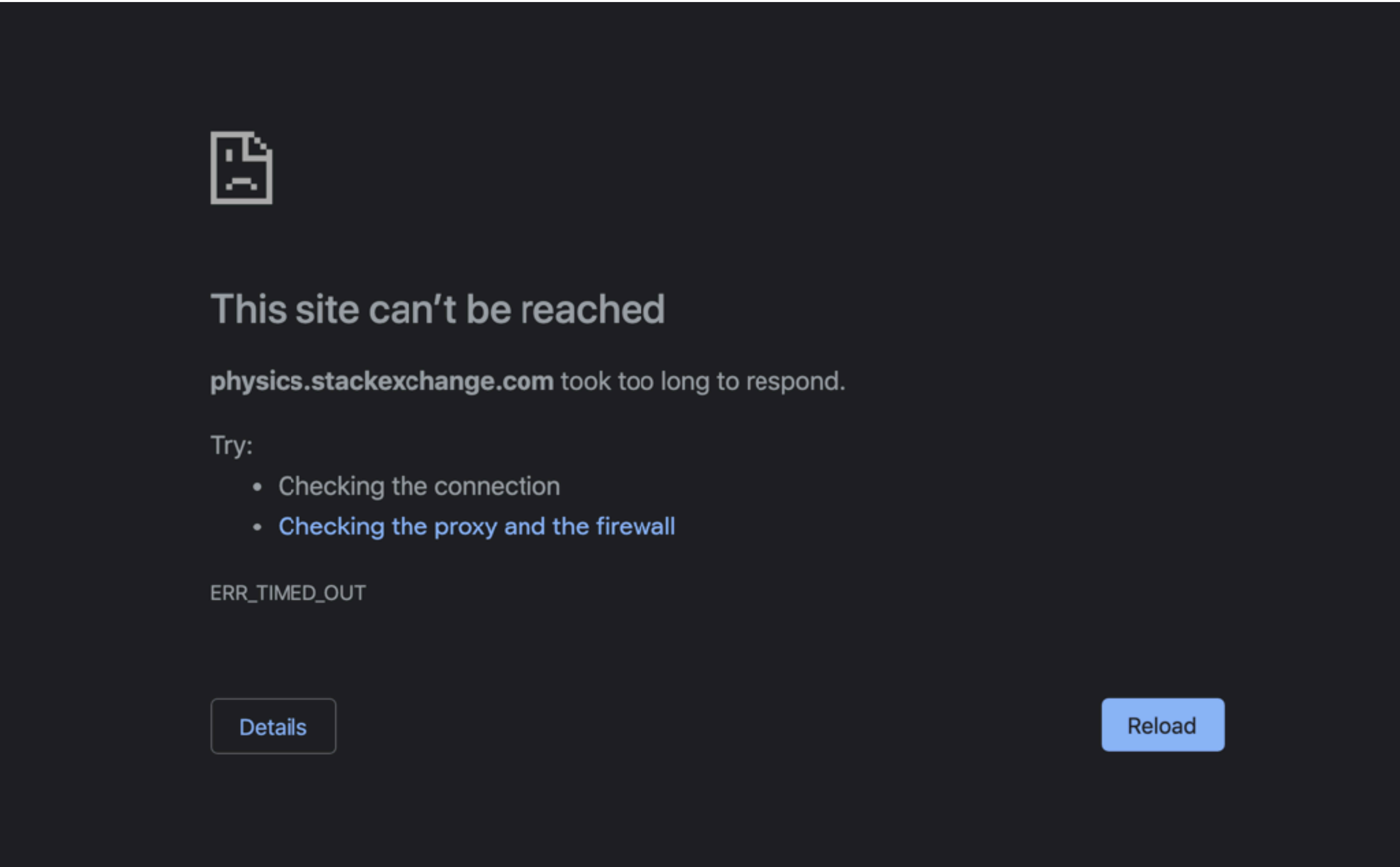
Astrophysics

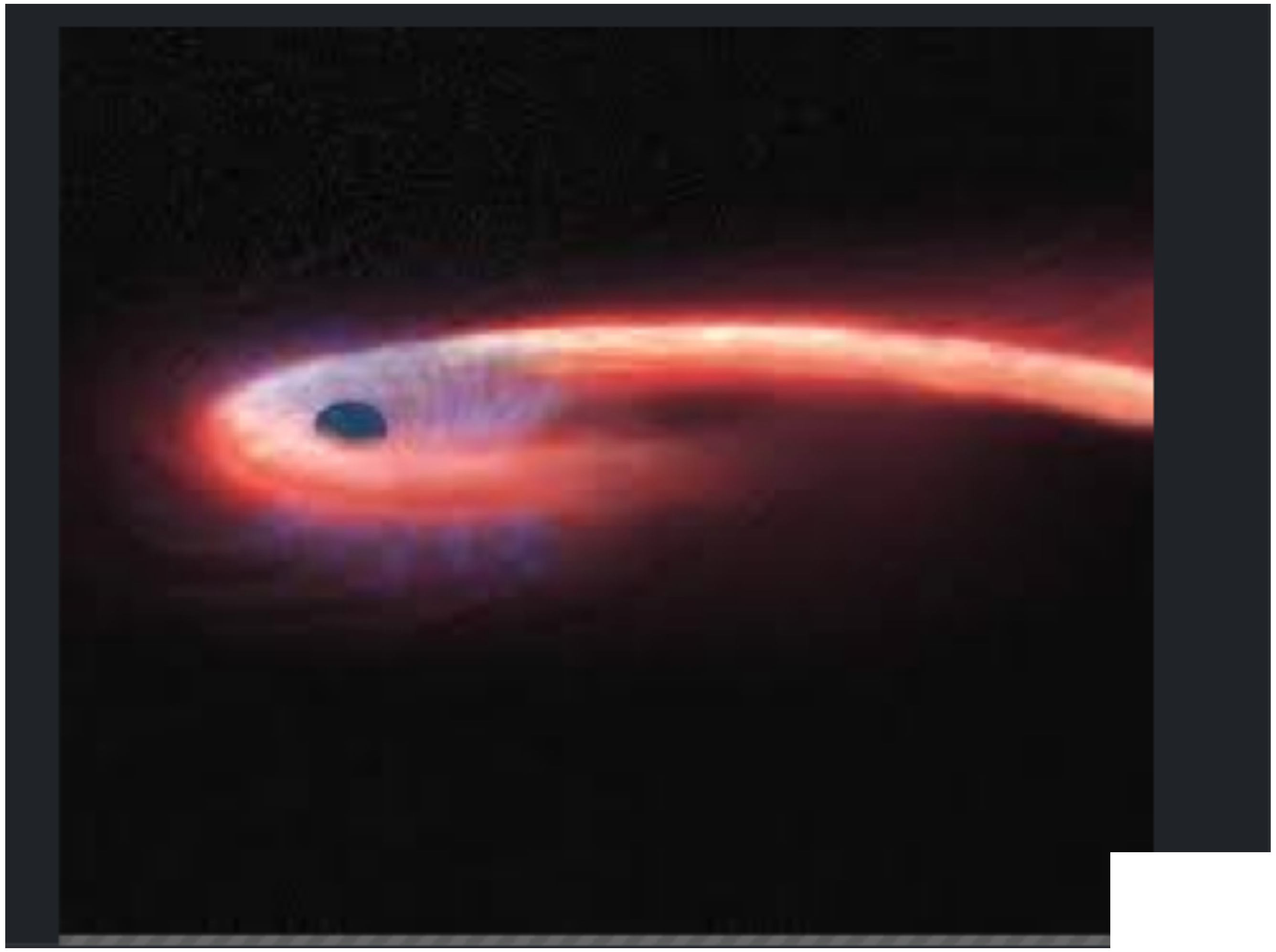
Guang-Xing Li 李广兴

Disclaimer: not enough time

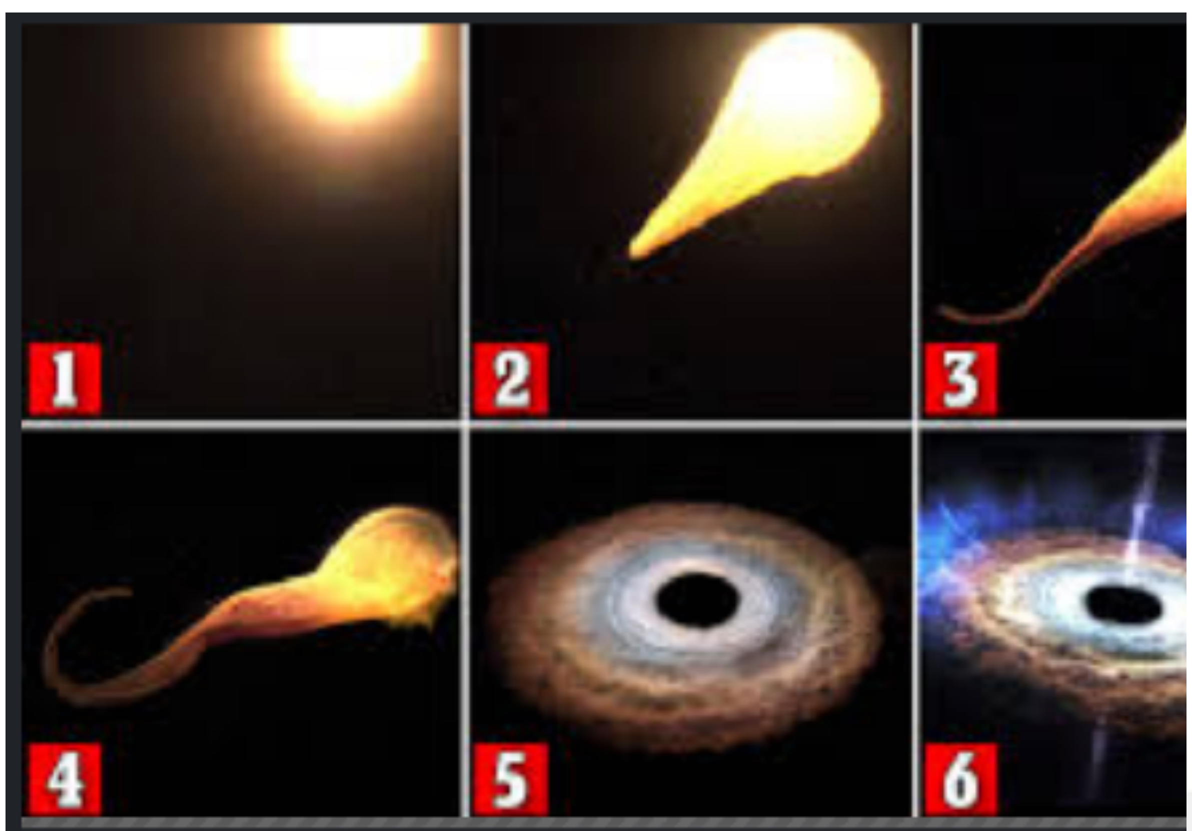


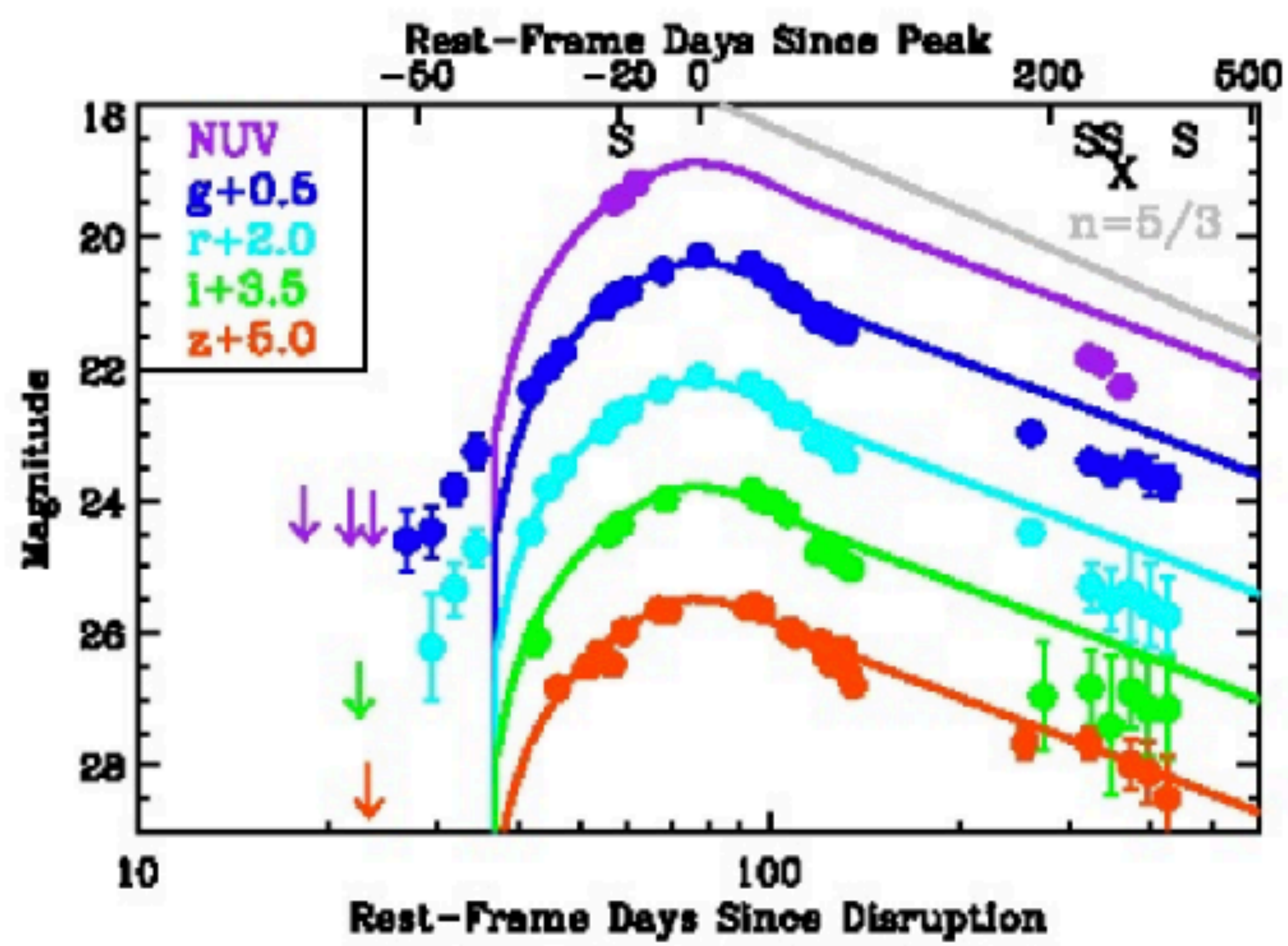
Tidal-disruption Events

Tidal disruption events



An ultraviolet-optical flare from the tidal disruption of a helium rich stellar core





General relativity: recap

arxiv.org/abs/gr-qc/9712019

Cornell University

We gratefully acknowledge support from the Simons Foundation and member institutions.

arXiv.org > gr-qc > arXiv:gr-qc/9712019

Search... All fields Search

Help | Advanced Search

General Relativity and Quantum Cosmology

Lecture Notes on General Relativity

Sean M. Carroll

(Submitted on 3 Dec 1997)

These notes represent approximately one semester's worth of lectures on introductory general relativity for beginning graduate students in physics. Topics include manifolds, Riemannian geometry, Einstein's equations, and three applications: gravitational radiation, black holes, and cosmology.

Comments:	238 pages, numerous figures. Individual chapters, and potentially updated versions, can be found at this http URL
Subjects:	**General Relativity and Quantum Cosmology (gr-qc)**; Astrophysics (astro-ph); High Energy Physics - Theory (hep-th)
Report number:	NSF-ITP/97-147
Cite as:	arXiv:gr-qc/9712019
	(or arXiv:gr-qc/9712019v1 for this version)

Bibliographic data

Select data provider: **Semantic Scholar** | INSPIRE HEP | NASA ADS | Prophy [Disable Bibex(What is Bibex?)]

References (5) Citations (80)

Download:

- PDF
- PostScript
- Other formats

(license)

Current browse context:

gr-qc

< prev | next >

new | recent | 9712

References & Citations

- INSPIRE HEP (refers to | cited by)
- NASA ADS

7 blog links (what is this?)

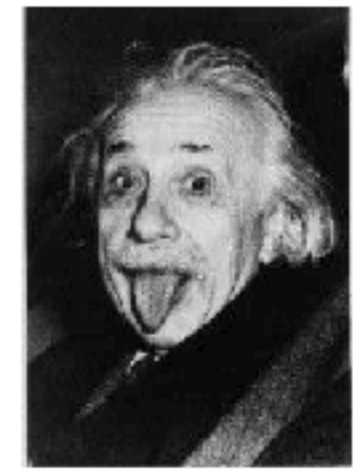
Lecture Notes on General Relativity...

- Sean Michael Carroll

Bibex: **Semantic Scholar**

Bookmark

http://www.blau.itp.unibe.ch/newlecturesGR.pdf



Lecture Notes on General Relativity

Matthias Blau

Albert Einstein Center for Fundamental Physics

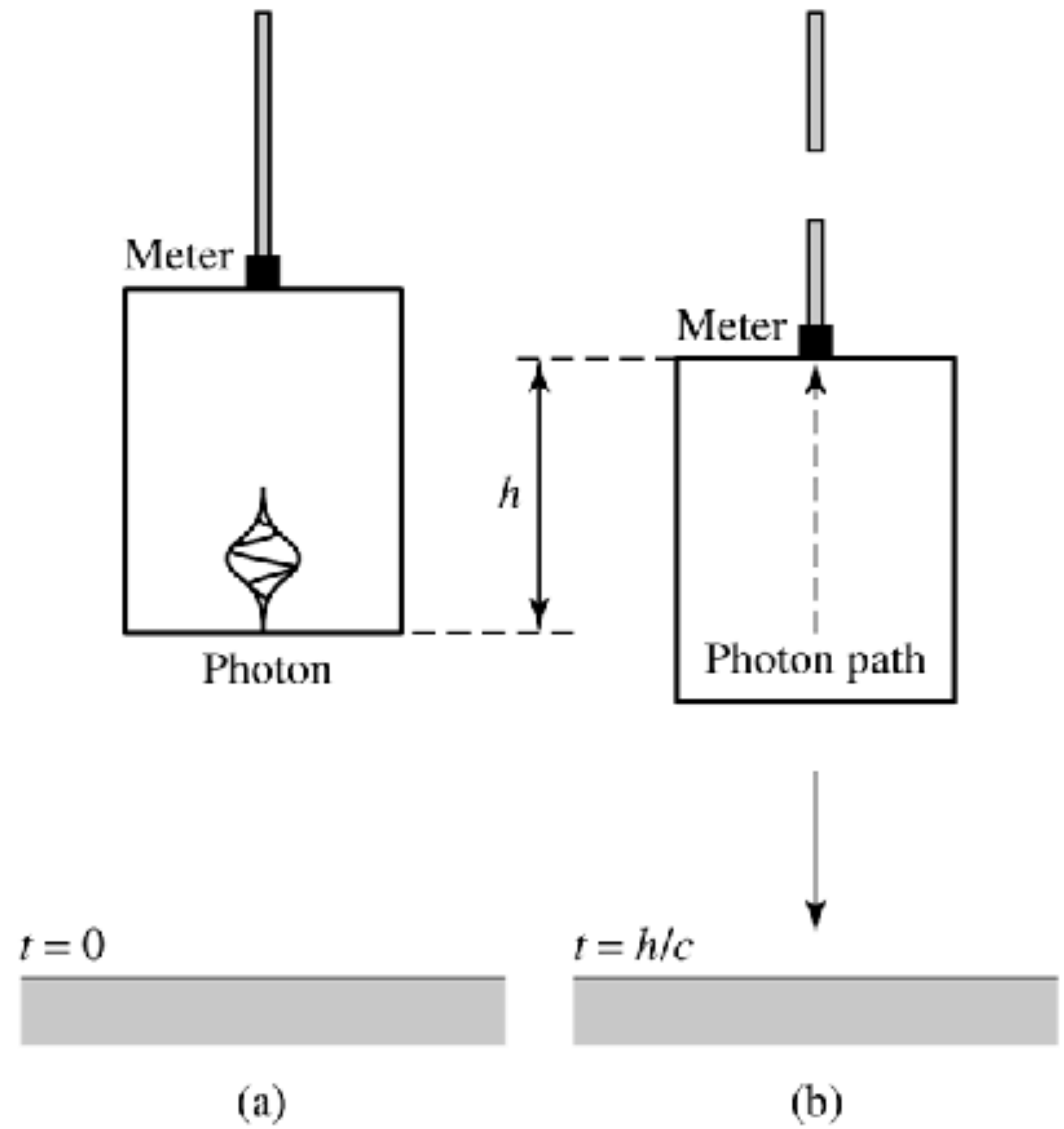
Institut für Theoretische Physik

Universität Bern

CH-3012 Bern, Switzerland

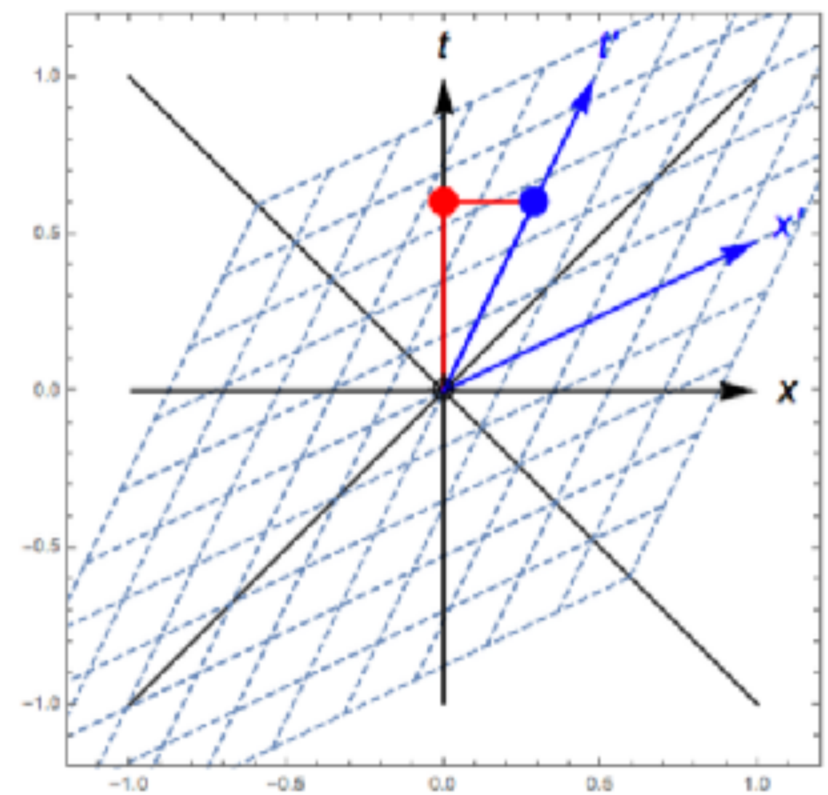
Gravitational redshift

$$\frac{\Delta \nu}{\nu_0} = -\frac{v}{c} = -\frac{gh}{c^2}.$$



Redshift in Special Relativity

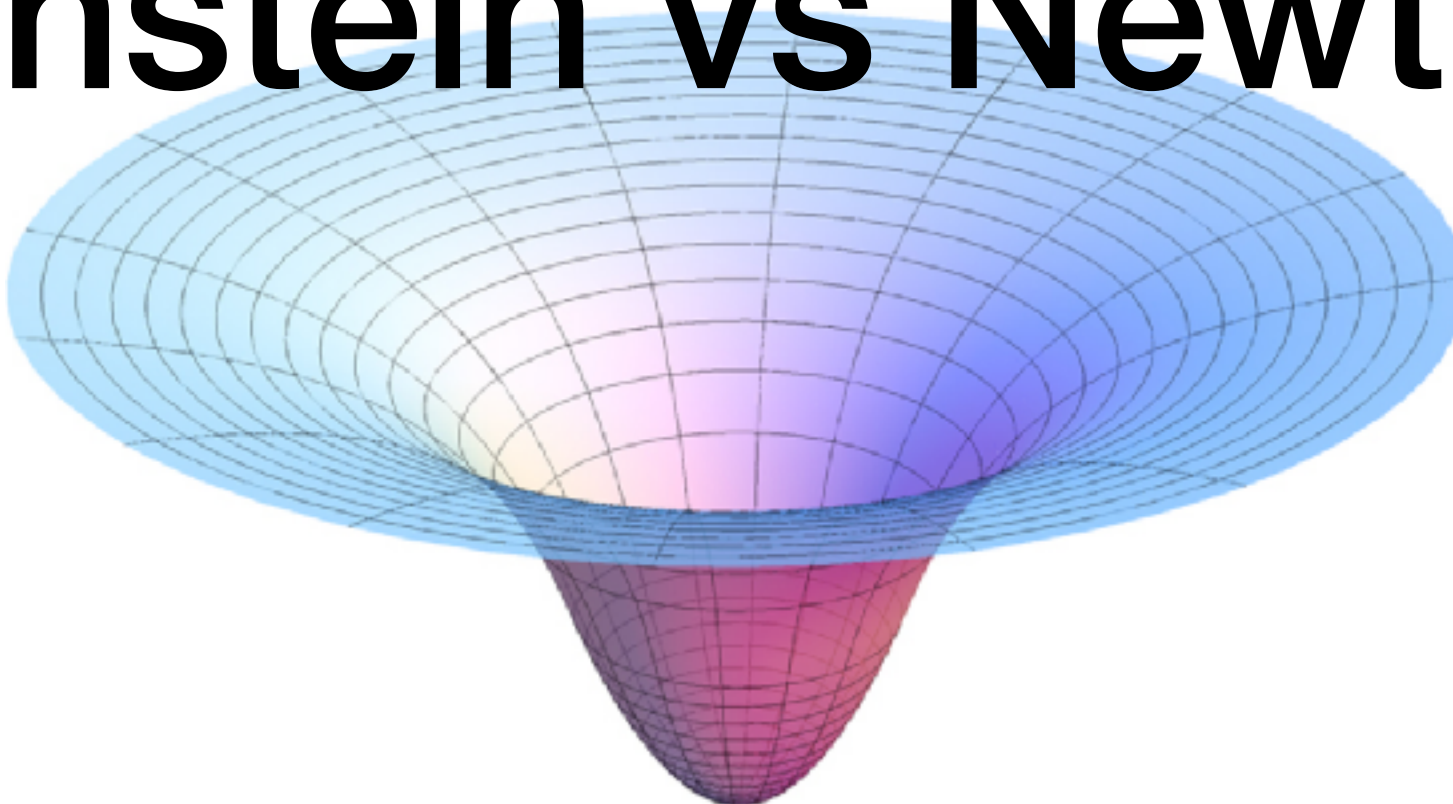
$$\Lambda^{\mu}{}_{\nu} = \begin{bmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$



$$p = (p^0, p^1, p^2, p^3) = \left(\frac{E}{c}, p_x, p_y, p_z\right)$$

(1) Find the four-momentum
(2) Multiple by the Lorentz matrix
(3) Get the new four-momentum
(4) Take the first element -> Energy

