

Stochastic Methods

Assignment 1

Sumeet Gyanchandani

1 Bayes Rule

If 5% of men and 0.25% of women are color-blind. A person is chosen at random and that person is color-blind. What is the probability that the person is male? (if males and females are in equal numbers).

By Bayes Rule:

$$\begin{aligned}
 \mathbb{P}(\text{male} | \text{colorblind}) &= \frac{\mathbb{P}(\text{colorblind} | \text{male}) \mathbb{P}(\text{male})}{\mathbb{P}(\text{colorblind})} \\
 &= \frac{\mathbb{P}(\text{colorblind} | \text{male}) \mathbb{P}(\text{male})}{\mathbb{P}(\text{colorblind} | \text{male}) \mathbb{P}(\text{male}) + \mathbb{P}(\text{colorblind} | \text{female}) \mathbb{P}(\text{female})} \quad [\text{By Total Probability}] \\
 &= \frac{0.05 \times 0.5}{0.05 \times 0.5 + 0.0025 \times 0.5} \\
 &= 0.9524
 \end{aligned}$$

2 Probability Density Functions

We know that when a Probability Density Functions is integrated over it's whole domain it is equal to 1.

Therefore,

$$\begin{aligned}
 \int_0^{\pi/2} a \sin x &= 1 \\
 a \int_0^{\pi/2} \sin x &= 1 \\
 a[-\cos(\pi/2) - (-\cos(0))] &= 1 \\
 a[0 - (-1)] &= 1 \\
 a &= 1
 \end{aligned}$$

Hence, for $a = 1$ the function f is the probability density function.

3 Matlab tutorial

Listing 1. Matlab Script

```
%(a) Set seed to 0 using command rng

rng(0);
seed = rng;

%(b) Create a Gaussian random sample X of 1000 observations with mean  $\mu=2$  and standard deviation  $\leftrightarrow$ 
 $\sigma=1.5$  using command randn.

sigma = 1.5;
mu = 2;
X = sigma.*randn(1000,1) + mu;
stats = [mean(X) std(X) var(X)];

%(c) Visualize X using command plot

figure; plot(X);
figure; plot(X, 'b+');

%(d) Show the normalized histogram of X using command hist and bar

figure; bar(hist(X) ./ sum(hist(X)));

%(e) Fit the normal distribution to X and obtain estimated  $\hat{\mu}$  and  $\hat{\sigma}$  using command normfit.

[muhat,sigmahat] = normfit(X);

%(f) Compute corresponding pdf values evaluated at X using  $\hat{\mu}$  and  $\hat{\sigma}$  with command normpdf.

Y = normpdf(X,muhat,sigmahat);

%(g) Plot obtained pdf values over normalized histogram graph using command scatter.

figure; scatter(X, Y);
```

