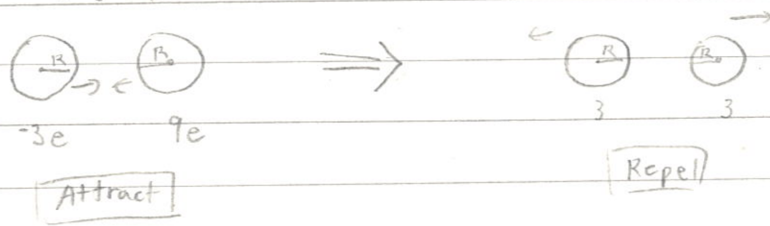
E & M review

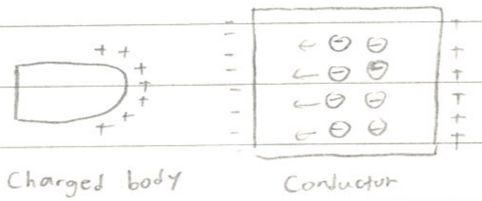
Part 1

* Electric fields

- Charging by contact



- Electrostatic induction



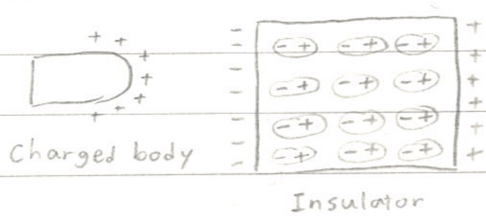
Electrons move, charge forms.

Free electrons in Conductor

*Protons don't move, but pos. charge due to lack of electrons

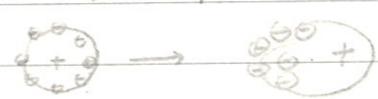
*i.e. positive charges don't move, unless moving of mass/object.

- Polarization in insulator



Atoms now, not free electrons

Atoms form dipole



*Polarization: electrostatic induction in insulators

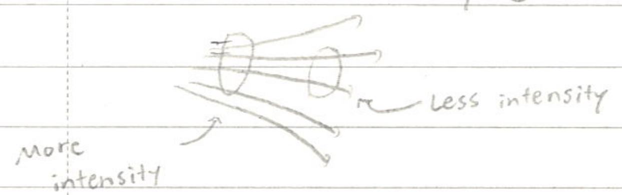
- Electric force: $F = \frac{kQq}{r^2}$, $k = 9 \times 10^9$

- Electric field: $E = \frac{F}{q} = \frac{kQ}{r^2}$

- Properties of electric field lines
  - ↳ Don't cross or separate

aren't allowed.

  - ↳ Electric flux density = electric field intensity

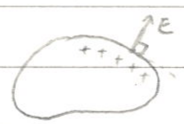


$$\boxed{E = \frac{\Phi}{A}}$$

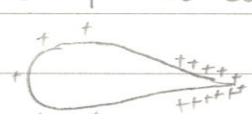
  - ↳ Electric field vectors are tangent to the field line

$\vec{F} = q\vec{E}$

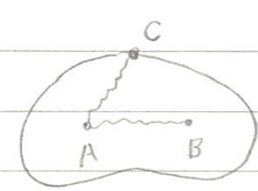
- Conductors
  - ↳ Equilibrium when no current (movement of charge) inside.
  - ↳ Net charge is distributed on the surface
  - ↳ Electric field is 0 inside conductor
  - ↳ Electric field on surface of conductor is ⊥ to the surface



  - ↳ Sharper the conductor → greater electric charge density



  - ↳ Electric potential is same in the interior & on the surface



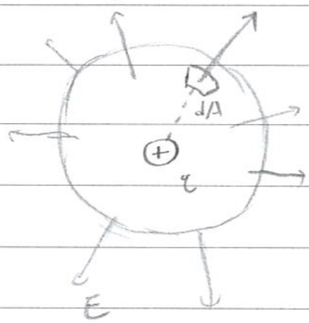
$V_A - V_B = Ed$, $E = 0$ so $V_A = V_B$

