# The pumping of a swing from the standing position

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mental quantum world structure. Most of the disparate views are reactions to the problem of measurement. Some of these views are cited in Ref. 39. For a critical survey of some of the recent alternative interpretations, see

46. **Quantum Mechanics and Experience**, D. Albert (Harvard U. P., Cambridge, MA, 1992). (E,I)

Note particularly the Ghirardi–Rimini–Weber (GRW) "spontaneous collapse" treatment of quantum evolution (pp. 92–112), which incorporates a nonobservation-driven indeterminism into the theory.

### *3. Quantum determinism: Bohm*

In 1952 David Bohm proposed a reformulation of quantum mechanics that preserved deterministically evolving space-time trajectories for quantum particles. Roughly, Bohm's approach was to reexpress the Schrödinger equation in a form resembling Newton's second law of motion, a form naturally interpreted in terms of particle trajectories. Particle behavior, while uniquely determined, is controlled by both a classical energy potential and a novel "quantum potential" whose novelty lies chiefly in that its value depends on the shape of the quantum mechanical state but not its magnitude. Thus, for instance, the quantum potential need not fall off with distance in the way the classical energy potential does.

This nonlocality in the Bohm account is one of the factors that has led it to be largely ignored by physicists. Reference 39 cites renewed interest in it as associated with discussions of nonlocality in the context of Bell's theorem. For a thorough examination of Bohm's original proposal and its reception in the physics community see

47. **Quantum Mechanics: Historical Contingency and the Copenhagen Hegemony**, J. Cushing (Univ. of Chicago, Chicago, 1994). (I,A)

For a comprehensive treatment of Bohmian quantum mechanics, with links to a wide variety of topics in quantum theory, see

48. **The Quantum Theory of Motion: An Account of the de Broglie-Bohm Causal Interpretation of Quantum Mechanics**, P. R. Holland (Cambridge U. P., Cambridge, 1993). (I,A)

Reference 46 also provides an accessible account (Chap. 7). A wide-ranging assessment of Bohm's approach, including such topics as its implications for the formulation of a quantum analog to classical chaos, is provided in

49. **Bohmian Mechanics and Quantum Theory: An Appraisal**, edited by J. Cushing, A. Fine, and S. Goldstein (Kluwer, Dordrecht, 1995). (E,I,A)

Bohm's own popular account of his formulation, set in the context of considerations of causality in general, is his classic

50. **Causality and Chance in Modern Physics**, D. Bohm (Harper, New York, 1957). (E,I)

Bohm's book provides an entree to many of the issues discussed in this section of this Resource Letter.

## III. FINAL WORD

To me and many others, the most exciting areas of physics from the standpoint of our understanding of causality and determinism lie in the study of the nature and role of singularities in general relativity and in the foundational treatment and experimental investigation of correlated quantum systems. In both of these areas there is the imminent and practical potential for the realization of Hamlet's timeless caution that there is more in heaven and earth than we ever dreamed of in our philosophy.

# The pumping of a swing from the standing position

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The pumping of a swing from a standing position is modeled as a rigid object forced to rotate back and forth at the lower ends of supporting ropes. This model after some approximations leads to a harmonic oscillator with driving and parametric terms. It is then argued that in the regime of the common playground swing the driving terms dominate and the pumping of a swing in the standing position is best characterized as a driven oscillator. Examination of the relative phase of the swinger and the swing also supports this conclusion. This model is compared with earlier work which claimed that the swing pumped by a standing swinger is characterized as a parametric oscillator. Simple demonstrations of both mechanisms are described. A comparison of pumping while standing and seated is also made. © *1996 American Association of Physics Teachers.*

## I. INTRODUCTION

This paper is a study of the mechanism of pumping a swing from the standing position. In an earlier paper,[1] referred to as SW1 in the current discussion, the corresponding mechanism for a seated swinger was analyzed. In that paper it was shown that the pumping mechanism is an example of a driven harmonic oscillator, where the second-order equation of motion contains a term periodic in time and independent of the variable describing the motion. Prior to that it was widely believed that this system was an example of a parametric oscillator where the time-dependent term is multiplied by the variable representing the motion. The present paper contains a similar analysis of the pumping of a swing from a standing position and reaches a similar conclusion.