Einstein vs Newton



Einstein	Newton
Field Equation $$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$	**Poisson Equation** $$\nabla^2\phi = 4\pi G\rho.$$
Metric g_{mn} = diag(1 + 2U, −1 + 2U, −1 + 2U, −1 + 2U).	**Gravitational potential** $U = -GM / c^2r$.
Geodesics equation $$\frac{d^2x^\mu}{ds^2} + \Gamma^\mu{}_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds} = 0$$	**Newton's law** $$\mathbf{F} = m\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = m\mathbf{a},$$

Schwarzschild Solution (Black Holes)

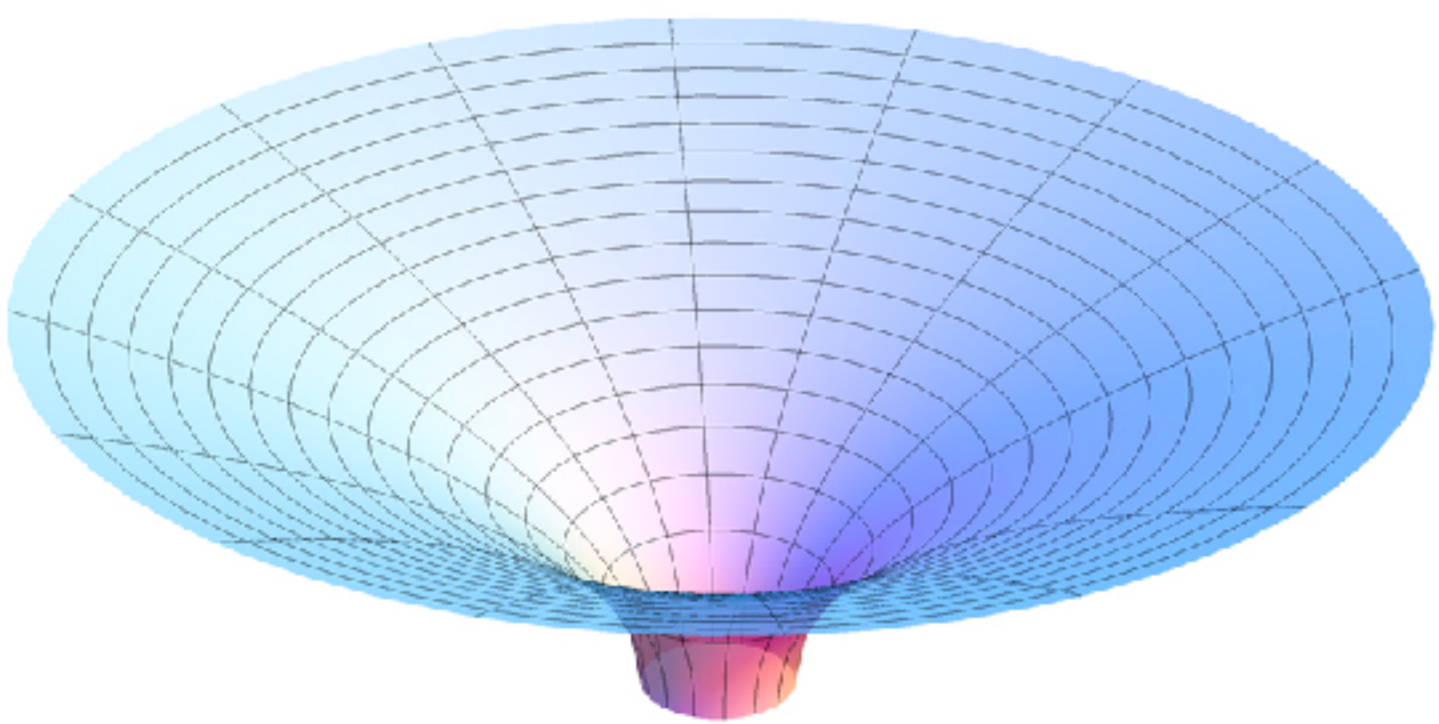
In Schwarzschild coordinates (t, r, θ, ϕ) the Schwarzschild metric (or equivalently, the line element for proper time) has the form

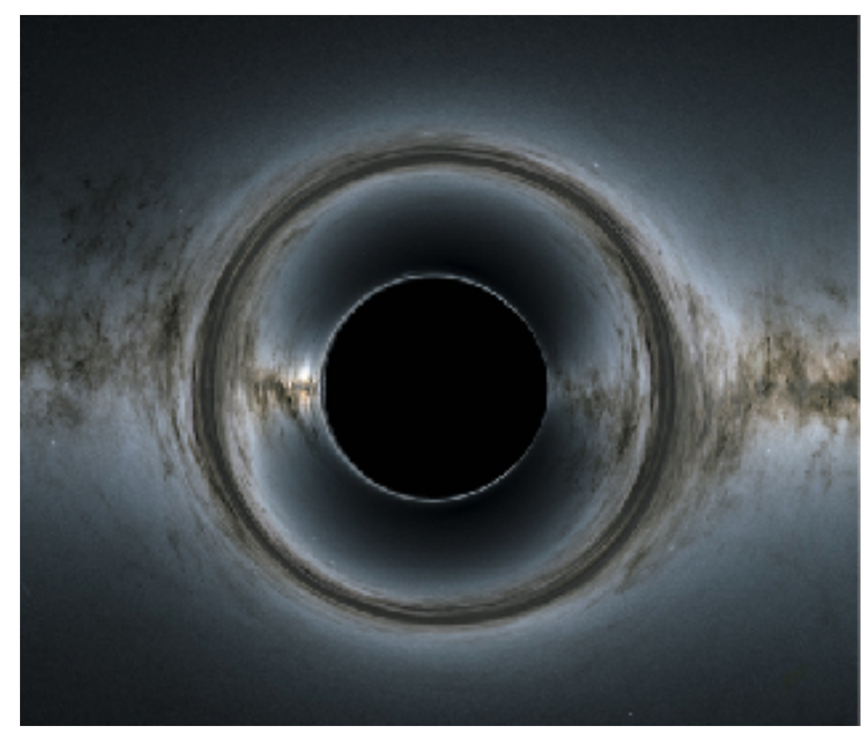
$$g = -c^2\, d\tau^2 = -\left(1 - \frac{r_s}{r}\right) c^2\, dt^2 + \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 + r^2 g_\Omega,$$

$$R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} = \frac{12 G^2 M^2}{r^6}\,.$$

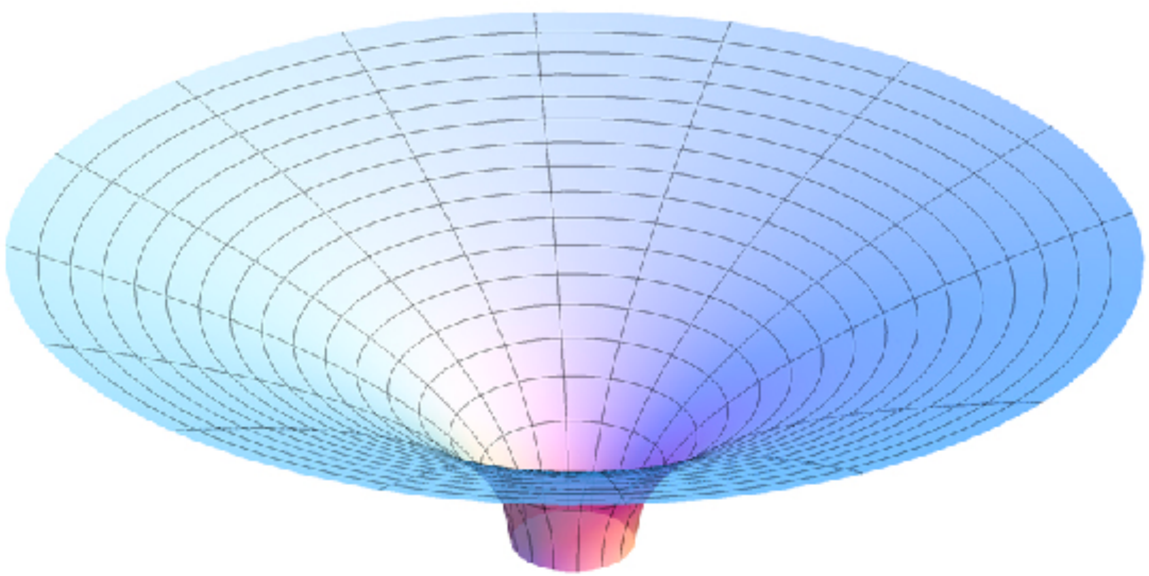
$$g_\Omega = \left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$$

$$r_s = \frac{2GM}{c^2}$$





Photos travel to event horizon



Photons have ds^2 = 0

$$ds^2 = 0 = -\left(1 - \frac{2GM}{r}\right) \mathrm{d}t^2 + \left(1 - \frac{2GM}{r}\right)^{-1} \mathrm{d}r^2 \ ,$$

$$\frac{dt}{dr} = \pm\left(1 - \frac{2GM}{r}\right)^{-1} \ .$$

$$ds^2 = -(1 - 2m/r)dt^2 + (1 - 2m/r)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \ .$$

Metric of curved space-time

- Cartesian
  - $ds^2 = dx^2 + dy^2$
  - $ds^2 = dr^2 + r^2 d\theta^2$
- Curved
  - $ds^2 = dr^2 + r^2 f(r) d\theta^2$

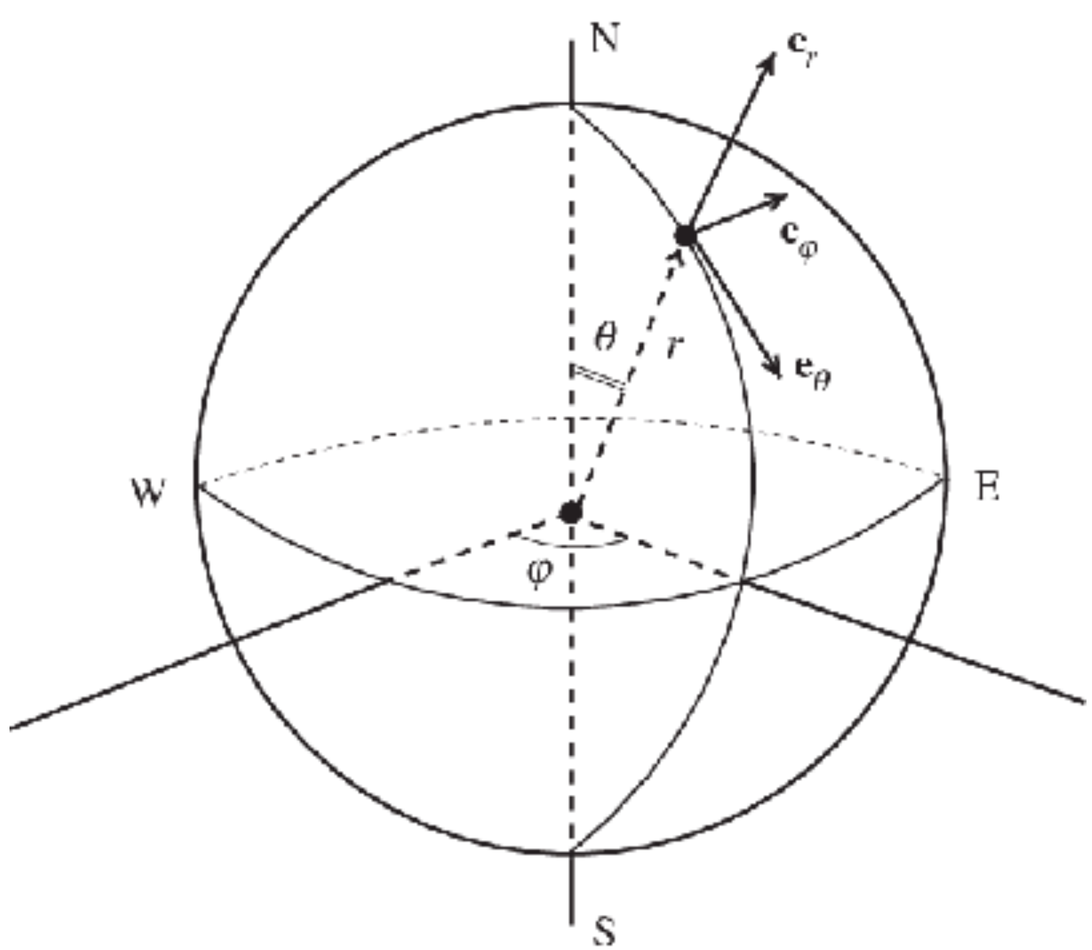
- Special Relativity
  - $ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$
- Curved e.g. around a black hole

$$g = -c^2\,d\tau^2 = -\left(1 - \frac{r_s}{r}\right) c^2\,dt^2 + \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 + r^2 g_\Omega,$$

where g_Ω is the metric on the two sphere, i.e. $g_\Omega = \left(d\theta^2 + \sin^2\theta\,d\varphi^2\right)$.

Local coordinates

$$ds^2(S^2_R) = R^2(d\theta^2 + \sin^2\theta d\phi^2) \equiv R^2 d\Omega^2 \quad .$$



$$E_1 = R^{-1}\partial_\theta \quad , \quad E_2 = (R\sin\theta)^{-1}\partial_\phi \quad ;$$

Raising and lowering indices

Vectors (order-1 tensors) [edit]

Multiplying by the *contravariant* metric tensor g^{ij} and contracting produces another tensor with an upper index:

$$g^{ij}A_j = B^i \ ,$$

The same base symbol is typically used to denote this new tensor, and repositioning the index is typically understood in this context to refer to this new tensor, and is called *raising the index*, which would be written

$$g^{ij}A_j = A^i \ .$$

Similarly, multiplying by the *covariant* metric tensor and contracting *lowers* an index (with the same understanding about the reuse of the base symbol):

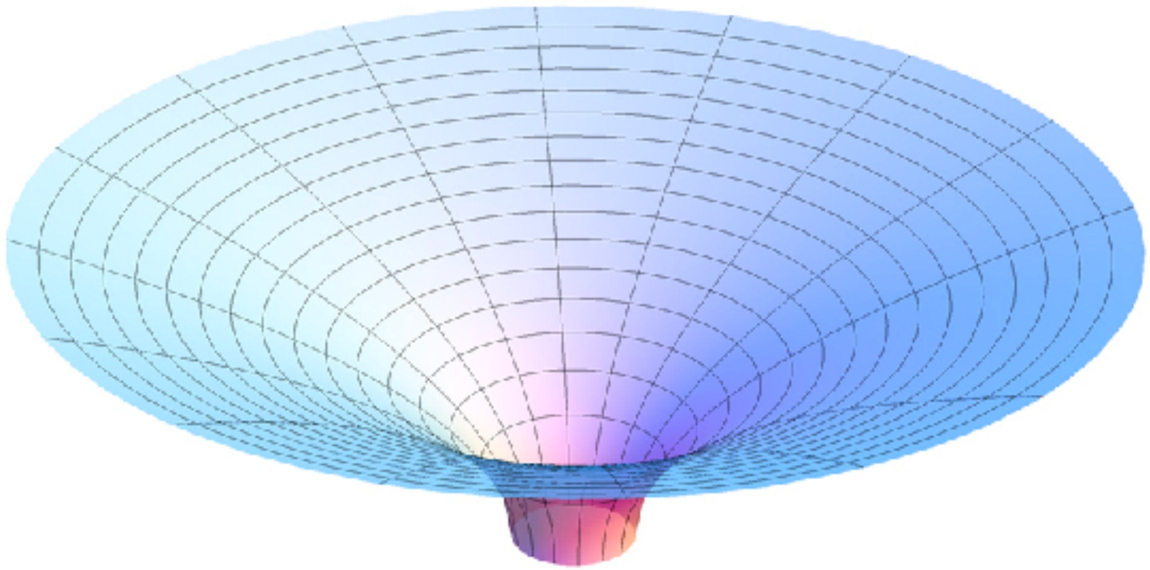
$$g_{ij}A^j = A_i \ .$$

The form g_{ij} need not be nonsingular to lower an index, but to get the inverse (and thus raise an index) it must be nonsingular.

Raising and then lowering the same index (or conversely) are inverse operations, which is reflected in the covariant and contravariant metric tensors being inverse to each other:

$$g^{ij}g_{jk} = g_{kj}g^{ji} = \delta^i{}_k = \delta_k{}^i$$

Local coordinates



$$ds^2 = -(1 - 2m/r)dt^2 + (1 - 2m/r)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad .$$

$$(e^0, e^1, e^2, e^3) = \left((1 - \frac{2m}{r})^{1/2}dt, (1 - \frac{2m}{r})^{-1/2}dr, rd\theta, r\sin\theta d\phi\right) \qquad (4.136)$$

Redshift : $(1 - 2m/r)^{1/2}$

The quantum limit of General Relativity

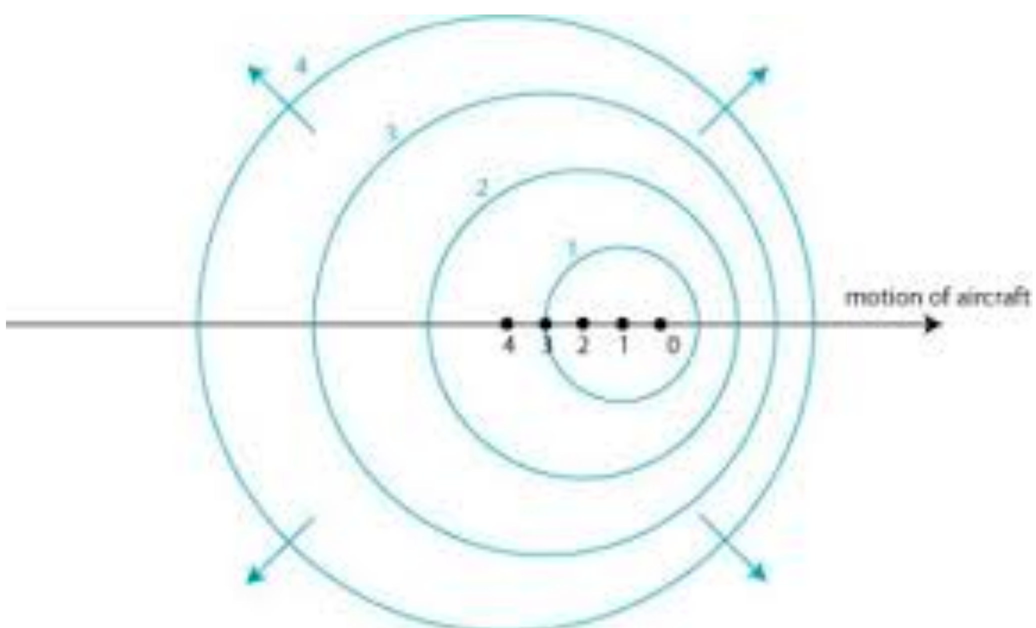
ICECUBE Neutrino Experiment

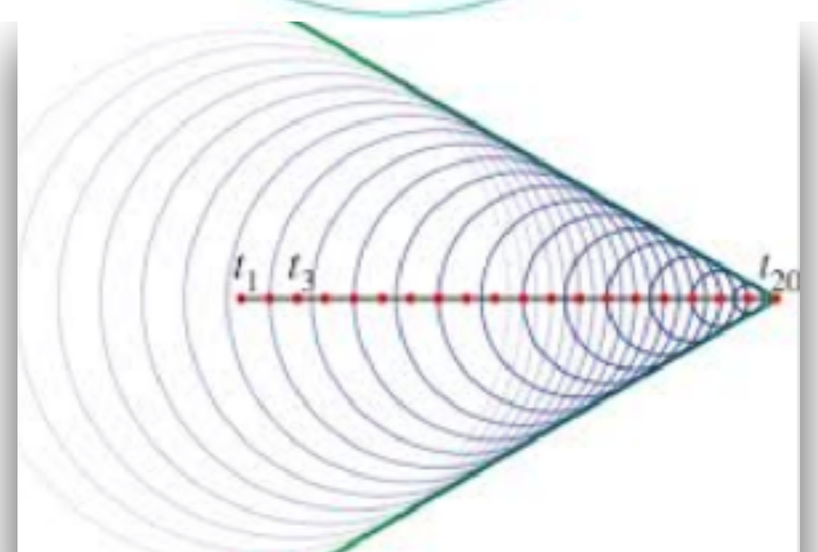
Beta decay

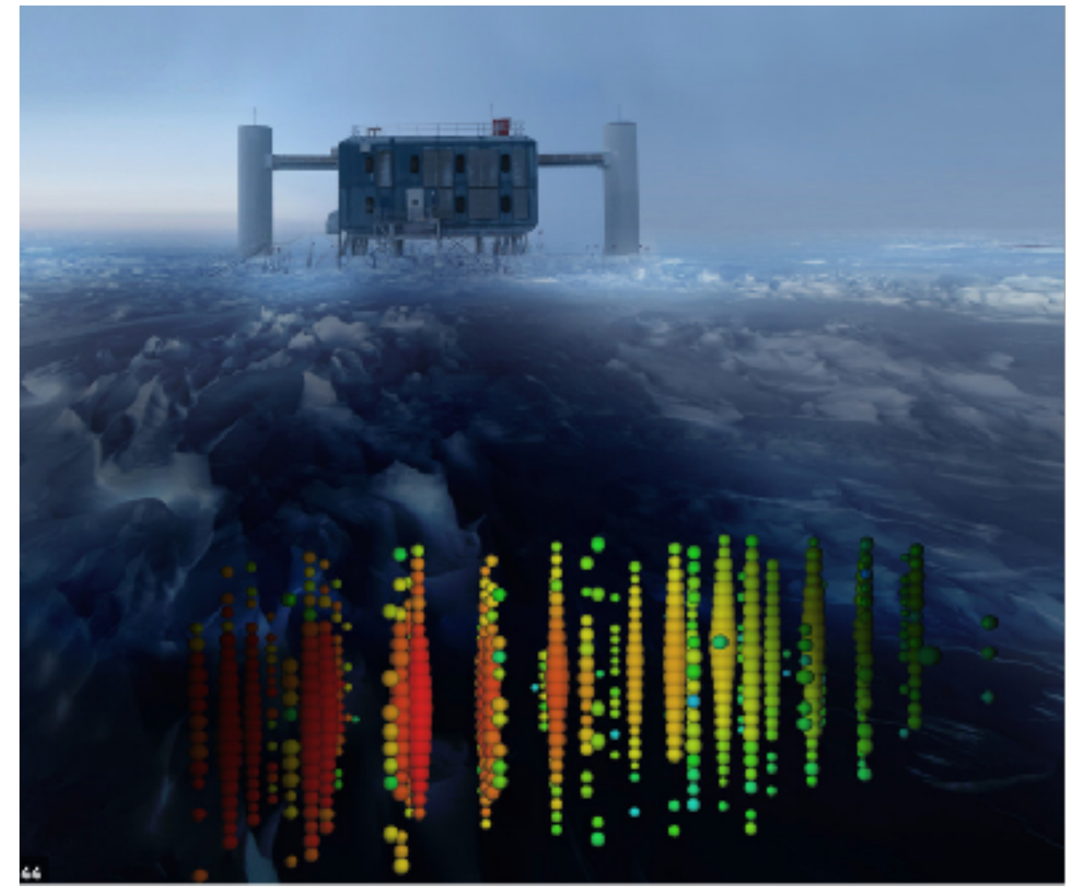
$$n^0 \rightarrow p^+ + e^- + \bar{\nu}_e^0$$

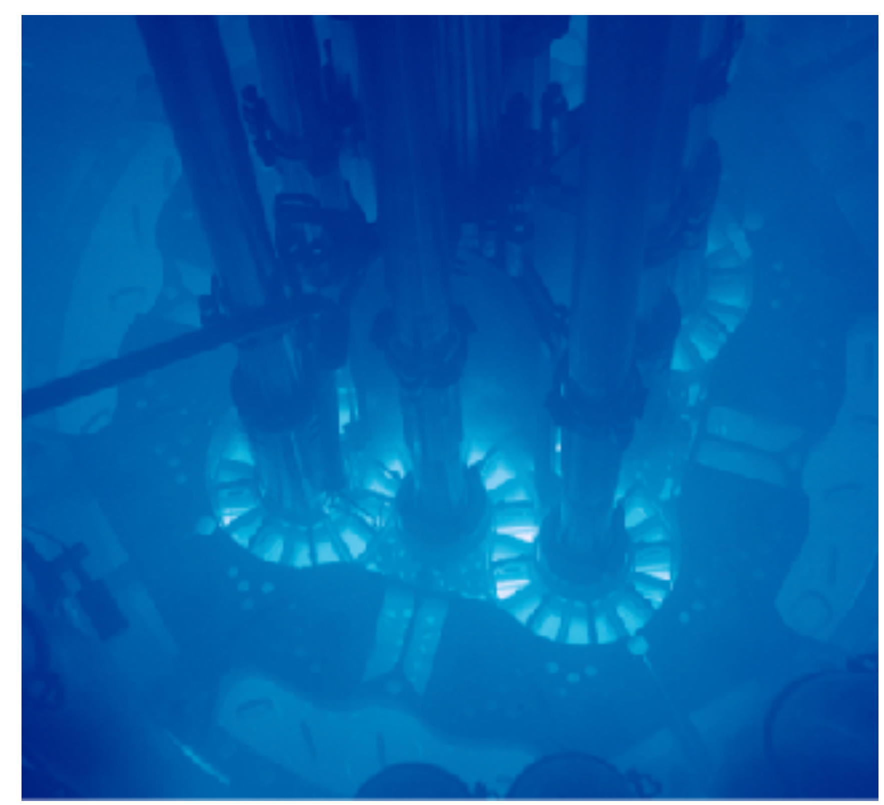
Inverse decay

$$\bar{\nu}_e + p \rightarrow e^+ + n,$$ [2][3][4]





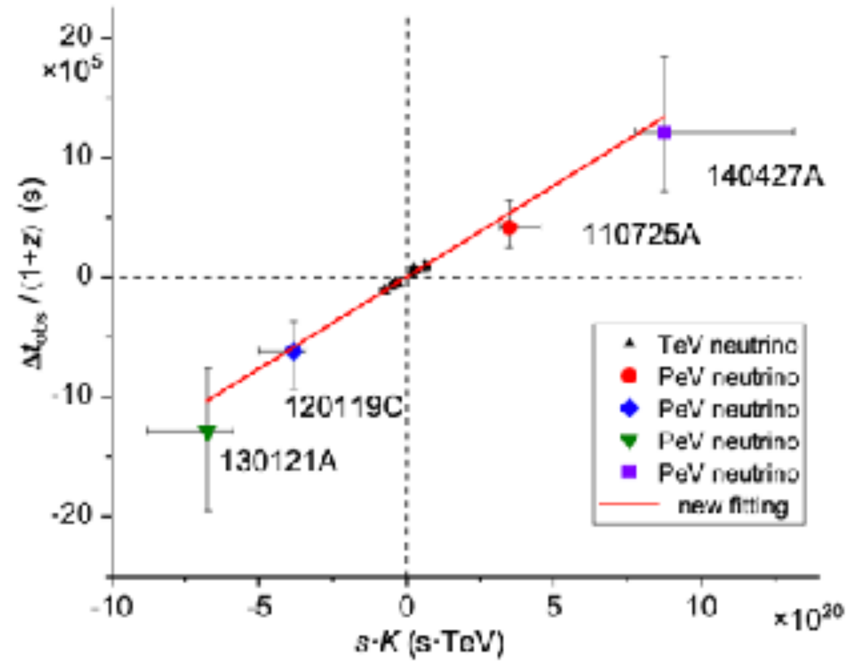




Superluminal neutrino?

