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Control of Autonomous Motion of Two-Wheel Bicycle with Gyroscopic Stabilisation

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Abstract

A bicycle with a gyroscopic stabilisation capable of autonomous motion along a straight line as well as along a curve is described. The stabilisation unit consists of two coupled gyroscopes spinning in opposite directions. It makes use of a gyroscopic torque due to the precession of gyroscopes. This torque counteracts the destabilising torque due to gravity forces. The control law of the actuator drive making the gyroscopes to precess is described.

1 Introduction

A bicycle equipped with a system of gyroscopic stabilisation of its upright position has been designed at the Institute of Mechanics of the Moscow Lomonosov State University (Fig. 1). The scheme of stabilisation is similar to that of Sherle-Shilovsky proposed in 1909 for stabilisation of a monorail car [1, 2]. The stabiliser contains two gyroscopes with rotors spinned in opposite directions. A torque delivered by a DC motor may be applied about the axes of the gyroscope frames. This control torque is the function of the following variables: the declination angle of the bicycle from the vertical, the orientation angle of the gyroscopes and its derivative.

Brown and Yangsheng [3] previously described a single-wheel vehicle with an internal gyroscope.

A control is presented here that provides the stability of the vertical position of the bicycle during its rectilinear motion and of an inclined position when moving over a curve. In comparison to the control in [1, 2], the feedback also incorporates the steering angle and the velocity of the bicycle. Because of this, the bicycle is able to perform turns with radii approaching its length.

In a conventional bicycle the front wheel not only controls the direction, but also contributes to the stability of the bicycle. To prevent a fall, the fork must be turned to the side of inclination. When riding without hands, the handle bar turns to the correct side by itself due to: 1) the gyroscopic effect because of the rotation of the front wheel, 2) the steering axis being inclined with respect to vertical and 3) the centre of the front wheel being placed slightly forward of the steering axis [1].

To the contrary of a conventional bike, this one is stabilised by a special gyroscopic stabiliser and does not make use of the factors mentioned above. Furthermore, to eliminate their influence, the steering tube is made vertical and the centre of the front wheel lies on its continuation (Fig. 1, 2). Another major difference is that the bicycle has the front wheel drive.

Three control loops control the motion of the bicycle along a curved path and stabilise it. Each loop has its own task.

The first loop controls the steering angle, that is, the direction of motion. An incremental encoder and a rotary drive are installed on the steering tube to measure and control the orientation of the fork.

The second servo loop controls the angular velocity of the front wheel, that is, the speed of the bicycle. Another encoder is installed on the axis of the wheel to measure its angular position and velocity. The rear wheel is passive.

The third loop controls the precession of the gyroscopes, preventing the bicycle from falling over. Two more encoders are related to this particular actuator: one, to measure the orientation of gyroscope frames, and another, to measure the declination angle of the bicycle from the vertical. To accomplish this, a free gyroscope with three DOF's is used, and its orientation relative to the frame is measured. The control law in this loop takes into account signals from all four mentioned encoders.

2 Gyroscopic Stabiliser

The stabiliser consists of two identical gyroscopes located between the wheels (Fig. 2). Each rotor is sealed within a frame, that can pivot relative to the chassis about the axis perpendicular to the sagittal plane of the bicycle (the centres of the wheels and the steering axis lie in this plane). In a normal situation, when this plane is vertical, this axis is horizontal and the rotor axis is vertical; the rotation axes of the rotor frames, parallel to each other, lie within the horizontal plane.

The rotations of gyroscope frames are not independent: they are interlinked by a gear train such that when one frame turns by some angle β, the other turns to the opposite direction by the same angle (Fig. 2).

The rotors spin with the same speed in opposite directions; hence their vectors of kinetic momentum H point in opposite directions. If a precession about the frame axes takes place, a gyroscopic torque is developed that counteracts the disbalancing torque due to the gravity.

