Homework 2

Mat 271E Probability and Statistics

March 22, 2023

1 Q1

If A is the set of families owning a computer and B is the set of families taking vacation, then it is given that:

$$n(A \cap B') = 28$$

$$n(A \cap B) = 87$$

$$n(A' \cap B) = 14$$

$$n(A' \cap B') = 17$$

1.a Find the probability that a randomly selected family is not taking a summer vacation this year

In the question, given that the number of families who are not taking summer vacation is 45. To calculate the probability of E, the frequency of an event E must be divided by total frequency. So, the frequency of families who are not taking summer vacation is 45. Total frequency is 146. Hence,

$$P(B') = \frac{28 + 17}{146} = \frac{45}{146} = 0.3$$

1.b Find the probability that a randomly selected family owns a computer

Similar to part 1.a, the frequency of families owning a computer is 115 among 146. Hence,

$$P(A) = \frac{115}{146} = 0.78$$

1.c Find the probability that a randomly selected family is taking a summer vacation this year, given that they own a computer

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{87}{146}}{\frac{115}{146}} = \frac{87}{115} = 0.78$$

1.d Find the probability that a randomly selected family is taking a summer vacation this year and owns a computer

$$P(A \cap B) = \frac{115}{146} = 0.78$$

1.e Are the events "owning a computer" and "taking a summer vacation this year" independent or dependent events? Explain using mathematics

Two events are independent if the occurrence of one event does not affect the probability of the other event. In this case, owning a computer and taking a summer vacation are independent events. Owning a computer does not affect the probability of taking a summer vacation. Mathematically, two events A and B are independent if:

$$P(B|A) = P(B)$$

or if and only if:

$$P(A \cap B) = P(A) \times P(B)$$

where P(A) is the probability of event A occurring and P(B) is the probability of event B occurring.

$$P(B|A) = 0.78$$

$$P(B) = \frac{101}{146} = 0.69$$

$$P(B|A) \neq P(B)$$

Because those equations do not hold, A and B are dependent events.

2 Q2

If the dice score is 1 or 2, one ball is picked from box A which contains 3 green and 1 yellow balls. Therefore, the probability of picking a yellow ball from box A is:

$$P(yellow\ ball\ from\ box\ A) = P(A_s) = \frac{1}{4} = 0.25$$

If the dice score is 3 or 4 or 5, one ball is picked from box B which contains 2 green and 2 yellow balls. Therefore, the probability of picking a yellow ball from box B is:

$$P(yellow\ ball\ from\ box\ B) = P(B_s) = \frac{2}{4} = 0.5$$

If the dice score is 6, one ball is picked from box C which contains 1 green and 3 yellow balls. Therefore, the probability of picking a yellow ball from box C is:

$$P(yellow\ ball\ from\ box\ C) = P(C_s) = \frac{3}{4} = 0.75$$

The total probability of picking a yellow ball can be calculated using the law of total probability:

$$P(yellow\ ball) = P(dice\ score = 1\ or\ 2) \times P(A_s) + P(dice\ score = 3\ or\ 4\ or\ 5) \times P(B_s) + P(dice\ score = 6) \times P(C_s)$$

$$P(yellow\ ball) = (1/3)(1/4) + (1/2)(1/2) + (1/6)(3/4)$$

$$P(yellow\ ball) = \frac{11}{24} = 0.45$$

Now let's calculate the probability of picking a green ball.

$$P(green\ ball\ from\ box\ A) = P(A_g) = \frac{3}{4} = 0.75$$

$$P(green\ ball\ from\ box\ B) = P(B_g) = \frac{2}{4} = 0.5$$

$$P(green\ ball\ from\ box\ C) = P(C_g) = \frac{1}{4} = 0.25$$

The total probability of picking a green ball can be calculated using the law of total probability:

$$P(greenball) = P(dice\ score = 1\ or\ 2) \times P(A_g) + P(dice\ score = 3\ or\ 4\ or\ 5) \times P(B_g) + P(dice\ score = 6) \times P(C_g)$$

$$P(green\ ball) = (1/3)(3/4) + (1/2)(1/2) + (1/6)(1/4)$$

$$P(green\ ball) = \frac{1}{4} + \frac{1}{4} + \frac{1}{24} = \frac{13}{24} = 0.55$$

Therefore, the probabilities of picking a yellow and a green balls are respectively:

Probability of picking a yellow: $\frac{11}{24}$

Probability of picking a green: $\frac{13}{24}$

3 Q3

Bayes' theorem can be used to calculate the probability that a person who tests positive actually has the disease.

Let B be the event that a person has the disease and A be the event that a person tests positive.

We are given:

P(B) = 0.02 (the probability that a person has the disease) P(A—B') = 0.01 (the probability of a false positive, i.e., testing positive when not infected) P(A'—B) = 0.10 (the probability of a false negative, i.e., testing negative when infected) We want to find P(A—B), which is the probability that a person has the disease given that they test positive.

Using Bayes' theorem:

$$P(B|A) = \frac{P(A|B) \times P(B)}{P(A|B) \times P(B) + P(A|B') \times P(B')}$$

$$= \frac{0.90 \times 0.02}{0.90 \times 0.02 + 0.01 \times 0.98} = 0.64$$

Therefore, if a person tests positive, there is a 64% chance that they actually have the disease.