Extending the Quantum Prisoner's Dilemma to the Optional Prisoner's Dilemma

Gene Yang

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Abstract

Prisoner's Dilemma is a common example in game theory. The players, Alice and Bob, can choose either to cooperate with each other or defect. Eisert's 1999 paper extended Prisoner's Dilemma to quantum circuits. He found that a quantum player with access to the full set of unitary gates could consistently gain an advantage over a classical player. In this paper, Eisert's circuit is extended to the Optional Prisoner's Dilemma, which has an extra choice: abstaining. A classical player that abstains can remove the quantum player's advantage. However, if the payoffs are modified to slightly penalize abstaining, introducing an imbalance to the table, the quantum player's advantage returns.

Contents

1 The 2x2 Prisoner's Dilemma

1.1 Classical version

In the classical Prisoner's Dilemma, there are two players: Alice and Bob. Both players are rational and have no knowledge of the other's choice. Their only options are to cooperate with the other player, or defect. Alice and Bob seek to maximize their individual payoffs, which we can model depending on their choices C (cooperate) or D (defect):

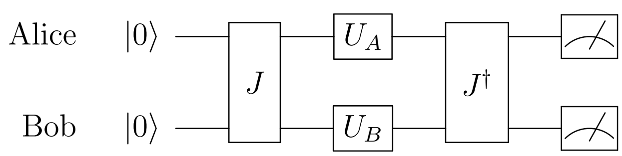
		Bob	
		C	D
Alice	C	(3, 3)	(0, 5)
	D	(5, 0)	(1, 1)

The format is $(\$_A, \$_B)$, where $\$_A$ is Alice's payoff and $\$_B$ is Bob's payoff. The ideal choice for both Alice and Bob is to cooperate, with a payoff of $(3, 3)$. However, cooperating has the risk of winning nothing if the other person defects. Thus the rational choice is to always defect, with a payoff of $(1, 1)$ [1].

The classical Prisoner's Dilemma has many applications in economics, biology, psychology, and more [1].

1.2 Quantum version

Eisert's original paper extended the classical Prisoner's Dilemma to a quantum version. The situation is modeled as a quantum circuit with 2 qubits [1]:



Alice and Bob's qubits are initially set to $|0\rangle$. They are entangled with a unitary operator J. Next, Alice and Bob modify their qubits by applying a unitary operator U. Finally, the effect of the J gate is reversed with $J^\dagger$, and the qubits are measured to determine the payoffs.

A measurement of $|0\rangle$ is associated with "cooperate" and $|1\rangle$ with "defect" [1]. Thus, a player can cooperate by setting $U = I$ or defect by setting $U = X$ (neglecting phases).

The general 2x2 unitary gate U is parametrized below, with $\theta \in [0, \pi/2]$ and $\alpha, \beta \in [-\pi, \pi]$. $U(0, 0, 0) = I$ is "cooperate" and $U(\pi/2, 0, 0) = iX$ is "defect" [2].

$$U(\theta, \alpha, \beta) = \begin{bmatrix} e^{i\alpha}\cos\theta & ie^{i\beta}\sin\theta \\ ie^{-i\beta}\sin\theta & e^{-i\alpha}\cos\theta \end{bmatrix}$$

The entangling gate J is [2]:

$$J = \frac{1}{\sqrt{2}}\left(I^{\otimes 2} + iX^{\otimes 2}\right)$$

1.3 Quantum player's advantage

The final state of the circuit is given by

$$|\psi_f\rangle = J^\dagger (U_A \otimes U_B) J |00\rangle$$

Using coefficients from the classical PD table, Alice and Bob's expected payoffs are [1]

$$\begin{aligned}
\langle \$_A \rangle &= 3 \left|\langle 00|\psi_f\rangle\right|^2 + 0 \left|\langle 01|\psi_f\rangle\right|^2 + 5 \left|\langle 10|\psi_f\rangle\right|^2 + 1 \left|\langle 11|\psi_f\rangle\right|^2 \\
\langle \$_B \rangle &= 3 \left|\langle 00|\psi_f\rangle\right|^2 + 5 \left|\langle 01|\psi_f\rangle\right|^2 + 0 \left|\langle 10|\psi_f\rangle\right|^2 + 1 \left|\langle 11|\psi_f\rangle\right|^2
\end{aligned}$$

If Alice is a classical player using $U_A(\theta_A, 0, 0)$ restricted to $\theta_A \in \{0, \pi/2\}$, while Bob has the full set of strategies $U_b(\theta_B, \alpha_B, \beta_B)$, Eisert found that Bob has an advantage over Alice in all cases. Bob can play the "miracle move" M [1]

$$M = U(\pi/4, \pi/4, 0) = \frac{i}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

which ensures that Bob's expected payoff is $3 + 2\sin(2\theta_A)$, while Alice's expected payoff is $1/2(1 - \sin(2\theta_A))$ [2]. Thus, if Alice is restricted to $\theta_A \in \{0, \pi/2\}$, her expected payoff is always 1/2, while Bob's expected payoff is always 3.

2 The 3x3 Optional Prisoner's Dilemma

2.1 Payoff table

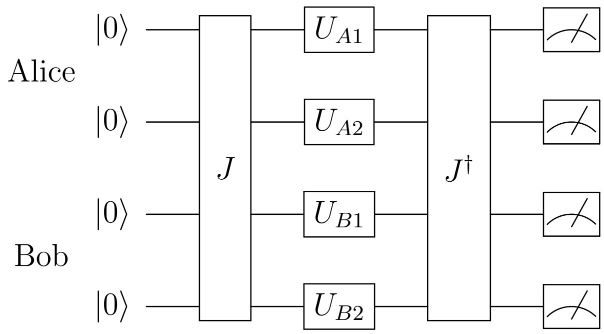
Eisert's quantum circuit for the Prisoner's Dilemma can be extended to a 3x3 Optional Prisoner's Dilemma. In the Optional Prisoner's Dilemma, a third choice is available to Alice and Bob: abstaining [3]. This accounts for real-life situations where a decision can be avoided. The new payoff table with C (cooperate), D (defect), and A (abstain) is

		Bob		
		C	D	A
	C	(3, 3)	(0, 5)	(2, 2)
Alice	D	(5, 0)	(1, 1)	(2, 2)
	A	(2, 2)	(2, 2)	(2, 2)

The format is $(\$_A, \$_B)$, where $\$_A$ is Alice's payoff and $\$_B$ is Bob's payoff. The classically best strategy is to abstain, guaranteeing a payoff of 2 no matter what. A question we can ask is if Bob, the quantum player, still has an advantage.

