Homework4

1. Using the sat data set, fit a model with the total SAT score as the response and expend, salary, ratio and takers as predictors. Perform regression diagnostics on this model to answer the questions below. Display any plots that are relevant but do not provide plots about which you have nothing to say. Suggest possible improvements or corrections where appropriate.

a. Check the constant variance assumption for the errors;

In [1]:
```
import pandas as pd
from sklearn import linear_model
import scipy.stats as stats
import matplotlib.pyplot as plt
import statsmodels.api as sm
from statsmodels.stats.outliers_influence import OLSInfluence as influence
import numpy as np
```

In [2]:
```
sat = pd.read_csv('sat.csv')
X = sat[['expend', 'salary','ratio','takers']]
y = sat['total']
```

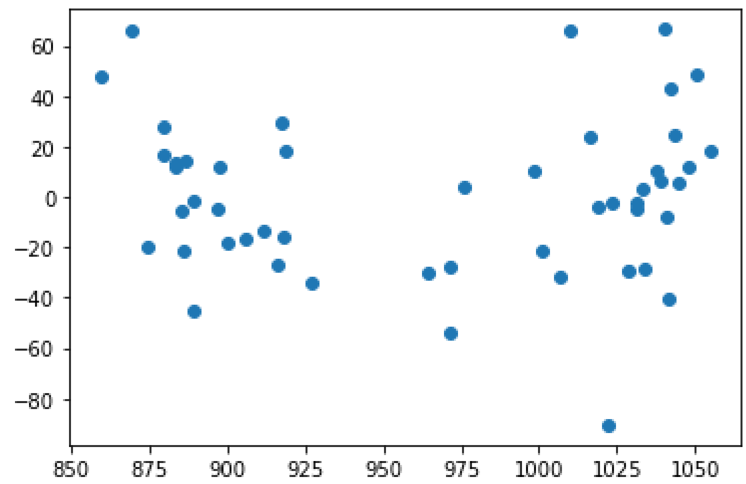
In [3]:
```
X_c = sm.add_constant(X)
model = sm.OLS(y, X_c).fit()
y_pred = model.predict(X_c)
model.summary()
```

Date:	Fri, 06 May 2022	**Prob (F-statistic):**	1.92e-16
Time:	09:28:06	**Log-Likelihood:**	-242.68
No. Observations:	50	**AIC:**	495.4
Df Residuals:	45	**BIC:**	504.9
Df Model:	4		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	1045.9715	52.870	19.784	0.000	939.486	1152.457
expend	4.4626	10.547	0.423	0.674	-16.779	25.704
salary	1.6379	2.387	0.686	0.496	-3.170	6.446
ratio	-3.6242	3.215	-1.127	0.266	-10.100	2.852
takers	-2.9045	0.231	-12.559	0.000	-3.370	-2.439

In [4]:
```
error = (y-y_pred).to_numpy()

plt.scatter(y_pred,error)
```

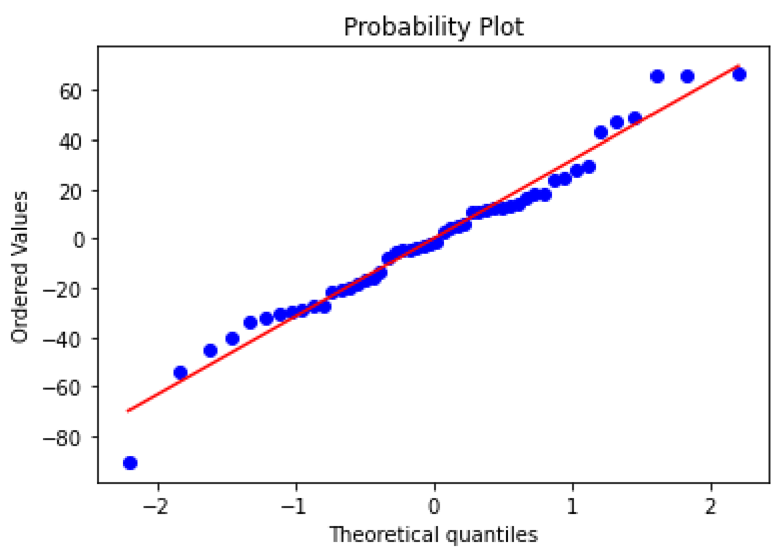
Out[4]: <matplotlib.collections.PathCollection at 0x7fbf42320b80>



the constant variance assumption for the errors is not met since the residual plot is a curve. The mean of the residual is not 0. Residuals are positive or negative depending on the value of x.

