18.03 Problems Worksheet 42

1. Consider the system $x' = 2x - 3xy$

$y' = -y + 2xy$

(i) Find the critical points

(ii) Linearize at each critical point. Say what this implies about the nonlinear system.

(iii) On the xy-plane show each critical point and its linear approximation.

(iv) Tie the points in (iii) together into a phase portrait of the system.

Note one of the critical points is not structurally stable so you will have to consider several possibilities for this type.

2. In Problem 1 of sheet 41 we worked with the system

$x' = 3x - x^2 - xy$

$y' = y - y^2 + xy.$

a) Which is the predator? Which is the prey?

b) If there is no y $(y = 0)$. What is the model for x? Analyze it; what is it called?

c) We sketched this yesterday, does the linearized system match the non-linear one?