Introduction to Elementary Particles

6/12/2025 (pp. 1-35)

✳ Introduction

- Experimental observations
  1. Scattering events
  2. Decays
  3. Bound states (particles stick together)

- Fast ↓

Classical mechanics	Quantum mechanics
Relativistic mechanics	Quantum field theory

Small →

Elementary particle physics

- Elementary particle physics uses relativistic concepts
  - ↳ Energy & momentum conserved, mass not always
  - ↳ e.g. $\Delta \to p + \pi$

- But also quantum mechanics
  - ↳ Uncertainty baked into particle interactions
  - ↳ Existence of antiparticles (???)

- eV: energy acquired by electron when accelerated through 1 Volt potential difference, 1.6×10^{-19} J

- Coulomb's Law in Heaviside-Lorentz system / Gaussian system
  - ↳ $F = \frac{1}{4\pi} \frac{q_1 q_2}{r^2}$ [HL]
  - ↳ $F = \frac{q_1 q_2}{r^2}$ [G]

$$\left.\right\} \quad q_{HL} = \sqrt{4\pi}\, q_G = \frac{1}{\sqrt{\epsilon_0}}\, q_{SI}$$

※ Fine-structure constant: $\alpha = \frac{e^2}{\hbar c} = \frac{1}{137.036}$

↳ e = Charge of electron in Gaussian units

*History

- What holds the nucleus together?
  - ↳ Strong force : powerful, short range
  - ↳ Yukawa: proton & neutron attracted by a field
    - ↳ Field should be quantized
    - ↳ Effects of field are possible through exchange of particles, or its quantum
    - ↳ Short range of strong force $\longrightarrow$ heavy particle?

- Leptons (light-weight): electrons, muons (μ)

"Hadrons"
- Mesons (middle-weight): Yukawa's particle - pions (π)
- Baryons (heavy-weight): protons, neutrons

- Beta decay: $A \rightarrow B + e^-$
  - ↳ Radioactive nucleus $A \rightarrow$ lighter nucleus B + electron
  - ↳ Cons. energy in CM frame

$$E = \left(\frac{m_A^2 - m_B^2 + m_C^2}{2m_A}\right)c^2$$

  - ↳ E is fixed once 3 masses are specified, but actual experiments show that E of emitted e^- varies.
  - ↳ Pauli: another particle emitted along e^- (neutrino, ν)

- Fundamental beta decay: $n \longrightarrow p^+ + e^- + \bar{\nu}$
  - ↳ We now call it the antineutrino, ν
  - ↳ N.B. n = neutron, $\bar{\nu}$ = antineutrino

- Charged pion decays: $\begin{cases} \pi^- \rightarrow \mu^- + \bar{\nu} \\ \pi^+ \rightarrow \mu^+ + \bar{\nu} \end{cases}$

- Muon decays: $\begin{cases} \mu^- \rightarrow e^- + \nu + \bar{\nu} \\ \mu^+ \rightarrow e^+ + \nu + \bar{\nu} \end{cases}$

keep reading, though...

- What distinguishes ν and $\bar{\nu}$?
  ↳ Lepton number: +1 for ν, −1 for $\bar{\nu}$
  ↳ Lepton number is conserved in physical processes

  ↳ But: $\mu^- \not\rightarrow e^- + \gamma$ is never observed, yet conserves charge & lepton number
    ↳ Different kinds of neutrinos, one for electron (ν_e) and one for muon (ν_μ)
    ↳ Muon number (L_μ): +1 for μ^- and ν_μ; −1 for μ^+ and $\bar{\nu}_\mu$
    ↳ Electron number (L_e): +1 for e^- and ν_e; −1 for e^+ and $\bar{\nu}_e$

- Conservation of lepton number:
  1. Conservation of electron number
  2. Conservation of muon number

※ Revised decays
- Beta decay: $n \rightarrow p^+ + e^- + \bar{\nu}_e$
- Pion decay: $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$
  $\pi^+ \rightarrow \mu^+ + \nu_\mu$
- Muon decay: $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$
  $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$

-

		Lepton #	Electron #	Muon #
Leptons	e^-	1	1	0
	ν_e	1	1	0
	μ^-	1	0	1
	ν_μ	1	0	1
Antileptons	e^+	−1	−1	0
	$\bar{\nu}_e$	−1	−1	0
	μ^+	−1	0	−1
	$\bar{\nu}_\mu$	−1	0	−1

• More on hadrons (mesons & baryons)

↳ Strongly interacting

↳ Kaons: $\left.\begin{array}{ll} K^0 \rightarrow \pi^+ + \pi^- & \text{(neutral)} \\ K^+ \rightarrow \pi^+ + \pi^+ + \pi^- & \text{(charged)} \end{array}\right\}$ mesons

↳ Lambda baryon: $\Lambda \rightarrow p^+ + \pi^-$ } Baryons

※ Called "strange" particles

↳ Produced copiously

↳ Decay slowly

↳ Produced by strong force, decay by weak force

↳ Produced in pairs (associated production)

※ Conservation of "strangeness": only conserved in strong interaction:

e.g. $\pi^- + p^+ \rightarrow K^+ + \Sigma^-$, K has strangeness $S = +1$

Σ has strangeness $S = -1$

Note how strange particles come in pairs.

Strangeness not conserved in weak interactions, like decay:

e.g. $\Lambda \rightarrow p^+ + \pi^-$

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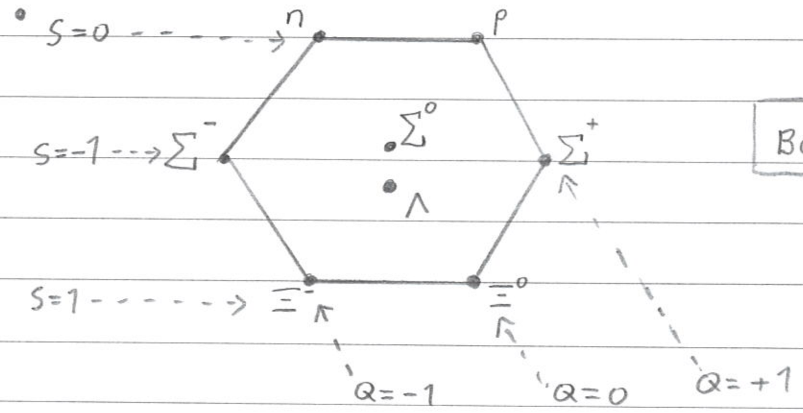
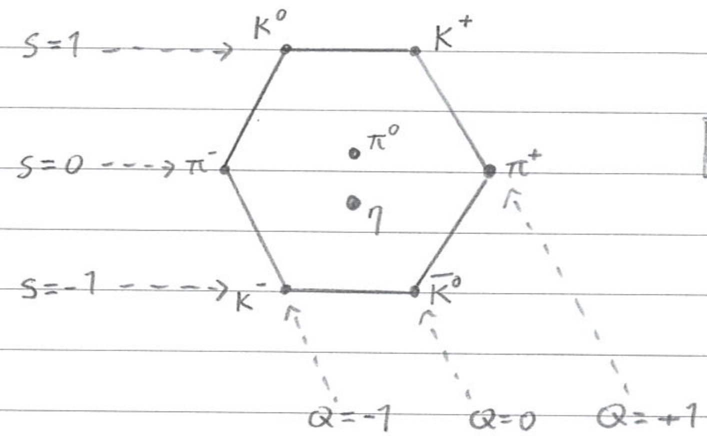
* The Eightfold way – "periodic table" of particle physics



Baryon octet

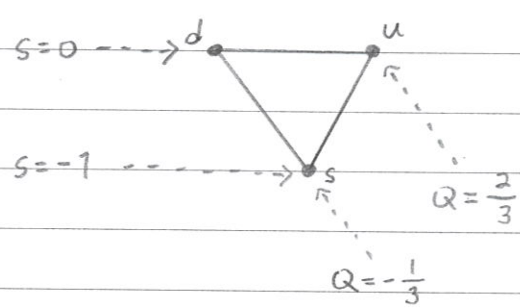


Meson octet

- Not limited to hexagons, e.g. baryon decuplet
- But why do hadrons fit into these patterns?

* The Quark Model

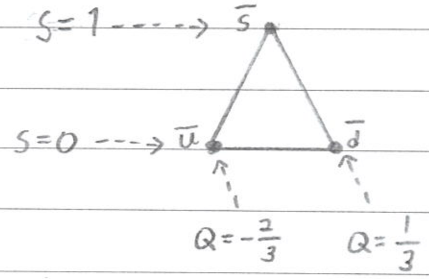
- Quarks: even more elementary constituents that make up hadrons
  - ↳ Come in 3 "flavors":



Q = Charge

S = Strangeness

- Also antiquarks, with opposite charge & strangeness



- Composition rules

  1. Every baryon = 3 quarks

     Every antibaryon = 3 antiquarks

  2. Every meson = 1 quark + 1 antiquark

- With quarks, we can reconstruct the baryon decuplet:

↳

qqq	Q	S	Baryon
uuu	2	0	Δ^{++}
uud	1	0	Δ^{+}
udd	0	0	Δ^{0}
ddd	-1	0	Δ^{-}
uus	1	-1	Σ^{*+}
uds	0	-1	Σ^{*0}
dds	-1	-1	Σ^{*-}
uss	0	-2	Ξ^{*0}
dss	-1	-2	Ξ^{*-}
sss	-1	-3	Ω^{-}

- We can also reconstruct the meson nonet:

↳

$q\bar{q}$	Q	S	Meson
$u\bar{u}$	0	0	π^0
$u\bar{d}$	1	0	π^+
$d\bar{u}$	-1	0	π^-
$d\bar{d}$	0	0	η
$u\bar{s}$	1	1	K^+
$d\bar{s}$	0	1	K^0
$s\bar{u}$	-1	-1	K^-
$s\bar{d}$	0	-1	$\bar{K}^0$
$s\bar{s}$	0	0	η'

※ 1 Problem: we've never observed a quark 

※ Doesn't this model violate Pauli exclusion principle?

↳ PEP: no 2 electrons can occupy same state. Also applies to particles of half-integer spin.

↳ Greenberg: quarks come in 3 flavors (u, d, s), but also 3 colors (red, blue, green)

$\boxed{\text{All naturally occurring particles are colorless}}$

↳ Either total of each color is 0, or each color is present in equal amounts.

↳ Explains why you can't make a particle out of 2 or 4 quarks

↳ Only combinations: $q\bar{q}$ (mesons), qqq (baryons), $\bar{q}\bar{q}\bar{q}$ (antibaryons)

* The Standard Model

- 3 Elementary Particles: Leptons, Quarks, Mediators
- Leptons:

		Q	L_e	L_μ	L_τ
First generation	e	-1	1	0	0
	ν_e	0	1	0	0
Second generation	μ	-1	0	1	0
	ν_μ	0	0	1	0
Third generation	τ	-1	0	0	1
	ν_τ	0	0	0	1

↳ Classified according to Charge (Q), electron # (L_e), muon # (L_μ), and tau # (L_τ)

↳ Antileptons have reversed signs → $\boxed{\text{total 12 leptons}}$

- Quarks:

		Q	D	U	S	C	B	T
First gen.	d	-1/3	-1	0	0	0	0	0
	u	2/3	0	1	0	0	0	0
Second gen.	s	-1/3	0	0	-1	0	0	0
	c	2/3	0	0	0	1	0	0
Third gen.	b	-1/3	0	0	0	0	-1	0
	t	2/3	0	0	0	0	0	1

↳ Classified according to charge (Q), down/up-ness (D, U), Strangeness (S), Charm (C), beauty (B), truth (T) ← $\boxed{\text{weird names}}$

↳ Signs reversed on antiquarks + each comes in 3 Colors → $\boxed{\text{total 36 quarks}}$

- Mediators:
  - ↳ For electromagnetic force: photon
  - ↳ For weak force: two W's and a Z
  - ↳ For strong force: gluons (8 exist in Standard Model)
  - ↳ For gravity: graviton (?)

※ Total 12 mediators + 1 graviton

- Final tally
  - ↳ 12 leptons
  - ↳ 36 quarks
  - ↳ 12 mediators (gravity not covered in Standard Model)

  - ↳ 1+ Higgs particles from Glashow-Weinberg-Salam Theory

※ At least 61 elementary particles

Some may be composites of even more elementary particles

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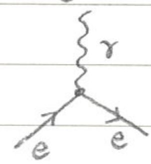
*Elementary Particle Dynamics

- The 4 forces:

Force	Strength	Theory	Mediator
Strong	10	Chromodynamics	Gluon
Electromagnetic	10^{-2}	Electrodynamics	photon
weak	10^{-13}	Flavordynamics	W and Z
Gravitational	10^{-42}	Geometrodynamics	Graviton

- Quantum Electrodynamics (QED)

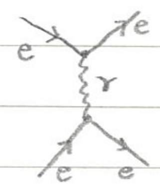
↳ All electromagnetic phenomena are reducible to



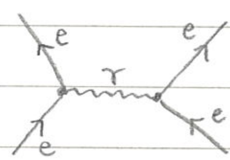
Time ⟶

Charged particle (e) enters, emits/absorbs photon, leaves

↳ Møller scattering: Coulomb repulsion of charges mediated by exchange of photon

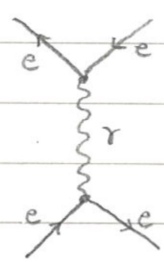


Time ⟶



Particle line going left means an antiparticle going forward.

↳ Bhabha scattering: Coulomb attraction, opposite charges



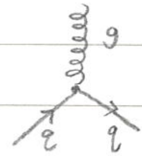
Electron & Positron go in, exchange photon → electron & Positron come out again

※ Coupling constant: $\alpha = 1/137$ by each vertex

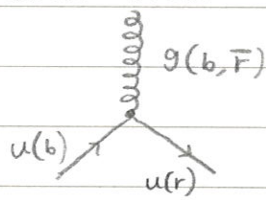
• Quantum Chromodynamics (QCD)

↳ Color plays the role of charge

↳ Fundamental process is quark → quark + gluon (cf. $e \to e + \gamma$)



↳ Color must be conserved, though



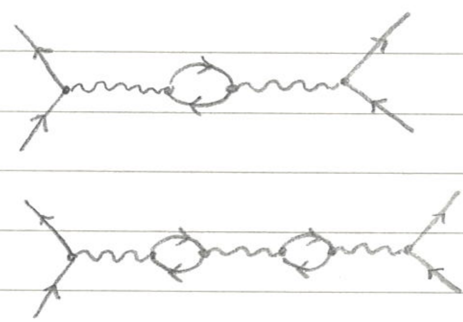
Blue up-quark → Red up-quark
Gluon carries away diff: 1 unit of blueness, minus 1 unit of redness

↳ Since gluons are bicolored, they can couple to other gluons:

※ Coupling constant for strong forces (α_s) is > 1

↳ Not really "constant" – depends on separation dist. of particles

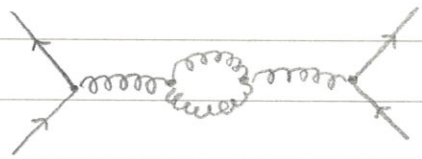
↳ In QCD, vacuum behaves like dielectric:

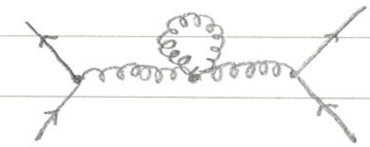


Sprouts positron/electron pairs.

↳ Virtual electron in bubble attracted to q, virtual positron repelled → vacuum polarization screens charge & reduces its field

※ Also must consider gluon-gluon vertices





↳ Quark polarization (prev.) drives α_s up at short distances, but gluon polarization drives it down.

$$\boxed{a \equiv 2f - 11n}$$

↳ Quark polarization: depends on # of flavors (f)

↳ Gluon polarization: depends on # of colors (n)

$$\left.\begin{array}{l}\text{↳ If } a<0\text{: effective coupling decreases} \\ \text{If } a>0\text{: eff. coupling increases}\end{array}\right\} \text{ at short dist.}$$

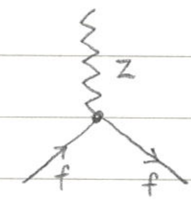
↳ In Standard Model, $f=6$, $n=3$, $a=-21$

QCD coupling decreases at short dist.

- Weak Interactions

1. Neutral (mediated by Z)

↳ Fundamental vertex:

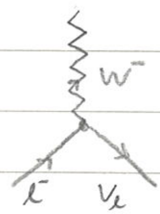


$f =$ any lepton or quark

2. Charged (mediated by Ws)

↳ Change flavor

↳ Fundamental charged vertex:



$\ell^- \to \nu_\ell + W^-$

Emission of W^-/absorption of W^+

※ What about quarks?

↳ Leptonic weak vertices connect members of same generation (e^- and ν_e ; μ^- and μ^- e.g.)

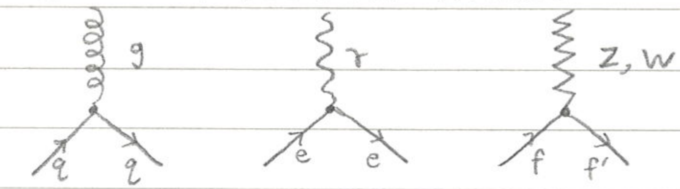
↳ Quark vertices also mostly operate within each gen. But not always, e.g. $\Lambda \to p^+ + \pi^-$ (change strangeness)

↳ Solution: instead of pairing $\binom{u}{d}, \binom{c}{s}, \binom{t}{b}$, the weak force couples $\binom{u}{d'}, \binom{c}{s'}, \binom{t}{b'}$, given by

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

✳ What's conserved?

- Follows from fundamental vertices



- Conserved:

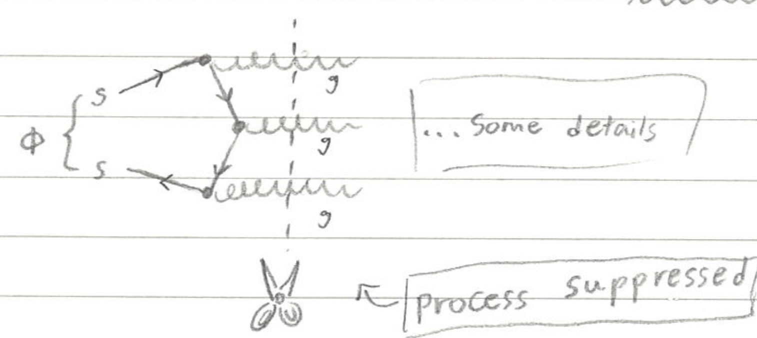
1. Charge (in weak interactions, W carries away difference)
2. Color (in strong interactions, gluon carries away diff.)
3. Baryon number
4. Lepton number

5. Flavor (?)

↳ Not conserved at a weak vertex, but weak forces are very weak indeed, so flavor is approximately conserved.

✂ The OZI rule: if diagram can be cut in 2 by slicing only gluon lines, the process is suppressed.

↳ e.g.



Introduction to Elementary Particles

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*Relativistic Kinematics

- Skimming a bit
- Lorentz transformations

  i. $x' = \gamma(x - vt)$ iii. $z' = z$

  ii. $y' = y$ iv. $t' = \gamma\left(t - \frac{v}{c^2}x\right)$

  where $\gamma = \dfrac{1}{\sqrt{1 - v^2/c^2}}$

- Consequences

  1. Relativity of simultaneity: events simultaneous in one frame aren't in others
  2. Lorentz contraction: moving obj. is shortened, $L = L'/\gamma$
  3. Time dilation: moving clocks run slow, $t = \gamma t'$
  4. Velocity addition: $u = \dfrac{u' + v}{1 + (u'v/c^2)}$

     ↳ u = speed in S, u' = speed in S', v = speed of S' rel. to S

- With 4-vectors, $x^0 = ct$, $x^1 = x$, $x^2 = y$, $x^3 = z$

  ↳ $x^{\mu'} = \sum_{\nu=0}^{3} \Lambda^{\mu}_{\nu} x^{\nu}$, $\mu = 0, 1, 2, 3$

  ↳ $\Lambda = \begin{bmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

  ↳ with Einstein's summation convention, $x^{\mu'} = \Lambda^{\mu}_{\nu} x^{\nu}$

- The metric: $g = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

  ↳ The invariant (under Lorentz transformations):

$$I = \sum_{\mu=0}^{3} \sum_{\nu=0}^{3} g_{\mu\nu} x^{\mu} x^{\nu} = g_{\mu\nu} x^{\mu} x^{\nu}$$

↳ Let the covariant 4-vector be $x_\mu \equiv g_{\mu\nu} x^\nu$

And the original contravariant 4-vector x^μ

Then $\boxed{I = x_\mu x^\mu}$

- How to transform:

↳ $a^{\mu'} = \Lambda^\mu_\nu a^\nu$

↳ $\left.\begin{array}{l} a_\mu = g_{\mu\nu} a^\nu \\ a^\mu = g^{\mu\nu} a_\nu \end{array}\right\}$

- The scalar product: $a \cdot b \equiv a_\mu b^\mu = a^0 b^0 - a^1 b^1 - a^2 b^2 - a^3 b^3$

↳ If $a^2 > 0$, a^μ is timelike

$a^2 < 0$, a^μ is spacelike

$a^2 = 0$, a^μ is light like

- Tensors

↳ A 2nd-rank tensor ($s^{\mu\nu}$) has 2 indices, $4^2 = 16$ components,

transforms w/ 2 factors of Λ: $s^{\mu\nu'} = \Lambda^\mu_\kappa \Lambda^\nu_\sigma s^{\kappa\sigma}$

↳ Similarly for 3rd-rank ($t^{\mu\nu\lambda}$): $t^{\mu\nu\lambda'} = \Lambda^\mu_\kappa \Lambda^\nu_\sigma \Lambda^\lambda_\tau t^{\kappa\sigma\tau}$

- Momentum & energy
  - ↳ Proper time: measured by clock at rest rel. to observer

$$d\tau = \frac{dt}{\gamma}$$

  - ↳ Proper velocity: $\eta \equiv \frac{dx}{d\tau} = \gamma v$

$$\eta^\mu = \gamma(c, v_x, v_y, v_z)$$

  - ↳ Relativistic momentum: $p \equiv m\eta$

$$p^\mu = m\eta^\mu = \left(\frac{E}{c}, p_x, p_y, p_z\right)$$

  - ↳ Relativistic energy: $E \equiv \gamma mc^2$
    - ↳ Rest energy $R \equiv mc^2$
    - Relativistic KE $T \equiv (\gamma - 1)mc^2$

※ For massless particles moving at c, $E = pc$

- Relativistic Collisions
  - ↳ Energy & momentum are always conserved.
  - ↳ Mass doesn't have to be conserved.

Introduction to Elementary Particles

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✱ Symmetries

- Noether's theorem: Symmetries $\longleftrightarrow$ Conservation laws
  ↳ Every conservation law reflects an underlying symmetry

- Symmetry: operation you can perform on a system that leaves it invariant

- Set of symmetry operations on system:
  1. Closure: if R_i & R_j are in set, R_iR_j is in set
  2. Identity: there is an element I such that $IR_i = R_iI = R_i$
  3. Inverse: For every R_i, there's an inverse R_i^{-1} such that $R_iR_i^{-1} = R_i^{-1}R_i = I$
  4. Associativity: $R_i(R_jR_k) = (R_iR_j)R_k$

  ※ These 4 are defining properties of a mathematical group
  ※ If group elements commute ($R_iR_j = R_jR_i$) $\longrightarrow$ Abelian group

- Reformulating groups as matrices:

Group name	Dimension	Matrices in group
$U(n)$	$n \times n$	Unitary ($U^*U = I$)
$SU(n)$	$n \times n$	Unitary, det 1
$O(n)$	$n \times n$	Orthogonal ($O^TO = I$)
$SO(n)$	$n \times n$	orthogonal, det 1

* Angular Momentum

- Orbital: $L = rmv$
- Spin: $L = I\omega$

- Classically: we can measure components of $\vec{L} = \vec{r} \times m\vec{v}$ to any precision

- QM: impossible to measure all 3 components simultaneously
  - ↳ Instead, measure $L^2 = L \cdot L$ and 1 component (L_z)
  - ↳ Measurement of L^2 always yields $\ell(\ell+1)\hbar^2$, with $\ell = 0, 1, 2, 3 \ldots$
  - ↳ Measurement of L_z always yields $m_\ell \hbar$, where m_ℓ is integer in range $-\ell$ to $+\ell$: $-\ell, -\ell+1, -\ell+2 \ldots \ell-1, \ell$

  - ↳ Same for spin: measurement of $S^2 = S \cdot S$ always yields $s(s+1)\hbar^2$
    But s can be a half integer: $s = 0, \frac{1}{2}, 1, \frac{3}{2}, 2 \ldots$

  - ↳ Measurement of S_z yields $m_s \hbar$, where m_s is integer/half-int between $-s$ and s: $-s, -s+1, \ldots, s-1, s$

※ A given particle can have any orbital angular mom. ℓ
But s (the spin) is fixed:

	Bosons (integer spin)		Fermions (half-int spin)	
	Spin 0	Spin 1	Spin $\frac{1}{2}$	Spin $\frac{3}{2}$
Elementary →	-	Mediators	Quarks/Leptons	-
Composite →	Pseudoscalar Mesons	Vector Mesons	Baryon octet	Baryon decuplet

orbital state ↓ spin state ↓

- Adding angular momenta
  - ↳ AM states represented by kets: $|\ell \; m_\ell\rangle$ or $|s \; m_s\rangle$
  - ↳ How to add 2 AM? $J = J_1 + J_2$
    - ↳ Let $J_1 = |j_1 \; m_1\rangle$ and $J_2 = |j_2 \; m_2\rangle$
    - ↳ Z-components add: $m = m_1 + m_2$
    - ↳ Magnitude spans every j from j_1+j_2 to $|j_1-j_2|$ in int.

$$j = |j_1-j_2| \, , \; |j_1-j_2|+1 \ldots (j_1+j_2)-1, \; j_1+j_2$$

* More on the Spin $\frac{1}{2}$ Particles

- Proton, neutron, electron, all quarks, all leptons
- A particle w/ spin-$\frac{1}{2}$ can have $m_s = \frac{1}{2}$ (spin up) or $-\frac{1}{2}$ (spin down)

$$|\tfrac{1}{2} \; \tfrac{1}{2}\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |\tfrac{1}{2} \; -\tfrac{1}{2}\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

※ The general state is $\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $|\alpha|^2 + |\beta|^2 = 1$

But measurement of S_z can only return $-\frac{1}{2}\hbar$ or $+\frac{1}{2}\hbar$

- Pauli spin matrices:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\boxed{\hat{S} = \frac{\hbar}{2}\sigma}$$

* Flavor Symmetries

- Heisenberg: regard neutron & proton as 2 states of 1 particle, called the nucleon
  - ↳ Nucleon: $N = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$, with $p = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $n = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$
  - ↳ Introduce Isospin $\vec{I}$. Components I_1, I_2, I_3
    - ↳ Nucleon carries isospin $\frac{1}{2}$
    - ↳ I_3 has eigenvalues $+\frac{1}{2}$ (proton) and $-\frac{1}{2}$ (neutron)

※ Isospin is conserved in all strong interactions

↳ "Strong interactions are invariant under internal symmetry group $SU(2)$"

- Examples

↳ For pions, $I=1$

$\pi^+ = |1\ 1\rangle$, $\pi^0 = |1\ 0\rangle$, $\pi^- = |1\ -1\rangle$

↳ For Λ, $I=0$

$\Lambda = |0\ 0\rangle$

↳ For Δ, $I=\frac{3}{2}$

$\Delta^{++} = |\frac{3}{2}\ \frac{3}{2}\rangle$, $\Delta^{+} = |\frac{3}{2}\ \frac{1}{2}\rangle$, $\Delta^{0} = |\frac{3}{2}\ -\frac{1}{2}\rangle$, $\Delta^{-} = |\frac{3}{2}\ -\frac{3}{2}\rangle$

※ I_3 ranges from $-I$ to I

So # of particles in multiplet (multiplicity) is $2I+1$

Introduction to Elementary Particles

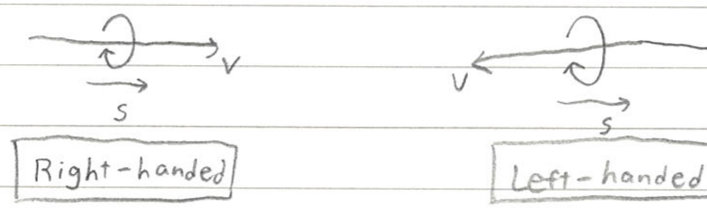
6/17/2025 (pp. 136–151)

* Discrete Symmetries

- "Mirror image of a physical process represents a perfectly possible physical process"
  - ↳ Lots of evidence for strong & electromagnetic processes
  - ↳ What about weak interactions?

- Parity violation is signature of weak force (Lee & Wu)

- Helicity: m_s/s, with direction of motion as z-axis
  - ↳ Right-handed: helicity +1 ($m_s = \frac{1}{2}$)
  - ↳ Left-handed: helicity −1 ($m_s = -\frac{1}{2}$)



※ In gen. not Lorentz invariant: Can convert RH to LH by changing direction of motion

※ For massless particles, Lorentz invariant

↓

Neutrinos are left-handed

Antineutrinos are right-handed

- Terminology
  - ↳ Inversions: $(x, y, z) \to (-x, -y, -z)$
  - ↳ P = inversion, "parity operator"

  - ↳ Polar vectors: change sign under P, $P(a) = -a$
  - ↳ Pseudo/axial vectors: don't change sign under P

    e.g. for $c = a \times b$, $P(c) = c$

※ In theory w/ Parity invariance, never add a vector to a pseudovector.

↳ e.g. $F = q[E + (v \times B)/c]$

↳ V = vector, B = pseudovector → $V \times B$ = vector, can add to E

↳ But $E + B$ is impossible.

Scalar	$P(s) = s$
Pseudoscalar	$P(p) = -p$
Vector (polar vector)	$P(v) = -v$
Pseudovector (axial vector)	$P(a) = a$

- Parity of fermions (half-int spin) is opposite of its antiparticle
- Parity of bosons (int. spin) is same as its antiparticle
- Quarks have positive intrinsic parity / antiquarks have negative

- Parity of composite system is product of its constituent parities

※ Remember: parity is conserved in strong & electromagnetic processes, but not weak processes.

✱ Charge Conjugation

- Operator that converts each particle to antiparticle
  ↳ $C|p\rangle = |\bar{p}\rangle$
  ↳ Changes sign of internal quantum numbers (charge, baryon number, lepton number, strangeness, etc.)
  ↳ Doesn't change mass, energy, momentum, spin

- Charge conjugation is multiplicative (like parity)
  Conserved in strong/electromagnetic interactions

Introduction to Elementary Particles

6/17/2025 (pp. 159-193)

* Bound States

- Analysis is easier if constituents are nonrelativistic
- If binding energy is small compared to rest of the constituents' energies $\rightarrow$ system is nonrelativistic

- For nonrelativistic QM, Schrödinger's equation:

$$\boxed{\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\Psi = i\hbar\frac{\partial}{\partial t}\Psi}$$

↳ where $\int |\Psi|^2 d^3r = 1$ (wavefunction is normalized)

↳ If V is independent of t, $\Psi(r,t) = \psi(r)e^{-iEt/\hbar}$

↳ where $\psi(r)$ satisfies time-independent SE:

$$\boxed{\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi(r) = E\,\psi(r)}$$

↳ Let Hamiltonian $H \equiv -\frac{\hbar^2}{2m}\nabla^2 + V$

∴ Time-independent SE: $\boxed{\hat{H}\psi = E\psi}$

- For spherically symmetric potential, SE in spherical coord:

$$\boxed{\psi(r,\theta,\phi) = \frac{u(r)}{r}Y_l^{m_l}(\theta,\phi)}$$

↳ Y is a spherical harmonic

↳ l and m_l are orbital angular momentum quantum numbers

↳ u(r) satisfies radial SE:

$$\boxed{-\frac{\hbar^2}{2m}\frac{d^2u}{dr^2} + \left[V(r) + \underbrace{\frac{\hbar^2}{2m}\frac{l(l+1)}{r^2}}\right]u = Eu}$$

Centrifugal barrier $\frac{\hbar^2}{2m}\frac{l(l+1)}{r^2}$

Augments V(r), the potential

- The Hydrogen atom (e^- and p^+)
  - ↳ Not elementary particle, but serves as model
  - ↳ Wavefunction concerns e^- (proton just sits at center)
  - ↳ $V(r) = -\frac{e^2}{r}$, Solutions occur at $\boxed{E_n = -\alpha^2 mc^2 \left(\frac{1}{2n^2}\right)}$
  - ↳ $n = 1, 2, 3 \ldots$ and $\boxed{\alpha \equiv \frac{e^2}{\hbar c} = \frac{1}{137.036}}$
  - ↳ Emitted wavelength as e^- moves between orbitals is given by

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right), \quad R = \frac{me^4}{4\pi\hbar^3 c} = 1.09737 \times 10^5 / cm$$

Rydberg formula

- Fine structure
  - ↳ Small departures from spectral lines predicted by Rydberg formula
  - ↳ Explanations:
    1. Relativistic correction: use of classical KE in Hamiltonian
    2. Spin-orbit coupling: spinning electron contributes small magnet, with dipole moment $\mu_e = -\frac{e}{mc} S$

※ Result: perturbation of n-th energy level by amount

$$\Delta E_{fs} = -\alpha^4 mc^2 \frac{1}{4n^4}\left(\frac{2n}{j + \frac{1}{2}} - \frac{3}{2}\right)$$

$$j = l \pm \frac{1}{2}$$

Can be any int. from 0 to $n-1$

Total angular momentum (spin + orbital)

- Magnetic moments of baryons
  - ↳ In absence of orbital motion, $\mu = \mu_1 + \mu_2 + \mu_3$
  - ↳ Vector sum of magnetic moments of 3 Constituent quarks
  - ↳ Depends on quark flavor & spin config

  - ↳ For spin-$\frac{1}{2}$ point particle w/ charge q & mass m,
    $\vec{\mu} = \frac{q}{mc}\vec{S}$, magnitude $\mu = \frac{q\hbar}{2mc}$

$$\boxed{\mu_u = \frac{2}{3}\frac{e\hbar}{2m_u c} \qquad \mu_d = -\frac{1}{3}\frac{e\hbar}{2m_d c} \qquad \mu_s = -\frac{1}{3}\frac{e\hbar}{2m_s c}}$$

∴ Magnetic moment of baryon B is

$$\mu_B = \langle B\uparrow | (\mu_1 + \mu_2 + \mu_3)_z | B\uparrow \rangle = \frac{2}{\hbar}\sum_{i=1}^{3} \langle B\uparrow | (\mu_i \vec{S}_i)_z | B\uparrow \rangle$$

- Baryon masses
  - ↳ Need to account for spin-spin contributions

$$M(\text{baryon}) = m_1 + m_2 + m_3 + A'\left[\frac{S_1 \cdot S_2}{m_1 m_2} + \frac{S_1 \cdot S_3}{m_1 m_3} + \frac{S_2 \cdot S_3}{m_2 m_3}\right]$$

$A' =$ constant

Introduction to Elementary Particles

6/18/2025 (pp. 197–221)

* The Feynman Calculus

- Decay Rates
  - ↳ We want the average lifetime of particles in a large sample
  - ↳ Decay rate (Γ): probability per unit time that particle disintegrates
    - ↳ For $N(t)$ muons, $dN = -\Gamma N\,dt$

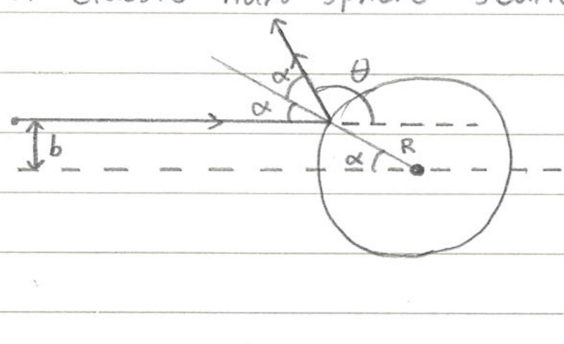
$$\boxed{N(t) = N(0)e^{-\Gamma t}}$$

  - ↳ Mean lifetime is $\boxed{\tau = \frac{1}{\Gamma}}$
  - ↳ Some particles can decay through different routes

    e.g. $\pi^+ \to \mu^+ + \nu_\mu$, $\pi^+ \to e^+ + \nu_e$, etc.

  - ↳ Total decay rate is $\boxed{\Gamma_{tot} = \sum_{i=1}^{n} \Gamma_i}$
  - ↳ Branching ratio for i-th decay mode is $\boxed{\Gamma_i / \Gamma_{tot}}$

- Cross Sections
  - ↳ For scattering effects, e.g. electron beam vs. hydrogen
  - ↳ Depends on particles involved
  - ↳ "Total" cross section: $\sigma_{tot} = \sum_{i=1}^{n} \sigma_i$
  - ↳ For classic hard-sphere scattering,

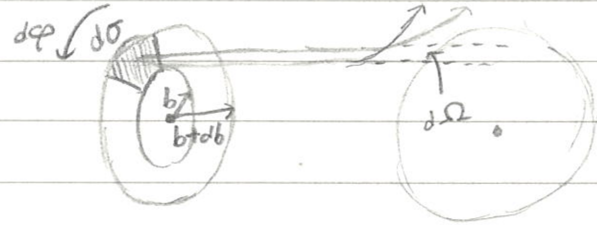


$b = R\sin(\alpha)$, $2\alpha + \theta = \pi$

↓

$$\begin{cases} b = R\cos(\theta/2) \\ \theta = 2\cos^{-1}(b/R) \end{cases}$$

↳ If particle comes in w/ impact parameter between b and $b+db$, scattering angle is between θ and $d\theta$.



↳ i.e. if it passes through area $d\sigma$, it scatters w/ angle $d\Omega$

$$\boxed{d\sigma = D(\theta)\, d\Omega}$$

↳ $D(\theta)$ = differential (scattering) cross-section

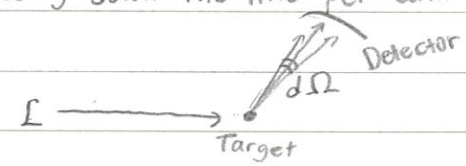
↳ Calculating, $d\sigma = |b\, db\, d\varphi|$, $d\Omega = |\sin(\theta)\, d\theta\, d\varphi|$

$$\boxed{D(\theta) = \frac{d\sigma}{d\Omega} = \left| \frac{b}{\sin\theta}\left(\frac{db}{d\theta}\right) \right|}$$

※ For hard-sphere scattering, $\frac{db}{d\theta} = -\frac{R}{2}\sin\left(\frac{\theta}{2}\right)$, $D(\theta) = \frac{R^2}{4}$

Total cross section $= \int d\sigma = \int \frac{R^2}{4}\, d\Omega = \pi R^2$

↳ Assume beam of particles w/ luminosity $\mathcal{L}$ (# of particles passing down the line per unit time & unit area)



↳ $dN = \mathcal{L}\, d\sigma = \mathcal{L}\, D(\theta)\, d\Omega$ (# of particles through $d\sigma$ per unit time)

↳ Event rate: # of particles per unit time reaching detector (dN)

※ "Event rate is $\underbrace{\text{cross section}}$ times luminosity"

$$D(\theta) = \frac{d\sigma}{d\Omega} = \frac{dN}{\mathcal{L}\, d\Omega}$$

⁕ The Golden Rule

- Decay rates & cross sections: the ingredients

  i) Amplitude ($\mathcal{M}$) of process — dynamical

  ii) Phase space available — kinematic

- Fermi's Golden Rule: transition rate is given by product of phase space & square of amplitude

- For decays:

  ↳ If particle 1 decays as $1 \to 2+3+\ldots+n$

  ↳ Decay rate is $$\Gamma = \frac{S}{2\hbar m_1}\int |\mathcal{M}|^2 (2\pi)^4 \delta^4(p_1 - p_2 - \ldots - p_n) \times \prod_{j=2}^{n} 2\pi\delta(p_j^2 - m_j^2c^2)\theta(p_j^0)\frac{d^4 p_j}{(2\pi)^4}$$

    ↳ m_i = mass of i-th particle, p_i = its 4-momentum

    ↳ S = factor to account for double-counting

    ↳ Dynamics of process contained in amplitude $\mathcal{M}(p_1, p_2 \ldots p_n)$

    Calculate using Feynman diagrams

    ↳ Rest of eqn is phase space (integrate over 4-momenta)

    1. Each outgoing particle lies on its mass shell: $p_j^2 = m_j^2 c^2$
    2. Each outgoing energy > 0 ; $p_j^0 = E_j/c > 0$
    3. Energy & momentum conserved: $p_1 = p_2 + p_3 + \ldots + p_n$

    ⁂ Every δ gets a factor of 2π, every d gets $\frac{1}{2\pi}$

  ⁂ Two-particle decays: $\boxed{\Gamma = \frac{S|\vec{p}|}{8\pi\hbar m_1^2 c}|\mathcal{M}|^2}$

- For scattering:
  - ↳ If particles 1 & 2 collide to produce $1+2 \to 3+4+\ldots+n$
  - ↳ Scattering cross section is

$$\sigma = \frac{S\hbar^2}{4\sqrt{(p_1 \cdot p_2)^2 - (m_1 m_2 c^2)^2}} \int |\mathcal{M}|^2 (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4 \ldots - p_n)$$

$$\underbrace{\times \prod_{j=3}^{n} 2\pi\delta(p_j^2 - m_j^2c^2)\,\theta(p_j^0)\frac{d^4p_j}{(2\pi)^4}}_{\prod_{j=3}^{n} \frac{1}{2\sqrt{\vec{p}_j^{\,2} + m_j^2c^2}} \frac{d^3p_j}{(2\pi)^3} \quad \text{with } p_j^0 = \sqrt{\vec{p}_j^{\,2} + m_j^2c^2}}$$

※ For two-body scattering: $\frac{d\sigma}{d\Omega} = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{S|\mathcal{M}|^2}{(E_1+E_2)^2} \frac{|\vec{p}_f|}{|\vec{p}_i|}$

↳ $1+2 \to 3+4$

↳ $|\vec{p}_f|$ is mag. of either outgoing momentum, similar for $|\vec{p}_i|$

↳ $\mathcal{M}$ has dimensions of $(mc)^{4-n}$, where n is the # of incoming + outgoing lines

✳ Feynman Rules

- How to determine $\mathcal{M}$
- Rules

0. Construct Feynman diagram
1. Label incoming & outgoing 4-momenta w/ p_i
   Label internal momenta w/ q_i
   Use arrows: forward in time for external lines, arbitrary for internal lines
2. For each vertex, write factor $-ig$ (g = coupling constant)
3. For each internal line, write factor $\frac{i}{q_j^2 - m_j^2c^2}$
   ↳ q_j is 4-momentum of line
   ↳ m_j is mass of particle

4. For each vertex, write $(2\pi)^4 \delta^4(k_1+k_2+k_3)$

↳ k are 3 4-momenta coming into vertex (negative if outgoing)

↳ Conserves energy & momenta: $\delta = 0$ unless sum of incoming = sum of outgoing

5. For each internal line, write $\frac{1}{(2\pi)^4} d^4 q_j$ and integrate over all internal momenta

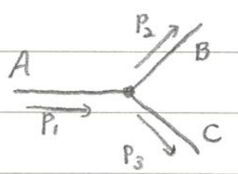
6. The result will include delta function $(2\pi)^4 \delta^4(p_1+p_2+\ldots-p_n)$

↳ Erase this factor

↳ Multiply by i

∴ The result is $\mathcal{M}$

• Ex. 1: $A \to B + C$



→ Vertex picks up $-ig$

→ Vertex picks up $(2\pi)^4 \delta^4(p_1 - p_2 - p_3)$

→ Discarded

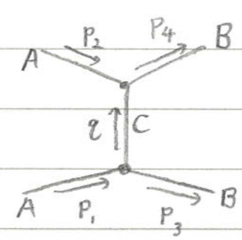
→ Multiply by i: $\mathcal{M} = g$

∴ Decay rate $= \Gamma = \frac{g^2 |\vec{p}|}{8\pi \hbar m_A^2 c}$, lifetime $= \tau = \frac{1}{\Gamma}$

↳ where $|\vec{p}| = \frac{c}{2m_A}\sqrt{m_A^4 + m_B^4 + m_C^4 - 2m_A^2 m_B^2 - 2m_A^2 m_C^2 - 2m_B^2 m_C^2}$

• Ex. 2: $A + A \to B + B$

↳ Lowest-order diagram

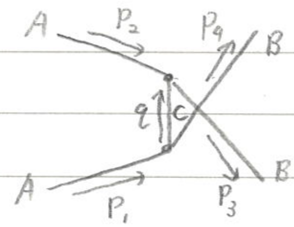


↳ Rules 1-5: $-i(2\pi)^4 g^2 \int \frac{1}{q^2 - m_C^2 c^2} \delta^4(p_1 - p_3 - q)\, \delta^4(p_2 + q - p_4)\, d^4 q$

↳ $q \to p_4 - p_2$ by δ, erase other δ and mult. by i

$$\boxed{\mathcal{M} = \frac{g}{(p_4 - p_2)^2 - m_C^2 c^2}}$$

↳ Another lowest-order diagram



↳ only differs by $P_3 \leftrightarrow P_4$

∴ Total amplitude: $\boxed{\mathcal{M} = \dfrac{g^2}{(P_4 - P_2)^2 - m_C^2 c^2} + \dfrac{g^2}{(P_3 - P_2)^2 - m_C^2 c^2}}$

※ Higher-order diagrams can be added to make $\mathcal{M}$ more accurate

Introduction to Elementary Particles

6/19/2025 (pp. 225 – 267)

* The Dirac Equation

- Relativistic QM
  - ↳ Particles of spin 0 described by Klein-Gordon eqn.
  - ↳ Particles of spin $\frac{1}{2}$ described by Dirac eqn.
  - ↳ Particles of spin 1 described by Proca eqn.

- The Dirac Equation: $i\hbar\gamma^\mu\partial_\mu\psi - mc\psi = 0$
  - ↳ $\psi = \begin{bmatrix}\psi_1\\ \psi_2\\ \psi_3\\ \psi_4\end{bmatrix}$, $\gamma^i = \begin{bmatrix}0 & \sigma^i\\ -\sigma^i & 0\end{bmatrix}$ where σ^i is the Pauli matrix

    $\gamma^0 = \begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$

- Solutions to the Dirac equation
  - ↳ Assume ψ is independent of position
  - ↳ Reduce to $\frac{i\hbar}{c}\gamma^0\frac{\partial\psi}{\partial t} - mc\psi = 0$

  ※ $\partial_\mu \equiv \frac{\partial}{\partial x^\mu}$, so $\partial_0 = \frac{1}{c}\frac{\partial}{\partial t}$, $\partial_1 = \frac{\partial}{\partial x}$, $\partial_2 = \frac{\partial}{\partial y}$, $\partial_3 = \frac{\partial}{\partial z}$

  - ↳ 4 solutions: $\psi^{(1)} = e^{-i(mc^2/\hbar)t}\begin{bmatrix}1\\0\\0\\0\end{bmatrix}$, $\psi^{(2)} = e^{-i(mc^2/\hbar)t}\begin{bmatrix}0\\1\\0\\0\end{bmatrix}$

    $\psi^{(3)} = e^{+i(mc^2/\hbar)t}\begin{bmatrix}0\\0\\1\\0\end{bmatrix}$, $\psi^{(4)} = e^{+(mc^2/\hbar)t}\begin{bmatrix}0\\0\\0\\1\end{bmatrix}$

    - ↳ $\psi^{(1)}$ = electron w/ spin up

      $\psi^{(2)}$ = electron w/ spin down
    - ↳ $\psi^{(3)}$ = positron w/ spin down

      $\psi^{(4)}$ = positron w/ spin up

※ Particle states satisfy momentum-space Dirac equation,
$(\gamma^\mu p_\mu - mc)u = 0$

※ Antiparticle states satisfy $(\gamma^\mu p_\mu + mc)v = 0$

※ 4 Canonical plane-wave solutions — $\psi(x) = a e^{-ik\cdot x} u(k)$

$$u^{(1)} = N \begin{bmatrix} 1 \\ 0 \\ \frac{cp_z}{E+mc^2} \\ \frac{c(p_x+ip_y)}{E+mc^2} \end{bmatrix} \qquad u^{(2)} = N \begin{bmatrix} 0 \\ 1 \\ \frac{c(p_x-ip_y)}{E+mc^2} \\ \frac{c(-p_z)}{E+mc^2} \end{bmatrix}$$

$$v^{(1)} = N \begin{bmatrix} \frac{c(p_x-ip_y)}{E+mc^2} \\ \frac{c(-p_z)}{E+mc^2} \\ 0 \\ 1 \end{bmatrix} \qquad v^{(2)} = N \begin{bmatrix} \frac{cp_z}{E+mc^2} \\ \frac{c(p_x+ip_y)}{E+mc^2} \\ 1 \\ 0 \end{bmatrix}$$

$\psi = a e^{-ip\cdot x/\hbar} u$ (particles)

$\psi = a e^{ip\cdot x/\hbar} v$ (antiparticles)

* Electrodynamics with fields

- Define the field strength tensor $F^{\mu\nu} = \begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{bmatrix}$

↳ $\left.\begin{array}{l} \nabla\cdot E = 4\pi\rho \\ \nabla\times B - \frac{1}{c}\frac{\partial E}{\partial t} = \frac{4\pi}{c}\vec{J} \end{array}\right\} \boxed{\partial_\mu F^{\mu\nu} = \frac{4\pi}{c} J^\nu}$

↳ $\nabla\cdot B = 0 \rightarrow$ means $B = \nabla\times A$ for some $A \rightarrow \boxed{F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu}$

$\therefore \boxed{\partial_\mu \partial^\mu A^\nu - \partial^\nu(\partial_\mu A^\mu) = \frac{4\pi}{c} J^\nu}$ ← inhomogeneous Maxwell Equations

- Lorentz Condition: $\partial_\mu A^\mu = 0$

↳ Result: $\boxed{\square A^\mu = \frac{4\pi}{c} J^\mu}$, where $\square \equiv \partial^\mu \partial_\mu = \underbrace{\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2}_{\text{Relativistic Laplacian "d'Alembertian"}}$

* Feynman Rules for QED

- Wavefunctions
  - ↳ Electrons: $\psi(x) = a e^{-(i/\hbar) p \cdot x} u^{(s)}(p)$
  - ↳ Positrons: $\psi(x) = a e^{(i/\hbar) p \cdot x} v^{(s)}(p)$
  - ↳ Photons: $A_\mu(x) = a e^{-(i/\hbar) p \cdot x} \epsilon_\mu^{(s)}$
    - ↳ $s = 1, 2$ for 2 spin states
    - ↳ $\epsilon_\mu^{(s)}$ = polarization vector, satisfies $p^\mu \epsilon_\mu^{(s)} = 0$

- Feynman Rules to find amplitude ($\mathcal{M}$)

1. To each external line, momentum P_i. For each internal line, q_i
   - ↳ P_i in positive direction (forward in time)
   - ↳ q_i has arbitrary direction

2. External lines contribute factors
   - ↳ Electrons $\begin{cases} \text{Incoming } (\rightarrow\bullet): & u \\ \text{outgoing } (\bullet\rightarrow): & \bar{u} \end{cases}$

     Positrons $\begin{cases} \text{Incoming } (\leftarrow\bullet): & \bar{v} \\ \text{Outgoing } (\bullet\leftarrow): & v \end{cases}$

     Photons $\begin{cases} \text{Incoming } (\sim\sim\bullet): & \epsilon_\mu \\ \text{Outgoing } (\bullet\sim\sim) & \epsilon_\mu^* \end{cases}$

3. Each vertex contributes factor $i g_e \gamma^\mu$, $g_e = e\sqrt{4\pi/\hbar c} = \sqrt{4\pi a}$

4. Propagators: each internal line contributes
   - ↳ Electrons & Positrons: $\dfrac{i(\gamma^\mu q_\mu + mc)}{q^2 - m^2c^2}$

     photons : $\dfrac{-i g_{\mu\nu}}{q^2}$

5. For each vertex, write $(2\pi)^4 \delta^4(k_1 + k_2 + k_3)$
   - ↳ To conserve energy & momentum
   - ↳ k = positive if incoming, negative if outgoing

6. Integrate over internal momenta, for each q_i, write

$\frac{d^4 q}{(2\pi)^4}$ and integrate.

7. Cancel δ function $(2\pi)^4 \delta^4(p_1 + p_2 + \ldots - p_n)$

Multiply by i, result is $\mathcal{M}$

8. (New rule for fermions) Antisymmetrization: include minus sign between diagrams that only differ in interchange of 2 incoming/outgoing electrons/positrons, or of incoming electron w/ outgoing positron (and vice versa)

※ Add up $\mathcal{M}$ for all diagrams to get $\mathcal{M}_{total}$, then use Golden Rule to find decay rate / scattering cross section, etc.

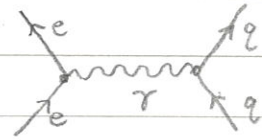
※ Most examples went over my head so I might come back to them later.

Introduction to Elementary Particles

6/20/2025 (pp. 275 - 301)

✳ Electrodynamics & Chromodynamics of Quarks

- Hadron Production in e^+e^- collisions

↳ 

→ But quarks join in combinations, produce mesons & baryons "Hadronization"

↳ $e^+ + e^- \rightarrow$ hadrons

↳ 1st stage: $e^+ + e^- \rightarrow \gamma \rightarrow q + \bar{q}$

↳ Amplitude $\mathcal{M} = \frac{Q g_e^2}{(p_1 + p_2)^2} [\bar{v}(p_2)\gamma^\mu u(p_1)][\bar{u}(p_3)\gamma_\mu v(p_4)]$

↳ Use Casimir's trick and express in terms of incident electron energy (E) and θ between incoming e^- and outgoing q

$$\langle|\mathcal{M}|^2\rangle = Q^2 g_e^4 \left\{1 + \left(\frac{mc^2}{E}\right)^2 + \left(\frac{Mc^2}{E}\right)^2 + \left[1 - \left(\frac{mc^2}{E}\right)^2\right]\left[1 + \left(\frac{Mc^2}{E}\right)^2\right]\cos^2\theta\right\}$$

※ Substituting into cross-section eqn and holding $E > Mc^2 \gg mc^2$,

$$\boxed{\sigma = \frac{\pi}{3}\left(\frac{\hbar Q c a}{E}\right)^2}$$

↳ As beam energy increases, we get a succession of threshold energies for quark flavors.

※ Ratio of hadron production rate vs. muon production rate:

$$R \equiv \frac{\sigma(e^+e^- \rightarrow hadrons)}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$ ← includes all quark-antiquark events

↓

$$\boxed{R(E) = 3\sum_i Q_i^2}$$

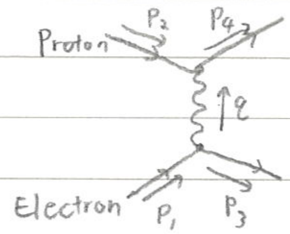
↳ 3 in front: 3 colors for every flavor

↳ e.g. at low energies w/ only u, d, s:

$$R = 3\left[\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2\right] = 2$$

• Elastic electron-proton scattering

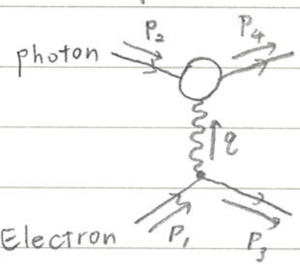
↳ Something like this?



$$\langle |\mathcal{M}|^2 \rangle = \frac{g_e^{\,4}}{q^4} L^{\mu\nu}_{\text{electron}} L_{\mu\nu\ \text{proton}}, \quad q = p_1 - p_3$$

$$L^{\mu\nu}_{\text{electron}} = 2\{p_1^{\mu} p_3^{\nu} + p_1^{\nu} p_3^{\mu} + g^{\mu\nu}[(mc)^2 - (p_1 \cdot p_3)]\}$$

※ But proton isn't a simple point charge, so



Blob b.c. we don't know how virtual photon interacts w/ photon

$$\langle |\mathcal{M}|^2 \rangle = \frac{g_e^{\,4}}{q^4} L^{\mu\nu}_{\text{electron}} K_{\mu\nu\ \text{photon}}$$

↳ $K_{\mu\nu}$ = Some quantity describing photon-photon vertex

* Feynman Rules for Chromodynamics

• QED: interactions of charged particles

↳ Mediated by photons

↳ Coupling const. $g_e = \sqrt{4\pi a}$

• QCD: interactions of colored particles

↳ Mediated by gluons

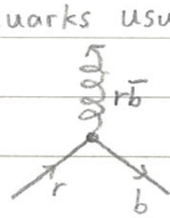
↳ Coupling const. $g_s = \sqrt{4\pi a_s}$

↳ Requires Dirac spinor (momentum & spin): $u^{(s)}(p)$

↳ and 3-vector for color

$C = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ for red, $C = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ for blue, $C = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ for green

※ Quarks usually change color at quark-gluon vertex:



Difference carried away by gluon

- 8 Types of gluons

$|1\rangle = (r\bar{b} + b\bar{r})/\sqrt{2}$ $|5\rangle = -i(r\bar{g} - g\bar{r})/\sqrt{2}$

$|2\rangle = -i(r\bar{b} - b\bar{r})/\sqrt{2}$ $|6\rangle = (b\bar{g} + g\bar{b})/\sqrt{2}$

$|3\rangle = (r\bar{r} - b\bar{b})/\sqrt{2}$ $|7\rangle = -i(b\bar{g} - g\bar{b})/\sqrt{2}$

$|4\rangle = (r\bar{g} + g\bar{r})/\sqrt{2}$ $|8\rangle = (r\bar{r} + b\bar{b} - 2g\bar{g})/\sqrt{6}$

↳ and a "color singlet" (colorless): $|9\rangle = (r\bar{r} + b\bar{b} + g\bar{g})/\sqrt{3}$

- Gluons have polarization vector ϵ^μ

↳ Orthogonal to gluon momentum: $\epsilon^\mu p_\mu = 0$ (Lorentz Condition)

↳ Coulomb gauge: $\epsilon^0 = 0$, $\epsilon \cdot p = 0$

↳ To describe color state of gluon, use 8-vector a

↳ $a^\alpha = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$, $a^\beta = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$, etc w/ superscripts $\underbrace{\alpha, \beta, \gamma \ldots}_{\text{1–8 states}}$

- Gell-Mann λ-matrices

↳ These are to SU(3) what Pauli matrices are to SU(2)

$$\lambda^1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \lambda^2 = \begin{bmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \lambda^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\lambda^4 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad \lambda^5 = \begin{bmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{bmatrix} \quad \lambda^6 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\lambda^7 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{bmatrix} \quad \lambda^8 = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

※ Commutators define fine-structure constants ($f^{\alpha\beta\gamma}$) of SU(3)

↳ $[\lambda^\alpha, \lambda^\beta] = 2if^{\alpha\beta\gamma}\lambda^\gamma$ → Repeated indices indicate summation over γ, 1–8

※ That was a buttload of math

- Feynman Rules for QCD

1. For external quarks w/ momentum p, spin s, color c

↳ Quark $\begin{cases} \text{Incoming } (\to\bullet) : u^{(s)}(p)c \\ \text{Outgoing } (\bullet\to) : \bar{u}^{(s)}(p)c^{\dagger} \end{cases}$ $\boxed{c^{\dagger} = \bar{c}^{*}}$

Antiquark $\begin{cases} \text{Incoming } (\leftarrow\bullet) : \bar{v}^{(s)}(p)c^{\dagger} \\ \text{Outgoing } (\bullet\leftarrow) : v^{(s)}(p)c \end{cases}$

For external gluons w/ momentum p, polarization ϵ, and color a

↳ Gluon $\begin{cases} \text{Incoming } (\xrightarrow{\alpha,\mu}\bullet) : \epsilon_{\mu}(p)a^{\alpha} \\ \text{Outgoing } (\bullet\xleftarrow{\alpha,\mu}) : \epsilon^{*}_{\mu}(p)a^{\alpha *} \end{cases}$

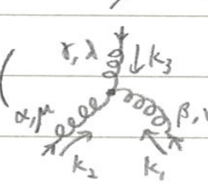
2. Propagators: each internal line contributes factor

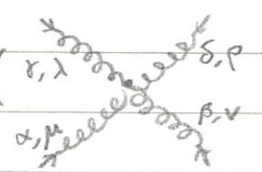
↳ Quarks & antiquarks $(\bullet\xrightarrow{q}\bullet)$: $\dfrac{i(\not{q}+mc)}{q^2-m^2c^2}$

↳ Gluons $(\underset{\alpha,\mu}{\bullet}\overset{q}{\sim\sim}\underset{\beta,\nu}{\bullet})$: $\dfrac{-ig_{\mu\nu}\delta^{\alpha\beta}}{q^2}$

3. Vertices: each vertex introduces factor

↳ Quark-gluon (): $\dfrac{-ig_s}{2}\lambda^{\alpha}\gamma^{\mu}$

↳ 3-gluon (): $-g_s f^{\alpha\beta\gamma}\left[g_{\mu\nu}(k_1-k_2)_{\lambda} + g_{\nu\lambda}(k_2-k_3)_{\mu} + g_{\lambda\mu}(k_3-k_1)_{\nu}\right]$

↳ 4-gluon (): $-ig_s^2\left[f^{\alpha\beta\eta}f^{\gamma\delta\eta}(g_{\mu\lambda}g_{\nu\rho}-g_{\mu\rho}g_{\nu\lambda}) + f^{\alpha\delta\eta}f^{\beta\gamma\eta}(g_{\mu\nu}g_{\lambda\rho}-g_{\mu\lambda}g_{\nu\rho}) + f^{\alpha\gamma\eta}f^{\delta\beta\eta}(g_{\mu\rho}g_{\nu\lambda}-g_{\mu\nu}g_{\lambda\rho})\right]$

↳ Sum over η

※ Gluon (k_1, k_2, k_3) positive if pointing into vertex, negative if pointing out

※ Rest same as QED: Cons. energy & momenta at each vertex → erase overall δ function, multiply by i to get $\mathcal{M}$

Introduction to Elementary Particles

6/21/2025 (pp. 307–346)

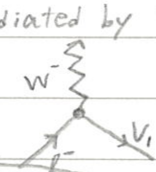
*Weak Interactions

- Mediators:
  - ↳ W^+, W^- } Heavy compared to photon & gluon (which are massless)
  - ↳ Z^0

- Massive particle w/ spin 1 has 3 allowed polarization states ($m_s = -1, 0, 1$)
- Massless free particle has 2 ($m_s = -1, 1$)
  - ↳ For photons & gluons, Lorentz condition $\epsilon^\mu p_\mu = 0$ and Coulomb gauge $\epsilon^0 = 0$
  - ↳ But for Ws and Z, we don't impose Coulomb gauge.

- Propagator for W & Z: $\frac{-i(g_{\mu\nu} - q_\mu q_\nu / M^2c^2)}{q^2 - M^2c^2} \approx \boxed{\frac{ig_{\mu\nu}}{(Mc)^2}, \quad q^2 \ll (Mc)^2}$
  - ↳ cf. $\frac{-ig_{\mu\nu}}{q^2}$ previously

- Charged weak interactions (mediated by Ws)
  - ↳ Fundamental leptonic vertex:



    - ↳ l^- = lepton (e.g. muon, e^-)
    - ↳ weak vertex factor: $\boxed{\frac{-ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)}$
      - ↳ $g_w = \sqrt{4\pi a_w}$

※ Weak fine structure constant: $\alpha_w = \frac{g_w^2}{4\pi} = \frac{1}{29.5}$
  - ↳ Larger than EM fine structure const.
  - ↳ Weak interactions are feeble b.c. mediators are so massive, not b.c. intrinsic coupling is small

- Charged weak interaction of quarks
  ↳ For leptons, coupling to $W^\pm$ takes place strictly within a particular generation

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \leftarrow \text{lepton generations}$$

  ↳ e.g.: $e^- \to \nu_e + W^-$, $\mu^- \to \nu_\mu + W^-$

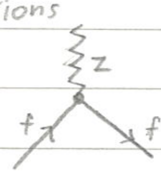
  ↳ Not so simple for quarks: $\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$
    ↳ W's couple to Caribbo-rotated states, not original pairs
    ↳ "Weak interaction generation" related to physical quark states by CKM matrix:

$$\begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix}$$

※ V_{ud} specifies coupling of u to d: $d \to u + W^-$

※ Values: $|V_{ij}| = \begin{bmatrix} 0.9738 & 0.2272 & 0.0040 \\ 0.2271 & 0.9730 & 0.0422 \\ 0.0081 & 0.0416 & 0.9991 \end{bmatrix}$

- Neutral weak interactions
  ↳ Mediated by Z^0:



← f = any lepton or quark
Same fermion that went in comes out.

  ↳ Recall: coupling of quarks/leptons to $W^\pm$ has vertex factor

$$\frac{-ig_w}{2\sqrt{2}} \gamma^\mu (1-\gamma^5)$$

  ↳ For coupling with Z^0: Vertex factor is $\boxed{\frac{-ig_z}{2} \gamma^\mu (c_V^f - c_A^f \gamma^5)}$

↳ where

f	C_V	C_A
ν_e, ν_μ, ν_τ	$\frac{1}{2}$	$\frac{1}{2}$
e^-, μ^-, τ^-	$-\frac{1}{2}+2\sin^2\theta_w$	$-\frac{1}{2}$
u, c, t	$\frac{1}{2}-\frac{4}{3}\sin^2\theta_w$	$\frac{1}{2}$
d, s, b	$-\frac{1}{2}+\frac{2}{3}\sin^2\theta_w$	$-\frac{1}{2}$

↳ and g_z is neutral coupling const.

↳ $g_w = \frac{g_e}{\sin\theta_w}$, $g_z = \frac{g_e}{\sin\theta_w \cos\theta_w}$, $g_e = e\sqrt{4\pi/\hbar c}$

↑ charge of electron

↳ $W^\pm$ and Z^0 mass relationship: $M_w = M_z \cos\theta_w$

※ $\boxed{\theta_w = 28.75°}$ from experiments

↳ Z^0 propagator modified: $\frac{1}{q^2-(M_z c)^2} \longrightarrow \boxed{\frac{1}{q^2-(M_z c)^2+i\hbar M_z \Gamma_z}}$

↳ b.c. Z^0 isn't stable

↳ "Smear out" mass accross lifetime w/ decay rate Γ_z

* Electroweak Unification

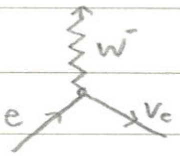
- How to unify weak & electromagnetic interactions?
- Problems

1. Strength disparity ← b.c. mediators for weak interactions have mass
2. Why do $W^\pm$ and Z^0 have mass? ← Higgs mechanism!

- Chiral spinors:

Particles	Antiparticles
$u_L = \frac{1}{2}(1-\gamma^5)u$	$v_L = \frac{1}{2}(1+\gamma^5)v$
$u_R = \frac{1}{2}(1+\gamma^5)u$	$v_R = \frac{1}{2}(1-\gamma^5)v$
$\bar{u}_L = \bar{u}\,\frac{1}{2}(1+\gamma^5)$	$\bar{v}_L = \bar{v}\,\frac{1}{2}(1-\gamma^5)$
$\bar{u}_R = \bar{u}\,\frac{1}{2}(1-\gamma^5)$	$\bar{v}_R = \bar{v}\,\frac{1}{2}(1+\gamma^5)$

• Ex. $e^- \to \nu_e + W^-$



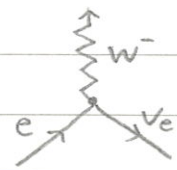
Contrib. to $\mathcal{M}$ from vertex is $j_\mu^- = \bar{\nu}\gamma_\mu\left(\frac{1-\gamma^5}{2}\right)e$

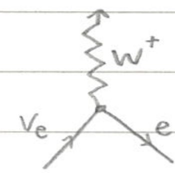
with spinors, $\boxed{j_\mu^- = \bar{\nu}_L \gamma_\mu e_L}$

Couples left-handed electrons to left-handed neutrinos

• Weak isospin & hypercharge

↳





Negatively charged weak current

$\boxed{j_\mu^- = \bar{\nu}_L \gamma_\mu e_L}$

Positively charged weak current

$\boxed{j_\mu^+ = \bar{e}_L \gamma_\mu \nu_L}$

↳ New notation: $\chi_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$, $\tau^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, $\tau^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$

↳ Then $\boxed{j_\mu^\pm = \bar{\chi}_L \gamma_\mu \tau^\pm \chi_L}$

↳ Recall hypercharge (Y); $Q = I^3 + \frac{1}{2}Y$ (Gell-Mann-Nishijima)

↳ Q = Charge, I_3 = 3rd comp. of isospin

↳ Introduce weak hypercharge current:

$$\boxed{j_\mu^Y = 2j_\mu^{em} - 2j_\mu^3 = -2\bar{e}_R\gamma_\mu e_R - \bar{e}_L\gamma_\mu e_L - \bar{\nu}_L\gamma_\mu\nu_L}$$

↳ Invariant construct

↳ Underlying symmetry $\underbrace{SU(2)}_{\text{weak isospin}} \otimes \underbrace{U(1)}_{\text{weak hypercharge}}$

(both chiralities)

※ e.g. for left-handed doublets; $\chi_L \to \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L, \begin{pmatrix} u \\ d' \end{pmatrix}_L, \begin{pmatrix} c \\ s' \end{pmatrix}_L, \begin{pmatrix} t \\ b' \end{pmatrix}_L$

↳ 3 Weak isospin currents $j_\mu = \frac{1}{2}\bar{\chi}_L\gamma_\mu\tau\chi_L$

↳ weak hypercharge current $j_\mu^Y = 2j_\mu^{em} - 2j_\mu^3$

↳ j_μ^{em} = electric current, $\sum_{i=1}^{2} Q_i(\bar{u}_{iL}\gamma_\mu u_{iL} + \bar{u}_{iR}\gamma_\mu u_{iR})$

Introduction to Elementary Particles

6/22/2025 (pp. 353 – 381)

*Lagrangian Formulation of Classical Particle Mechanics

- $F = ma \longrightarrow F = -\nabla U$
  - ↳ If force is conservative
  - ↳ Gradient of scalar potential energy function

- Lagrangian: $L = T - U$, $T = \frac{1}{2}mv^2$
  - ↳ Function of coordinates q_i and time derivatives $\dot{q}_i$
  - ↳ Fundamental law of motion is Euler-Lagrange eqn:

$$\boxed{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) = \frac{\partial L}{\partial q_i}} \quad \leftarrow i = 1, 2, 3$$

- Lagrangians in relativistic field theory
  - ↳ Particle is localized → we want to find position as func. of time
  - ↳ Field occupies region of space → find functions in terms of position and time

∴ Notation: $\partial_\mu \varphi_i \equiv \frac{\partial \varphi_i}{\partial x^\mu}$

1. Klein-Gordon Lagrangian for Scalar (spin-0) field
   - ↳ $\partial_\mu \partial^\mu \varphi + \left(\frac{mc}{\hbar}\right)^2 \varphi = 0$
   - ↳ $\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)(\partial^\mu \varphi) - \frac{1}{2}\left(\frac{mc}{\hbar}\right)^2 \varphi^2$

2. Dirac Lagrangian for spinor (spin-$\frac{1}{2}$) field
   - ↳ $i\gamma^\mu \partial_\mu \psi - \left(\frac{mc}{\hbar}\right)\psi = 0$
   - ↳ $\mathcal{L} = i(\hbar c)\bar{\psi}\gamma^\mu \partial_\mu \psi - (mc^2)\bar{\psi}\psi$

3. Proca Lagrangian for vector (spin-1) field
   - ↳ $\partial_\mu F^{\mu\nu} + \left(\frac{mc}{\hbar}\right)^2 A^\nu = 0$
   - ↳ $\mathcal{L} = -\frac{1}{16}F^{\mu\nu}F_{\mu\nu} + \frac{1}{8\pi}\left(\frac{mc}{\hbar}\right)^2 A^\nu A_\nu$

∴ $F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu$

- Local gauge invariance
  - ↳ We require complete Lagrangian to be invariant under local phase transformations
  - ↳ Introduce massless vector field A^μ:

$$\mathcal{L} = [i\hbar c\bar{\Psi}\gamma^\mu\partial_\mu\Psi - mc^2\bar{\Psi}\Psi] - \left[\frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu}\right] - (q\bar{\Psi}\gamma^\mu\Psi)A_\mu$$

  - ↳ A^μ is electromagnetic potential
  - ↳ Gauge invariance: $A_\mu \to A_\mu + \partial_\mu\lambda$

※ Generates all electrodynamics, specifies current produced by Dirac particles

※ Minimal coupling rule: substitute $\mathcal{D}_\mu$ for ∂_μ
  - ↳ $\mathcal{D}_\mu \equiv \partial_\mu + i\frac{q}{\hbar c}A_\mu$
  - ↳ Converts globally invariant Lagrangian into locally invariant one

- Yang-Mills Theory
  - ↳ Assume 2 spin-$\frac{1}{2}$ fields Ψ_1 and Ψ_2
  - ↳ $\Psi \equiv \begin{pmatrix}\Psi_1 \\ \Psi_2\end{pmatrix}$, $\bar{\Psi} = (\bar{\Psi}_1, \bar{\Psi}_2)$
  - ↳ Lagrangian: $\mathcal{L} = i\hbar c\bar{\Psi}\gamma^\mu\partial_\mu\Psi - c^2\bar{\Psi}M\Psi$, $M = \begin{pmatrix} m_1 & 0 \\ 0 & m_2\end{pmatrix}$
  - ↳ Assume $m_1 = m_2$, and global $SU(2)$ invariance of Lagrangian to be held locally
  - ↳ Yang-Mills Lagrangian:

$$\mathcal{L} = [i\hbar c\bar{\Psi}\gamma^\mu\partial_\mu\Psi - mc^2\bar{\Psi}\Psi] - \frac{1}{16\pi}F^{\mu\nu}\cdot F_{\mu\nu} - (q\bar{\Psi}\gamma^\mu\tau\Psi)\cdot A_\mu$$

  - ↳ $F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu - \frac{2q}{\hbar c}(A^\mu \times A^\nu)$
  - ↳ τ_1, τ_2, τ_3 are the Pauli matrices

※ "Dirac field generates 3 currents $J^\mu \equiv cq(\bar{\Psi}\gamma^\mu\tau\Psi)$ which act as sources for gauge fields: $\mathcal{L} = -\frac{1}{16\pi}F^{\mu\nu}\cdot F_{\mu\nu} - \frac{1}{c}J^\mu\cdot A_\mu$"

• Chromodynamics

↳ Free lagrangian for a flavor: $\mathcal{L} = i\hbar c \bar{\Psi}\gamma^\mu \partial_\mu \Psi - mc^2 \bar{\Psi}\Psi$

↳ $\Psi = \begin{pmatrix} \Psi_r \\ \Psi_b \\ \Psi_g \end{pmatrix}$, $\bar{\Psi} = (\bar{\Psi}_r, \bar{\Psi}_b, \bar{\Psi}_g)$ for 3 colors

↳ Invariant under $U(3)$: $\Psi \to U\Psi$ ($\bar{\Psi} \to \bar{\Psi}U^\dagger$)

where U is 3×3 unitary, $U^\dagger U = I$

※ Any unitary matrix can be written as exponentiated Hermitian

$U = e^{iH}$, $H^\dagger = H$

※ Any 3×3 Hermitian matrix can be written as

$H = \theta I_3 + \lambda \cdot a$

↳ $\lambda \cdot a \equiv \lambda_1 a_1 + \lambda_2 a_2 + \ldots + \lambda_8 a_8$, λ_i are Gell-Mann matrices

a_i are real numbers

$\therefore$ $\boxed{U = e^{i\theta} e^{i\lambda \cdot a}}$

↳ Modify $\mathcal{L}$ to make it invariant under local $SU(3)$ gauge transformations

↳ Make replacement $\mathcal{D}_\mu \equiv \partial_\mu + i\frac{q}{\hbar c}\lambda \cdot A_\mu$

↳ Assign to gauge fields A_μ a rule $\mathcal{D}_\mu \Psi \to S(\mathcal{D}_\mu \Psi)$

↳ After some math,

$$\mathcal{L} = [i\hbar c \bar{\Psi}\gamma^\mu \partial_\mu \Psi - mc^2 \bar{\Psi}\Psi] - \frac{1}{16\pi} F^{\mu\nu} \cdot F_{\mu\nu} - (q\bar{\Psi}\gamma^\mu \lambda \Psi) \cdot A_\mu$$

↳ Complete Lagrangian for Chromodynamics

↳ Interaction of 3 equal-mass Dirac fields (3 colors) w/ 8 massless vector fields (gluons)

↳ Global $SU(3)$ symmetry holds locally ☺

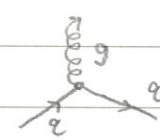
* Feynman Rules

- Each $\mathcal{L}$ consists of free Lagrangian (for each participating field) and interaction terms ($\mathcal{L}_{int}$)
  - ↳ Free Lagrangian → Propagator
  - ↳ Interaction terms → vertex factors

- Propagators
  - ↳ Spin-0: $\dfrac{i}{p^2-(mc)^2}$
  - ↳ Spin-1: $\dfrac{-i}{p^2-(mc)^2}\left[g_{\mu\nu}-\dfrac{P_\mu P_\nu}{(mc)^2}\right]$
  - ↳ Spin-$\frac{1}{2}$: $\dfrac{i(\not{p}+mc)}{p^2-(mc)^2}$
  - ↳ Massless Spin-1: $-i\dfrac{g_{\mu\nu}}{p^2}$

  $\boxed{\text{e.g. photon}}$

- Vertex factors
  - ↳ write $i\mathcal{L}_{int}$ in momentum space ($i\hbar\partial_\mu \to p_\mu$) and "rub out" involved field variables
  - ↳ e.g. QED: $i\mathcal{L}_{int} = -i(q\bar{\Psi}\gamma^\mu\Psi)A_\mu \longrightarrow -i\sqrt{\dfrac{4\pi}{\hbar c}}\,q\gamma^\mu = ig_e\gamma^\mu$
    - ↳ Rubbed out $\bar{\Psi}, \Psi, \sqrt{\hbar c/4\pi}\,A_\mu$ (QED vertex factor)
  - ↳ Chromodynamics quark-gluon coupling: $-i\dfrac{g_s}{2}\gamma^\mu\lambda$



  - ↳ etc.

※ Higgs mechanism

- ↳ Imparts mass to gauge fields
- ↳ Responsible for masses of $W^\pm$ and Z^0
- ↳ All fundamental interactions (weak, strong, electromagnetic) can be described by local gauge theories.

Introduction to Elementary Particles

6/22/2025 (pp. 387-421)

* Neutrino Oscillations

- Basically quantum mechanics of mixed states
  - ↳ Linear combinations: $\nu_1 = \cos\theta\, \nu_\mu - \sin\theta\, \nu_e$
    $\nu_2 = \sin\theta\, \nu_\mu + \cos\theta\, \nu_e$
  - ↳ Simple time dependence $e^{-iE_1 t/\hbar}$: $\nu_1(t) = \nu_1(0) e^{-iE_1 t/\hbar}$
    $\nu_2(t) = \nu_2(0) e^{-iE_2 t/\hbar}$

- E.g. if particle starts as ν_e
  - ↳ $\nu_e(0) = 1$, $\nu_\mu(0) = 0$
  - ↳ $\nu_1(0) = -\sin\theta$, $\nu_2(0) = \cos\theta$
  - ↳ Then $\nu_1(t) = -\sin\theta\, e^{-iE_1 t/\hbar}$, $\nu_2(t) = \cos\theta\, e^{-iE_2 t/\hbar}$
  - ↳ $\nu_\mu(t) = \sin\theta\cos\theta\left(-e^{-iE_1 t/\hbar} + e^{-iE_2 t/\hbar}\right)$

$$\therefore \boxed{P_{\nu_e\to\nu_\mu} = |\nu_\mu(t)|^2 = \left[\sin(2\theta)\sin\left(\frac{E_2-E_1}{2\hbar}t\right)\right]^2}$$

Probability that $\nu_e \to \nu_\mu$

  - ↳ At relativistic conditions, $E_2 - E_1 \approx \frac{(m_2^2 - m_1^2)}{2E}c^4$

$$\boxed{P_{\nu_e\to\nu_\mu} = \left\{\sin(2\theta)\sin\left[\frac{(m_2^2-m_1^2)c^4}{4\hbar E}t\right]\right\}^2}$$

※ Requirements for neutrino oscillations:

1. There must be mixing (θ)
2. Masses must be unequal

※ Chp. 12 (Afterward) skimmed