A torque Q is applied to the axis of frame rotation by a DC motor through a reductor. The control voltage is constructed as a function of the angle ψ of the declination from the vertical (angle of bank), precession angle β, its angular velocity $\dot{\beta}$, steering angle δ, and the linear speed of the center of the front wheel V. The angle of bank ψ is measured by the free 3-DOF gyroscope - gyrovertical which is installed on the bicycle chassis.

3 Mathematical Model

In the sketch of the bicycle in the Fig. 3, the angle of bank ψ and wheel-to-ground contact points K_1 and K_2 are shown. The plane of the front wheel intersects the support surface over some straight line; for small angles of bank ψ the steering angle δ is approximately equal to the angle between this line and the segment K_1K_2 (Fig. 4). The following kinematic relationship may be derived from this figure:

$$l\dot{\alpha} = V \sin\delta \qquad (1)$$

where $l = K_1K_2$, α is the angle between the segment K_1K_2 and axis OX (Fig. 4). Within the linear approximation the distance l does not depend on angles ψ and δ.

The oscillation of the bicycle on the angle of bank and the precession of the gyroscopes are described by a rather complicated set of equations. We show here only linear equations. They were obtained by linearisation of complete equations around state $\psi = \beta = \dot{\psi} = \dot{\beta} = 0$. This way we obtain the following:

$$B\ddot{\beta} - 2H\dot{\psi} = Q \qquad (2)$$

$$D\ddot{\psi} + 2H\dot{\beta} - Eg\psi = E(\ddot{y}\cos\alpha - \ddot{x}\sin\alpha) \qquad (3)$$

Here B and D are the moments of inertia of the rotors of the gyroscopes together with their frames and of the whole bicycle about the appropriate axes, $H \approx 10\ kg{\cdot}m^2/s$ is the kinetic momentum of the rotor (it is considered constant), Q is the torque, developed by the drive, $E = mb$, where $m \approx 20\ kg$ is the total mass of the bicycle and $b \approx 0.2\ m$ is the distance between the common centre of mass and the segment K_1K_2, x, y are the co-ordinates of middle point of this segment, where the centre of mass is assumed to map. The distance between the wheel axes or, otherwise said, the length l of the segment K_1K_2 is approximately $0.75\ m$, the radius of the wheels is $0.15\ m$. The term $Eg\psi$ describes the destabilising torque component due to the gravity, the term $2H\dot{\beta}$ is a gyroscopic torque impeding the fall of the bicycle, and the right-side term in (3) is the moment of inertia forces emerging during the manoeuvres of the bicycle.

In Eq. (2) the torque due to friction in the axis of precession is neglected. We will also neglect the back emf in the DC motor and the inductance of the motor winding.

This way in composing the mathematical model (1), (2), (3), the motion of the bicycle is decomposed into two parts. Eq. (1) describes the motion of the axis K_1K_2 (this axis contains the points of wheel contacts with the support surface), which is the motion along the trajectory. Given this motion, Eqs. (2) and (3) describe the motion (inclination) of the bicycle relative to this axis.

Using relation (1), we rewrite Eq. (3) as follows:

$$D\ddot{\psi} + 2H\dot{\beta} - Eg\psi = \frac{1}{2}E\left[\frac{V^2}{l}\sin 2\delta + \frac{d}{dt}(V\sin\delta)\right] \tag{4}$$

The right-hand side of this equation depends only on the trajectory and velocity of the front wheel motion.

Let us represent the desired trajectory of the bicycle and the velocity of the front wheel V in the following way:

$$\sigma = \sigma(s), \quad \frac{ds}{dt} = V \tag{5}$$

where s is the natural parameter of the trajectory (current path length), σ is the angle between a tangent to the trajectory and some fixed direction, and the velocity V is function of time t, path s or of some other variable. If the front wheel moves exactly along the prescribed trajectory, the angle δ changes according to the following relation:

$$\delta = \sigma - \alpha \tag{6}$$

Solving (1), (5), (6), we get the functions $\delta(t)$ and $V(t)$. Substituting them into (2) and (4), it is possible for a known Q to solve them for bank angle oscillations and for the motion of the gyroscopes.