$V_A - V_C = Ed$, $E = 0$ so $V_A = V_C$

E&M Review

* Gauss's Law

- Diagram:



$$\boxed{\oint E \cdot dA = \frac{q_{enc}}{\varepsilon_0}}$$

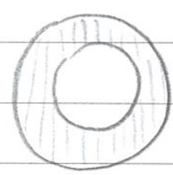
$\varepsilon_0 = 4\pi k$

Alternatively, $\boxed{\Phi_E = EA = \frac{q_{enc}}{\varepsilon_0}}$

※ Conductors: all charge at surface

Insulators: charge distributed evenly

※ For thick conducting shell:



Charge Q

Q is distributed both on inner & outer surfaces

$Q_{inner} + Q_{outer} = Q$

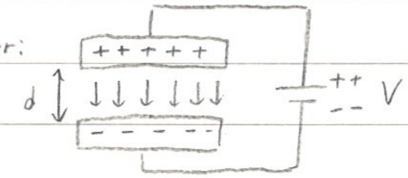
All charge on surface

※ $E = 0$ inside conductor

E & M Review

* Capacitance

- Capacitor:



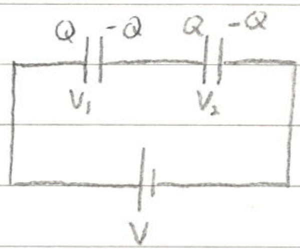
↳ $Q = CV$

↳ Capacitance $= C = \frac{\varepsilon_0 A}{d}$ (ε_0 = electric permittivity)

↳ Capacitance measured in Farads

↳ Electrostatic energy (stored as electric field): $u = \frac{1}{2} C V^2$

- In series:

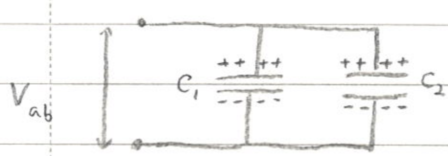


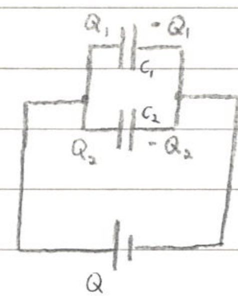
$V = V_1 + V_2$

$\frac{Q}{C} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$, Q are equal

$$\boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}}$$

- In parallel:





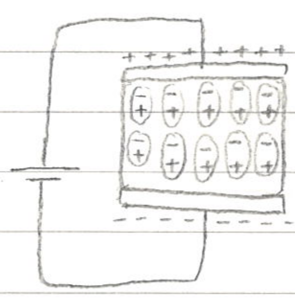
$Q = Q_1 + Q_2$ (Charges divided)

$CV = C_1 V_1 + C_2 V_2$

But $V_1 = V_2 = V$, b.c. voltage in parallel circuit is same.

$$\boxed{C = C_1 + C_2}$$

- Dielectric (insulator) & capacitance



K = Dielectric constant

$\varepsilon = K\varepsilon_0$, electric permittivity in material

$$\boxed{C = \frac{K\varepsilon_0 A}{d}}$$

• Charging capacitance

1) V fixed, $d \to 2d$ (battery connected)

$C = \frac{\varepsilon_0 A}{d}$, $C \to \frac{1}{2}C$

$Q = CV$, V const. so $Q \to \frac{1}{2}Q$

$u = \frac{1}{2}CV^2$, so $u \to \frac{1}{2}u$

* $V = Ed$, so $E \to \frac{1}{2}E$ to make V const.

2) Q fixed, $d \to 2d$ (battery disconnected → charge isolated)

$C = \frac{\varepsilon_0 A}{d}$, $C \to \frac{1}{2}C$

$Q = CV$, $V = Ed$, so $V \to 2V$ and E is const.

