curvature
gμν is the metric tensor
~~Λ is the cosmological constant~~
Tμν is the stress–energy tensor

Curvature of Spacetime ~ Matter

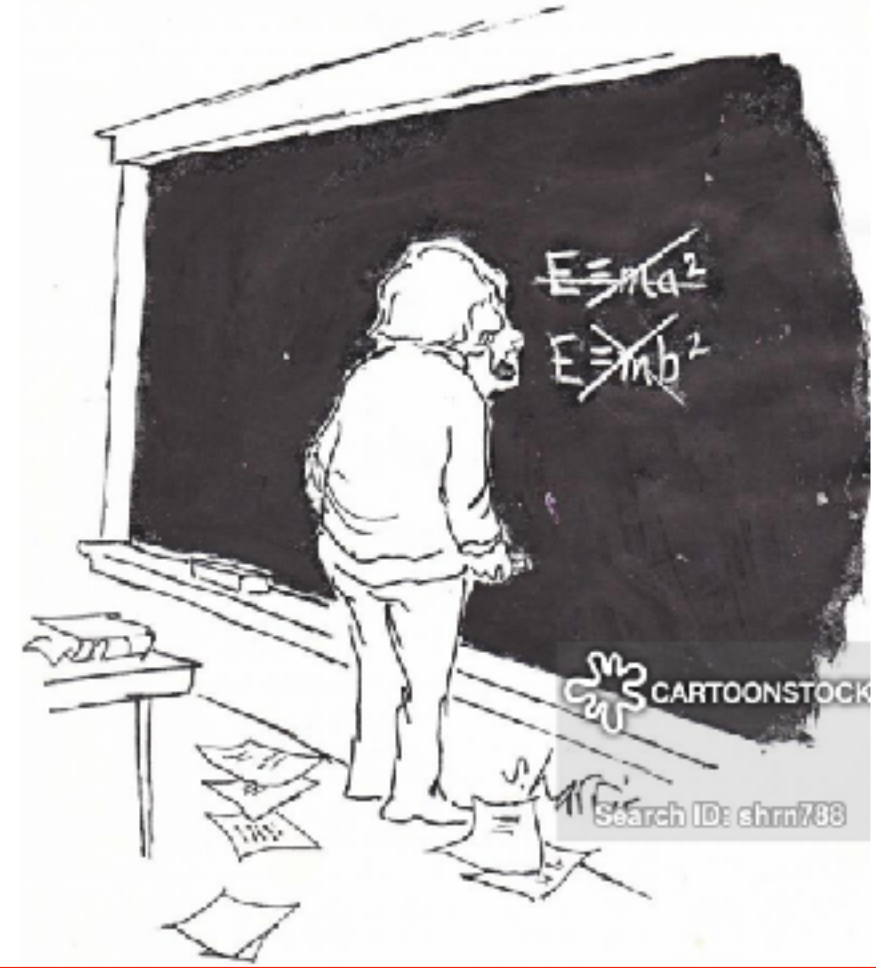


Derivation

$$\boxed{R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \cancel{\Lambda g_{\mu\nu}} = \frac{8\pi G}{c^4} T_{\mu\nu}}$$

Rμν is the Ricci curvature tensor
R is the scalar curvature
gμν is the metric tensor
~~*Λ is the cosmological constant*~~
Tμν is the stress–energy tensor

Curvature of Spacetime ~ Matter



$$G^{\alpha\beta} \equiv R^{\alpha\beta} - \frac{1}{2} g^{\alpha\beta} R$$

$$G^{\alpha\beta}{}_{;\beta} = 0.$$

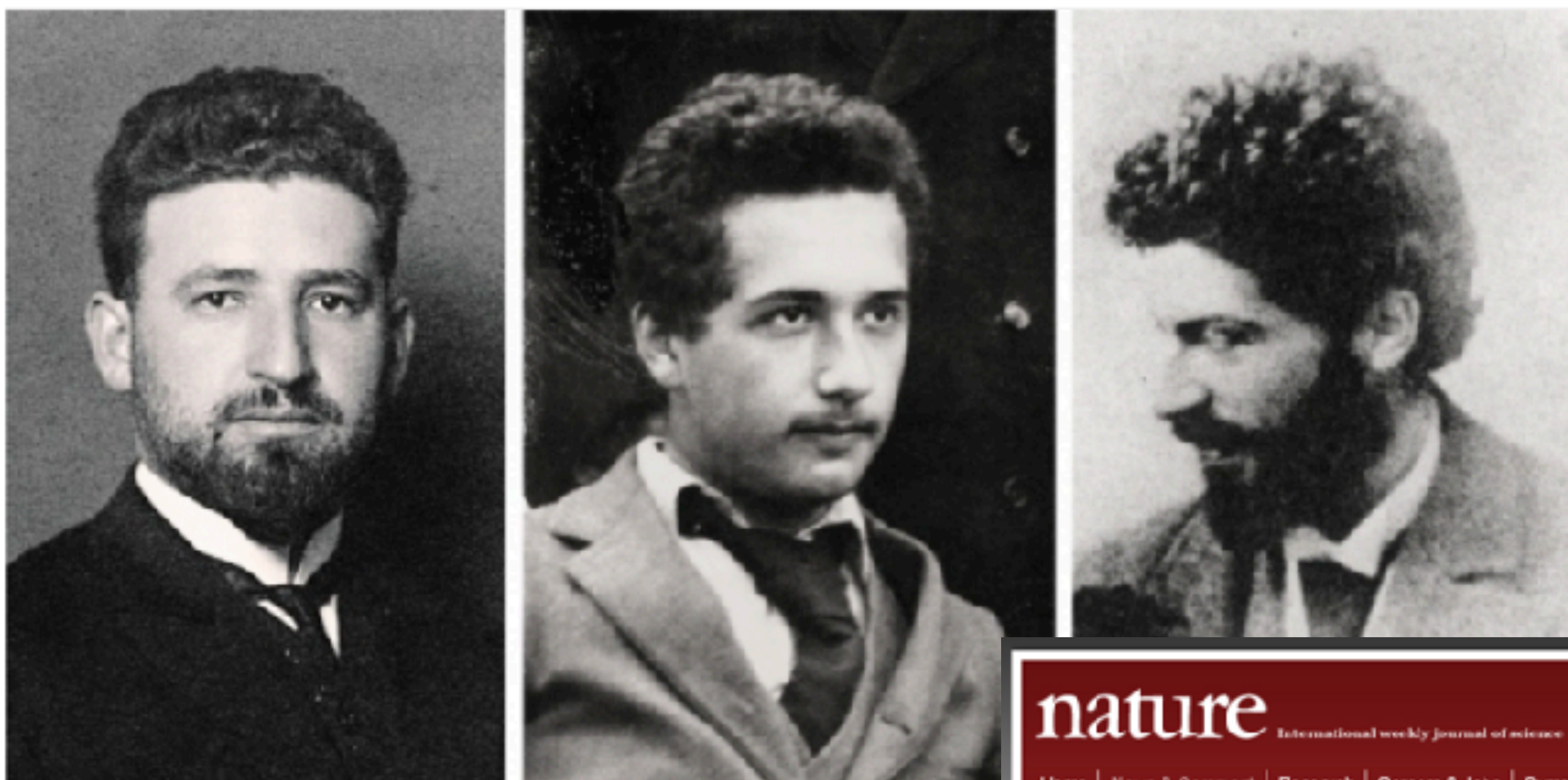
$$T^{\alpha\beta}{}_{;\beta} \equiv 0,$$

?

$$\Rightarrow \quad G^{\alpha\beta} = 8\pi T^{\alpha\beta}$$

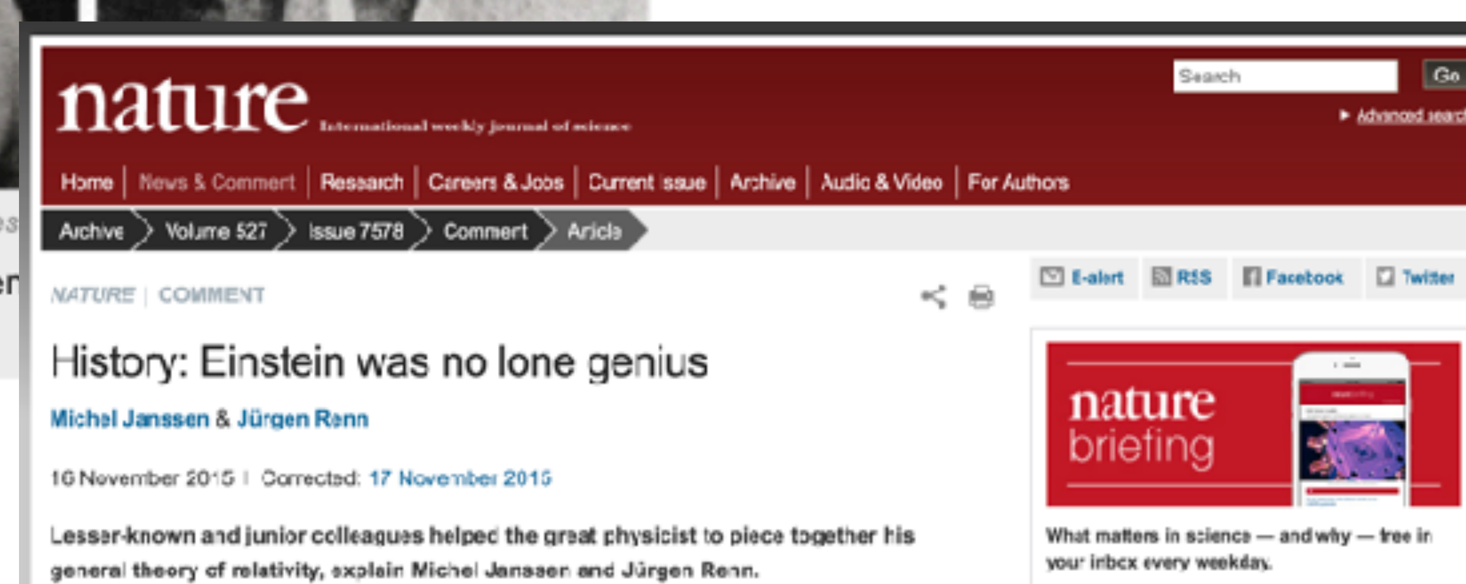
Einstein was not alone

"Outline of a Generalized Theory of Relativity and of a Theory of Gravitation"
Einstein: Physics
Grossman: Mathematics



Grossmann, Einstein: ETH-Bibliothek Zürich/Bildarchiv; Bes

Marcel Grossmann (left) and Michele Besso (right), university frien
made important contributions to general relativity.



