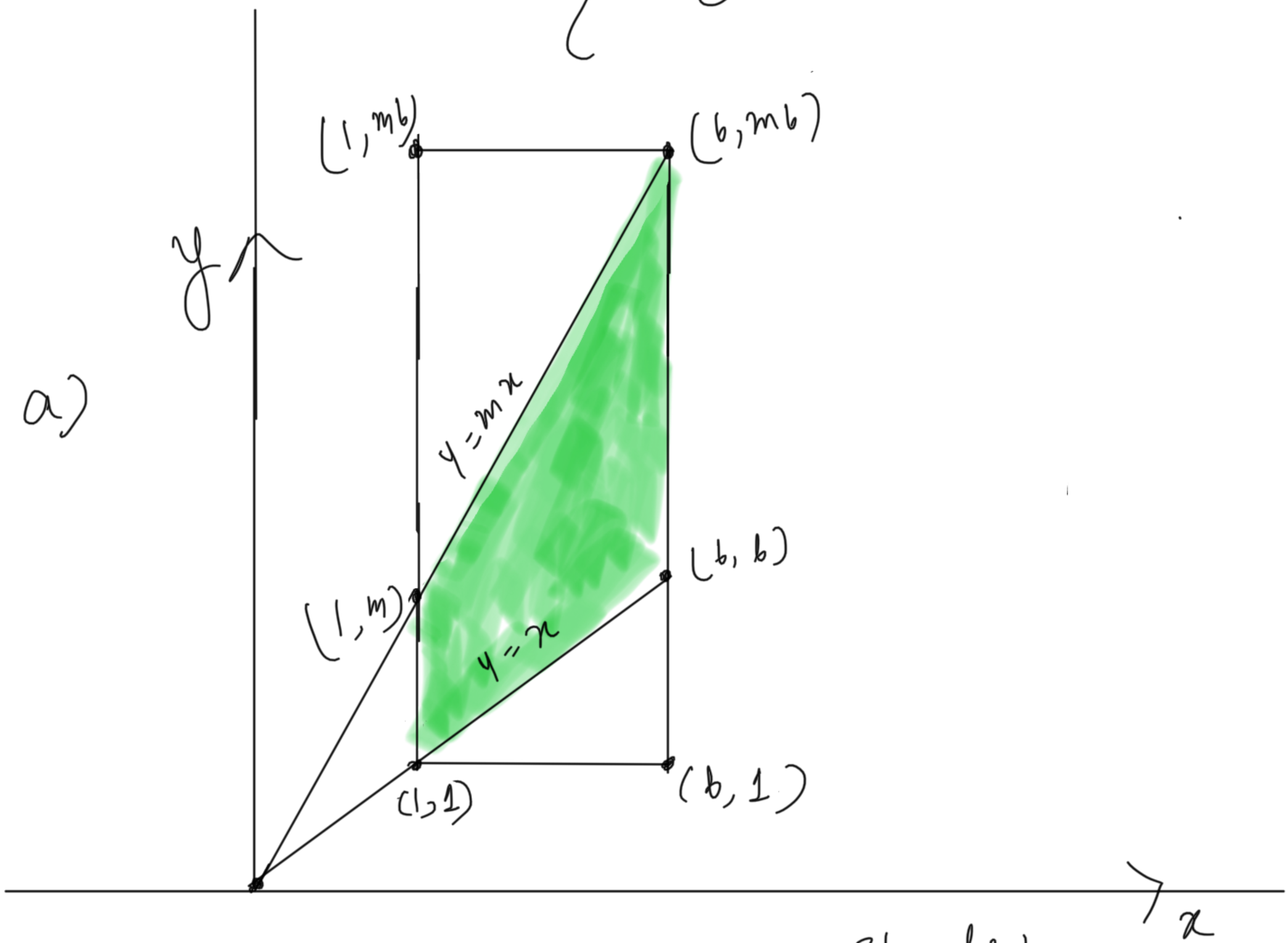
Quiz 5th Feb (grading) :-

$R = [1, b] \times [1, mb]$, with $1 < m$.

Qn 1.

$$f(x,y) = \begin{cases} \dfrac{1}{(x+y)^2}, & x \le y \le m.x \\ 0 & \text{otherwise.} \end{cases}$$

a)



Ans: f is non-zero at green-shaded region: $1 \le x \le b$, $x \le y \le mx$.

Grading: If students identify the region bdd by $y = x$ & $y = mx$, give $\boxed{2 \text{ marks.}}$

- If they donot lebex end pts or intersecting pts, donot need to cut marks.
- If there is still further confusion, keep the copy separate and let me know.

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b) f is integrable over $R = [1, b] \times [1, mb]$ because

- f is bounded.
- f can have discontinuity only along lines $y = x$ and $y = mx$.

So, the set of discontinuity of f is of "Content zero".

[For students, it is enough if they mention that the discontinuity of f can occur only along two st lines $y = x$ and $y = mx$, [or along the graphs of continuous fun^c s.)]

So, f is integrable.

