Light & Heat

week 7 Notes

* Oscillatory Motion

- Simple Harmonic Motion (SHM) requires restoring force
  - ↳ Restoring force most easily generated by linear force.

    $F(x) \sim -(x-x_0)$ stems from quadratic potential

    $$F(x) = \frac{-dU}{dx} \sim -x \rightarrow U(x) \sim \frac{1}{2}x^2$$

- Stable equilibrium point can be approximated by quadratic potential

  $$F(x=x_0) = -\frac{dU(x)}{dx}\Big|_{x=x_0} = 0$$

  $$U(x) = \cancel{U(x=x_0)} + \cancel{\frac{dU(x)}{dx}\Big|_{x=x_0} x} + \boxed{\frac{1}{2!}\frac{d^2U(x)}{dx^2}\Big|_{x=x_0} x^2} + \frac{1}{3!}\frac{d^3U(x)}{dx^3}\Big|_{x=x_0} x^3 \ldots$$

  $$U(x) \approx \frac{1}{2!}\frac{d^2U(x)}{dx^2}\Big|_{x=x_0} x^2$$

- Transverse waves: oscillation of media is perpendicular to direction of wave travel.
- Longitudinal waves: oscillation of media is parallel to direction of wave travel.

* Mathematical description of waves

- Displacement of media from equilibrium described via wavefunction

$$f(x, t)$$

※ 1-D: e.g. wave on string, $y(x,t)$

2-D: e.g. ripples from pond, $z(x, y, t)$

3-D: e.g. waves from point source; $P(x, y, z, t)$

※ Displacements can have many dimensions too.

↳ e.g. string oscillating in yz plane if wave pulse travels in x direction

$$\vec{f}(x, t) = y(x,t)\hat{j} + z(x,t)\hat{k}$$

- Harmonic wave: sinusoidally varying waves

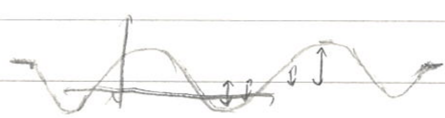
$$y(x,t) = A\sin\left(2\pi\left(\frac{x}{\lambda} - ft\right)\right) = A\sin\left(2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right)$$

Light & Heat

Week 8 Notes

* Waves

- 1-D wave equation for waves on a string
  - ↳ Tension & mass density uniform
  - ↳ Small displacement

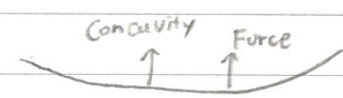


$$\frac{\partial^2 y(x,t)}{\partial t^2} = \frac{T}{\mu}\frac{\partial^2 y(x,t)}{\partial^2 x}$$

- 1-D wave equation
  - ↳ Mathematical shape $\Psi(x)$
  - ↳ Move in x-direction by $x \pm vt$

$$\frac{\partial^2 \Psi}{\partial t^2} = v^2 \frac{\partial^2 \Psi}{\partial x^2}$$

- Wavefunction must satisfy wave equation $\frac{\partial^2 y(x,t)}{\partial t^2} = \frac{T}{\mu}\frac{\partial^2 y(x,t)}{\partial x^2}$
  - ↳ $\frac{\partial^2 y}{\partial t^2}$: Acceleration
  - ↳ $\frac{\partial^2 y}{\partial x^2}$: Concavity



- Solutions of wave equation
  - ↳ Recall harmonic wave function $y(x,t) = A\sin\left(2\pi\left(\frac{x}{\lambda} - ft\right)\right)$

equal

$$\begin{cases} \frac{\partial^2 y}{\partial t^2} = -A(2\pi f)^2 \sin\left(2\pi\left(\frac{x}{\lambda} - ft\right)\right) \\ \frac{T}{\mu}\frac{\partial^2 y}{\partial x^2} = -A\frac{T}{\mu}\left(\frac{2\pi}{\lambda}\right)^2 \sin\left(2\pi\left(\frac{x}{\lambda} - ft\right)\right) \end{cases}$$

$$\downarrow$$

$$\frac{T}{\mu}\frac{1}{\lambda^2} = f^2 \rightarrow \frac{T}{\mu} = (f\lambda)^2$$

But $v = f\lambda$, so $\frac{T}{\mu} = v^2$, $\boxed{\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}}$

General form for 1D wave eq'n.

※ If a wave solution can be expressed as $f(x - vt)$, then it will satisfy the 1D wave equation.

- Define:
  - ↳ Wavenumber $k = \frac{2\pi}{\lambda}$
  - ↳ Angular frequency $\omega = 2\pi f$

∴ Then wavefunction is $y(x,t) = A\sin(kx - \omega t)$

✱ Linear Superposition

- Given 2 solutions to wave equation $\Psi_1(x,t)$ and $\Psi_2(x,t)$, a linear combo of two solutions is also a solution,

$$\Psi_3(x,t) = A\Psi_1(x,t) + B\Psi_2(x,t)$$

- Waves reflecting off boundaries
  - ↳ Left-moving wave $f(x - vt)$
  - ↳ Right-moving wave $g(x + vt)$

General

1. String attached to rigid wall

$$y(x,t) = Af(x - vt) + Bg(x + vt)$$

$$y(0,t) = 0 = Af(-vt) + Bg(vt) \longrightarrow f(-vt) = -g(vt)$$

(A&B are constants)

$$\therefore g(x + vt) = -f(-x - vt)$$

$$\boxed{y(x,t) = f(x - vt) - f(-x - vt)}$$

2. String attached to pole, free to move

$$y(x,t) = f(x - vt) + f(-x - vt)$$

(wave reflects same way - not flipped)

- Waves reflecting off boundaries

1. Fixed boundaries

Harmonic wavefunction

$$y(x,t) = A\sin(kx-\omega t) - A\sin(-kx-\omega t)$$
$$= A(\sin(kx-\omega t) + \sin(kx+\omega t))$$

Use $2\sin(\alpha)\cos(\beta) = \sin(\alpha+\beta) + \sin(\alpha-\beta)$

$$y(x,t) = 2A\sin(kx)\cos(\omega t)$$

Amplitude $2A\cos(\omega t)$

※ A <u>standing wave</u> $y(x,t) = A\sin\left(\frac{n\pi}{L}x\right)\cos(\omega t)$

Node = places where amplitude is 0, $\sin(kx) = 0$

Antinode = places where amplitude is max, $\sin(kx) = 1$

Only certain frequencies can exist for standing waves.

Other freq's will interfere & dampen out

Possible L for standing wave: $\sin(kL) = 0$, $\lambda = \frac{2L}{n}$, $f = \frac{nv}{2L}$

↳ Discrete spectrum of allowed freq's = modes (n)

※ General motion of string can be decomposed into linear superposition of modes

$$y(x,t) = \sum_{n=1}^{\infty} 2A_n \sin(k_n x)\cos(\omega_n t + \phi_n)$$

＊ Energy transmission via waves

- For wave traveling right w/ speed v, a segment of string has transverse velocity

$$v_{trans} = \frac{\partial y(x,t)}{\partial t} = -A\omega\cos(kx-\omega t)$$

$$\text{Power} = P = \vec{T}\cdot\vec{v}_{trans} = T\sin\theta\left[-A\omega\cos(kx-\omega t)\right]$$

For small displacements, $\sin\theta \approx \tan\theta \approx \partial y/\partial x$

$$P \approx T\frac{\partial y}{\partial x}\left[-A\omega\cos(kx-\omega t)\right] = TA^2 k\omega\cos^2(kx-\omega t)$$

Substitute $v = \sqrt{\frac{T}{\mu}} \rightarrow T = v^2\mu$ and $v = \frac{\omega}{k}$

$$P \approx A^2\mu v\omega^2\cos^2(kx-\omega t)$$

Find average P:

$$P_{avg} = \frac{1}{T}\int_0^T P\,dt = A^2\mu v\omega^2\frac{1}{T}\underbrace{\int_0^T\cos^2(kx-\omega t)\,dt}_{\frac{T}{2}}$$

$$= \frac{1}{2}\mu v\omega^2 A^2$$

- Avg. E within length Δx:

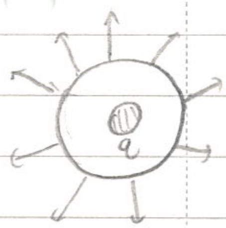
$$\Delta E_{avg} = P_{avg}\Delta t = \frac{1}{2}\mu v\omega^2 A^2\Delta x$$

※ Power transmitted & energy per unit length is proportional to the amplitude of the wave squared.

Light & Heat

week 9 Notes

* Maxwell's Equations



- Gauss' Law: $\oint_S E_n \, dA = \oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{inside}}{\epsilon_0}$
  - ↳ Net flux through Gaussian surface is proportional to charge enclosed

- No magnetic monopoles: $\oint_S \vec{B} \cdot d\vec{A} = 0$
  - ↳ Same thing but with magnetic fields

- Faraday's Law: $\oint_C E \cdot d\vec{l} = -\frac{\partial}{\partial t} \int_S \vec{B} \cdot d\vec{A}$
  - ↳ Time derivative of magnetic flux is related to amount of emf produced.

- Ampere's Generalized Law: $\oint_C B \cdot d\vec{l} = \mu_0 I + \mu_0 \epsilon_0 \frac{\partial}{\partial t} \int_S E_n \, dA$
  - ↳ Magnetic field along line integral of Amperian closed loop is proportional to the enclosed current.

