Efficient Massively Parallel Methods for Dynamic Programming

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STOC 2017

Spark

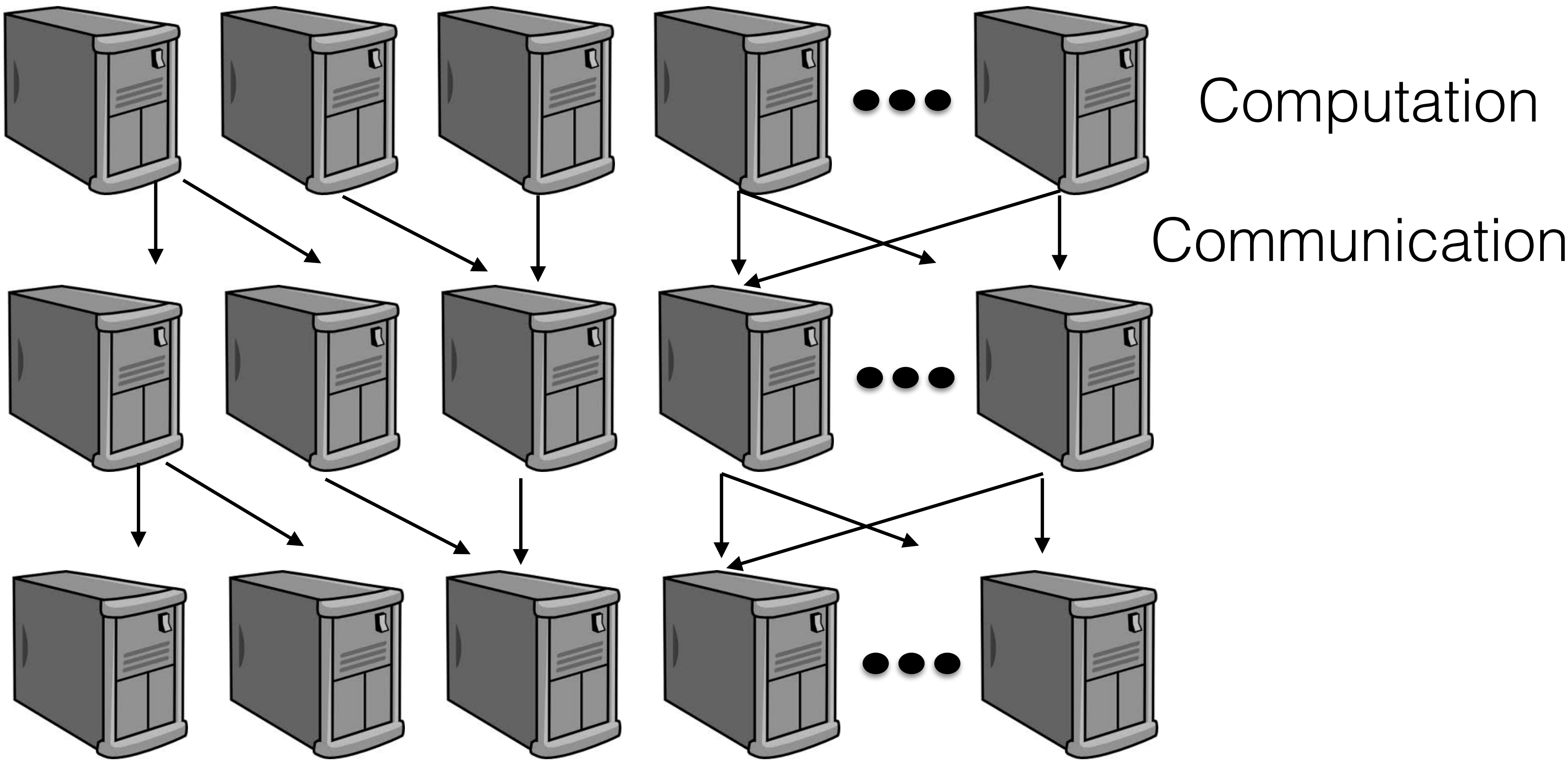
Lightning-Fast Cluster Computing

hadoop

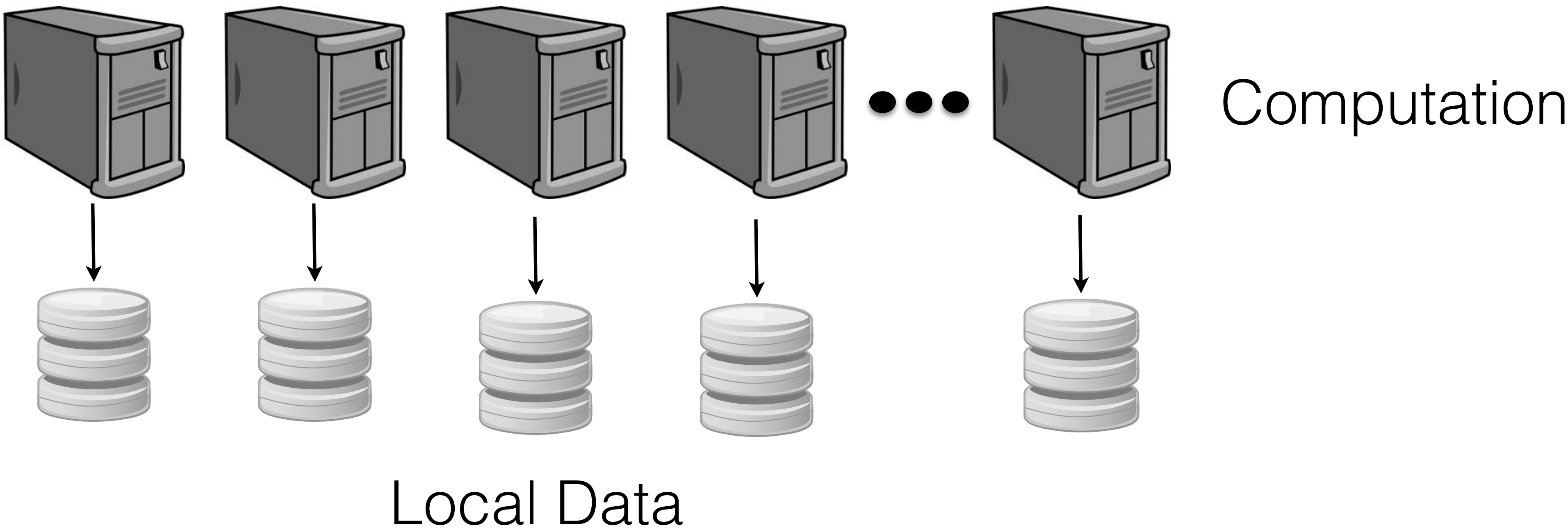
Revolution in Scalability

Breakthroughs Driven by Practitioners

Massively Parallel Computation



Massively Parallel Computation



Models

- Models first motivated by MapReduce
- Refined and parameterized
  - Capture most modern massively parallel environments

Dean-Ghemawat OSDI 2004,
Feldman-Muthukrishnan-Sidiropoulos-Stein-Svitkina SODA 2008 (TALG 2010),
Karloff-Suri-Vassilvitskii SODA 2010,
Goodrich-Sitchinava-Zhang ISAAC 2011,
Pietracaprina-Pucci-Riondato-Silvestri-Upfal ICS 2012
Goel-Mungagala ArXiv 2012,
Beame-Joutris-Suciu PODS 2013,
Andoni-Nokolov-Onak-Yaroslavtev STOC 2014,
Roughgarden-Vassilvitiskii SPAA 2016