4 Stationary Regimes and Control

The equations (2) and (4) describe a system with two degrees of freedom, but having only one control parameter Q. Let $\delta = 0$, i.e., the bicycle moves straight ahead. The Eq. (4) becomes homogeneous. The verification of the Kalman controllability criterion [4] gives that the system is quite controllable if and only if $H \neq 0$ and $b \neq 0$. This is, of course, true in our case, and the equilibrium position

$$\psi = \beta = \dot{\psi} = \dot{\beta} = 0 \tag{7}$$

corresponding to the desired upright position of the bicycle and of the axes of gyroscope rotors, can be stabilised by a linear feedback including all four state variables ψ, β, $\dot{\psi}$, $\dot{\beta}$. In [1, 2], it is shown, that the state (7) can also be stabilised by a feedback including only three variables:

$$Q = k_\beta\beta - k\dot{\beta} + k_\psi\psi \tag{8}$$

In other words, it is shown that for a sufficiently large kinetic momentum H the coefficients k_β, k, k_ψ can be selected such that the Hurwitz criterion of the asymptotic stability of solution (7) is satisfied. The coefficient k_β proves to be positive in accordance with the Kelvin's theorem, stating that an unstable system can be stabilised by gyroscopic forces only if the number of unstable degrees of freedom is even [2, 5].

If $k_\beta > 0$, the torque $k_\beta\beta$ is directed so as to turn the gyroscopes over, making them statically unstable similar to the Lagrange's gyroscope with the upper location of the mass center.

That is, the position and derivative term gains in the control (8) have opposite signs.

Assume now that the bicycle is turning with a constant velocity and a constant steering angle, that is,

$$\dot{V} = 0, \quad \dot{\delta} = 0 \tag{9}$$

Under these conditions the system (2), (4), (8) has a stationary solution

$$\psi = \psi_s = \frac{V^2}{2gl}\sin 2\delta, \quad Q = 0, \quad \beta = \beta_s = -\frac{k_\psi\psi_s}{k_\beta} \tag{10}$$

As it follows from (10), the stationary precession angle is different from zero. But it is known that the gyroscopes are most effective around the zero precession angle. This statement cannot be proven within the scope of linear model (2), (4) as it follows from the consideration of nonlinear model. This is because instead of the terms $2H\dot{\psi}$ and $2H\dot{\beta}$, describing the components of the stabilising gyroscopic torque in the linear equations (2) and (4), the nonlinear equations contain terms $2H\dot{\psi}\cos\beta$ and $2H\dot{\beta}\cos\beta$. They attain their maxima at $\beta=0$ and go to zero as $|\beta| \to \pi/2$. It would be possible to reduce the value of β_s by increasing the coefficient k_β, but its upper limit is imposed by the conditions of stability. For this reason we replace the control (8) by the following:

$$Q = k_\beta \beta - k\dot{\beta} + k_\psi(\psi - \psi_s) \tag{11}$$

In this case the stationary value of β becomes zero: $\beta = \beta_s = 0$.

Assume that the angle of bank is measured by the gyrovertical with some error $\Delta\psi$. In this case, the sum $\psi + \Delta\psi$ instead of ψ is used in the feedback (11). Let $\Delta\psi = const$. This error may be caused by slightly inaccurate measurement of initial position of the gyrovertical and/or of the bicycle. This constant error would result in a stationary value

$$\beta = \beta_s = -\frac{k_\psi \Delta\psi}{k_\beta} \tag{12}$$

To get rid of the offset (12) in the precession angle β, consider instead of (11) the following control, containing the integral of the precession angle:

$$Q = k_\beta \beta - k\dot{\beta} + k_\psi(\psi + \Delta\psi - \psi_s) + k_\sigma \sigma, \quad \dot{\sigma} = \beta \tag{13}$$

