# SC 617

## Quiz-Week3

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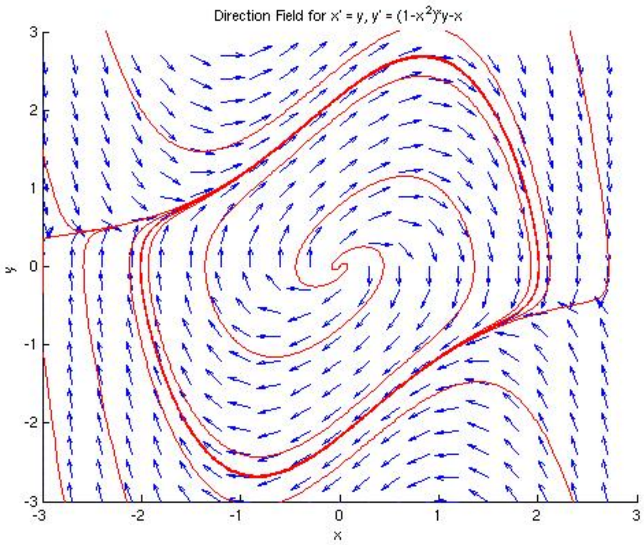
1. Discuss stability of Vander Pol Oscillator equation




Ref1.

Here the case which we have taken is for $\mu = 1$ , we know that origin is the equilibrium point. but if look the direction of fields, all trajectories are moving outside even if we distort a little from equilibrium position which is the origin here. So it's unstable

2. i. Prove ES $\implies$ UAS
   ii. Prove GES $\implies$ GUAS [1]

i. Assume $x_e = 0$
ES if there exists an $\alpha > 0$ , and for every $\epsilon > 0$ there exists a $\delta(\epsilon) > 0$ such that

$$||x(t)|| \le \epsilon e^{-\alpha(t-t_0)}, \forall t \ge t_0$$

whenever $||x_0|| < \delta(\epsilon)$.
UAS = US + UA
So, for UA, we already have $\delta$ to be independent of $t_0$

$$lim_{t\to\infty}||x(t)|| \le \epsilon * lim_{t\to\infty} e^{-\alpha(t-t_0)} = 0$$

For US, we already have $\delta$ to be independent of $t_0$

$$||x(t)|| \le \epsilon e^{-\alpha(t-t_0)} \le \epsilon$$

[1]Ref. P. Ioannou and J. Sun, Robust Adaptive Control, Upper Saddle River, NJ: Prentice Hall 1996

ii. GES if there exists an $\alpha > 0$ , and for any $\beta > 0$ there exists a $k(\beta) > 0$ such that

$$||x(t)|| \leq k(\beta)e^{-\alpha(t-t_0)}, \forall t \geq t_0$$

whenever $||x_0|| < \beta$. $\forall x_0 \in \mathbb{R}^n$
GUAS = US + GUA
For GUA, since we have $\forall t_0, x_0$ above definition

$$lim_{t\to\infty}||x(t)|| \leq k(\beta)lim_{t\to\infty}e^{-\alpha(t-t_0)} = 0$$

For US, we already have $k$ to be independent of $t_0$, choose $k(\beta) = \beta$

$$||x(t)|| \leq k(\beta)e^{-\alpha(t-t_0)} = \beta e^{-\alpha(t-t_0)} \leq \beta$$

whenever $||x_0|| \leq k(\beta)$

3. Prove the GAS for $\dot{x} = -\sigma x^3$ system [2]

Assuming $\sigma > 0$ , GAS i.e Stability + Globally Attractive
On solving the equation with $x = x_0, at, t = t_0$ we get

$$x(t) = \frac{\pm 1}{\sqrt{2\sigma(t-t_0) + \frac{1}{x_0^2}}}$$

Here the $x_e = 0$, we need to prove stability, for arbitrary $t_0$ and $\epsilon > 0$ there exists a $\delta(\epsilon, t_0)$ such that $|x_0 - x_e| = |x_0| < \delta \implies |x(t) - x_e| = |x(t)| < \epsilon, \forall t \geq t_0$
Choose $\delta = \epsilon$

$$|x(t)| = \left|\frac{\pm 1}{\sqrt{2\sigma(t-t_0) + \frac{1}{x_0^2}}}\right| \leq \left|\frac{1}{\sqrt{\frac{1}{x_0^2}}}\right| = |x_0| < \epsilon$$

To prove it as global attractive, we need to show as $t \to \infty, x(t) \to x_e = 0, \forall x_0 \in \mathbb{R}^n$

$$lim_{t\to\infty}x(t) = lim_{t\to\infty}\frac{\pm 1}{\sqrt{2\sigma(t-t_0) + \frac{1}{x_0^2}}} = 0$$

[2]Ref. P. Ioannou and J. Sun, Robust Adaptive Control, Upper Saddle River, NJ: Prentice Hall 1996