2.2 Circuit setup

The circuit is similar to the classical 2x2 Prisoner's Dilemma. However, since there are 3 choices, each player must have 2 qubits to represent all their possible choices.



Alice can manipulate her two qubits with U_{A1} and U_{A2}, while Bob can manipulate his two qubits with U_{B1} and U_{B2}. Previously, a measurement of $|0\rangle$ was associated with "cooperate" and $|1\rangle$ with "defect." Now we associate a measurement of $|00\rangle$ with "cooperate" and $|11\rangle$ with "defect." For symmetry, both $|01\rangle$ or $|10\rangle$ is associated with "abstain."

The entangling gate J is similar to before, but modified for 4 qubits [2]:

$$J = \frac{1}{\sqrt{2}}\left(I^{\otimes 4} + iX^{\otimes 4}\right)$$

The general 2x2 unitary gate is parametrized in the same way as before [2]:

$$U(\theta, \alpha, \beta) = \begin{bmatrix} e^{i\alpha}\cos\theta & ie^{i\beta}\sin\theta \\ ie^{-i\beta}\sin\theta & e^{-i\alpha}\cos\theta \end{bmatrix}$$

2.3 Payoff calculations

The final state of the circuit is

$$|\psi_f\rangle = J^\dagger(U_{A1} \otimes U_{A2} \otimes U_{B1} \otimes U_{B2})J\,|0000\rangle$$

Alice and Bob's payoffs depend on the final measurement of $|\psi_f\rangle$:

$$\begin{aligned}\langle \$_A\rangle = 3\,|\langle 0000|\psi_f\rangle|^2 + 0\,|\langle 0011|\psi_f\rangle|^2 + 2\,|\langle 0010|\psi_f\rangle|^2 + 2\,|\langle 0001|\psi_f\rangle|^2 +\\ 5\,|\langle 1100|\psi_f\rangle|^2 + 1\,|\langle 1111|\psi_f\rangle|^2 + 2\,|\langle 1110|\psi_f\rangle|^2 + 2\,|\langle 1101|\psi_f\rangle|^2 +\\ 2\,|\langle 0100|\psi_f\rangle|^2 + 2\,|\langle 0111|\psi_f\rangle|^2 + 2\,|\langle 0110|\psi_f\rangle|^2 + 2\,|\langle 0101|\psi_f\rangle|^2 +\\ 2\,|\langle 1000|\psi_f\rangle|^2 + 2\,|\langle 1011|\psi_f\rangle|^2 + 2\,|\langle 1010|\psi_f\rangle|^2 + 2\,|\langle 1001|\psi_f\rangle|^2\end{aligned}$$

$$\begin{aligned}\langle \$_B\rangle = 3\,|\langle 0000|\psi_f\rangle|^2 + 5\,|\langle 0011|\psi_f\rangle|^2 + 2\,|\langle 0010|\psi_f\rangle|^2 + 2\,|\langle 0001|\psi_f\rangle|^2 +\\ 0\,|\langle 1100|\psi_f\rangle|^2 + 1\,|\langle 1111|\psi_f\rangle|^2 + 2\,|\langle 1110|\psi_f\rangle|^2 + 2\,|\langle 1101|\psi_f\rangle|^2 +\\ 2\,|\langle 0100|\psi_f\rangle|^2 + 2\,|\langle 0111|\psi_f\rangle|^2 + 2\,|\langle 0110|\psi_f\rangle|^2 + 2\,|\langle 0101|\psi_f\rangle|^2 +\\ 2\,|\langle 1000|\psi_f\rangle|^2 + 2\,|\langle 1011|\psi_f\rangle|^2 + 2\,|\langle 1010|\psi_f\rangle|^2 + 2\,|\langle 1001|\psi_f\rangle|^2\end{aligned}$$

As before, Alice is the classical player, restricted to $U(\theta, 0, 0)$ with $\theta \in \{0, \pi/2\}$, Bob, the quantum player, has access to all $U(\theta, \alpha, \beta)$ with $\theta \in [0, \pi/2]$ and $\alpha, \beta \in [-\pi, \pi]$ [2].

2.4 Optimal quantum strategy

Let $U_{A1} = U(\theta_1, 0, 0)$, $U_{A2} = U(\theta_2, 0, 0)$, $U_{B1} = U(\gamma_1, \alpha_1, \beta_1)$, and $U_{B2} = U(\gamma_2, \alpha_1, \beta_2)$. Carrying out the tensor products and simplifying exponentials for $|\psi_f\rangle$ gives the equations in Appendix A.

Using those equations, we can apply casework to Bob's payoff $\langle \$_B \rangle$ depending on each of Alice's choices. If Alice cooperates, she sets $(\theta_1, \theta_2) = (0, 0)$, so that $(U_{A1} \otimes U_{A2}) |00\rangle = |00\rangle$. Similarly, if Alice defects, she sets $(\theta_1, \theta_2) = (\pi/2, \pi/2)$. If Alice abstains, she sets $(\theta_1, \theta_2) = (0, \pi/2)$ or $(\theta_1, \theta_2) = (\pi/2, 0)$.

