Homography

- General, $(n+1)$-dimension case
  - P^n: n-dimensional space
  - $R^{(n+1)}$ extended space of P^n
  - transformation $P^n \to P^d$ is a homography, it is a linear transformation $R^{(n+1)}$
  - It is applied using homogeneous coordinates:

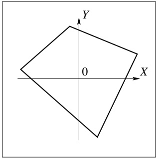
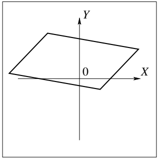
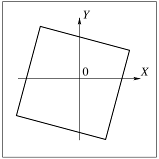
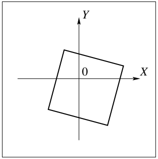
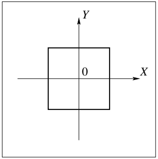
$$\mathbf{u}' \sim H\mathbf{u}$$

$\rightarrow$ H is a non-singular $(n+1) \times (n+1)$ matrix

- 3D case: $n + 1 = 3$
  - P^2 is a plane in R^3
  - A homography is a projective transormation between two planes

  $\rightarrow$ it is unequivocal
  - Lines remain lines after homographic transformation.

Special cases of a homography

projective	affine	similarity	metric	identity
				
$\det H \neq 0$	$H = \begin{bmatrix} A & \mathbf{t} \\ \mathbf{0}^T & 1 \end{bmatrix}$	$H = \begin{bmatrix} sR & -R\mathbf{t} \\ \mathbf{0}^T & 1 \end{bmatrix}$	$H = \begin{bmatrix} R & -R\mathbf{t} \\ \mathbf{0}^T & 1 \end{bmatrix}$	$H = E$
	$\det A \neq 0$	$R^T R = E$	$R^T R = E$	
	$\det R = 1$	$\det R = 1$		
		$s > 0$		

Plane-plane homography (1)

- A planar pattern is given in 3D space.
- Two different images are taken:

$$\mathbf{u} = \lambda_1 \mathbf{P}_1 \mathbf{X} \quad \mathbf{u}' = \lambda_2 \mathbf{P}_2 \mathbf{X}$$

- Origin and oriantation of coordinate system can be freely selected
  - Let plane $Z = 0$ be the plane of the pattern
  - Then an arbitrary point within the pattern is $\mathbf{X_i} = [X_i, Y_i, 0, 1]^T$.
- Projection is more simple:

$$\mathbf{u}_i = \lambda_1 \tilde{\mathbf{P}}_1 \tilde{\mathbf{X}}_i \quad \mathbf{u}'_i = \lambda_2 \tilde{\mathbf{P}}_2 \tilde{\mathbf{X}}_i$$

- where $\tilde{\mathbf{X}}_i = [X_i, Y_i, 1]^T$. Matrices $\tilde{\mathbf{P}}_1$ and $\tilde{\mathbf{P}}_2$ is the original $\mathbf{P_1}$ and $\mathbf{P_2}$ matrices, removing the third column.

Application of homography (1)

- $\tilde{\mathbf{P}}_1$ and $\tilde{\mathbf{P}}_2$ are 3×3 square matrices
  - They are invertible.
- Spatial points:

$$\frac{1}{\lambda_1}\tilde{\mathbf{P}}_1^{-1}\mathbf{u} = \mathbf{X} \quad \frac{1}{\lambda_2}\tilde{\mathbf{P}}_2^{-1}\mathbf{u}' = \mathbf{X}$$

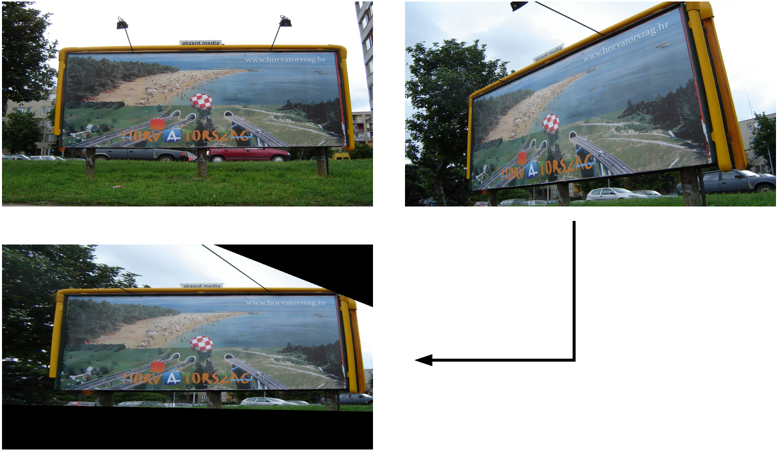
- Image coordinates can be computed from each other:

$$\mathbf{u}' = \frac{\lambda_2}{\lambda_1}\tilde{\mathbf{P}}_2\tilde{\mathbf{P}}_1^{-1}\mathbf{u}$$

- Transformation is given by the 3×3 matrix $\frac{\lambda_2}{\lambda_1}\tilde{\mathbf{P}}_2\tilde{\mathbf{P}}_1^{-1}$.

  $\rightarrow$ This is a homography.

Application of homography (1) : transformation of planar patterns



Application of homography (1/b) : Inverse Perspective Mapping

- For vision system of autonomous vehicles, the road is one of the main focuses of attention.
  - The road is a planar surface.
  - $\rightarrow$ It can be rectified by a homography.
- Objects can be more accurately detected in rectified images.
  - The distances can also be measured and visualized.

Application of homography (1/b) : Inverse Perspective Mapping

Left: Original image. Right: Rectified image.

Application of homography (2)

- A 3D world is given, two images are taken from the same focal point. Only camera orientations differ.
  - $\rightarrow$ Input for a panoramic image.
- The origin is selected as the common focal points of the images.
- Camera projection matrices: $\mathbf{P}_1 = \mathbf{K}_1[\mathbf{R}_1|0]$ and $\mathbf{P}_2 = \mathbf{K}_2[\mathbf{R}_2|0]$
- Projection: $\mathbf{u} = \mathbf{K}[\mathbf{R}|0][X, Y, Z, 1]^T$. Homogeneous (last) coordinate does not effect result.
- Transformation between two corresponding image locations:

$$\mathbf{u}' = \mathbf{K}_2\mathbf{R}_2\mathbf{R}_1{}^T\mathbf{K}_1{}^{-1}\mathbf{u}$$

- Transformation is represented by 3×3 matrix $\mathbf{K}_2\mathbf{R}_2\mathbf{R}_1{}^T\mathbf{K}_1{}^{-1}$.
  - $\rightarrow$ This is a homography as well.

Application of homography (2) : panoramic imaging