The pumping of a swing is an almost ideal example of a physical system. It is simple and transparent enough to be analyzed with confidence and complicated enough to produce almost magical results. When pumped from the standing position, the pumped swing is widely believed to be an example of a parametric instability.[2–7] The swinger is viewed as flexing and straightening his knees thereby lowering and raising his center of mass along the length of the swing. As a result the relevant parameter, the distance from the point of support of the swing to the center of mass of the swinger, consists of a constant plus a periodically varying length. The analysis of this motion, which will be repeated here in Sec. III, leads to the standard equation characteristic of a parametric instability. One can go to the playground, use the method of flexing and straightening the knees and successfully pump a swing in a standing position. There is, however, a problem. Aside from being the oldest kid at the playground, you find that you are the only one performing this purely vertical motion. All of the other "kids" are doing a lot more leaning forward and back, much more rotation. To make matters worse, the other kids are more successful at pumping the swings, and their growth of amplitude per cycle is greater than yours. The only time I have seen anyone swing according to the "physicist's method" was when watching a young girl who was trying to copy my strange movements.

We will examine the effects of leaning back and forth of the body for the standing swinger. Some authors have considered this motion but only as a mechanism to initiate the motion of the swing.[8–10] Their analyses would not allow them to see past the initial stages and they failed to appreciate the fundamental role it plays in the typical pumping of the swing.

In the analysis carried out here, the swinger will be treated as a rigid body rotating about the lower point of the ends of the supporting ropes of the swing. The swinger uses his[11] arms to force his body to lean forward and back. The equations of motion, which are obtained in Sec. II, contain driving as well as parametric terms. It is then argued that the driving term dominates in the regime typical of swinging. To compare this mechanism with earlier arguments, a system where the center of mass moves along the supporting ropes is analyzed in Sec. III and compared with the results of Sec. II. Demonstrations which show the mechanisms of Secs. II and III are described in Sec. IV. Section V gives a closing discussion and a comparison of the results for swinging in a standing position and in a seated position.

## II. DERIVATION AND ANALYSIS OF THE EQUATION OF MOTION

We model the swinger as a rigid body of mass $m$ and a moment of inertia about the center of mass of $mR^2$ (this is the moment about the axis perpendicular to the plane of the swinging motion). The length of the ropes supporting the swinger is $l$. The angle between these supporting ropes and the vertical is $\phi$. The rigid body rotates about the lower end of the ropes. The center of mass of the swinger is at a distance $s$ from the lower ends of the ropes and the angular

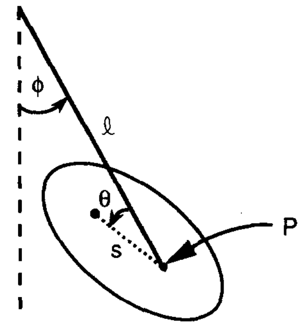


Fig. 1. Model of the swinger discussed in Sec. II. Here the swinger pumps the swing by rocking or leaning forward and backward.

displacement from the center of mass to the line of the rope is $\theta$ in the sense indicated in Fig. 1. The swing is driven by the swinger forcing the angle $\theta$ to vary as

$$\theta=\theta_0 \cos \omega t. \tag{1}$$

Since the point $P$ of the swinger is fixed at the lower ends of the ropes, the swinger rotates with an angular velocity of $\dot\phi+\dot\theta$. The Lagrangian for the angle $\phi$ is given by,

$$L=\tfrac{1}{2}m[l^2\dot\phi^2-2sl\dot\phi(\dot\phi+\dot\theta)\cos\theta+(s^2+R^2)(\dot\phi+\dot\theta)^2] +mg[l\cos\phi-s\cos(\phi+\theta)].$$