Grading: • If any gives the full ans, give $\boxed{\text{2 marks.}}$

• If anyone gives partial ans. (i.e, mention bdd, but not clear about discontinuity) give $\boxed{\text{1 marks.}}$

To compute: $\iint_R f(x,y)\, dx\, dy$

Fubini: (Since f is integrable ... exist).

Applying Fubini's ... iterated integrals also ...
and iterated integrals

$$\iint_R f(x,y)\,dx\,dy = \int_{x=1}^{b}\left(\int_{y=x}^{mx} f(x,y)\,dy\right)dx.$$

$$= \int_{x=1}^{b}\left(\int_{y=x}^{mx} \frac{1}{(x+y)^2}\,dy\right)dx$$

Grading: For writing ~~the~~ above steps, give $\boxed{2 \text{ marks.}}$

Even, if they donot mention the Fubini, but write the iterated integral correctly give $\boxed{\text{2 marks (no need to cut marks)}}$

$$= \int_{x=1}^{b} \frac{(x+y)^{-2+1}}{-2+1}\Bigg|_{y=x}^{mx} dx$$

$$\int^{b} \Big[\quad \frac{1}{\quad} \Big] dx$$

$$= \int_{x=1} \left[\frac{1}{2x} - \frac{1}{(m+1)x} \right]$$

$$= \left[\frac{1}{2} \log x \Big|_{x=1}^{6} - \frac{1}{m+1} \log x \Big|_{x=1}^{6} \right]$$

$$= \frac{1}{2} \log 6 - \frac{1}{m+1} \log 6$$

$$= \frac{m+1-2}{2(m+1)} \log 6$$

$$= \frac{m-1}{2(m+1)} \log 6.$$

Grading: For finishing the computation correctly, give

$\boxed{1 \text{ marks.}}$

So on 1:-

$2+2+2+1.$

Qn 2. Compute:

$$I = \int_{x=0}^{\sqrt{b}} \int_{y=0}^{x} \int_{z=0}^{b-x^2} \frac{\sin 2z}{b-z} \, dz \, dy \, dx.$$

- For, $b = \pi/2$, and $b = \pi$,

$$f(x,y,z) = \frac{\sin 2z}{b-z} \quad \text{is bounded}$$

and Continuous over

$$W = \left\{ (x,y,z) \in \mathbb{R}^3 : \begin{array}{l} 0 \le x \le \sqrt{b}, \\ 0 \le y \le x, \quad 0 \le z \le b - x^2 \end{array} \right\}$$

So integrable over W.

Also, it can be observed that all iterated integrals exist.

So, changing the order of integration:- $0 \le z \le b$,

$$0 \le x \le \sqrt{b-z},$$

$$0 \le y \le x,$$

$$I = \int_{z=0}^{b} \left(\int_{x=0}^{\sqrt{b-z}} \left(\int_{y=0}^{x} \frac{\sin 2z}{(b-z)}\, dy \right) dx \right) dz$$

$$= \int_{z=0}^{b} \left(\int_{x=0}^{\sqrt{b-z}} x \cdot \frac{\sin 2z}{(b-z)}\, dx \right) dz$$

$$= \int_{z=0}^{b} \frac{\sin 2z}{b-z} \cdot \frac{x^2}{2} \Bigg|_{x=0}^{\sqrt{b-z}} dz$$

$$= \frac{1}{2} \int_{z=0}^{b} \frac{\sin(2z)}{\cancel{b-z}} \cdot \cancel{(b-z)}\, dz$$

$$= -\frac{1}{2} \cdot \frac{\cos(2z)}{2} \Bigg|_{z=0}^{b}$$

$$= -\frac{1}{4} \left[\cos(2b) - 1 \right]$$

$$= \frac{1}{4} \left[1 - \cos 2b \right].$$

Grading :-

- For writing the iterated integral after changing the order of integration and obtaining the upper and lower limits for y, x, z variables correctly, give $\boxed{4 \text{ marks}}$.
(Do not need to bother if they justify Fubini or not).

However, If anyone writes the iterated integral ~~wrongly~~ but mention that they are using Fubini/ or some justification given, give $\boxed{2 \text{ marks}}$.

- Then computing iterated integral correctly give $\boxed{2 \text{ marks}}$.

in any

If they do mistakes in σ step, just cut $\boxed{1 \text{ mark}}$.

So grading $\boxed{4+2}$.

4 marks for writing iterated integral correctly and 2 marks for computation.

- For $b=1$, and $b=1/2$, f is not bdd and not-triple integrable. and Fubini may not be applicable.

However, some of the iterated integrals exist.

For this quiz, if any one (without any justification), do the computation with changing the integration and the order of the

Find the value correctly for

$$\int_{z=0}^{b}\left(\int_{x=0}^{\sqrt{1-z}}\left(\int_{y=0}^{x} f(x,y,z)\,dy\right)dx\right)dz,$$

We shall give full marks.

- If they do mistake to write the upper / lower limit of integration or do mistake in computation, the grading rules are same as mentioned for $b = \pi, \pi/2$ above.
- If they mention that the func is unbounded and do not proceed further, then also we give $\boxed{6 \text{ marks}}$.

cases

Please, keep these cases

separately and forward to

me.