Note about this deck

- We covered a bunch of this stuff last week
- Last week’s copy of this slide deck had a bunch of Greedy Algorithm stuff (major optimism that we would get to it)
- That stuff is OUT of this deck now
  - It’s moved to “Lecture 9” deck today
- But we have one topic at the end of this deck to cover (TopoSort algo #2)

Lecture 8

COMP 3760

Solving problems with graph algorithms

Topological Sorting (Text chapter 4.2)

Greedy Algorithms (Text chapter 9)

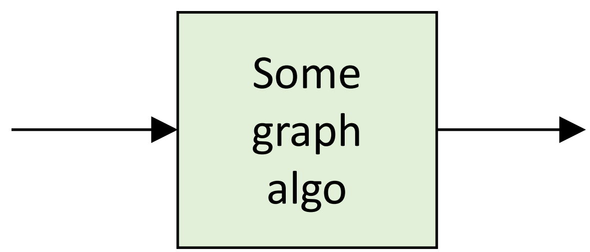
Solving problems with graph algorithms

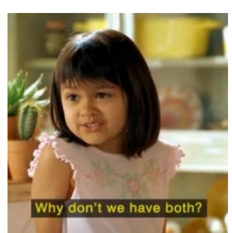
How can we use graph algorithms to solve problems

Two strategies

1. Modify a known graph algorithm
   - Technically we have already done this
   - DFS and BFS in class notes did not perform output
   - But we tracked it in the examples

2. Use a known graph algorithm as a black box
   - Black box needs input
   - Black box gives output



- Bonus strategy: 

Example: Connected components

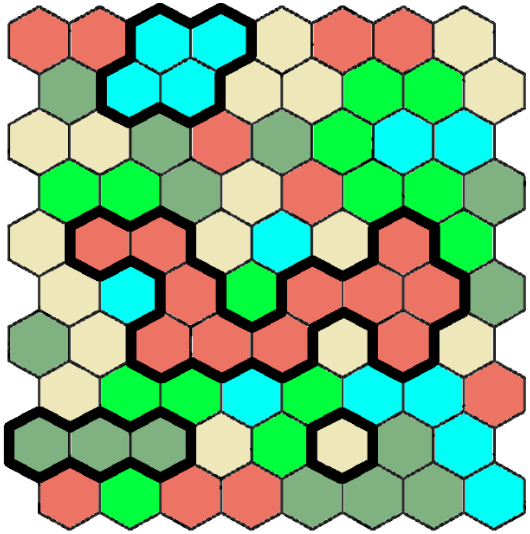
- Problem: Given a graph G, how many *connected components* does G have?

- Strategy 1: *Modify a known graph algorithm*

- Solution idea: Use either DFS or BFS
  - Add a counter to the "main loop"
  - Count how many times (from main) the helper function is called
  - Return the counter

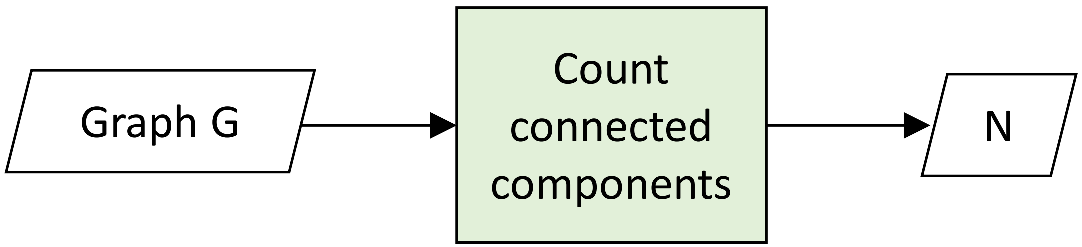
Example 2: Count map regions

- A board game is played on hex tiles.
- Tiles are drawn at random from a large supply.
- Adjacent tiles of the same colour make up a *region*.
- Input data is a list of the colours for all tiles:
  - colour(0,0) = red
  - colour(4,4) = aqua
- Problem: determine how many regions are on the map.
  - This map has 38 regions →



Solution idea

- Strategy 2: Use the “connected components” algorithm as a black box

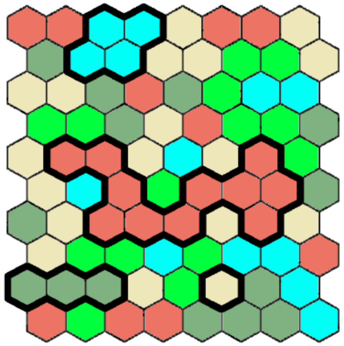


- We need to construct a clever graph – *just the right graph*
  - It will encapsulate or model or represent our input problem in some way

Finding the right graph

We are given a list of colour/point data:

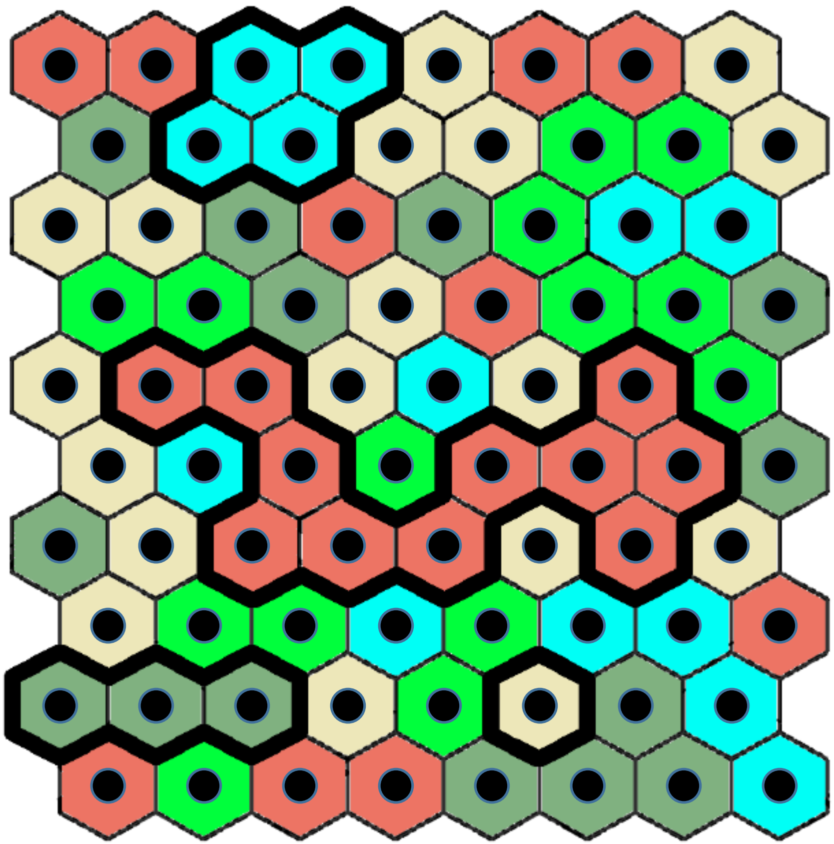
colour(0,0) = red
colour(0,1) = red
colour(0,2) = aqua
colour(0,3) = aqua
colour(0,4) = beige
...
colour(9,7) = aqua



- For a graph we need vertices and edges
- Vertices represent *things* and edges represent *relationships between things*

- The things we have are tiles
  - Idea: represent each tile as a vertex
  - Every vertex "label" will be a point (the grid location)

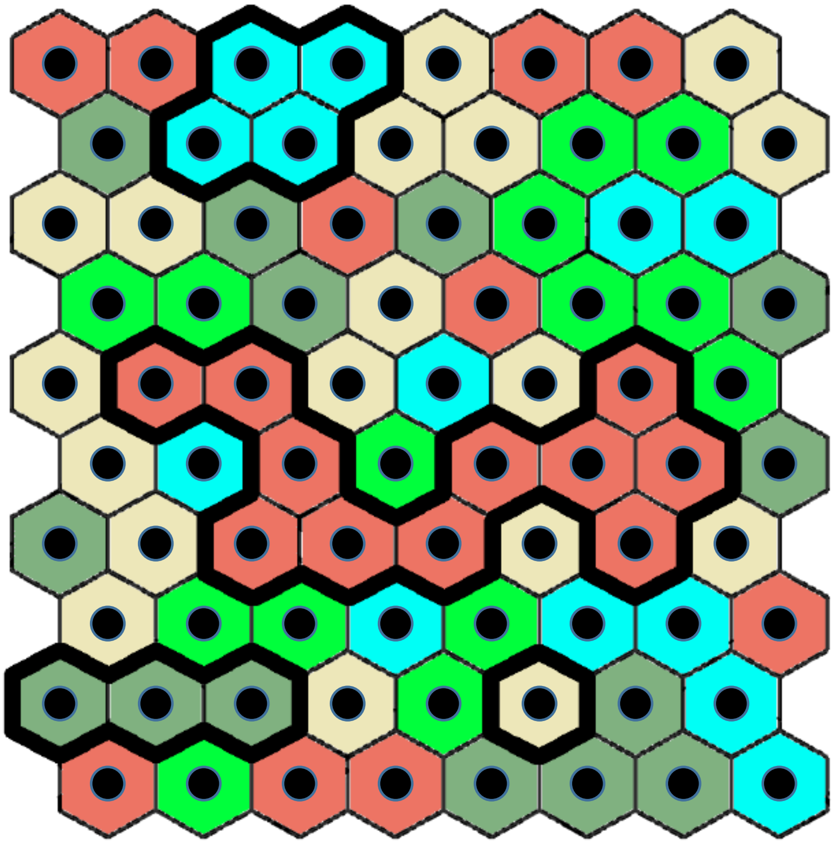
Every tile is a vertex



V = {(0,0), (0,1), (0,2), (0,3), (0,4), (0,5), (0,6), (0,7), (1,0), (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (1,7), (2,0), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (2,7), (3,0), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (3,7), (4,0), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (4,7), (5,0), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (5,7), (6,0), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6), (6,7), (7,0), (7,1), (7,2), (7,3), (7,4), (7,5), (7,6), (7,7), (8,0), (8,1), (8,2), (8,3), (8,4), (8,5), (8,6), (8,7), (9,0), (9,1), (9,2), (9,3), (9,4), (9,5), (9,6), (9,7) }