Superluminal Neutrino

$$\delta t/t = \delta v/v$$



Superluminal Neutrino

$$(\delta v/v) \approx f(E/E_{\rm planck}) \approx A_1 \times (E/E_{\rm planck} + \dots)$$

Note:

$$1\ E_{\rm P} \approx 1.956 \times 10^{9}\ {\rm J} \approx 1.2209 \times 10^{19}\ {\rm GeV}$$

$$E_{\rm P} = \sqrt{\frac{\hbar c^5}{G}},$$

Not Secure | astro.ucla.edu/~wright/CosmoCalc.html

Enter values, hit a button

69.6 H_o
0.286 $Omega_M$
1 z
Open Flat
0.714 $Omega_{vac}$
General

Open sets $Omega_{vac} = 0$ giving an open Universe [if you entered $Omega_M < 1$]
Flat sets $Omega_{vac} = 1-Omega_M$ giving a flat Universe.
General uses the $Omega_{vac}$ that you entered.
Source for the default parameters.

For $H_o = 69.6$, $Omega_M = 0.286$, $Omega_{vac} = 0.714$, $z = 1.000$

- It is now 13.721 Gyr since the Big Bang.
- The age at redshift z was 5.903 Gyr.
- The light travel time was 7.817 Gyr.
- The comoving radial distance, which goes into Hubble's law, is 3350.7 Mpc or 10.928 Gly.
- The comoving volume within redshift z is 157.569 Gpc^3.
- The angular size distance D_A is 1675.3 Mpc or 5.4642 Gly.
- This gives a scale of 8.122 kpc/".
- The luminosity distance D_L is 6701.2 Mpc or 21.857 Gly.

1 Gly = 1,000,000,000 light years or 9.461×10^{26} cm.
1 Gyr = 1,000,000,000 years.
1 Mpc = 1,000,000 parsecs = 3.08568×10^{24} cm, or 3,261,566 light years.

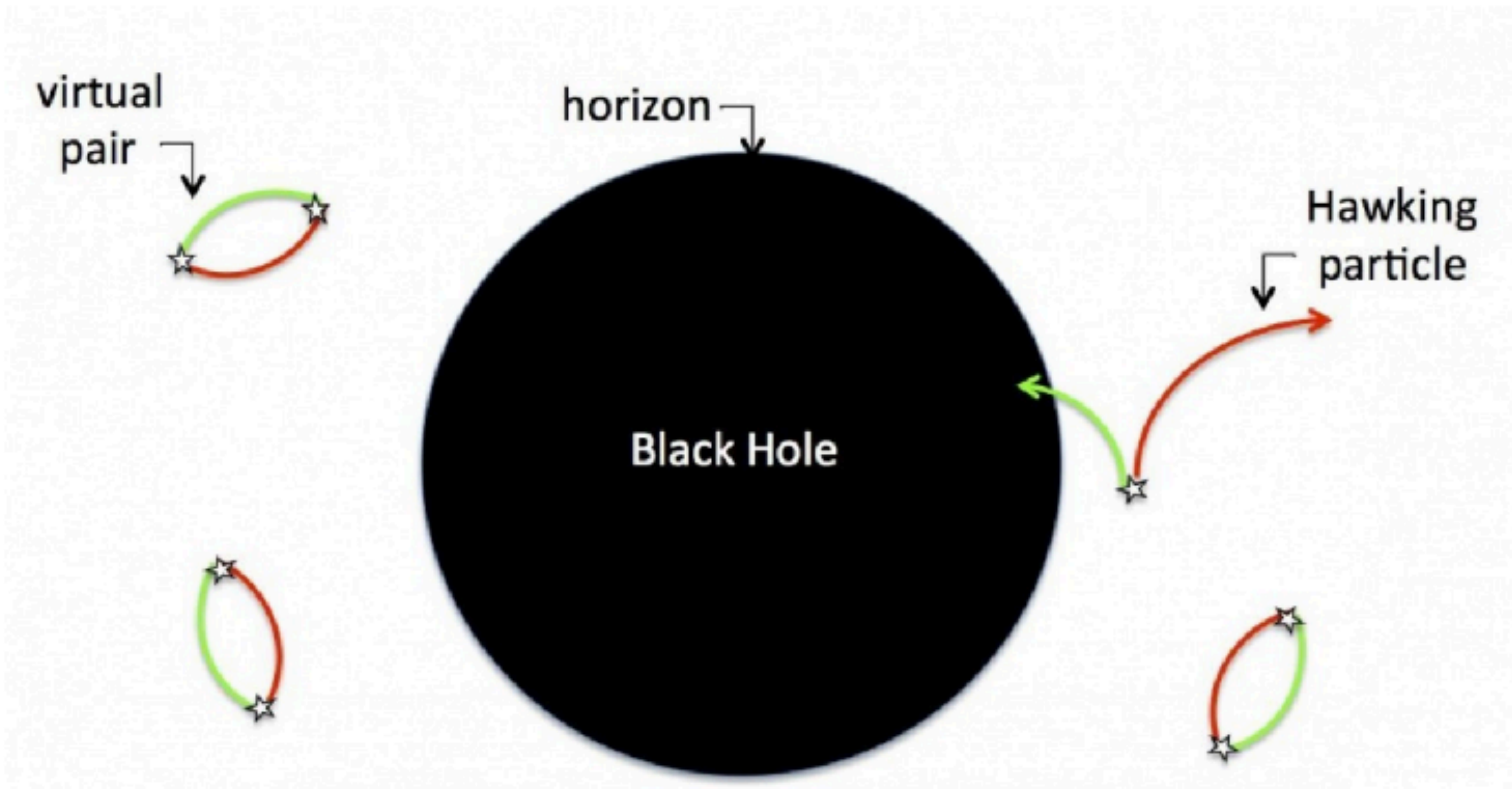
Tutorial: Part 1 | Part 2 | Part 3 | Part 4
FAQ | Age | Distances | Bibliography | Relativity

See the advanced and light travel time versions of the calculator.

James Schombert has written a Python version of this calculator.

Ned Wright's home page

Hawking radiation



Hawking radiation

Image Source

Unruh effect

The Unruh effect was first described by Stephen Fulling in 1973, Paul Davies in 1975 and W. G. Unruh in 1976.[1][2][3]

$$k_{\mathrm{B}} T = \frac{\hbar a}{2\pi c}.$$

Other implications [edit]

The Unruh effect would also cause the decay rate of accelerating particles to differ from inertial particles. Stable particles like the electron could have nonzero transition rates to higher mass states when accelerating at a high enough rate.[10][11][12]

Hawking radiation

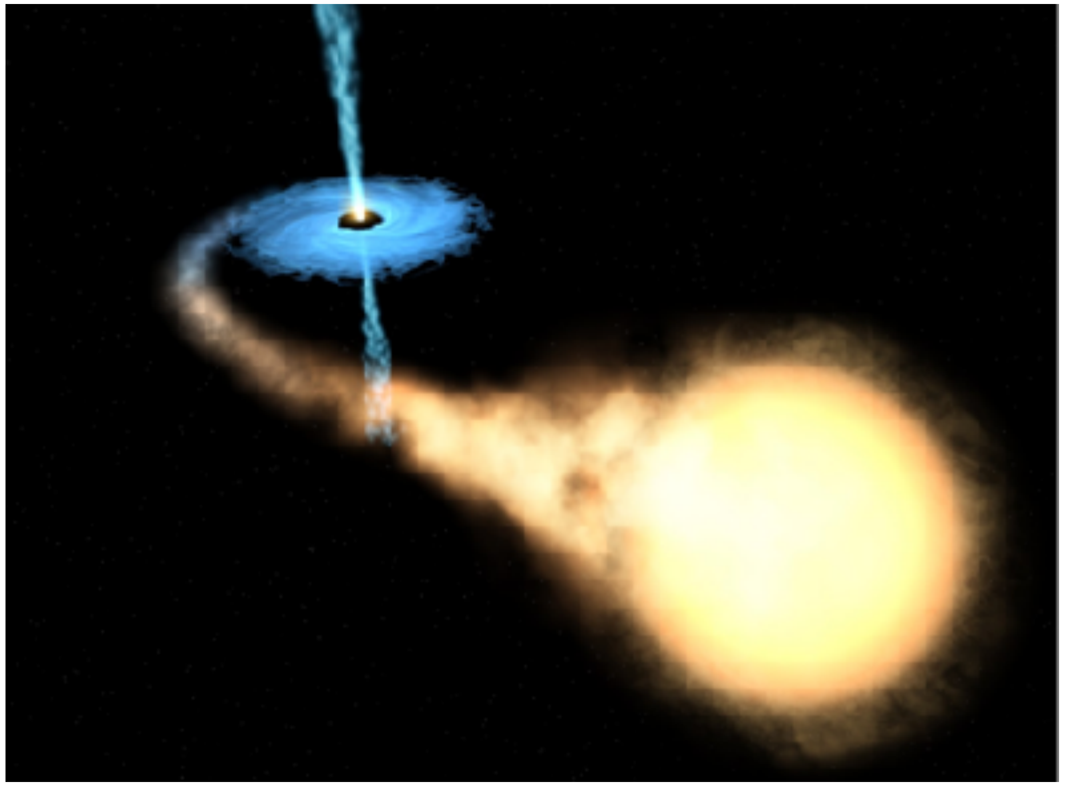
Discovery [edit]

Hawking's discovery followed a visit to Moscow in 1973 where the Soviet scientists Yakov Zel'dovich and Alexei Starobinsky convinced him that rotating black holes ought to create and emit particles. When Hawking did the calculation, he found to his surprise that even non-rotating black holes produce radiation.[10]

$$T = \frac{\hbar c^3}{8 \pi G k_B M} \approx 1.227 \times 10^{+23} \mathrm{K \cdot kg} \times \frac{1}{M} = 6.169 \times 10^{-8} \mathrm{K} \times \frac{M_\odot}{M},$$

Binary star evolution
& Compact stars

Accretion disk theory

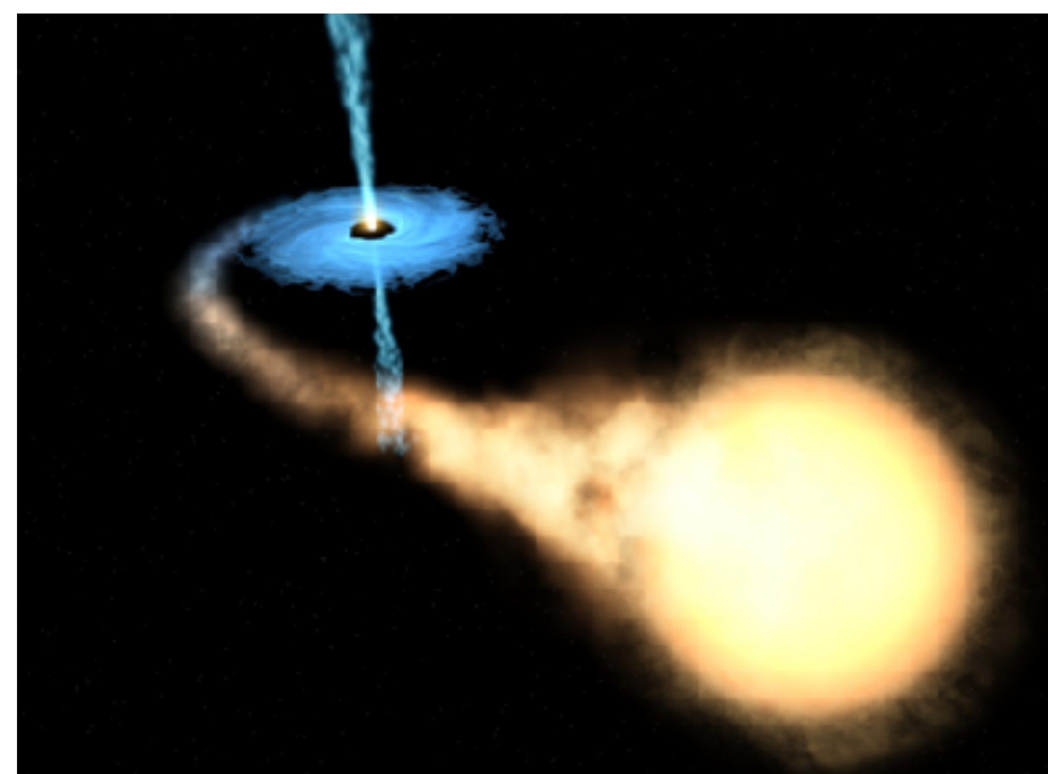


Accretion disk theory

Rashid
Sunyaev

Physics researcher

Rashid Alievich Sunyaev is a Soviet and Russian astrophysicist of Tatar descent. He was educated at the Moscow Institute of Physics and Technology. He became a professor at MIPT in 1974. Wikipedia



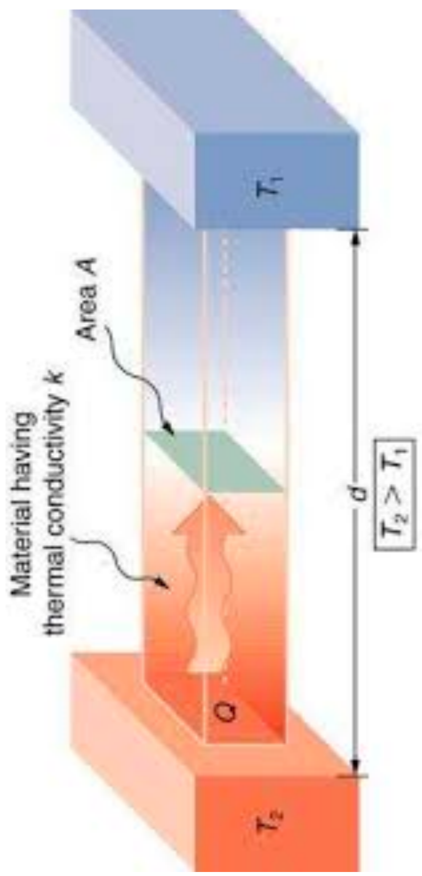

Morgan Stanley

Heat conduction

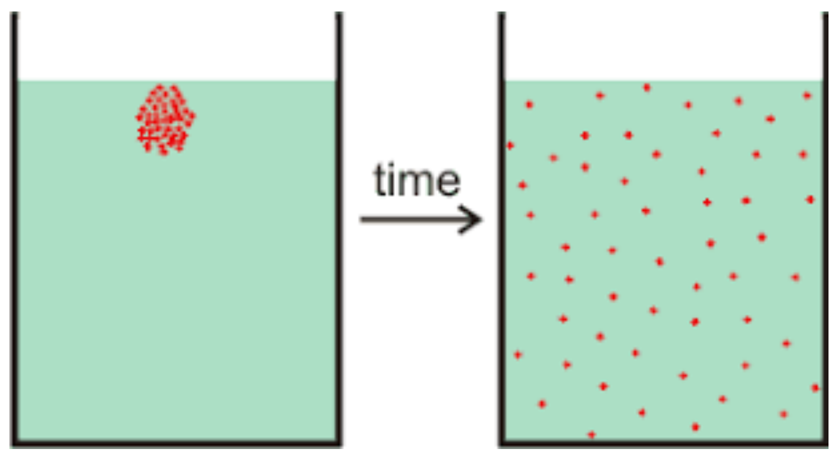
$$\frac{\partial \Theta}{\partial t} = D \frac{\partial^2 \Theta}{\partial x^2},$$

?

Heat conduction



Heat conduction

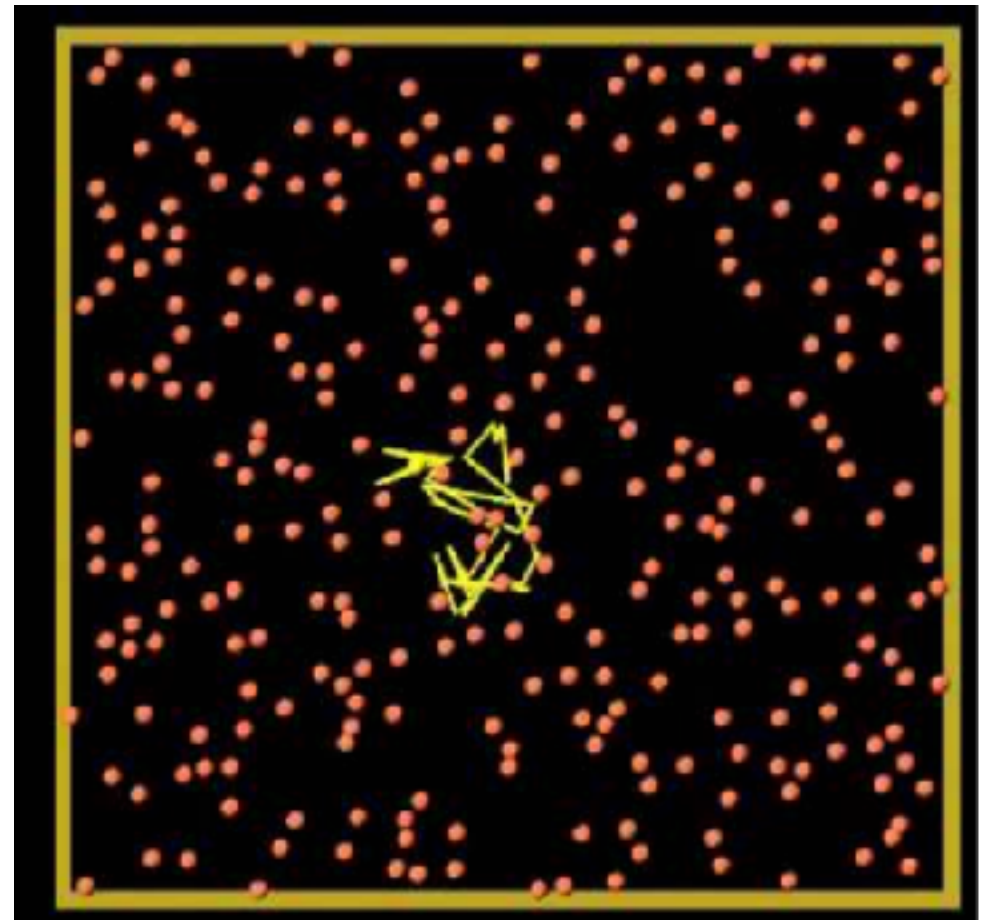


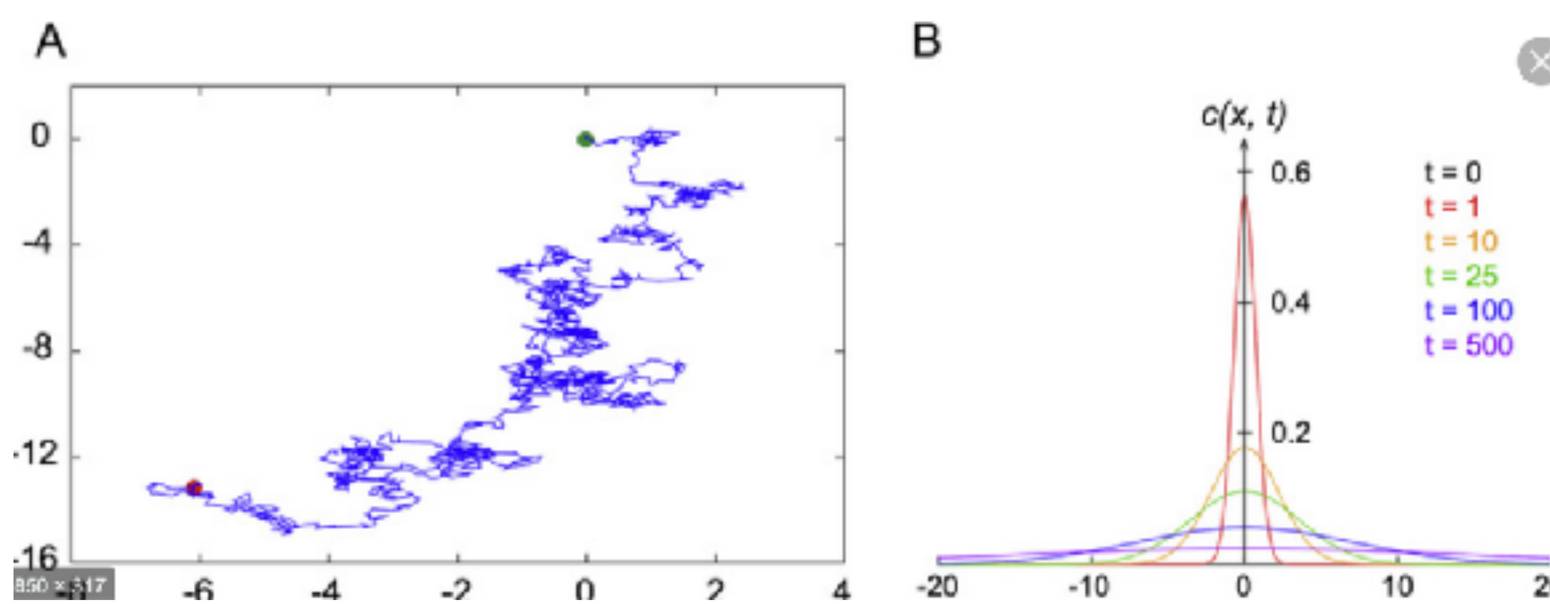
Chemical diffusion

Image smoothing

Heat conduction

$$\frac{\partial \Theta}{\partial t} = D \frac{\partial^2 \Theta}{\partial x^2},$$





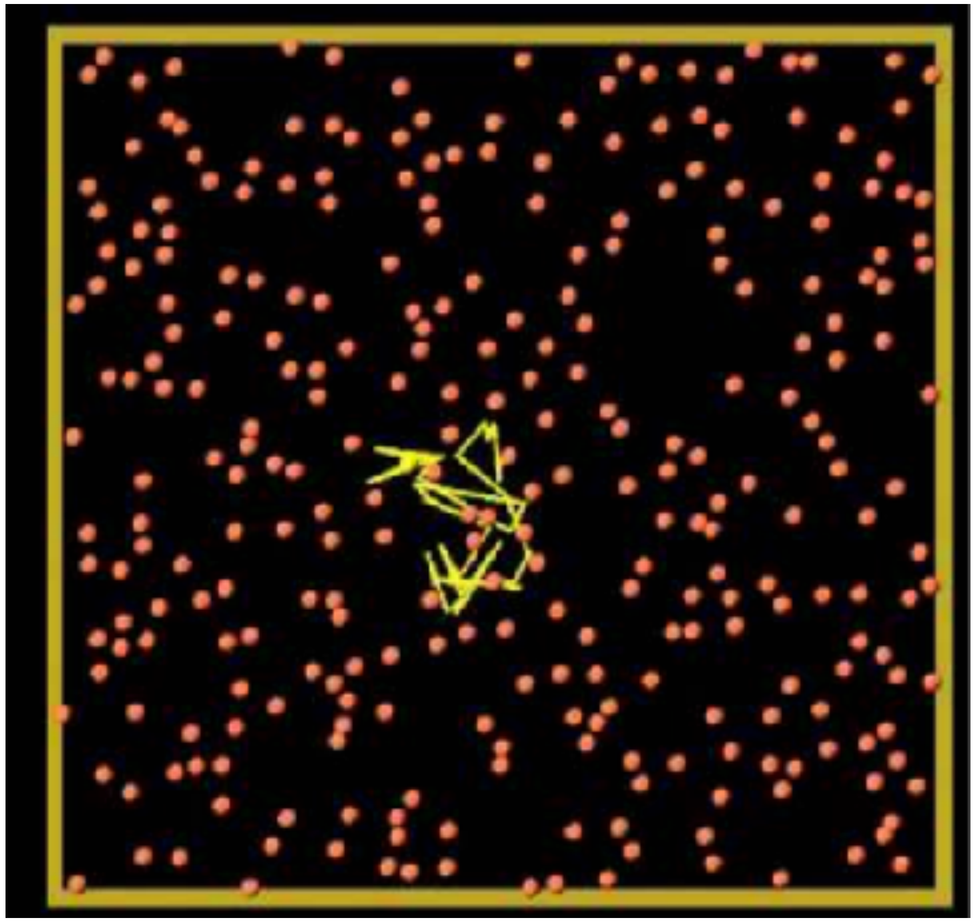
Mean free path

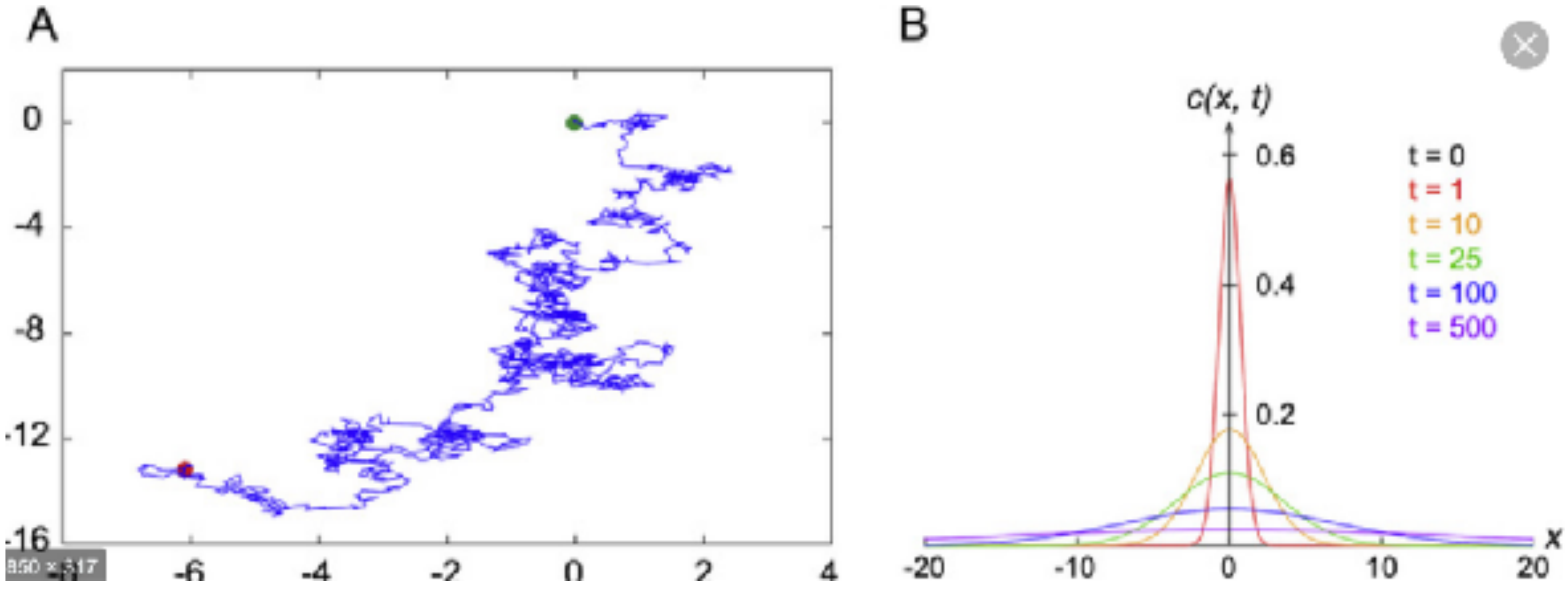
Vacuum range	Pressure in hPa (mbar)	Pressure in mmHg (Torr)	number density (Molecules / cm^3)	number density (Molecules / m^3)	Mean free path
Ambient pressure	1013	759.8	2.7×10^{19}	2.7×10^{25}	68 nm[2]
Low vacuum	300 – 1	$220 – 8\times10^{-1}$	$10^{19} – 10^{16}$	$10^{25} – 10^{22}$	0.1 – 100 μm
Medium vacuum	$1 – 10^{-3}$	$8\times10^{-1} – 8\times10^{-4}$	$10^{16} – 10^{13}$	$10^{22} – 10^{19}$	0.1 – 100 mm
High vacuum	$10^{-3} – 10^{-7}$	$8\times10^{-4} – 8\times10^{-8}$	$10^{13} – 10^{9}$	$10^{19} – 10^{15}$	10 cm – 1 km
Ultra-high vacuum	$10^{-7} – 10^{-12}$	$8\times10^{-8} – 8\times10^{-13}$	$10^{9} – 10^{4}$	$10^{15} – 10^{10}$	$1 km – 10^{5} km$
Extremely high vacuum	$<10^{-12}$	$<8\times10^{-13}$	$<10^{4}$	$<10^{10}$	$>10^{5}$ km