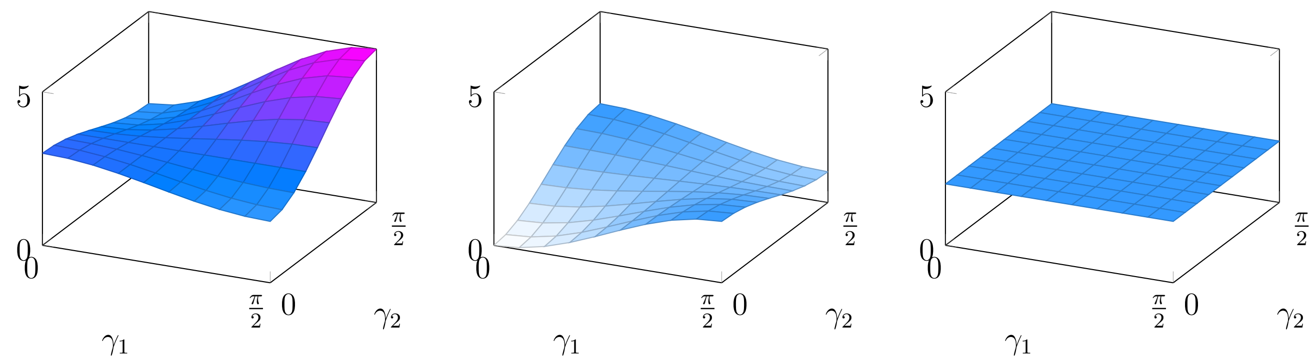


Figure 1: (From left to right) Bob's payoff as a function of his γ_1, γ_2 if Alice 1) cooperates, 2) defects, or 3) abstains. Here, Bob sets $\alpha_1 = \alpha_2 = 0$, $\beta_1 = \beta_2 = 0$

To minimize Bob's expected losses over all of Alice's strategies, we can set $\alpha_1 = \alpha_2 = \pi/4$, $\beta_1 = \beta_2 = 0$.

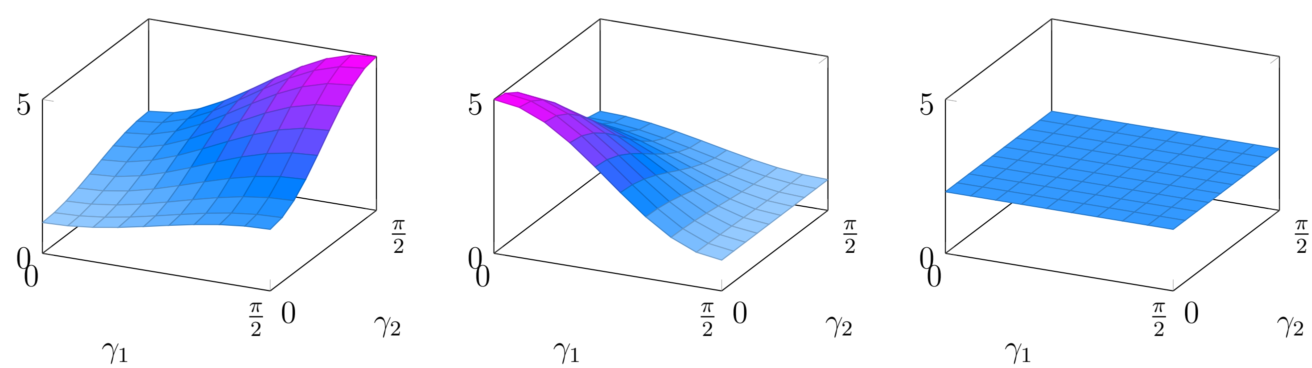


Figure 2: (From left to right) Bob's payoff as a function of his γ_1, γ_2 if Alice 1) cooperates, 2) defects, or 3) abstains. Here, Bob sets $\alpha_1 = \alpha_2 = \pi/4$, $\beta_1 = \beta_2 = 0$

If Bob sets $\gamma_1 = \gamma_2 = \pi/4$, he can gain a reasonable advantage when Alice cooperates, and also when she defects. Thus Bob's optimal move might be $U_{B1} = U(\pi/4, \pi/4, 0)$ and $U_{B2} = U(\pi/4, \pi/4, 0)$. These are exactly the same as Eisert's "miracle move" M in the 2x2 Prisoner's Dilemma. So in the Optional Prisoner's Dilemma, Bob's best strategy is to apply $M \otimes M$ on his two qubits.

If Bob plays $M \otimes M$, the graphs of his and Alice's expected payoffs become

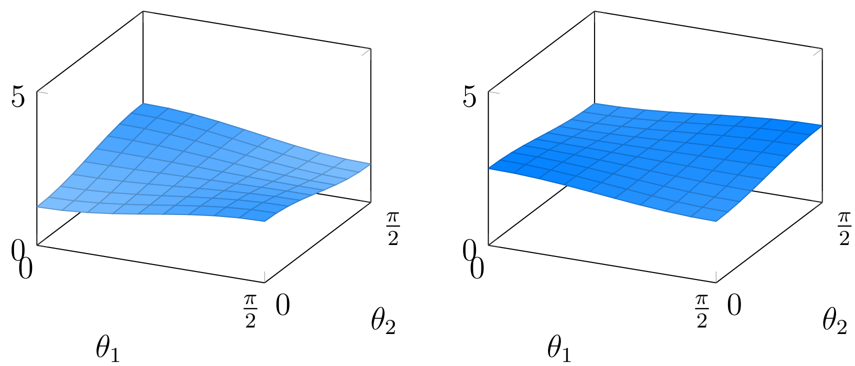


Figure 3: Left: Alice's payoff as a function of her θ_1, θ_2. Right: Bob's payoff as a function of Alice's θ_1, θ_2. Assuming Bob plays $M \otimes M$.

Since Alice is restricted to $\theta_1, \theta_2 \in \{0, \pi/2\}$, it's sufficient to look at the four cases of (θ_1, θ_2), i.e. the corners: $(0, 0), (0, \pi/2), (\pi/2, 0), (\pi/2, \pi/2)$.

(θ_1, θ_2), Alice's play	Alice's payoff	Bob's payoff
$(0, 0)$ Alice cooperates	1.25	2.5
$(0, \pi/2)$ Alice abstains	2	2
$(\pi/2, 0)$ Alice abstains	2	2
$(\pi/2, \pi/2)$ Alice defects	1.25	2.5

Bob gains an advantage over Alice if she cooperates or defects. However, if Alice abstains, Bob no longer has an advantage. Abstaining is also the classically best strategy. Thus, in the Optional Prisoner's Dilemma, Bob has no quantum advantage against a rational player.