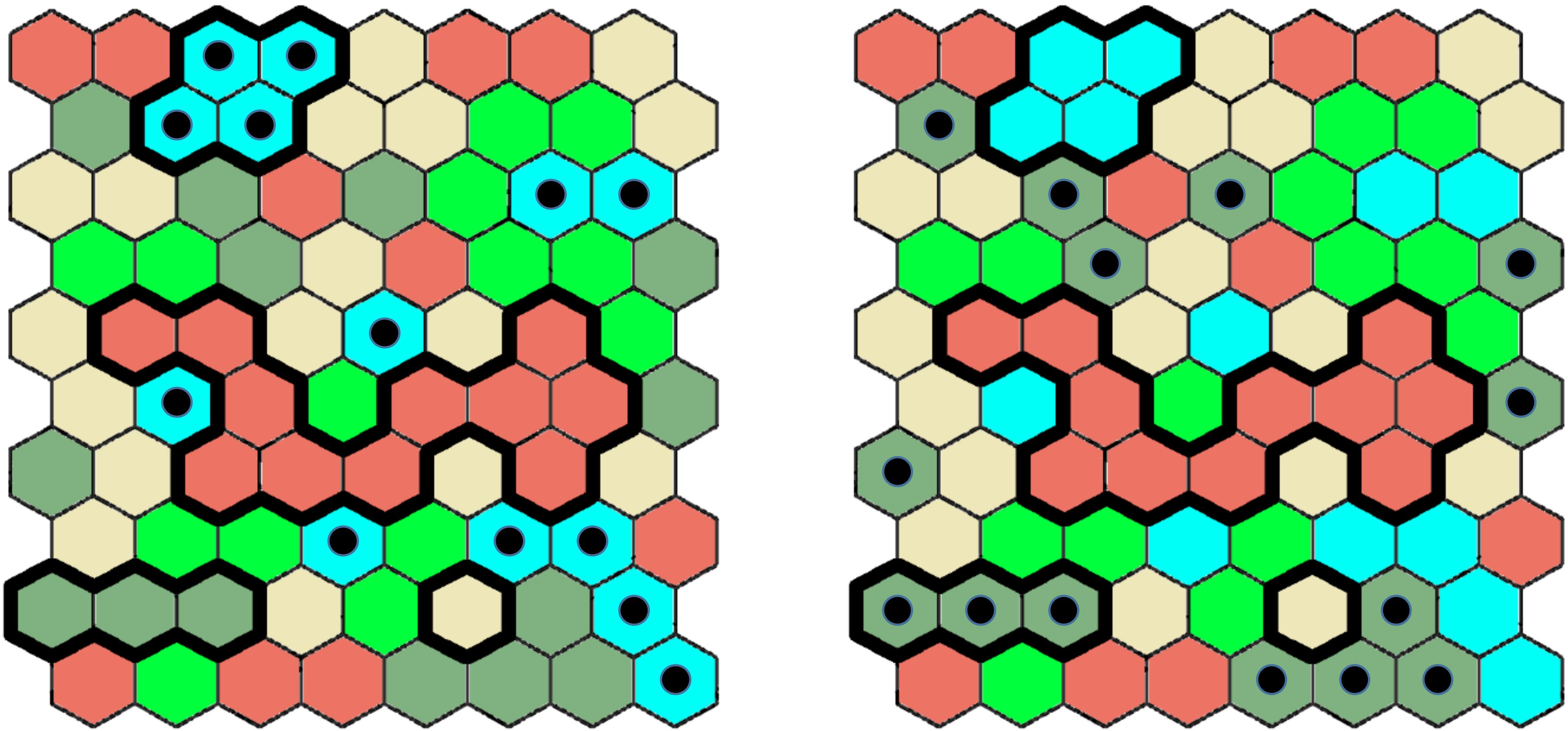
Now how about edges?

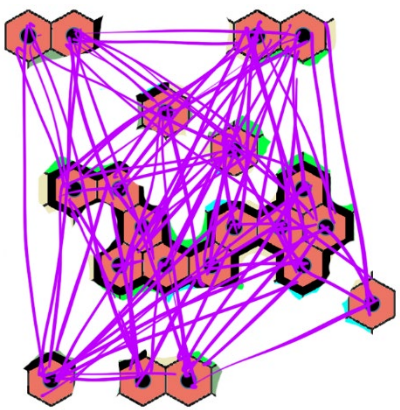
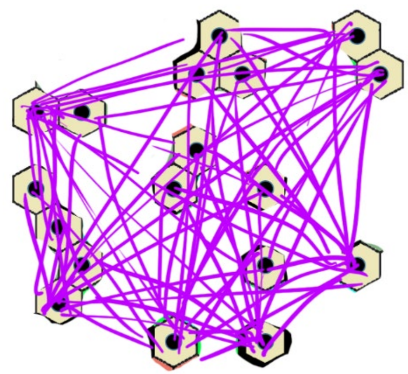
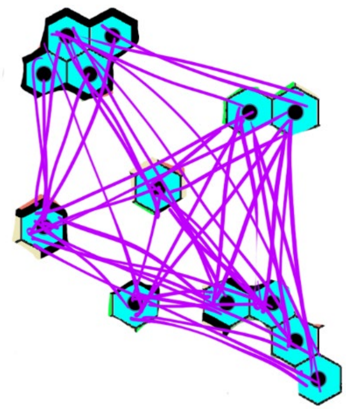
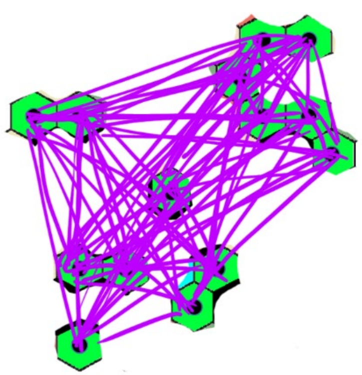
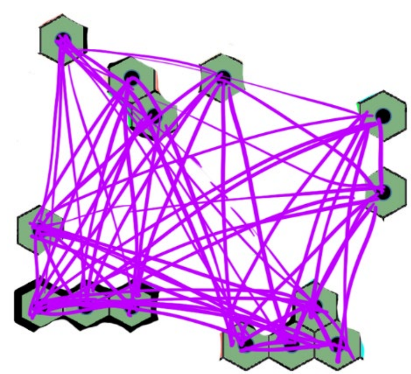
- Idea: Connect vertices of the same colour



