Robust model fitting by RANSAC

Lecturer: Dániel Baráth

Talk outline

- Standard Model Fitting Problem
- Robust Model Fitting Problem
- RANdom SAmple Consensus (RANSAC)
  - Sampling techniques (Uniform sampling, NAPSAC, PROSAC)
  - Model quality calculation (RANSAC and MSAC quality functions)
  - Local optimization (LO, LO^+ and GC-RANSAC)
  - Techniques to speed-up RANSAC (WaldSAC)

Some example problems

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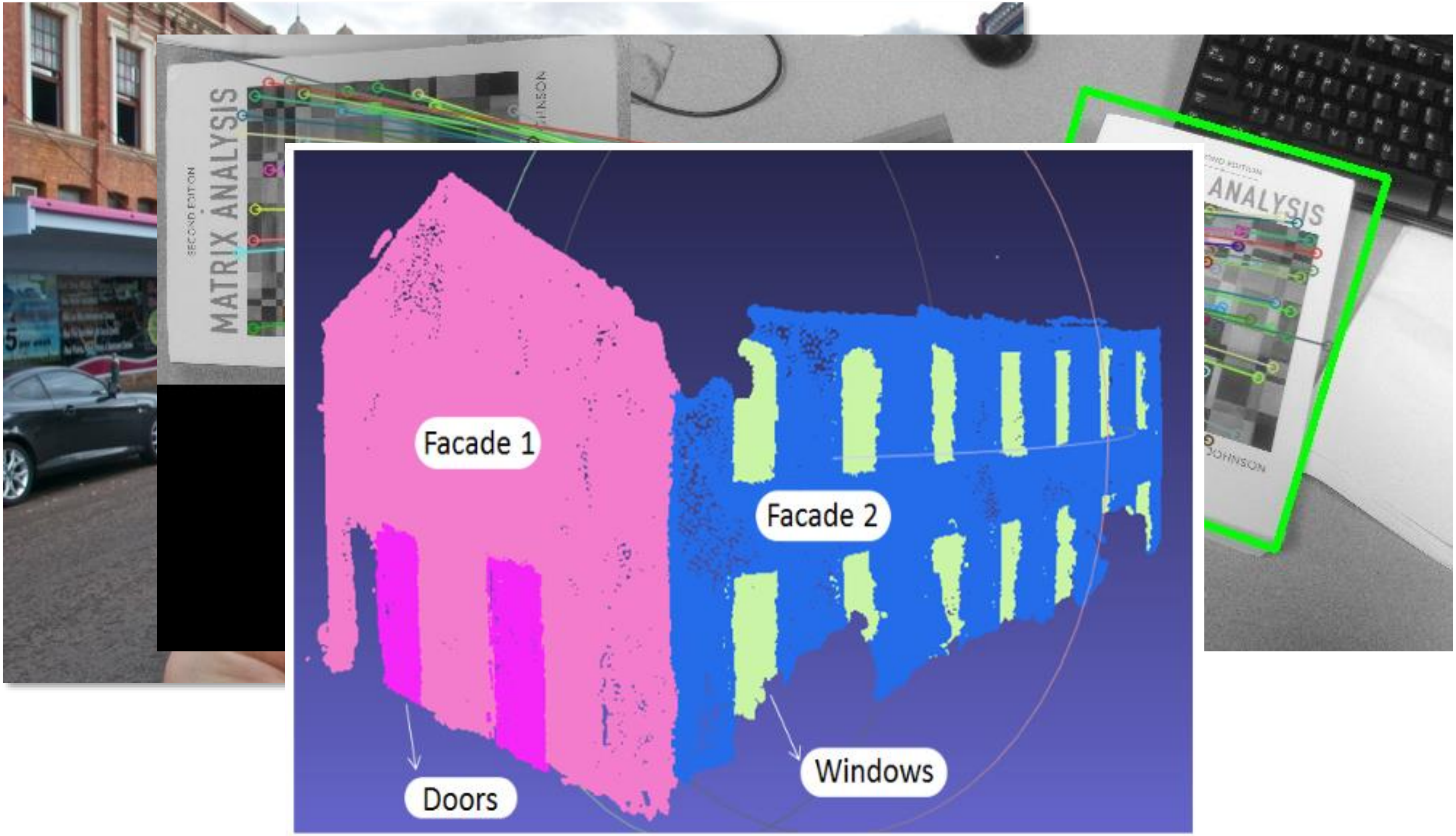


Standard Model Fitting – Example (Line Fitting)

Data points

$$\mathcal{X} = \{\mathbf{x}_j, j = 1, 2, ..., N_p\}$$
$$(\mathbf{x}_j \in \mathbb{R}^2)$$

Find the line which

"best fits" these points.

Finding Line: Line Parametrization

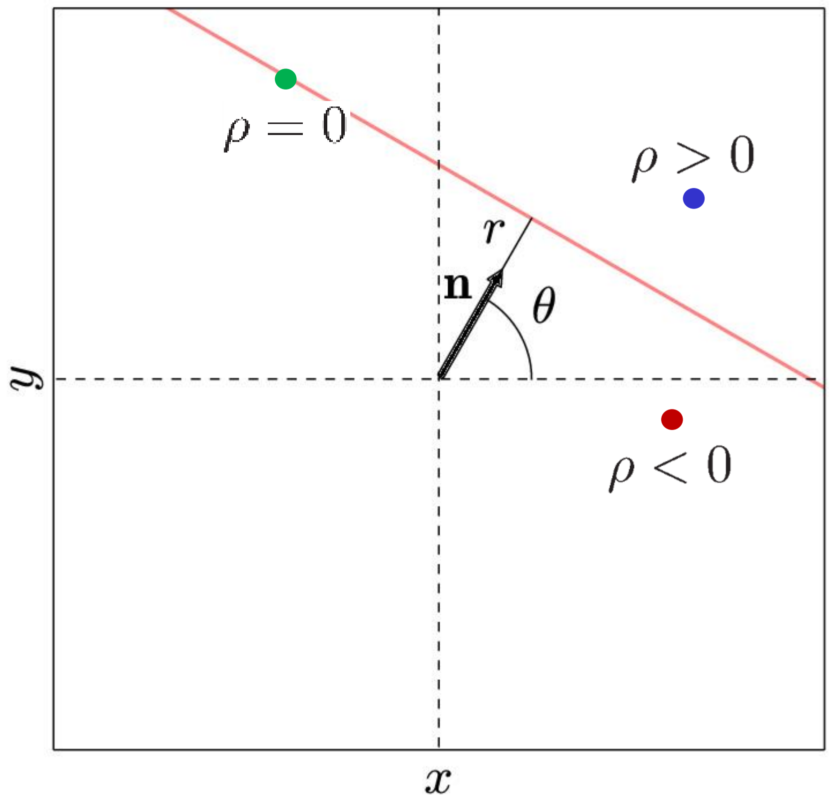
- Line parametrization (homogeneous):

$$ax + by + c = 0, \qquad (a \neq 0 \vee b \neq 0) \tag{1}$$
$$a, b, c \in \mathbb{R} : \text{ line parameters} \tag{2}$$
$$(x, y) : \text{ point coordinates} \tag{3}$$

- Line parametrization (radial):

$$x \cos\theta + y \sin\theta = r, \tag{4}$$
$$\theta \in [0, \pi[, \ r \in \mathbb{R} : \text{ line parameters} \tag{5}$$

Find Line: Line Parametrization and Residuals



Note: $\mathbf{n} = (\cos\theta, \sin\theta)$ (thus $\|\mathbf{n}\| = 1$)

- Line parameters: $\theta \in [0, \pi[$, $r \in \mathbb{R}$
- Point $\mathbf{x} = (x, y)$ on the line:

$$x \cos\theta + y \sin\theta = r$$
$$\Leftrightarrow \mathbf{x} \cdot (\cos\theta, \sin\theta) = r$$

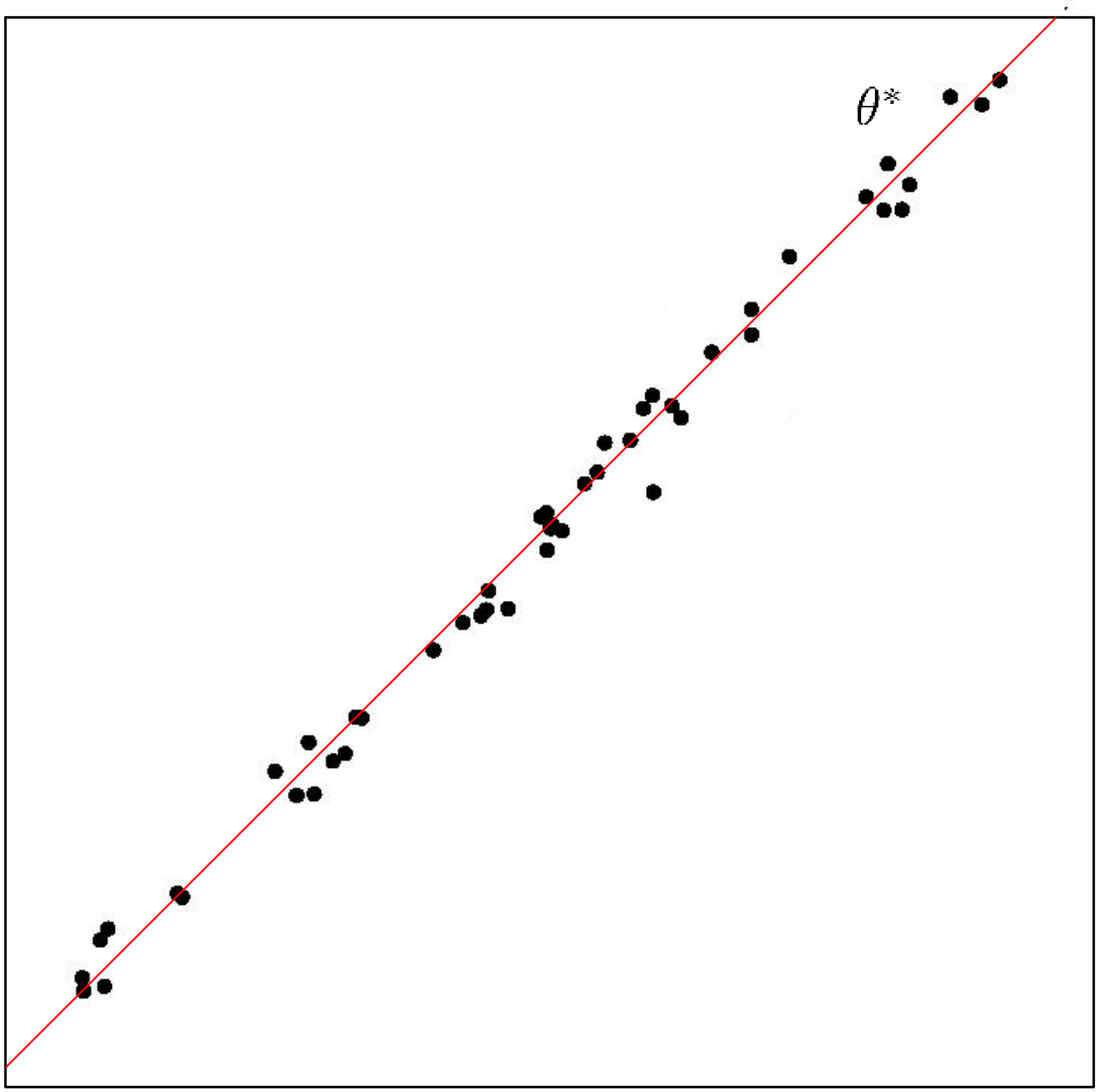
- Point $\mathbf{x} = (x, y)$ not on the line:

$$\mathbf{x} \cdot (\cos\theta, \sin\theta) \neq r$$

  - Signed distance $\rho(\mathbf{x})$ from line:

$$\rho(\mathbf{x}) = \mathbf{x} \cdot (\cos\theta, \sin\theta) - r$$

Standard Model Fitting – Example (Line Fitting)



Data points

$$\mathcal{X} = \{\mathbf{x}_j, j = 1, 2, ..., N_p\}$$
$$(\mathbf{x}_j \in \mathbb{R}^2)$$

Find the line which "best fits" the points.

As optimization: Find best line with parameters θ^* as

$$\theta^* = \underset{\theta}{\text{argmin}} \sum_{\mathbf{x} \in \mathcal{X}} f(\mathbf{x}, \theta)$$

For $f_{LSQ}(\mathbf{x}, \theta) = [\rho(\mathbf{x})]^2$

this is easily solvable by Singular Value Decomposition (SVD).

Standard Model Fitting - Formulation

Given:

- $\mathcal{X} = \{x_j\}_{j=1}^{N_p}$ set of data points

Find:

- θ^* instance

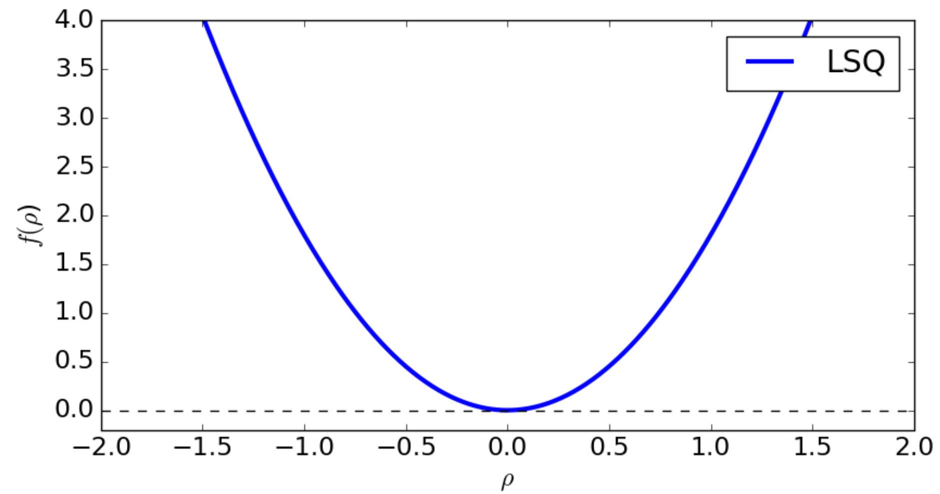
  such that:

$$\theta^* = \arg\min_{\theta} \sum_{\mathbf{x} \in \mathcal{X}} f(\mathbf{x}, \theta)$$

Line fitting example (Least Squares)

$$\theta = (r, \phi)$$

$$f_{LSQ}(\mathbf{x}, \theta) = [\rho(\mathbf{x})]^2$$

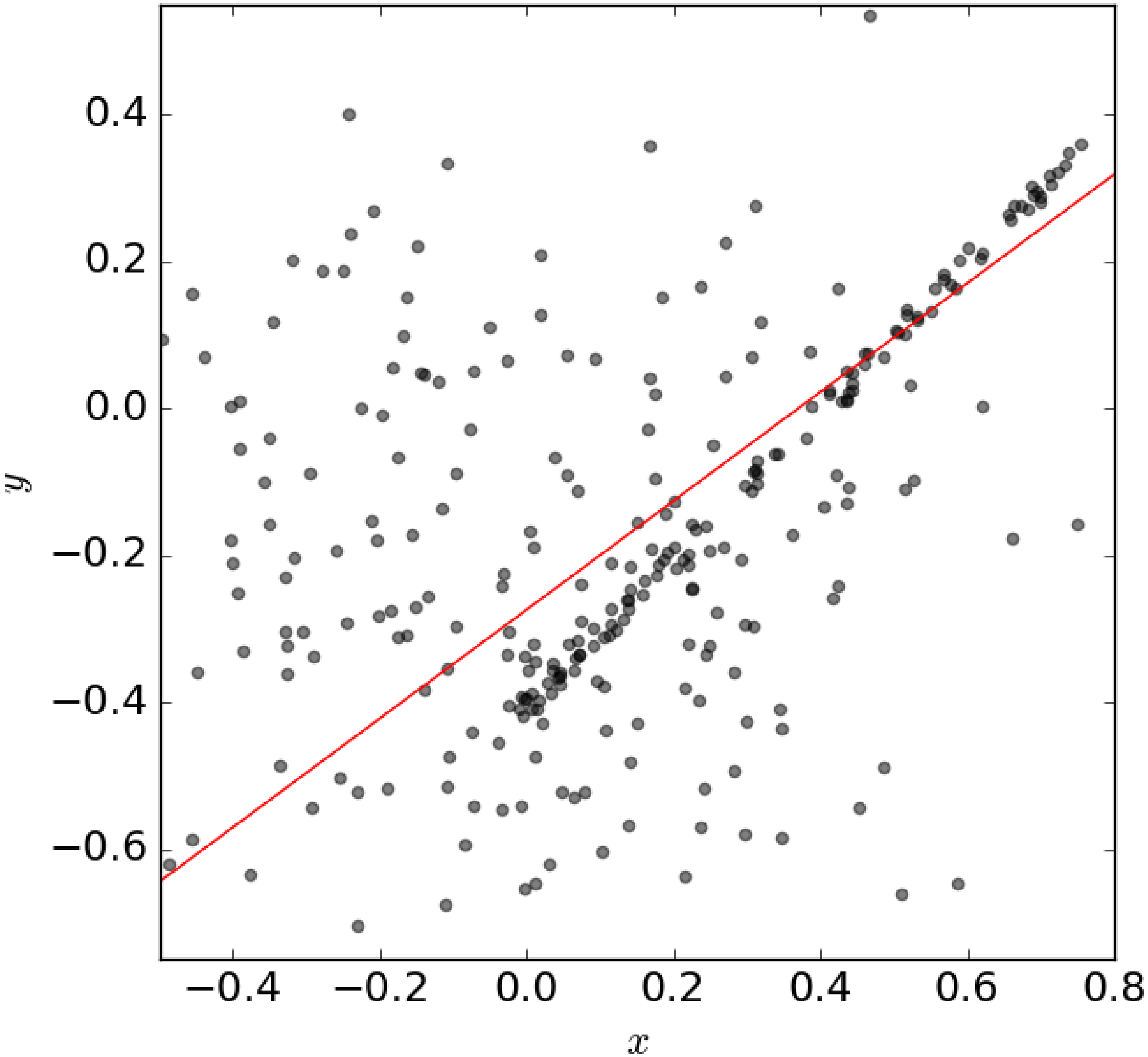


Notes:

- justified as maximum-likelihood method for noise with normal distribution
- used by Gauss ~ 1800
- Laplace considered, in the late 1700's, the sum of absolute differences
  $f(x,\, \theta) = |\rho(\mathbf{x})|$

Line Fitting with Outliers

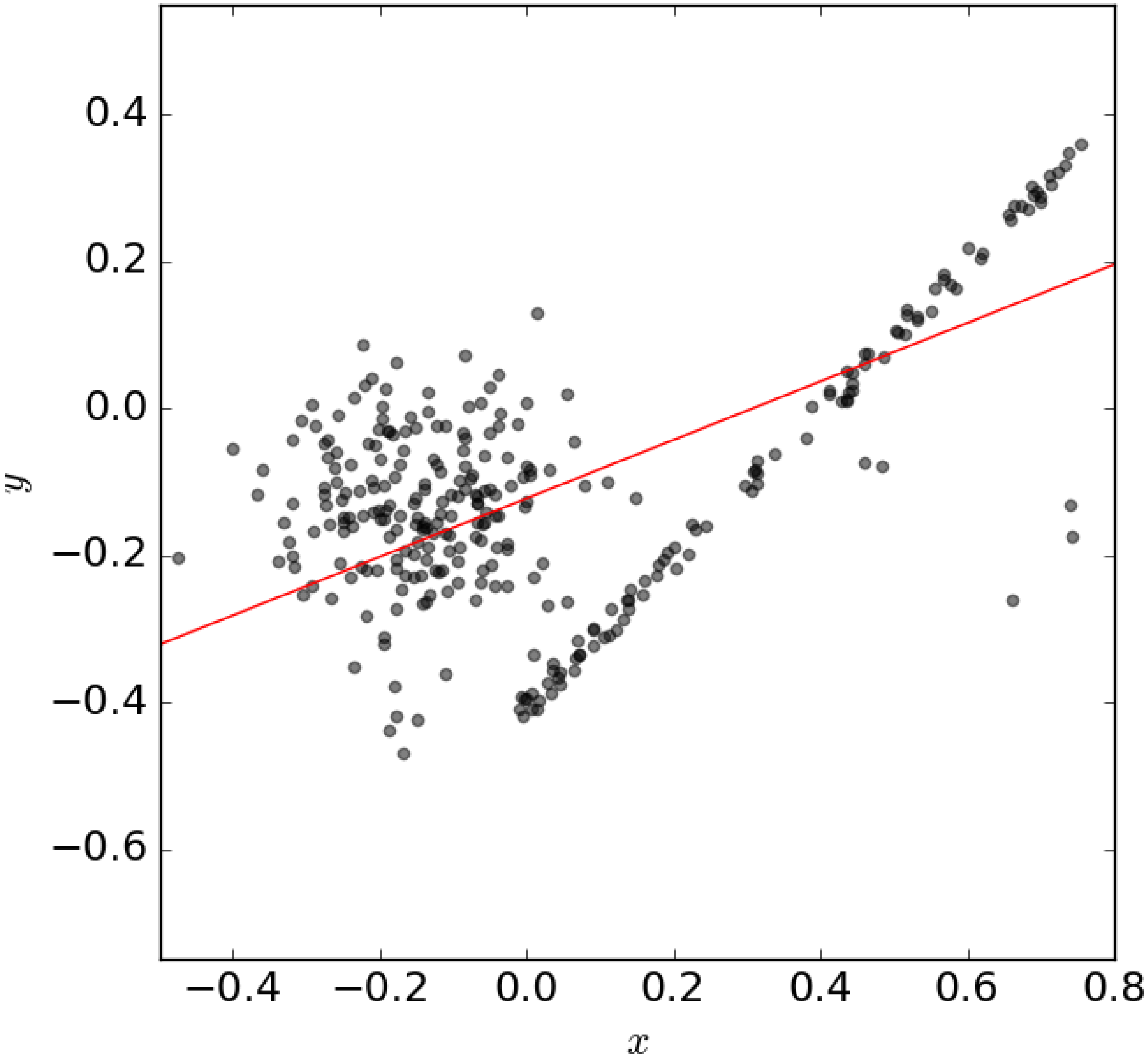
Example 1



Least squares fit

Line Fitting with Outliers

Example 2



Least squares fit

Robust Model Fitting – Robust Loss

Given:

- $\mathcal{X} = \{x_j\}_{j=1}^{N_p}$ set of data points

Find:

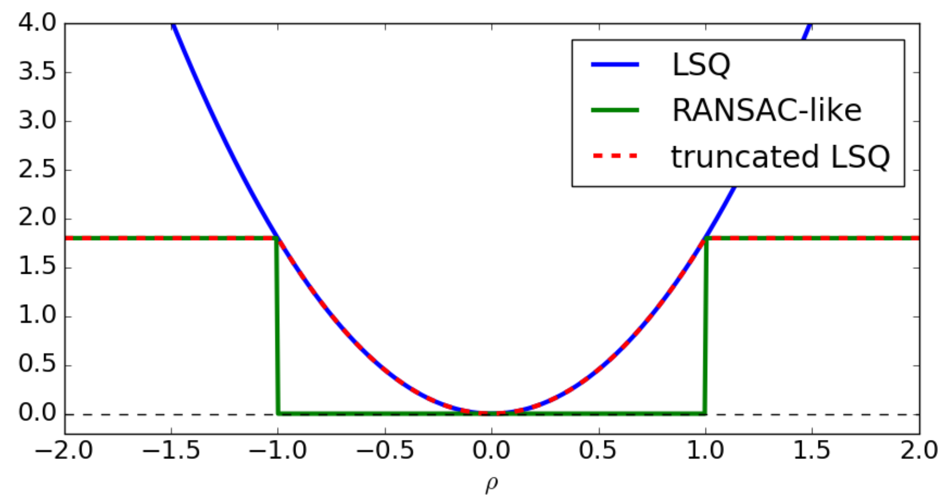
- θ^* instance

 such that:

$$\theta^* = \arg\min_{\theta} \sum_{\mathbf{x} \in \mathcal{X}} f(\mathbf{x}, \theta)$$

Line fitting example (Robust)

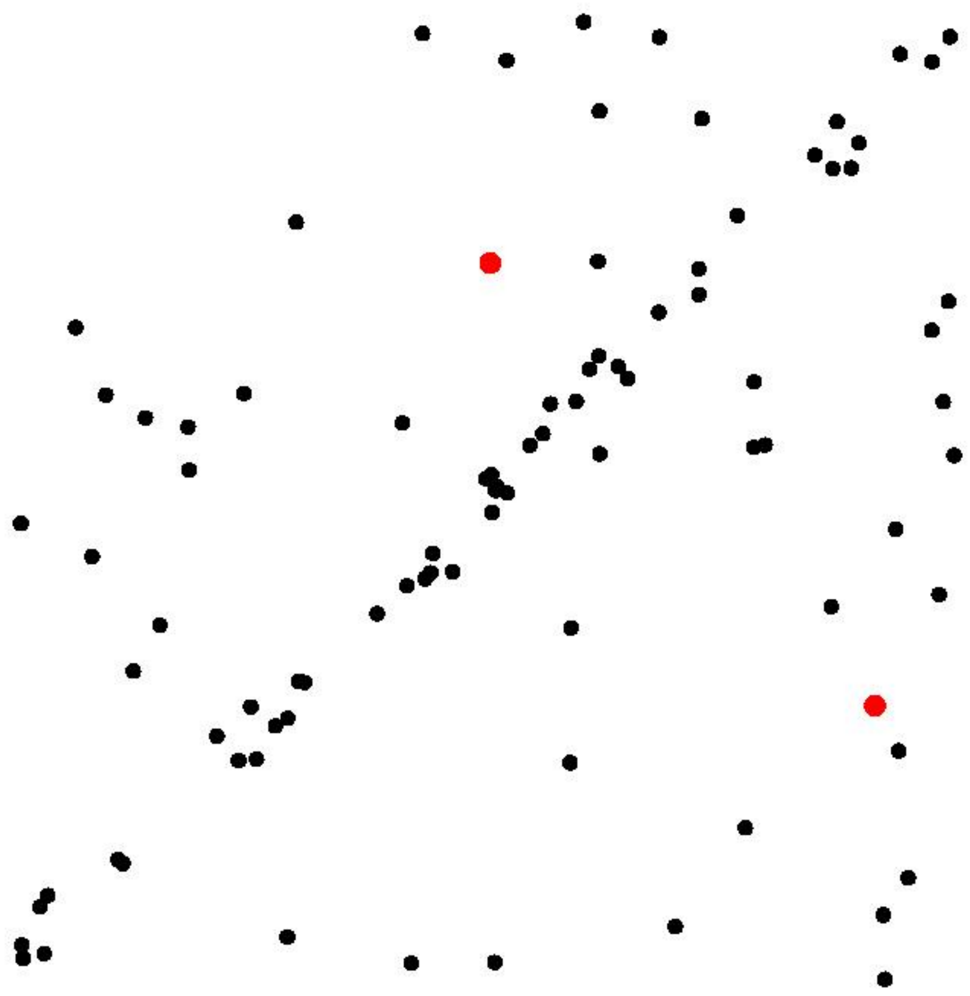
$$f_{\text{RANSAC}}(\mathbf{x}, \theta) = \begin{cases} 0, & \text{if } \rho(\mathbf{x}) \leq \text{threshold } T \\ c, & \text{otherwise} \end{cases}$$