This gives the equation of motion,

$$[l^2-2ls\cos\theta+s^2+R^2]\ddot\phi =-gl\sin\phi+gs\sin(\phi+\theta)-ls\sin\theta\,\dot\theta^2+(ls\cos\theta -s^2-R^2)\ddot\theta-2ls\sin\theta\,\dot\theta\dot\phi.$$

The dependence on $\phi$ is expanded in powers of $\phi$ and only the zeroth- and first-order terms are kept. This is justified for swinging at small to modest amplitudes; thus the result will be correct as the motion starts but will become less accurate as the amplitude grows. The dependence on the angle $\theta$ is also expanded in powers of $\theta$ and terms to second order are kept. We anticipate that $\theta$ will be about 0.5 rad. With these approximations the equation of motion becomes,

$$[(l-s)^2+R^2]\ddot\phi=-g(l-s)\phi+gs\theta+[s(l-s)-R^2]\ddot\theta -\left(\frac{gs}{2}\right)\theta^2\phi-2ls\theta\dot\theta\dot\phi-ls\theta^2\ddot\phi. \tag{2}$$

This expansion leaves us with a harmonic oscillator, two driving terms, and three terms with $\phi$, $\dot\phi$, and $\ddot\phi$ times a time-varying coefficient. Keeping terms to second order in $\theta$ allows all classes of possible terms linear in $\phi$ and its derivatives to be represented. Higher-order expansions in $\theta$ would simply lead to small modifications to terms of these same types. When the expression for $\theta$ given in Eq. (1) is substituted in Eq. (2) we have

$$\ddot\phi+\omega_0^2\phi=F\cos\omega t+A\cos(2\omega t)\phi+B\sin(2\omega t)\dot\phi +C\cos(2\omega t)\ddot\phi, \tag{3}$$

where

$$\omega^2=\omega_0^2=\frac{g(l-s)}{R'^2},$$

$$F=\frac{\omega^2 lR^2\theta_0}{(l-s)R'^2},$$

$$A=-\frac{\omega^2 s\theta_0^2}{4(l-s)},$$

$$B=\frac{\omega l s\theta_0^2}{R'^2},$$

$$C=-\frac{ls\theta_0^2}{2R'^2},$$

$$R'^2=(l-s)^2+R^2.$$

In obtaining these expressions we have neglected $\theta_0^2/2$ compared to 1. In addition, we anticipate the fact that the swinger will pump at the natural frequency of the swing and set $\omega=\omega_0$.

This is a harmonic oscillator with a driving term, $F\cos\omega t$, and three parametric terms,[12] $A\cos 2\omega t\phi + B\sin 2\omega t\dot{\phi}+C\cos 2\omega t\ddot{\phi}$. Any of the last four terms are capable of driving the swing. If the amplitude of the motion of the swing is sufficiently small, the contributions from the parametric terms are negligible, the driving term dominates, and the swing is properly considered a driven oscillator. For motion at a sufficiently great amplitude the situation is reversed, the parametric terms dominate and the swing is properly considered a parametric oscillator. In the small amplitude regime the equation of motion is approximately

$$\ddot{\phi}+\omega_0^2\phi=F\cos\omega_0 t, \tag{4}$$

where we have assumed that the motion of the swinger is at the natural frequency of the swing. Assuming that the swing is started from rest with $\phi=0$ the solution of Eq. (4), and Eq. (3) for small amplitudes, is

$$\phi=(Ft/2\omega_0)\sin\omega_0 t. \tag{5}$$

In this regime the growth of the amplitude is linear with time and the growth per cycle is

$$\Gamma_D=\frac{\pi F}{\omega_0^2}=\frac{\pi lR^2\theta_0}{(l-s)R'^2}. \tag{6}$$