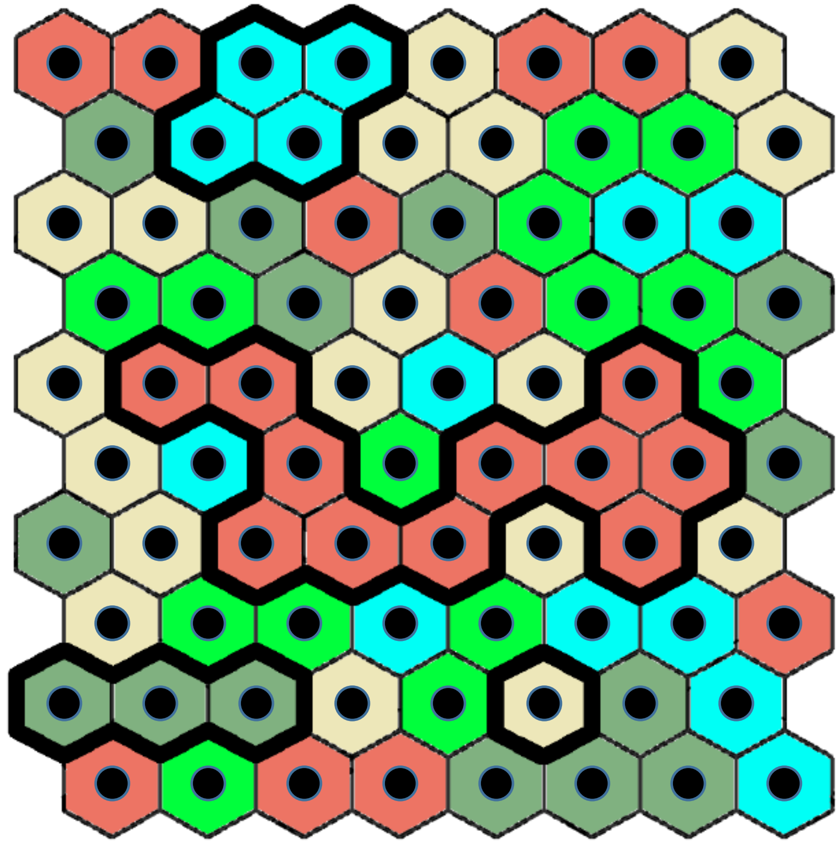
Now how about edges?

- Idea: Connect vertices of the same colour

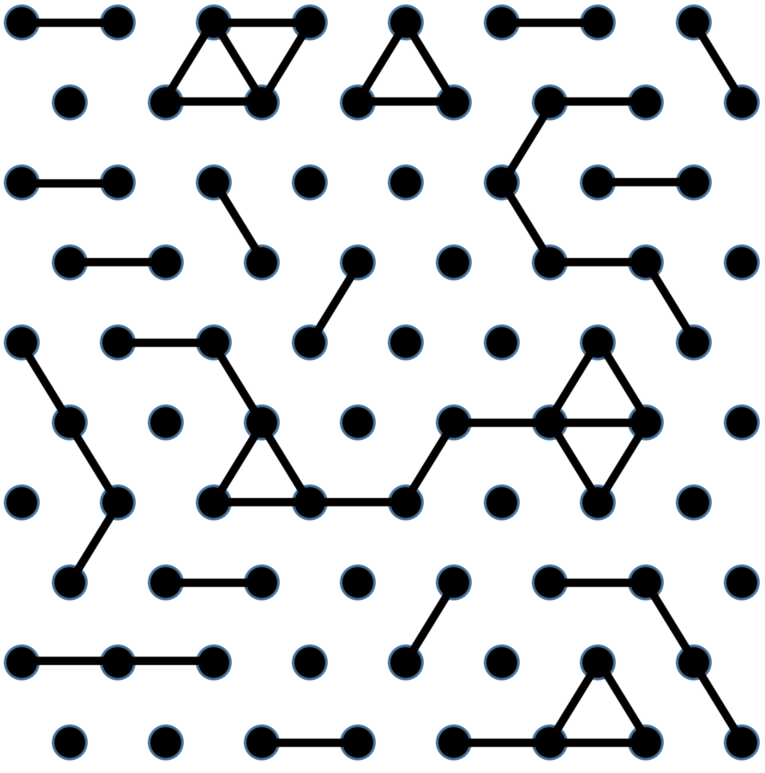


Idea 2: Same colour AND adjacent



Idea 2: Same colour AND adjacent



The Solution

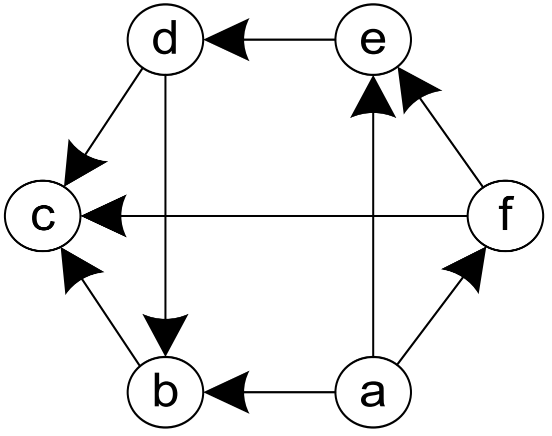
- Step 1: Define a Graph G=(V, E) as follows:
  - The vertices of G are the tiles of the map; each is represented by a grid location aka point (0,0) to (m,n)
  - There is an edge between two vertices u and v iff
    - u and v have the same colour AND
    - u and v are in adjacent locations
- Step 2: Run “Count Connected Components” on G
- Step 3: The output of step 2 is the final answer

Graph Algorithms: Topological sorting

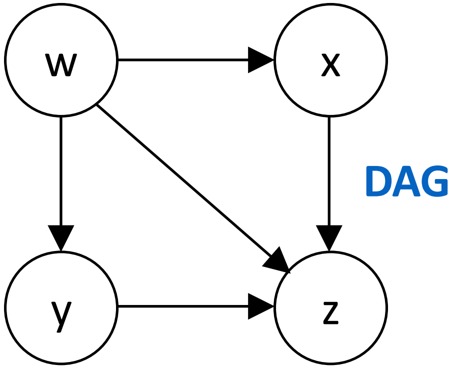
Textbook: Chapter 4.2

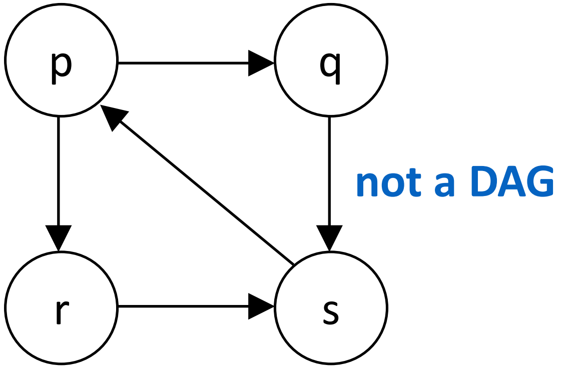
Directed acyclic graphs (DAGs)

