Quiz-2

Quiz 2 for Regular Students

1.
(i) Define when a square matrix with entries in $\mathbb{C}$ is said to be diagonalizable.

(ii) Is it true that every square matrix with entries in $\mathbb{C}$ is diagonalizable? Justify your answer.

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2.
Consider the linear map $T : \mathbb{R}^3 \to \mathbb{R}^4$ defined by $T(\mathbf{x}) = \mathbf{A}\mathbf{x}$ for $\mathbf{x} \in \mathbb{R}^3 = \mathbb{R}^{3\times 1}$, where

$$\mathbf{A} = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 3 & -2 \\ 0 & 1 & 3 \\ 1 & -1 & 1 \end{bmatrix}.$$

Let $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ and $(\mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_3, \mathbf{f}_4)$ denote the standard bases of $\mathbb{R}^3$ and $\mathbb{R}^4$ respectively. Also, let a and b denote the last two digits of your roll number (for example, if your roll number is 200010059, then $a = 5$ and $b = 9$). Find the matrix of T with respect to the ordered basis $(\mathbf{e}_3, a\,\mathbf{e}_1 + 2\,\mathbf{e}_2 + b\,\mathbf{e}_3, \mathbf{e}_1)$ of $\mathbb{R}^3$ and the ordered basis $(\mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_3, \mathbf{f}_4)$ of $\mathbb{R}^4$.

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3.

Let a and b denote the last two digits of your roll number (for example, if your roll number is 200010059, then $a = 5$ and $b = 9$). Consider the matrix

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 3 & 4 & a & b \end{bmatrix}.$$

$\lambda^4 - b\lambda^3 - a\lambda^2 - 4\lambda - 3 = 0$

Calculate the characteristic polynomial of $\mathbf{A}$.

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4.

Let a and b denote the last two digits of your roll number (for example, if your roll number is 200010059, then $a = 5$ and $b = 9$). Consider the vectors

$$\mathbf{v}_1 = [2\ 0\ 0]^\top, \quad \mathbf{v}_2 = [a\ 3\ 0]^\top, \quad \mathbf{v}_3 = [b\ 2\ 1]^\top$$

in $\mathbb{R}^3 = \mathbb{R}^{3\times 1}$ and let V be the subspace of $\mathbb{R}^3$ spanned by $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$. Use the Gram-Schmidt orthonormalization process to find an orthonormal basis of V.

e_1, e_2, e_3

Marks: 2

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