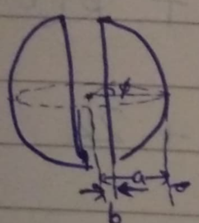
Short Quiz 3

1. $W : \{(x,y,z) \mid x^2+y^2+z^2 \le a^2 \text{ and } x^2+y^2 \ge b^2\}$

Use Cylindrical Coordinates $x = \rho\cos\phi$; $y = \rho\sin\phi$; $z = z$ so that

$$J_{xyz}(\rho,\phi,z) = \rho$$

$$I = \iiint_W \frac{1}{\sqrt{3}\pi}\,dx\,dy\,dz = \iiint_{W'} \frac{1}{\sqrt{3}\pi}\,|J_{xyz}(\rho,\phi,z)|\,d\rho\,d\phi\,dz$$



Clearly $\rho \in [b, a]$

For some given ρ,

$$x^2+y^2+z^2 \le a^2 \Rightarrow z^2 \le a^2-\rho^2$$

$$\Rightarrow z \in \left[-\sqrt{a^2-\rho^2}, \sqrt{a^2-\rho^2}\right]$$

$\phi \in [0, 2\pi]$

$$\therefore I = \int_0^{2\pi}\int_b^a\int_{-\sqrt{a^2-\rho^2}}^{\sqrt{a^2-\rho^2}} \frac{\rho}{\sqrt{3}\pi}\,dz\,d\rho\,d\phi$$

$$\Rightarrow I = \int_0^{2\pi}\int_b^a \frac{2\rho\sqrt{a^2-\rho^2}}{\sqrt{3}\,\pi}\,dz\,d\rho\,d\phi$$

$$\Rightarrow I = \int_0^{2\pi} \frac{2}{3\sqrt{3}}\frac{(a^2-b^2)^{3/2}}{\pi}\,d\phi \Rightarrow \boxed{I = \frac{4}{3\sqrt{3}}(a^2-b^2)^{3/2}}$$

$a^2 = 1$	$b^2 = 1/4$	$\Rightarrow I = 0.5$
$a^2 = 4$	$b^2 = 1$	$\Rightarrow I = 4$
$a^2 = 16$	$b^2 = 4$	$\Rightarrow I = 32$

2. $\bar{S} = (1,0,0)$ to $(3,0,k\pi)$

$\bar{F} = (\pi \sin z, \ \cos\sqrt{y}, \ x/\pi)$

Parameterise $\bar{S}$ as

$\bar{s} = (2t+1, 0, kt\pi), \ t \in [0,1]$

$\Rightarrow d\bar{s} = [(2, 0, k\pi)]dt$

$\bar{F} = (\pi \sin(kt\pi), 1, \frac{2t+1}{\pi})$

$\Rightarrow \bar{F} \cdot d\bar{s} = [2\pi \sin(kt\pi) + (2t+1)k]\,dt$

$\Rightarrow \int_C \bar{F} \cdot d\bar{s} = \int_0^1 2\pi \sin(kt\pi)\,dt + \int_0^1 k(2t+1)\,dt$

$\Rightarrow \int_C \bar{F} \cdot d\bar{s} = -\frac{2}{k} \cos(kt\pi)\Big|_0^1 + k(t^2+t)\Big|_0^1$

$\Rightarrow \int_C \bar{F} \cdot d\bar{s} = -\frac{2}{k}[\cos(k\pi) - 1] + 2k$

$\Rightarrow \boxed{\int_C \bar{F} \cdot d\bar{s} = \frac{2}{k}[1 - \cos(k\pi)] + 2k}$

$\Rightarrow \boxed{\int_C \bar{F} \cdot d\bar{s} = \begin{cases} 6 & k = 1 \\ 5 & k = 1/2 \\ 4 & k = 2 \end{cases}}$