- A **<u>directed graph</u>** is a graph whose edges are directional or one-way



- A **<u>directed acyclic graph</u>** is a directed graph that contains no cycles





Topological sort problem

- Given a set of tasks with dependencies (precedence constraints), *e.g.*, “task A must be completed before task B”, …
- … find a linear ordering of the tasks that satisfies all dependencies

Example: Getting dressed

- Suppose you need to wear all these items:

  - Belt
  - Jacket
  - Pants
  - Shirt
  - Shoes
  - Socks
  - Suspenders
  - Tie
  - Underwear

- Some of these items must come before others

Example: Getting dressed

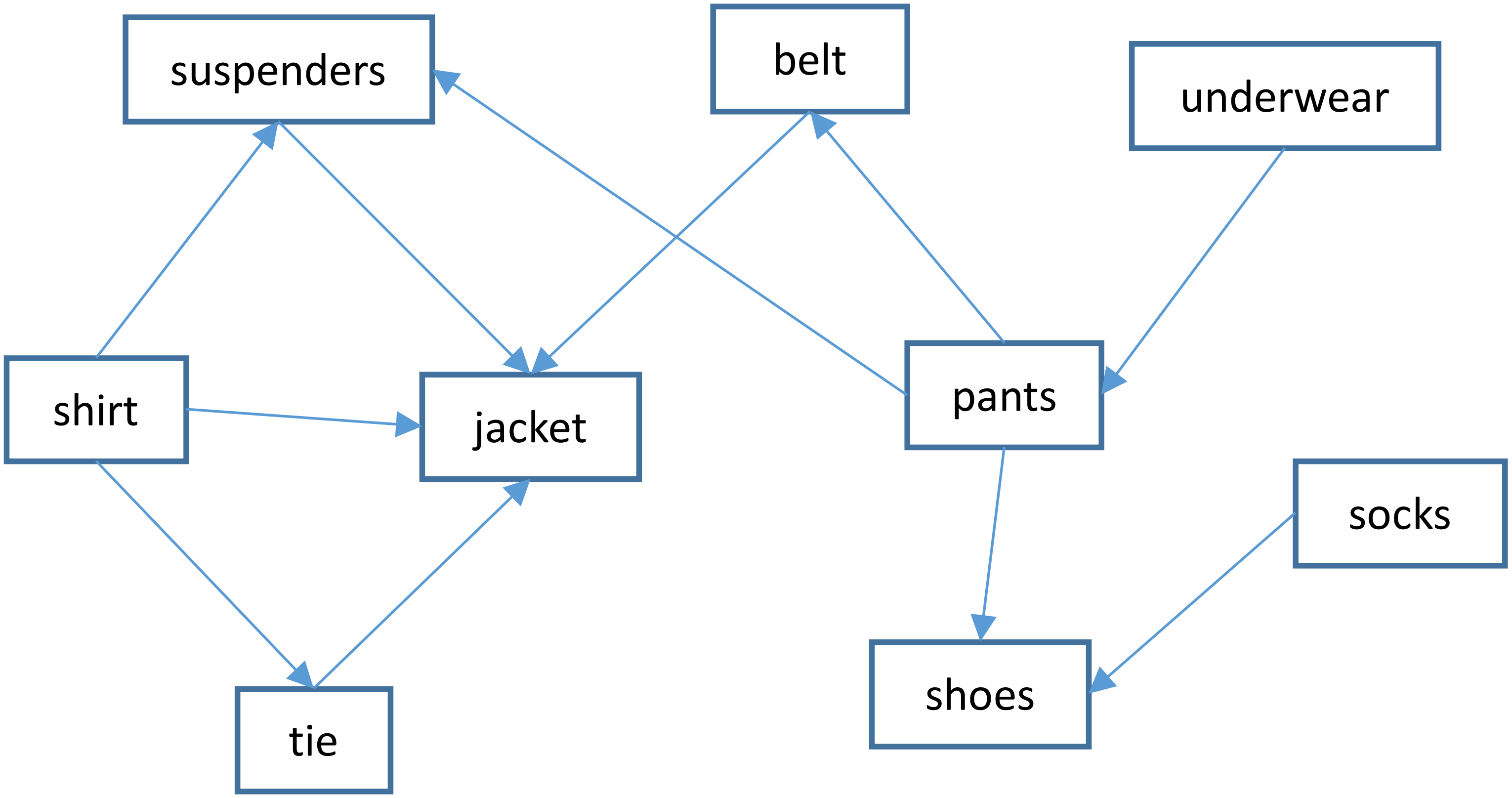
- Socks before shoes
- Shirt before suspenders
- Pants before suspenders
- Pants before shoes
- Pants before belt
- Shirt before tie
- Shirt before jacket
- Suspenders before jacket
- Belt before jacket
- Tie before jacket
- Underwear before pants