Unifying Models

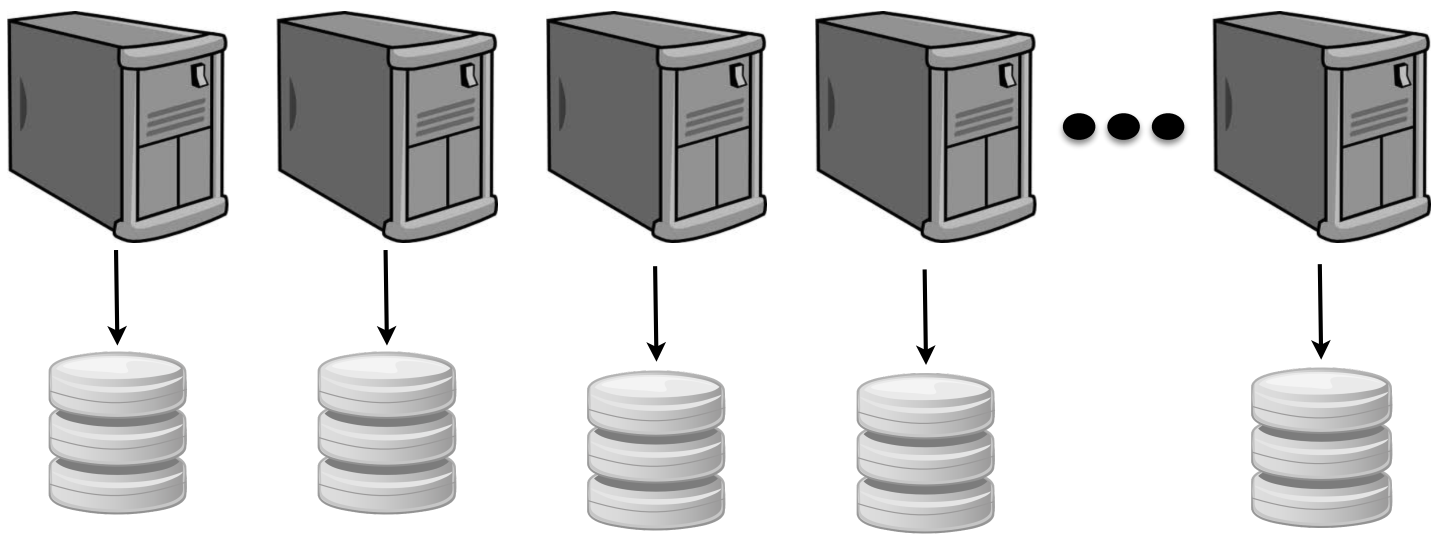
- $O(n)$ input size
- $O(m)$ number of machines
- $\tilde{O}(n/m)$ memory on the machines

- Minimize rounds

Unifying Models

- $O(n)$ input size
- $O(m)$ number of machines
- $\tilde{O}(n/m)$ memory on the machines
    - $\tilde{O}(n^{1+\frac{1}{2}\epsilon}/m)$ ‘extra’ memory setting
- Minimize rounds

Algorithmic Challenges



Dynamic Programming

Principled Framework

- Identify two key properties enabling simulation
    - Monotonicity
    - Decomposability
- $(1+\epsilon)$-approximations in $O(\frac{1}{\delta})$ rounds using $\tilde{O}(n)$ aggregate memory, $O(n^\delta)$ memory per machine for any constants $\delta, \epsilon > 0$
    - Optimal binary search tree
    - Longest increasing subsequence
    - Weighted interval selection

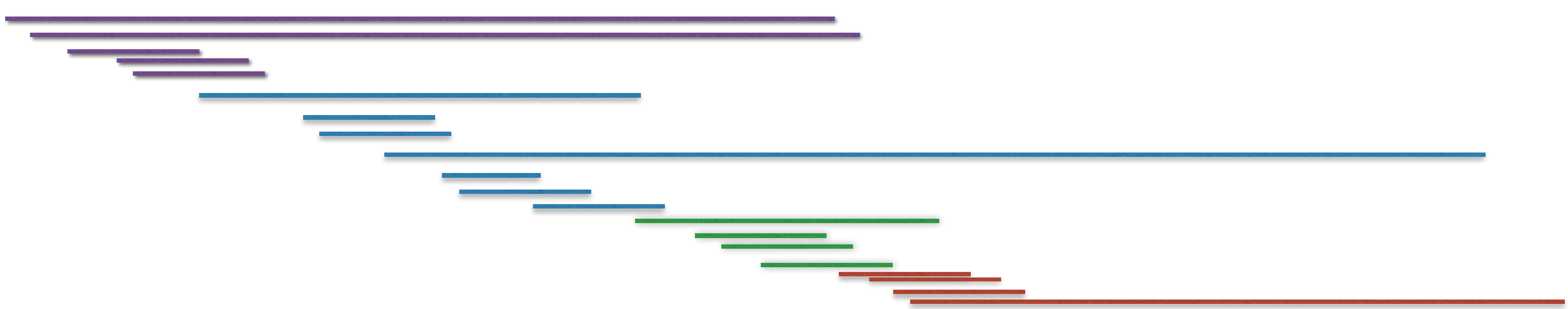
Weighted Interval Selection

- Collection of intervals $I_i = (s_i, e_i)$ with weight (profit) w_i
- Select a maximum weight independent set of intervals