$u = \frac{1}{2}CV^2$, $u = 2u$

3) V fixed, insert dielectric (battery connected)

$C \to KC$

$Q = CV$, V const. so $Q \to KQ$

$u = \frac{1}{2}CV^2$, $u \to Ku$

4) Q fixed, insert dielectric (battery disconnected → charge isolated)

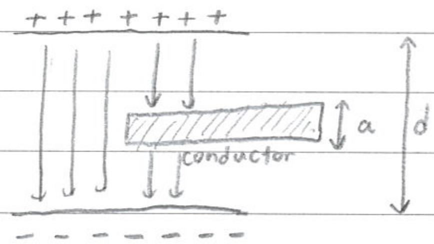
$C \to KC$

$Q = CV$, $V = Ed$, Q fixed so $V \to \frac{V}{K}$ and $E \to \frac{E}{K}$

$u = \frac{1}{2}CV^2$, $u \to \frac{u}{K}$

- Examples of Capacitance

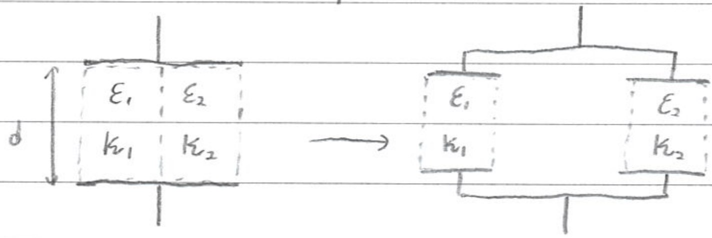
1) Insert Conductor



$$C_0 = \frac{\varepsilon_0 A}{d}$$

$$\boxed{C_{new} = \frac{\varepsilon_0 A}{d - a}}$$

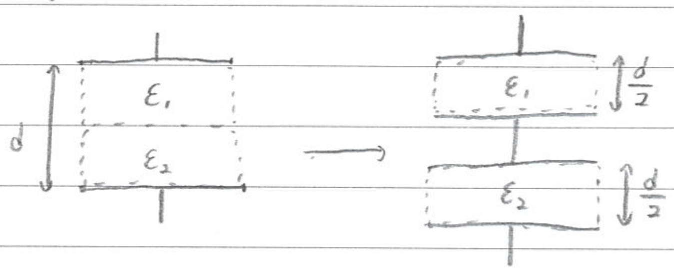
2) Dielectric in parallel



$$C = C_1 + C_2 = \frac{\kappa_1 \varepsilon_0 \frac{A}{2}}{d} + \frac{\kappa_2 \varepsilon_0 \frac{A}{2}}{d}$$

$$\boxed{C = \frac{\kappa_1 + \kappa_2}{2}\left(\frac{\varepsilon_0 A}{d}\right)}$$

3) Dielectrics in series



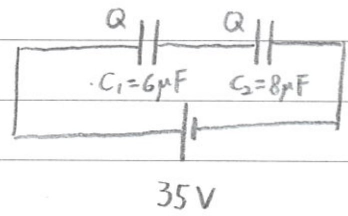
$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{\frac{d}{2}}{\kappa_1 \varepsilon_0 A} + \frac{\frac{d}{2}}{\kappa_2 \varepsilon_0 A}$$

$$\frac{1}{C} = \frac{d}{\varepsilon_0 A} \frac{\kappa_1 + \kappa_2}{2\kappa_1 \kappa_2}$$

$$C = \frac{2\kappa_1 \kappa_2}{\kappa_1 + \kappa_2}\left(\frac{\varepsilon_0 A}{d}\right)$$

• Voltage & Energy in each Capacitor

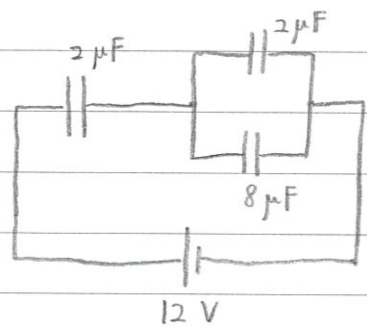
1) Series



$Q=CV$, Q const. in series circuit

	C	V	Q	$u=\frac{1}{2}CV^2=\frac{1}{2}QV$
C_1	6μ	20	120	1200 J
C_2	8μ	15	120	900 J

2) Parallel (example)



Parallel circuit can be reduced to single capacitor of $10\mu F$.