When the amplitude is large the equation of motion is approximately

$$\ddot{\phi}+\omega_0^2\phi=A\cos(2\omega_0 t)\phi+B\sin(2\omega_0 t)\dot{\phi}+C\cos(2\omega_0 t)\ddot{\phi}, \tag{7}$$

where we have again assumed that the swinger will pump at the natural frequency and set $\omega$ equal to $\omega_0$. The solution of this equation is

$$\phi=\phi_0 e^{\lambda t}[\cos\omega_0 t-\sin\omega_0 t] = \sqrt{2}\phi_0 e^{\lambda t}\cos(\omega_0 t+\pi/4),$$

where

$$\lambda=\pm(A-\omega_0 B-\omega_0^2 C)/(4\omega_0). \tag{8}$$

It should be noted that $\phi_0$ may be positive or negative. Using the expressions given in Eq. (3),

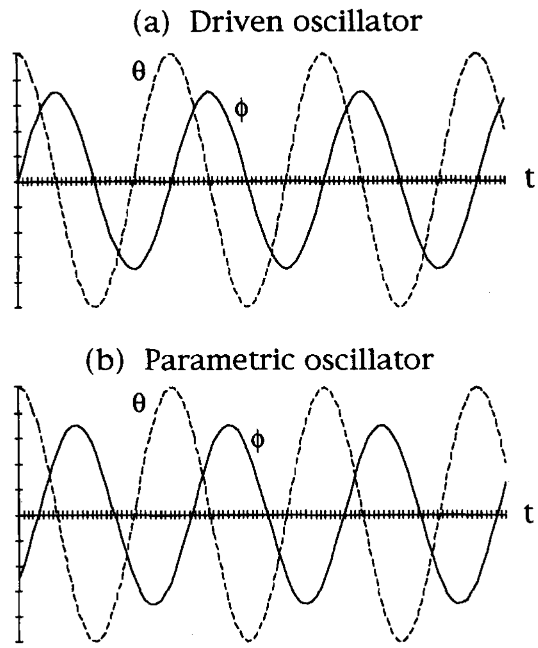


Fig. 2. Graphs of $\phi$ (solid line) and $\theta$ (dashed line) vs time for the driven oscillator regime (a) and for the parametric oscillator regime (b). The purpose of the graphs is to display the relative phases of $\phi$ and $\theta$ for the two regimes. For $\phi$ positive values correspond to the swing forward. For $\theta$ positive values correspond to the swinger leaning back (see Fig. 1). The units on the scales are arbitrary.

$$\lambda=\pm\frac{s\omega\theta_0^2}{16(l-s)R'^2}[R^2+(3l-s)(l-s)]$$

and the growth per cycle is

$$\Gamma_p=\frac{2\pi\lambda}{\omega_0}\bar{\phi}_0=\frac{\pi s\theta_0^2}{8(l-s)R'^2}[R^2+(3l-s)(l-s)]\bar{\phi}_0, \tag{9}$$

where $\bar{\phi}_0$ is the average amplitude during the cycle.

A more detailed derivation of the result given in Eq. (8) is given in SW1 but the results are not hard to show and can most likely be carried out in less time than that required to locate the earlier reference. Thus in this regime one has exponential growth. The other clear distinction between the two regimes is the different relative phase between the motion of the swing, described by $\phi$, and the motion of the swinger.

Plots of $\phi$ and $\theta$ for the two regimes, the driven oscillator and parametric oscillator, are shown in Figs. 2(a) and (b). To relate these graphs to the motion of the swinger one must note that $\phi$ positive corresponds to the swing being forward and $\theta$ positive represents the swinger leaning back. Thus for the driven oscillator regime, the swinger leans back as the swinger swings forward ($\phi$ goes from negative to positive) and leans forward as the swing goes from forward to back. The swinger is relatively quiet (all the way forward or all the way back) as the swing passes through the lowest point ($\phi=0$). Most of the motion on the part of the swinger (changing of $\theta$) occurs near the maximum values of $\phi$. A moment's reflection should lead the reader to see that this is consistent with what one sees at playgrounds. The parametric oscillator regime and graph 2(b), give a rather different picture. Here the swinger raises his center of mass ($\theta$ goes from $+\theta_0$ or

$-\theta_0$ toward $\theta=0$) as the swing passes through the lowest point ($\phi=0$). This gives a motion where the swinger is quite active (changing $\theta$) as he passes through the lowest point. Again a moment's reflection should lead the reader to conclude that this is at odds with common experience. In plotting Fig. 2(b) the $\phi_0$ of Eq. (8) was taken as a negative since this motion most closely matches common experience. Taking $\phi_0$ as a positive gives a motion which is even further from what one sees. Viewed another way, there are two motions, $\phi_0$ positive or negative, which are both equally effective as far as the parametric mechanism is concerned. The absence of these two motions in common playground practice must seem as another strike against the parametric oscillator explanation of the pumping of the swing.

If the swing obeying Eq. (3) is started from rest at $\phi=0$, the amplitudes grow linearly with time in the driven oscillator regime. As the amplitude grows, a transition is made to the parametric regime. In this regime the growth is exponential. Numerical solutions of Eq. (3) verify the correctness of these conclusions, and show a smooth transition between regimes. These numerical results are given in SW1. The gradual transition between regimes occurs in the region where the amplitude growth per cycle for the driven oscillator analysis, equals that from the parametric analysis.

Equating the expressions given in Eqs. (6) and (9) gives the critical amplitude, $\phi_c$, where this transition occurs,

$$\phi_c=F/(2\lambda\omega_0). \tag{10}$$

Using Eqs. (3), (8)–(10) we find

$$\phi_c=\frac{8l}{\theta_0 s}\frac{R^2}{(R^2+(3l-s)(l-s))}.$$

For the standing swinger of height $h$, assuming the body can be represented as a homogeneous rod, we have $s=\frac{1}{2}h$ and $R^2=\frac{1}{12}h^2$. Taking $l=2.5$ m, $\theta_0=0.5$, and $h=2$ m gives $\Gamma_D=0.34$, $\Gamma_p=0.26\phi_0$, and $\phi_c\sim 1$ rad. These values will change somewhat if different values are taken for the parameters but unless the swing is very long, its value will remain close to 1 or perhaps larger. Indeed at this value of $\phi_c$ one would question the small angle approximation used to obtain Eqs. (2) and (3). In any case we are forced to conclude that the swing pumped by a standing swinger is a driven oscillator. This is again consistent with our earlier observations about the relative motion of the swinger and the motion of the swing.

## III. PUMPING THE SWING BY RAISING AND LOWERING THE CENTER OF MASS WITH THE KNEES

One criticism of the above model would be that it does not allow for simple motion of the center of mass of the swinger along the supporting ropes by flexing of the knees. A model to represent this is shown in Fig. 3. The variables are also defined in the figure. The corresponding Lagrangian is given by

$$L'=(1/2)m(l'^2\dot{\phi}^2+\dot{l}'^2)+mgl'\cos\phi \tag{11}$$

and the equation of motion is,

$$l'\ddot{\phi}+2\dot{l}'\dot{\phi}=-g\sin\phi.$$

When the motion of the swinger is represented as $l'=l_0'+\Delta\cos 2\omega t$ and $\phi$ is assumed small, we have the parametric equation,

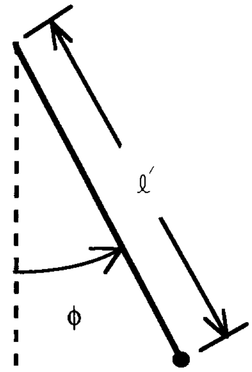


Fig. 3. Model of the swinger discussed in Sec. III. Here the swinger pumps the swing by raising and lowering his center of mass using his knees.

$$\ddot{\phi}+(g/l_0')\phi=(4\Delta\omega/l_0')\sin(2\omega t)\dot{\phi}-(\Delta/l_0')\cos(2\omega t)\ddot{\phi}$$

and from Eq. (8) a growth rate of

$$\lambda=3\Delta\omega_0/(4l_0'). \tag{12}$$

The growth per cycle for this system[13] is

$$\Gamma_p=\frac{3\pi\Delta}{2l_0'}\phi_0.$$

Taking $\Delta=0.08$ m (full range of motion 0.16 m) and $l_0'$ as 2.5 m our new mechanism gives $\lambda\simeq.024$ and $\Gamma_p=0.15\phi_0$. Thus we conclude that this new mechanism is not a more effective way to pump a swing parametrically than the motion of the previous section and indeed may be worse. My own attempts at pumping a swing by this method have been successful, but as I look around the playground very few others are using it. The motion is somewhat awkward; the rocking motion in addition to being more effective is more pleasant and somehow better suited to the human physiology.

## IV. DEMONSTRATIONS AND OBSERVATIONS OF THE MECHANISMS

Demonstrations of both mechanisms are easy to perform. For the parametric instability as described in Sec. III one need only pass the string of a pendulum over a horizontal bar as shown in Fig. 4. When the string is periodically pulled and released at twice the natural frequency of the pendulum, the mass is raised and lowered and small swinging motions of

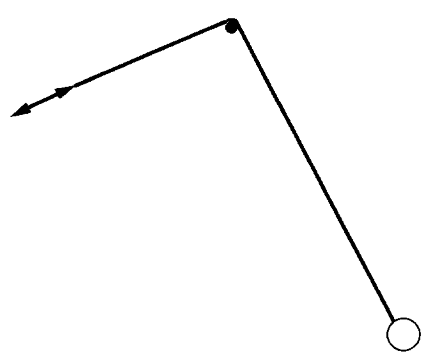

Fig. 4. Demonstration of parametric oscillator. Models system of Sec. III.

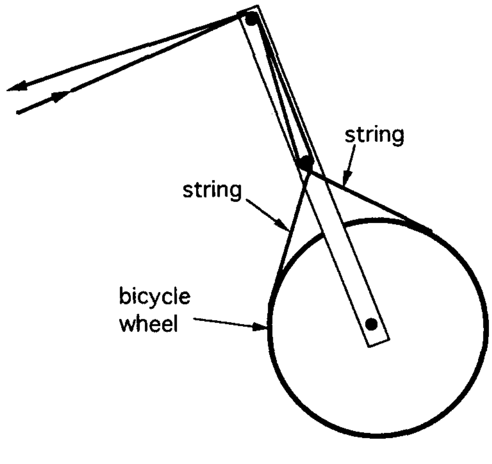


Fig. 5. Demonstration of driven oscillator. Models system of Sec. II in the driven oscillator regime.

the pendulum will grow. Maximum growth is achieved by raising the mass as the pendulum passes through the lowest point of its arc and lowering it at the ends. This well-known demonstration offers considerable insight into parametric instability.

The driven oscillator mechanism as presented in Sec. II can be demonstrated by attaching a bicycle wheel, preferably the loaded wheel that is commonly used to demonstrate gyroscopic motion, at the lower end of a lever arm which is supported and free to rotate at the top. In practice the lever arm can be made with a pair of standard laboratory rods and clamps. The arrangement is shown in Fig. 5. Two strings are attached to the rim of the wheel and passed over the support axis at the top as shown in the figure. The strings are now pulled at regular intervals so as to cause the wheel to rotate clockwise and counterclockwise. The torque on the arm and wheel is given only by gravity thus the change of angular momentum about the upper point of support is given by,

$$\frac{dM}{dt} = -mgl \sin \phi. \tag{13}$$

It should be noted that the strings apply no external torque as long as the radius of the upper support over which the strings pass is small.[14] The angular momentum can be written as the sum of angular momentum due to motion of the center of mass plus angular momentum due to rotation about the center of mass or,