- Dynamic program
  - Sort intervals by starting points in increasing order
  - $A(i)$ optimal solution of $I_i, I_{i+1}, \ldots, I_n$
  - $A(i) = \max\{A(i+1), w_i + A(j)\}$ where $j = \texttt{argmin}_{j'} e_i < s'_j$

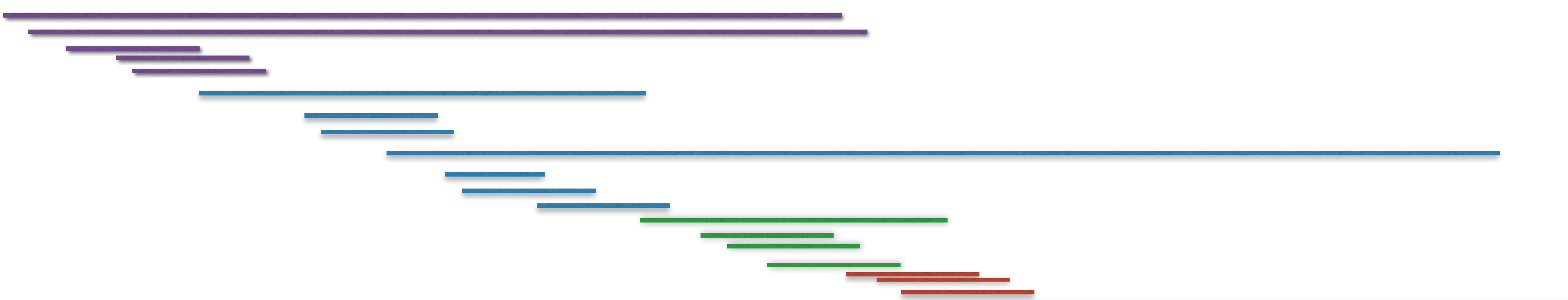
Distributed Algorithm

- m machines
- Distribute intervals to machines in sorted order
  - n/m intervals per machine



Distributed Algorithm

- Compute subproblems locally, but which ones?
  - $B(i,j)$ start no earlier than s_i and end by s_j
- Combine on a single machine
  - Super-linear space!



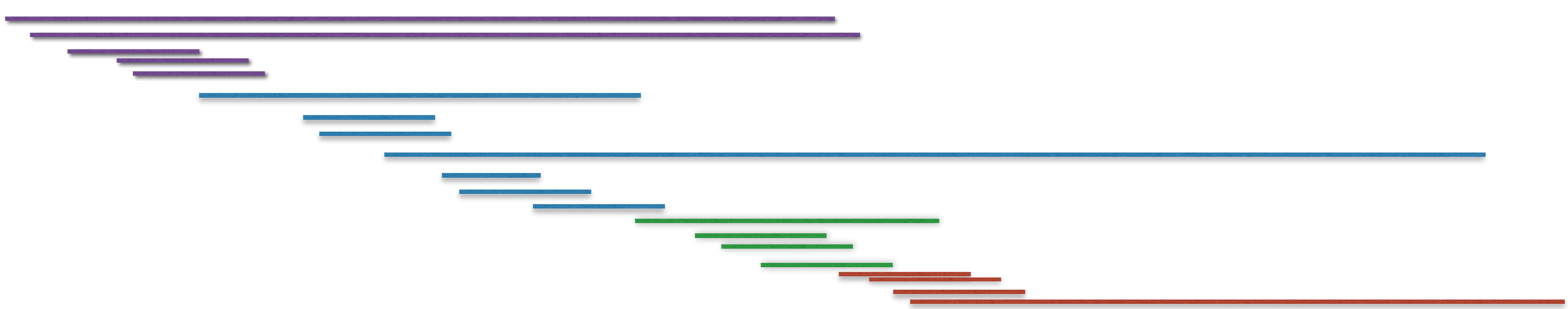
Logarithmic Rounds using Monotonicity

- Any subproblem has less optimum value for a maximization objective

- $B(i,j)$ satisfies monotonicity, $B(i',j') \geq B(i,j)$ for all $i' \leq i, j' \geq j$

