Indian Institute of Technology Bombay

MA 106 Linear Algebra

Spring 2021 SRG/DP

Common Quiz 1

Date: March 24, 2021 Max. Marks: 10
Time: 8.30 AM – 9.15 AM

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1. Consider the 4×5 matrix

$$A = \begin{bmatrix} 1 & 0 & 2 & 1 & a \\ 1 & 1 & 5 & 2 & b \\ 1 & 2 & 8 & 4 & 12 \\ 3 & 4 & 18 & 8 & 27 \end{bmatrix},$$

where a and b denote the last two digits of your roll number (e.g., if your roll number is 200010059, then $a = 5$ and $b = 9$). Determine:

(i) The row canonical form of A.

(ii) The nullity of A. [3 marks]

2. Let $r_1, \ldots, r_6$ denote the last six digits of your roll number (so that r_6 is the last digit, r_5 the second last digit, and so on). Consider the matrices

$$\mathbf{a} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \end{bmatrix}, \quad \mathbf{b} = [r_6\ r_5\ r_4\ r_3\ r_2\ r_1] \quad \text{and} \quad \mathbf{A} = \mathbf{ab}.$$

of sizes 6×1, 1×6, and 6×6, respectively. Compute the rank of $\mathbf{A}$ and write down a basis for the column space of $\mathbf{A}$. [3 marks]

3. Let V denote the subspace of $\mathbb{R}^{1\times 4}$ spanned by $\mathbf{a}_1, \mathbf{a}_2$ and $\mathbf{a}_3$, where

$$\mathbf{a}_1 = [-1\ \ 0\ \ 1\ \ 2], \quad \mathbf{a}_2 = [3\ \ 4\ \ -2\ \ 5], \quad \text{and} \quad \mathbf{a}_3 = [1\ \ 4\ \ 0\ \ 9].$$

Find the dimension of V. Further let $\mathbf{A}$ be the 3×4 matrix whose row vectors given by $\mathbf{a}_1, \mathbf{a}_2$ and $\mathbf{a}_3$, and let $\mathbf{c}$ be the 3×1 column vector

$$\mathbf{c} = \begin{bmatrix} 0 \\ a \\ b \end{bmatrix},$$

where a and b denote the last two digits of your roll number (e.g., if your roll number is 200010059, then $a = 5$ and $b = 9$). Then determine if the linear system $\mathbf{Ax} = \mathbf{c}$ has (i) no solution, (ii) unique solution, or (iii) infinitely many solutions. [4 marks]