(Q)1 $I = \iint_R x\cos xy \, dx\,dy$

$$I_1 = \int_0^{12\pi}\underbrace{\int_0^{2} x\cos xy \, dy}\,dx$$

$\forall x \in [0, 12\pi]$

$\int_0^2 x\cos xy\,dy$ exists

($\because$ the function $g(x) = k\cos kx$ is continuous and Bounded for finite k)

$\therefore$ By Fubini's Theorem

I exists and $I = I_1$

$$I = \int_0^{12\pi}\int_0^2 x\cos xy \, dy\,dx$$

$$= \int_0^{12\pi} \cancel{x}\,\frac{\sin xy}{\cancel{x}}\Big|_0^2 dx$$

$$= \int_0^{12\pi} \sin 2x \, dx$$

$$= 0$$

(Q)2 :- $f(x,y) = x + xye^y \geq 0$
$\forall (x,y) \in [2,7] \times [0,1]$

$$\iint_R f(x,y)\,dx\,dy$$

gives volume enclosed between planes
$x=2, x=7, y=0, y=1, z=0$ & $z = x + xye^y$

$$\therefore \text{volume} = \int_0^1\int_2^7 x + xye^y\,dx\,dy$$

$$= \int_0^1 \frac{x^2}{2}(1+ye^y)\Big|_2^7\,dy$$

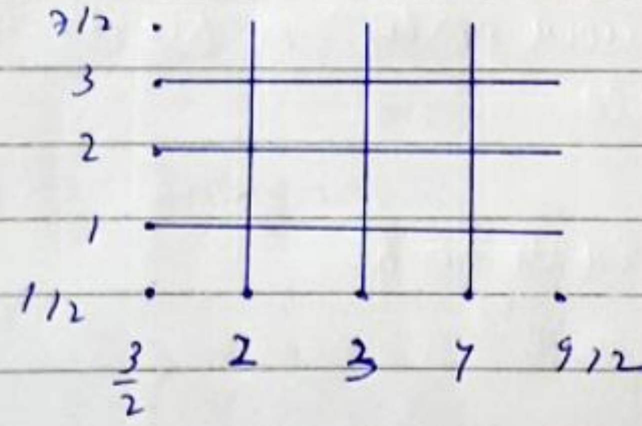
$$= \frac{45}{2}\int_0^1 1+ye^y\,dy$$

$$= \frac{45}{2}\left[1 + ye^y\Big|_0^1 - e^y\Big|_0^1\right]$$

$$= \frac{45}{2}\left[1 + e - (e-1)\right]$$

$$= 45$$

(Q) 3 $R = [3/2, 9/2] \times [1/2, 7/2]$



$f(x, y) = 1$ when $x \in \mathbb{Z}$ or $y \in \mathbb{Z}$
$= 0$ otherwise

$\therefore$ $f(x, y)$ is 1 on any point lying on Blue Lines.
$\because$ Every Point on the Blue Line is Discontinuous and there a Infinitely many such Points

$\therefore$ $f(x, y)$ is discontinuous on infinite set of Points in R.

$\forall (x,y) \in R$

$0 \le f(x,y) < 1$

$\therefore$ f is a Bounded function on R

$\because$ f is Bounded and Continuous Except on a finite number of graphs of continuous function

Namely

$$\left.\begin{array}{l} x = 2 \\ x = 3 \\ x = 4 \end{array}\right\} y \in [1/2, 7/2]$$

$$\left.\begin{array}{l} y = 1 \\ y = 2 \\ y = 3 \end{array}\right\} x \in [3/2, 9/2]$$

$\therefore$ f is Integrable.

$$\cancel{I_1 = \int_c^d \int_a^b f(x,y)\,dx\,dy} \qquad I_2 = \int_a^b \int_c^d f(x,y)\,dy\,dx$$

Suppose we fix $x = x_1$

if $x_1 \in Z$

$$f(x_1, y) = 1 \quad \forall y [c,d]$$

A Constant function

Hence Integrable.

if $x_1 \notin Z$

$$f(x_1, y) = 1 \quad \text{if } y \in \{1, 2, 3\}$$
$$= 0 \quad \text{otherwise}$$

$\because$ It is discontinuous only at 3 (finite) Points

Hence Integrable

$\therefore I_2$ exists on R

$$I_1 = \int_c^d \int_a^b f(x,y)\,dx\,dy$$

Suppose we fix $y = y_1$

if $y_1 \in \mathbb{Z}$

$$f(x, y_1) = 1 \quad \forall x \in [a,b]$$

constant function

Hence Integrable

if $y_1 \notin \mathbb{Z}$

$$f(x,y_1) = \begin{cases} 1 & \text{if } x \in \{2,3,4\} \\ 0 & \text{otherwise} \end{cases}$$

$\therefore$ it is discontinuous only on finite (3) points

it is integrable

$\therefore I_2$ exists on R