Swap: A use of Monotonicity

- Define a new recurrence $C(i,w) = \min_{j' \mid B(i,j') \geq w} j'$
  - Increasing due to monotonicity
- Set $C(i,w) = \min j_2$ over all w_1, w_2 where $w_1 + w_2 = w$, $C(i, w_1) = j_1$ and $C(j_1, w_2) = j_2$



Sketching the Recurrence

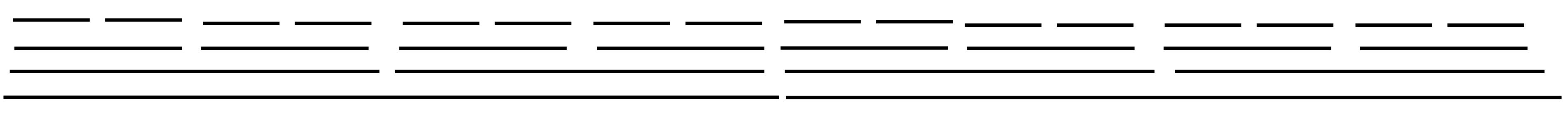
- Approximately sketch the recurrence
  - Compute $C(i,w)$ where w is of the form $(1+\frac{\epsilon}{10\log n})^k$ to ensure polylogarithmic space
  - Need a new approximate recurrence $C'(i,w)$
- Recursively compute by considering w_1, w_2 where $w \simeq w_1 + w_2$
- Set $C'(i,w) = \min j_2$ such that $C'(i,w_1) = j_1$ and $C'(j_1,w_2) = j_2$
- Loose $(1+\frac{\epsilon}{10\log n})$ factor each iteration however for small k

$$(1+\frac{\epsilon}{10\log n})^k \leq 1+\epsilon$$

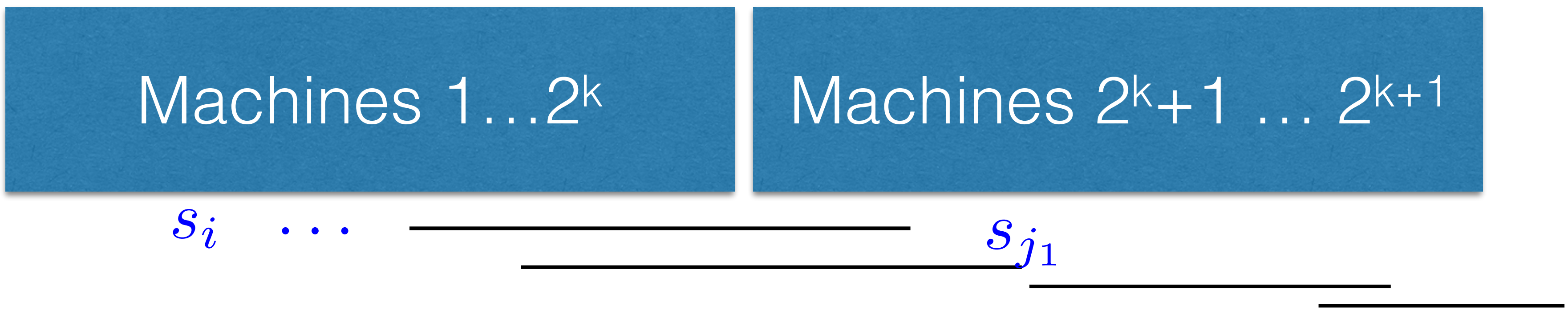
Computing the Sketch

- Initialize $C'(i,w)$ by considering only intervals on the machine i is assigned to

- In the kth round $C'(i,w)$ will contain the optimal value amongst intervals on machines up to 2^k away



Computing the Sketch



- Updating $C'(i, w)$
- Consider all pairs w_1, w_2 where $w_1 + w_2 \simeq w$
- When $C'(i, w_1) = j_1$ send the value $C'(j_1, w_2) = j_2$ to the machine i is stored on
  - Computed at the beginning of the round
- Update $C'(i, w)$ to be the minimum such j_2

Generalizing the Idea

- Framework extends to other problems
- Combining with decomposability requires new ideas

Future Work

- General methods
- Lower bounds
  [Sarma-Afrati-Salihoglu-Ullman; Fish-Kun-Lelkes-Reyzin-Turaan; Jacob-Lieber-Sitchinava; Beame-Koutris-Suciu; Roughgarden-Vassilvitskii-Wang]

Thank you!

Questions?