NATURE | COMMENT

History: Einstein was no lone genius

Michel Janssen & Jürgen Renn

16 November 2015 | Corrected: 17 November 2015

Lesser-known and junior colleagues helped the great physicist to piece together his general theory of relativity, explain Michel Janssen and Jürgen Renn.

Solutions to Einstein Eq.

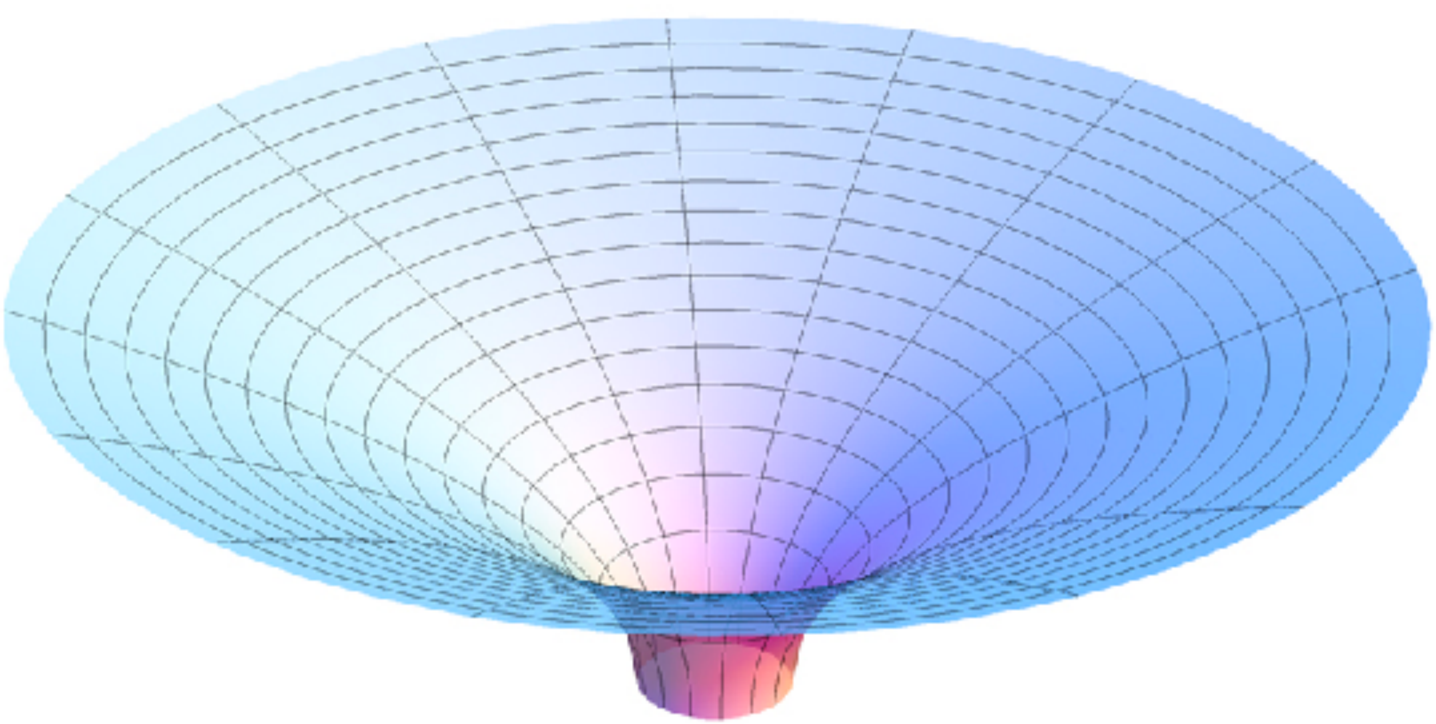
Do not try it yourself yet..

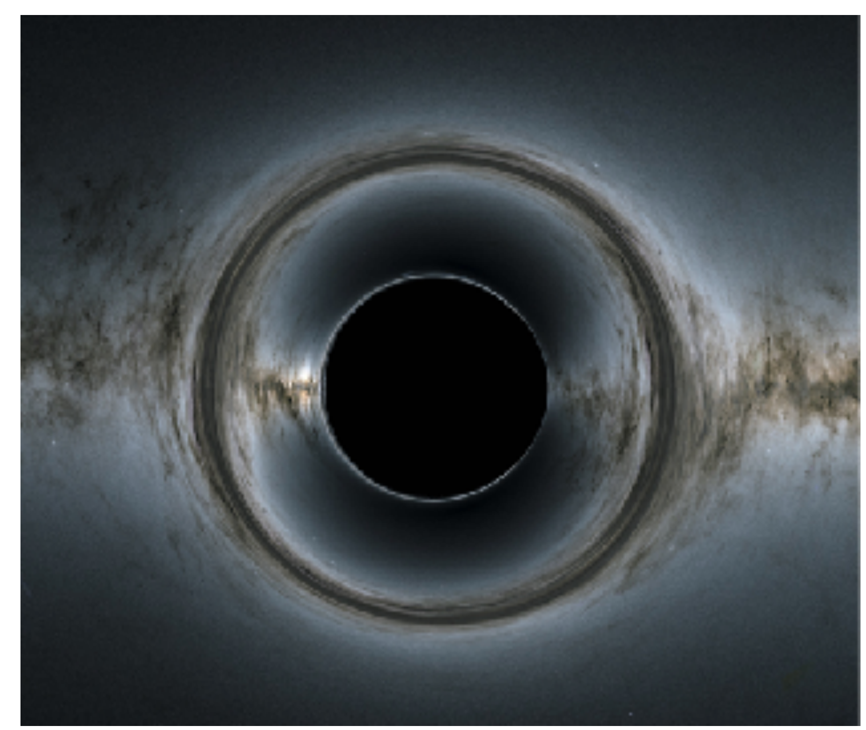
Schwarzschild Solution (Black Holes)

In Schwarzschild coordinates (t, r, θ, ϕ) the Schwarzschild metric (or equivalently, the line element for proper time) has the form

$$g = -c^2\,d\tau^2 = -\left(1 - \frac{r_s}{r}\right)c^2\,dt^2 + \left(1 - \frac{r_s}{r}\right)^{-1}dr^2 + r^2 g_\Omega,$$

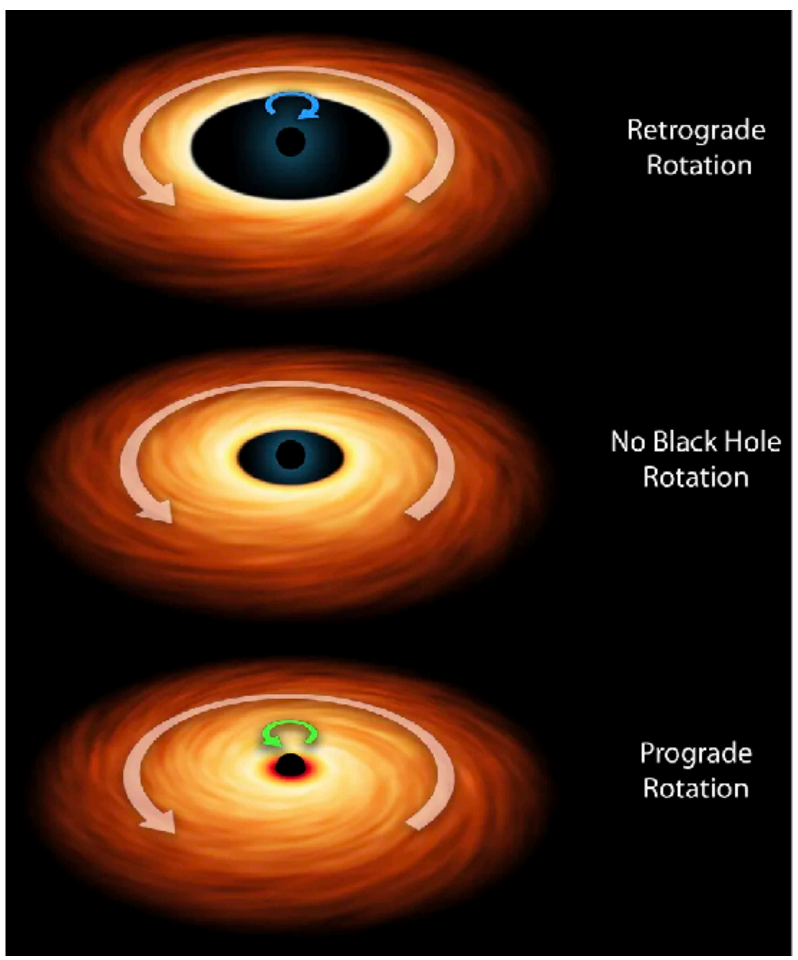
$$g_\Omega = \left(d\theta^2 + \sin^2\theta\,d\varphi^2\right) \qquad r_s = \frac{2GM}{c^2}$$