Represent the problem as a graph

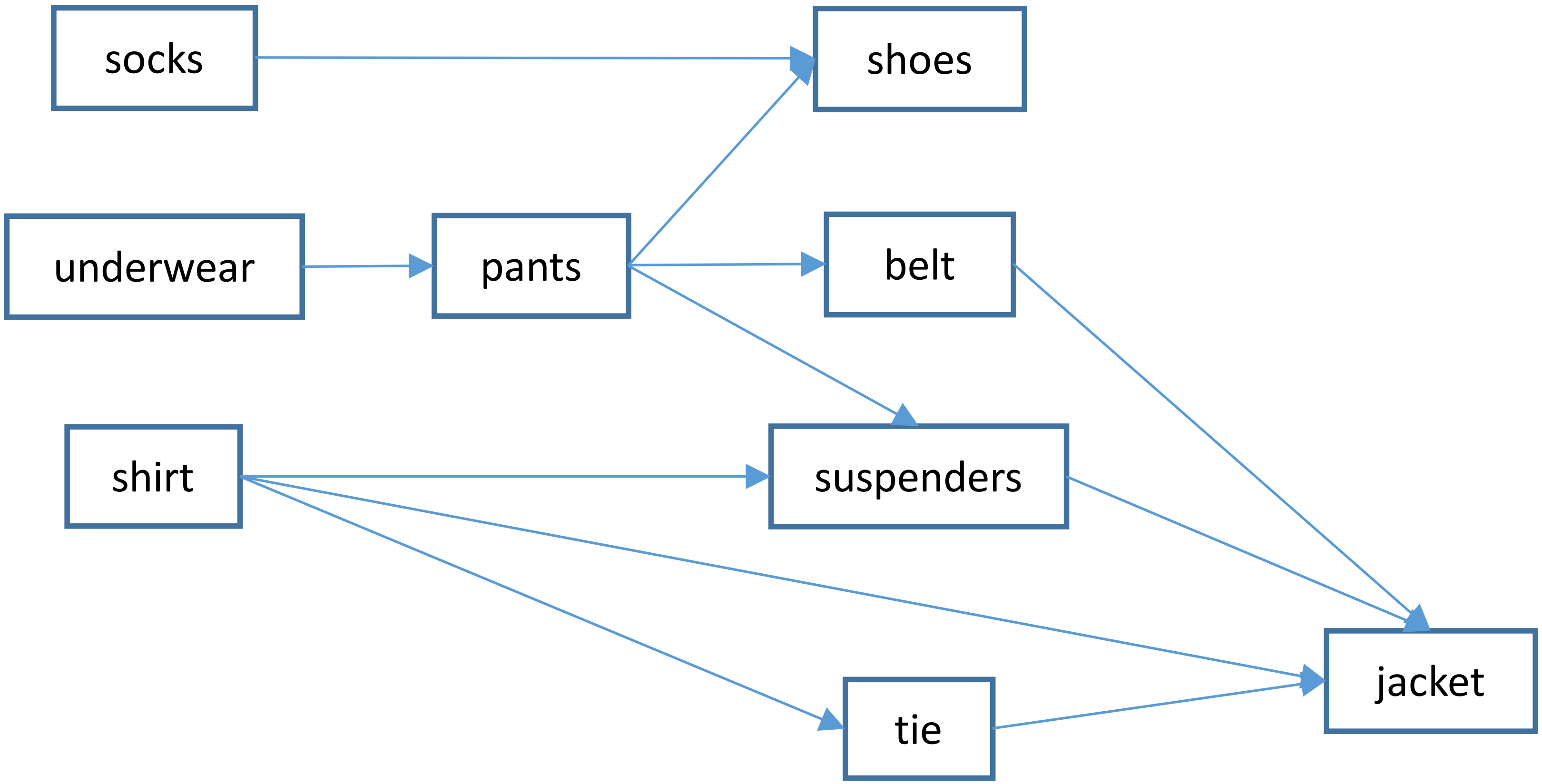
1. V = vertices are the items (tasks)
2. E = edges are the dependencies (constraints) between tasks

- an edge ($v \rightarrow w$) means:
  - w is dependent on v, OR (in other words)
  - Task v comes before task w

Clothing graph



Eyeballing a solution



socks … underwear … shirt … pants … shoes … belt … suspenders … tie … jacket

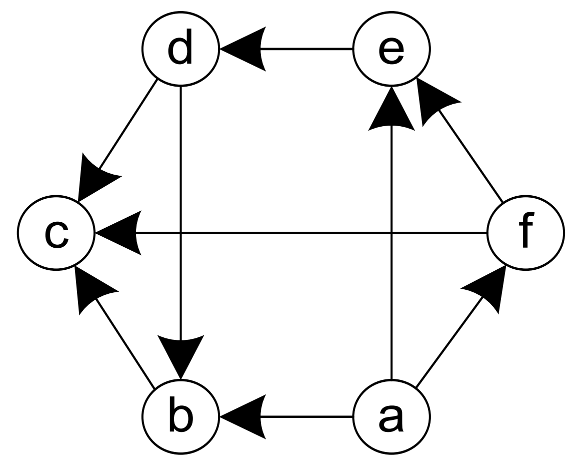
Topological Sort Algo 1: Use Depth First Search

1. Apply DFS to G
   - Starting at any vertex
   - No, really: ANY vertex
2. The order in which vertices become dead ends is the *reverse* of a topological sort order
   - Why?

Example 1

- Assume you have a set of 6 tasks (a, b, c, d, e, f) with the following dependencies:
  - a must be done before b, e, f
  - b must be done before c
  - d must be done before b and c
  - e must be done before d
  - f must be done before c and e

- Step 1: Construct a directed graph to represent the problem (verify it is a DAG)

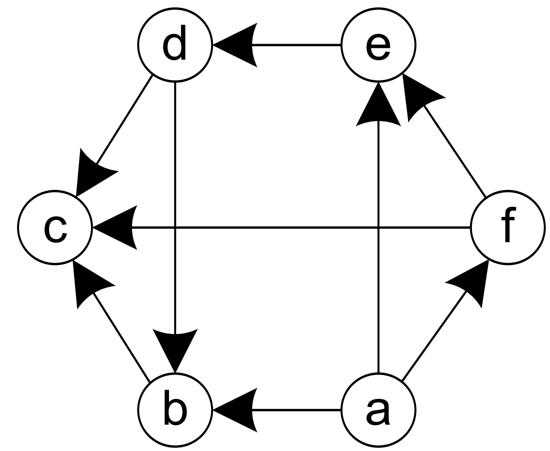


Example 1 (cont)