3 Modified 3x3 Optional Prisoner's Dilemma

3.1 Modified payoff table

A modification to the payoff table in the Optional Prisoner's Dilemma causes Bob's quantum advantage to return. We do this by imbalancing the payoffs: if one or more players abstain, the abstaining player receives a 2, while a cooperating or defecting player receives a 2.5. Realistically, we're penalizing indecision. An indecisive government, for example, may be less favored than a decisive one that makes mistakes. The new payoff table with C (cooperate), D (defect), and A (abstain) is

		Bob		
		C	D	A
	C	(3, 3)	(0, 5)	(2.5, 2)
Alice	D	(5, 0)	(1, 1)	(2.5, 2)
	A	(2, 2.5)	(2, 2.5)	(2, 2)

The format is $(\$_A, \$_B)$, where $\$_A$ is Alice's payoff and $\$_B$ is Bob's payoff. Abstaining is still the classically optimal strategy, with a guaranteed payoff of at least 2.

3.2 Payoff calculations

The quantum setup is the same as for the Optional Prisoner's Dilemma in section 2.2. Alice and Bob each have two qubits they act on with unitary gates. Measuring $|00\rangle$ is associated with "cooperate," $|11\rangle$ with "defect," and $|01\rangle$ or $|10\rangle$ with "abstain." The general unitary gate is parametrized as $U(\theta, \alpha, \beta)$.

The final state of the circuit is

$$|\psi_f\rangle = J^\dagger(U_{A1} \otimes U_{A2} \otimes U_{B1} \otimes U_{B2})J\,|0000\rangle$$

Alice and Bob's payoffs depend on the final measurement of this state:

$$\begin{aligned}\langle \$_A\rangle = {} & 3\,|\langle 0000|\psi_f\rangle|^2 + 0\,|\langle 0011|\psi_f\rangle|^2 + 2.5\,|\langle 0010|\psi_f\rangle|^2 + 2.5\,|\langle 0001|\psi_f\rangle|^2 + \\ & 5\,|\langle 1100|\psi_f\rangle|^2 + 1\,|\langle 1111|\psi_f\rangle|^2 + 2.5\,|\langle 1110|\psi_f\rangle|^2 + 2.5\,|\langle 1101|\psi_f\rangle|^2 + \\ & 2\,|\langle 0100|\psi_f\rangle|^2 + 2\,|\langle 0111|\psi_f\rangle|^2 + 2\,|\langle 0110|\psi_f\rangle|^2 + 2\,|\langle 0101|\psi_f\rangle|^2 + \\ & 2\,|\langle 1000|\psi_f\rangle|^2 + 2\,|\langle 1011|\psi_f\rangle|^2 + 2\,|\langle 1010|\psi_f\rangle|^2 + 2\,|\langle 1001|\psi_f\rangle|^2\end{aligned}$$

$$\begin{aligned}\langle \$_B\rangle = {} & 3\,|\langle 0000|\psi_f\rangle|^2 + 5\,|\langle 0011|\psi_f\rangle|^2 + 2\,|\langle 0010|\psi_f\rangle|^2 + 2\,|\langle 0001|\psi_f\rangle|^2 + \\ & 0\,|\langle 1100|\psi_f\rangle|^2 + 1\,|\langle 1111|\psi_f\rangle|^2 + 2\,|\langle 1110|\psi_f\rangle|^2 + 2\,|\langle 1101|\psi_f\rangle|^2 + \\ & 2.5\,|\langle 0100|\psi_f\rangle|^2 + 2.5\,|\langle 0111|\psi_f\rangle|^2 + 2\,|\langle 0110|\psi_f\rangle|^2 + 2\,|\langle 0101|\psi_f\rangle|^2 + \\ & 2.5\,|\langle 1000|\psi_f\rangle|^2 + 2.5\,|\langle 1011|\psi_f\rangle|^2 + 2\,|\langle 1010|\psi_f\rangle|^2 + 2\,|\langle 1001|\psi_f\rangle|^2\end{aligned}$$

Alice, the classical player, is restricted to $U(\theta, 0, 0)$ with $\theta \in \{0, \pi/2\}$, Bob, the quantum player, has access to all $U(\theta, \alpha, \beta)$ with $\theta \in [0, \pi/2]$ and $\alpha, \beta \in [-\pi, \pi]$.

3.3 Optimal quantum strategy

Let $U_{A1} = U(\theta_1, 0, 0)$, $U_{A2} = U(\theta_2, 0, 0)$, $U_{B1} = U(\gamma_1, \alpha_1, \beta_1)$, and $U_{B2} = U(\gamma_2, \alpha_1, \beta_2)$. Carrying out the calculations for $|\psi_f\rangle$ gives the equations in Appendix B. This is similar to the equations for the unmodified Optional Prisoner's Dilemma in Appendix A, but with different coefficients.

As before, we use these equations and apply casework to Bob's payoff $\langle \$_B \rangle$ depending on each of Alice's choices. It turns out that Bob's optimal strategy is the same as before: Bob sets $\gamma_1 = \gamma_2 = \pi/4$, $\alpha_1 = \alpha_2 = \pi/4$, and $\beta_1 = \beta_2 = 0$. Thus, Bob's optimal strategy is still to apply $M \otimes M$ on his two qubits, where M is Eisert's "miracle move."

If Bob plays $M \otimes M$, the graphs of his and Alice's expected payoffs become

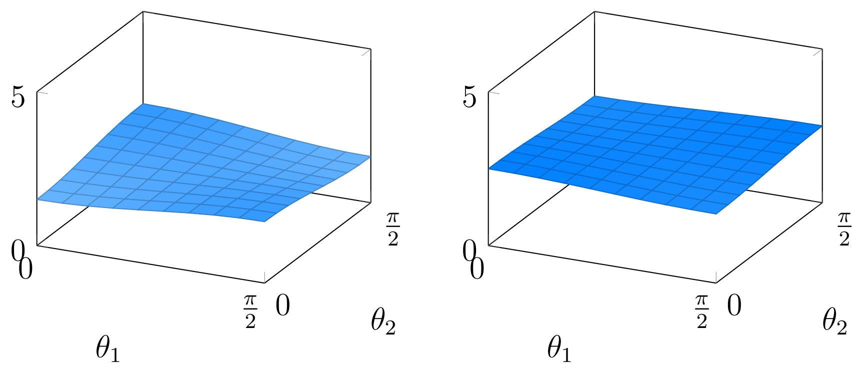


Figure 4: Left: Alice's payoff as a function of her θ_1, θ_2. Right: Bob's payoff as a function of Alice's θ_1, θ_2. Assuming Bob plays $M \otimes M$.

As the classical player, Alice is restricted to $\theta_1, \theta_2 \in \{0, \pi/2\}$. Therefore, it's sufficient to look at the four corners: $(0,0), (0,\pi/2), (\pi/2,0), (\pi/2,\pi/2)$.

(θ_1, θ_2), Alice's play	Alice's payoff	Bob's payoff
$(0,0)$ Alice cooperates	1.5	2.5
$(0,\pi/2)$ Alice abstains	2	2.25
$(\pi/2,0)$ Alice abstains	2	2.25
$(\pi/2,\pi/2)$ Alice defects	1.5	2.5

Unlike before in the Optional Prisoner's Dilemma, Bob has an advantage in all cases. This makes sense, as Bob's quantum strategy can exploit the imbalances in the payoff table to guarantee higher expected returns. However, this advantage disappears if one player can force a fixed payoff, e.g. by abstaining in the unmodified Optional Prisoner's Dilemma.

Given the applicability of Prisoner's Dilemma in several fields, generalizing it to quantum probabilities is an interesting take [1]. Maybe the Optional Prisoner's Dilemma, which can account for indecision, will also be useful in that regard.

References

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[2] Adrian P. Flitney and Derek Abbott. *An introduction to quantum game theory*. 2002. arXiv: quant-ph/0208069 [quant-ph]. URL: https://arxiv.org/abs/quant-ph/0208069.

[3] Zhao Song and The Anh Han. *Emergence of Cooperation and Commitment in Optional Prisoner's Dilemma*. 2025. arXiv: 2508.06702 [cs.GT]. URL: https://arxiv.org/abs/2508.06702.

4 Appendix A: Alice and Bob's Payoff in the OPD

$$
\begin{aligned}
\langle \$_A \rangle = \frac{3}{4}\Big|&2\cos\theta_1\cos\theta_2\cos\gamma_1\cos\gamma_2\cos(\alpha_1+\alpha_2)+2i\sin\theta_1\sin\theta_2\sin\gamma_1\sin\gamma_2\sin(\beta_1+\beta_2)\Big|^2+\\
\frac{0}{4}\Big|&-2\cos\theta_1\cos\theta_2\sin\gamma_1\sin\gamma_2\cos(\beta_1+\beta_2)+2i\sin\theta_1\sin\theta_2\cos\gamma_1\cos\gamma_2\sin(\alpha_1+\alpha_2)\Big|^2+\\
\frac{2}{4}\Big|&2i\cos\theta_1\cos\theta_2\cos\gamma_1\sin\gamma_2\cos(\alpha_1-\beta_2)+2\sin\theta_1\sin\theta_2\sin\gamma_1\cos\gamma_2\sin(\beta_1-\alpha_2)\Big|^2+\\
\frac{2}{4}\Big|&2i\cos\theta_1\cos\theta_2\sin\gamma_1\cos\gamma_2\cos(\beta_1-\alpha_2)-2\sin\theta_1\sin\theta_2\cos\gamma_1\sin\gamma_2\sin(\alpha_1-\beta_2)\Big|^2+
\end{aligned}
$$

$$
\begin{aligned}
\frac{5}{4}\Big|&-2\sin\theta_1\sin\theta_2\cos\gamma_1\cos\gamma_2\cos(\alpha_1+\alpha_2)-2i\cos\theta_1\cos\theta_2\sin\gamma_1\sin\gamma_2\sin(\beta_1+\beta_2)\Big|^2+\\
\frac{1}{4}\Big|&2\sin\theta_1\sin\theta_2\sin\gamma_1\sin\gamma_2\cos(\beta_1+\beta_2)-2i\cos\theta_1\cos\theta_2\cos\gamma_1\cos\gamma_2\sin(\alpha_1+\alpha_2)\Big|^2+\\
\frac{2}{4}\Big|&-2i\sin\theta_1\sin\theta_2\cos\gamma_1\sin\gamma_2\cos(\alpha_1-\beta_2)-2\cos\theta_1\cos\theta_2\sin\gamma_1\cos\gamma_2\sin(\beta_1-\alpha_2)\Big|^2+\\
\frac{2}{4}\Big|&-2i\sin\theta_1\sin\theta_2\sin\gamma_1\cos\gamma_2\cos(\beta_1-\alpha_2)+2\cos\theta_1\cos\theta_2\cos\gamma_1\sin\gamma_2\sin(\alpha_1-\beta_2)\Big|^2+
\end{aligned}
$$

$$
\begin{aligned}
\frac{2}{4}\Big|&2i\cos\theta_1\cos\theta_2\cos\gamma_1\cos\gamma_2\cos(\alpha_1+\alpha_2)+2\sin\theta_1\cos\theta_2\sin\gamma_1\sin\gamma_2\sin(\beta_1+\beta_2)\Big|^2+\\
\frac{2}{4}\Big|&-2i\cos\theta_1\sin\theta_2\sin\gamma_1\sin\gamma_2\cos(\beta_1+\beta_2)+2\sin\theta_1\cos\theta_2\cos\gamma_1\cos\gamma_2\sin(\alpha_1+\alpha_2)\Big|^2+\\
\frac{2}{4}\Big|&-2\cos\theta_1\sin\theta_2\cos\gamma_1\sin\gamma_2\cos(\alpha_1-\beta_2)-2i\sin\theta_1\cos\theta_2\sin\gamma_1\cos\gamma_2\sin(\beta_1-\alpha_2)\Big|^2+\\
\frac{2}{4}\Big|&-2\cos\theta_1\sin\theta_2\sin\gamma_1\cos\gamma_2\cos(\beta_1-\alpha_2)+2i\sin\theta_1\cos\theta_2\cos\gamma_1\sin\gamma_2\sin(\alpha_1-\beta_2)\Big|^2+
\end{aligned}
$$