$Q=CV$, Q const. in series

2:10 10:2

C	V	Q	$u=\frac{1}{2}QV$
2μ	10	20μ	$100\ \mu J$
2μ	2	4μ	$4\ \mu J$
8μ	2	16μ	$16\ \mu J$

Parallel

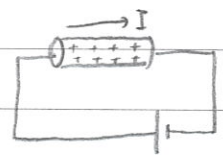
* Q const in series circuit
  V const in parallel circuit

"Par V"

E & M Review

* Resistors

- Resistance: measure of opposition to current flow
- Diagram:

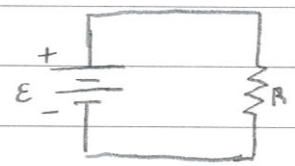


$\boxed{I = \frac{Q}{t}}$

↳ Electric current: ⊕ → ⊖

Electrons: ⊖ → ⊕

Unit: C/s (Ampere)



$\mathcal{E}$ = electromotive force (emf)

Voltage of battery

- Resistance & voltage

↳ Ohm's Law: $\boxed{V = IR}$ → Even w/ same voltage, if resistance is high, current can't flow as well.

↳ Resistance: $\boxed{R = \rho \frac{l}{A}}$ * ρ = resistivity
l = length
A = area

- Electric power delivered by battery: $\boxed{P = IV}$
- Electric power dissipated by R: $\boxed{P_R = I^2 R}$ from $V = IR$
- Energy: $\boxed{E = Pt}$

↳ Not Electric field

- Connection

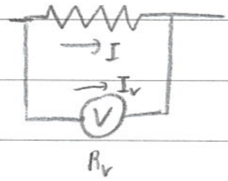
1) In series: $V = V_1 + V_2 \rightarrow IR = I_1 R_1 + I_2 R_2$

$\boxed{R = R_1 + R_2}$

2) In parallel: $I = I_1 + I_2 \rightarrow \frac{V}{R} = \frac{V_1}{R_1} + \frac{V_2}{R_2}$

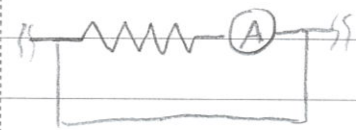
$\boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}}$

- Voltmeter



Measure voltage

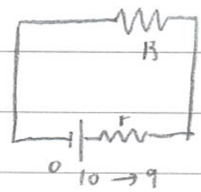
- Ammeter



Measure current

(Internal resistance must be close to 0)

- Batteries



10 V
(emf)

$V = \mathcal{E} - Ir$

$= 9$ in this case if $I = 1$

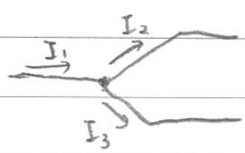
This is the terminal voltage (max V possible)

r = internal resistance (inherent in battery)

* Kirchhoff's Rules

1) Junction rule (charge conservation)

↳ Sum of currents into any junction is 0



$I_1 = I_2 + I_3$

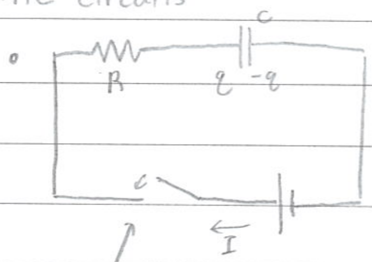
2) Loop rule (energy conservation)