$$M = ml^2\dot{\phi} + I_0(\dot{\phi} + \dot{\theta}), \tag{14}$$

where $m$ is the mass of the wheel, $l$ is the length of the arm, $I_0$ is the moment of inertia of the wheel about its axis, $\phi$ is the angle between the arm and the vertical, and $\theta$ is the angle between the orientation of the wheel and the arm. The mass of the arm has been neglected. When $\theta$ is replaced by $\theta_0 \cos \omega t$ in Eqs. (13) and (14) we have,

$$(ml^2 + I_0)\ddot{\phi} = -mgl\phi - I_0\theta_0\omega \cos \omega t$$

or a driven harmonic oscillator. The driving mechanism, achieved via an exchange of angular momentum due to rotation about the center of mass for that due to motion of the center of mass about the point of support, is of course the same in Sec. II as here. In the present system it is not possible to raise and lower the center of mass with the rotation of the wheel as it is with the rotation of the rod/body of Sec. II so the parametric terms do not appear in the present system. In truth this is a better representation of the seated swinger as described in SW1 but in either case the driven oscillator mechanism is the same.

Once the apparatus is constructed, one first demonstrates the angular momentum exchange by starting from rest with the lever hanging straight down where the external torque due to gravity is zero. One then pulls a string, say the one which makes the wheel turn clockwise. Immediately afterward the arm will swing in a counterclockwise direction about the point of support based on conservation of total angular momentum. This motion of course will not continue since it will soon be spoiled by the gravitational torque which grows with $\phi$.

Once the lever starts swinging the growth of amplitude can be demonstrated. The strings are pulled regularly so that $\omega$ matches the natural frequency of the system. It is easiest to use the arm as a metronome. The pulling of the strings should occur near the ends of the motion.

## V. DISCUSSION AND CONCLUSIONS

Let us review the main points of the paper. The motion of a swinger pumping a swing while standing has been approximated by a rigid body rotating about the lower end of a swing. This, with some reasonable approximations, gives a second-order differential equation containing parametric as well as driving terms. The conclusion that the pumped swing is a driven oscillator and not a parametric oscillator is based on two observations. The first is to note that the relative phase relation between the motion of the swinger and the motion of the swing is quite different for the driven oscillator regime and for the parametric oscillator regime. What is commonly seen in practice is in far better agreement with the driven oscillator mechanism. The second is the greater effectiveness of the driven oscillator mechanism for the typical playground swing. This was demonstrated by showing that the transition between the driven oscillator regime and the parametric regime, $\phi_c$, occurred at an angle that was greater than that commonly encountered for the swinging. This model was then compared with the earlier model of the swinger where the motion of the swinger was purely vertical. Although it is certainly possible to pump a swing with the purely vertical motion one finds that this is not the mechanism of choice at the playground. The same two criticisms of the parametric oscillator mechanism raised earlier can be applied to this model as well. The phase relation between the motion of the swinger and the swing is wrong and the mechanism is not that effective. Thus combining the current conclusions with those of SW1, we conclude that the common pumping of a swing is an example of a driven oscillator whether the swinger is standing or seated.

The analysis of Sec. II also allows us to compare the effectiveness of the driven oscillator mechanism for pumping the swing in the standing position with pumping in the seated position. We consider the ratios of growth per cycle, $\Gamma_D$, for both where the extra subscript $T$ and $E$ stand for "standing" and "seated," respectively. Using Eq. (6), we have

$$\frac{\Gamma_{DT}}{\Gamma_{DE}} = \left(\frac{\theta_{OT}}{\theta_{OE}}\right)\left(\frac{l}{l-S_T}\right)^3,$$

where the $l$'s have been taken as the same in both cases, $S_E$ is taken as zero, and it is assumed that $R \ll l - s_D$ and $l$. Although one expects that, since one can rock farther while seated than while standing, $\theta_{OT}/\theta_{OE}$ will be somewhat greater than 1, the dominant factor is $l/(l-S_T)$ since it occurs to the third power. Using our earlier typical values $l=2.5$, $S_T=1$, this factor is 4.6. Thus we expect that the growth per cycle is a few times larger for the standing swinger than for the seated swinger. This is again consistent with common experience.

