MA 108
Tutorial 4 Solutions

D1 T5

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QUESTION 4.1

QUESTION 4.7

QUESTION 4.10

QUESTION 4.11

QUESTION 4.14

QUESTION 5.1

QUESTION 5.7

QUESTION 4.1

You can find homogeneous solution by putting $y = x^m$

$$m^2 - 2m = 0$$

$$m_1 = 0, m_2 = 2$$

$$y_{hs} = c_1 x^{m_1} + c_2 x^{m_2} = c_1 + c_2 x^2$$

Divide the equations by x, we get $y'' - y'/x = (x+3)x^2 e^x$
So $r(x) = (x+3)x^2 e^x$ now what we can do ??

By method of variation of parameter to find particular solution

$$y_{ps} = -y_1 \int \frac{y_2 r(x)}{W(y_1, y_2)} + y_2 \int \frac{y_1 r(x)}{W(y_1, y_2)}$$

where $y_1 = 1$, $y_2 = x^2$ and $W(y_1, y_2) = 2x$

$$y_{ps} = \frac{x^2 e^x (x^2 - 1)}{2}$$

QUESTION 4.7

Observation: if y_1, y_2 are two solutions of a non homogeneous linear equation, then $y_1 - y_2$ is a solution of the homogeneous part.
Applying this here we get $y_1 - y_2$, $y_2 - y_3$ are solutions of the homogeneous part.
One can check that these two solutions are independent (by computing the Wronskian, for instance or otherwise).
Therefore, the general solution of the given DE may be written as

$$c_1(y_1 - y_2) + c_2(y_2 - y_3) + y_1$$

$$c_1(e^{x^2}(1 - x)) + c_2(2 - e^{x^2}) + 1 + e^{x^2}$$

QUESTION 4.10

QUESTION 4.10.i

For homogenoeus part (complementary fn) : $y_{hs} = c_1 e^{ix} + c_2 e^{-ix} + c_3 x e^{ix} + c_4 x e^{-ix}$
For particular solution:(Use of annhilator I have shown you can also solve via basic too)

The annihilator for r(x) = sin x is $A = D^2 + 1$

$$\implies AL(y) = (D^2+1)(D^4+2D^2+1) = (D^2+1)^3$$

$$\implies D = \pm i, repeated`thrice$$

The general solution of AL(y) = 0 is

$$c_1e^{ix} + c_2e^{-ix} + c_3xe^{ix} + c_4xe^{-ix} + c_5x^2e^{ix} + c_6x^2e^{-ix}$$

Here the first four terms are annihilated, so we have a solution of the form $c_5x^2e^{ix} + c_6x^2e^{-ix}$ or $ax^2sinx + bx^2cosx$
Plugging this in the ODE, toget the values of a and b by comparing coefficients we get a=-1/8 and b=0

QUESTION 4.11

You can find homogeneous solution by putting $y = x^m$

$$m^4 + 2m^3 + 3m^2 + 2m + 1 = 0$$

$$m^4 + m^2 + 1 + 2m^3 + 2m^2 + 2m = 0 = (m^2 + m + 1)^2$$

$$m_1 = w, m_2 = w, m_3 = \overline{w}, m_4 = \overline{w}$$

$$y_{hs} = c_1 x^{m_1} + c_2 lnx x^{m_2} + c_3 x^{m_3} + c_4 lnx x^{m_4} = c_1 x^{w} + c_2 lnx x^{w} + c_3 x^{\overline{w}} + c_4 lnx x^{\overline{w}}$$

Now take the particular soln to be of form $y_{ps} = ax^3$
Put this in ODE we get a=1/121
So

$$y = c_1 x^w + c_2 lnxx^w + c_3 x^{\overline{w}} + c_4 lnxx^{\overline{w}} + x^3/121$$

Thanks to Vaibhav for pointing out :)

QUESTION 4.14

QUESTION 4.14

One of the way to prove, we know it would be a second order homogeneous DE in general form (GDE) as $y'' + p(x)y' + q(x)y = 0$

Lets have $y_1 = x^2e^x$ and $y_2 = x^3e^x$

The wronskian for this is $W(y_1, y_2) = x^4e^{2x}$

So $W = e^{-\int pdx}$ we get p(x) = -2-4/x

Substituting y_1 or y_2 in GDE we get $q(x) = 1 + 6/x^2$

Any constant coefficient homogeneous differential equation with x^2e^x as a solution must also have e^x and xe^x as solutions.

Hence it cannot have them as solution

QUESTION 5.1

QUESTION 5.1.iv

Assume a to be *real*[1]

Extra info : [The result is true for complex a but the proof given does not go through. One must use Cauchy's integral formula to deform the path of the integral into the complex domain. The result however may be used even for complex a].

If you don't know the bracket part you can skip it not needed

Put t(s + a) = u in the defining integral,

$$\mathcal{L}(t^2 e^{-at}) = \int_0^\infty t^2 e^{-t(s+a)} dt = \frac{1}{(s+a)^3} \int_0^\infty u^2 e^{-u} du = \frac{2}{(s+a)^3}$$

QUESTION 5.1.iv

More generally for complex a and b the following holds:

$$\mathcal{L}(t^a e^{-bt}) = \frac{\Gamma(a+1)}{(s+b)^{a+1}}$$

,

QUESTION 5.7

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$$\mathcal{L}(f) = \sum_{n=1}^{\infty} \int_{n-1}^{n} ne^{-st}dt = \frac{e^s - 1}{s} \sum_{1}^{\infty} ne^{-ns} = \frac{e^s}{s(e^s - 1)}$$

Questions?

Contact me via 190100051@iitb.ac.in

Thank You