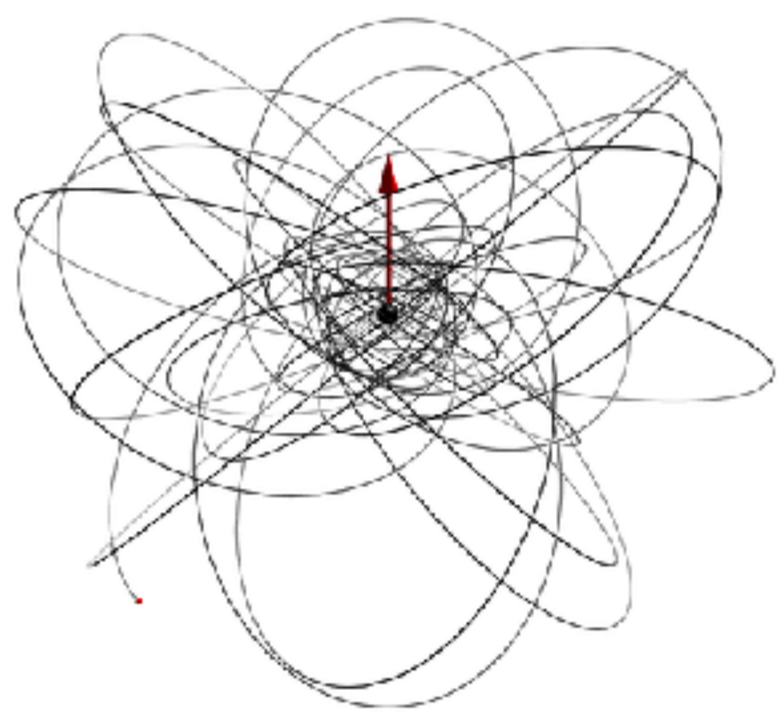


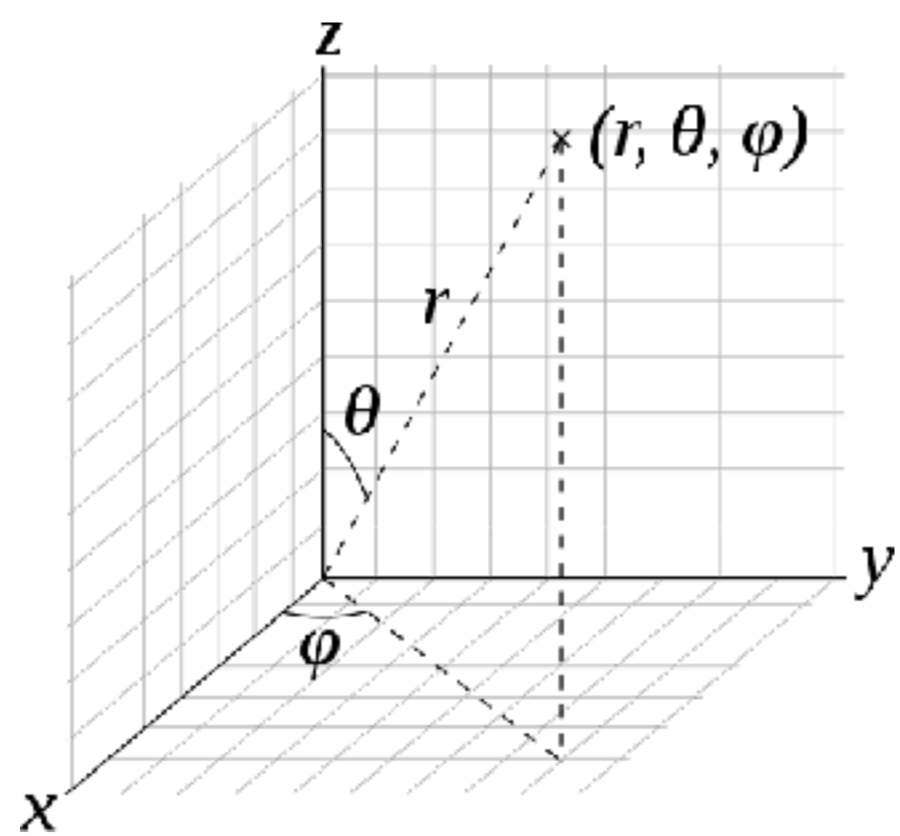
Kerr Solutions (Rotating Black Holes)

$$g^{\mu\nu}\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x^\nu} = \frac{1}{c^2\Delta}\left(r^2+a^2+\frac{r_s r a^2}{\Sigma}\sin^2\theta\right)\left(\frac{\partial}{\partial t}\right)^2 + \frac{2r_s r a}{c\Sigma\Delta}\frac{\partial}{\partial\phi}\frac{\partial}{\partial t} - \frac{1}{\Delta\sin^2\theta}\left(1-\frac{r_s r}{\Sigma}\right)\left(\frac{\partial}{\partial\phi}\right)^2 - \frac{\Delta}{\Sigma}\left(\frac{\partial}{\partial r}\right)^2 - \frac{1}{\Sigma}\left(\frac{\partial}{\partial\theta}\right)^2$$



Frame-dragging





Robertson-Walker (Cosmology)

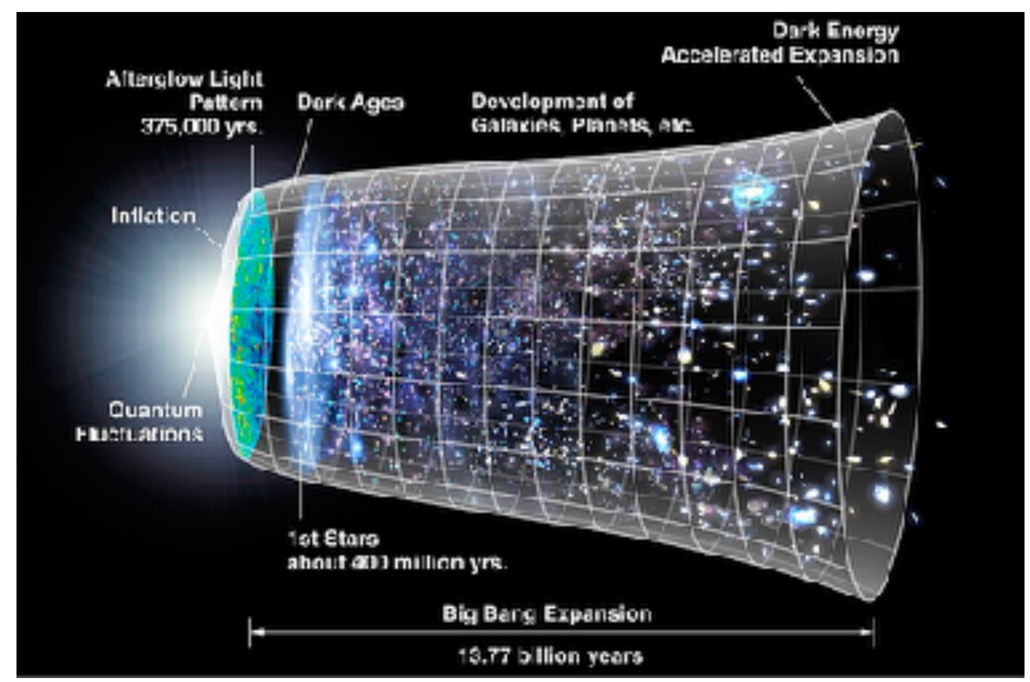
Friedmann–Lemaître–Robertson–Walker metric

$$-c^2\,\mathrm{d}\tau^2 = -c^2\,\mathrm{d}t^2 + a(t)^2\,\mathrm{d}\mathbf{\Sigma}^2$$

$$\mathrm{d}\mathbf{\Sigma}^2 = \frac{\mathrm{d}r^2}{1-kr^2} + r^2\,\mathrm{d}\mathbf{\Omega}^2, \quad \text{where } \mathrm{d}\mathbf{\Omega}^2 = \mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2.$$

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2} - \frac{\Lambda c^2}{3} = \frac{8\pi G}{3}\rho$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2} - \Lambda c^2 = -\frac{8\pi G}{c^2}p.$$



Relativistic Effects

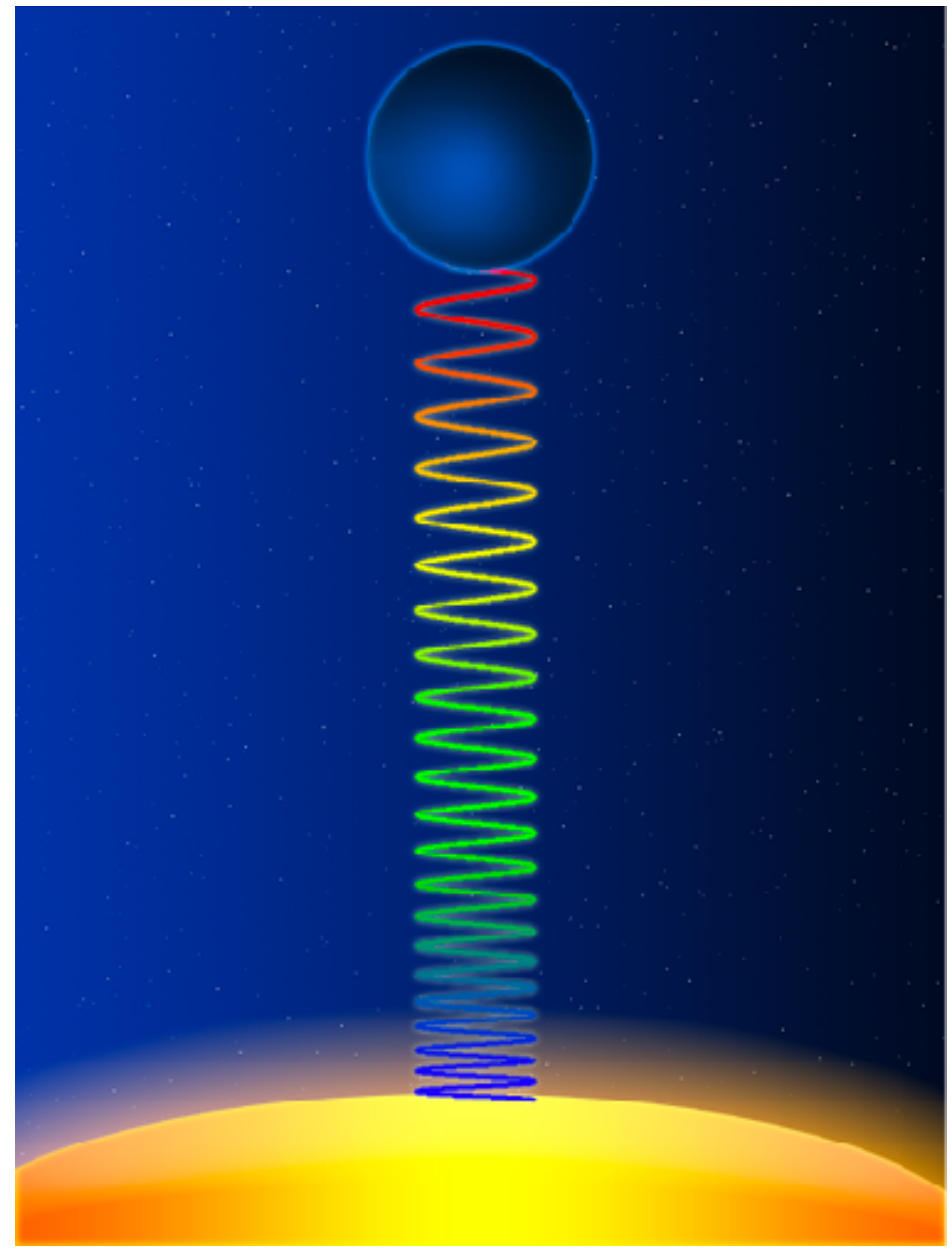
Gravitational redshift

Redshift (红移)