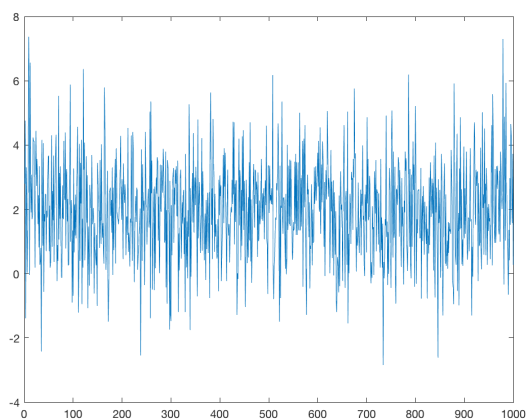


Figure 1. Visualization of X using command plot

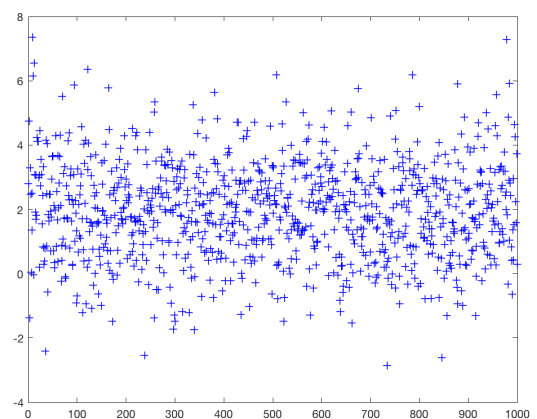


Figure 2. X using command plot with + markers

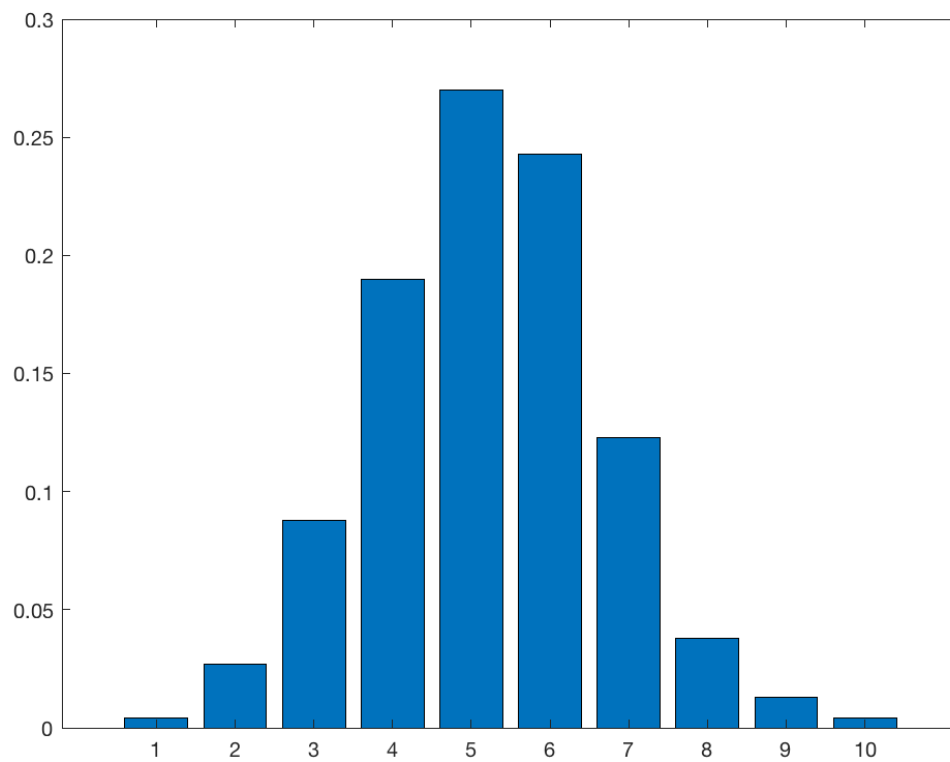


Figure 3. The normalized histogram of X using command `hist` and `bar`

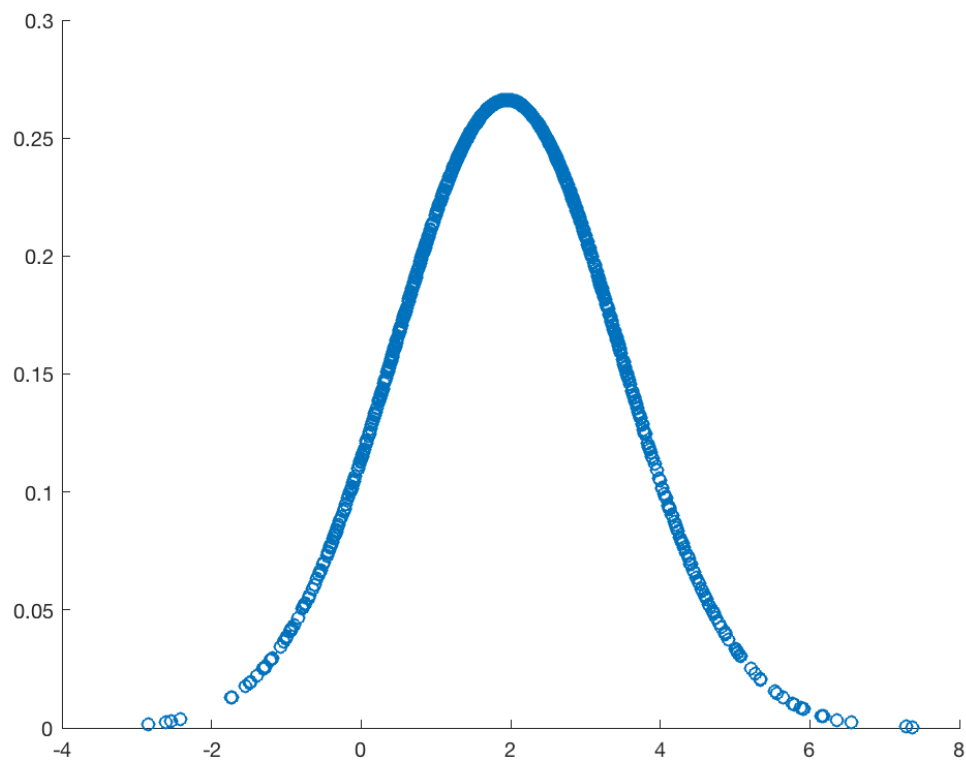


Figure 4. Plot of pdf values over normalized histogram graph using command `scatter`