Calibration by a spatial object

- Given m point correspondence between 3D scene and images plane $\mathbf{X}_i \rightarrow \mathbf{u}_i$: $\mathbf{u}_i \sim P\mathbf{X}_i, \quad i = 1, \ldots, m$
- Task: estimation of $P = KR\,[I| - t]$.
  - At least 6 correspondences required
  - Over-determined system
- Wrong correspondence $\longrightarrow$ robust methods
  - Many correspondences $\longrightarrow$ outlier detection

Calibration by Cartesian coordinates

$$\alpha \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix},$$

where $\alpha \neq 0$ is an arbitrary scale factor.

Equations can be rewritten as

$$u = \frac{P_{11}X + P_{12}Y + P_{13}Z + P_{14}}{P_{31}X + P_{32}Y + P_{33}Z + P_{34}} = \frac{\mathbf{p}_1^T\mathbf{X}}{\mathbf{p}_3^T\mathbf{X}}, \tag{22}$$

$$v = \frac{P_{21}X + P_{22}Y + P_{23}Z + P_{24}}{P_{31}X + P_{32}Y + P_{33}Z + P_{34}} = \frac{\mathbf{p}_2^T\mathbf{X}}{\mathbf{p}_3^T\mathbf{X}}, \tag{23}$$

where $\mathbf{p}_i$ is the i-th row of projection matrix P.

Linear estimation of a projection matrix 1/2

Equations are multiplied by the common denominator:

$$(P_{31}X + P_{32}Y + P_{33}Z + P_{34})u = P_{11}X + P_{12}Y + P_{13}Z + P_{14} \tag{24}$$
$$(P_{31}X + P_{32}Y + P_{33}Z + P_{34})v = P_{21}X + P_{22}Y + P_{23}Z + P_{24} \tag{25}$$

For the i-th point, $A_i\mathbf{p} = 0$, where

$$A_i = \begin{bmatrix} X_i & Y_i & Z_i & 1 & 0 & 0 & 0 & 0 & -u_iX_i & -u_iY_i & -u_iZ_i & -u_i \\ 0 & 0 & 0 & 0 & X_i & Y_i & Z_i & 1 & -v_iX_i & -v_iY_i & -v_iZ_i & -v_i \end{bmatrix} \tag{26}$$

$$\mathbf{p} = [P_{11}, P_{12}, P_{13}, P_{14}, P_{21}, P_{22}, P_{23}, P_{24}, P_{31}, P_{32}, P_{33}, P_{34}]^{\mathrm{T}} \tag{27}$$

For all the points, a homogeneous linear system of equation obtained in the form $A\mathbf{p} = \mathbf{0}$ where

$$A = [A_1, A_2, \ldots, A_m]^{\mathrm{T}}$$

Linear estimation of a projection matrix 2/2

- $\mathbf{p} = \mathbf{0}$ trivial solution omitted.
  - estimation obtained up to a scale
  - $\rightarrow$ norm is fixed as $\|\mathbf{p}\| = 1$
- For noiseless case
  - rank of $\mathbf{A}$ is 11, perfect solution is obtained
- For over-determined and noisy case,
  - only estimation can be computed
  - minimization of $\|A\mathbf{p}\|$ subject to: $\|\mathbf{p}\| = 1$.
  - $\rightarrow$ optimal solution if the eigenvector of $A^T A$ corresponding to the least eigenvalue.
  - solution can be obtained by Singular Value Decomposition (SVD) as well.

Decomposition of a projection matrix

- Structure of a projection matrix:

$$\mathbf{P} = \mathbf{KR}\left[\mathbf{I}| - \mathbf{t}\right] \tag{28}$$

- First three columns of matrix $\mathbf{P}$: $\mathbf{P}_{3\times 3} = \mathbf{KR}$
  - Decomposition can be obtained by RQ - decomposition
  - It decomposes $\mathbf{P}$ into product of an upper triangular and an othonormal matrices
- Last column of matrix $\mathbf{P}$:

$$\mathbf{p}_4 = -\mathbf{KRt} \tag{29}$$

- Thus,

$$\mathbf{t} = -\mathbf{R}^T\mathbf{K}^{-1}\mathbf{p}_4 \tag{30}$$

Data normalization

- Point coordinates can be normalized similarly to homography estimation
- Original transformation: $[u, v, 1]^T \sim \mathbf{P}[X, Y, Z, 1]^T$
- Normalizing transformations: $[u', v', 1]^T = \mathbf{T}_{2D}[u, v, 1]^T$ and $[X', Y', Z', 1]^T = \mathbf{T}_{3D}[X, Y, Z, 1]^T$
  - $\mathbf{T}_{2D}$ 2D transformation(s) (size: 3×3)
  - $\mathbf{T}_{3D}$ 3D transformation(s) (size: 4×4)
- Projection by normalized coordinates: $[u', v', 1]^T \sim \mathbf{P}'[X', Y', Z', 1]^T$
- Solution applied normalized coordinates:
  - $\mathbf{P} = \mathbf{T}_{2D}^{-1}\mathbf{P}'\mathbf{T}_{3D}$ or $\mathbf{P}' = \mathbf{T}_{2D}\mathbf{P}\mathbf{T}_{3D}^{-1}$.