Notes:

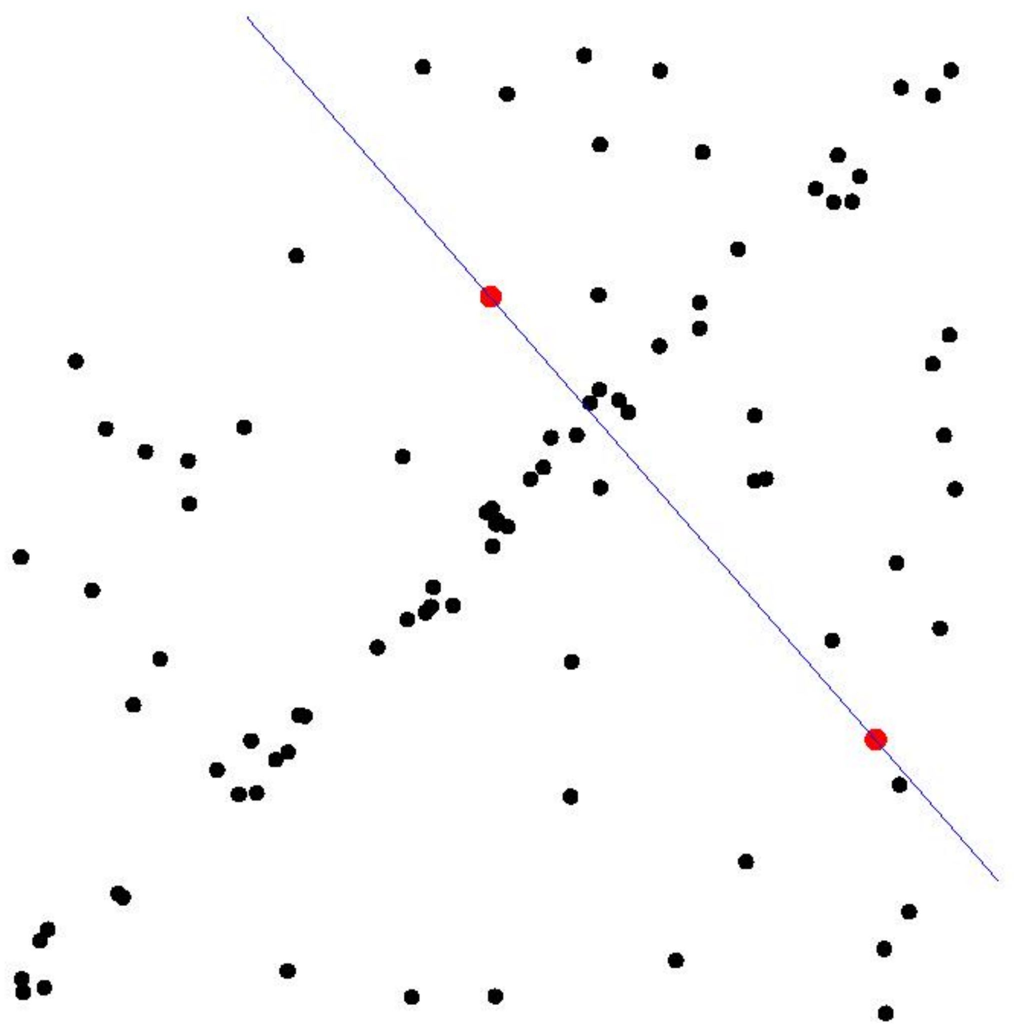
- Fischler and Bolles, 1981 – RANSAC (computer vision community)
- Rouseeuw, 1984 – Least Median of Squares (statistics community)