↳ Sum of potential diff. in any loop is 0

↳ Potential flow: high → low

E&M Review

*RC circuits



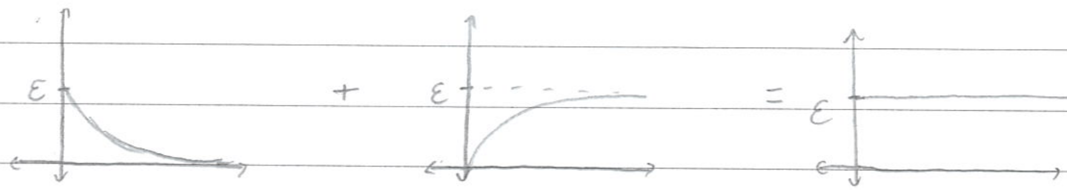
close switch to charge

$\mathcal{E} - IR = \frac{q}{C}$ (when charging)

$RI = \mathcal{E} - \frac{q}{C}$

$$\boxed{R\frac{dq}{dt} = \mathcal{E} - \frac{q}{C}}$$

V_R (resistor) $= IR$ + V_C (capacitor) $= \frac{q}{C}$ = $\mathcal{E}$ (battery voltage)



Charging:

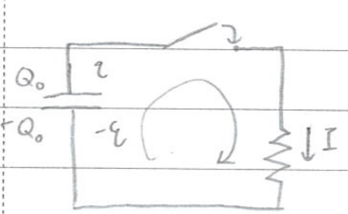
- $t=0$: $q=0$, $I=\frac{\mathcal{E}}{R}$ (capacitor acts like wire ——)
- $t=\infty$: $I=0$, $q=C\mathcal{E}$ (capacitor acts like broken wire —— ——)
- $t=\tau$:

$\boxed{q = C\mathcal{E}\left(1-e^{-t/\tau}\right)}$ $\boxed{I = \frac{\mathcal{E}}{R}e^{-t/\tau}}$ $\tau = RC$

*$t = -RC\ln\left(1-\frac{q}{C\mathcal{E}}\right)$ or just solve from equations

discharging:

- $q = q_0 e^{-t/\tau}$
- $I = I_0 e^{-t/RC}$

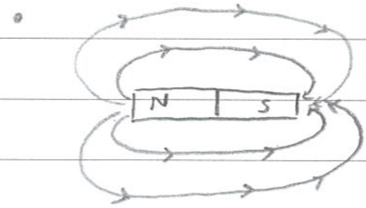


$\frac{q}{C} - IR = 0$

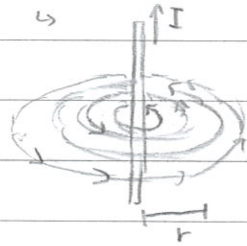
$\frac{q}{C} + R\frac{dq}{dt} = 0$

E & M Review

* Magnetic Field

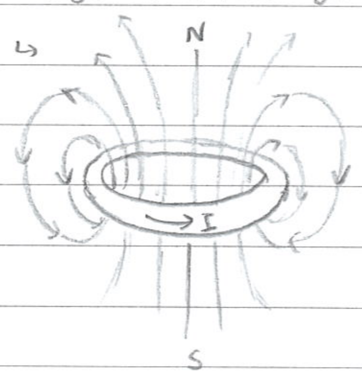
- Magnetic permeability: $\mu_0 = 4\pi \times 10^{-7}$ (vacuum), unit = T [Tesla]
- 

  1. North → South
  2. No splitting of field lines
  3. Magnetic field intensity = magnetic flux density

- Magnetic field due to straight wire
  ↳ Right-hand rule w/ current
  ↳ 

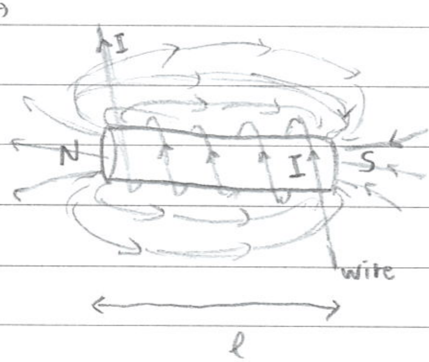
  $B = k\frac{I}{r}$, $k = 2\times 10^{-7}$

