MA111 (IIT Bombay) Quiz, 5th February,2021-subjective part

Subjective: The detailed grading instructions along with the outline of answers are given in the handwritten note.

1 Qn 1

1. Let f be defined on the rectangle $R = [1, b] \times [1, mb]$ with $1 < m$ as follows:

$$f(x, y) = \begin{cases} \frac{1}{(x+y)^2} & \text{if } x \leq y \leq mx \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Sketch the region in xy plane where f is non-zero. [2]

   (b) Give a reasoning why the function f is integrable on R? Find the integral of f on R. [2+2+1]

With different values of b and m

- (Upload answer) Let f be defined on the rectangle $R = [1, 2] \times [1, 4]$ as follows:

$$f(x, y) = \begin{cases} \dfrac{1}{(x+y)^2} & \text{if } x \leq y \leq 2x \\ 0 & \text{otherwise.} \end{cases}$$

  1. Sketch the region in xy plane where f is non-zero.
  2. Give a reasoning why the function f is integrable on R? Find the integral of f on R. (Answers can be in terms of logarithm or trigonometric functions.)

  [2+2+2+1]

  Ans. $\dfrac{\log 2}{6}$.

- (Upload answer)Let f be defined on the rectangle $R = [1, 3] \times [1, 6]$ as follows:

$$f(x, y) = \begin{cases} \dfrac{1}{(x+y)^2} & \text{if } x \leq y \leq 2x \\ 0 & \text{otherwise.} \end{cases}$$

  1. Sketch the region in xy plane where f is non-zero.
  2. Give a reasoning why the function f is integrable on R? Find the integral of f on R. (Answers can be in terms of logarithm or trigonometric functions.)

  [2+2+2+1]

  Ans. $\dfrac{\log 3}{6}$

- (Upload answer)Let f be defined on the rectangle $R = [1, 2] \times [1, 6]$ as follows:

$$f(x, y) = \begin{cases} \dfrac{1}{(x+y)^2} & \text{if } x \leq y \leq 3x \\ 0 & \text{otherwise.} \end{cases}$$

  1. Sketch the region in xy plane where f is non-zero.

2. Give a reasoning why the function f is integrable on R? Find the integral of f on R. (Answers can be in terms of logarithm or trigonometric functions.)

[2+2+2+1]

Ans. $\dfrac{\log 2}{4}$

- (Upload answer)Let f be defined on the rectangle $R = [1,4] \times [1,12]$ as follows:

$$f(x,y) = \begin{cases} \dfrac{1}{(x+y)^2} & \text{if } x \le y \le 3x \\ 0 \quad \text{otherwise.} \end{cases}$$

1. Sketch the region in xy plane where f is non-zero.
2. Give a reasoning why the function f is integrable on R? Find the integral of f on R. (Answers can be in terms of logarithm or trigonometric functions.)

[2+2+2+1]

Ans. $\dfrac{\log 4}{4}$

2 Qn 2

1. Compute $\displaystyle\int_{x=0}^{\sqrt{b}} \int_{y=0}^{x} \int_{z=0}^{b-x^2} \frac{\sin 2z}{b-z} dz\ dy\ dx$, for $b > 0$. Write solution with justifications and Upload your answer. [4+2]

With different values of

- Compute $\displaystyle\int_{x=0}^{\sqrt{\frac{\pi}{2}}} \int_{y=0}^{x} \int_{z=0}^{\frac{\pi}{2}-x^2} \frac{\sin 2z}{\frac{\pi}{2}-z} dz\ dy\ dx$. Write solution with justifications and Upload your answer. [4+2]

  Ans. $\frac{1}{2}$

- Compute $\displaystyle\int_{x=0}^{\sqrt{\pi}} \int_{y=0}^{x} \int_{z=0}^{\pi-x^2} \frac{\sin 2z}{\pi-z} dz\ dy\ dx$. Write solution with justifications and Upload your answer. [4+2]

  Ans. 0.

- Compute $\displaystyle\int_{x=0}^{1} \int_{y=0}^{x} \int_{z=0}^{1-x^2} \frac{\sin 2z}{1-z} dz\ dy\ dx$. Write solution with justifications and Upload your answer. [6]

  Ans. The function is unbounded at $z = 1$, so it is not triple integrable and the Fubini theorem may not be applicable.

  However (if anyone calculates in case), check the iterated integral

$$\int_{z=0}^{1} \int_{x=0}^{\sqrt{1-z}} \int_{y=0}^{x} \frac{\sin 2z}{1-z} dz\ dy\ dx = ?$$

$\frac{1}{4}[1 - \cos 2]$

- Compute $\displaystyle\int_{x=0}^{\sqrt{\frac{1}{2}}}\int_{y=0}^{x}\int_{z=0}^{\frac{1}{2}-x^2}\frac{\sin 2z}{\frac{1}{2}-z}\;dy\;dx\;dz$. Write solution with justifications and Upload your answer. [6]

  Ans. The function is unbounded at $z = 1/2$, so it is not triple integrable and the Fubini theorem may not be applicable.

  However (if anyone calculates in case), check the iterated integral

$$\int_{z=0}^{1/2}\int_{x=0}^{\sqrt{1/2-z}}\int_{y=0}^{x}\frac{\sin 2z}{1/2-z}\;dy\;dx\;dz =?$$

  $\frac{1}{4}[1-\cos 1]$