Random Sample Consensus - RANSAC



Select sample of m points at random

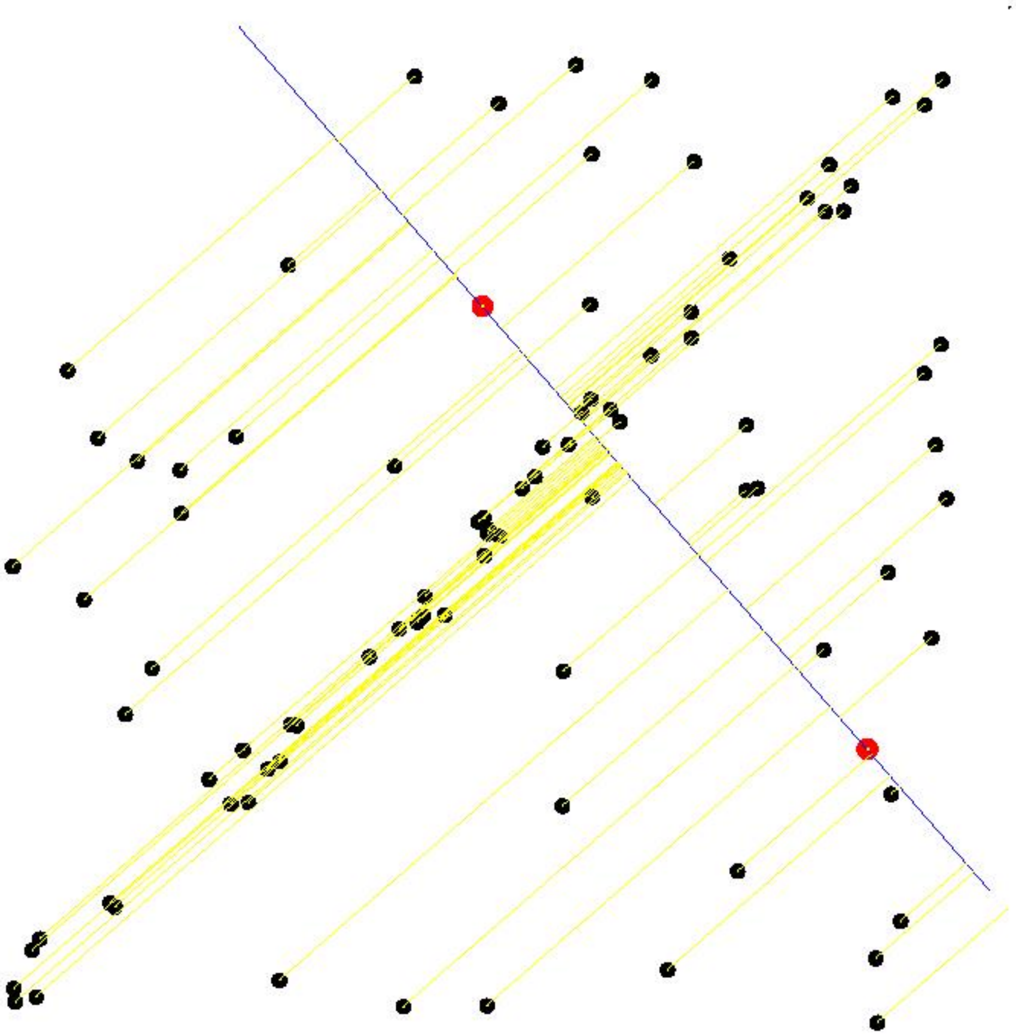
RANSAC



Select sample of m points at random

Estimate model parameters from the data in the sample

RANSAC

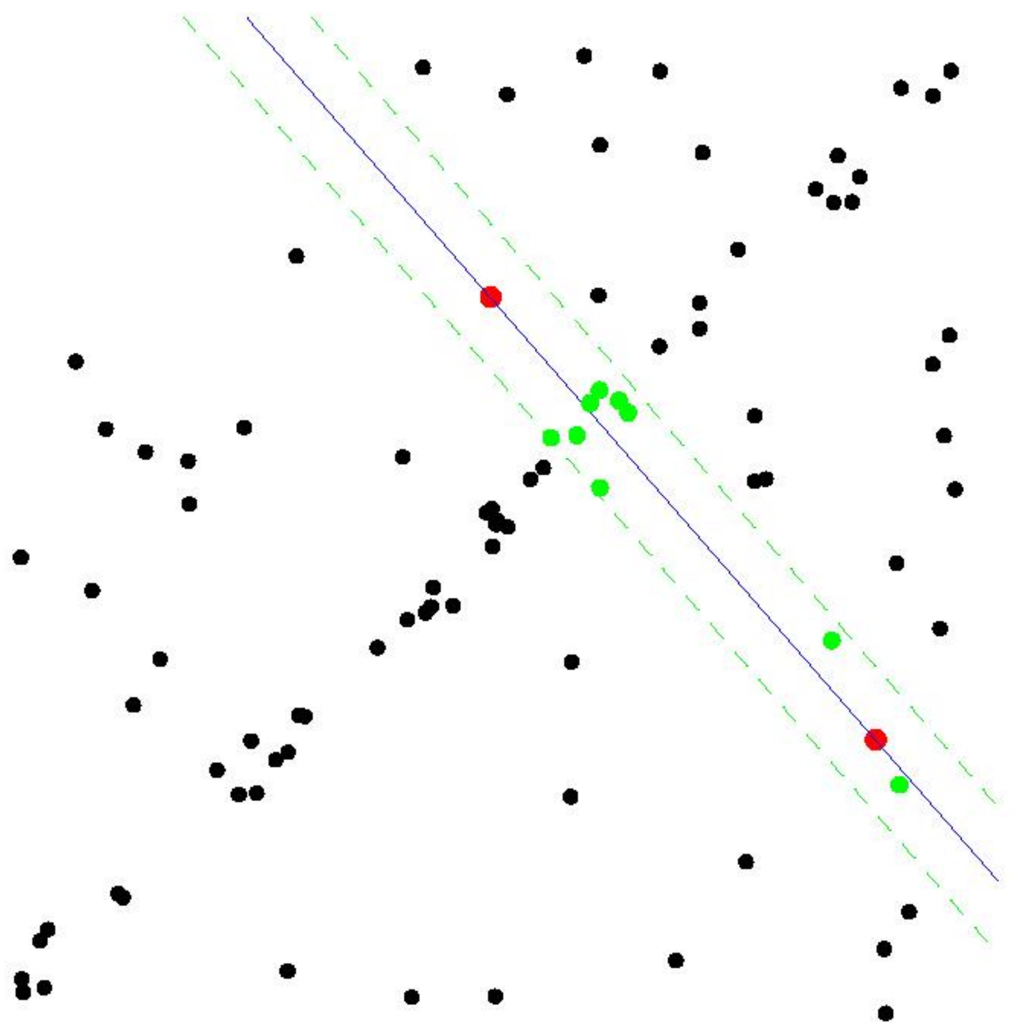


Select sample of m points at random

Estimate model parameters from the data in the sample

Evaluate the error (residual) for each data point