$$
\begin{aligned}
\frac{2}{4}\Big|&2i\sin\theta_1\cos\theta_2\cos\gamma_1\cos\gamma_2\cos(\alpha_1+\alpha_2)+2\cos\theta_1\sin\theta_2\sin\gamma_1\sin\gamma_2\sin(\beta_1+\beta_2)\Big|^2+\\
\frac{2}{4}\Big|&-2i\sin\theta_1\cos\theta_2\sin\gamma_1\sin\gamma_2\cos(\beta_1+\beta_2)+2\cos\theta_1\sin\theta_2\cos\gamma_1\cos\gamma_2\sin(\alpha_1+\alpha_2)\Big|^2+\\
\frac{2}{4}\Big|&-2\sin\theta_1\cos\theta_2\cos\gamma_1\sin\gamma_2\cos(\alpha_1-\beta_2)-2i\cos\theta_1\sin\theta_2\sin\gamma_1\cos\gamma_2\sin(\beta_1-\alpha_2)\Big|^2+\\
\frac{2}{4}\Big|&-2\sin\theta_1\cos\theta_2\sin\gamma_1\cos\gamma_2\cos(\beta_1-\alpha_2)+2i\cos\theta_1\sin\theta_2\cos\gamma_1\sin\gamma_2\sin(\alpha_1-\beta_2)\Big|^2
\end{aligned}
$$

$$
\begin{aligned}
\langle \$_B \rangle = \ & \frac{3}{4}\Big|2\cos\theta_1\cos\theta_2\cos\gamma_1\cos\gamma_2\cos(\alpha_1+\alpha_2)+2i\sin\theta_1\sin\theta_2\sin\gamma_1\sin\gamma_2\sin(\beta_1+\beta_2)\Big|^2+\\
& \frac{5}{4}\Big|-2\cos\theta_1\cos\theta_2\sin\gamma_1\sin\gamma_2\cos(\beta_1+\beta_2)+2i\sin\theta_1\sin\theta_2\cos\gamma_1\cos\gamma_2\sin(\alpha_1+\alpha_2)\Big|^2+\\
& \frac{2}{4}\Big|2i\cos\theta_1\cos\theta_2\cos\gamma_1\sin\gamma_2\cos(\alpha_1-\beta_2)+2\sin\theta_1\sin\theta_2\sin\gamma_1\cos\gamma_2\sin(\beta_1-\alpha_2)\Big|^2+\\
& \frac{2}{4}\Big|2i\cos\theta_1\cos\theta_2\sin\gamma_1\cos\gamma_2\cos(\beta_1-\alpha_2)-2\sin\theta_1\sin\theta_2\cos\gamma_1\sin\gamma_2\sin(\alpha_1-\beta_2)\Big|^2+\\
& \\
& \frac{0}{4}\Big|-2\sin\theta_1\sin\theta_2\cos\gamma_1\cos\gamma_2\cos(\alpha_1+\alpha_2)-2i\cos\theta_1\cos\theta_2\sin\gamma_1\sin\gamma_2\sin(\beta_1+\beta_2)\Big|^2+\\
& \frac{1}{4}\Big|2\sin\theta_1\sin\theta_2\sin\gamma_1\sin\gamma_2\cos(\beta_1+\beta_2)-2i\cos\theta_1\cos\theta_2\cos\gamma_1\cos\gamma_2\sin(\alpha_1+\alpha_2)\Big|^2+\\
& \frac{2}{4}\Big|-2i\sin\theta_1\sin\theta_2\cos\gamma_1\sin\gamma_2\cos(\alpha_1-\beta_2)-2\cos\theta_1\cos\theta_2\sin\gamma_1\cos\gamma_2\sin(\beta_1-\alpha_2)\Big|^2+\\
& \frac{2}{4}\Big|-2i\sin\theta_1\sin\theta_2\sin\gamma_1\cos\gamma_2\cos(\beta_1-\alpha_2)+2\cos\theta_1\cos\theta_2\cos\gamma_1\sin\gamma_2\sin(\alpha_1-\beta_2)\Big|^2+\\
& \\
& \frac{2}{4}\Big|2i\cos\theta_1\cos\theta_2\cos\gamma_1\cos\gamma_2\cos(\alpha_1+\alpha_2)+2\sin\theta_1\cos\theta_2\sin\gamma_1\sin\gamma_2\sin(\beta_1+\beta_2)\Big|^2+\\
& \frac{2}{4}\Big|-2i\cos\theta_1\sin\theta_2\sin\gamma_1\sin\gamma_2\cos(\beta_1+\beta_2)+2\sin\theta_1\cos\theta_2\cos\gamma_1\cos\gamma_2\sin(\alpha_1+\alpha_2)\Big|^2+\\
& \frac{2}{4}\Big|-2\cos\theta_1\sin\theta_2\cos\gamma_1\sin\gamma_2\cos(\alpha_1-\beta_2)-2i\sin\theta_1\cos\theta_2\sin\gamma_1\cos\gamma_2\sin(\beta_1-\alpha_2)\Big|^2+\\
& \frac{2}{4}\Big|-2\cos\theta_1\sin\theta_2\sin\gamma_1\cos\gamma_2\cos(\beta_1-\alpha_2)+2i\sin\theta_1\cos\theta_2\cos\gamma_1\sin\gamma_2\sin(\alpha_1-\beta_2)\Big|^2+\\
& \\
& \frac{2}{4}\Big|2i\sin\theta_1\cos\theta_2\cos\gamma_1\cos\gamma_2\cos(\alpha_1+\alpha_2)+2\cos\theta_1\sin\theta_2\sin\gamma_1\sin\gamma_2\sin(\beta_1+\beta_2)\Big|^2+\\
& \frac{2}{4}\Big|-2i\sin\theta_1\cos\theta_2\sin\gamma_1\sin\gamma_2\cos(\beta_1+\beta_2)+2\cos\theta_1\sin\theta_2\cos\gamma_1\cos\gamma_2\sin(\alpha_1+\alpha_2)\Big|^2+\\
& \frac{2}{4}\Big|-2\sin\theta_1\cos\theta_2\cos\gamma_1\sin\gamma_2\cos(\alpha_1-\beta_2)-2i\cos\theta_1\sin\theta_2\sin\gamma_1\cos\gamma_2\sin(\beta_1-\alpha_2)\Big|^2+\\
& \frac{2}{4}\Big|-2\sin\theta_1\cos\theta_2\sin\gamma_1\cos\gamma_2\cos(\beta_1-\alpha_2)+2i\cos\theta_1\sin\theta_2\cos\gamma_1\sin\gamma_2\sin(\alpha_1-\beta_2)\Big|^2
\end{aligned}
$$

