MA 106
Tutorial 2 Solutions

D1 T5

GYANDEV GUPTA

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IIT BOMBAY

QUESTION 1

QUESTION 2

QUESTION 3

QUESTION 4

QUESTION 5

QUESTION 6

QUESTION 7

QUESTION 1

QUESTION 1

Row1 Pivot1 = 1

Swap R_2 and R_3

$$\begin{bmatrix} 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$R_2 := R_2 - R_1$

$$\begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & -1 & 1 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

Row2 Pivot2 = -1

QUESTION 1(continue)

$R_2 := R_2/(-1)$

$$\begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$R_1 := R_1 - 2R_2$

$$\begin{bmatrix} 1 & 0 & 3 & -1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

Row3 Pivot3= 1

QUESTION 1(continue)

$R_1 := R_1 - 3R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$R_2 := R_2 + R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

Above Matrix is the row canonical form of the given Matrix.

$$\begin{bmatrix} & & \end{bmatrix}\begin{bmatrix} & A & \end{bmatrix}$$

QUESTION 2

$$\begin{array}{c} A \\ \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix} \end{array} \xrightarrow{(R_2 \to R_2 - R_1)} \begin{array}{c} A' \\ \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \end{array}$$

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$$A' = E\,A$$

QUESTION 2(i)

Each row operation is represented by E_i matrices. Let's take $E_1, E_2, ...E_k$ be elementary row transformation matrix such that $E = \underline{E_1E_2.....E_kI}$ so we get

$$A' = E_1E_2.....E_kA$$

Finally

$$\boxed{A' = EA}$$

$E_k \cdots E_2 E_1 A = I$

Earlier we got to know that $\boxed{\mathbf{E}} = \mathbf{E_1E_2.....E_kI}$, here we can see that E_i are elementary matrices which are invertible and hence the product of all such $\mathbf{E_i}$ are invertible. We can get the inverse by

$$\mathbf{E}^{-1} = (\mathbf{E_1E_2.....E_k})^{-1}$$

$$\mathbf{E}^{-1} = \mathbf{E}_k^{-1}\mathbf{E}_{k-1}^{-1}.....\mathbf{E}_1^{-1}$$

Think how can you prove part3 on the basis of first part and second part

QUESTION 2(iii)

A square matrix **A** is invertible if and only if you can row reduce **A** to an identity matrix **I**
Let's take the forward case so we have been given matrix is invertible .So on performing k row operations we obtain **I**

$$E_1E_2.....E_kA = \mathbf{I}$$

$$\boxed{A = E_k^{-1}E_{k-1}^{-1}.....E_1^{-1}}$$

Hence its proved

Lets take the backward case so we have been given that A is represented a product of elementary matrices

$$A = E_1 E_2 .....E_k$$

we claim that inverse exist. Think (Why ?)

$$A^{-1} = (E_1 E_2 .....E_k)^{-1}$$

$$A^{-1} = E_k^{-1} E_{k-1}^{-1} .....E_1^{-1}$$

QUESTION ~~3~~ 2

QUESTION 3

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right]$$

QUESTION 4

QUESTION 4

Let $S = [v_1, v_2, ...v_s]$. Since $S \subset T$ let $T = [v_1, v_2, ...v_s, u_1, u_2, ..u_t]$. Now suppose if S is **Linearly dependant** then $\exists \alpha_1, \alpha_2 ... \alpha_s$ such that $\alpha_1 v_1 + \alpha_2 v_2 ... + \alpha_s v_s = 0$ and not all α_i are zero. Now let $\beta_1 v_1 + \beta_2 v_2 + .. + \beta_s v_s + \beta_{s+1} u_1 + \beta_{s+2} u_2 + ... \beta_{s+t} u_t = 0$. Put $\beta_{s+i} = 0$ where $i \geq 1$ and $\beta_i = \alpha_i$ for $i \leq s$. So this tuple value of β isnt zero hence T is **Linearly dependant.**

QUESTION 4

$\beta_{s+1}, \beta_{s+2}, \beta_{s+t} \neq 0$

If T is Linearly independant then the only solution for $\beta_1 v_1 + \beta_2 v_2 + .. + \beta_s v_s + \beta_{s+1} u_1 + \beta_{s+2} u_2 + ... \beta_{s+t} u_t = 0$ is $\beta_i = 0$. Suppose if S is **Linearly dependant** then it means $\exists \alpha_1, \alpha_2 ... \alpha_s$ such that $\alpha_1 v_1 + \alpha_2 v_2 ... + \alpha_s v_s = 0$. Sp put $\beta_i = \alpha_i$ for $i \leq s$ and $\beta_{s+i} = 0$. This tuple satisfies the above equation yet $\beta \neq 0$. So this contradicts that T is Linearly independant. Hence S is **Linearly independant**

Think about the converse? Is it true or false?

QUESTION 5

QUESTION 5

QUESTION 6

QUESTION 6

$c_1a_1 + c_2a_2 + ...c_ia_i + ..c_ja_j... + c_sa_s = 0$. Since these vectors are linearly independant, $\forall_k\ c_k = 0$. Now consider
$\beta_1a_1 + \beta_2a_2 + ...\beta_i(a_i + \alpha a_j) + ..\beta_ja_j... + \beta_sa_s = 0$. So
$\beta_1a_1 + \beta_2a_2 + ...\beta_ia_i + ..(\beta_j + \beta_i\alpha)a_j... + \beta_sa_s = 0$. So
$\beta_1 = \beta_2 = ..\beta i.. = \beta_s = 0, \beta_j + \alpha\beta_i = 0$. Hence $\forall_k \beta_k = 0$. So this set of vectors is also linearly independant.

$(\beta_1 - c_1)a_1 \dots (\beta_i - c_i)a_i \dots (\beta_j + \beta_i\alpha - c_j)a_j \dots (\beta_s - c_s)a_s = 0$

QUESTION 7

$$\begin{bmatrix} 8 & -4 \\ -2 & 1 \\ 6 & -3 \end{bmatrix} \quad S \quad \begin{bmatrix} -2 & 1 \\ 8 & -4 \\ 6 & -3 \end{bmatrix} \xrightarrow{R_2 \to R_2 + 4R_1} \begin{bmatrix} \boxed{-2} & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\boxed{\text{Row rank} = 1}$$

$$R_3 \to R_3 + 3R_1$$

$$\begin{bmatrix} \boxed{1} & 2 & 0 \\ 0 & 8 & -1 \\ 0 & 0 & 3 \\ 0 & 4 & 5 \end{bmatrix} \quad \text{Swap}(R_3, R_4) \quad R_3 \to R_3 - \frac{1}{2}R_2 \quad \begin{bmatrix} \boxed{1} & 2 & 0 \\ 0 & \boxed{8} & -1 \\ 0 & 0 & \boxed{5.5} \\ 0 & 0 & 3 \end{bmatrix} \quad R_4 \to R_4 - \frac{3}{5.5}R_3$$

$$\boxed{3}$$

Questions?

Contact me via 190100051@iitb.ac.in

Thank You