  $$\boxed{B = \frac{\mu_0}{2\pi}\frac{I}{r}}$$

- Magnetic field due to circular wire
  ↳ Right-hand rule again
  ↳ 

  $B = k'\frac{I}{r}$, $k' = 2\pi \times 10^{-7} = \frac{\mu_0}{2}$

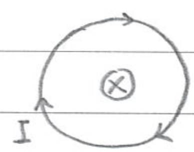
  $$\boxed{B = \frac{\mu_0}{2}\frac{I}{r}}$$

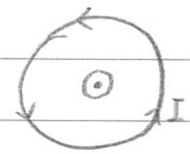
- Magnetic field due to solenoid
  ↳ 

  $n = \frac{N}{\ell} = \frac{\text{\# of loops}}{\ell}$

  $$\boxed{B = \mu_0 n I}$$

- Conventions

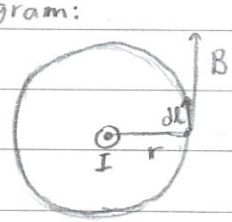




B into page   B out of page

* Ampere's Law

- Diagram:

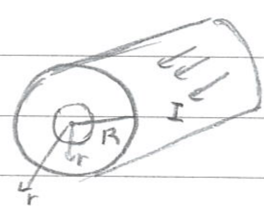


dl = Small segment of loop

$$\oint B\,dl = B\oint dl = \left(k\frac{I}{r}\right)(2\pi r) = \underbrace{2\pi k}_{\mu_0} I$$

- Ampere's Law: $\boxed{\oint \vec{B}\cdot d\vec{l} = \mu_0 I}$
  - ↳ Line integral of $\vec{B}$ over amperian loop depends on the current contained within it.

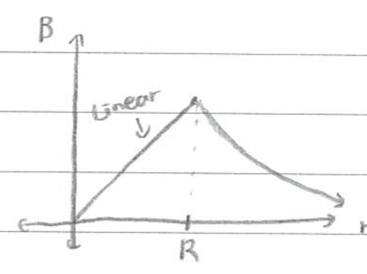
- Around a wire:



$$\oint B\cdot dl = \mu_0 I_{contained}$$

$$2\pi r B = \mu_0 I_{contained}$$

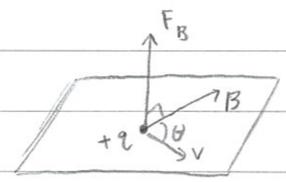
$$2\pi r B = \begin{cases} \mu_0 I & r > R \\ \mu_0 I \dfrac{\pi r^2}{\pi R^2} & r < R \end{cases}$$



$$B = \begin{cases} \dfrac{\mu_0 I}{2\pi r} & r > R \\ \dfrac{\mu_0 I}{2\pi R^2} r & r < R \end{cases}$$

*Magnetic Force

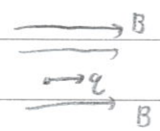
- Electromagnetic force on moving charge in magnetic field



$$\boxed{\begin{gathered}\vec{F_B} = q\vec{v}\times\vec{B} \\ |F_B| = qvB\sin\theta\end{gathered}}$$

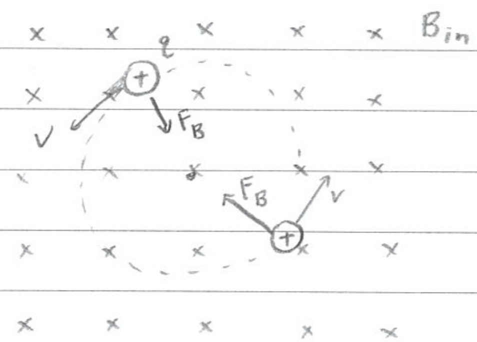
- If projected in same dir. as B: $\sin\theta = 0$, $F_B = 0$