- Some math operators
  - ↳ $\vec{\nabla}$: $\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$
  - ↳ Gradient: $\nabla f = \hat{i}\frac{\partial f}{\partial x} + \hat{j}\frac{\partial f}{\partial y} + \hat{k}\frac{\partial f}{\partial z}$ (scalar func → vector func)
  - ↳ Divergence: $\nabla \cdot \vec{f} = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z}$ (vector func → scalar func)
  - ↳ Curl: $\nabla \times \vec{f} = \left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right] \times \left[\hat{i} f_x(x,y,z) + \hat{j} f_y(x,y,z) + \hat{k} f_z(x,y,z)\right]$

    (vector func → vector func)

- Differential forms
  - ↳ Gauss' Law: $\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$
  - ↳ No magnetic monopoles: $\nabla \cdot \vec{B} = 0$
  - ↳ Faraday's Law: $\nabla \times \vec{E} = -\frac{\partial}{\partial t}\vec{B}$
  - ↳ Ampere's generalized law: $\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial}{\partial t}\vec{E}$

displacement vector

- Maxwell's Equations in free space (vacuum Maxwell equations)
  - ↳ No sources: no charge or current
  - ↳ $\nabla \cdot \vec{E} = 0$

$\nabla \cdot \vec{B} = 0$

$\nabla \times \vec{E} = -\frac{\partial}{\partial t}\vec{B}$

$$\nabla \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial}{\partial t}\vec{E} \xrightarrow{\text{Time deriv.}} \nabla \times \frac{\partial \vec{B}}{\partial t} = \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2}\vec{E}$$

$$-\nabla \times [\nabla \times \vec{E}] = \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2}\vec{E}$$

Limit to x,t: $\vec{E}(x,t) = \hat{i}E_x(x,t) + \hat{j}E_y(x,t) + \hat{k}E_z(x,t)$

$$\partial_y \vec{E} = \partial_z \vec{E} = 0$$

$$\nabla \times \vec{E} = \hat{i}\partial_x \times [\hat{i}E_x + \hat{j}E_y + \hat{k}E_z] = \hat{k}\partial_x E_y - \hat{j}\partial_x E_z$$

$$\nabla \times \nabla \times \vec{E} = \hat{i}\partial_x \times [\hat{k}\partial_x E_y - \hat{j}\partial_x E_z] = -\hat{j}\partial_x^2 E_y - \hat{k}\partial_x^2 E_z$$

$$\hat{j}\frac{\partial^2 E_y}{\partial x^2} + \hat{k}\frac{\partial^2 E_z}{\partial x^2} = \mu_0 \epsilon_0 \left(\hat{i}\frac{\partial^2 E_x}{\partial t^2} + \hat{j}\frac{\partial^2 E_y}{\partial t^2} + \hat{k}\frac{\partial^2 E_z}{\partial t^2}\right)$$

Each component independent, $\frac{\partial^2 E_x}{\partial t^2} = 0$

※ Maxwell's wave equation (for electric field)

$$\frac{\partial^2 E_y}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_y}{\partial t^2}$$

$\vec{E}(x,t)$, represents wave traveling in x direction oscillating transversely.

$$V_{EM\ wave} = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \approx 3 \times 10^8 \ \frac{m}{s} = c$$

- Takeaways
  - ↳ EM wave travels at same speed as light
  - ↳ Transverse oscillations
  - ↳ Harmonic solutions of form $E(x,t) = E_0 \sin(kx - \omega t)$
  - ↳ Now $c = \frac{\omega}{k}$

※ The wave is not an oscillation of some medium (waves travel through empty space)

- Consider EM wave traveling in x-direction and $\vec{E} = \hat{j} E_y(x,t)$ only.

$$\nabla \times \vec{E} = -\frac{\partial}{\partial t}\vec{B}$$

$$\hat{i}\partial_x \times \hat{j} E_y = \hat{k}\,\partial_x E_y = -\frac{\partial}{\partial t}\vec{B}$$

Thus $\vec{B} \sim \hat{k}$

For $\vec{E} = \hat{j} E_0 \cos(kx - \omega t)$ and $\vec{B} = \hat{k} B_0 \cos(kx - \omega t)$,

$$\hat{k}\,\partial_x E_y = -\hat{k}(-\omega) B_0 \sin(kx - \omega t)$$

$$\hat{k} E_0 \sin(kx - \omega t) = \omega B_0 \sin(kx - \omega t)$$

$$E_0 = \frac{\omega}{k} B_0 = c B_0$$

Relates magnitudes of $\vec{E}$ and $\vec{B}$

* Energy & Momentum of EM waves

- For parallel plate capacitors:

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{\epsilon_0 A}{d}\right)(Ed)^2 = \frac{1}{2}\epsilon_0 E^2 (Ad)$$

Per unit volume, $u_E = \frac{U}{V} = \frac{1}{2}\epsilon_0 E^2$

- For an inductor:

$$U = \frac{1}{2}LI^2 = \frac{1}{2}(\mu_0 n^2 A l)\left(\frac{B}{\mu_0 n}\right)^2 = \frac{B^2}{2\mu_0} A l$$

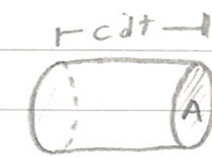
Per unit volume, $u_B = \frac{U}{V} = \frac{B^2}{2\mu_0}$

- For an EM wave, $|E| = c|B|$

$$u_B = \frac{B^2}{2\mu_0} = \frac{E^2}{2c^2\mu_0} = \frac{\mu_0\epsilon_0 E^2}{2\mu_0} = \frac{1}{2}\epsilon_0 E^2 = u_E$$