RANSAC



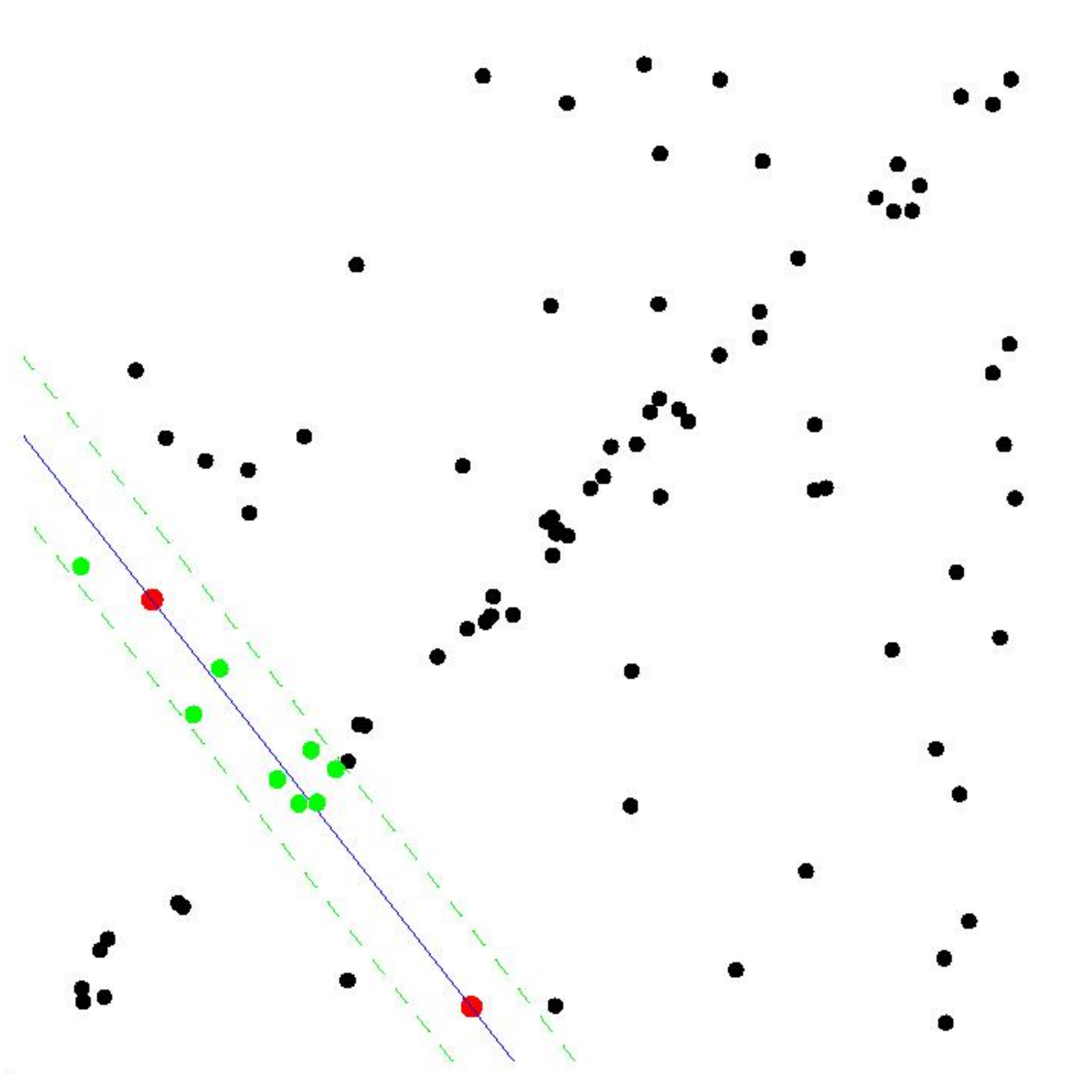
Select sample of m points at random

Estimate model parameters from the data in the sample

Evaluate the error (residual) for each data point

Select data that support the current hypothesis

RANSAC



Select sample of m points at random

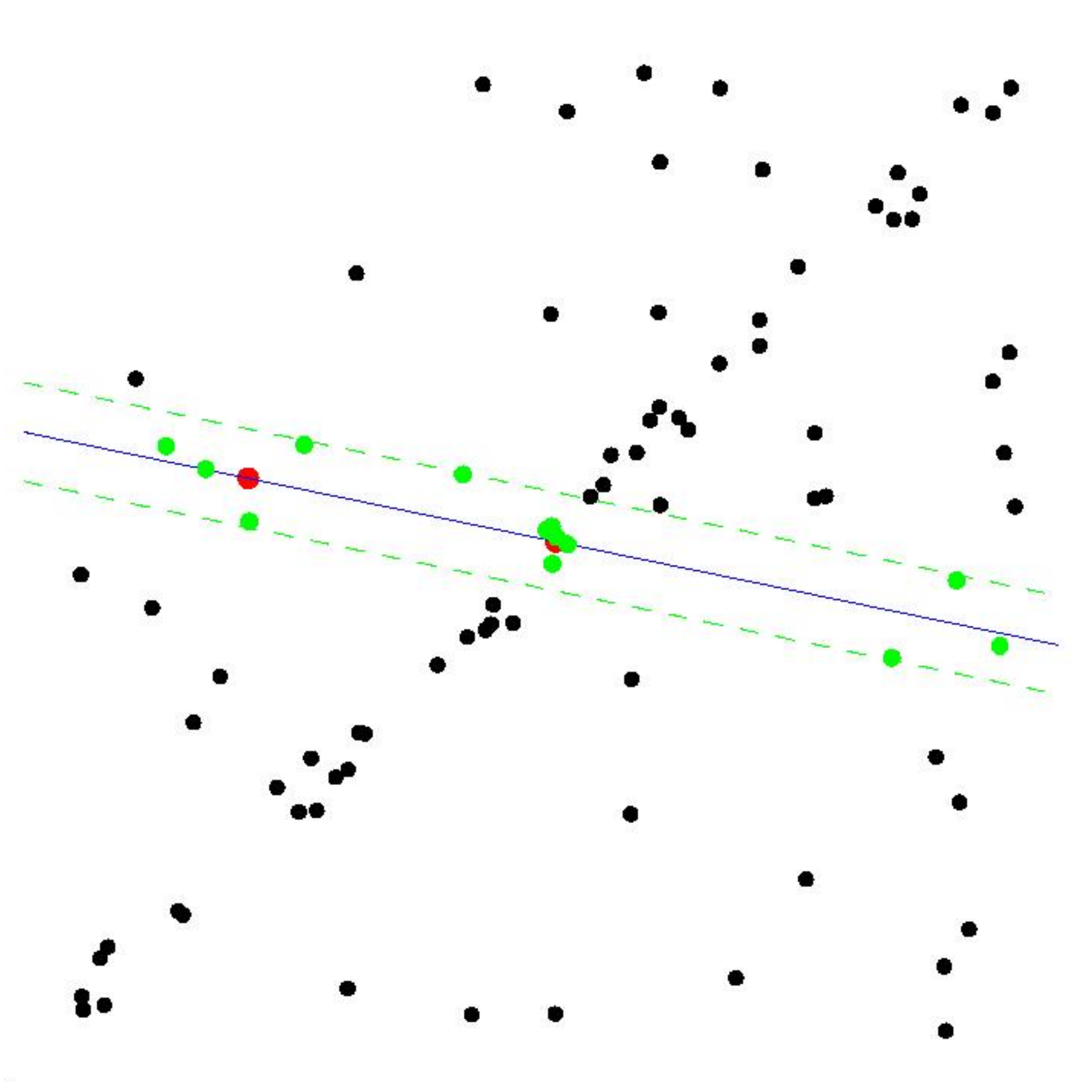
Estimate model parameters from the data in the sample