- <u>Step 2</u>: Apply DFS

  Order vertices
  become dead ends:

  c b d e f a



- <u>Step 3</u>: Reverse this to get topological sort order:

  a f e d b c

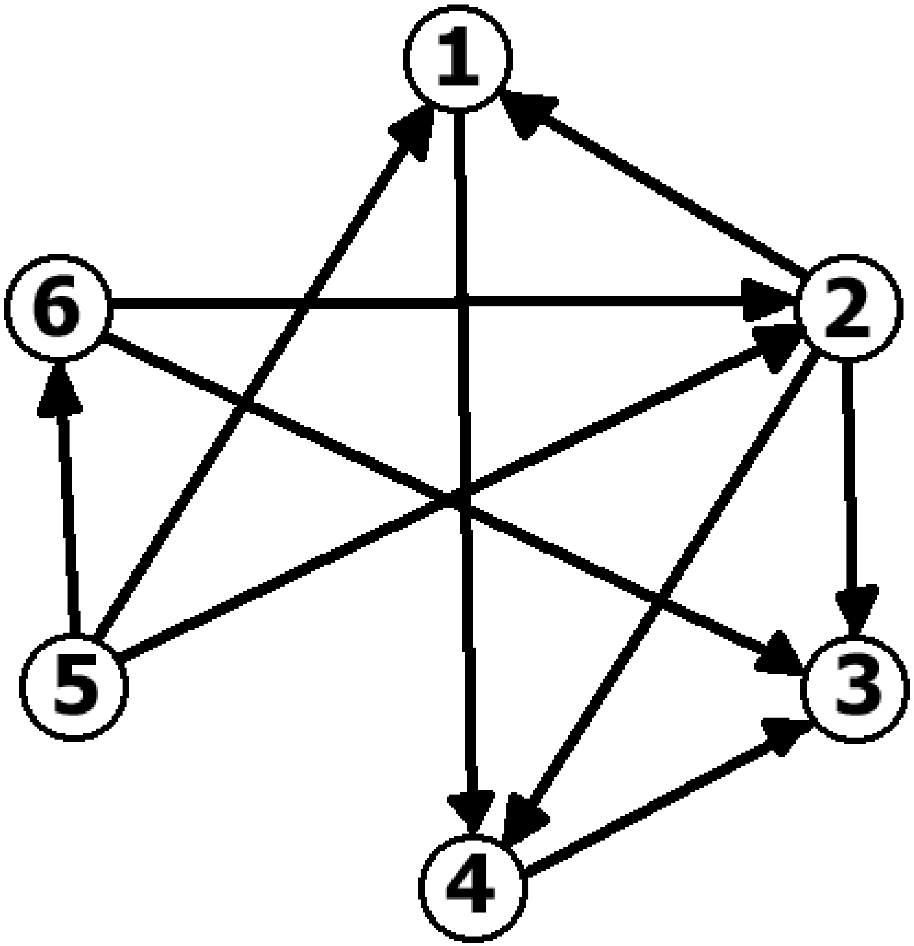
Example 2

2 1 (2 before 1)
1 4 (1 before 4)
2 3 (2 before 3)
5 6 (5 before 6)
2 4 (2 before 4)

4 3 (4 before 3)
5 2 (5 before 2)
5 1 (5 before 1)
6 3 (6 before 3)
6 2 (6 before 2)

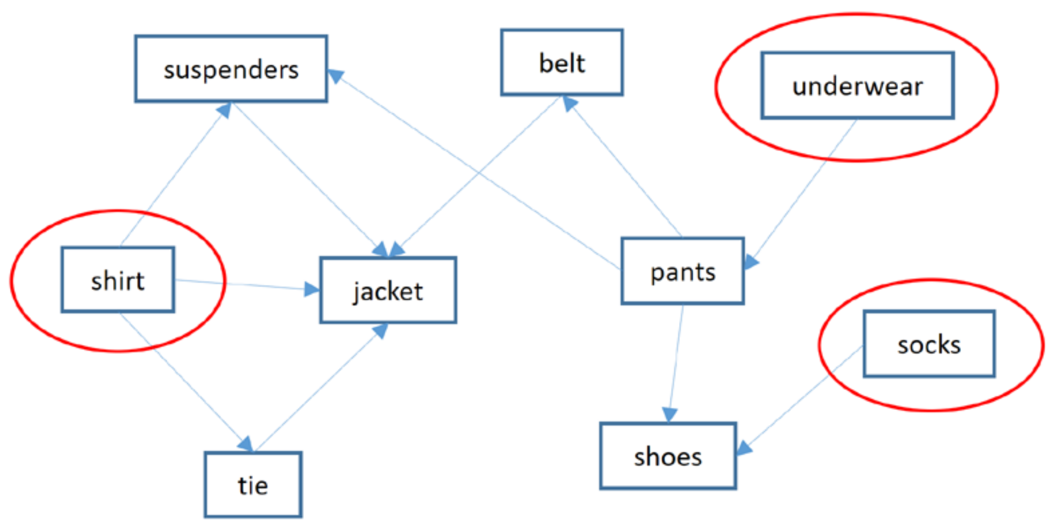
- Step 1: draw the graph (and verify it is a DAG)
- Step 2: apply DFS, get “dead-end” order
- Step 3: reverse this to get topological sort order

Example 2 (cont)



TopoSort Algorithm 2: Decrease (by 1) and conquer

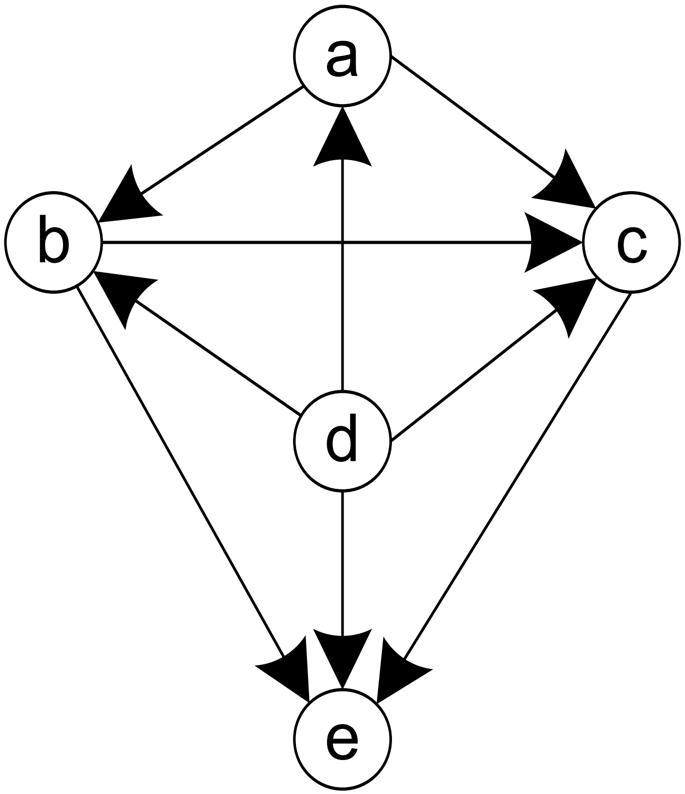
- Key observation:
  - If a vertex v in the dependency graph G has no incoming arrows (*i.e.* in-degree(v) == 0), then v does not have any dependencies
  - It follows that any v that does not have dependencies is a candidate to be visited next in topological order
  - *i.e.* any of these can go first →