Total energy in EM wave: $u = u_E + u_B = \epsilon_0 E^2 = \frac{B^2}{\mu_0}$ $\Big\}$ $\frac{EB}{\mu_0 c}$

- Total energy in EM wave



↳ Volume: $c\Delta t A$

↳ Energy inside volume: $u(c\Delta t A)$

↳ Power through Cross Section: $\frac{uc\Delta t A}{\Delta t}$

↳ Power flowing per unit cross-sectional area $= \frac{uc\Delta t A}{\Delta t A} = uc$

※ This is the magnitude of the Poynting vector.

$$|\vec{S}| = uc = \frac{EB}{\mu_0}$$

$$\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0} \quad \text{(power flows in dir. of propagation)}$$

※ Intensity: $I = \frac{1}{T}\int_0^T S\,dt = \frac{1}{\mu_0}E_0 B_0 \frac{1}{T}\int_0^T \cos^2(kx - \omega t)\,dt = \frac{1}{2\mu_0}E_0 B_0$

$$I = c\frac{\epsilon_0}{2}E_0^2$$

- Electromagnetic momentum (loose deriv.)
  - ↳ Assume energy U in volume $V = c\Delta t A$ is completely absorbed, then work done = energy transmitted,

$$U = W = \vec{F}\cdot\vec{d}$$

$$I = \frac{U}{\Delta t A} = \frac{\vec{F}\cdot\vec{d}}{\Delta t A} = \frac{Fc\Delta t}{\Delta t A} = c\frac{F}{A} \quad \text{(intensity)}$$

$$= cP \quad (P = \text{pressure})$$

$$P = \frac{I}{c} = \frac{\epsilon_0}{2}E_0^2$$

Use $F = \frac{dp}{dt} \longrightarrow I = c\frac{dp}{dtA} = \frac{U}{\Delta t A} \longrightarrow dp = \frac{U}{c}\frac{dt}{\Delta t}$

Integrate from 0 to Δt, $p = \frac{U}{c}$ (Electromagnetic momentum)

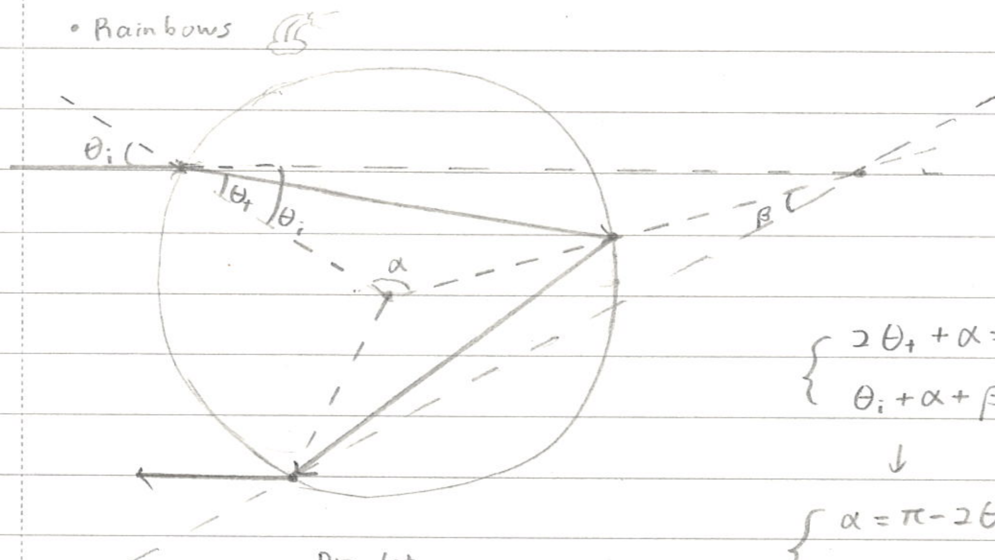
Light & Heat

Week 10 Notes

* Dispersion

- Index of refraction is dependent on wavelength in general

- Rainbows



Droplet

$$\begin{cases} 2\theta_t + \alpha = \pi \\ \theta_i + \alpha + \beta = \pi \end{cases}$$

$$\downarrow$$

$$\begin{cases} \alpha = \pi - 2\theta_t \\ \beta = 2\theta_t - \theta_i \end{cases}$$

↳ Snell's Law: $n \sin\theta_t = \sin\theta_i$

$$\boxed{\beta = 2\sin^{-1}\left(\frac{\sin\theta_i}{n}\right) - \theta_i}$$

※ Greatest irradiance is at maximum value of β

$$\frac{d\beta_{max}}{d\theta_i} = 0 = 2\frac{d}{d\theta_i}\left[\sin^{-1}\left(\frac{\sin\theta_i}{n}\right)\right] - 1$$

$$= \frac{2}{\sqrt{1 - \frac{\sin^2\theta_i}{n^2}}} \cdot \frac{d}{d\theta_i}\left[\frac{\sin\theta_i}{n}\right] - 1$$

$$= \frac{2}{\sqrt{1 - \frac{\sin^2\theta_i}{n^2}}} \frac{\cos\theta_i}{n} - 1$$

$$\therefore \frac{1}{2} = \sqrt{\frac{1 - \sin^2\theta_i}{n^2 - \sin^2\theta_i}} \longrightarrow \sin\theta_i = 0.8604 \text{ or } 59.4^\circ$$