Heat conduction

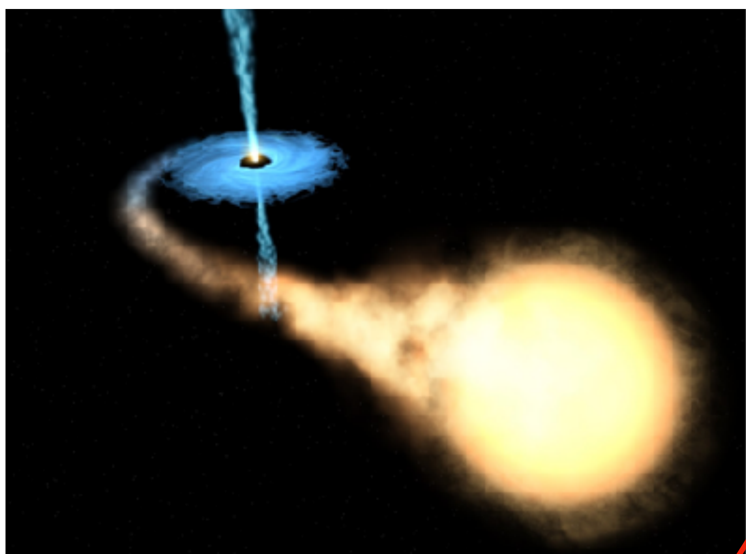
$$\frac{\partial \Theta}{\partial t} = D \frac{\partial^2 \Theta}{\partial x^2},$$

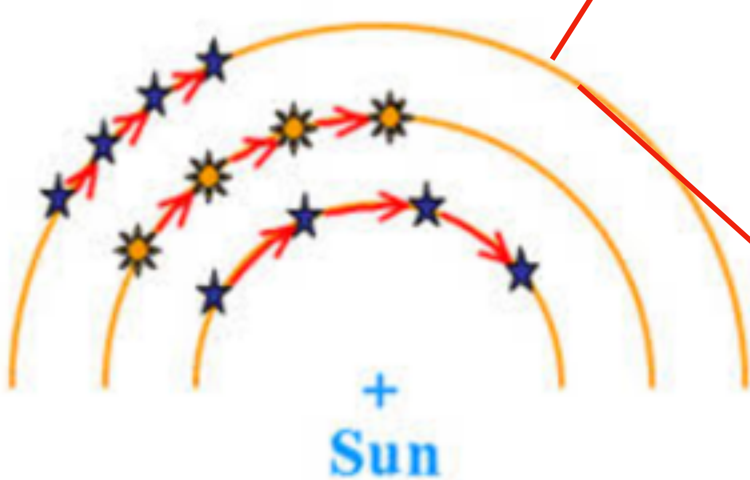
$$D = \lambda_{\text{mpf}} v$$

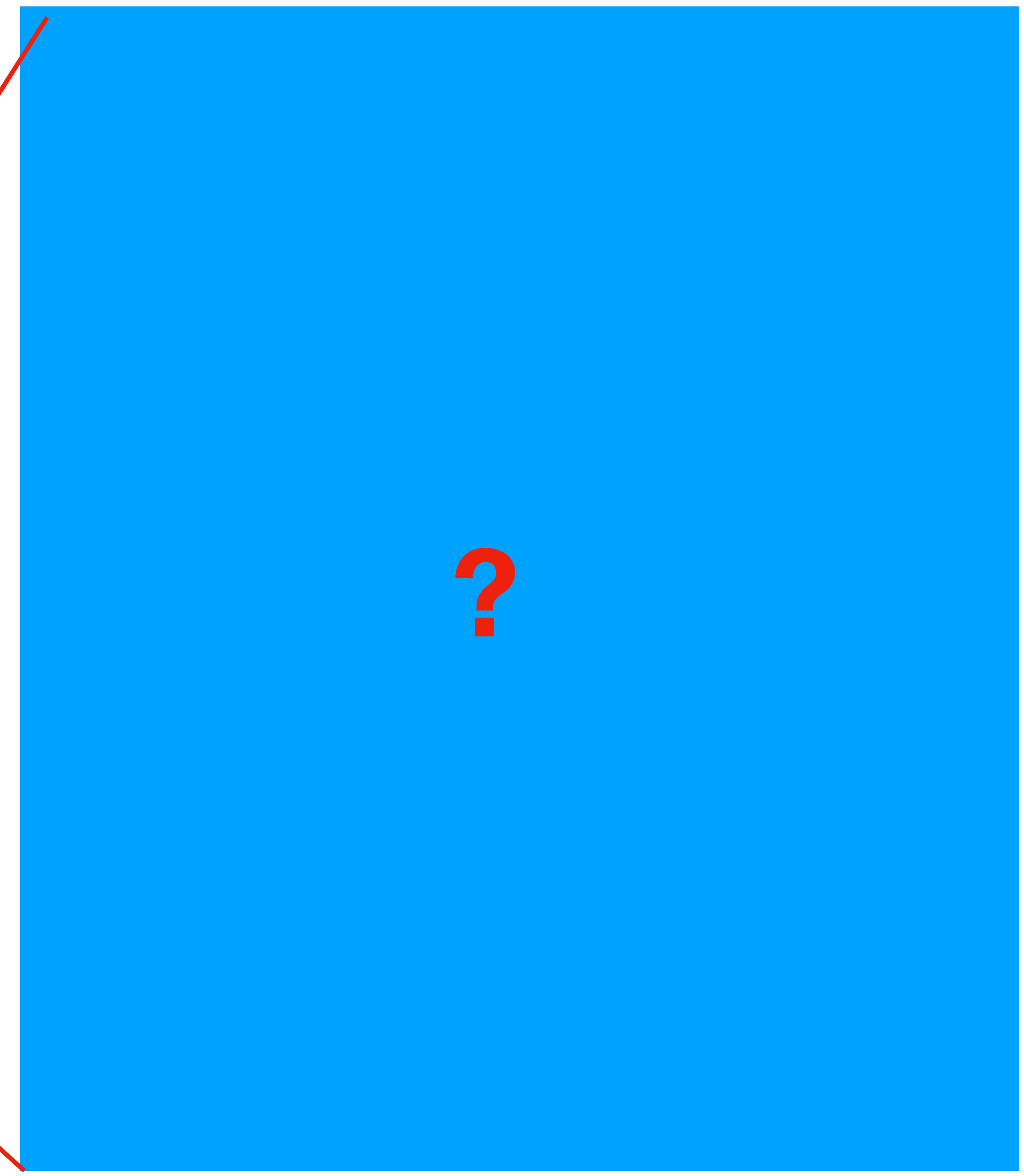


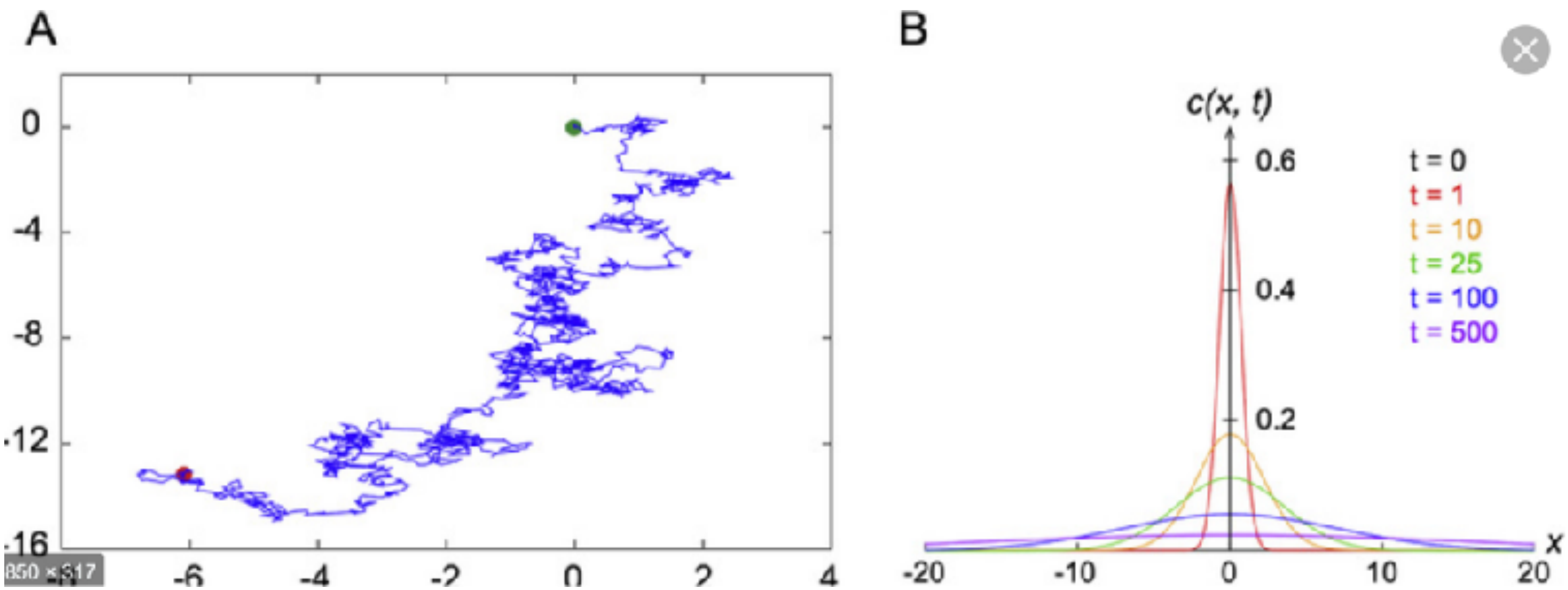


Accretion disk

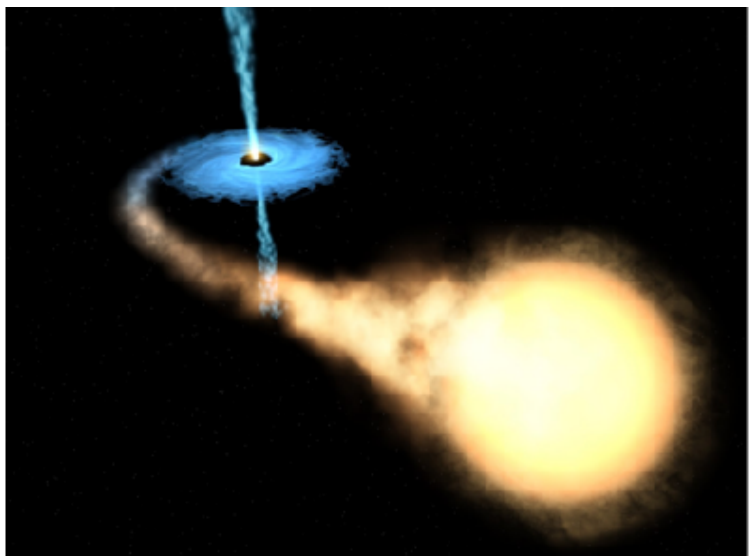


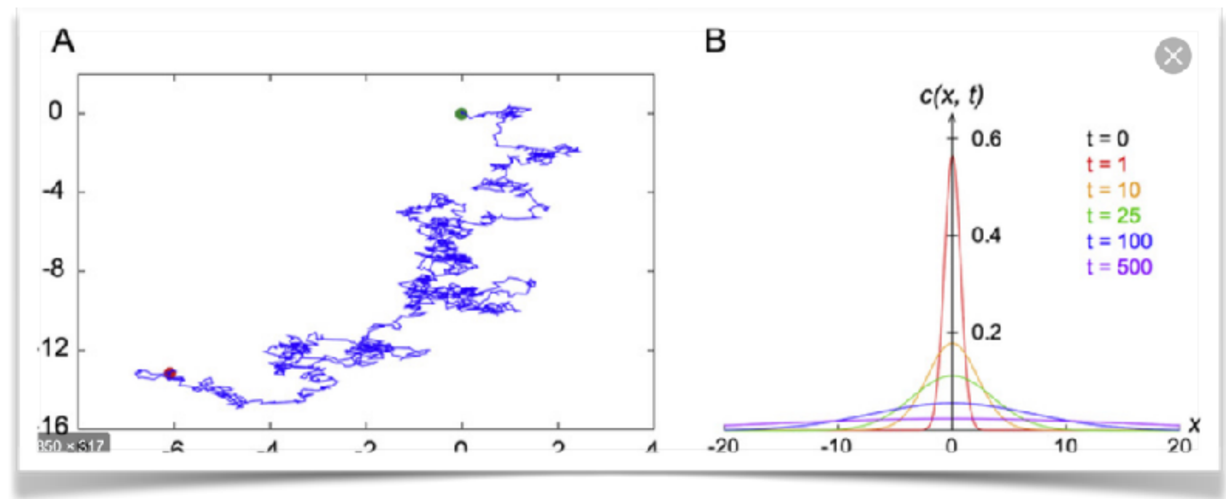


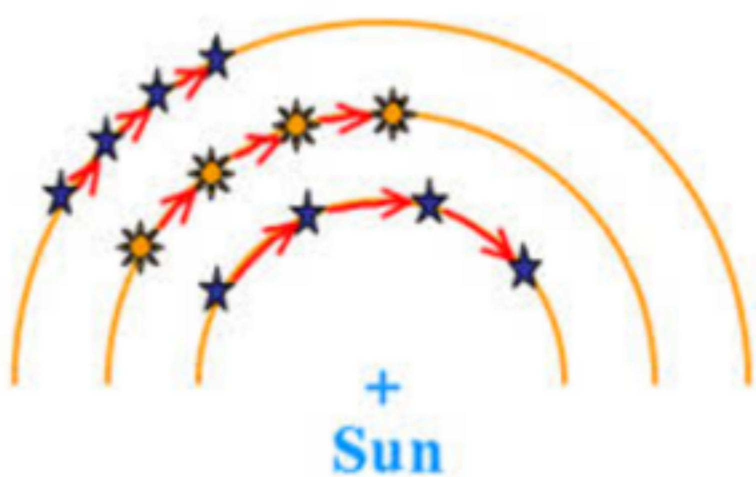


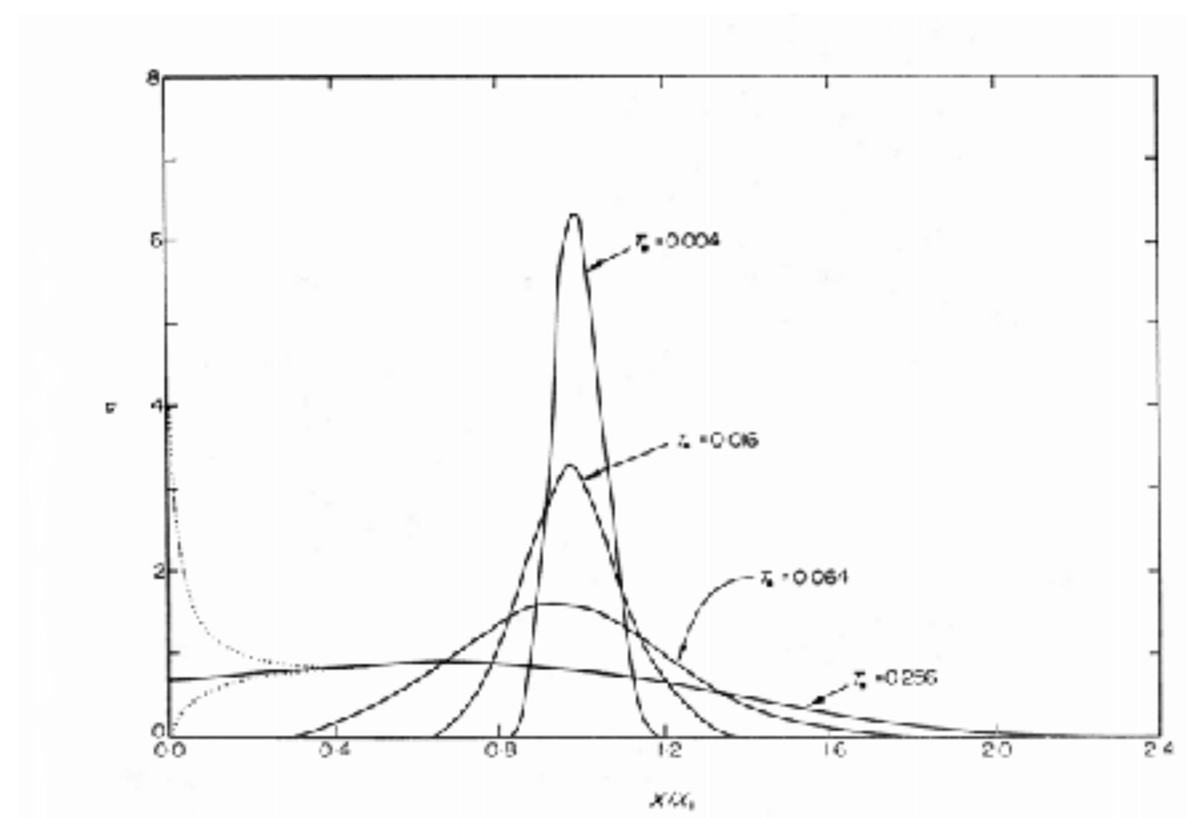

A
0
-4
-8
-12
-16
-6
-4
-2
0
2
4
B
c(x, t)
0.6
0.4
0.2
t = 0
t = 1
t = 10
t = 25
t = 100
t = 500
x
-20
-10
0
10
20