b. Check the normality assumption;

In [6]:
```
stats.probplot(error, dist="norm", plot=plt)
plt.show()
```



The approximately straight line verifies the normality for the errors.

c. Check for large leverage points;

In [7]:
```
influence = model.get_influence()
leverage = influence.hat_matrix_diag
#point with high leverage = (2k+2)/n = 10/50= 0.2
high_leverage = sat[abs(leverage) > 0.2]
high_leverage
```

Out[7]:

	Unnamed: 0	expend	ratio	salary	takers	verbal	math	total
4	California	4.992	24.0	41.078	45	417	485	902
6	Connecticut	8.817	14.4	50.045	81	431	477	908
29	New Jersey	9.774	13.8	46.087	70	420	478	898
43	Utah	3.656	24.3	29.082	4	513	563	1076

leverages of the states

In [8]:
```
leverage
```

Out[8]:
```
array([0.09537668, 0.18030612, 0.04931612, 0.05382878, 0.28211791,
       0.03014533, 0.22545191, 0.05823786, 0.14068586, 0.09418039,
       0.05164378, 0.05847998, 0.1348354 , 0.05031937, 0.05882595,
       0.10383439, 0.0410646 , 0.08754545, 0.139736  , 0.05372121,
       0.08871286, 0.16536832, 0.06069775, 0.06245712, 0.05589831,
       0.12118024, 0.07953981, 0.04539029, 0.0828354 , 0.22209778,
       0.0453355 , 0.1915752 , 0.09063656, 0.08104629, 0.04477728,
       0.05949338, 0.09828873, 0.12277229, 0.06739042, 0.0967159 ,
       0.09658262, 0.09162128, 0.05405622, 0.2921128 , 0.08566501,
       0.15230581, 0.10816994, 0.06206536, 0.09807708, 0.08748135])
```

d. Check for outliers;

the outliers has predictor values that is three standard deviations away from the average residual.

In [9]:
```
student_resid = influence.resid_studentized_external
outliers = sat[abs(student_resid) > 3]
outliers
```

Out[9]:

	Unnamed: 0	expend	ratio	salary	takers	verbal	math	total
47	West Virginia	6.107	14.8	31.944	17	448	484	932

e. Check for influential points;

In [10]:
```
C, P = influence.cooks_distance
ind= np.argsort(C)[-5:]
influential_points = sat.loc[ind]
influential_points
```

Out[10]:

	Unnamed: 0	expend	ratio	salary	takers	verbal	math	total
6	Connecticut	8.817	14.4	50.045	81	431	477	908
33	North Dakota	4.775	15.3	26.327	5	515	592	1107
28	New Hampshire	5.859	15.6	34.720	70	444	491	935
47	West Virginia	6.107	14.8	31.944	17	448	484	932
43	Utah	3.656	24.3	29.082	4	513	563	1076

f. Check the structure of the relationship between the predictors and the response.

use stepwise regression to remove nonsignificant predictors from the model one at a time until all predictors have p value below a certain threshhold. Start with 4 factors in this case(takers. salary, ratio, and expense). We remove expense first since the model output indicates that expense is the leasy siginificant one. Repeat this process until all predictors are significant.

the effect of takers on total score is significant, and the effects of expand, ratio, and salary are not significant enough.

---

2. For the prostate data, fit a model with lpsa as the response and the other variables as the predictors. Answer the questions as in 1.

In [12]:
```
prostate = pd.read_csv('prostate.csv')
X = prostate[["lcavol","lweight","age","lbph","svi","lcp","gleason","pgg45"
y = prostate['lpsa']
```

In [13]:
```
X_c = sm.add_constant(X)
model2 = sm.OLS(y, X_c).fit()
y_pred = model2.predict(X_c)
model2.summary()
```

Out[13]:

OLS Regression Results

Dep. Variable:	lpsa	**R-squared:**	0.655
Model:	OLS	**Adj. R-squared:**	0.623
Method:	Least Squares	**F-statistic:**	20.86
Date:	Fri, 06 May 2022	**Prob (F-statistic):**	2.24e-17
Time:	09:33:47	**Log-Likelihood:**	-99.476
No. Observations:	97	**AIC:**	217.0
Df Residuals:	88	**BIC:**	240.1
Df Model:	8		
Covariance Type:	nonrobust		

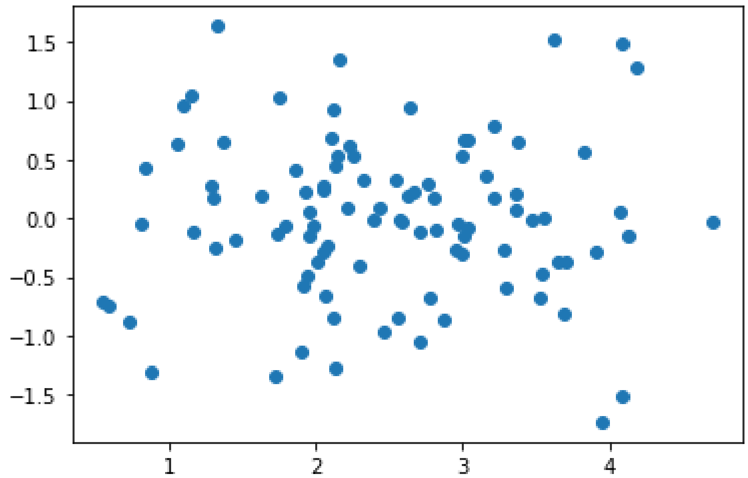
	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
const	0.6693	1.296	0.516	0.607	-1.907	3.246
lcavol	0.5870	0.088	6.677	0.000	0.412	0.762
lweight	0.4545	0.170	2.673	0.009	0.117	0.792
age	-0.0196	0.011	-1.758	0.082	-0.042	0.003
lbph	0.1071	0.058	1.832	0.070	-0.009	0.223
svi	0.7662	0.244	3.136	0.002	0.281	1.252
lcp	-0.1055	0.091	-1.159	0.250	-0.286	0.075
gleason	0.0451	0.157	0.287	0.775	-0.268	0.358
pgg45	0.0045	0.004	1.024	0.309	-0.004	0.013

Omnibus:	0.235	**Durbin-Watson:**	1.507
Prob(Omnibus):	0.889	**Jarque-Bera (JB):**	0.026
Skew:	-0.017	**Prob(JB):**	0.987
Kurtosis:	3.073	**Cond. No.**	1.28e+03

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 1.28e+03. This might indicate that there are
strong multicollinearity or other numerical problems.

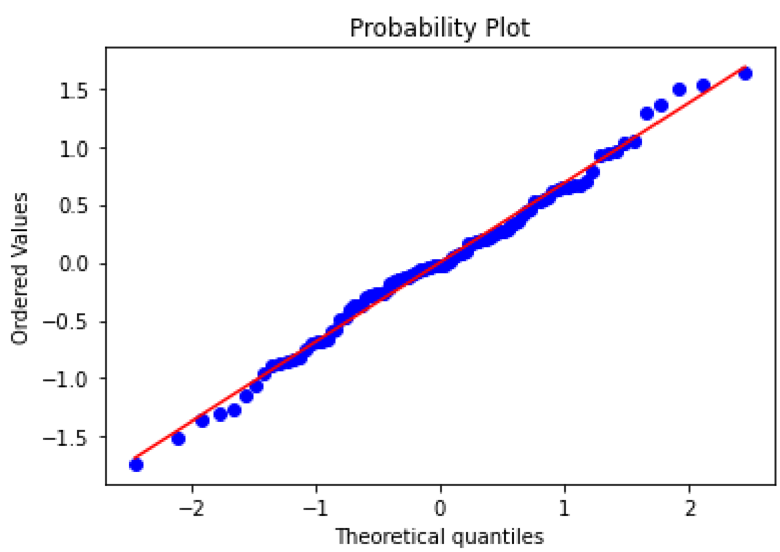
In [14]:
```
error = (y-y_pred).to_numpy()

plt.scatter(y_pred,error)
```

Out[14]: <matplotlib.collections.PathCollection at 0x7fbf425682b0>



the constant variance assumption for the errors is met

In [15]:
```
stats.probplot(error, dist="norm", plot=plt)
plt.show()
```



The approximately straight line verifies the normality for the errors.