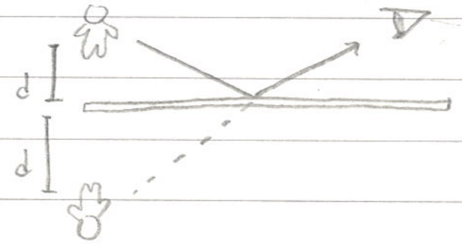
$\beta_{max} = 21.3^\circ$

$2\beta_{max} = 42^\circ$

↑ For a rainbow to be seen

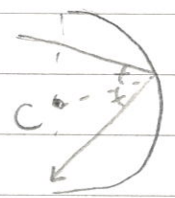
* Plane mirrors & spherical mirrors

- Law of reflection: angle of incidence = angle of reflection

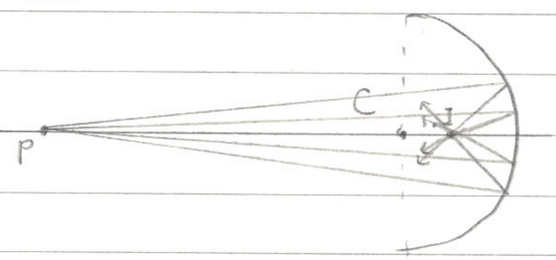


- Spherical mirrors

↳ All rays reflect about the normal, which passes through C



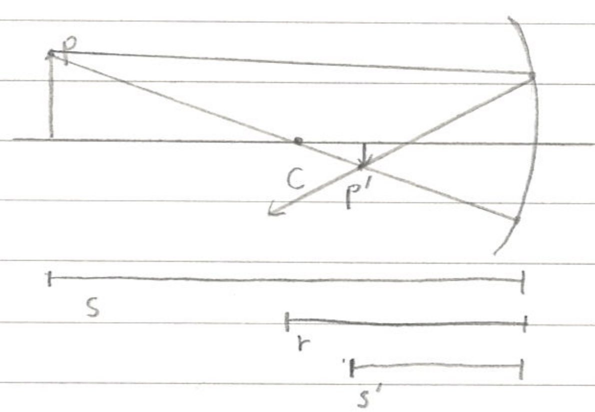
↳ Paraxial rays: emanate from object at P but close to central axis



Paraxial rays cross at image point I

But in general, rays will not all cross at same point

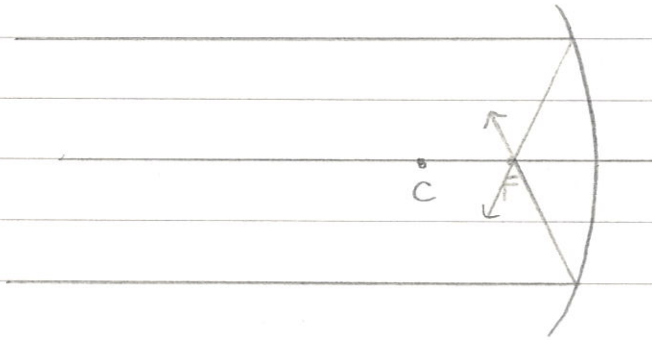
↳ Spherical aberration



$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{r}$$

for small angles

• For objects at ∞



All rays pass through $r/2$

F is focal point

focal length, $f = r/2$

$$\boxed{\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}}$$

↳ Lateral magnification: $m = \frac{y'}{y} = -\frac{s'}{s}$

(Just ratio of heights)

(Negative sign indicates inversion)

• How to draw ray diagrams of mirrors

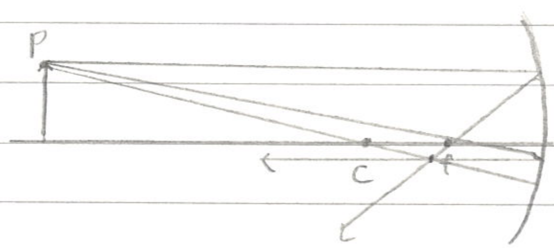
↳ Finding where image is

↳ Draw 3 rays:

Parallel ray: parallel to axis, passes through f

Focal ray: passes through f, reflects parallel to axis

Radial ray: passes through C



↳ Sign convention

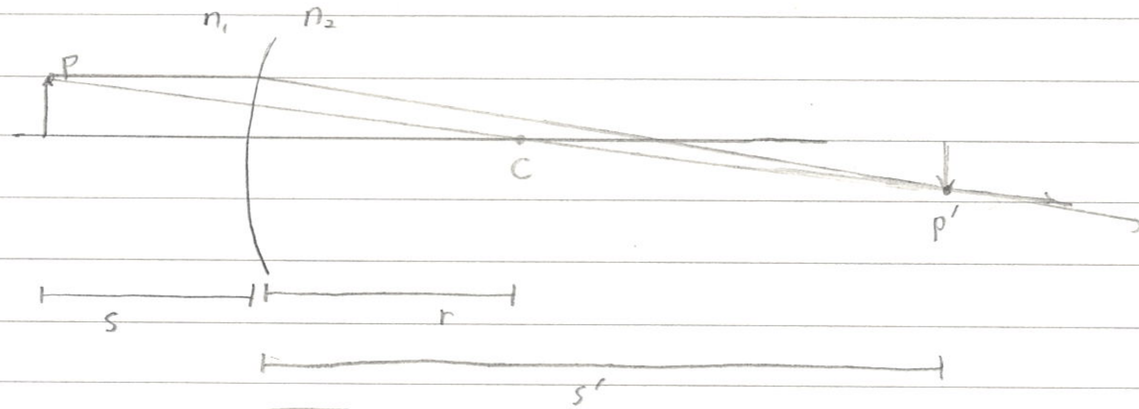
s is ⊕ if obj. is on incident-light side of mirror

s′ is ⊕ if image is on reflected-light side of mirror

f is ⊕ if C is on reflected-light side of mirror

(concave ⊕, convex ⊖)

- Images formed by refraction



$$\boxed{\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{r}}$$

↳ Sign convention

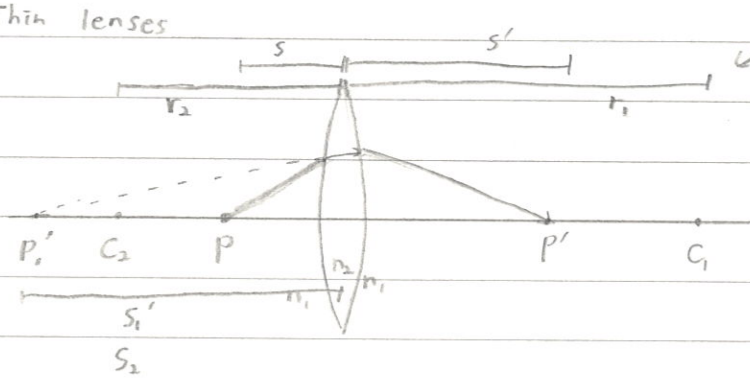
s is ⊕ if obj. is on incident-light side

s' is ⊕ if image is on refracted-light side

r is ⊕ if C is on refracted-light side

↳ Magnification: $m = \frac{y'}{y} = -\frac{n_1 s'}{n_2 s}$

- Thin lenses



← Radius of curvature

$r_2 > 0$ } here
$r_1 < 0$

(consider from refracted light side)

↳ Lens maker formula: $\boxed{\frac{1}{s} + \frac{1}{s'} = \frac{(n_2 - n_1)}{n_1}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]}$

↳ Thin lens equation: $\boxed{\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}}$

↳ Sign convention

s is $\oplus$ if on light-incident side

s' is $\oplus$ if on light-refracted side

f depends on r_1, r_2

r is $\oplus$ if C is on refracted light side

↳ Power of a lens: $P = \frac{1}{f}$ [measured in diopters, m^{-1}]

Light & Heat

Week 12 Notes

* Polarization

- Direction of oscillation of E-field component of an EM wave

- Recall:

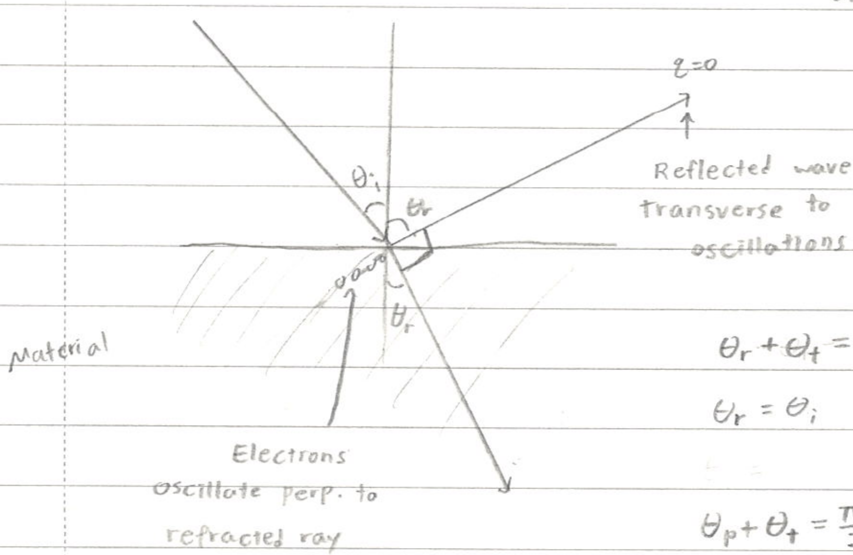
$$\nabla^2 \vec{E} = \frac{\partial^2 \vec{E}}{\partial x^2} + \frac{\partial^2 \vec{E}}{\partial y^2} + \frac{\partial^2 \vec{E}}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\vec{E} = E_0 \cos(kx - \omega t)$$