Accretion disk

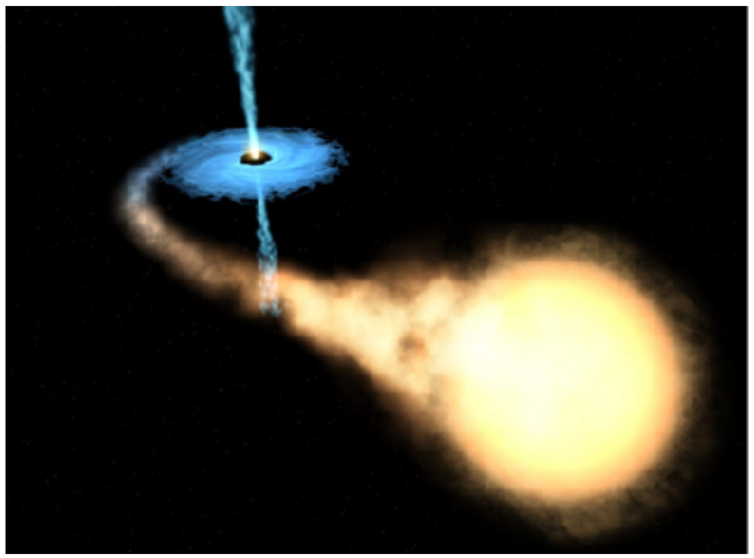








Accretion disk



$$t \approx \frac{x^2}{2D}$$

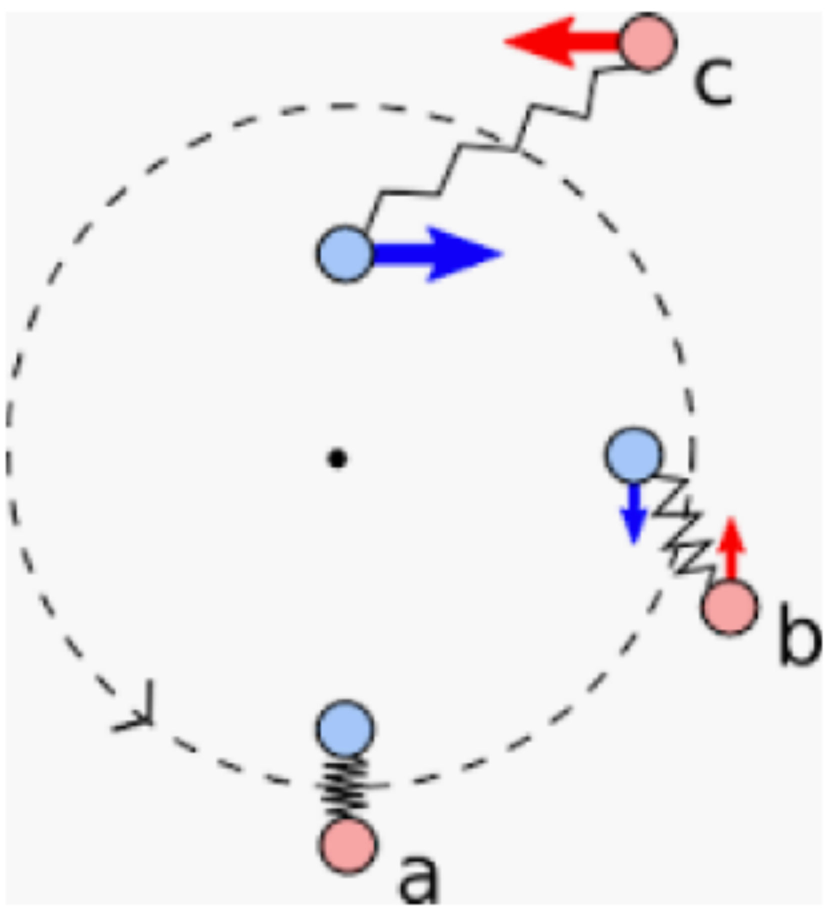
$$\lambda_{\rm mfp} \rightarrow H$$

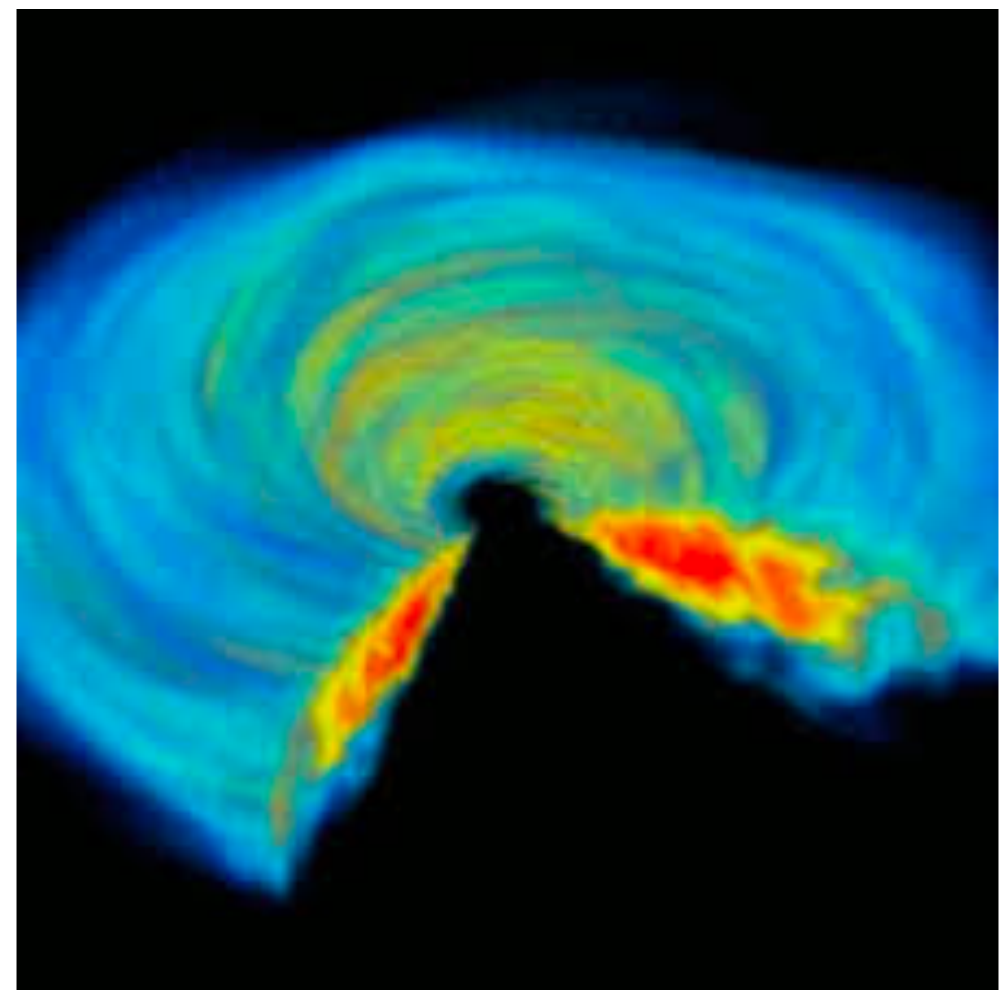
$$\nu = \alpha c_{\rm s} H$$

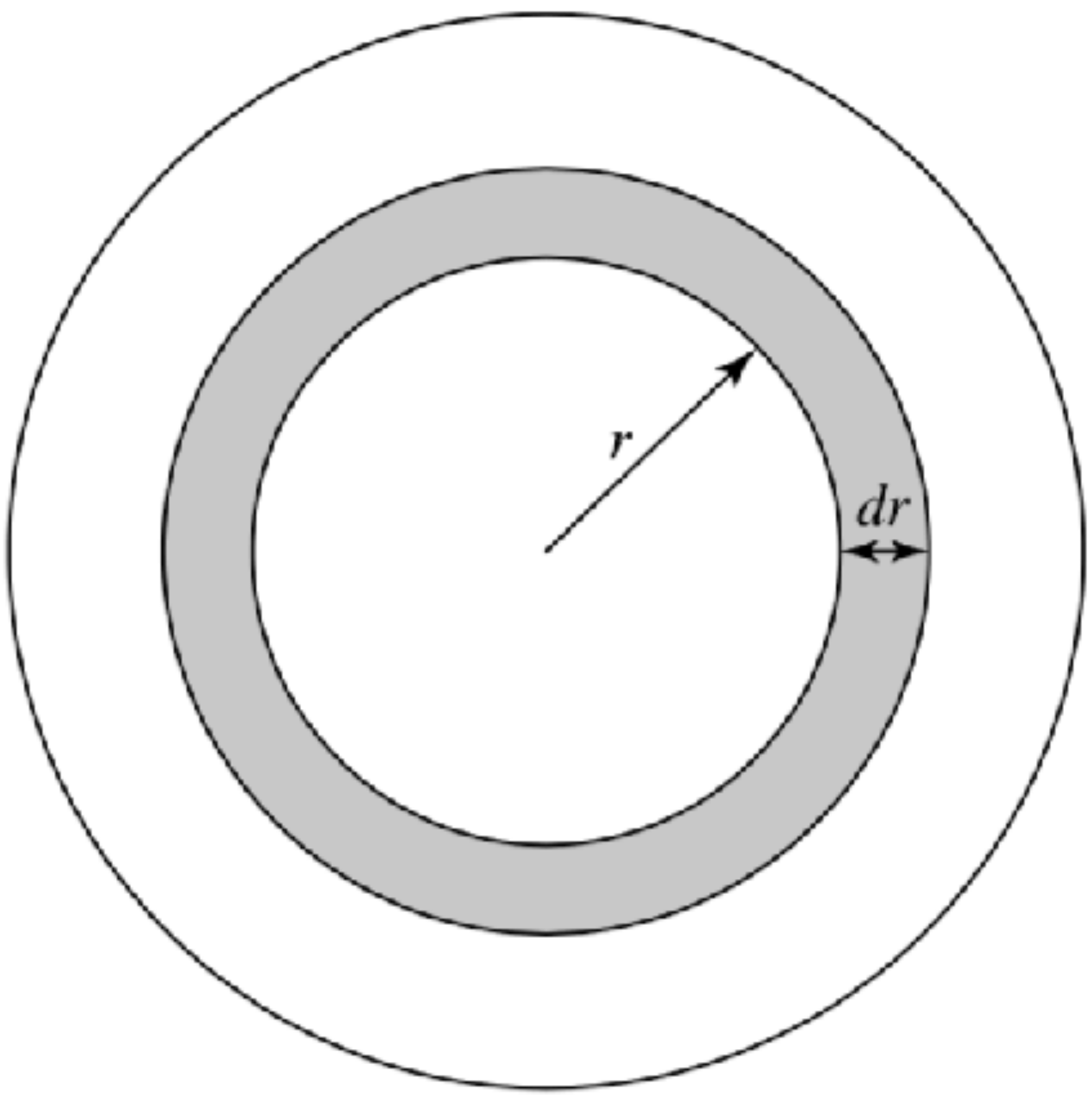
Larger Viscosity-> Faster Evolution

Mechanism: MRI Instability

Accretion disk: transfer of angular momentum







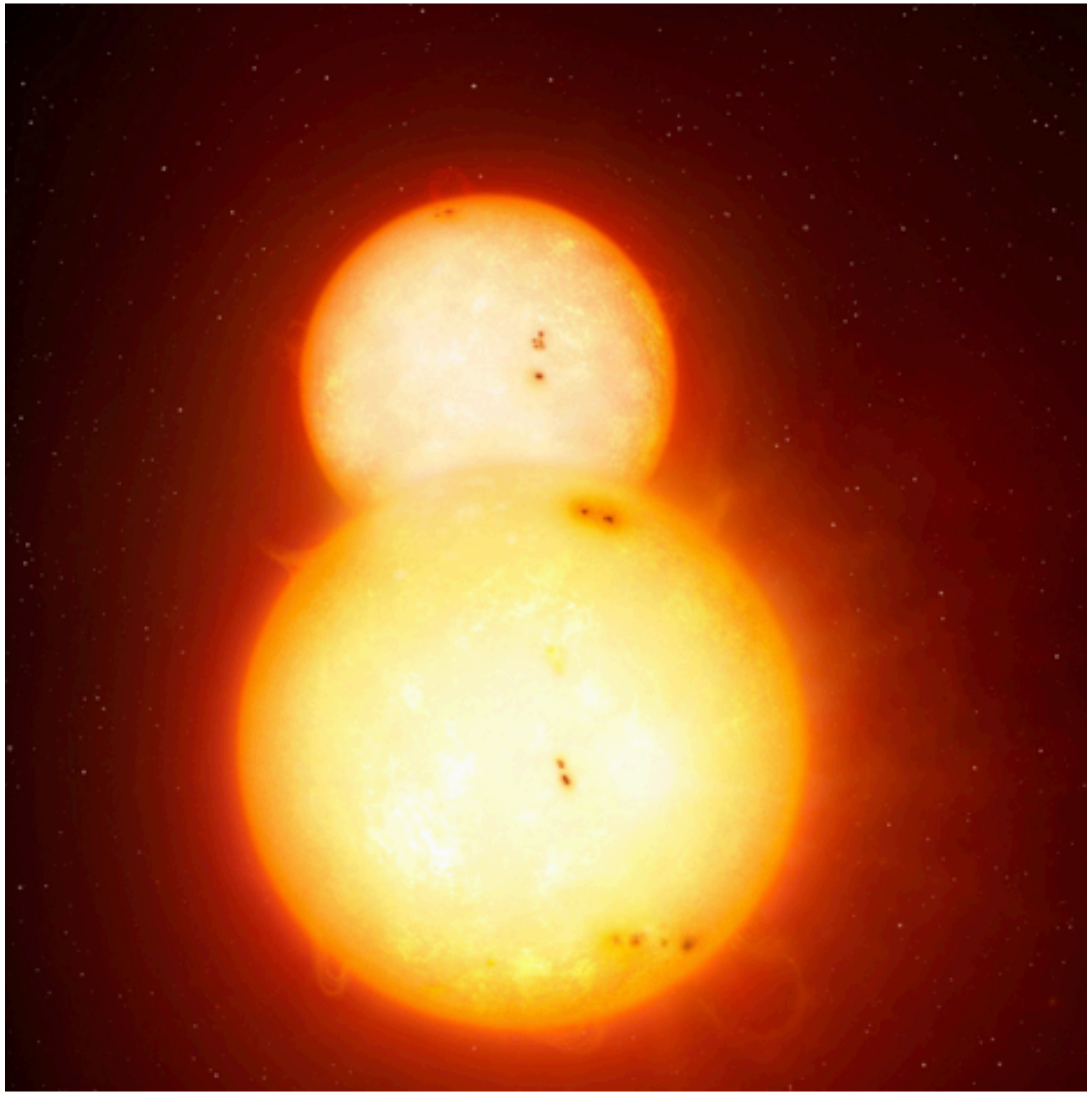
$$dL_{\text{ring}}t = dE = G\,\frac{M_1\dot{M}t}{2r^2}\,dr.$$

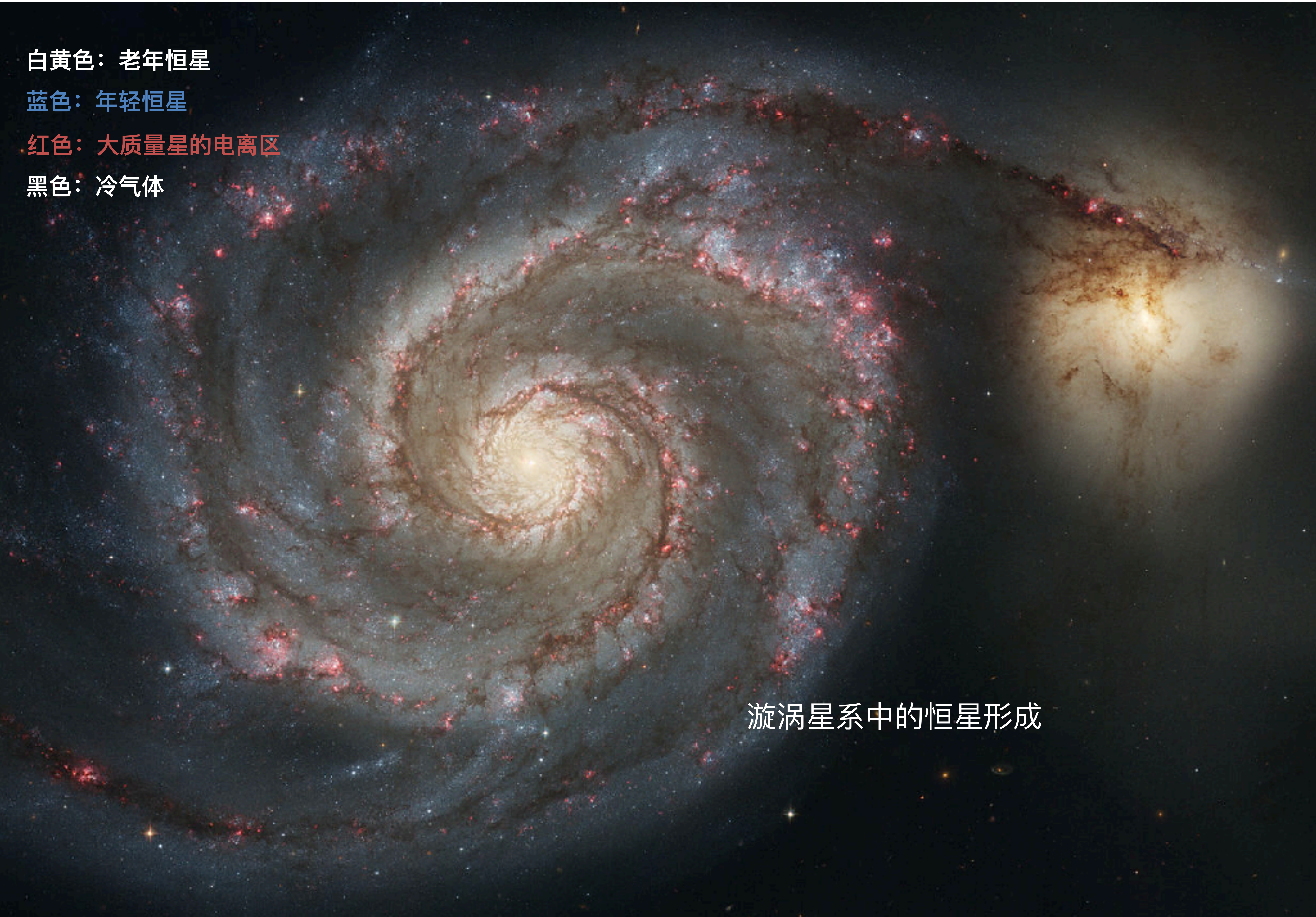
$$\begin{aligned} dL_{\text{ring}} &= 4\pi r\sigma T^4\,dr \\ &= G\,\frac{M_1\dot{M}}{2r^2}\,dr \end{aligned}$$

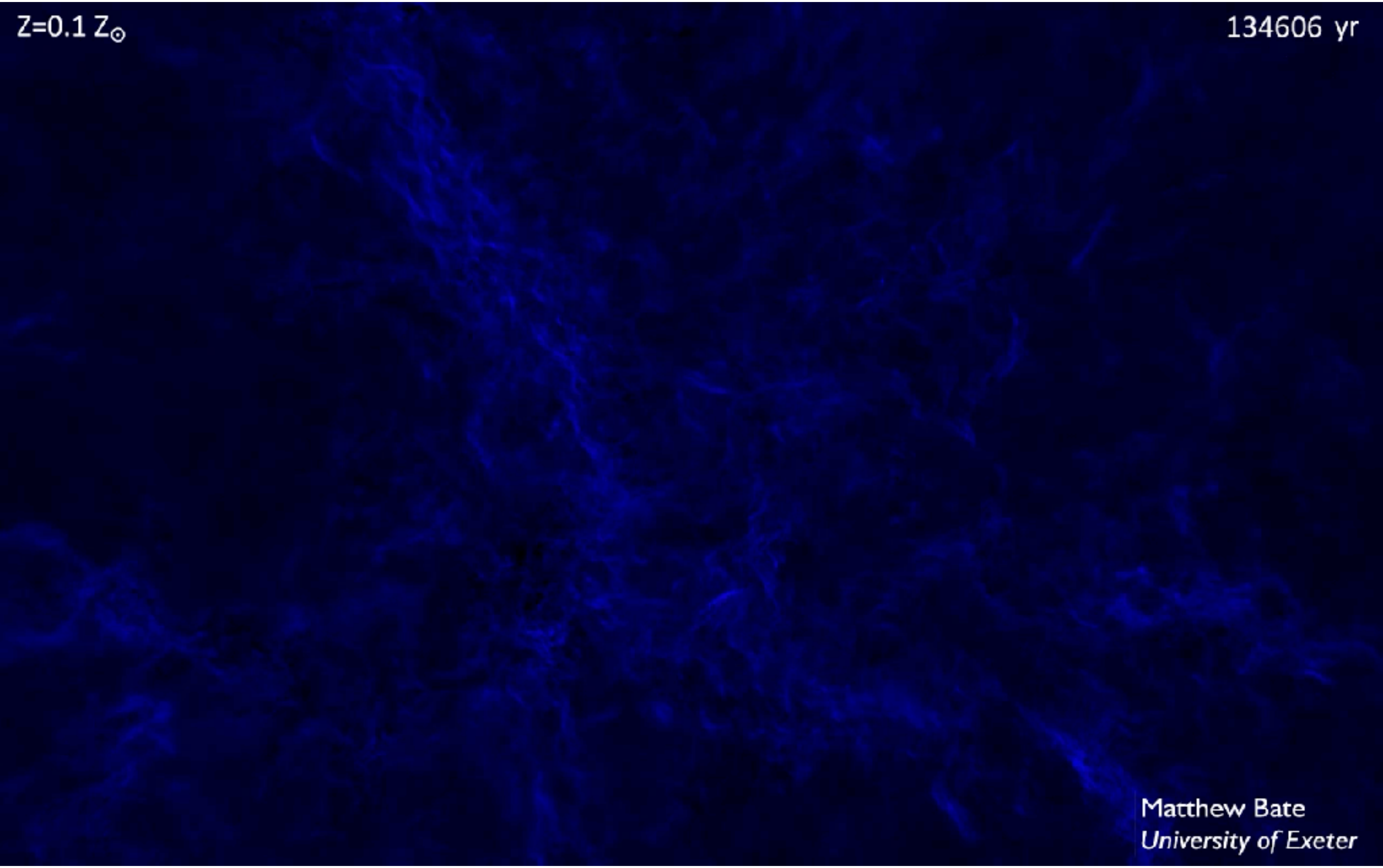
$$T = \left(\frac{GM\dot{M}}{8\pi\sigma R^3}\right)^{1/4}\left(\frac{R}{r}\right)^{3/4}.$$

Binary stars


80 × 1509

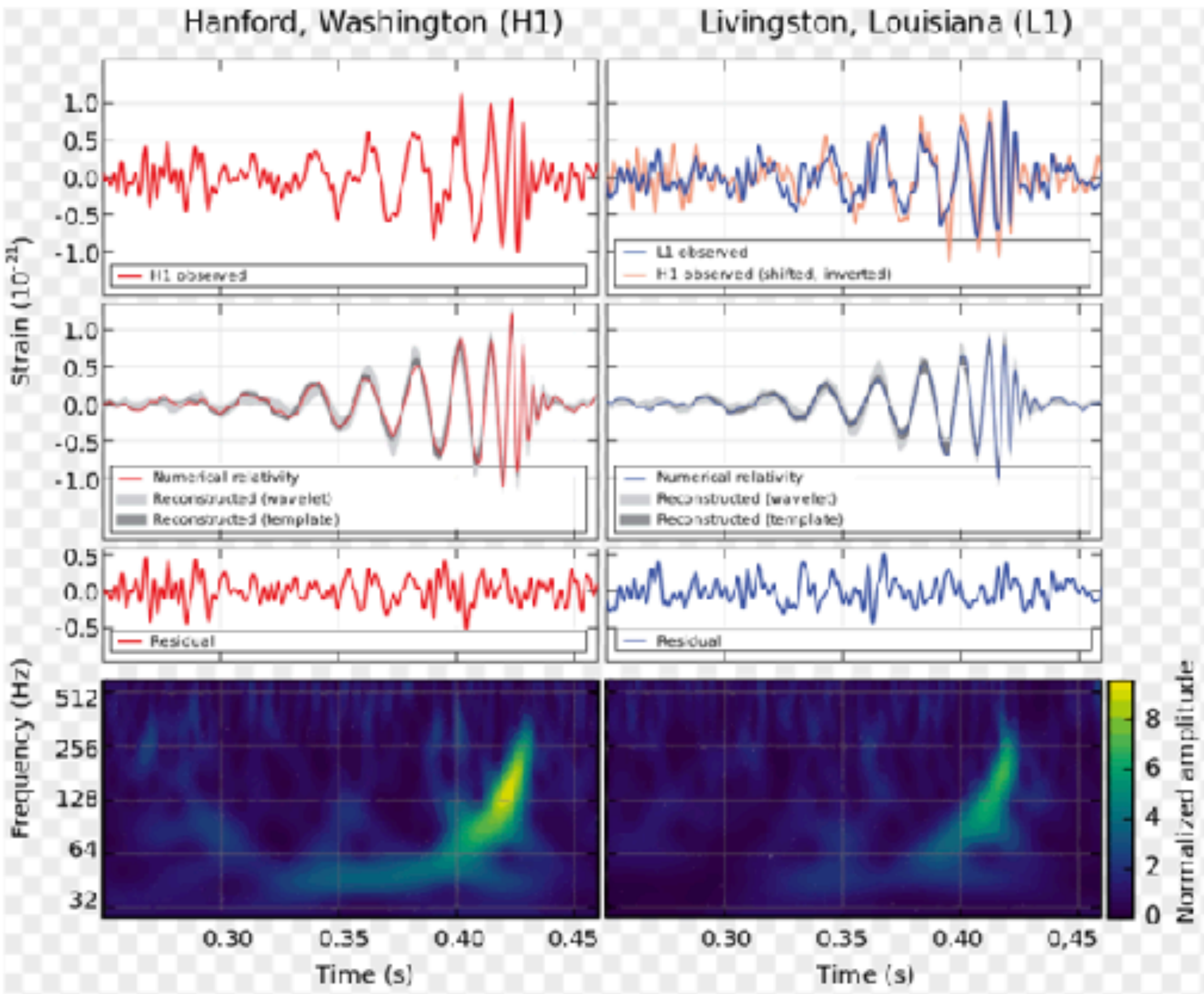


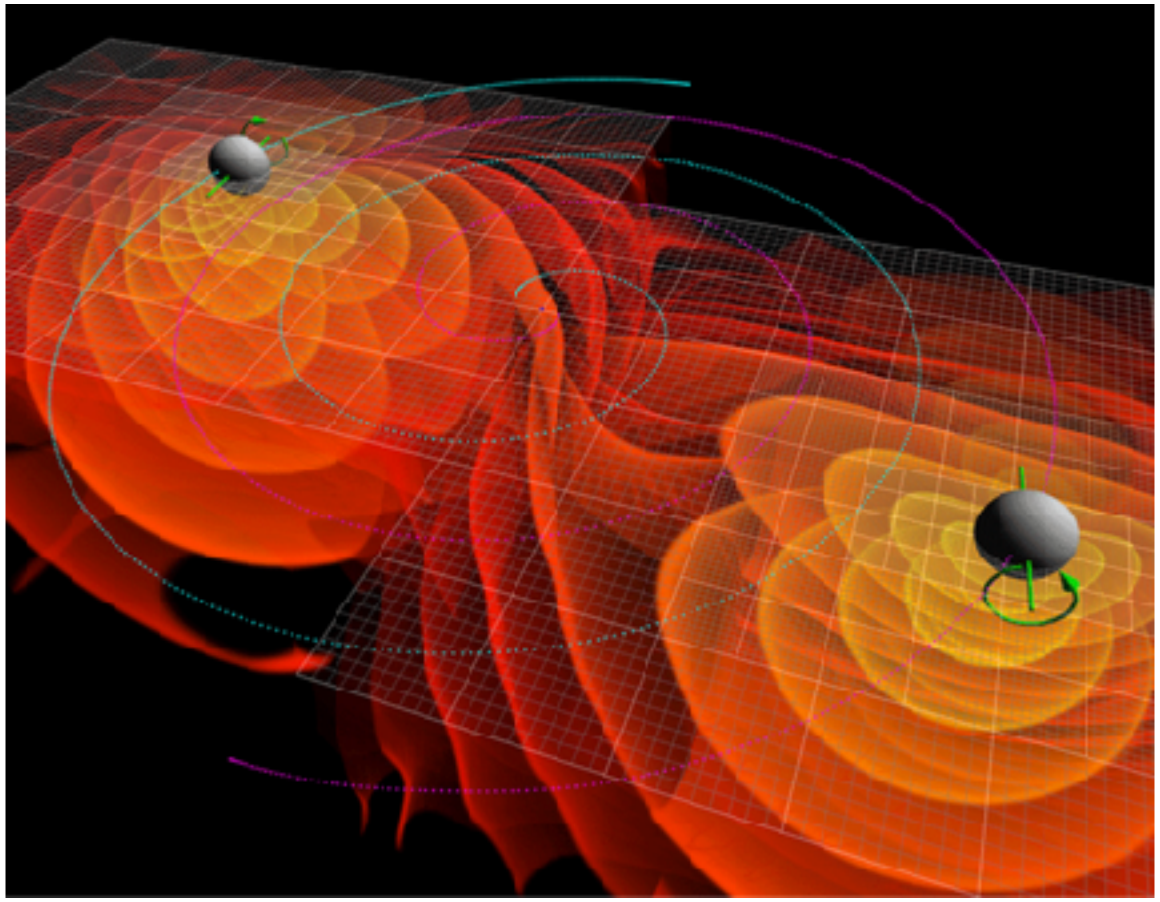

白黄色：老年恒星
蓝色：年轻恒星
红色：大质量星的电离区
黑色：冷气体
漩涡星系中的恒星形成


$Z=0.1\ Z_{\odot}$
134606 yr
Matthew Bate
University of Exeter

Binary stars

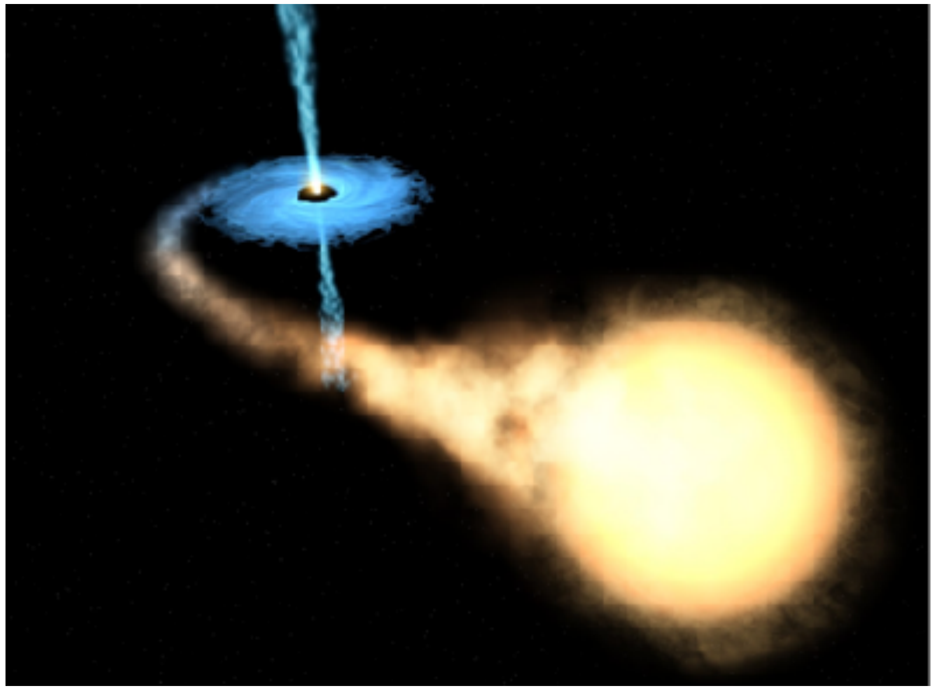
A wide star-black-hole binary system from radial-velocity measurements

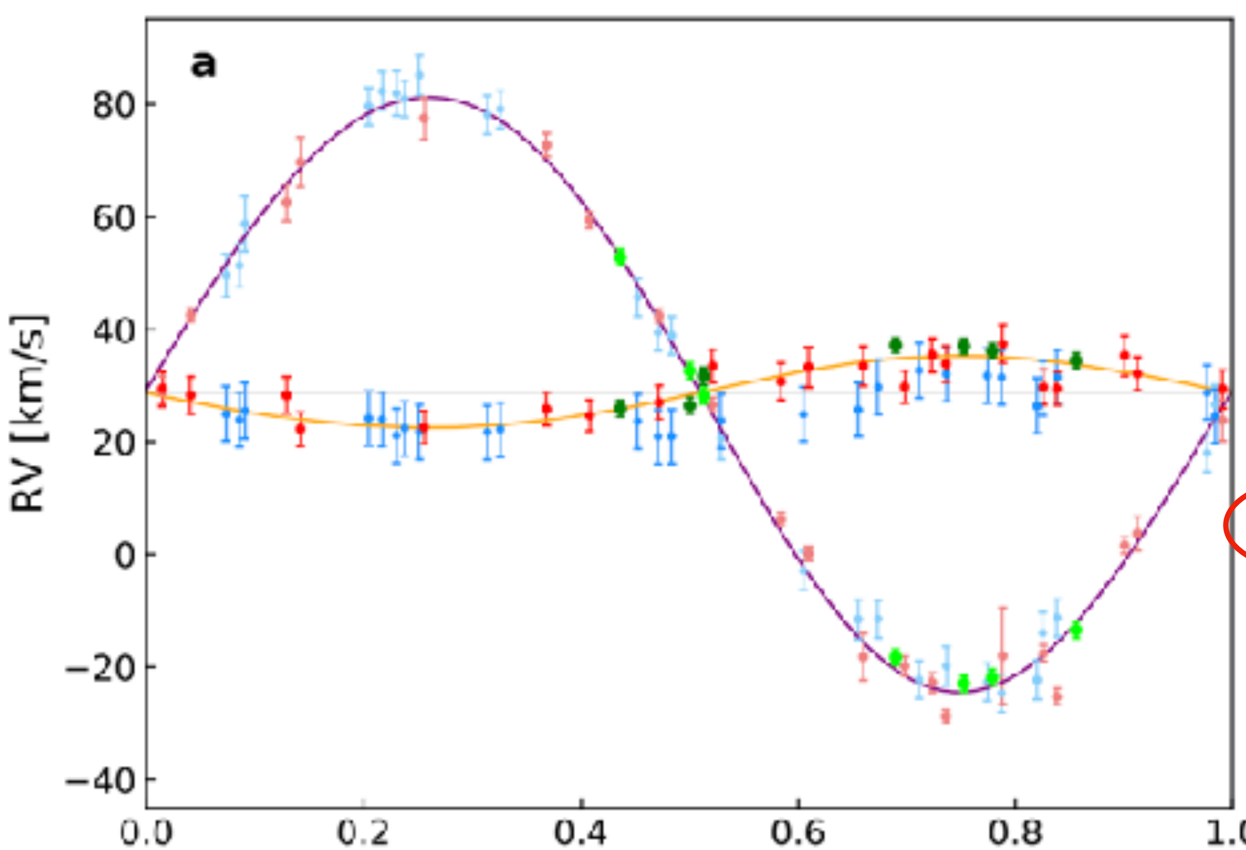




two **black holes** with masses of 35_{+5} times and 30_{+3} $_{-4}$ times the mass of the **Sun** (in the source frame),

