

Shortest Paths with BFS

Here is my own solution to “Quiz 7”. The problem was to modify BFS so that it finds *shortest paths* to all vertices from a given starting vertex (i.e. the SSSP problem).

The key changes to the algorithm are:

1. Change the function signature to include the starting vertex. (I also renamed the algorithm.)
2. Maintain a value for “distance” of every vertex, initialized to “infinity” for all vertices.
3. Eliminate the loop in the main part, instead calling helper just one time. (NOTE: At this point, since the helper function is not recursive, there’s almost no need to have a “helper” function at all. This code could all be put together in a single function. But I left it this way to preserve the underlying similarity to BFS.)
4. In the helper function:
 - a. Set distance of v to be 0 (this is the starting vertex)
 - b. Each time you visit a vertex (w), its distance is one step farther than the vertex you are “coming from” ($Q.head$)

Here is the algorithm with the locations of those changes marked:

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Algorithm Shortest_Paths_From_S(Graph G, vertex S) [1]  
// Graph G = {V,E}  
// vertex S = starting vertex  
    initialize visited to false for all vertices  
    initialize distance to “infinity” for all vertices [2]  
    // call the helper function just once, with S  
    bfs_helper(S) [3]  
END  
  
function bfs_helper(Vertex v)  
    visit node v  
    set distance(v) = 0 [4a]  
    initialize a queue Q  
    add v to Q  
    while Q is not empty  
        for each w adjacent to Q.head  
            if w has not been visited  
                visit node w  
                // w is one step farther away than the  
                // vertex we are currently processing  
                set distance(w) = distance(Q.head) + 1 [4b]  
                add w to Q  
            endif  
        endfor  
        Q.dequeue()  
    endwhile  
END  
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```