In a stationary case for the control (13) we will have

$$\psi = \psi_s = \frac{V^2}{2gl}\sin 2\delta, \quad \beta = \beta_s = 0,$$
$$Q = 0, \quad \sigma = \sigma_s = -\frac{k_\psi \Delta\psi}{k_\beta} \tag{14}$$

Therefore in the stationary solution (14) the precession angle β equals zero despite an unknown error $\Delta\psi$ in the measured inclination angle of the bicycle.

Using Hurwitz criterion it is possible to show that the following inequalities are the necessary conditions of the asymptotic stability of the solution (14):

$$k_\beta > 0, \quad k > 0, \quad k_\phi > 0, \quad k_\sigma > 0$$

That is, to the contrary of the common *PID*-controller the sign of the coefficient at the derivative $\dot{\beta}$ in the control (13) is opposite to that of the position and integral terms.

Finally we come to the following control:

$$Q = k_\beta \beta - k\dot{\beta} + k_\psi(\psi + \Delta\psi - \frac{V^2}{2gl}\sin 2\delta) +$$
$$+ k_\sigma \int_0^t \beta(\tau)d\tau \tag{15}$$

It is assumed here that the bicycle is moving along a circle with a constant speed. This type of control was used in simulations as well as in real experiments.

5 Implementation of the Control Algorithm

During the motion the bicycle is fully autonomous. The stabilising gyroscopes and gyro of the inclinometer (gyrovertical) are spinned up before the start using an external power supply. All control functions are accomplished by an on-board control system.

The control system consists of two programmable controllers built around Intel 80C196KC chips with a clock frequency of 20 MHz.

The controllers communicate via CAN bus, the software is downloaded from an external PC over the same bus.

The control programmes implement the servosystems described in the Introduction. One of them controls the motion of the bicycle along the track (this is a velocity servosystem on the speed V of the front wheel) as well as the steering speed servosystem depending on the shift from the track. Both servosystems are implemented within one loop with a period of *5 ms*.

The other controller drives the gyroscopes of the stabiliser. This is in fact a servosystem including multiple variables according to Eq. (15). The values β, $\dot{\beta}$, ψ are taken from the incremental encoders and V and δ are received from the first controller. The calculations are performed with a period of *1 ms*.

To reduce the maximal precession angle of the stabiliser gyroscopes when moving over an arc, the linear speed V is then somewhat reduced.

6 Experiments

The experiments were carried out on the track of the mobile robot contest of the International Festival of Sciences and Technologies (France, May 1997). The track is presented in Fig. 5. Each square is 2 by 2 m in size. To follow the designated track, the control system must be informed of the current bias of the front wheel from the track. A track sensor has been installed on the front wheel to produce this signal. It was used to program the steering angle of the front wheel. For the time being the bicycle can move along the track at a speed of about *1 m/s*.

Control (15) ensures asymptotic stability of the stationary motion along the straight line or circle. However, during the motion on the experimental track the bicycle does not quite reach a stationary mode with a constant steering angle δ. The control system works in a transient mode, but nevertheless, the control (15) ensures the stable motion of the bicycle, and the maximal precession angle is small.

To further improve the quality of stabilisation, we plan to make the control (15) more complex, taking into account not only the first "centrifugal" term from the right-hand side of the equation (4), but also the second one describing the torque of the inertia force because of the changing velocity V and/or steering angle δ.