In [16]:
```
influence = model2.get_influence()
leverage = influence.hat_matrix_diag
#point with high leverage = (2k+2)/n = 2*8 + 2 /97= 0.2
high_leverage = prostate[abs(leverage) > (18/97)]
high_leverage
```

Out[16]:

	Unnamed: 0	lcavol	lweight	age	lbph	svi	lcp	gleason	pgg45	lpsa
31	32	0.182322	6.1076	65	1.704748	0	-1.38629	6	0	2.00821
36	37	1.423108	3.6571	73	-0.579818	0	1.65823	8	15	2.15756
40	41	0.620577	3.1420	60	-1.386294	0	-1.38629	9	80	2.29757
73	74	1.838961	3.2367	60	0.438255	1	1.17865	9	90	3.07501
91	92	2.532903	3.6776	61	1.348073	1	-1.38629	7	15	4.12955

leverages

In [17]:
```
leverage
```

Out[17]:
```
array([0.07873101, 0.06758053, 0.13596177, 0.07766218, 0.03499946,
       0.08331908, 0.02989838, 0.0494461 , 0.0940149 , 0.04023404,
       0.04386826, 0.08925939, 0.04428928, 0.07318519, 0.05020755,
       0.06897432, 0.06664413, 0.08320122, 0.12212111, 0.04895576,
       0.03901634, 0.08400872, 0.04434074, 0.07206303, 0.04582684,
       0.06594655, 0.12048487, 0.06479337, 0.12707056, 0.14633177,
       0.05065029, 0.33047574, 0.09515819, 0.04280678, 0.05106283,
       0.06791041, 0.2184392 , 0.09801067, 0.06794996, 0.08106758,
       0.24100789, 0.06115256, 0.04674467, 0.09036588, 0.04262527,
       0.05037151, 0.1506595 , 0.03401242, 0.13512286, 0.05080725,
       0.09924342, 0.06415518, 0.09348895, 0.07187492, 0.13470108,
       0.05990394, 0.11665631, 0.08910835, 0.05105674, 0.05799578,
       0.07677022, 0.08328592, 0.18468066, 0.09024807, 0.06930978,
       0.03186343, 0.10275238, 0.06477415, 0.12851989, 0.10032173,
       0.07369386, 0.08242713, 0.10951482, 0.19121086, 0.09640539,
       0.08250756, 0.08575379, 0.11272985, 0.09614805, 0.08839341,
       0.04703294, 0.13546482, 0.09985996, 0.16486479, 0.05500489,
       0.07678173, 0.08402812, 0.07548214, 0.14356635, 0.12517373,
       0.15531867, 0.20924207, 0.07897648, 0.18454695, 0.14129097,
       0.11814056, 0.11689127])
```

In [18]:
```
student_resid = influence.resid_studentized_external
outliers = prostate[abs(student_resid) > 2]
outliers
```

Out[18]:

	Unnamed: 0	lcavol	lweight	age	lbph	svi	lcp	gleason	pgg45	lpsa
38	39	2.660959	4.0851	68	1.373716	1	1.83258	7	35	2.21375
46	47	2.727853	3.9954	79	1.879465	1	2.65676	9	100	2.56879
68	69	-0.446287	4.4085	69	-1.386294	0	-1.38629	6	0	2.96269
94	95	2.907447	3.3962	52	-1.386294	1	2.46385	7	10	5.14312
96	97	3.471967	3.9750	68	0.438255	1	2.90417	7	20	5.58293

In [19]:
```
C, P = influence.cooks_distance
ind= np.argsort(C)[-5:]
influential_points = prostate.loc[ind]
influential_points
```

Out[19]:

	Unnamed: 0	lcavol	lweight	age	lbph	svi	lcp	gleason	pgg45	lpsa
96	97	3.471967	3.9750	68	0.438255	1	2.90417	7	20	5.58293
94	95	2.907447	3.3962	52	-1.386294	1	2.46385	7	10	5.14312
68	69	-0.446287	4.4085	69	-1.386294	0	-1.38629	6	0	2.96269
46	47	2.727853	3.9954	79	1.879465	1	2.65676	9	100	2.56879
31	32	0.182322	6.1076	65	1.704748	0	-1.38629	6	0	2.00821

```
use stepwise regression to remove nonsignificant predictors from the model
one at a time until all predictors have p value below a certain
threshhold. Start with 8 factors in this case(lcavol, lweight, age, svi,
lbph, lcp, gleason, pgg45). We remove gleason first since the model output
indicates that gleason is the least siginificant one. Repeat the process
until all remaining predictors are significant.
```

---

3. For the swiss data, fit a model with Fertility as the response and the other variables as predictors. Answer the questions as in 1.

In [23]:
```
swiss = pd.read_csv('swiss.csv')
X = swiss[["Agriculture","Examination","Education","Catholic","Infant_Morta
y = swiss['Fertility']
```

In [24]:
```
X_c = sm.add_constant(X)
model = sm.OLS(y, X_c).fit()
y_pred = model.predict(X_c)
model.summary()
```

Out[24]:

OLS Regression Results

Dep. Variable:	Fertility	**R-squared:**	0.707
Model:	OLS	**Adj. R-squared:**	0.671
Method:	Least Squares	**F-statistic:**	19.76
Date:	Fri, 06 May 2022	**Prob (F-statistic):**	5.59e-10
Time:	09:36:37	**Log-Likelihood:**	-156.04
No. Observations:	47	**AIC:**	324.1
Df Residuals:	41	**BIC:**	335.2
Df Model:	5		
Covariance Type:	nonrobust		

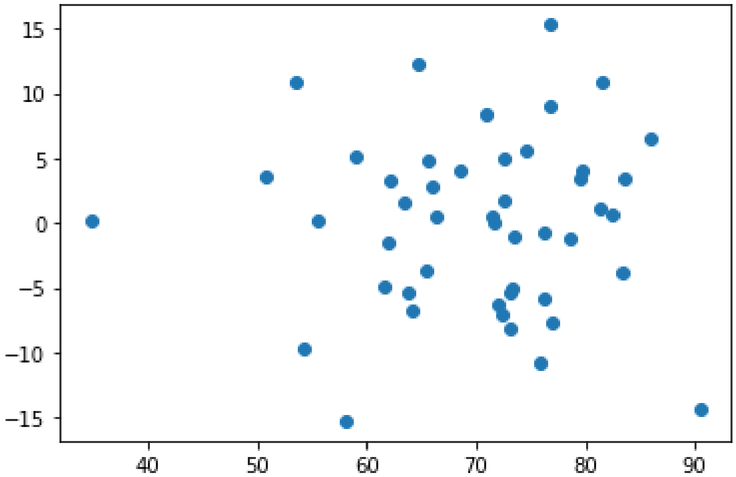
	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
const	66.9152	10.706	6.250	0.000	45.294	88.536
Agriculture	-0.1721	0.070	-2.448	0.019	-0.314	-0.030
Examination	-0.2580	0.254	-1.016	0.315	-0.771	0.255
Education	-0.8709	0.183	-4.758	0.000	-1.241	-0.501
Catholic	0.1041	0.035	2.953	0.005	0.033	0.175
Infant_Mortality	1.0770	0.382	2.822	0.007	0.306	1.848

Omnibus:	0.058	**Durbin-Watson:**	1.454
Prob(Omnibus):	0.971	**Jarque-Bera (JB):**	0.155
Skew:	-0.077	**Prob(JB):**	0.925
Kurtosis:	2.764	**Cond. No.**	807.

