SC 617

Quiz-Week5

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1. Define absolute continuity ? [1]

> A function f : $[a, b] \to \mathbb{R}$ is absolutely continuous on [a, b] iff, for any $\epsilon > 0$, there is a $\delta > 0$ such that
>
> $$\sum_{i=1}^{n} |f(\alpha_i) - f(\beta_i)| < \epsilon$$
>
> for any finite collection of subintervals (α_i, β_i) of [a, b] with $\sum_{i=1}^{n} |\alpha_i - \beta_i| < \delta$

2. You have $\dot{x}_1 = x_2$ and $\dot{x}_2 = -\lambda_1 x_1 - \lambda_2 x_2$
   $\lambda = [\lambda_1, \lambda_2]^T$ and $\lambda \in D$ where $D = \{(\lambda_1, \lambda_2) \in \mathbb{R}^2 | \lambda_1, \lambda_2 > 0\}$ Use integration lemma to prove λ as ULES

> $x(t_0) = x_0$, x $= [x_1, x_2]^T$
> the solution is given by x(t) $= exp(At)x(t_0)$
> here the matrix $A = [[0, 1], [-\lambda_1, -\lambda_2]]$ and matrix -A is positive definite so the matrix A is negative definite and t is a positive no. so $||exp(At)||$ is bounded above take it to be c
>
> $$||x(t)||_p = ||exp(At)x(t_0)|| \leq c||x_0||$$
>
> hence so by the integration lemma we can say that for all $x_0 \in B_r$ and all $t_0 > 0$
> λ is ULES

[1] Ref. P. Ioannou and J. Sun, Robust Adaptive Control, Upper Saddle River, NJ: Prentice Hall 1996