A wide star-black-hole binary system from radial-velocity measurements

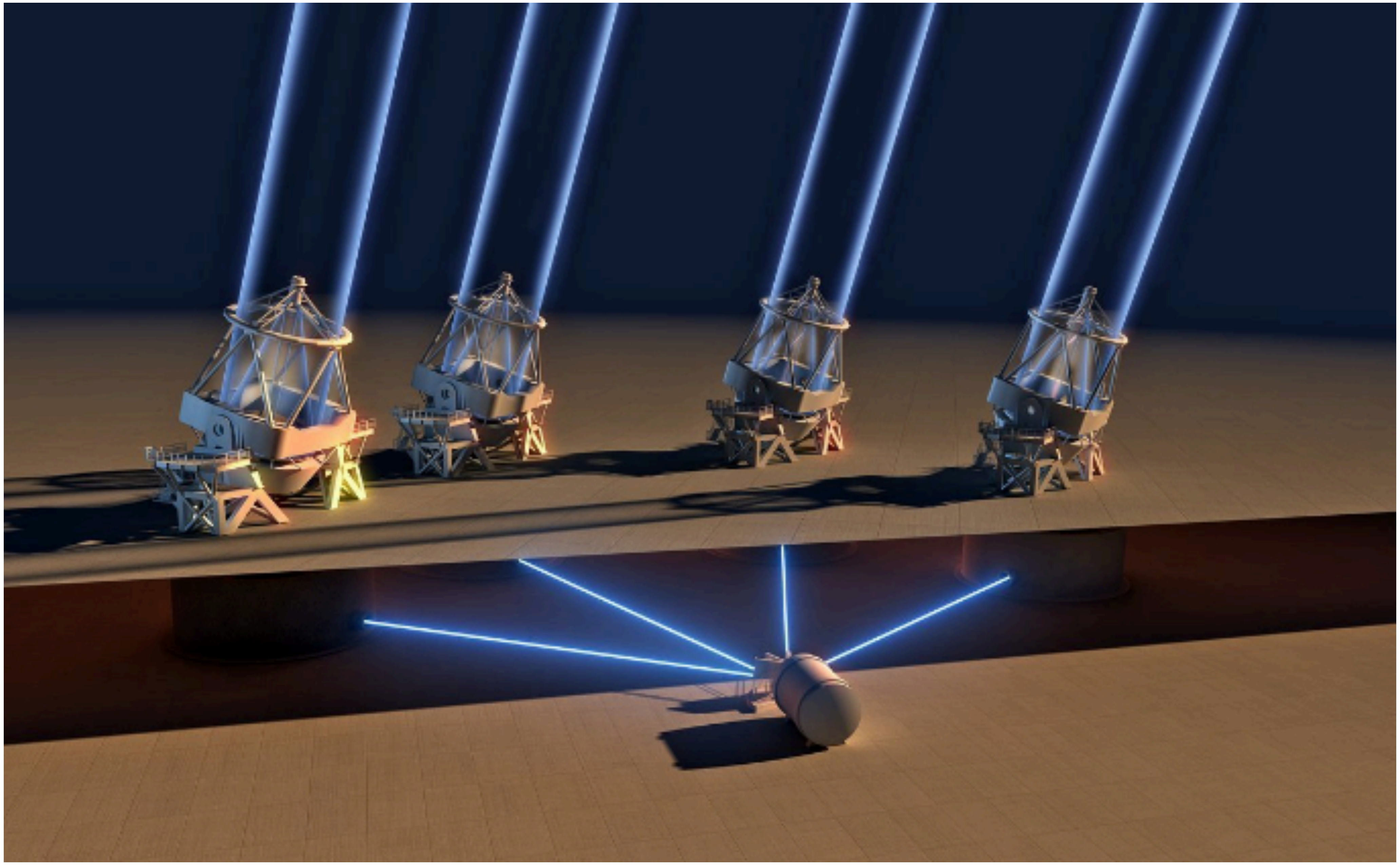




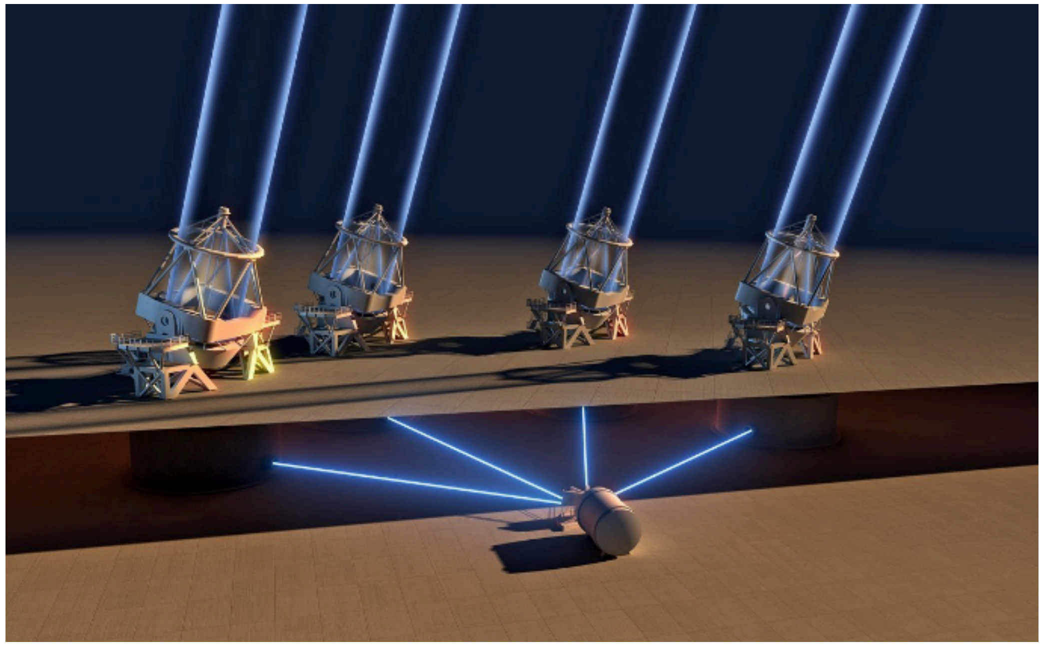
which is a B-type star, taken over two years. We find that the motion of the B-star and an accompanying Hα emission line require the presence of a dark companion with a mass of

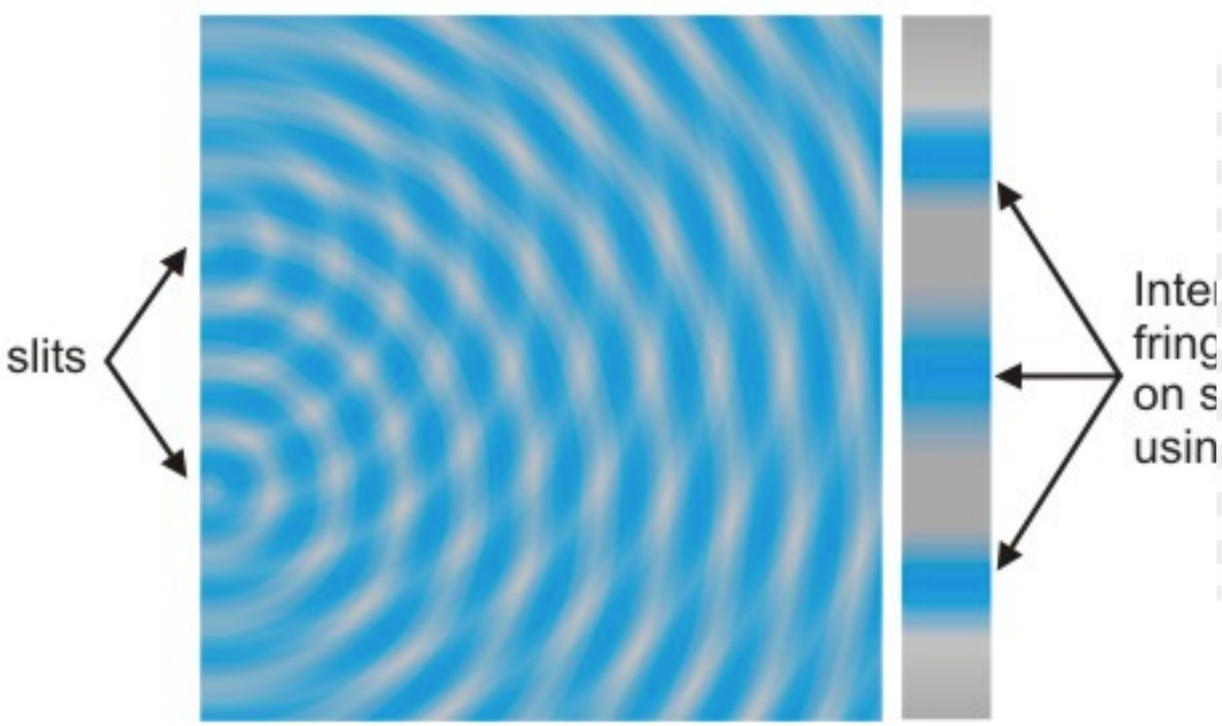
68^{+11}_{-13} $M_\odot$, which can only be a black hole. The long orbital period of 78.9 days shows that this is a wide binary system. The gravitational wave experiments have detected similarly massive black holes[7,8], but forming such massive ones in a high-metallicity environment would be extremely challenging to current stellar evolution theories[9–11].

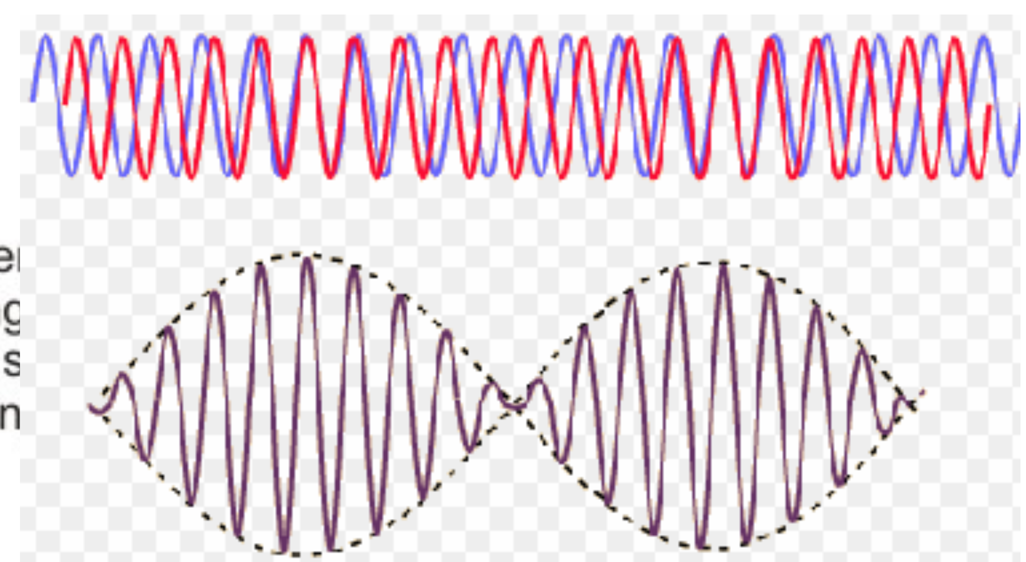
Binary stars

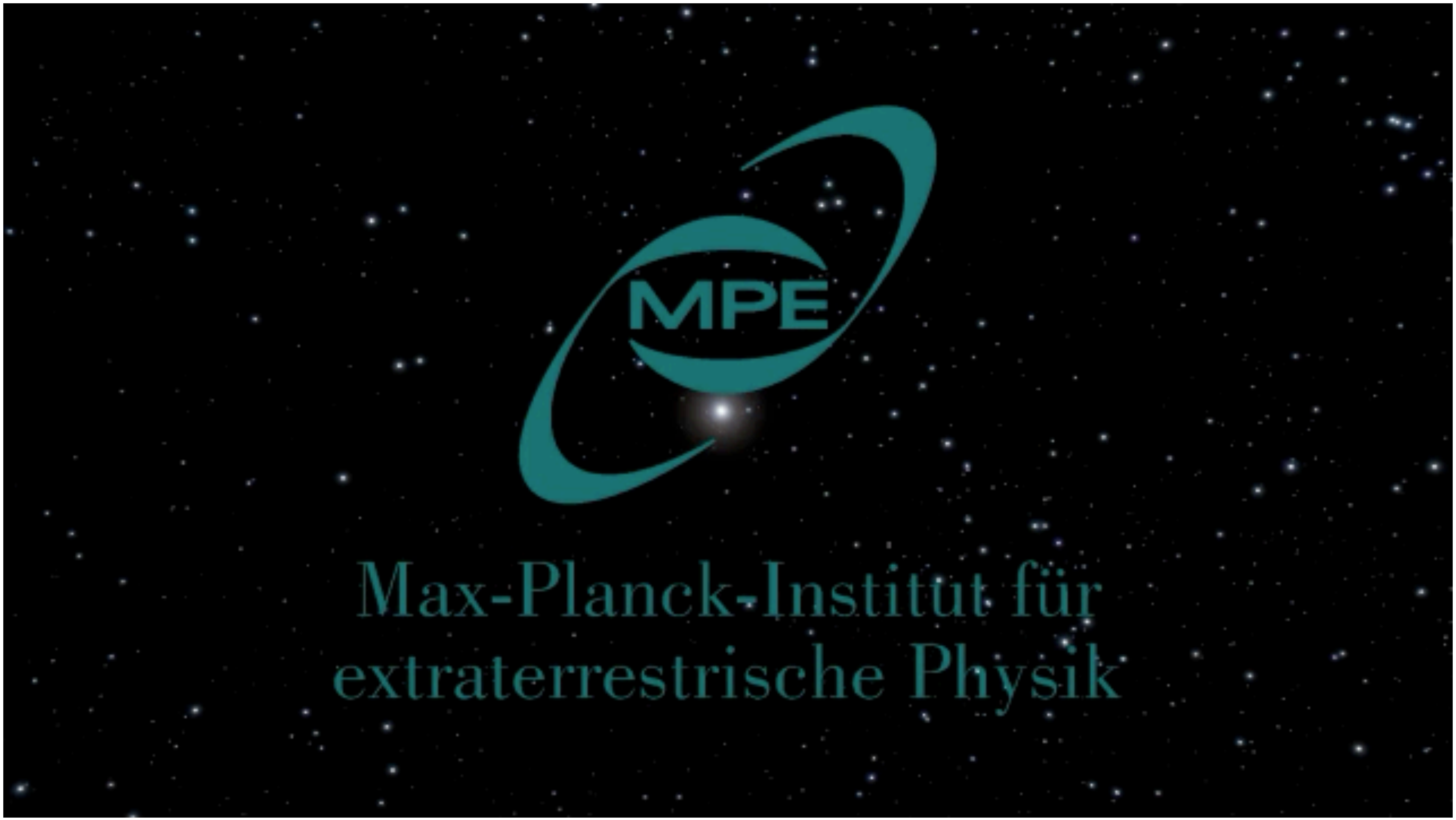


www.eso.org



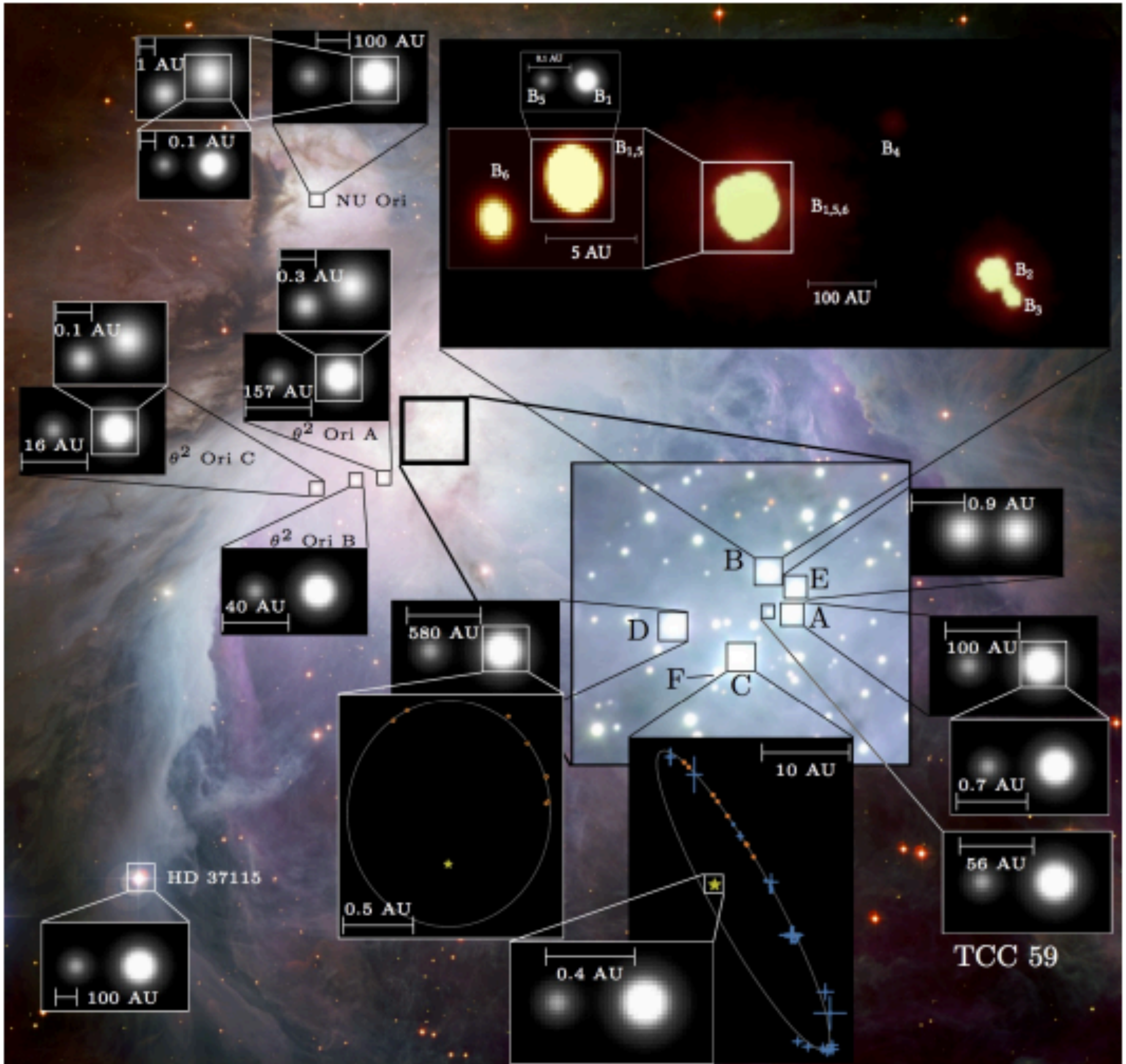

slits



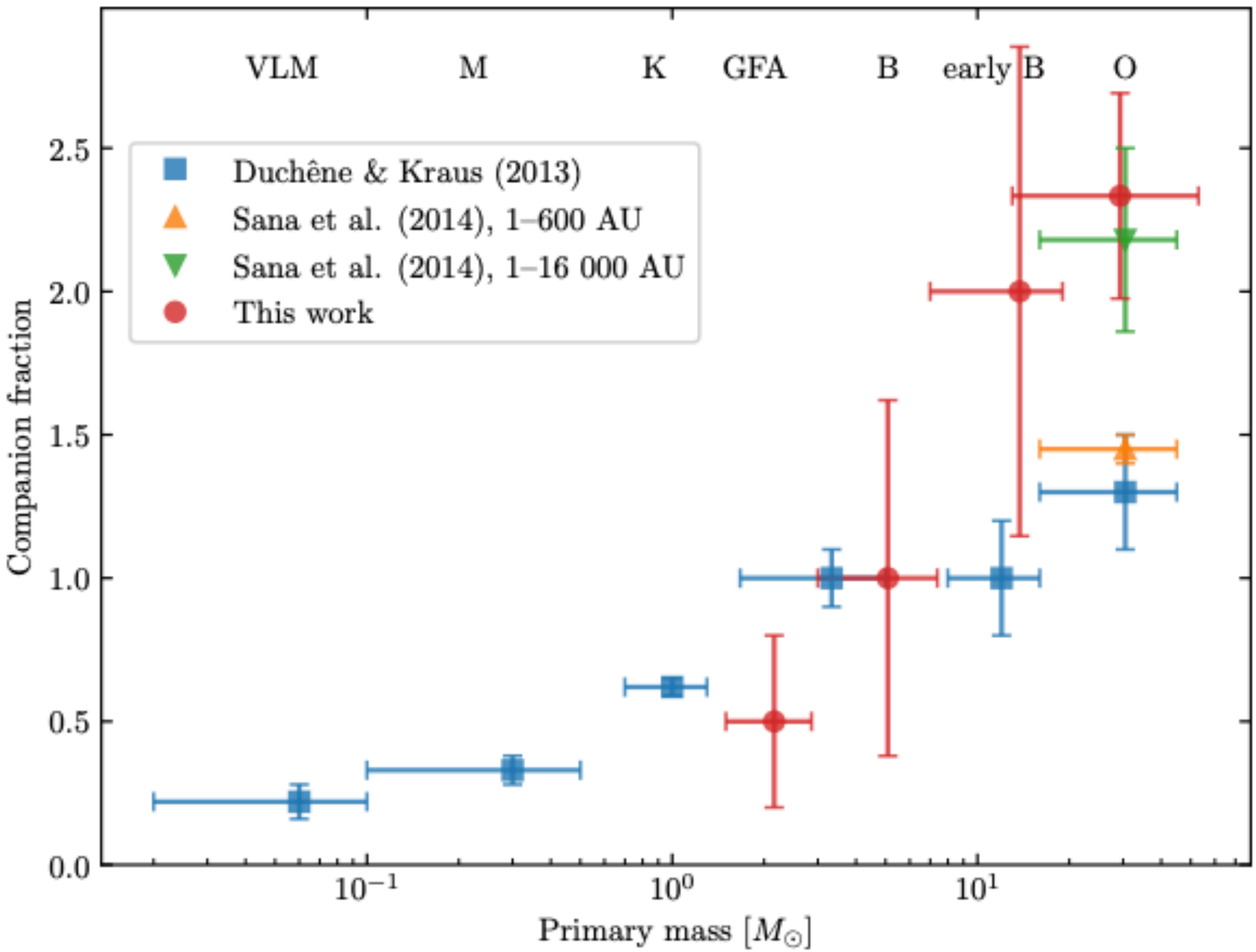

MPE
Max-Planck-Institut für
extraterrestrische Physik

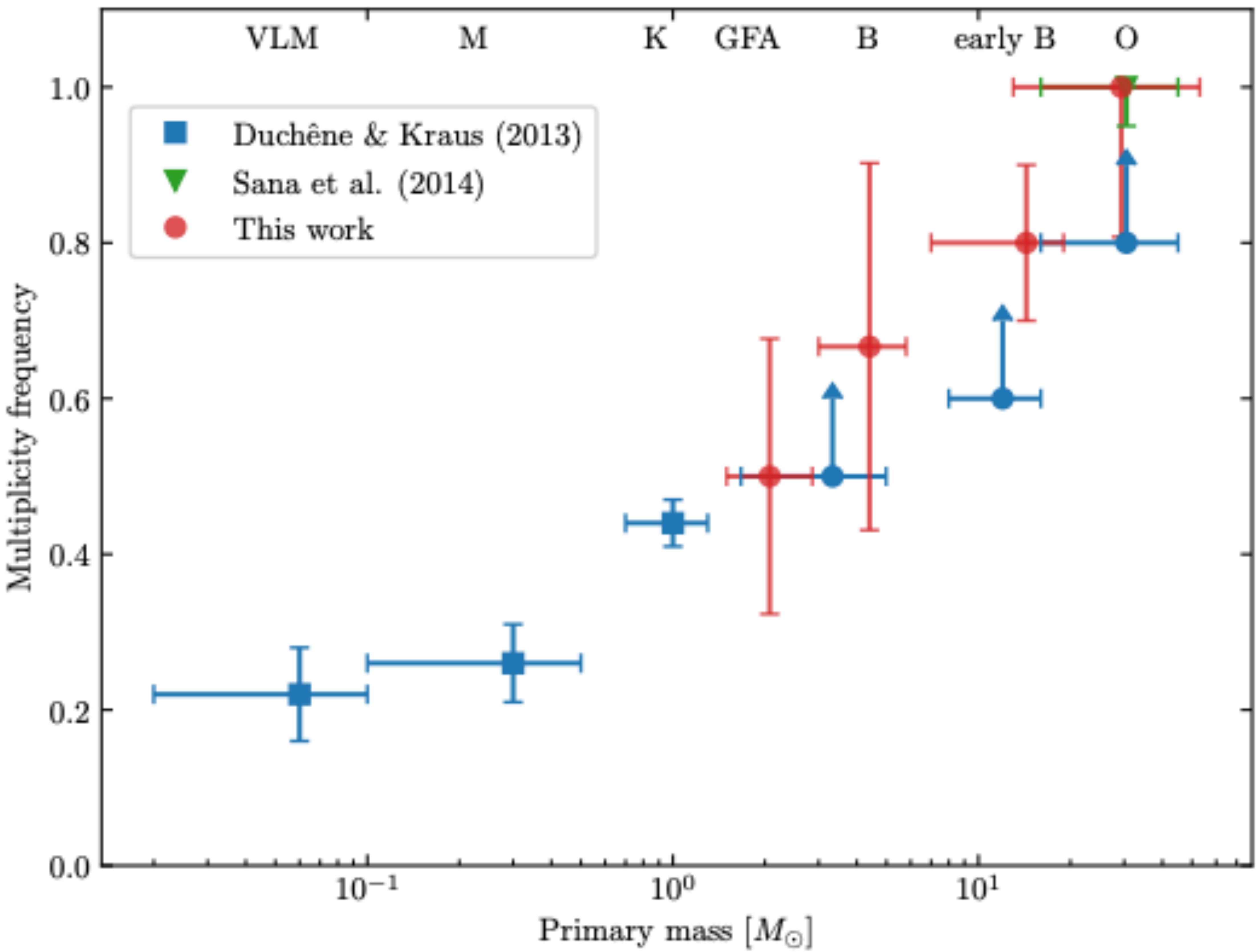
Multiple Star Systems in the Orion Nebula