- For an E-field traveling in z-dir
  - ↳ Right-circularly polarized: $\hat{i} E_0 \cos(kz-\omega t) + \hat{j} E_0 \sin(kz-\omega t)$
  - ↳ Left-circularly polarized: same with $\ominus$
  - ↳ Elliptical polarized: $\vec{E}(z,t) = \hat{i} E_{0x} \cos(kz-\omega t) + \hat{j} E_{0y} \cos(kz - \omega t + \epsilon)$
    - ↳ $E_{0x} \neq E_{0y}$
    - $\epsilon \neq 0, \pm \frac{n}{2}\pi, \pi$
  - ↳ Unpolarized light: randomly fluctuating polarization

$$\vec{E}(z,t) = \hat{i} E_0 \cos(kz - \omega t + \epsilon_x) + \hat{j} E_0 \cos(kz - \omega t + \epsilon_y)$$

Randomly oscillating



$$\theta_r + \theta_t = \frac{\pi}{2}$$

$\theta_r = \theta_i$ (Law of reflection)

$\theta_p + \theta_t = \frac{\pi}{2}$ (Brewster's angle)

$\tan(\theta_p) = \frac{n_t}{n_i}$ (Brewster's Law)

Reflected light is maximally polarized ⊥ to plane of incidence

- Polarized filters

  ↳ Malus's Law: $\boxed{I(\theta) = I_0 \cos^2\theta}$

  Irradiance after passing through

* Bonus: mathematical representation of polarization

- Use $e^{i\theta} = \cos\theta + i\sin\theta$

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}, \quad \sin\theta = \frac{e^{i\theta} + e^{-i\theta}}{2i}$$

- Define $\vec{\mathcal{E}}_H(z,t) = \hat{i}\,\mathcal{E}_0 e^{i(kz-\omega t)}$, $\vec{\mathcal{E}}_V(z,t) = \hat{j}\,\mathcal{E}_0 e^{i(kz-\omega t)}$

- Then e.g. for right & left circularly polarized

$$\vec{\mathcal{E}}_R(z,t) = \frac{\mathcal{E}_0}{\sqrt{2}}\left(\hat{i}e^{i(kz-\omega t)} + i\hat{j}e^{i(kz-\omega t)}\right)$$

$$\vec{\mathcal{E}}_L(z,t) = \frac{\mathcal{E}_0}{\sqrt{2}}\left(\hat{i}e^{i(kz-\omega t)} - i\hat{j}e^{i(kz-\omega t)}\right)$$

- Convert back by taking real component: $\vec{E}(z,t) = \mathrm{Re}\left(\vec{\mathcal{E}}(z,t)\right)$

Light & Heat

week 13 Notes

* Coherent Light (independent light sources)

- Consider 2 harmonic sol'ns to wave eqn.

$$E_1(x,t) = E_{01}\sin(\omega_1 t - k_1 x - \epsilon_1)$$
$$E_2(x,t) = E_{02}\sin(\omega_2 t - k_2 x - \epsilon_2)$$

↳ Monochromatic if $\omega_1 = \omega_2$

↳ Equal amplitude if $E_{01} = E_{02}$

↳ phase difference $= \epsilon_1 - \epsilon_2$

↳ Coherent if phase difference is const. in time

- Linear superposition

$$E_3(x,t) = E_1(x,t) + E_2(x,t)$$
$$= \underbrace{2E_0\cos\left(\frac{k(x_1-x_2)+(\epsilon_1-\epsilon_2)}{2}\right)}_{\text{Amplitude}}\sin\left(\omega t - \frac{k(x_1+x_2)}{2} - \frac{(\epsilon_1-\epsilon_2)}{2}\right)$$

$$= 2\tilde{E}_0\sin(\omega t - k\tilde{x} - \tilde{\epsilon}) \quad , \quad \tilde{E}_0 = 2E_0\cos\left(\frac{\delta}{2}\right)$$

↳ Same frequency ω, different amplitude & phase

↳ when $\delta = 0, \pm 2\pi, \pm 4\pi \ldots$ max amplitude

$= \pm\pi, \pm 3\pi, \pm 5\pi \ldots$ min amplitude

(phase diff.) $\quad \delta \equiv (kx_1 + \epsilon_1) - (kx_2 + \epsilon_2) = \frac{2\pi}{\lambda}(x_1 - x_2) + (\epsilon_1 - \epsilon_2)$

If waves are in phase at emitters,

$$\delta = \frac{2\pi}{\lambda}(x_1 - x_2) = \frac{2\pi}{\lambda_0}n(x_1 - x_2)$$

(optical path diff.) $\quad (x_1 - x_2) \equiv \Delta r$

*• Atoms radiate in ~10ns pulses

Coherence time $\tau \sim 10\text{ns}$ (time over which phase relationship is const.)

