MA 106 Tutorial 2 Solutions

D1 T5

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QUESTION 1 QUESTION 2 QUESTION 5 QUESTION 6





Row1 Pivot1 = 1
Swap
$$R_2$$
 and R_3

$$R_2 := R_2 - R_1$$

Row2 Pivot2 =
$$-1$$

$$\left[\begin{array}{ccccc}
1 & 2 & 1 & 1 \\
1 & 1 & 2 & 0 \\
0 & 0 & 1 & -1
\end{array}\right]$$

$$\left[\begin{array}{ccccccc}
1 & 2 & 1 & 1 \\
0 & -1 & 1 & -1 \\
0 & 0 & 1 & -1
\end{array}\right]$$

QUESTION 1(continue)

$$R_2 := R_2/(-1)$$

$$R_1:=R_1-2R_2$$

$$\begin{bmatrix} 7 & 2 & 1 & 1 \\ 0 & 7 & -1 & 1 \\ 0 & 0 & 7 & -1 \end{bmatrix}$$

$$\begin{bmatrix}
1 & 0 & 3 & -1 \\
0 & 1 & -1 & 1 \\
0 & 0 & 1 & -1
\end{bmatrix}$$

QUESTION 1(continue)

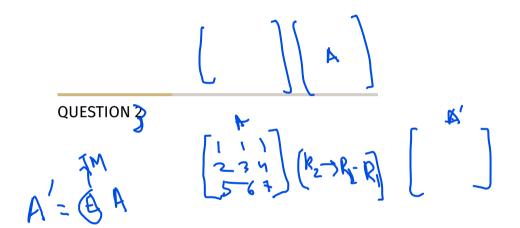
$$R_1 := R_1 - 3R_3$$

$$\left[\begin{array}{ccccc}
1 & 0 & 0 & 2 \\
0 & 1 & -1 & 1 \\
0 & 0 & 1 & -1
\end{array}\right]$$

$$R_2 := R_2 + R_3$$

$$\left[\begin{array}{ccccc}
1 & 0 & 0 & 2 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & -1
\end{array}\right]$$

Above Matrix is the row canonical form of the given Matrix.





QUESTION 2(i)

Each row operation is represented by E_i matrices. Let's take E_1, E_2,E_k be elementary row transformation matrix such that $E = E_1 E_2E_k I$ so we get

$$A'=E_1E_2.....E_kA$$

Finally

$$\mathsf{A}' = \mathsf{E}\mathsf{A}$$



QUESTION 2(ii)

Earlier we got to know that $E_i = E_1 E_2 \dots E_k I$, here we can see that E_i are elementary matrices which are invertible and hence the product of all such E_i are invertible. We can get the inverse by

$$E^{-1} = (E_1 E_2 E_k)^{-1}$$

$$E^{-1} = E_k^{-1} E_{k-1}^{-1} E_1^{-1}$$

Think how can you prove part3 on the basis of first part and second part

QUESTION 2(iii)

A square matrix ${\bf A}$ is invertible if and only if you can row reduce ${\bf A}$ to an identity matrix ${\bf I}$

Let's take the forward case so we have been given matrix is invertible .So on performing k row operations we obtain I

$$\begin{aligned} E_1 E_2 E_k A &= I \\ A &= E_k^{-1} E_{k-1}^{-1} E_1^{-1} \end{aligned}$$

Hence its proved



QUESTION 2(iii)

Lets take the backward case so we have been given that A is represented a product of elementary matrices

$$A=E_1E_2.....E_k$$

we claim that inverse exist. Think (Why?)

$$A^{-1} = (E_1 E_2 E_k)^{-1}$$

$$A^{-1} = E_k^{-1} E_{k-1}^{-1} E_1^{-1}$$



QUESTION 3 2







Let $S = [v_1, v_2, ...v_s]$. Since $S \subset T$ let $T = [v_1, v_2, ...v_s, u_1, u_2, ...u_t]$. Now suppose if S is **Linearly dependant** then $\exists \alpha_1, \alpha_2 ... \alpha_s$ such that $\alpha_1 v_1 + \alpha_2 v_2 ... + \alpha_s v_s = 0$ and not all α_i are zero. Now let $\beta_1 v_1 + \beta_2 v_2 + ... + \beta_s v_s + \beta_{s+1} u_1 + \beta_{s+2} u_2 + ... \beta_{s+t} u_t = 0$. Put $\beta_{s+i} = 0$ where $i \geq 1$ and $\beta_i = \alpha_i$ for $i \leq s$. So this tuple value of β isnt zero hence T is **Linearly dependant**.



If T is Linearly independant then the only solution for

 $\beta_1 v_1 + \beta_2 v_2 + ... + \beta_s v_s + \beta_{s+1} u_1 + \beta_{s+2} u_2 + ... \beta_{s+t} u_t = 0$ is $\beta_i = 0$. Suppose if S is **Linearly dependant** then it means $\exists \alpha_1, \alpha_2 ... \alpha_s$ such that $\alpha_1 v_1 + \alpha_2 v_2 ... + \alpha_s v_s = 0$. Sp put $\beta_i = \alpha_i$ for $i \leq s$ and $\beta_{s+i} = 0$. This tuple satisfies the above equation yet $\beta \neq 0$. So this contradicts that T is Linearly independant. Hence S is **Linearly independant**

Think about the converse? Is it true or false?









OUESTION 6

$$c_1a_1 + c_2a_2 + ...c_ia_i + ...c_ja_j... + c_sa_s = 0$$
. Since these vectors are linearly independent, $\forall_k c_k = 0$. Now consider

$$\beta_1 a_1 + \beta_2 a_2 + ... \beta_i (a_i + \alpha a_j) + ... \beta_j a_j ... + \beta_s a_s = 0.$$
 So

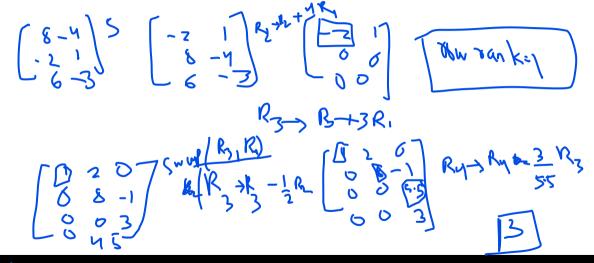
$$\beta_1 a_1 + \beta_2 a_2 + ... \beta_i a_i + ... (\beta_j + \beta_i \alpha) a_j ... + \beta_s a_s = 0.$$
 So
$$\beta_1 = \beta_2 = ... \beta_i ... = \beta_s = 0, \beta_i + \alpha \beta_i = 0.$$
 Hence $\forall_k \beta_k = 0.$ So this set of vectors is

also linearly independent.

(B'-c)), ... (b!-c) vi -- (P!+bx-c), ... (A 2-42), Eo









QUESTIONS?

Contact me via 190100051@iitb.ac.in THANK YOU