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

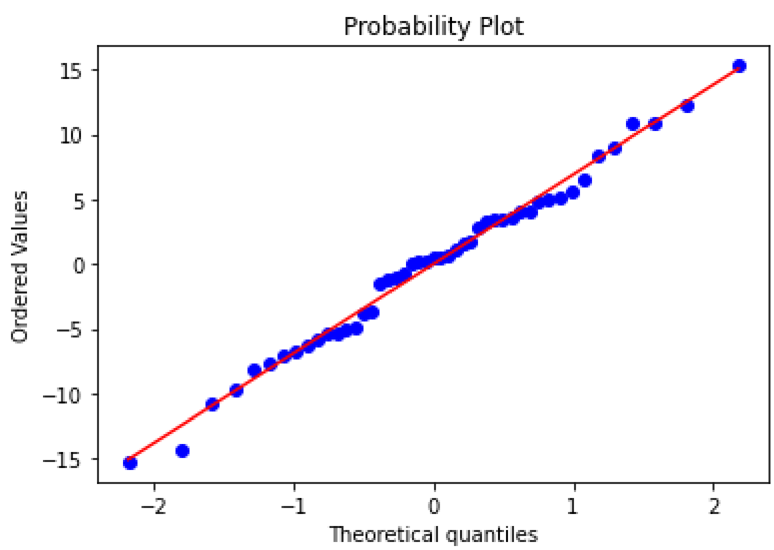
In [25]:
```
error = (y-y_pred).to_numpy()

plt.scatter(y_pred,error)
```

Out[25]: <matplotlib.collections.PathCollection at 0x7fbf42ad1850>



the constant variance assumption for the errors is met

In [26]:
```
stats.probplot(error, dist="norm", plot=plt)
plt.show()
```



The approximately straight line verifies the normality for the errors.

In [27]:
```
influence = model.get_influence()
leverage = influence.hat_matrix_diag
#point with high leverage = (2k+2)/n = 12/47
high_leverage = swiss[abs(leverage) > (12/47)]
high_leverage
```

Out[27]:

	Unnamed: 0	Fertility	Agriculture	Examination	Education	Catholic	Infant_Mortality
18	La Vallee	54.3	15.2	31	20	2.15	10.8
44	V. De Geneve	35.0	1.2	37	53	42.34	18.0

leverages

In [28]:
```
leverage
```

Out[28]:
```
array([0.15681744, 0.12258494, 0.17368296, 0.07961648, 0.07219003,
       0.19833238, 0.14308241, 0.14145774, 0.07994004, 0.10682273,
       0.13676942, 0.08319314, 0.08392561, 0.10990923, 0.12551221,
       0.106312  , 0.06853488, 0.10175031, 0.3512078 , 0.11137547,
       0.07425778, 0.08277061, 0.06410496, 0.1092143 , 0.10036242,
       0.12569646, 0.18059095, 0.07905125, 0.0532821 , 0.07706245,
       0.1733592 , 0.09204746, 0.1083415 , 0.09800616, 0.07675938,
       0.09177212, 0.1424621 , 0.081257  , 0.07683116, 0.22629672,
       0.09981583, 0.20532241, 0.07366702, 0.17219053, 0.45583631,
       0.21067014, 0.11595449])
```

In [29]:
```
student_resid = influence.resid_studentized_external
outliers = swiss[abs(student_resid) > 2]
outliers
```

Out[29]:

	Unnamed: 0	Fertility	Agriculture	Examination	Education	Catholic	Infant_Mortality
5	Porrentruy	76.1	35.3	9	7	90.57	26.6
36	Sierre	92.2	84.6	3	3	99.46	16.3
46	Rive Gauche	42.8	27.7	22	29	58.33	19.3

In [30]:
```
C, P = influence.cooks_distance
ind= np.argsort(C)[-5:]
influential_points = swiss.loc[ind]
influential_points
```

Out[30]:

	Unnamed: 0	Fertility	Agriculture	Examination	Education	Catholic	Infant_Mortality
45	Rive Droite	44.7	46.6	16	29	50.43	18.2
46	Rive Gauche	42.8	27.7	22	29	58.33	19.3
41	Neuchatel	64.4	17.6	35	32	16.92	23.0
36	Sierre	92.2	84.6	3	3	99.46	16.3
5	Porrentruy	76.1	35.3	9	7	90.57	26.6

use stepwise regression to remove nonsignificant predictors from the model one at a time until all predictors have p value below a certain threshhold. Start with 5 factors in this case(Agriculture, Examination, Education, Catholic, Infant Mortality). We remove infant mortality first since the model output indicates that infant mortality is the least siginificant one. Repeat the process until all remaining predictors are significant.

---

4. For divusa, data fit a model with divorce as the response and the other variables, except year as predictors. Check for serial correlation.