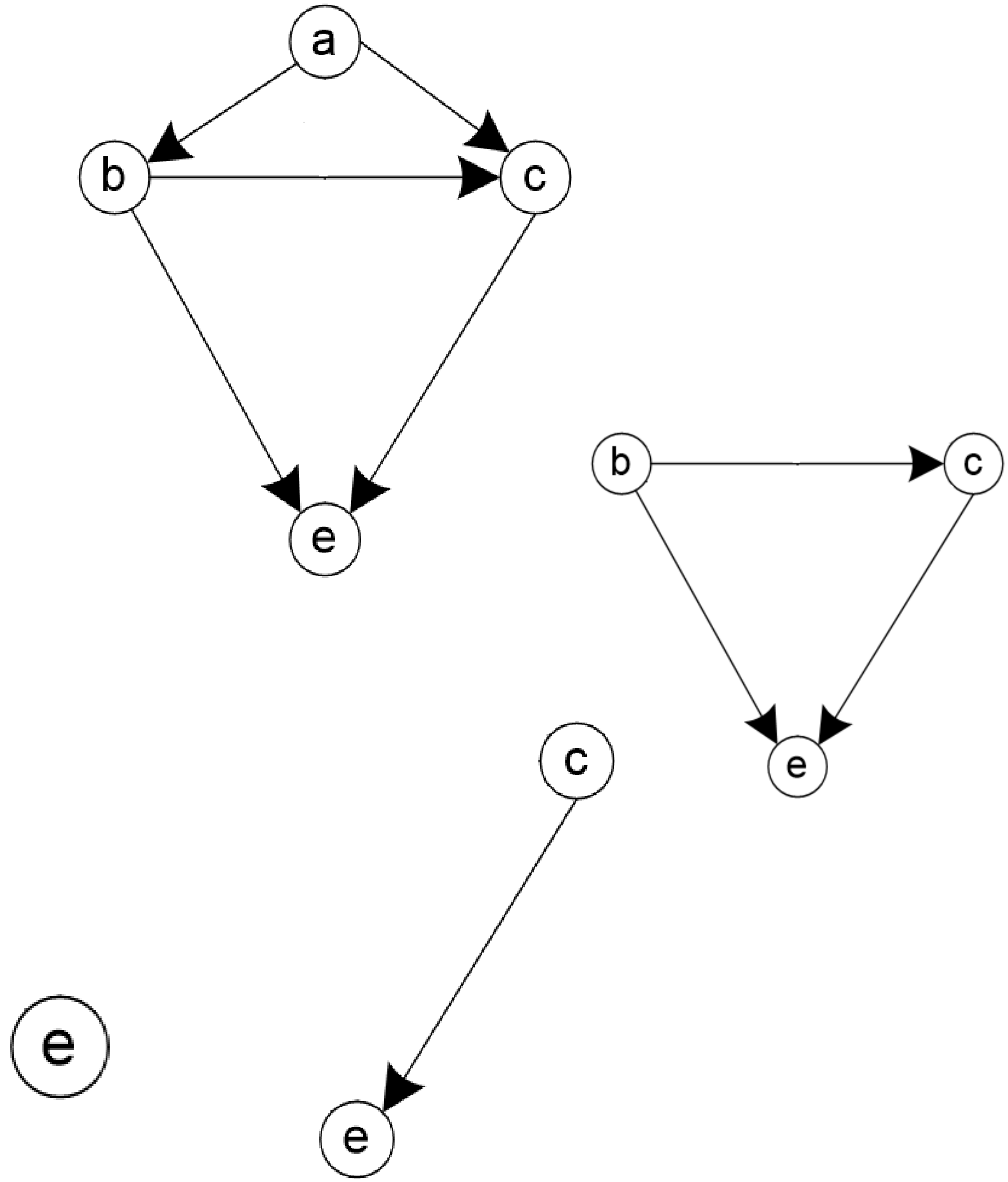


Idea of the algorithm

- Identify a $v \in V$ that has in-degree=0
- Delete v and all edges coming out of it
- Repeat until done
- The topological order is the order the vertices are deleted
- If there are $v \in V$, but no v has in-degree=0, the graph G is not a DAG (no feasible solution exists)

Example 3





Algorithm details

- Use a set to store the candidate vertices
  - *I.e.* the vertices with in-degree = 0
  - Any ordered set will do, e.g. TreeSet.
- Use an ordered list to store the delete order
  - Any list type will do, e.g. ArrayList, always adding to the end

TopoSort “Decrease by one” pseudocode

```
Algorithm TopoSort(G)
   create an empty ArrayList A
   create an empty TreeSet Candidates
   add all v with inDegree=0 to Candidates
   while Candidates is not empty
      v = Candidates.first()
      add v to A
      for each vertex w adjacent to v
         remove edge (v,w) from G
         if w has inDegree=0
            add w to Candidates
      remove vertex v from G
   if there are no vertices remaining in G
      solution is in A
   else
      no solution exists
```

Practice problems

- Chapter 4.2, page 142, question 1
- Chapter 5.3, page 185, questions 5 & 6