↳ Const. v



- If projected perpendicular to B:

↳ Circular motion

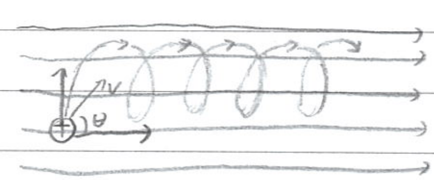


$$\frac{mv^2}{r} = qvB$$

$$\boxed{\begin{gathered}\text{Radius: } r = \frac{mv}{qB} \\ \text{Period: } \frac{2\pi r}{v} = \frac{2\pi m}{qB}\end{gathered}}$$

- If projected at angle to B:

↳ Helical motion



$v_{\parallel} = v\cos\theta$

$v_{\perp} = v\sin\theta$

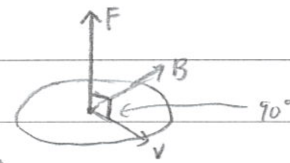
$$\boxed{l = v_{\parallel}T = v\sin\theta\,\frac{2\pi m}{qB}}$$

- No work on charged particle moving in pure mag. field

↳ $W_B = Fd\cos\theta$, $\theta = 90^\circ$, $W_B = 0$

↳ $\Delta K = W_B = 0$

↳ Direction changes, so $a \neq 0$, but $|v| = \text{const.}$

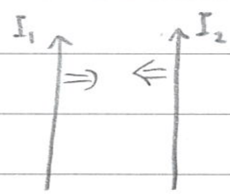


- $F = q\vec{v}\times\vec{B}$, $\vec{v} = \dfrac{\vec{l}}{t}$, $F = \dfrac{q}{t}\vec{l}\times\vec{B}$

$\boxed{\vec{F} = I\vec{l}\times\vec{B}}$ $\quad |F| = I l B \sin\theta$

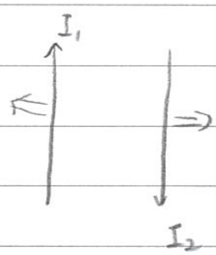
- Or more generally, $\vec{F} = \int I\, d\vec{l}\times\vec{B} = I\int d\vec{l}\times\vec{B}$

- Wires



$F_1 = B I_2 l = \dfrac{\mu_0 I_1}{2\pi r} I_2 l$

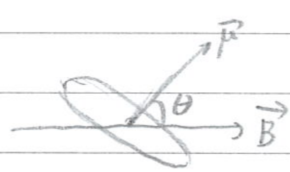
$$\boxed{F_1 = \frac{\mu_0 I_1 I_2 l}{2\pi r}}$$



$F = \dfrac{\mu_0 I_1 I_2 l}{2\pi r}$, same

- At angle (loop of wire)

↳



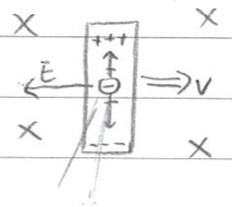
$\tau = \vec{\mu}\times\vec{B} = \mu B\sin\theta$

$= IAB\sin\theta$

E&M Notes

*Electromagnetic Induction

- Diagram:



Conducting bar moves w/ $\vec{V}$

Electrons move with $\vec{V}$, current flows in opposite direction.

↳ Now e^- gets force qvB to bottom

↳ Accumulation of e^- on bottom: electric field between top & bottom of conducting bar