5 Appendix B: Alice and Bob's Payoff in the Modified OPD

$$
\begin{aligned}
\langle \$_A \rangle = \; & \frac{3}{4}\Big|2\cos\theta_1\cos\theta_2\cos\gamma_1\cos\gamma_2\cos(\alpha_1+\alpha_2)+2i\sin\theta_1\sin\theta_2\sin\gamma_1\sin\gamma_2\sin(\beta_1+\beta_2)\Big|^2+\\
& \frac{0}{4}\Big|-2\cos\theta_1\cos\theta_2\sin\gamma_1\sin\gamma_2\cos(\beta_1+\beta_2)+2i\sin\theta_1\sin\theta_2\cos\gamma_1\cos\gamma_2\sin(\alpha_1+\alpha_2)\Big|^2+\\
& \frac{2.5}{4}\Big|2i\cos\theta_1\cos\theta_2\cos\gamma_1\sin\gamma_2\cos(\alpha_1-\beta_2)+2\sin\theta_1\sin\theta_2\sin\gamma_1\cos\gamma_2\sin(\beta_1-\alpha_2)\Big|^2+\\
& \frac{2.5}{4}\Big|2i\cos\theta_1\cos\theta_2\sin\gamma_1\cos\gamma_2\cos(\beta_1-\alpha_2)-2\sin\theta_1\sin\theta_2\cos\gamma_1\sin\gamma_2\sin(\alpha_1-\beta_2)\Big|^2+\\
& \\
& \frac{5}{4}\Big|-2\sin\theta_1\sin\theta_2\cos\gamma_1\cos\gamma_2\cos(\alpha_1+\alpha_2)-2i\cos\theta_1\cos\theta_2\sin\gamma_1\sin\gamma_2\sin(\beta_1+\beta_2)\Big|^2+\\
& \frac{1}{4}\Big|2\sin\theta_1\sin\theta_2\sin\gamma_1\sin\gamma_2\cos(\beta_1+\beta_2)-2i\cos\theta_1\cos\theta_2\cos\gamma_1\cos\gamma_2\sin(\alpha_1+\alpha_2)\Big|^2+\\
& \frac{2.5}{4}\Big|-2i\sin\theta_1\sin\theta_2\cos\gamma_1\sin\gamma_2\cos(\alpha_1-\beta_2)-2\cos\theta_1\cos\theta_2\sin\gamma_1\cos\gamma_2\sin(\beta_1-\alpha_2)\Big|^2+\\
& \frac{2.5}{4}\Big|-2i\sin\theta_1\sin\theta_2\sin\gamma_1\cos\gamma_2\cos(\beta_1-\alpha_2)+2\cos\theta_1\cos\theta_2\cos\gamma_1\sin\gamma_2\sin(\alpha_1-\beta_2)\Big|^2+\\
& \\
& \frac{2}{4}\Big|2i\cos\theta_1\cos\theta_2\cos\gamma_1\cos\gamma_2\cos(\alpha_1+\alpha_2)+2\sin\theta_1\cos\theta_2\sin\gamma_1\sin\gamma_2\sin(\beta_1+\beta_2)\Big|^2+\\
& \frac{2}{4}\Big|-2i\cos\theta_1\sin\theta_2\sin\gamma_1\sin\gamma_2\cos(\beta_1+\beta_2)+2\sin\theta_1\cos\theta_2\cos\gamma_1\cos\gamma_2\sin(\alpha_1+\alpha_2)\Big|^2+\\
& \frac{2}{4}\Big|-2\cos\theta_1\sin\theta_2\cos\gamma_1\sin\gamma_2\cos(\alpha_1-\beta_2)-2i\sin\theta_1\cos\theta_2\sin\gamma_1\cos\gamma_2\sin(\beta_1-\alpha_2)\Big|^2+\\
& \frac{2}{4}\Big|-2\cos\theta_1\sin\theta_2\sin\gamma_1\cos\gamma_2\cos(\beta_1-\alpha_2)+2i\sin\theta_1\cos\theta_2\cos\gamma_1\sin\gamma_2\sin(\alpha_1-\beta_2)\Big|^2+\\
& \\
& \frac{2}{4}\Big|2i\sin\theta_1\cos\theta_2\cos\gamma_1\cos\gamma_2\cos(\alpha_1+\alpha_2)+2\cos\theta_1\sin\theta_2\sin\gamma_1\sin\gamma_2\sin(\beta_1+\beta_2)\Big|^2+\\
& \frac{2}{4}\Big|-2i\sin\theta_1\cos\theta_2\sin\gamma_1\sin\gamma_2\cos(\beta_1+\beta_2)+2\cos\theta_1\sin\theta_2\cos\gamma_1\cos\gamma_2\sin(\alpha_1+\alpha_2)\Big|^2+\\
& \frac{2}{4}\Big|-2\sin\theta_1\cos\theta_2\cos\gamma_1\sin\gamma_2\cos(\alpha_1-\beta_2)-2i\cos\theta_1\sin\theta_2\sin\gamma_1\cos\gamma_2\sin(\beta_1-\alpha_2)\Big|^2+\\
& \frac{2}{4}\Big|-2\sin\theta_1\cos\theta_2\sin\gamma_1\cos\gamma_2\cos(\beta_1-\alpha_2)+2i\cos\theta_1\sin\theta_2\cos\gamma_1\sin\gamma_2\sin(\alpha_1-\beta_2)\Big|^2
\end{aligned}
$$