GRAVITY collaboration[*]: Martina Karl[**1], Oliver Pfuhl[***1], Frank Eisenhauer[1], Reinhard Genzel[1,2], Rebekka Grellmann[3], Maryam Habibi[1], Roberto Abuter[4], Matteo Accardo[4], António Amorim[5], Narsireddy Anugu[6], Gerardo Ávila[4], Myriam Benisty[7], Jean-Philippe Berger[7], Nicolas Blind[8], Henri Bonnet[4], Pierre Bourget[9], Wolfgang Brandner[10], Roland Brast[4], Alexander Buron[1], Alessio Caratti o Garatti[10,15], Frédéric Chapron[11], Yann Clénet[11], Claude Collin[11], Vincent Coudé du Foresto[11], Willem-Jan de Wit[9], Tim de Zeeuw[1,12], Casey Deen[1], Françoise Delplancke-Ströbele[4], Roderick Dembet[4], Frédéric Derie[4], Jason Dexter[1], Gilles Duvert[7], Monica Ebert[10], Andreas Eckart[3,13], Michael Esselborn[4], Pierre Fédou[11], Gert Finger[4], Paulo Garcia[6,9], Cesar Enrique Garcia Dabo[4], Rebeca Garcia Lopez[10,15], Feng Gao[1], Éric Gendron[11], Stefan Gillessen[1], Frédéric Gonté[4], Paulo Gordo[5], Ulrich Grözinger[10], Patricia Guajardo[9], Sylvain Guieu[7], Pierre Haguenauer[4], Oliver Hans[1], Xavier Haubois[9], Marcus Haug[1,4], Frank Haußmann[1], Thomas Henning[10], Stefan Hippler[10], Matthew Horrobin[3], Armin Huber[10], Zoltan

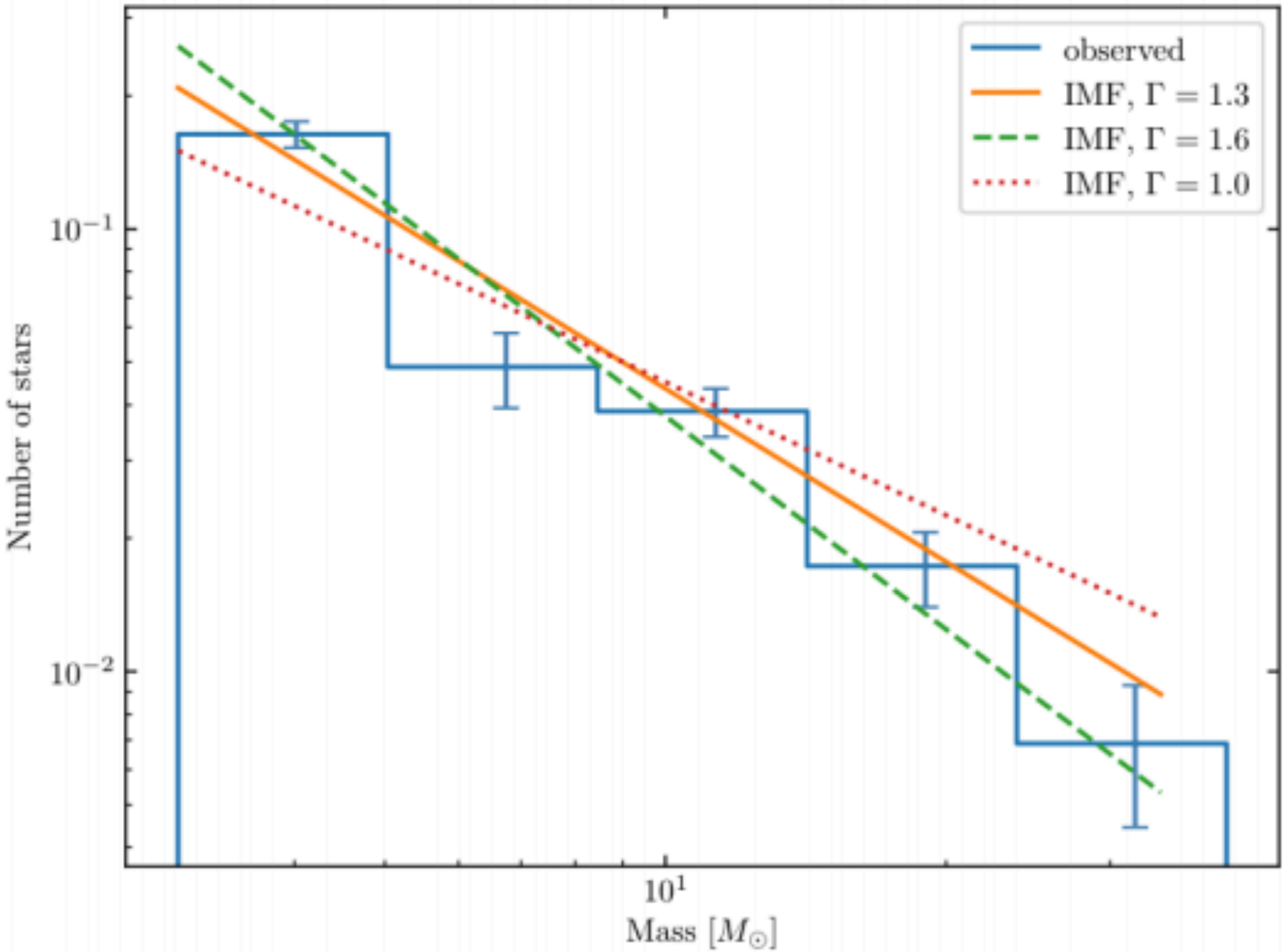


1 AU
100 AU
0.1 AU
NU Ori
0.3 AU
0.1 AU
157 AU
16 AU
θ^2 Ori A
θ^2 Ori C
θ^2 Ori B
40 AU
580 AU
0.5 AU
HD 37115
100 AU
0.4 AU
10 AU
B_5
B_1
$B_{1,5}$
B_6
$B_{1,5,6}$
B_4
5 AU
100 AU
B_2
B_3
0.9 AU
B
E
A
D
F
C
100 AU
0.7 AU
56 AU
TCC 59


VLM
M
K
GFA
B
early B
O
Duchêne & Kraus (2013)
Sana et al. (2014), 1–600 AU
Sana et al. (2014), 1–16 000 AU
This work
Companion fraction
0.0
0.5
1.0
1.5
2.0
2.5
10^{-1}
10^{0}
10^{1}
Primary mass [$M_\odot$]

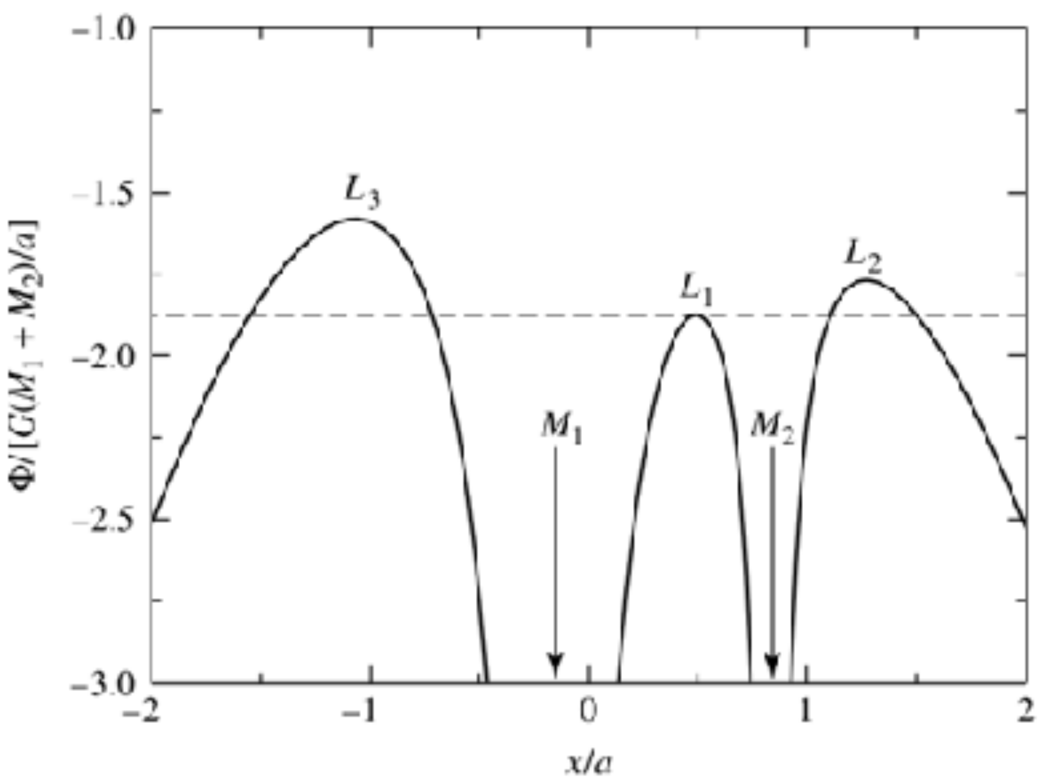


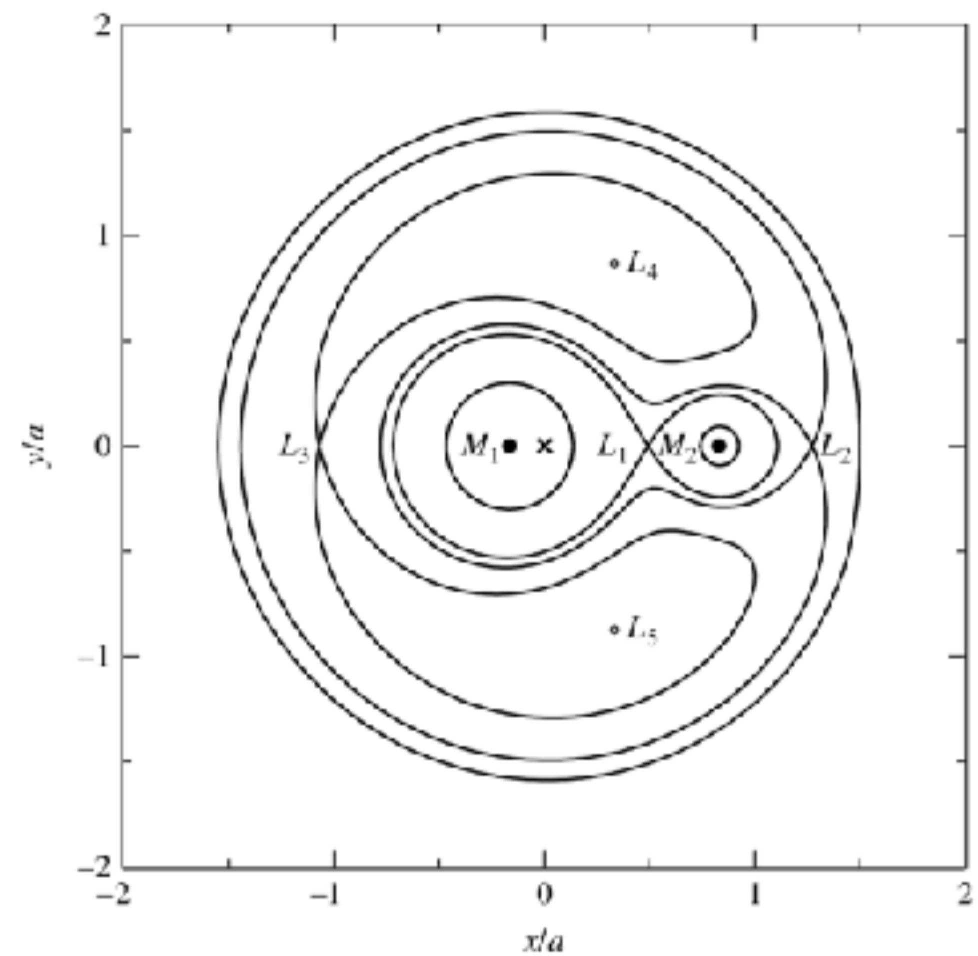
VLM
M
K
GFA
B
early B
O
Duchêne & Kraus (2013)
Sana et al. (2014)
This work
Multiplicity frequency
1.0
0.8
0.6
0.4
0.2
0.0
10^{-1}
10^{0}
10^{1}
Primary mass $[M_\odot]$


observed
IMF, $\Gamma = 1.3$
IMF, $\Gamma = 1.6$
IMF, $\Gamma = 1.0$
10^{-1}
10^{-2}
Number of stars
10^{1}
Mass [$M_\odot$]

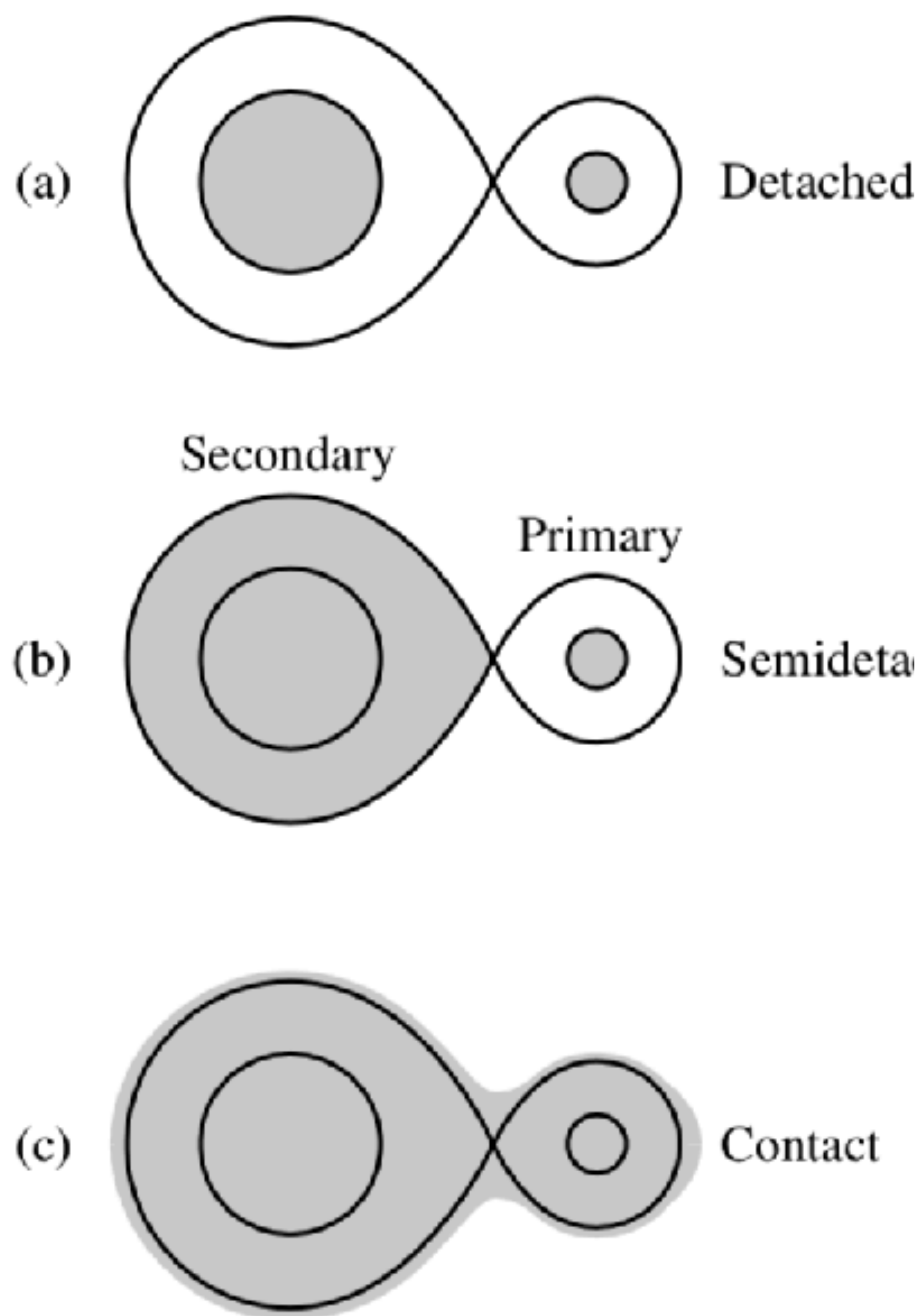
Binary stars

Temperature of Accretion Disk

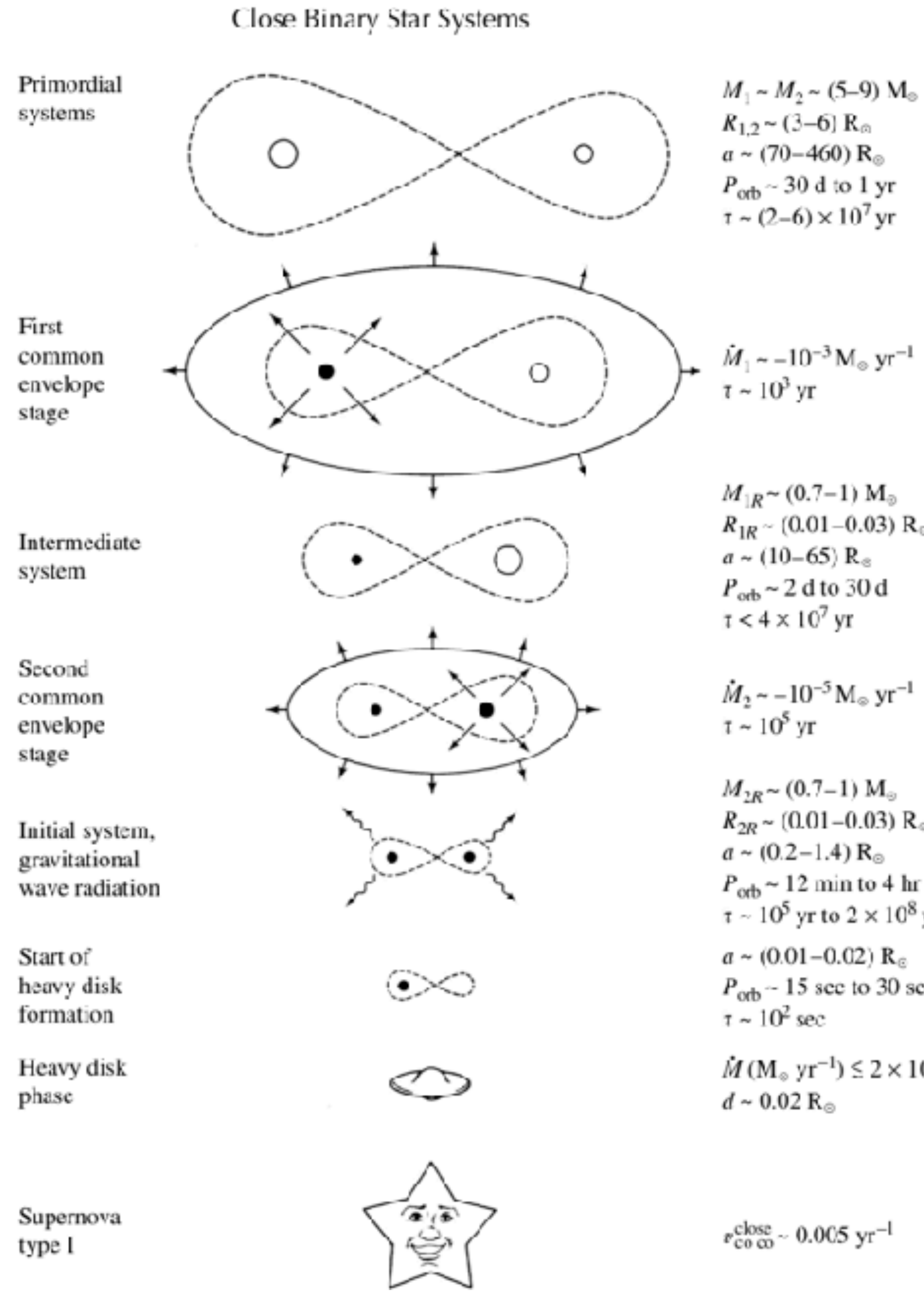




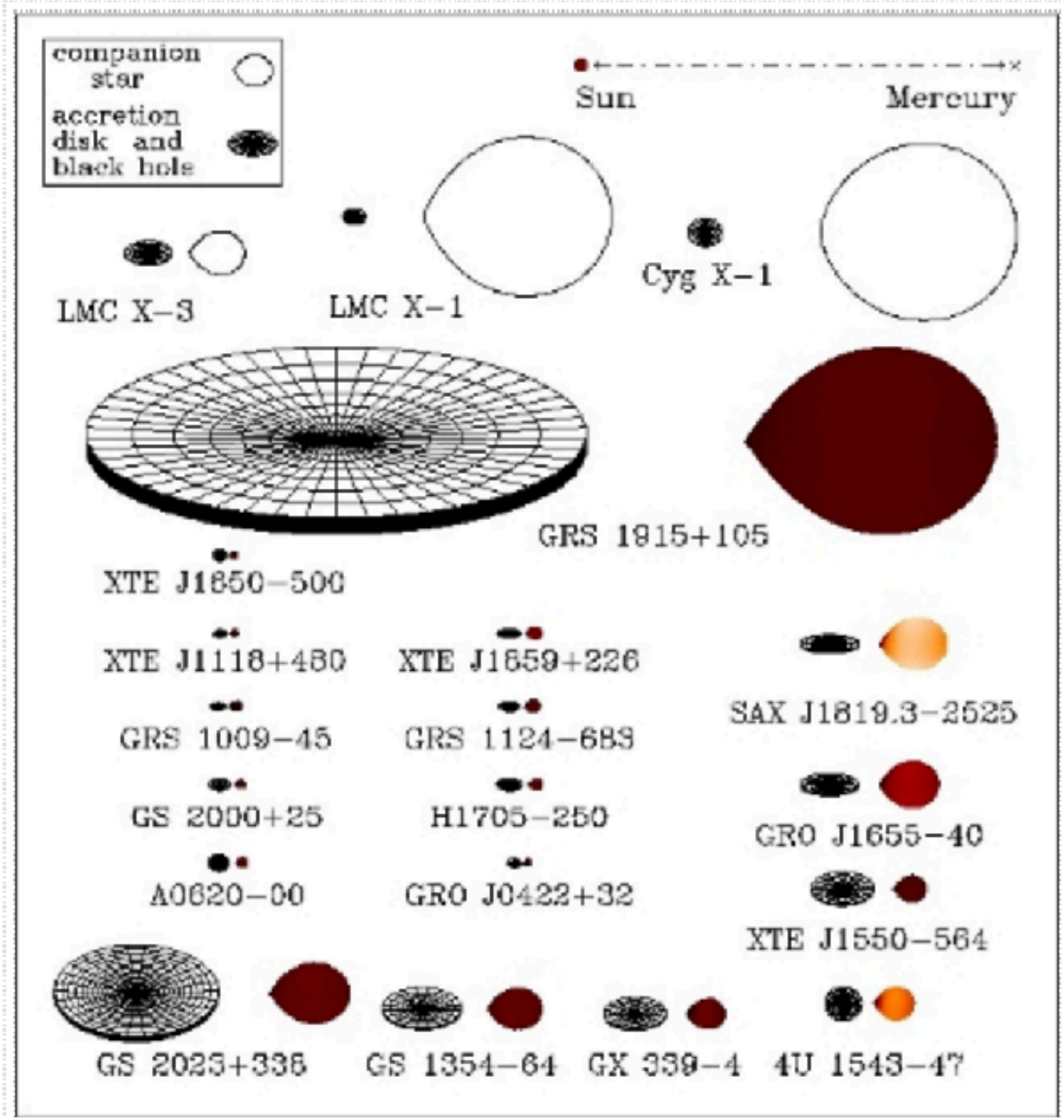
$$\boxed{\Phi = -G\left(\frac{M_1}{s_1}+\frac{M_2}{s_2}\right) + \frac{1}{2}\omega^2 r^2.}$$