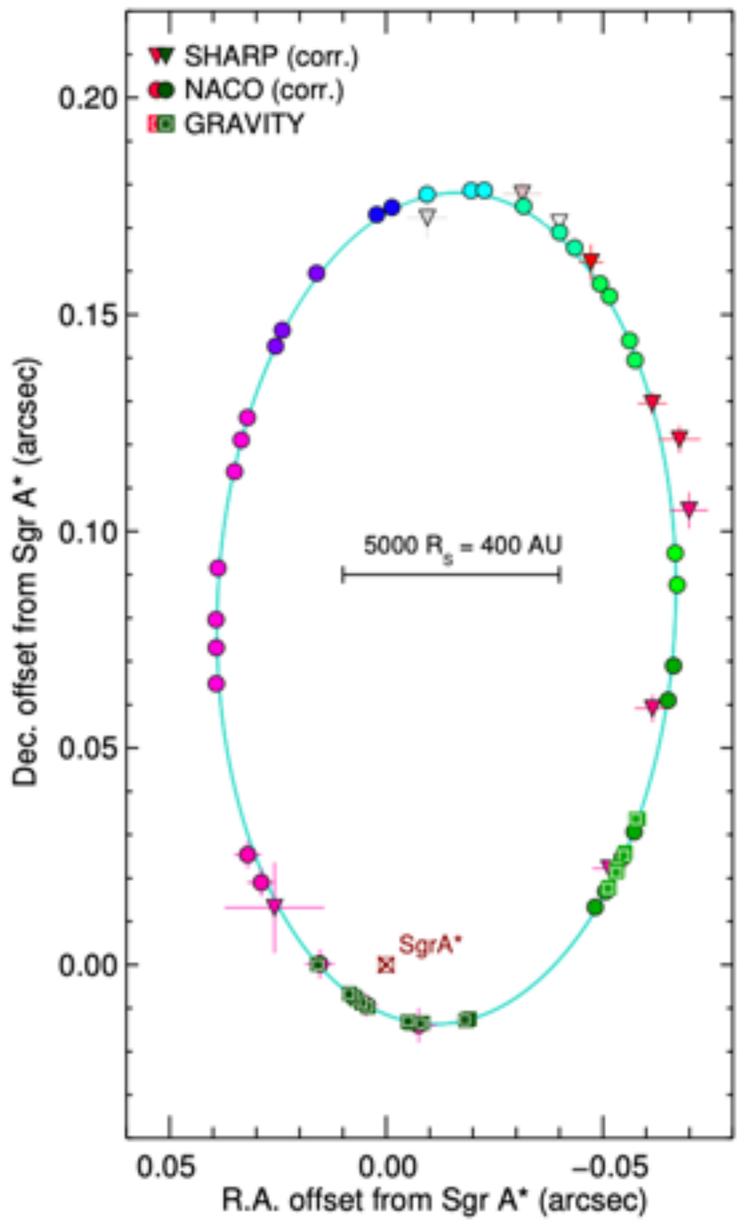
$$\frac{\lambda_\infty}{\lambda_\mathrm{e}} = \left(1 - \frac{r_\mathrm{S}}{R_\mathrm{e}}\right)^{-\frac{1}{2}},$$

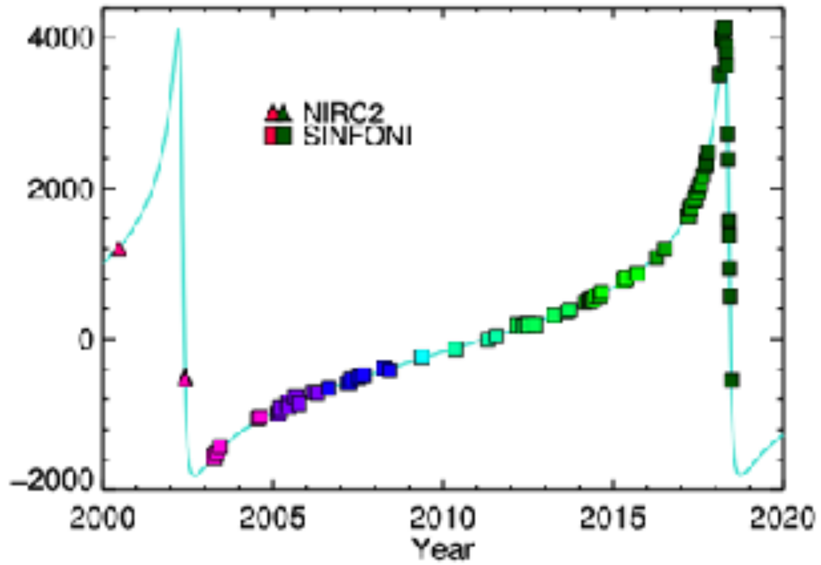
Time dilation (时钟变慢)

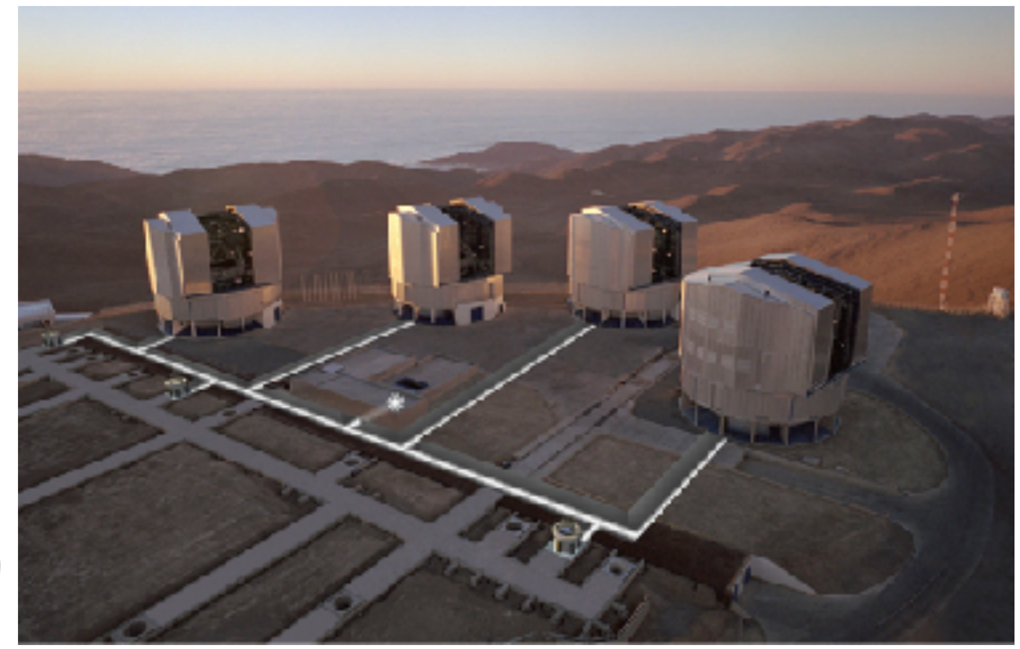
$$t_0 = t_f\sqrt{1 - \frac{2GM}{rc^2}} = t_f\sqrt{1 - \frac{r_s}{r}}$$

