MA111 Tutorial Sheet 4 – Final Answer Key

1. (a) open, path-connected, simply connected
   (c) path-connected, simply connected
   (d) open, path-connected
2. not conservative
3. $4/e$
4. not path-independent
5. $C_1(t) = (t\pi, 0)$ for $t \in [0,1]$; $C_2(t) = (t\pi/2, 0)$ for $t \in [0,1]$
6. (a) $f = ye^{xy} + K$
   (b) $f = ye^x + x\sin(y) + K$
   (c) $f = x^2y + xy^{-2} + K$
7. (a) $f(x,y,z) = x^2yz - \cos(x)$; $\displaystyle\int_{\mathbf{c}} \mathbf{F}\cdot\mathbf{ds} = 0$
   (b) $f(x,y) = xe^{xy}$; $\displaystyle\int_{\mathbf{c}} \mathbf{F}\cdot\mathbf{ds} = -1$
   (c) $f(x,y,z) = xyz + z^2$; $\displaystyle\int_{\mathbf{c}} \mathbf{F}\cdot\mathbf{ds} = 77$
8. $\phi(x,y,z) = x^2y + xz^3$; $\displaystyle\oint_C \mathbf{v}\cdot d\underline{s} = 0$
9. (b) 2π
   (c) not conservative
10. Hint: $\mathbf{F} = \nabla g$, where $g(r,\phi,\theta) = \displaystyle\int_0^r sf(s)\,ds$