↳ $qE = qvB$

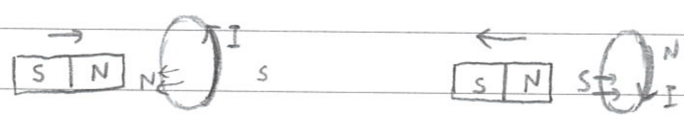
Voltage $V = E\ell = B\ell v$

(rod acts like battery!) ⟵ motional emf

*But only when rod moves $\perp$ to B, otherwise $q\vec{v} \times \vec{B} = 0$

- Lenz's Law

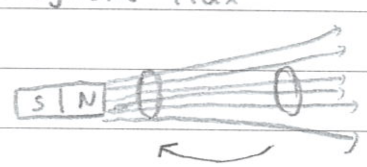
↳ Move magnet towards wire



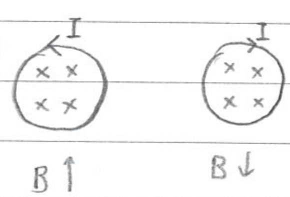
↳ Electromagnetic induction

↳ I = induced current

*emf is induced in the direction to oppose change in magnetic flux



*



⟵ use right-hand rule.

- Faraday's Law
  - ↳ Tells us magnitude of emf (Lenz tells us direction)
  - ↳ Induced voltage: $\mathcal{E} = -\frac{d\Phi}{dt}$
  - ↳ But if there are multiple coil loops, $\boxed{\mathcal{E} = -N\frac{d\Phi}{dt}}$
  - ↳ In the absence of a wire, the electric field is still produced.

$$\boxed{\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}}$$

Faraday's Law

E & M Review

*Electric Potential

- Electric potential energy: $\Delta U = W_{ex} = F_{ex} d = qEd$
- Electric potential difference: $\Delta V = \frac{\Delta U}{q} = E$

  ↳ or voltage

- Between 2 points

↳ Bring from $r = \infty$

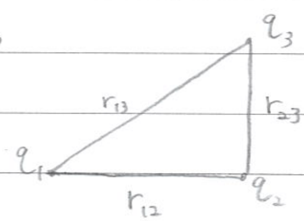
↳ $\Delta U = -W_{Ext} = -\int_{\infty}^{r} \frac{kQq}{r^2} dr = \frac{kQq}{r}\Big|_{\infty}^{r}$

↳ $\underbrace{U(r) - U(\infty)}_{0} = \frac{kQq}{r}$ ← Electric PE between 2 charges

↳ Electric potential due to Q: $V = \frac{kQ}{r}$

- Electric PE for multiple charges

↳



$U = U_{12} + U_{23} + U_{13}$

$= \frac{kq_1q_2}{r_{12}} + \frac{kq_2q_3}{r_{23}} + \frac{kq_1q_3}{r_{13}}$

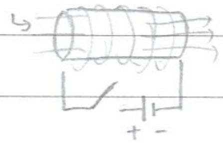
- Electric potential & electric field

↳ $\underbrace{F = -\frac{dU}{dr}}_{\text{Physical}} \rightarrow \underbrace{E = -\frac{dV}{dr}}_{\text{E \& M}}$

E & M Review

*RL circuits

- Self-induced emf opposes current: $\mathcal{E} = -N\frac{d\Phi}{dt}$, $\Phi = BA$, $B = \mu_0 \frac{N}{\ell} I$

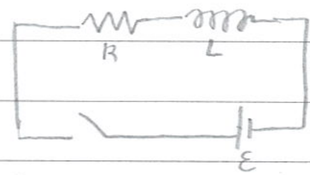


$$\boxed{\mathcal{E}_L = -L\frac{dI}{dt}}$$

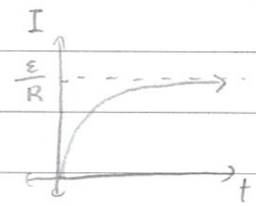
L: self-induction

$$\boxed{L = \mu_0 \frac{N^2}{\ell} A}$$

- RL Circuit



$$\mathcal{E} - IR - L\frac{dI}{dt} = 0 \quad \begin{cases} \mathcal{E} & \text{const} \\ I & \text{increases} \\ \frac{dI}{dt} & \text{decreases} \end{cases}$$



- Magnetic Energy: $u_B = \frac{1}{2}LI^2$

↳ Energy in inductor stored as magnetic field

- Inductive time const: $\tau = \frac{L}{R}$