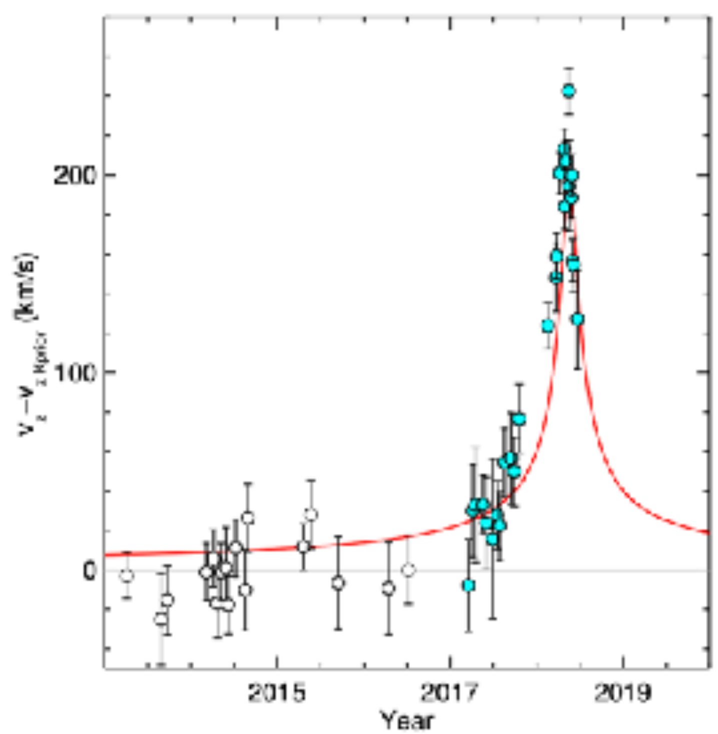


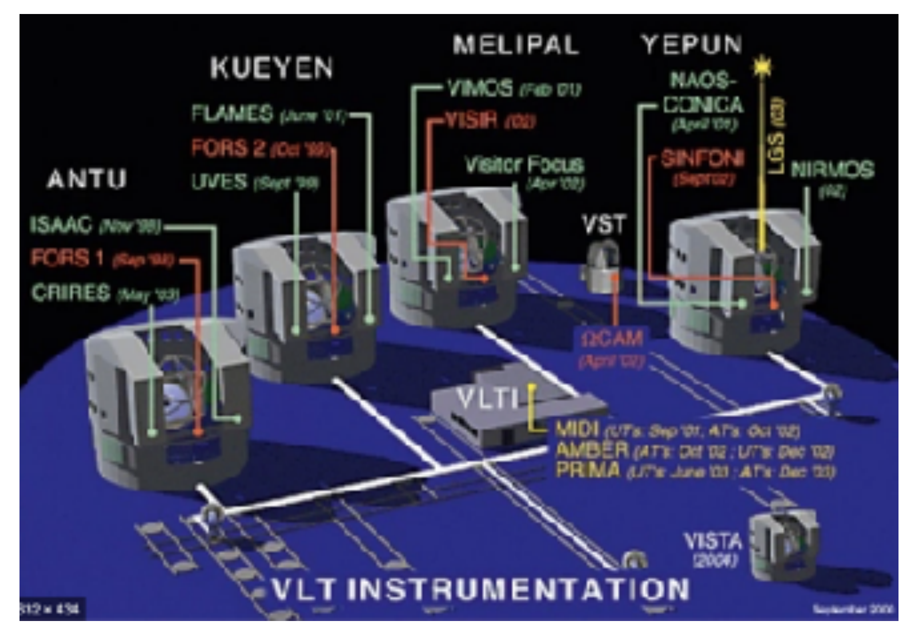
Gravitational redshift— Observation





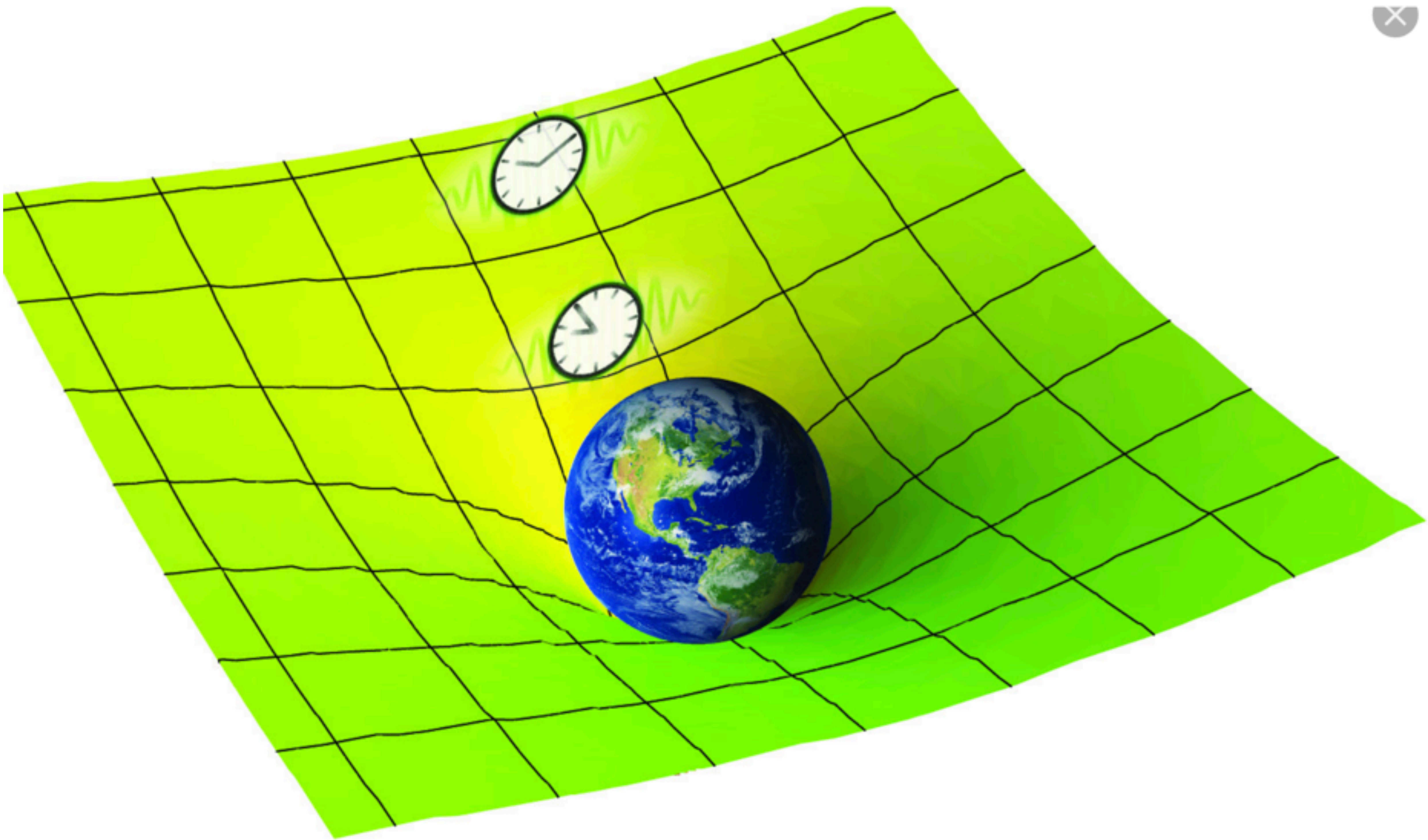






Gravitational time dilation (引力时间延迟)

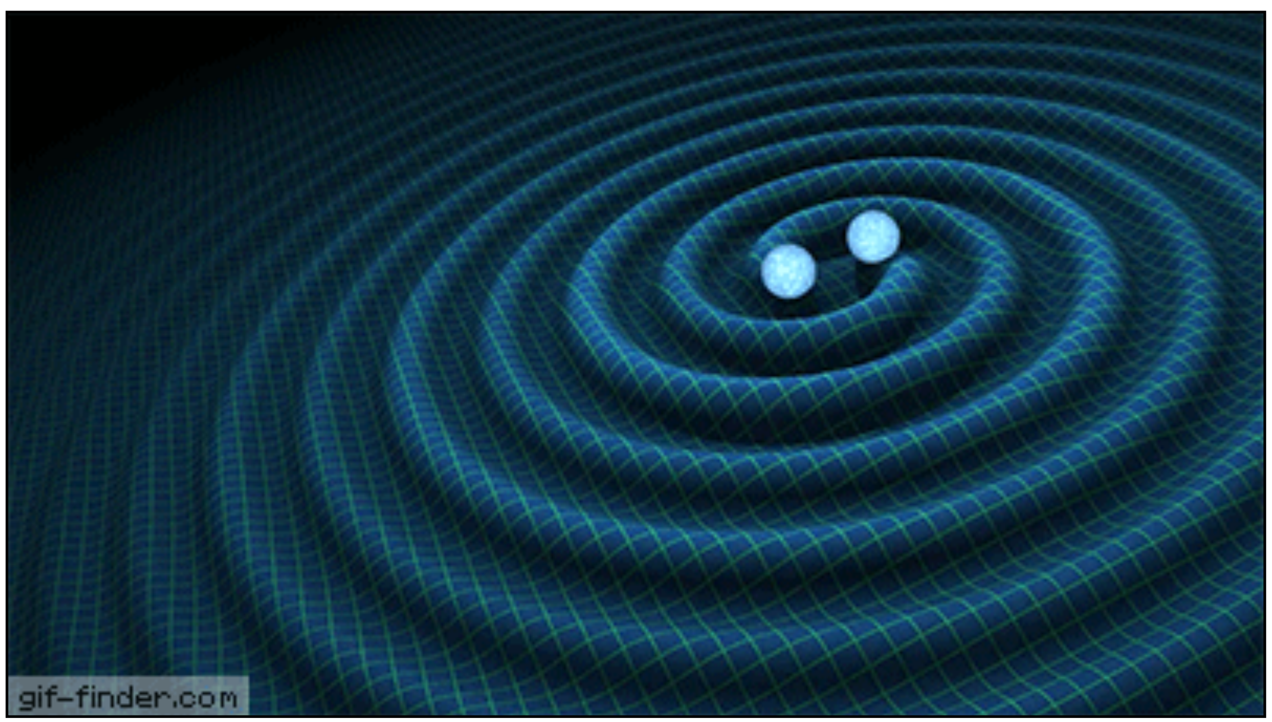
$$t_0 = t_f \sqrt{1 - \frac{2GM}{rc^2}} = t_f \sqrt{1 - \frac{r_s}{r}}$$

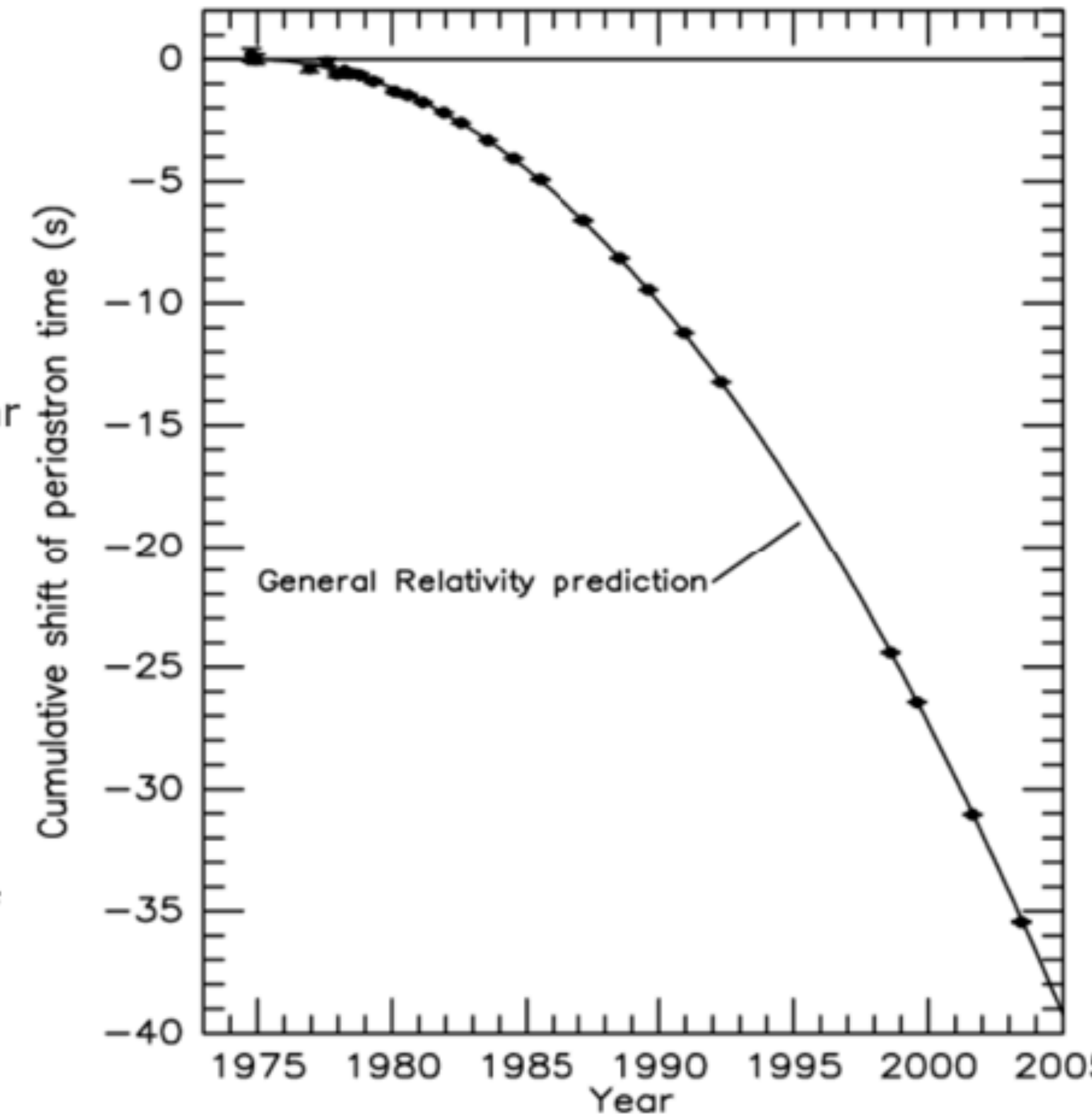


Part 5 Gravitational Wave

Gravitational wave radiation

Energy loss and orbital decay





Hulse and Taylor 1975.

