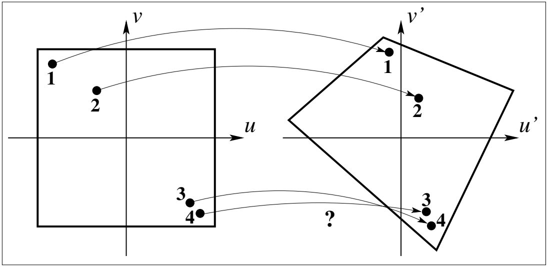
Point-correspondence-based homography estimation



- m point correspondences are given. $\mathbf{u}_i \rightarrow \mathbf{u}'_i$:
  $\mathbf{u}'_i \sim H\mathbf{u}_i, \quad i = 1, \ldots, m$
- Task: estimate H
  - at least $m = n + 2$ correspondences are required
    - planar homography: at least four points needed.
  - For more points, problem is over-determined.
- In case of outliers: robust estimation
  - Robustification requires more corresponences.

Homography written by point locations

$$\alpha \begin{bmatrix} u' \\ v' \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix},$$

where $\alpha \neq 0$ is an unknown scale factor.

Transformation yields

$$u' = \frac{h_{11}u + h_{12}v + h_{13}}{h_{31}u + h_{32}v + h_{33}} = \frac{\mathbf{h}_1^T \mathbf{u}}{\mathbf{h}_3^T \mathbf{u}}, \tag{16}$$

$$v' = \frac{h_{21}u + h_{22}v + h_{23}}{h_{31}u + h_{32}v + h_{33}} = \frac{\mathbf{h}_2^T \mathbf{u}}{\mathbf{h}_3^T \mathbf{u}}, \tag{17}$$

where $\mathbf{h}_i$ is the i-th row of matrix $\mathbf{H}$.

Linear estimation of planar homography 1/2

Equations are multiplied by the common denominator

$$(h_{31}u + h_{32}v + h_{33})u' = h_{11}u + h_{12}v + h_{13} \tag{18}$$

$$(h_{31}u + h_{32}v + h_{33})v' = h_{21}u + h_{22}v + h_{23} \tag{19}$$

For the i-th point, two homogeneous equations are obtained as $A_i\mathbf{h} = 0$, where

$$A_i = \begin{bmatrix} u_i & v_i & 1 & 0 & 0 & 0 & -u_iu_i' & -v_iu_i' & -u_i' \\ 0 & 0 & 0 & u_i & v_i & 1 & -u_iv_i' & -v_iv_i' & -v_i' \end{bmatrix}, \tag{20}$$

$$\mathbf{h} = [h_{11}, h_{12}, h_{13}, h_{21}, h_{22}, h_{23}, h_{31}, h_{32}, h_{33}]^T \tag{21}$$

For all points, $A\mathbf{h} = \mathbf{0}$ linear system of equations should be solved, where

$$A = [A_1, A_2, \dots, A_m]^T$$

Linear estimation of planar homography 2/2

- Trivial solution $\mathbf{h} = \mathbf{0}$ is discarded
  - $\mathbf{h}$ can be determined up to a scale
  - $\rightarrow$ the norm is fixed: $\|\mathbf{h}\| = 1$
- If there are $m = 4$ correspondences; or $m > 4$, but data are noisy-free
  - if rank of A equals 8, exact solution can be obtained.
- If $m > 4$ and data are contaminated
  - only an estimate can be computed,
  - by minimizing $\|A\mathbf{h}\|$, subject to $\|\mathbf{h}\| = 1$.
  - $\rightarrow$ Optimal solution in the least squares sense is the eigenvalue of $A^\mathrm{T} A$ corresponding to the smallest eigenvalue.

Properties of linear estimation 1/2

- Linear method
  $\rightarrow$ unequivocal, clear solution
- Low computational demand
  $\rightarrow$ fast execution
- The cost function of the estimation is determined by ϵ

$$\epsilon = \|\mathit{A}\mathbf{h}\|$$

- ϵ-is an algebraic distance
  - no direct geometric meaning

  $\rightarrow$ minimization of geometric distance(s) preferred

Properties of linear estimation 2/2

- Not robust
  - noisy correspondences
  - Works well if there are no outliers
  - $\rightarrow$ one outlier can destroy the good result
  - $\rightarrow$ (*breakdown point*) is very low
- Due to numerical computation, data normalization required
  - elements in coefficient should be in the same order of magnitude
  - $\rightarrow$ translation: origo should be at the center of gravity
  - $\rightarrow$ scale: spread should be set to $\sqrt{2}$
- Numerical optimization is usually applied, linear method yields initial value

Data normalization

- Coordinate system can be freely selected.
- Original homography: $[u_2, v_2, 1]^T \sim \mathbf{H}[u_1, v_1, 1]^T$
  - Modified coordinates: $[u'_1, v'_1, 1]^T = \mathbf{T}_1[u_1, v_1, 1]^T$ and $[u'_2, v'_2, 1]^T = \mathbf{T}_2[u_2, v_2, 1]^T$
    - where $\mathbf{T}_1$ and $\mathbf{T}_2$ are affine transformations (translation + scale)
- Projection by the modified homography: $[u'_2, v'_2, 1]^T \sim \mathbf{H'}[u'_1, v'_1, 1]^T$
- After substitution: $\mathbf{T}_2[u_2, v_2, 1]^T \sim \mathbf{H'T}_1[u_1, v_1, 1]^T$
- Then: (multiplication by $\mathbf{T}_2^{-1}$ from the left): $[u_2, v_2, 1]^T \sim \mathbf{T}_2^{-1}\mathbf{H'T}_1[u_1, v_1, 1]^T$
- Thus, $\mathbf{H} = \mathbf{T}_2^{-1}\mathbf{H}'\mathbf{T}_1$ or $\mathbf{H}' = \mathbf{T}_2\mathbf{H}\mathbf{T}_1^{-1}$: