MA 108
Tutorial 3 Solutions

D1 T5

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QUESTION 3.9

QUESTION 3.10

QUESTION 3.11

QUESTION 3.12

QUESTION 4.1

QUESTION 4.4

QUESTION 3.9

QUESTION 3.9.ix

We know y = $y_{hs} + y_{ps}$

Put y=e^{mx} in homogeneous part of eqn, we get

$$m^3 - m = 0$$

$$m = 0, 1, -1$$

$$y_{hs} = c_1e^{0x} + c_2e^{x} + c_3e^{-x}$$

Assume the $y_{ps} = (ax^3 + bx^2 + cx + d)e^x$, put this in ODE

$$(6a - 3ax^2 - 2bx - c)e^x = 2x^2e^x$$

$\implies$ a=-2/3 and c=-4 and b=0 where d is a free variable we can take it to be zero

$$y_{ps} = -(2/3)x^3 - 4x$$

$$y = c_1 + c_2e^x + c_3e^{-x} - (2/3)x^3 - 4x$$

QUESTION 3.9.x

We know y = $y_{hs} + y_{ps}$
Put y=e^{mx} in homogeneous part of eqn, we get

$$m^3 - 5m^2 + 8m - 4 = 0$$

$$m = 1, 2, 2$$

$$y_{hs} = c_1 e^{1x} + c_2 e^{2x} + c_3 x e^{2x}$$

Assume the $y_{ps} = (acosx + bsinx)e^x$, put this in ODE... cosx $\mapsto c, sinx \mapsto s$

$$(as - bc) - 5(-ac - bs) + 8(-as + bc) - 4(ac + bs) = 2c$$

QUESTION 3.9.x

$\implies$ 7b+a=2 and b=7a gives a=1/25 and b=7/25

$$y_{ps} = (\frac{1}{25}cosx + \frac{7}{25}sinx)e^x$$

$$y = c_1e^{1x} + c_2e^{2x} + c_3xe^{2x} + (\frac{1}{25}cosx + \frac{7}{25}sinx)e^x$$

QUESTION 3.10

QUESTION 3.10.iii

We know y = $y_{hs} + y_{ps}$

Put y=e^{mx} in homogeneous part of eqn, we get

$$m^2 - 4m + 3 = 0$$

$$m = 1, 3$$

$$y_{hs} = c_1 e^{1x} + c_2 e^{3x}$$

Assume the $y_{ps} = ae^{3x}$, put this in ODE...

$$(9a - 4 * 3a + 3a)e^{3x} = 4e^{3x} = 0$$

which is absurd

Lets take $y_{ps} = axe^{3x}$, put this in ODE...

QUESTION 3.10.iii

$$3a(3x+1)+3a-4a(3x+1)-4a+3ax=4$$

$\implies$ a=2

$$y_{ps}=2xe^{3x}$$

$$y=c_1e^{1x}+c_2e^{3x}+2xe^{3x}$$

Use the ICs y(0)=-1 and y'(0)=3 we get

-1 = c_1+c_2 and 3 = c_1+3c_2 +2 $\implies$ $c_1=-2$ and $c_2=1$

$$y=-2e^x+(1+2x)e^{3x}$$

QUESTION 3.11

QUESTION 3.11.iv

You can take the particular solution of the form

$$y_{ps} = (ax + b)[csin(x/\sqrt{2}) + dcos(x/\sqrt{2}]e^{x/\sqrt{2}}$$

QUESTION 3.12

QUESTION 3.12

You can find homogeneous solution by putting $y = x^m$

$$4m^2 + 4m + 1 = 0$$

$$m_1 = -1/2 = m_2$$

$$y_{hs} = c_1 x^{m_1} + c_2 lnx x^{m_2} = \frac{c_1 + c_2 lnx}{\sqrt{x}}$$

Divide the equations by x^2, we get $y'' + 2y'/x + y/4x^2 = 1/x^2\sqrt{x}$
So $r(x) = 1/x^2\sqrt{x}$ now what we can do ??

One can use method of variation of parameter to find particular solution

$$y_{ps} = -y_1 \int \frac{y_2 r(x)}{W(y_1, y_2)} + y_2 \int \frac{y_1 r(x)}{W(y_1, y_2)}$$

where $y_1 = x^{-1/2}$, $y_2 = lnx x^{-1/2}$
Or if one can guess $y_{ps} = A(lnx)^2/\sqrt{x}$ with A=1/2

QUESTION 4.1

QUESTION 4.1.ii

Put y=e^{mx} in homogeneous part of eqn, we get

$$m^2 + 1 = 0 \implies m = \pm i$$

$$y_{hs} = c_1 e^{ix} + c_2 e^{-ix} = acosx + bsinx$$

we get $y_1 = cosx$ and $y_2 = sinx$ and r(x) = tanx
$W(y_1, y_2) = y_1 y_2' - y_2 y_1' = 1$

$$y_{ps} = -y_1 \int \frac{y_2 r(x)}{W(y_1, y_2)} + y_2 \int \frac{y_1 r(x)}{W(y_1, y_2)}$$

$$y_{ps} = -cosx \int \frac{sinxtanxdx}{1} + sinx \int \frac{cosxtanxdx}{1}$$

QUESTION 4.1.ii

$$y_{ps} = -cosx \int \frac{sin^2xdx}{cosx} - sinxcosx$$

$$y_{ps} = -cosx \int \frac{1 - cos^2xdx}{cosx} - sinxcosx$$

$$y_{ps} = -cosx(ln(|secx + tanx|) - sinx) - sinxcosx$$

$$y_{ps} = -cosx(ln|secx + tanx|)$$

QUESTION 4.4

QUESTION 4.4.v

Refer to the result given in Q.3 or lecture slides

$y_2(x) = \psi(x)y_1(x)$ where $\psi(x) = \int_a^x \frac{exp[-\int_a^t p(u)du]dt}{y_1^2(t)}$

Here p(x) = $\frac{-x}{1-x^2}$

$$\psi(x) = \int_a^x \frac{t(1-t^2)^{-1/2}dt}{1-t^2}$$

$$\psi(x) = \int_a^x \frac{tdt}{(1-t^2)^{3/2}} = \frac{1}{(1-x^2)^{1/2}}$$

Refer to the result given in Q.3 or lecture slides

$y_2(x) = \psi(x)y_1(x)$ where $\psi(x) = \int_a^x \frac{exp[-\int_a^t p(u)du]dt}{y_1^2(t)}$

Here p(x) = $\frac{-xsinx}{sinx-xcosx}$

$$\psi(x) = \int_a^x \frac{(sint - tcost)dt}{t^2}$$

$$\psi(x) = \frac{-sinx}{x}$$

QUESTIONS?

Contact me via 190100051@iitb.ac.in

THANK YOU