$$
\begin{aligned}
\langle \$_B \rangle = \frac{3}{4}&\Big| 2\cos\theta_1\cos\theta_2\cos\gamma_1\cos\gamma_2\cos(\alpha_1+\alpha_2) + 2i\sin\theta_1\sin\theta_2\sin\gamma_1\sin\gamma_2\sin(\beta_1+\beta_2)\Big|^2 + \\
\frac{5}{4}&\Big| -2\cos\theta_1\cos\theta_2\sin\gamma_1\sin\gamma_2\cos(\beta_1+\beta_2) + 2i\sin\theta_1\sin\theta_2\cos\gamma_1\cos\gamma_2\sin(\alpha_1+\alpha_2)\Big|^2 + \\
\frac{2}{4}&\Big| 2i\cos\theta_1\cos\theta_2\cos\gamma_1\sin\gamma_2\cos(\alpha_1-\beta_2) + 2\sin\theta_1\sin\theta_2\sin\gamma_1\cos\gamma_2\sin(\beta_1-\alpha_2)\Big|^2 + \\
\frac{2}{4}&\Big| 2i\cos\theta_1\cos\theta_2\sin\gamma_1\cos\gamma_2\cos(\beta_1-\alpha_2) - 2\sin\theta_1\sin\theta_2\cos\gamma_1\sin\gamma_2\sin(\alpha_1-\beta_2)\Big|^2 +
\end{aligned}
$$

$$
\begin{aligned}
\frac{0}{4}&\Big| -2\sin\theta_1\sin\theta_2\cos\gamma_1\cos\gamma_2\cos(\alpha_1+\alpha_2) - 2i\cos\theta_1\cos\theta_2\sin\gamma_1\sin\gamma_2\sin(\beta_1+\beta_2)\Big|^2 + \\
\frac{1}{4}&\Big| 2\sin\theta_1\sin\theta_2\sin\gamma_1\sin\gamma_2\cos(\beta_1+\beta_2) - 2i\cos\theta_1\cos\theta_2\cos\gamma_1\cos\gamma_2\sin(\alpha_1+\alpha_2)\Big|^2 + \\
\frac{2}{4}&\Big| -2i\sin\theta_1\sin\theta_2\cos\gamma_1\sin\gamma_2\cos(\alpha_1-\beta_2) - 2\cos\theta_1\cos\theta_2\sin\gamma_1\cos\gamma_2\sin(\beta_1-\alpha_2)\Big|^2 + \\
\frac{2}{4}&\Big| -2i\sin\theta_1\sin\theta_2\sin\gamma_1\cos\gamma_2\cos(\beta_1-\alpha_2) + 2\cos\theta_1\cos\theta_2\cos\gamma_1\sin\gamma_2\sin(\alpha_1-\beta_2)\Big|^2 +
\end{aligned}
$$

$$
\begin{aligned}
\frac{2.5}{4}&\Big| 2i\cos\theta_1\cos\theta_2\cos\gamma_1\cos\gamma_2\cos(\alpha_1+\alpha_2) + 2\sin\theta_1\cos\theta_2\sin\gamma_1\sin\gamma_2\sin(\beta_1+\beta_2)\Big|^2 + \\
\frac{2.5}{4}&\Big| -2i\cos\theta_1\sin\theta_2\sin\gamma_1\sin\gamma_2\cos(\beta_1+\beta_2) + 2\sin\theta_1\cos\theta_2\cos\gamma_1\cos\gamma_2\sin(\alpha_1+\alpha_2)\Big|^2 + \\
\frac{2}{4}&\Big| -2\cos\theta_1\sin\theta_2\cos\gamma_1\sin\gamma_2\cos(\alpha_1-\beta_2) - 2i\sin\theta_1\cos\theta_2\sin\gamma_1\cos\gamma_2\sin(\beta_1-\alpha_2)\Big|^2 + \\
\frac{2}{4}&\Big| -2\cos\theta_1\sin\theta_2\sin\gamma_1\cos\gamma_2\cos(\beta_1-\alpha_2) + 2i\sin\theta_1\cos\theta_2\cos\gamma_1\sin\gamma_2\sin(\alpha_1-\beta_2)\Big|^2 +
\end{aligned}
$$

$$
\begin{aligned}
\frac{2.5}{4}&\Big| 2i\sin\theta_1\cos\theta_2\cos\gamma_1\cos\gamma_2\cos(\alpha_1+\alpha_2) + 2\cos\theta_1\sin\theta_2\sin\gamma_1\sin\gamma_2\sin(\beta_1+\beta_2)\Big|^2 + \\
\frac{2.5}{4}&\Big| -2i\sin\theta_1\cos\theta_2\sin\gamma_1\sin\gamma_2\cos(\beta_1+\beta_2) + 2\cos\theta_1\sin\theta_2\cos\gamma_1\cos\gamma_2\sin(\alpha_1+\alpha_2)\Big|^2 + \\
\frac{2}{4}&\Big| -2\sin\theta_1\cos\theta_2\cos\gamma_1\sin\gamma_2\cos(\alpha_1-\beta_2) - 2i\cos\theta_1\sin\theta_2\sin\gamma_1\cos\gamma_2\sin(\beta_1-\alpha_2)\Big|^2 + \\
\frac{2}{4}&\Big| -2\sin\theta_1\cos\theta_2\sin\gamma_1\cos\gamma_2\cos(\beta_1-\alpha_2) + 2i\cos\theta_1\sin\theta_2\cos\gamma_1\sin\gamma_2\sin(\alpha_1-\beta_2)\Big|^2
\end{aligned}
$$