SC 617

Quiz-Week6

Gyandev Satyaram Gupta, 190100051

1. For a stable linear system $\dot{x} = Ax + \phi(t)$ where A is hurwitz and $lim_{t\to\infty}\phi(t) = 0$, exponentially so prove that $x(t) \to 0$? [1]

For such a given system we can write the solution as,

$$x(t) = e^{(t-t_0)A}x(t_0) + \int_{t_0}^{t} e^{(t-\tau)A}\phi(\tau)d\tau$$

For a linear time-invariant system and with A being hurwitz ,and using the below bound, we have

$$||e^{(t-t_0)A}|| \leq ke^{-\lambda(t-t_0)}, \forall t \geq t_0 \geq 0, k, \&\lambda > 0$$

$$||x(t)|| \leq ke^{-\lambda(t-t_0)} + \int_{t_0}^{t} ke^{-\lambda(t-\tau)}||\phi(\tau)||d\tau$$

$$||x(t)|| \leq ke^{-\lambda(t-t_0)} + (k/\lambda)sup_{t_0\leq\tau\leq t}||\phi(\tau)||$$

Now we know that $\phi(t)$ is exponentially stable so $||\phi(t)|| \leq \gamma e^{-\beta(t-t_0)}$

$$||x(t)|| \leq lim_{t\to\infty}ke^{-\lambda(t-t_0)} + (k/\lambda)\gamma e^{-\beta(t-t_0)}$$

Choose $\min(\lambda, \beta) = \alpha$ and combine all other coefficient as ϵ

$$||x(t)|| \leq \epsilon e^{-\alpha(t-t_0)}, \implies lim_{t\to\infty}x(t) \to 0$$

2. You have $\dot{e}_1 = e_2$ and $\dot{e}_2 = \theta^* f(x,t) + u - \ddot{r}$
$u = -k_1e_1 - k_2e_2 - \theta^* f(x,t) + \ddot{r}$
Parameter estimate , $u = -k_1e_1 - k_2e_2 - \hat{\theta} f(x,t) + \ddot{r}$
Use Signal chasing ,to prove that as $t \to \infty, e_1 \to 0, and, e_2 \to \infty$

Assume $\tilde{\theta}, \alpha$ are bounded and positive
$k_2 - \alpha > 0$
Define an Energy Functional V(t) $= \frac{(e_2+\alpha e_1)^2}{2} + \frac{\tilde{\theta}}{2\sigma} \geq 0$
On choosing $\dot{\hat{\theta}} = \sigma(e_2 + \alpha e_1)f(x,t)$

$$\dot{\mathrm{V}} = -(k_2 - \alpha)(e_2 + \alpha e_1)^2 \leq 0$$

Lets use Barbalat's Lemma,to prove that as $t \to \infty, e_1 \to 0, and, e_2 \to 0$.
Proof:
1. Since V (t) is lower bounded ($V \geq 0$) and non-increasing ($\dot{V}(t) \leq 0$), implies that $V_\infty := lim_{t\to\infty}V(t) < \infty$ (i.e. the limit exists).
2. $V(t) \leq V(0) \implies V$ is bounded and other terms are also bounded $\implies$

[1] H. K. Khalil, Nonlinear Systems, Upper Saddle River, NJ: Prentice Hall 2002

$(e_2 + \alpha e_1)$ are bounded $\implies (e_2 + \alpha e_1) \in L_\infty$.
3. $\int_0^\infty \frac{dV}{dt} dt = \int_0^\infty -(k_2 - \alpha)(e_2 + \alpha e_1)^2 dt \implies V_\infty - V(0) = \int_0^\infty -(k_2 - \alpha)(e_2 + \alpha e_1)^2 dt \implies (e_2 + \alpha e_1) \in L_2$ which means that $e_1, e_2 \in L_2$
4. Since $\dot{(e_2 + \alpha e_1)} = -k_1 e_1 - k_2 e_2 + \tilde{\theta} f(x,t) + \alpha e_2$ and assume that $\tilde{\theta} f(x,t)$ is bounded, so all terms are bounded on rhs $\implies \dot{(e_2 + \alpha e_1)}$ is bounded $\implies \dot{(e_2 + \alpha e_1)} \in L_\infty$
Now, $(e_2 + \alpha e_1) \in L_\infty \cap L_2$ and $\dot{(e_2 + \alpha e_1)} \in L_\infty \implies (e_2 + \alpha e_1)$ is Uniformly Continuous, then, using the Corollary of Barbalat's Lemma, we can say that $(e_2 + \alpha e_1) \to 0$.
5. We have $(e_2 + \alpha e_1) \to 0$ as $t \to \infty \implies (\dot{e_1} + \alpha e_1) \to 0$ and now take Laplace transform and using final value theorem, $lim_{s \to 0} s E_1(s) = lim_{s \to 0} \frac{se(0)}{s+\alpha} = 0, \implies e_1 \to 0$ and we know $(e_2 + \alpha e_1) \to 0 \implies e_2 \to 0$