References

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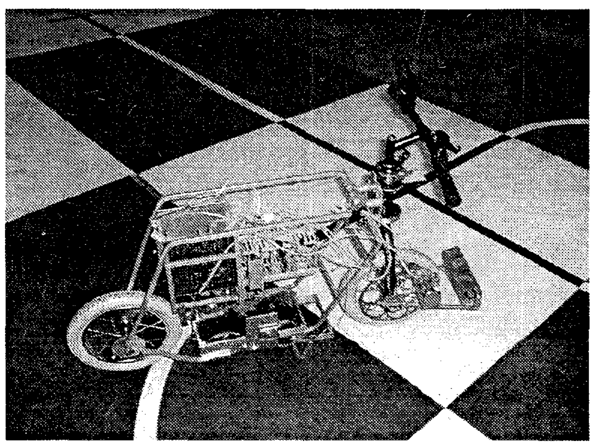

Figure 1. Photo of the bicycle.

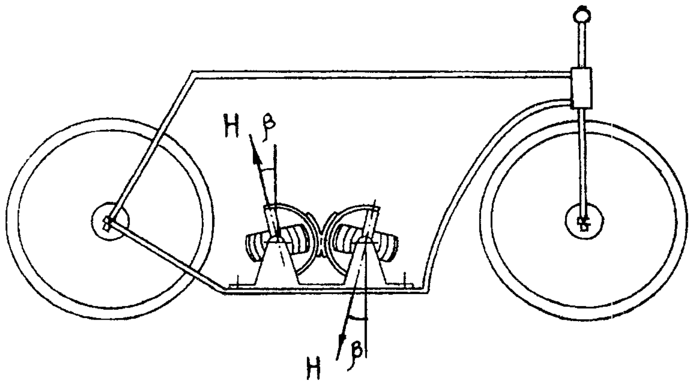

Figure 2. The scheme of the bicycle with gyroscopic stabiliser (side view).

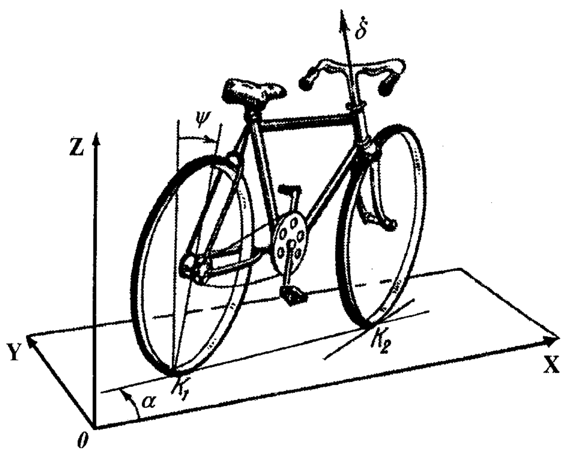

Figure 3. Sketch of the bicycle.

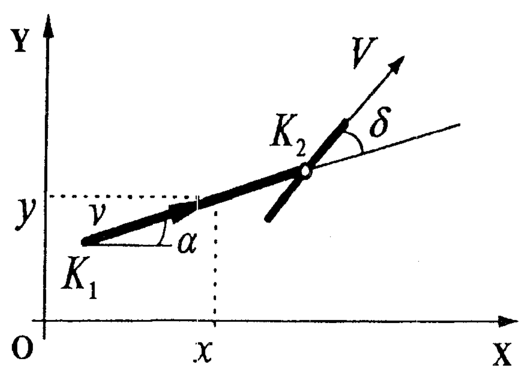

Figure 4. Kinematic scheme of the bicycle motion over the support surface.

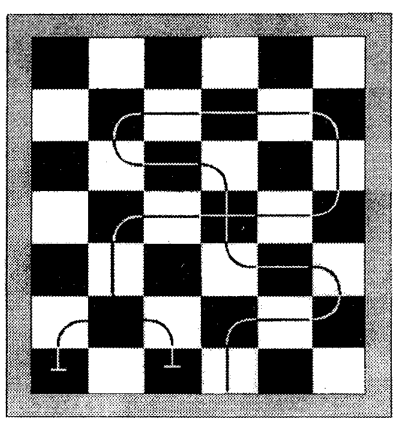

Figure 5. Experimental track.