(a)
Detached
Secondary
Primary
(b)
Semideta
(c)
Contact

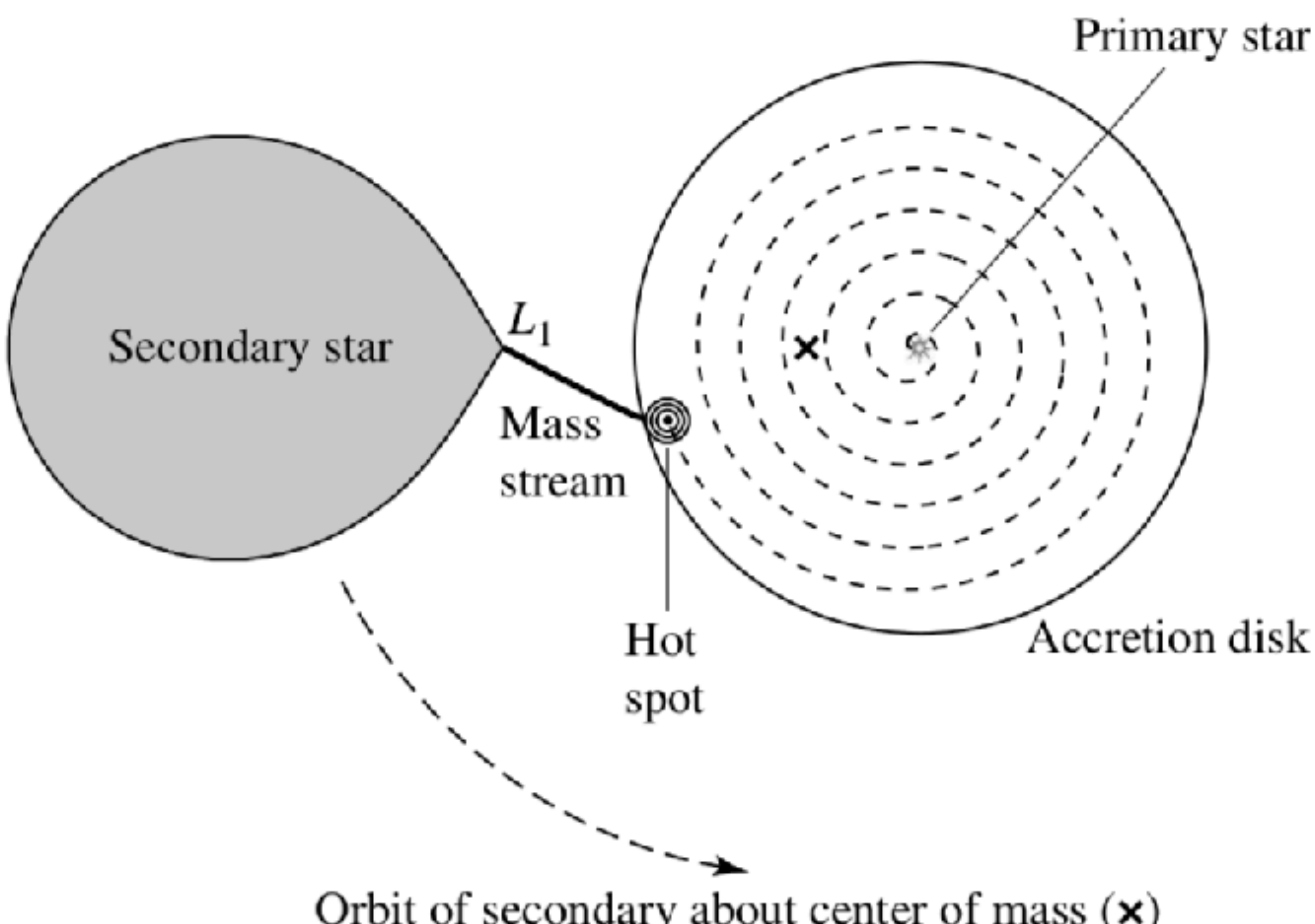


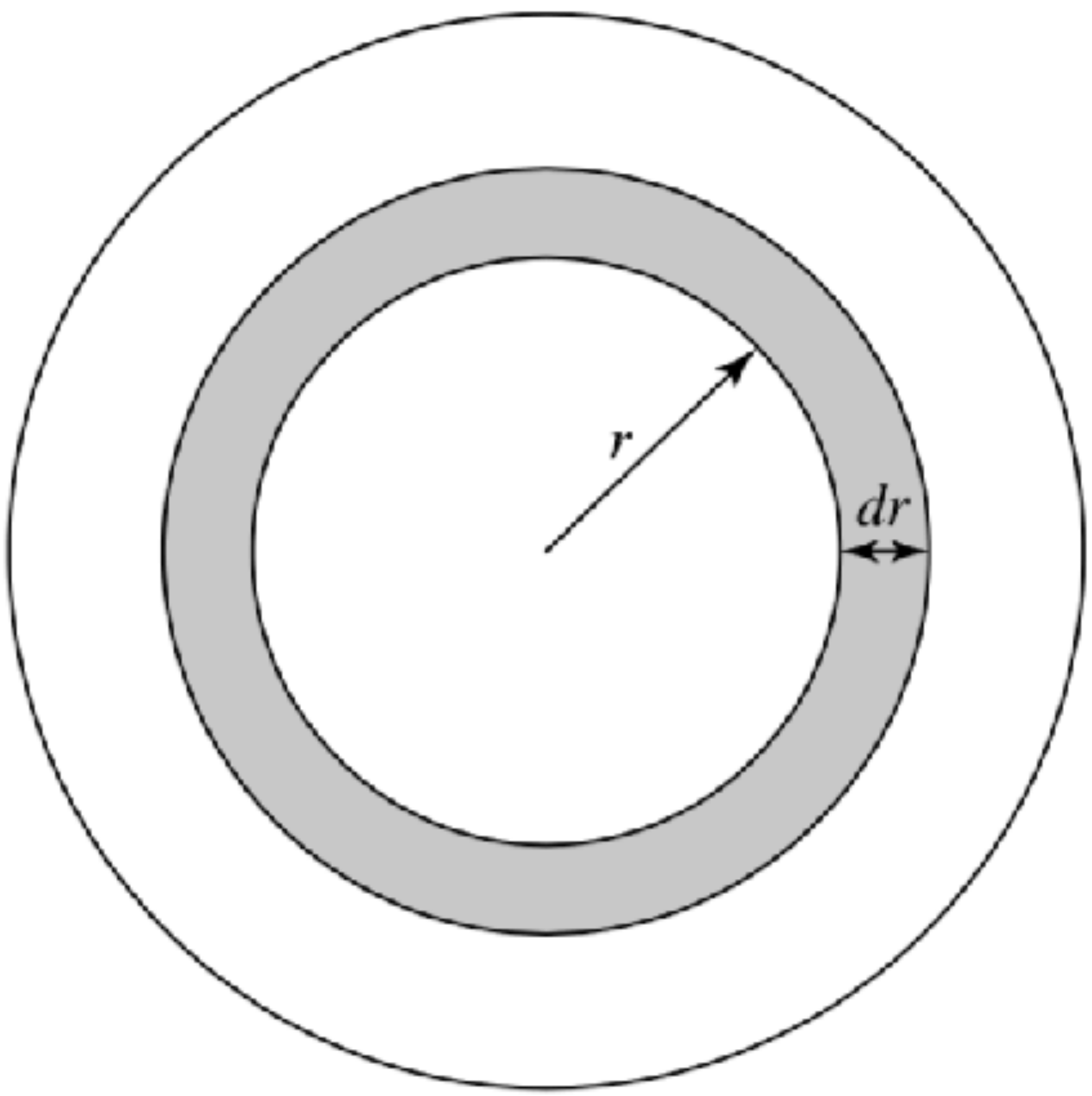
Close Binary Star Systems
Primordial systems
$M_1 \sim M_2 \sim (5\text{–}9)\ M_\odot$
$R_{1,2} \sim (3\text{–}6)\ R_\odot$
$a \sim (70\text{–}460)\ R_\odot$
$P_{orb} \sim 30$ d to 1 yr
$\tau \sim (2\text{–}6) \times 10^7$ yr
First common envelope stage
$\dot{M}_1 \sim -10^{-3}\ M_\odot\ yr^{-1}$
$\tau \sim 10^3$ yr
$M_{1R} \sim (0.7\text{–}1)\ M_\odot$
$R_{1R} \sim (0.01\text{–}0.03)$
Intermediate system
$a \sim (10\text{–}65)\ R_\odot$
$P_{orb} \sim 2$ d to 30 d
$\tau < 4 \times 10^7$ yr
Second common envelope stage
$\dot{M}_2 \sim -10^{-5}\ M_\odot\ yr^{-1}$
$\tau \sim 10^5$ yr
$M_{2R} \sim (0.7\text{–}1)\ M_\odot$
$R_{2R} \sim (0.01\text{–}0.03)$
Initial system, gravitational wave radiation
$a \sim (0.2\text{–}1.4)\ R_\odot$
$P_{orb} \sim 12$ min to 4 hr
$\tau \sim 10^5$ yr to 2×10^8
Start of heavy disk formation
$a \sim (0.01\text{–}0.02)\ R_\odot$
$P_{orb} \sim 15$ sec to 30 se
$\tau \sim 10^2$ sec
Heavy disk phase
$\dot{M}\ (M_\odot\ yr^{-1}) \leq 2 \times 1$
$d \sim 0.02\ R_\odot$
Supernova type I
$\nu_{co\ co}^{close} \sim 0.005\ yr^{-1}$



companion star
accretion disk and black hole
Sun
Mercury
LMC X-3
LMC X-1
Cyg X-1
GRS 1915+105
XTE J1650-500
XTE J1118+480
XTE J1859+226
SAX J1819.3-2525
GRS 1009-45
GRS 1124-683
GS 2000+25
H1705-250
GRO J1655-40
A0620-00
GRO J0422+32
XTE J1550-564
GS 2023+338
GS 1354-64
GX 339-4
4U 1543-47

Close Binary Star Systems



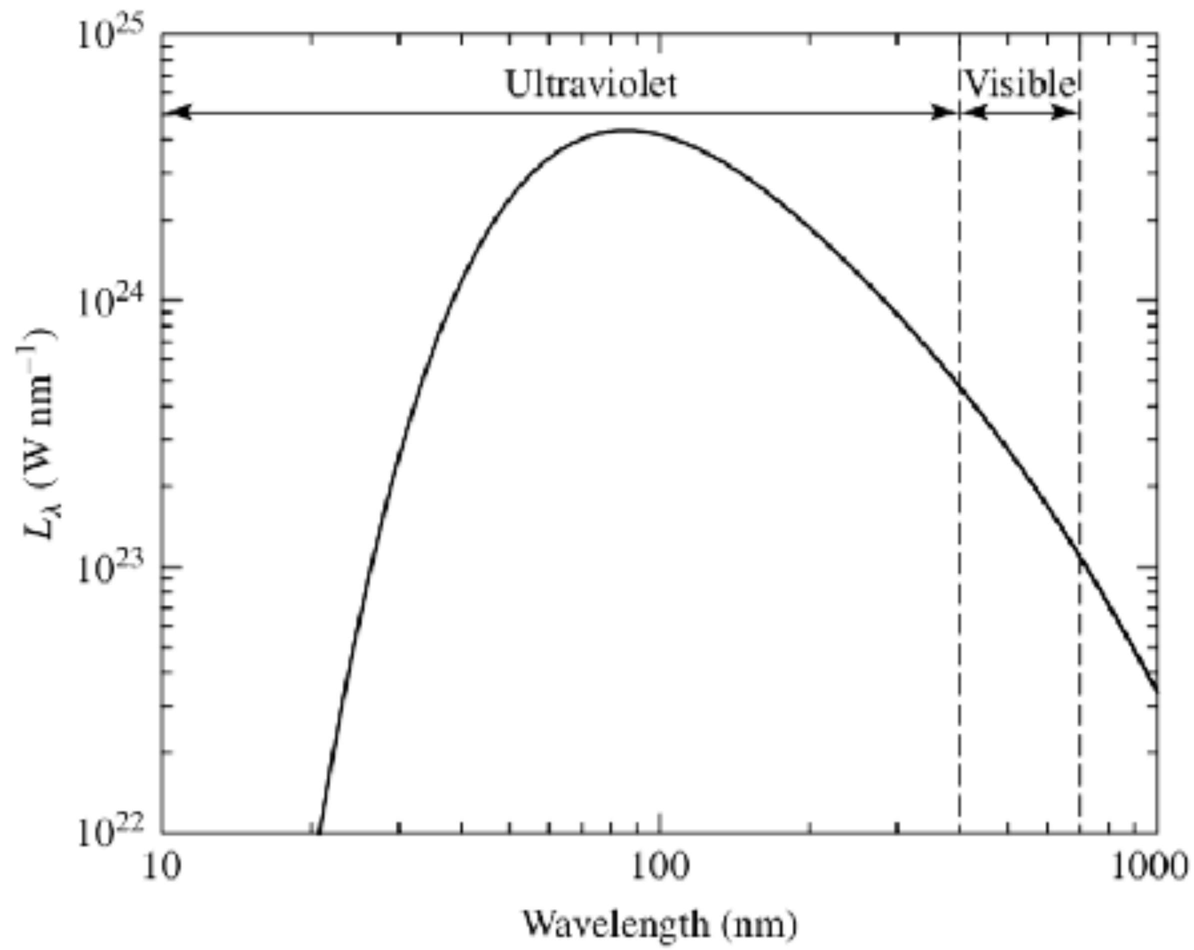


$$dL_{\text{ring}}t = dE = G\,\frac{M_1\dot{M}t}{2r^2}\,dr.$$

$$\begin{aligned} dL_{\text{ring}} &= 4\pi r\sigma T^4\,dr \\ &= G\,\frac{M_1\dot{M}}{2r^2}\,dr \end{aligned}$$

$$T = \left(\frac{GM\dot{M}}{8\pi\sigma R^3}\right)^{1/4}\left(\frac{R}{r}\right)^{3/4}.$$

WHITE DWARFS IN SEMIDETACHED BINARIES

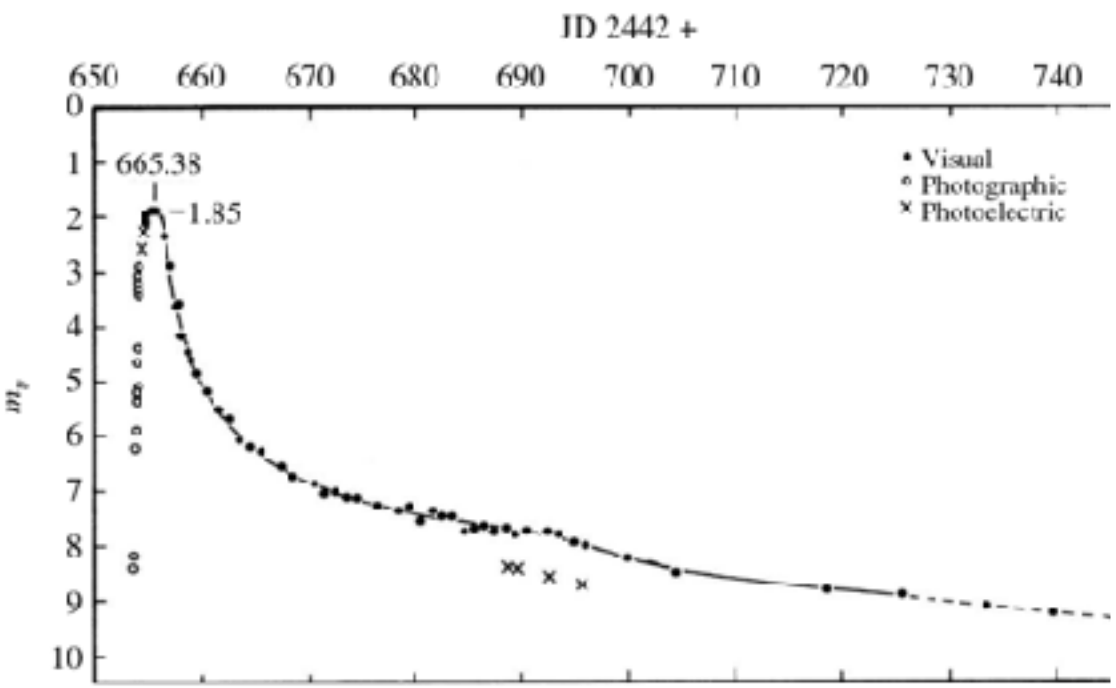


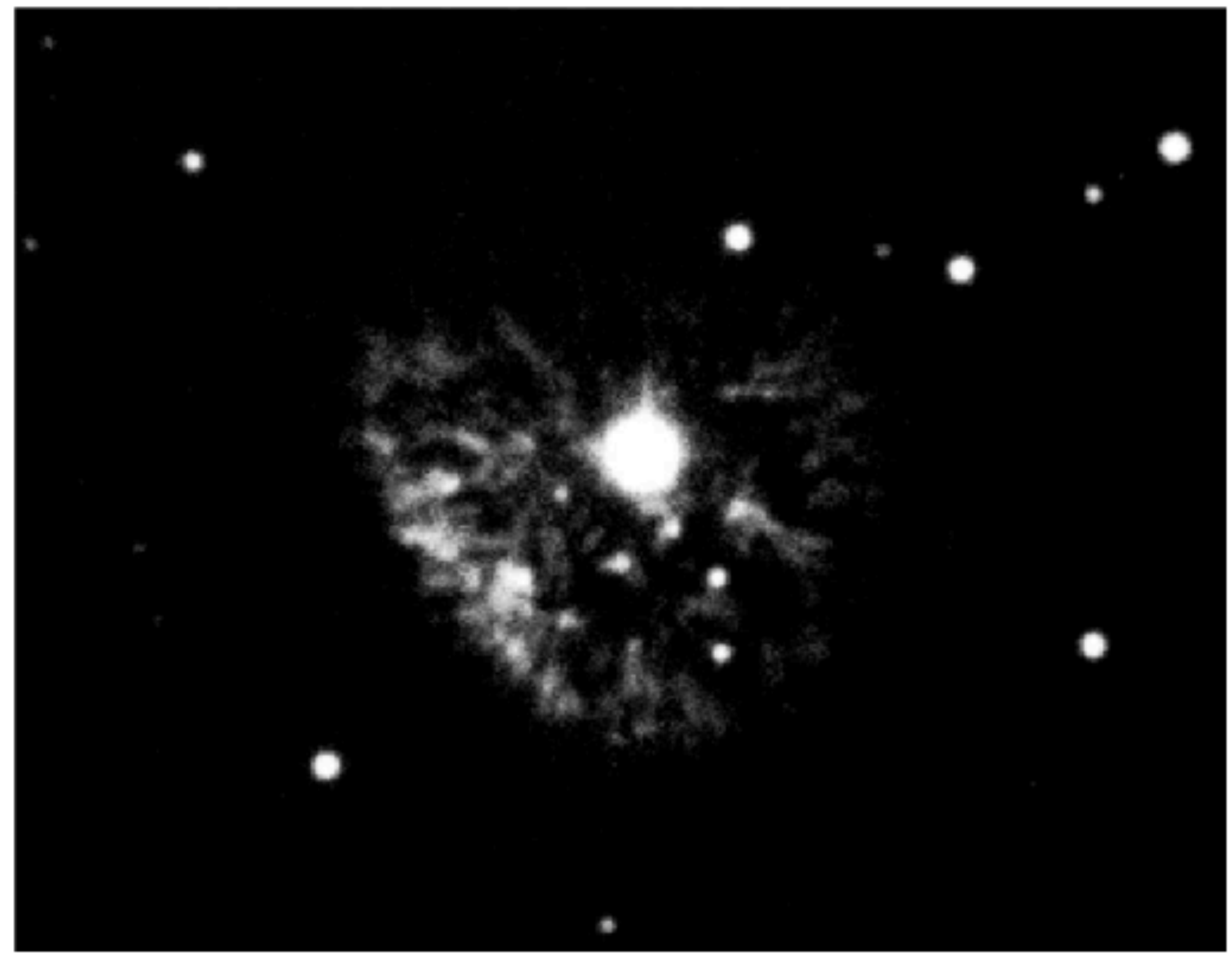
$$L_{\rm disk} = G\,\frac{M\dot{M}}{2R}$$

$$T_{\max} = 0.488\left(\frac{3GM\dot{M}}{8\pi\sigma R^3}\right)^{1/4} = 4.4\times10^4$$

$$\dot{M} \approx 10^{12} - 10^{13}\ \mathrm{kg\ s^{-1}} \approx 10^{-11} - 10^{-10}\ \mathrm{M_\odot\ yr^{-1}},$$

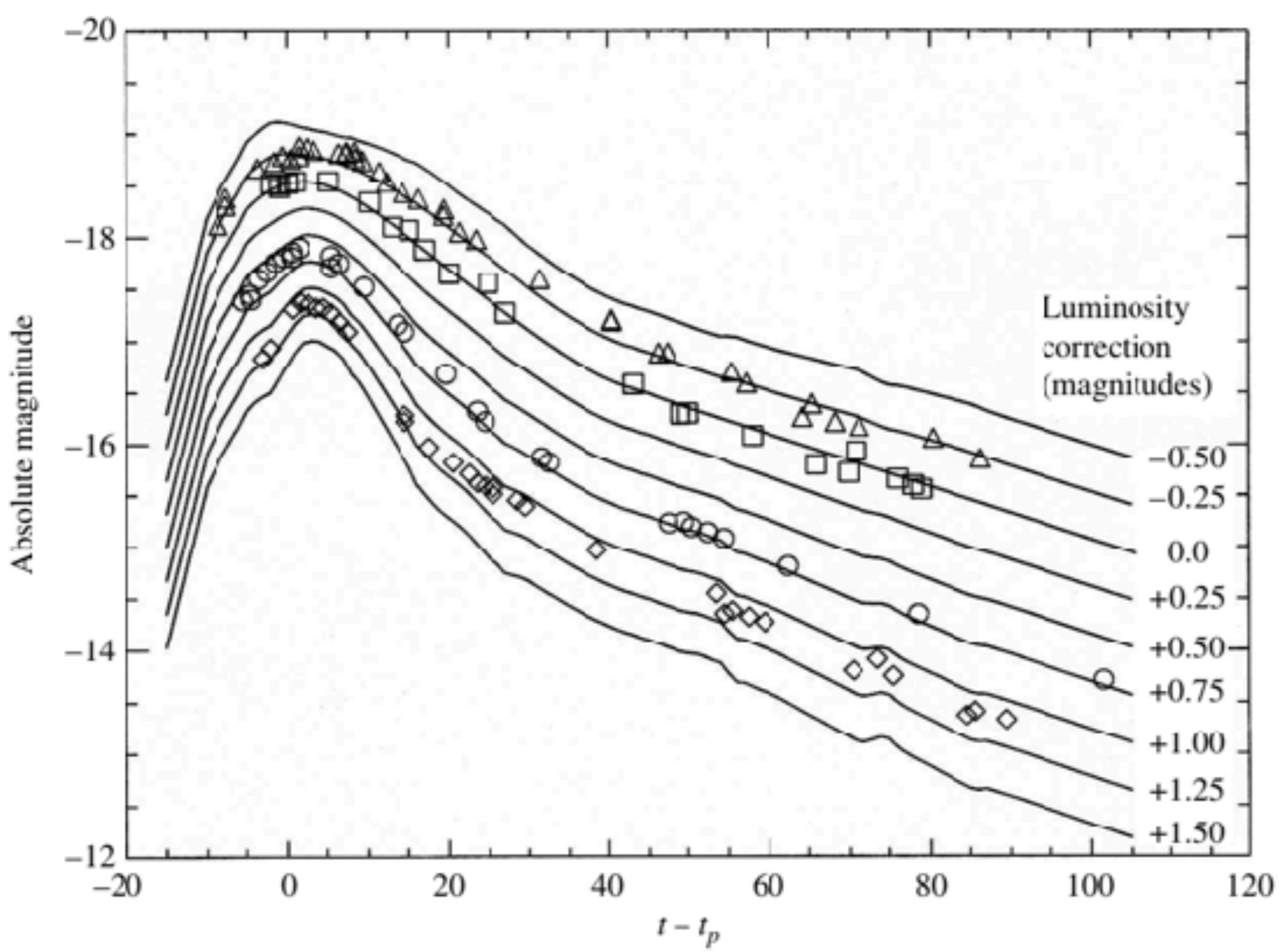
Thermonuclear Burning





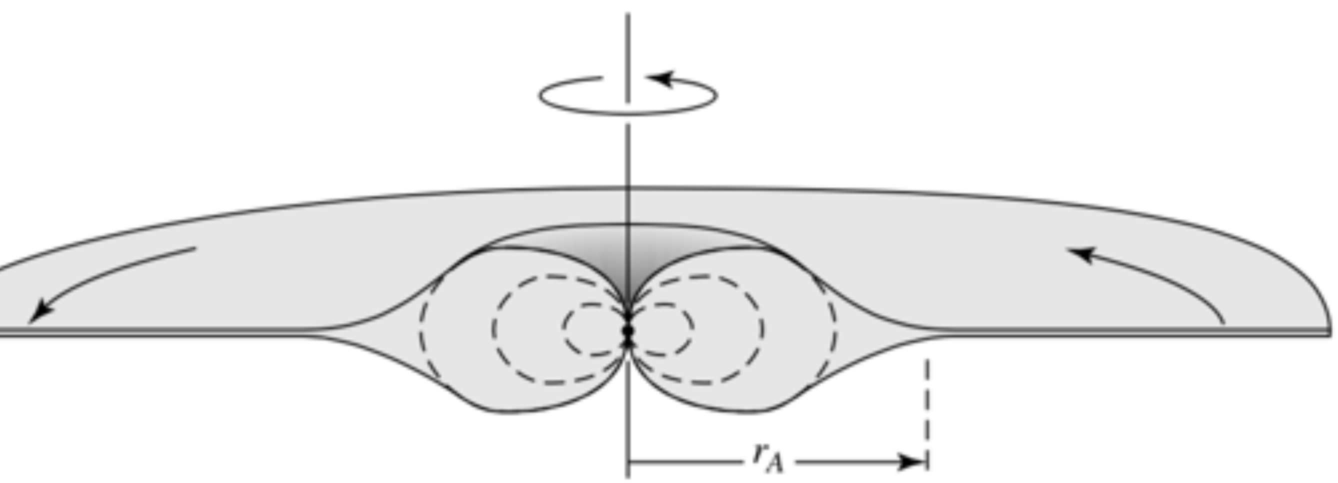
Models of Type Ia Supernovae

$$\langle M_B \rangle \simeq \langle M_V \rangle \simeq -19.3 \pm 0.03,$$



Binary X-Ray Pulsars

$$B(r) = B_s \left(\frac{R}{r}\right)^3 ,$$



Further confirmation that most X-ray pulsars are accreting neutron stars comes from the observation that the periods of these objects are slowly decreasing. As time passes, they

X-ray burst: low-B Neutron star

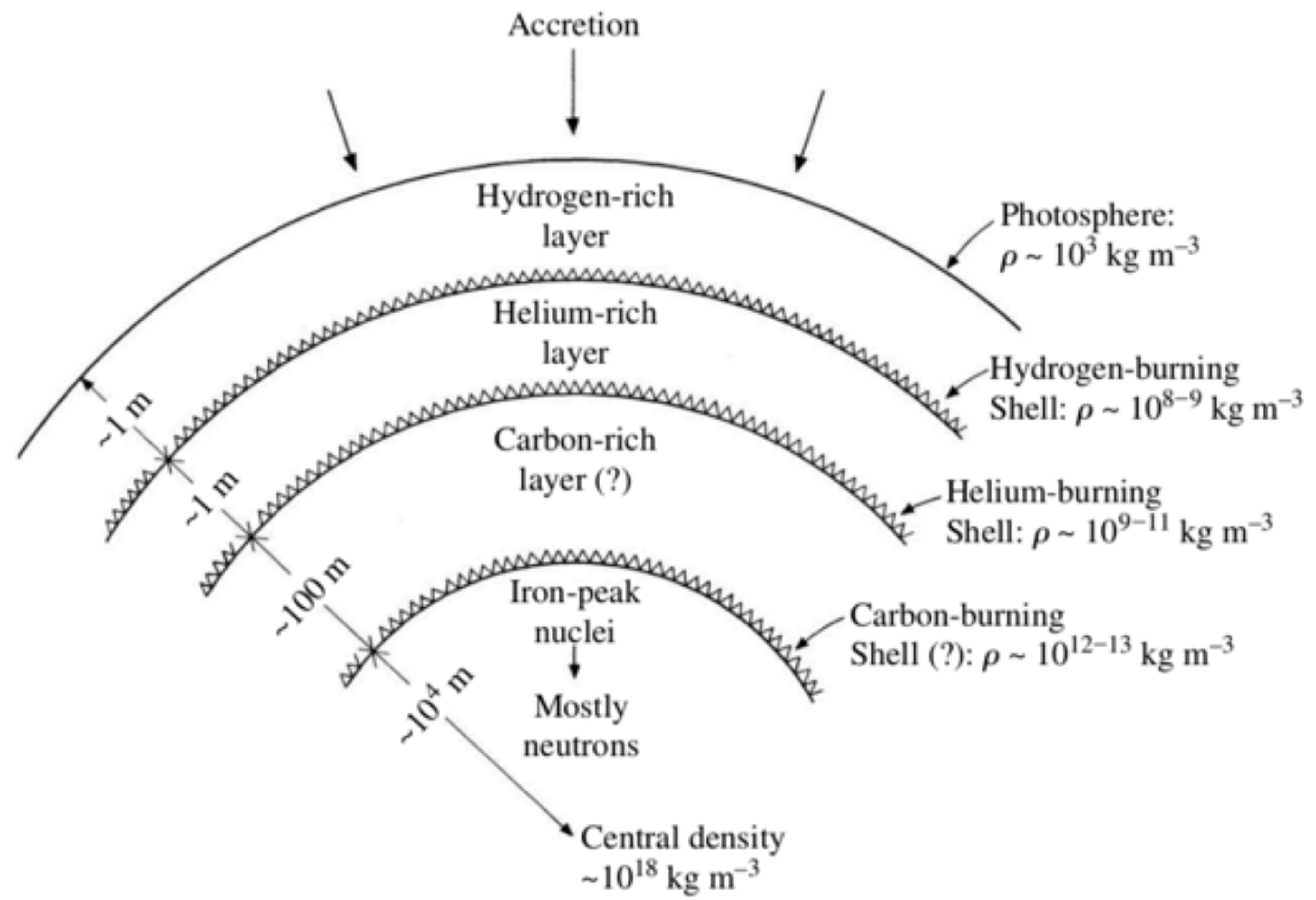


FIGURE 25 Surface layers on an accreting neutron star. (Figure adapted from Joss, *Comments Astrophys.*, *8*, 109, 1979.)