Gravitational wave

Measuring gravitational wave

Amplitude of Gravitational Wave

$$h = \frac{\delta l}{l}$$

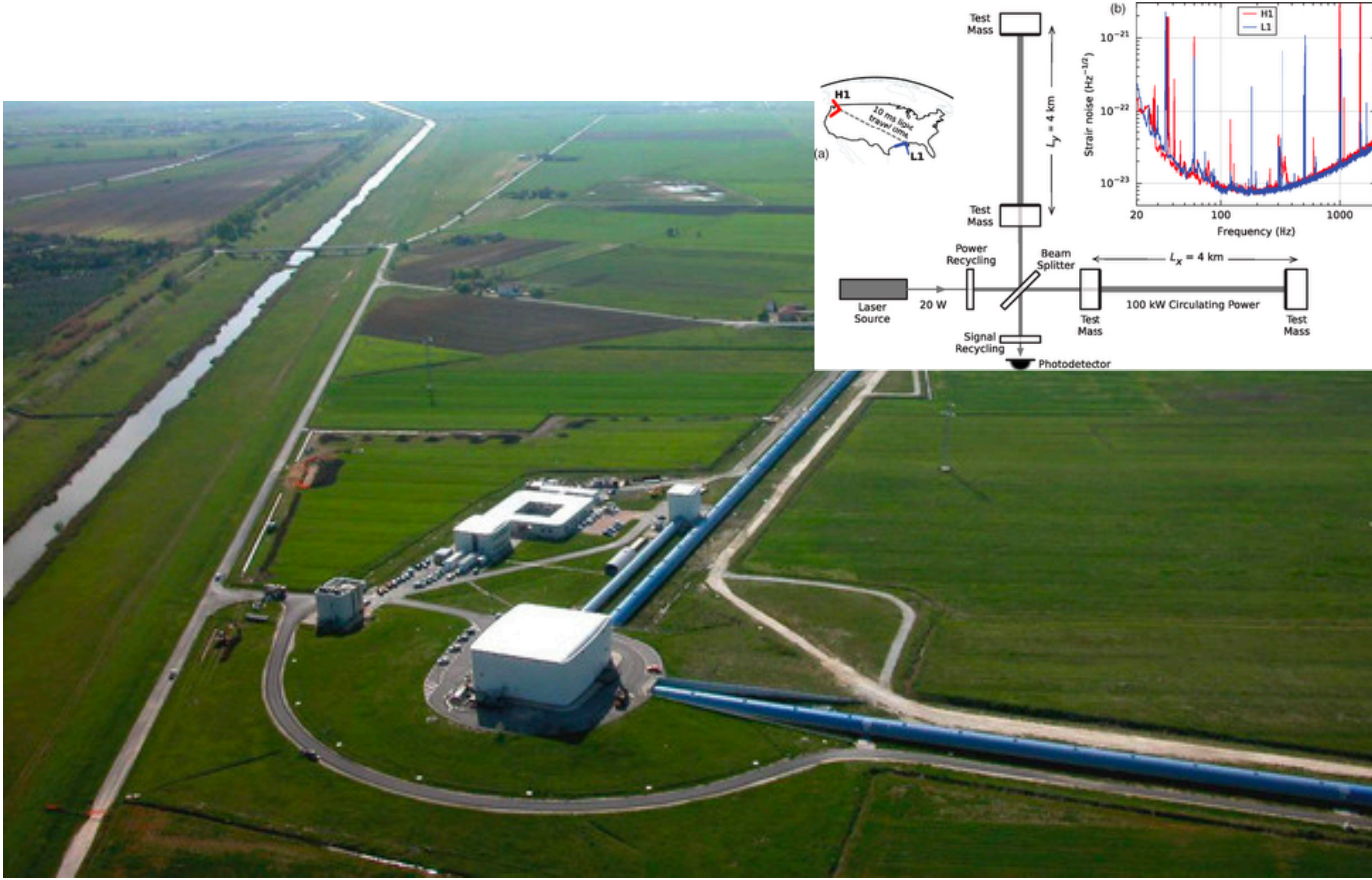
Where

$$h = \frac{2G}{c^4}\frac{1}{r}\frac{\partial^2 Q}{\partial t^2}$$

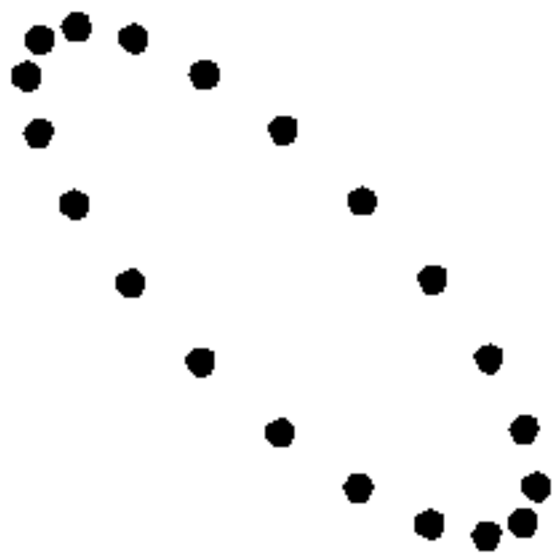
https://www.youtube.com/watch?v=4GbWfNHtHRg

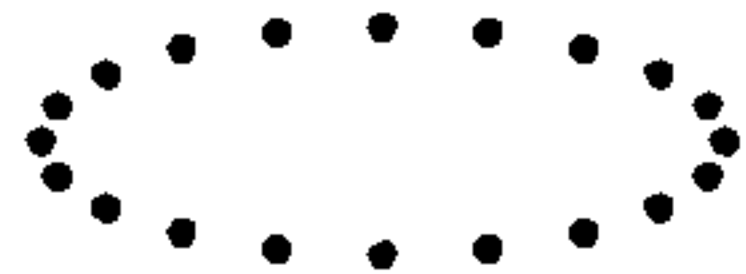
Part 5.1 LIGO Observatory

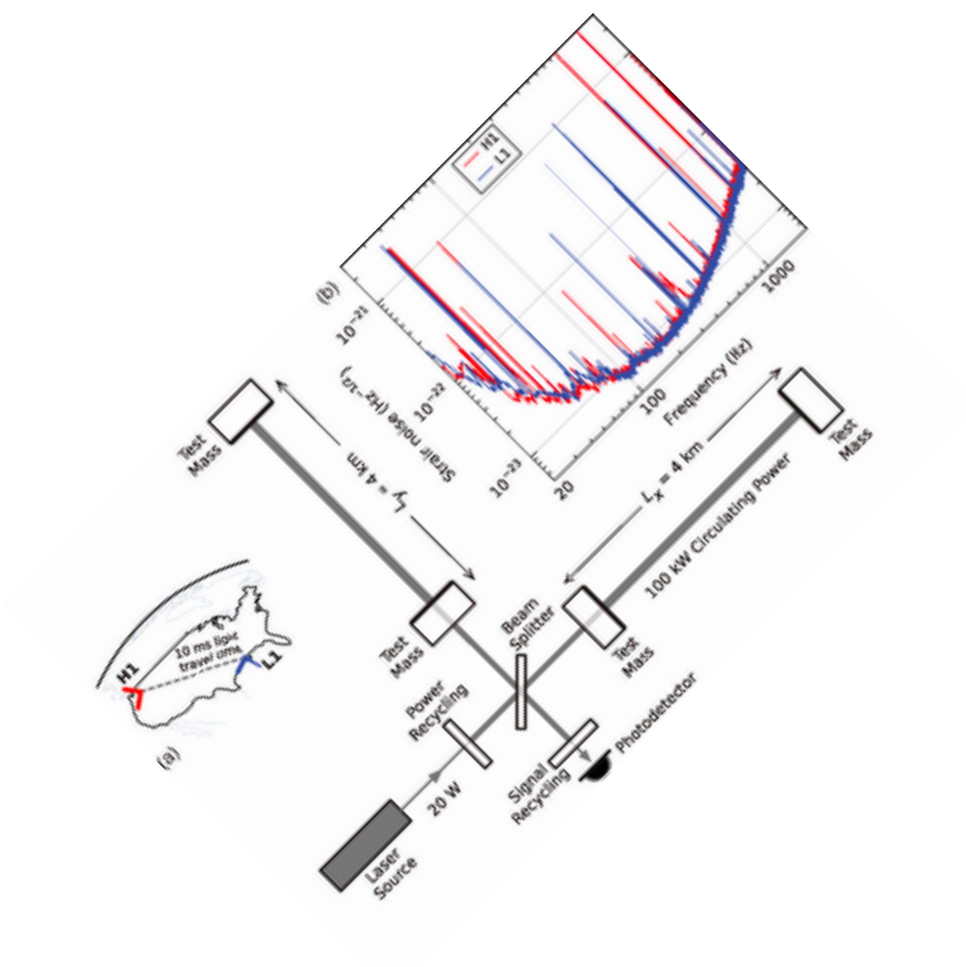
Part 5.2 Squeezed Laser and Nobel Prize

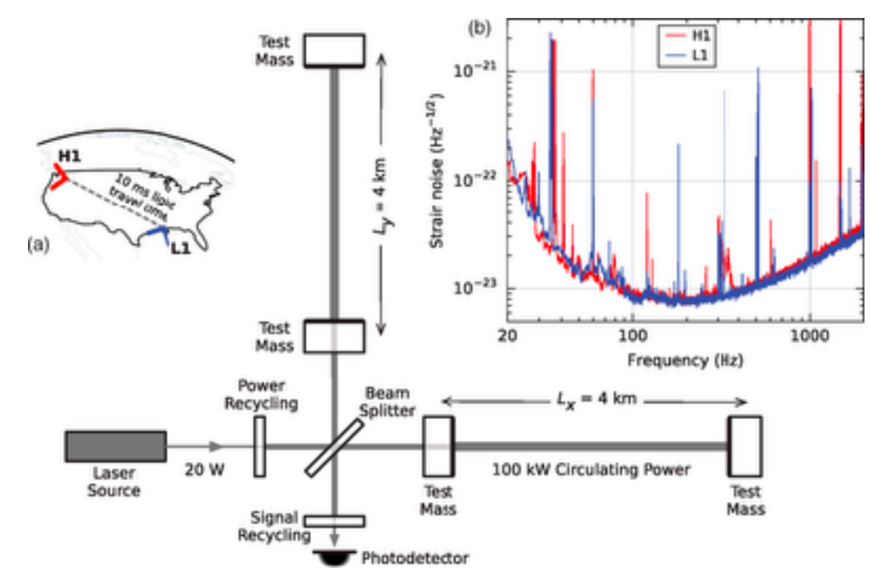


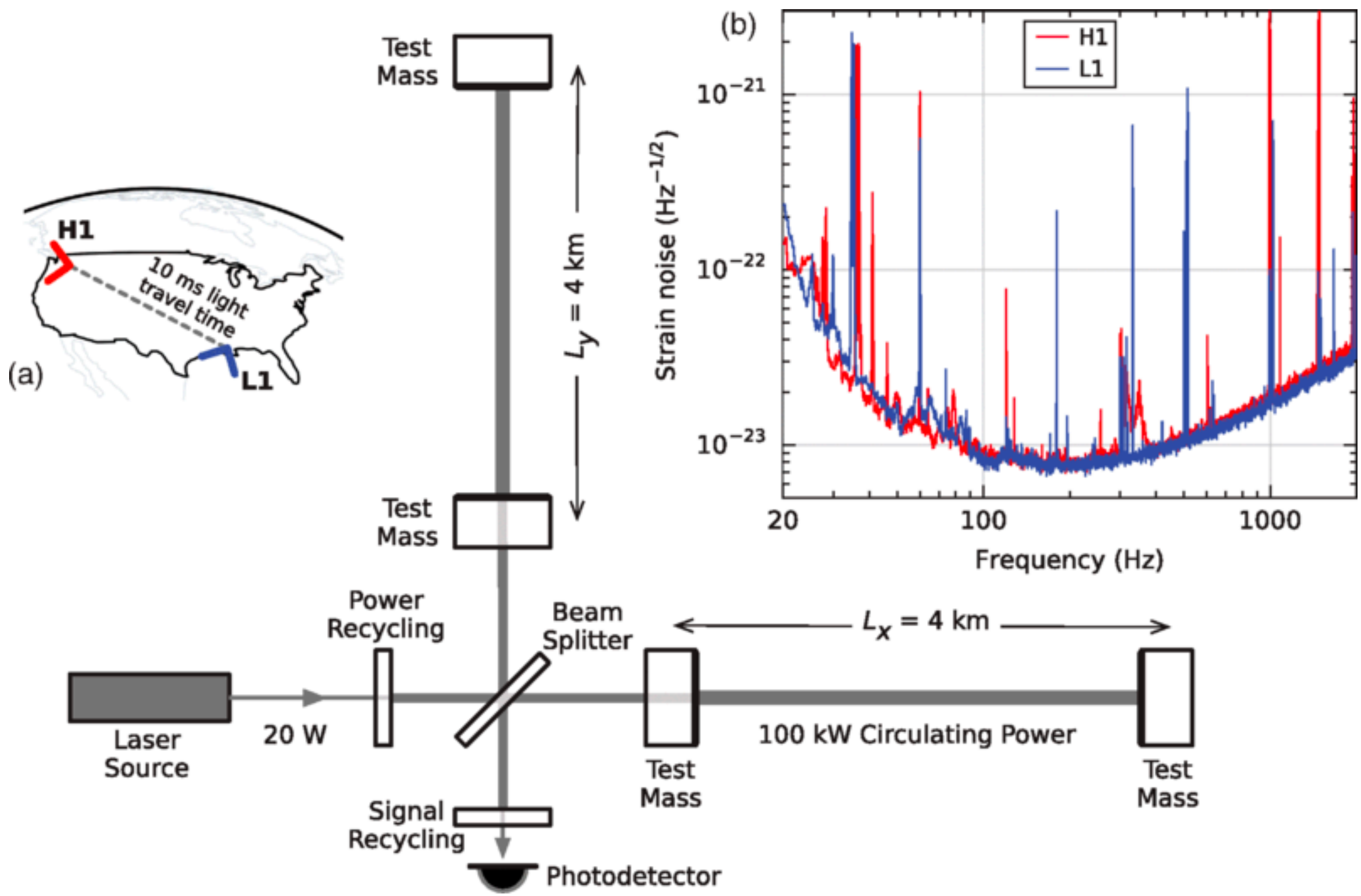
H1
10 ms light travel time
L1
(a)
Test Mass
Test Mass
L_y = 4 km
Power Recycling
Beam Splitter
L_x = 4 km
Laser Source
20 W
100 kW Circulating Power
Test Mass
Test Mass
Signal Recycling
Photodetector
(b)
H1
L1
Strain noise (Hz$^{-1/2}$)
10^{-21}
10^{-22}
10^{-23}
20
100
1000
Frequency (Hz)








(a)
H1
10 ms light travel time
L1
Test Mass
Test Mass
L_y = 4 km
Power Recycling
Beam Splitter
L_x = 4 km
Laser Source
20 W
Test Mass
100 kW Circulating Power
Test Mass
Signal Recycling
Photodetector
(b)
H1
L1
Strain noise (Hz$^{-1/2}$)
10^{-21}
10^{-22}
10^{-23}
20
100
1000
Frequency (Hz)



(a)
H1
L1
10 ms light travel time
Test Mass
Test Mass
L_y = 4 km
(b)
H1
L1
Strain noise (Hz$^{-1/2}$)
10^{-21}
10^{-22}
10^{-23}
20
100
1000
Frequency (Hz)
Power Recycling
Beam Splitter
L_x = 4 km
Laser Source
20 W
100 kW Circulating Power
Test Mass
Test Mass
Signal Recycling
Photodetector

LIGO Observatory

- 1970s Early work on gravitational-wave detection by laser interferometers
- 1999 LIGO inauguration ceremony
- 2006 LIGO design sensitivity achieved.
- 2010 Initial LIGO operations conclude; Advanced LIGO installation begins at the observatories.
- 2014 Advanced LIGO installation complete (Squeezed Laser)
- Sept 14, 2015 Advanced LIGO detects gravitational waves from collision of two black holes
- Aug 14, 2017 Gravitational waves from a binary black hole merger observed by LIGO and Virgo

Fundamental physics

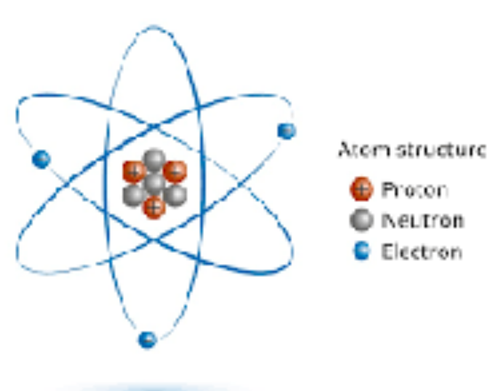
Planck constant:

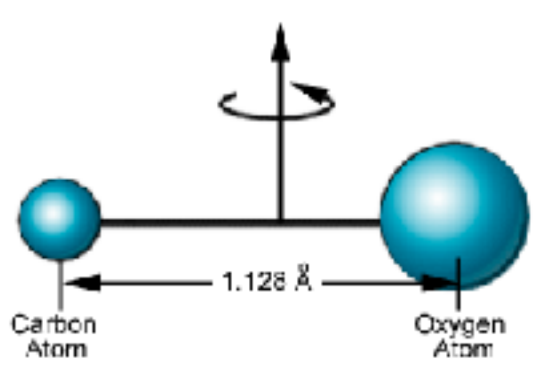
$$h = 6.626\,070\,15 \times 10^{-34}\ \mathrm{J{\cdot}s}.$$ [2][3]

$$E = hf.$$

Uncertainty Principle:

Position-Momentum	Angular momentum	Energy-time
$\Delta x\, \Delta p_x \geq \frac{\hbar}{2},$	$I\omega^2 = h\omega$	$\delta E\, \delta t \approx h$





?

Fundamental physics

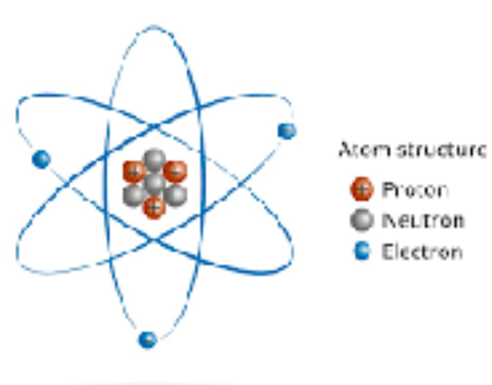
Planck constant:

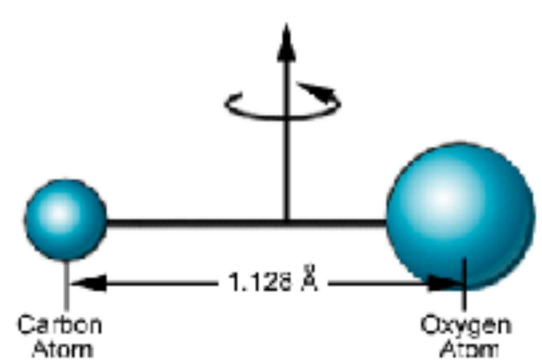
$$h = 6.626\,070\,15 \times 10^{-34}\ \mathrm{J \cdot s.}$$ [2][3]

$$E = hf.$$

Uncertainty Principle:

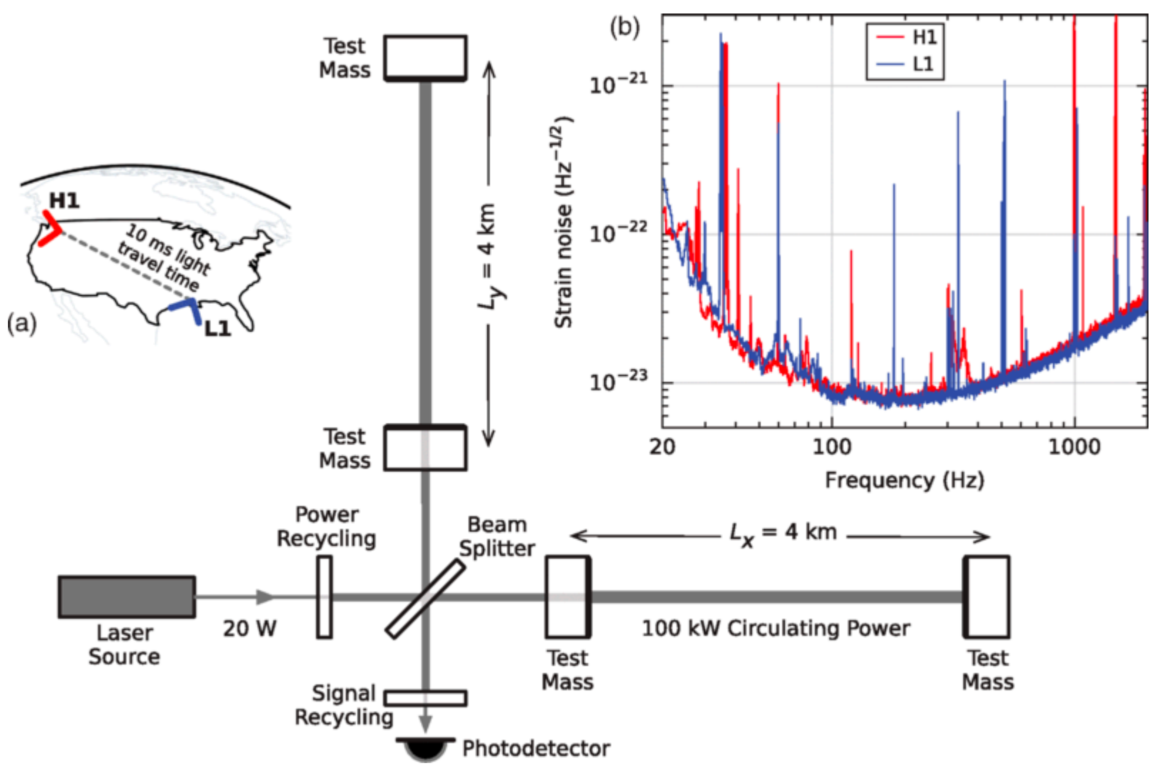
Position-Momentum	Angular momentum	Energy-time
$\Delta x\, \Delta p_x \geq \frac{\hbar}{2},$	$I\omega^2 = h\omega$	$\delta E\, \delta t \approx h$