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[10]J. T. McMullan, "On initiating motion in a swing," Am. J. Phys. **40**, 764–766 (1972).
[11]This journal's style would dictate that I should avoid using strictly masculine or feminine gender pronouns in this description. Since I am trying to construct a clear distinct image in the reader's mind, I would like to avoid asking the reader to ponder a nebulous person of unidentified gender. Although it certainly makes no difference as far as the physics is concerned, I have chosen one; I will use the masculine pronoun in this paper. This also avoids the very awkward "he/she" or "he or she" usage. For those readers who find this choice offensive I point out that in my earlier paper on the same subject, SW1, the swinger was a woman and the feminine pronoun was used throughout.
[12]Through the standard substitution $\phi=(1-B\cos(2\omega_0 t)/4\omega_0)y$, Eq. (3) can be written as $\ddot{y}+\omega_0^2 y\cong F\cos(\omega_0 t)+(A-\omega_0 B-\omega_0^2 C)\cos(2\omega_0 t)y$ after some approximations. Although this does lead to the same results, the approximations made at this stage make one uneasy.
[13]This is identical to the result given in Eq. (11) of J. A. Burns Ref. 3, above. References 2 and 4 give similar results.
[14]This can be verified by pulling on both strings simultaneously and attempting to drive the swing. This should be almost impossible.

# Measuring the speed of digital signals (speed of light)

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An odd number of digital not-gates connected in a closed loop circuit will produce a relatively stable free running oscillator. The frequency of oscillation is determined by the number and type of not-gates, the circuit supply voltage, and the total length of connection wire used between the not-gates. Measuring the frequency for various lengths of connection wire enables the speed of the digital signals in the wire to be calculated. The method is sensitive enough to reveal that signals traveling in 22 gauge insulated copper wire propagate about 5% slower than those in 22 gauge bare copper wire. The speed in insulated wire is $(2.87\pm0.03)\times10^8$ m/s and the speed in bare wire is $(3.03\pm0.02)\times10^8$ m/s. This experiment has been found to be a very popular undergraduate laboratory due to its low cost, straightforward theory, and relatively high ($\sim$1%) precision. The absence of a light source eliminates complicated optics and impresses students with the fact that visible light is not the *only* thing that travels with the speed of light! 

## I. INTRODUCTION

Measuring the speed of light has fascinated physicists for ages. It began, perhaps, with the ingenious astronomical method used by Ole Roemer[1] in 1676 and continues today with experiments employing sophisticated lasers and atomic clocks. A good summary of some of the more recent methods can be found in the papers by Vanderkooy[2] and Trudeau.[3] In 1983 the speed of light in vacuum was defined as 299 792 458 m/s and length measurements are now determined by how long it takes light to travel between two points. As many authors have pointed out, however, measuring such a fast moving phenomenon as light continues to be a challenging and educational exercise.

The experimental method presented here involves measuring the number of cycles of a digital voltage signal around a closed loop not-gate oscillator in 32 s. The frequency of the oscillator changes between 7 and 2 MHz as the total length of intergate connection wire is increased from 1.8 to 55.8 m. From a linear fit of the data (wire length versus times around the loop) both the speed of the digital signals in the wire and the delay time of the not-gates can be calculated.

Because light is not used (except, as James Clerk Maxwell reminds us, "to see the instruments"),[4] the problems associated with the conversion of light signals to electrical ones and vice versa are eliminated. A closed loop circuit allows the signal to circulate for as long as desired. In many laboratory-based speed-of-light experiments the pulse travels only a few meters and timers with nanosecond accuracy are required. In the current procedure the digital signal travels

  