

# SC 617

## Quiz-Week1

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1. The statement is false,  $\lim_{t \rightarrow \infty} \dot{f}(t) = 0$  implies that  $\lim_{t \rightarrow \infty} f(t)$  converges to a finite value. Provide a function to prove the given statement wrong

**Solution:** Let's take a simple function  $f(t) = \ln(t)$ , where  $t > 0$  so  $\dot{f}(t) = \frac{1}{t}$

$$\lim_{t \rightarrow \infty} \dot{f}(t) = \lim_{t \rightarrow \infty} \frac{1}{t} = 0$$

But,

$$\lim_{t \rightarrow \infty} f(t) = \lim_{t \rightarrow \infty} \ln(t) = \infty$$

doesn't converge to a finite value

So the statement is proven to be false.

2. What is the definition of supremum ?

Let  $S$  be a subset of  $\mathbb{R}$ . An element  $M \in \mathbb{R}$  is called a supremum or a least upper bound of the set  $S$  if

- (i)  $M$  is an upper bound of  $S$ , that is,  $x \leq M$  for all  $x \in S$ , and
- (ii)  $M \leq \alpha$  for every upper bound  $\alpha$  of  $S$ .

OR

Let  $S$  be a subset of  $\mathbb{R}$ . An element  $M \in \mathbb{R}$  is called a supremum or a least upper bound of the set  $S$  if and only if  $\forall \epsilon > 0$ ,  $M - \epsilon$  is not an upper bound.