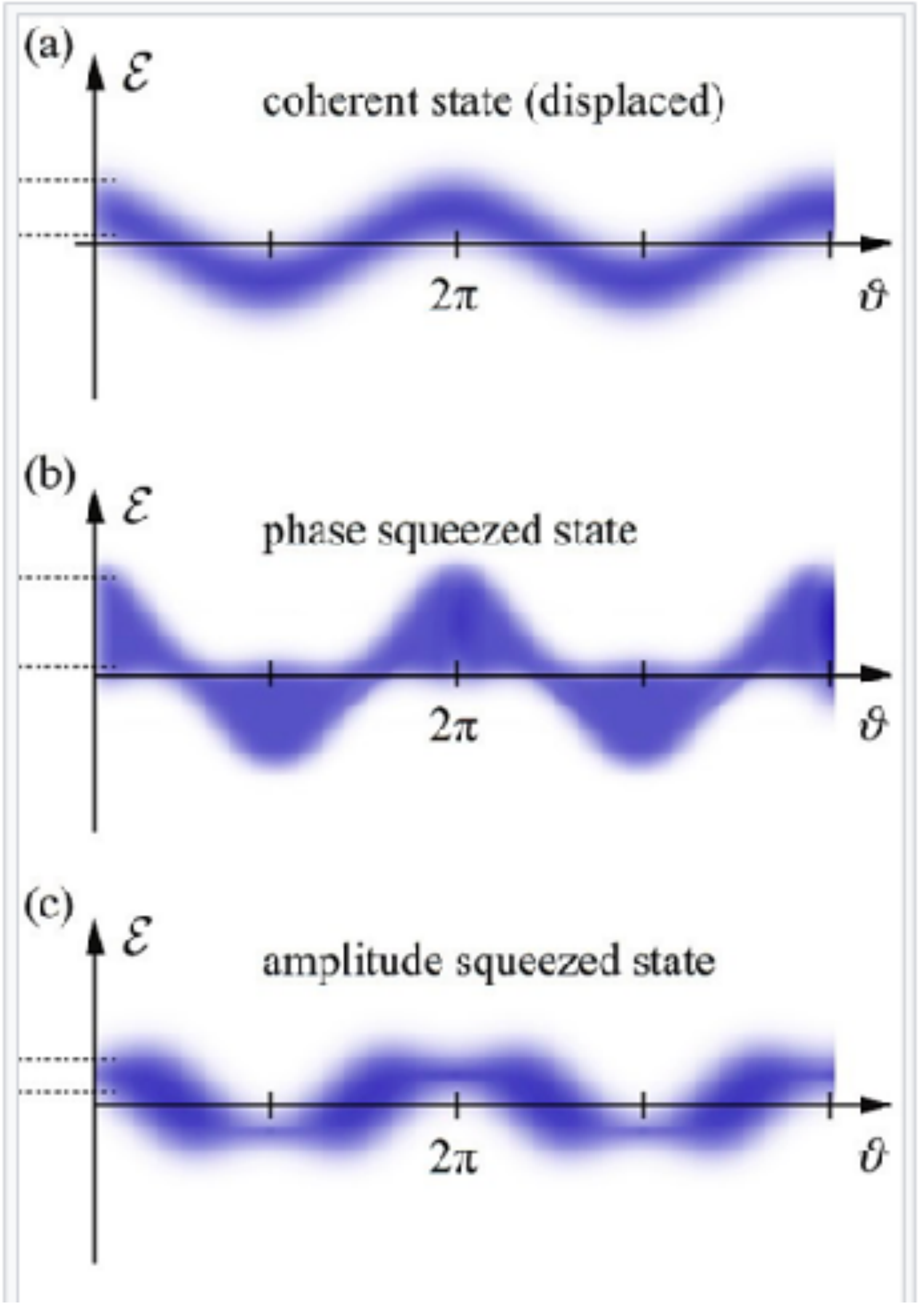




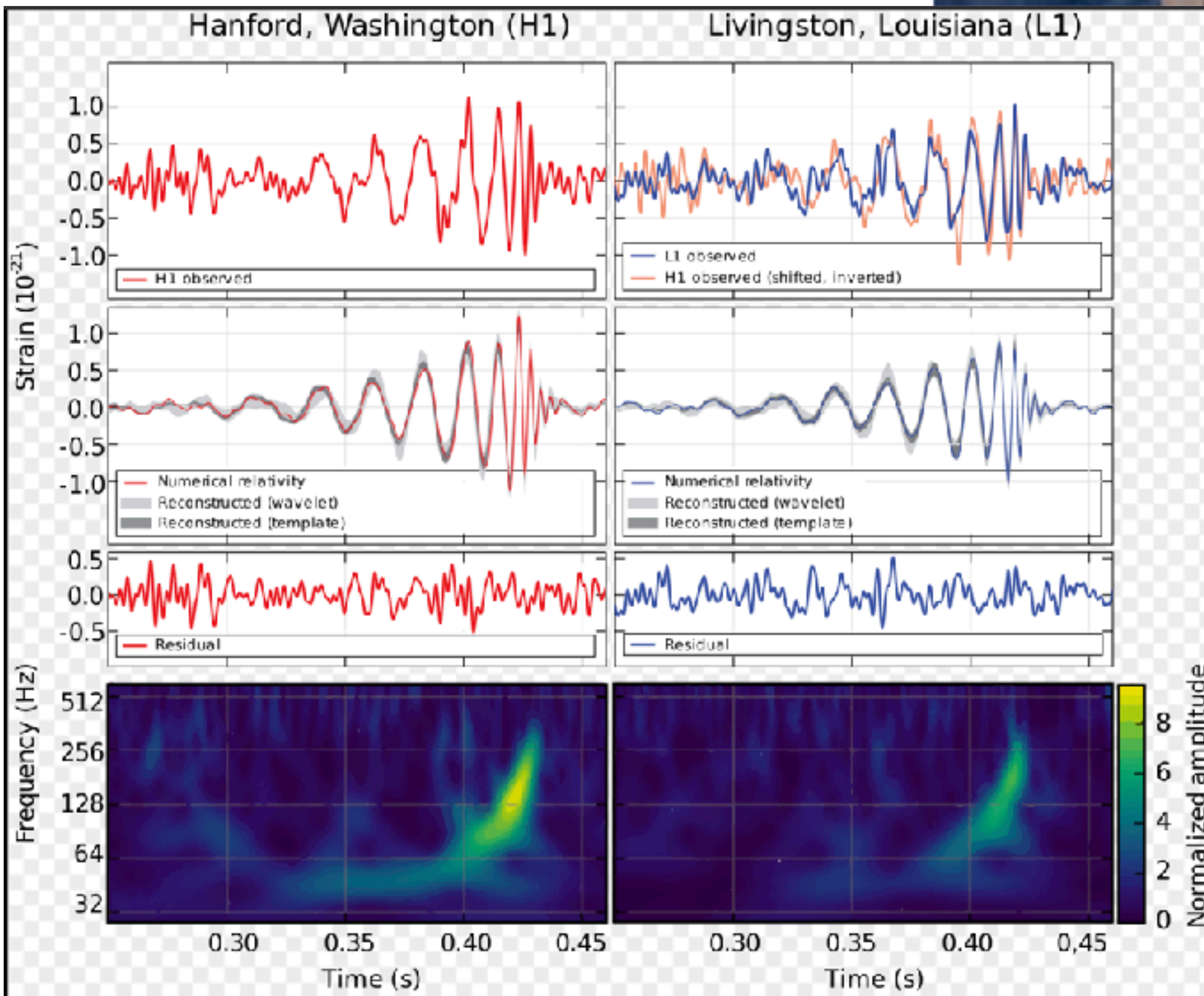
Phase-amplitude uncertainty

Squeezed states of light





Gravitational Wave Detection



el Prize for ...

References

- Good overview of General Physics
  - Malcolm Longair “Theoretical concepts in physics”, 物理中的理论概念
  - 

- General Relativity
  - 

Not logged in Talk Contributions Create account Log in

WIKIPEDIA
The Free Encyclopedia

Main page
Contents
Featured content
Current events
Random article
Donate to Wikipedia
Wikipedia store

Interaction

Help
About Wikipedia
Community portal
Recent changes
Contact page

Tools

What links here

Article Talk Read Edit View history Search Wikipedia

Chandrasekhar limit

From Wikipedia, the free encyclopedia

The **Chandrasekhar limit** (/tʃʌndrəˈseɪkər/) is the maximum mass of a stable white dwarf star. The currently accepted value of the Chandrasekhar limit is about 1.4 $M_{\odot}$ (2.765×10^{30} kg).[1][2][3]

White dwarfs resist gravitational collapse primarily through electron degeneracy pressure (compare main sequence stars, which resist collapse through thermal pressure). The Chandrasekhar limit is the mass above which electron degeneracy pressure in the star's core is insufficient to balance the star's own gravitational self-attraction. Consequently, a white dwarf with a mass greater than the limit is subject to further gravitational collapse, evolving into a different type of stellar remnant, such as a neutron star or black hole. Those with masses under the limit remain stable as white dwarfs.[4]

The limit was named after Subrahmanyan Chandrasekhar, an Indian astrophysicist who improved upon the accuracy of the calculation in 1930, at the age of 20, in India by calculating the limit for a polytrope model of a star in hydrostatic equilibrium, and comparing his limit to the earlier limit found by E. C. Stoner for a uniform density star. Importantly, the existence of a limit, based on the conceptual breakthrough of combining relativity with Fermi degeneracy, was indeed first established in separate papers published by Wilhelm Anderson and E. C. Stoner in 1929. The limit was initially ignored by the community of scientists because such a limit would logically require the existence of black holes, which were considered a scientific impossibility at the time. That the roles of Stoner and Anderson are often forgotten in the astronomy community has been noted.[5][6]