In [33]:
```
divusa = pd.read_csv('divusa.csv')
X = divusa[["unemployed","femlab","marriage","birth","military"]]
y = divusa['divorce']
```

In [34]:
```
X_c = sm.add_constant(X)
model = sm.OLS(y, X_c).fit()
y_pred = model.predict(X_c)
model.summary()
```

No. Observations:	77	AIC:	301.4
Df Residuals:	71	BIC:	315.5
Df Model:	5		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	2.4878	3.394	0.733	0.466	-4.279	9.255
unemployed	-0.1113	0.056	-1.989	0.051	-0.223	0.000
femlab	0.3836	0.031	12.543	0.000	0.323	0.445
marriage	0.1187	0.024	4.861	0.000	0.070	0.167
birth	-0.1300	0.016	-8.333	0.000	-0.161	-0.099
military	-0.0267	0.014	-1.876	0.065	-0.055	0.002

Omnibus:	0.098	Durbin-Watson:	0.300

Check for serial correlation

In [35]:
```
from statsmodels.stats.stattools import durbin_watson
durbin_watson(model.resid)
```

Out[35]: 0.29988344695130026

Since the dw statistics = 0.29988 is below 2, this means there is positive autocorrelation.

---

5. To show how two statistics that summarize how well a regression model fits, the F-ratio, and R2, the coefficient of determination relate: a. Write down R^2 in terms of both Error SSE and Regression SSR.

$$R^2 = \frac{SSR}{SST} = \frac{SSR}{SSR + SSE}$$

b. Write down F-ratio in terms of Error SS, Regression SS, k, and n.

$$F - ratio = \frac{MS_{regression}}{MS_{within}} = \frac{\frac{SS_{regression}}{k}}{\frac{SS_{error}}{n-k-1}} = (\frac{n-k-1}{k})\frac{SSR}{SSE}$$

c. Establish the algebraic relationship

$$1 - R^2 = \frac{SSE}{SSR + SSE}$$

$$\frac{R^2}{1 - R^2}\frac{n - (k+1)}{k} = (\frac{n-k-1}{k})\frac{\frac{SSR}{SSR+SSE}}{\frac{SSE}{SSR+SSE}} = (\frac{n-k-1}{k})\frac{SSR}{SSE} = F - ratio$$

d. Suppose that n = 40, k = 5, and R2 = 0.20. Calculate the FRatio. Perform the usual test of model adequacy to determine whether the five explanatory variables jointly and significantly affect the response variable.

In [41]:
```
n = 40
k = 5
R2 = 0.2
dfn = k-1
dfd = n-k
F_ratio = (R2/(1-R2))*((n-k-1)/k)
print('F_ratio: ', F_ratio)
```

```
F_ratio:  1.7
```

In [42]:
```

crit = stats.f.ppf(q=1-0.05, dfn= dfn, dfd= dfd)
p_value = 1- stats.f.cdf(F_ratio, dfn=dfn, dfd=dfd)
print('critical value: ', crit)
print('p-value: ', p_value)
```

```
critical value:  2.641465186128566
p-value:  0.17207414000385024
```

the F-ratio= 1.7 smaller than the critical value 2.64, and the p-value > 0.05 we fail to reject the null hypothesis that the five explanatory variables jointly does not significantly affect the response variable.

e. Suppose that n = 400 (not 40), k = 5, and R2 = 0.20. Calculate the FRatio. Perform the usual test of model adequacy to determine whether the five explanatory variables jointly and significantly affect the response variable.

In [44]:
```
n = 400
k = 5
R2 = 0.2
dfn = k-1
dfd = n-k
F_ratio = (R2/(1-R2))*((n-k-1)/k)
print('F_ratio: ', F_ratio)
crit = stats.f.ppf(q=1-0.05, dfn= dfn, dfd= dfd)
p_value = 1- stats.f.cdf(F_ratio, dfn=dfn, dfd=dfd)
print('critical value: ', crit)
print('p-value: ', p_value)
```

```
F_ratio:  19.7
critical value:  2.3945328958915275
p-value:  8.43769498715119e-15
```

The F-ratio = 19.7 > the critical value 2.39, and the p-value < 0.05. We can reject the null hypothesis that the five explanatory variables jointly does not significantly affect the response variable and conclude that the five explanatory variables jointly do significantly affect the response variable.

In [ ]: