ABSTRACT

Extending Quantum Prisoner's Dilemma to the Optional Prisoner's Dilemma

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Background

Prisoner's Dilemma is a common example in game theory, with extensions to biology, psychology, and even economics. The players Alice and Bob can choose to either cooperate or defect with each other, and their choice determines their potential payoffs. In 1999, Eisert et al. proposed a quantized version of Prisoner's Dilemma, where players manipulate qubits with unitary gates. The qubits are then measured to determine the payoffs. Eisert found that a quantum player with access to the full set of unitary gates could consistently gain an advantage over a classical player [1].

Purpose

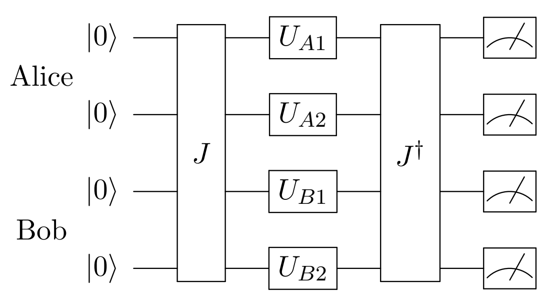
In this paper, we build off Eisert's paper to extend the 3x3 Optional Prisoner's Dilemma (OPD) to a quantum circuit. The 3x3 OPD is a rarer variant of the 2x2 Prisoner's Dilemma. In the OPD, players have an additional choice: abstaining. This more accurately models real-life situations, where abstaining from a choice is an implicit option (e.g. procrastination).

Methods

Here Bob is the quantum player, and Alice is the classical player. The players are given 1 extra qubit to account for all 3 choices. We use the same entangling gate J as Eisert, but adapted for 4 qubits. The new payoff table with C (cooperate), D (defect), and A (abstain) is

		Bob		
		C	D	A
	C	(3, 3)	(0, 5)	(2, 2)
Alice	D	(5, 0)	(1, 1)	(2, 2)
	A	(2, 2)	(2, 2)	(2, 2)

The circuit is similar to the classical 2x2 Prisoner's Dilemma. However, since there are 3 choices, each player must have 2 qubits to represent all their possible choices.



We associate a measurement of $|00\rangle$ with "cooperate" and $|11\rangle$ with "defect." For symmetry, both $|01\rangle$ or $|10\rangle$ is associated with "abstain." Bob has access to all 2x2 unitary gates, while Alice is limited [2]. Carrying out the calculations, the quantum player can gain an advantage over the classical player in all cases by using the appropriate unitary gates. However, the classical player can nullify any advantage by choosing to abstain.

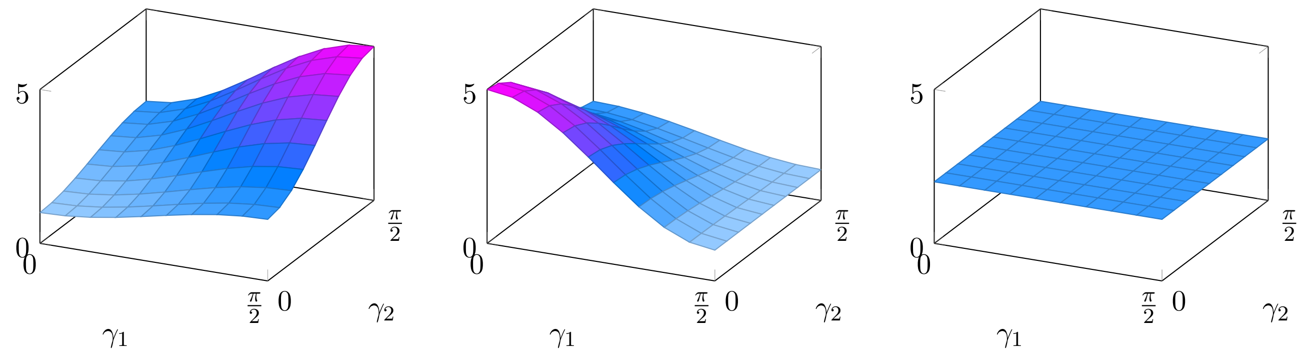


Figure 1: (From left to right) Bob's potential payoff as a function of his decision parameters if Alice 1) cooperates, 2) defects, or 3) abstains.

A modification to the payoff table causes Bob's quantum advantage to return in all cases. We do this by imbalancing the payoffs: if one or more players abstain, the abstaining player receives a 2, while a cooperating or defecting player receives a 2.5. Realistically, we're penalizing indecision. An indecisive government, for example, may be less favored than a decisive one that makes mistakes.

(θ_1, θ_2), Alice's play	Alice's payoff	Bob's payoff
$(0, 0)$ Alice cooperates	1.5	2.5
$(0, \pi/2)$ Alice abstains	2	2.25
$(\pi/2, 0)$ Alice abstains	2	2.25
$(\pi/2, \pi/2)$ Alice defects	1.5	2.5

Results

In an imbalanced OPD that penalizes abstaining, Bob wins in all cases. However, his advantage disappears if one player can force a fixed payoff, e.g. by abstaining in the unmodified OPD. It seems that an imbalance in the table may be required for the quantum player to have an advantage.

Given the applicability of Prisoner's Dilemma in several fields, generalizing it to quantum probabilities is an interesting take [1]. The Optional Prisoner's Dilemma, which can account for indecision, may be useful in that regard, modeling real-life situations more accurately.

References

[1] Jens Eisert, Martin Wilkens, and Maciej Lewenstein. "Quantum Games and Quantum Strategies". In: *Physical Review Letters* 83.15 (Oct. 1999), pp. 3077–3080. ISSN: 1079-7114. DOI: `10.1103/physrevlett.83.3077`. URL: `http://dx.doi.org/10.1103/PhysRevLett.83.3077`.

[2] Adrian P. Flitney and Derek Abbott. *An introduction to quantum game theory*. 2002. arXiv: `quant-ph/0208069 [quant-ph]`. URL: `https://arxiv.org/abs/quant-ph/0208069`.