Coherence length $\lambda_c = c\tau$

* Diffraction patterns

- Double slit interference

↳ $d \sin\theta = m\lambda$ interference at maximum

$d \sin\theta = (m - \frac{1}{2})\lambda$ interference at minimum

$m = 1, 2, 3, \ldots$

$\delta = kd\sin\theta = \frac{2\pi}{\lambda} d \sin\theta$

↳ $\delta = \pi, 3\pi, 5\pi \ldots$ min

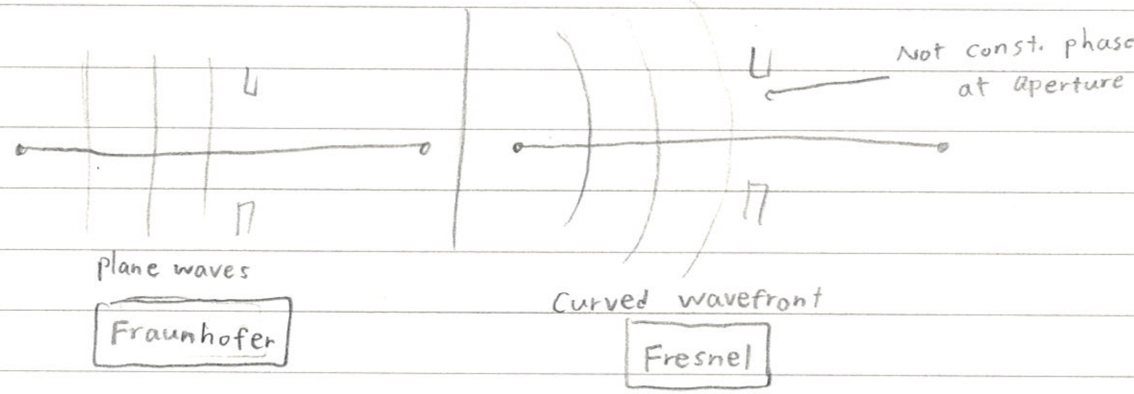
$= 2\pi, 0, 4\pi \ldots$ max

Light & Heat

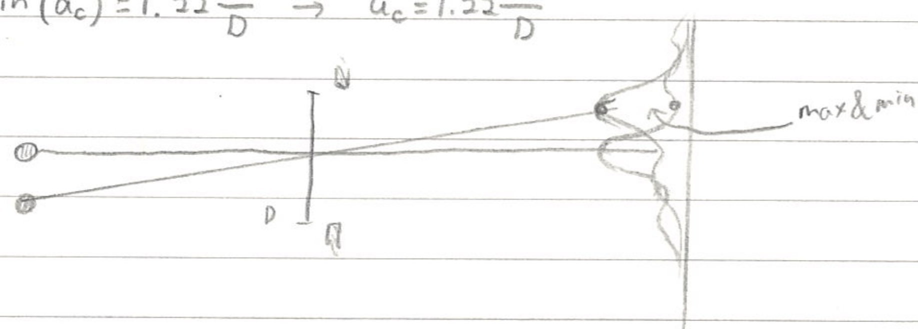
week 15 Notes

*Fresnel Diffraction

- Source & observer are close to aperture
- Fraunhofer diffraction: source & observer are far from aperture
- Fresnel number: $F = \frac{a^2}{R\lambda}$ $\begin{cases} F \ll 1 \text{; Fraunhofer} \\ F \geq 1 \text{, Fresnel} \end{cases}$



- Circular aperture
  ↳ Fraunhofer zone: irradiance described by Airy function
- Rayleigh's criterion for resolution
  ↳ when max. of one falls on first diffraction min. of other
  ↳ $\sin(a_c) = 1.22\frac{\lambda}{D} \rightarrow a_c = 1.22\frac{\lambda}{D}$



↳ when 2 far away sources can be resolved

• Sources of phase differences

↳ Emitted from source with different phase

↳ Travel different dist. to point: $\delta = \frac{2\pi}{\lambda} \Delta r$

↳ Pass through different materials: $\delta = \frac{2\pi}{\lambda_0}(n_1 - n_2)x$

↳ Phase change under reflection

Recall (String)	Now (Light)
If string end fixed → ref. wave flipped	If reflecting off denser medium → flipped
If string end free → ref. wave same	If reflecting off less dense medium → not flipped

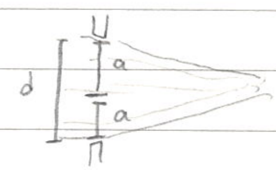
※ Single slit diffraction: $I = \frac{1}{2}\left[E_0(r)\frac{\sin\frac{N\delta}{2}}{\sin\frac{\delta}{2}}\right]^2$

↳ Messy derivation

↳ N=2 for double slit: $I = 4I_0 \cos^2\frac{\delta}{2}$

↳ N=1: $I = \frac{E_0^2(r)}{2} = I_0$

※ $I = I(0)\left[\frac{\sin\beta}{\beta}\right]^2$, $\beta = a\sin\theta$



↳ Use phase diff. $\delta = kd\sin\theta = \frac{2\pi}{\lambda}d\sin\theta$

Incorporate single slit diffraction into double slit pattern:

$$I = 4I_0\left[\frac{\sin\beta}{\beta}\right]^2\cos^2\frac{\delta}{2}$$

※ Diffraction: slit is wider, only one now ("aperture")

↳ Bending of waves around corners