Evaluate the error (residual) for each data point

Select data that support the current hypothesis

Repeat sampling

RANSAC



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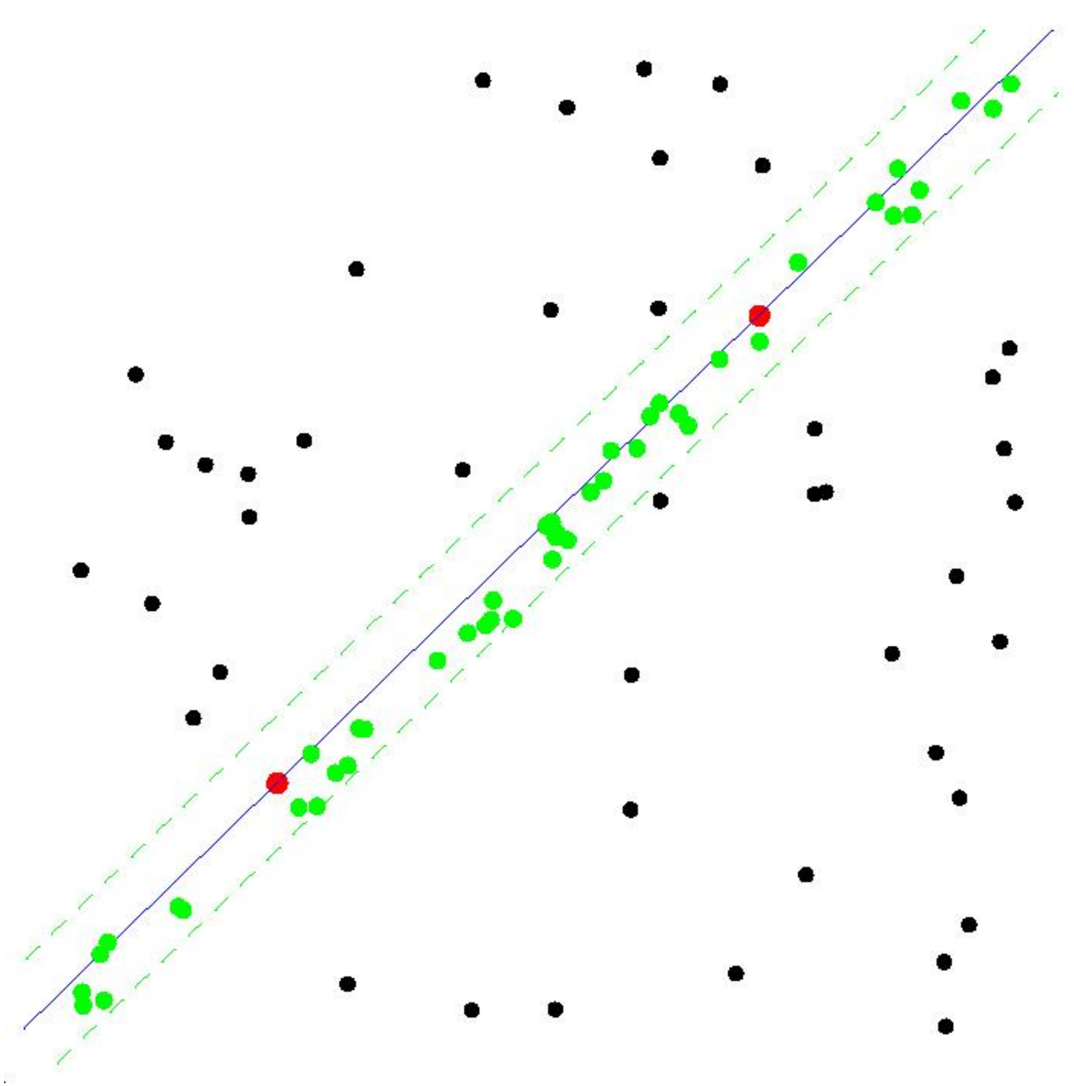

Select sample of m points at random

Estimate model parameters from the data in the sample

Evaluate the error (residual) for each data point

Select data that support the current hypothesis

Repeat sampling

RANSAC [Fischler and Bolles 1981]

Input: $\mathcal{X} = \{\mathbf{x}_j\}_{j=1}^{N}$ data points

$e(S) = \theta$ estimates *model parameters* θ given sample $S \subseteq \mathcal{X}$

$$f(\mathbf{x}, \theta) = \begin{cases} 0, & \text{if distance to model } \leq \text{ threshold } \sigma \\ 1, & \text{otherwise} \end{cases}$$

Cost function for single data point $\mathbf{x}$

$\Rightarrow J(\theta) = \sum_{\mathbf{x} \in \mathcal{X}} f(\mathbf{x}, \theta)$ is #outliers

η – required confidence in the solution, σ – outlier threshold

Output: θ^* parameter of the model minimizing the cost function

$iter \leftarrow 0,\ J^* \leftarrow \infty$
repeat
    Select *random* $S \subseteq \mathcal{X}$ (sample size $m = |S|$) SAMPLING
    Estimate parameters $\theta = e(S)$
    Evaluate $J(\theta) = \sum_{\mathbf{x} \in \mathcal{X}} f(\mathbf{x}, \theta)$ VERIFICATION
    If $J(\theta) < J^*$ then SO-FAR-THE-BEST
        $\theta^* \leftarrow \theta,\ J^* \leftarrow J(\theta)$
    $iter \leftarrow iter + 1$
until P(better solution exists) $= f(|\mathcal{X}|, J^*, iter) < \eta$
Compute θ^* from all inliers $\mathcal{X}_{in}$: $\theta^* \leftarrow \text{LocalOptimization}(\mathcal{X}_{in}, \theta^*)$

RANSAC – how many samples?

- N Number of points
- Q Number of inliers,
- m Size of sample
- $\epsilon = Q/N$ Inlier ratio

Probability of all-inlier (uncontaminated) sample:

$$P(\text{inlier sample}) = \frac{\binom{Q}{m}}{\binom{N}{m}} \approx \epsilon^m$$

Mean time for hitting all-inliers sample is proportional to $1/P$.

Finding the solution with confidence η requires

$$k \geq \log(1 - \eta) \;/\; \log\left(1 - \epsilon^m\right)$$

RANSAC termination - How many samples?

Inlier ratio $\epsilon = Q/N$ [%]

Size of the sample m

	15%	20%	30%	40%	50%	70%
2	132	73	32	17	10	4
4	5916	1871	368	116	46	11
7	$1.75 \cdot 10^6$	$2.34 \cdot 10^5$	$1.37 \cdot 10^4$	1827	382	35
8	$1.17 \cdot 10^7$	$1.17 \cdot 10^6$	$4.57 \cdot 10^4$	4570	765	50
12	$2.31 \cdot 10^{10}$	$7.31 \cdot 10^8$	$5.64 \cdot 10^6$	$1.79 \cdot 10^5$	$1.23 \cdot 10^4$	215
18	$2.08 \cdot 10^{15}$	$1.14 \cdot 10^{13}$	$7.73 \cdot 10^9$	$4.36 \cdot 10^7$	$7.85 \cdot 10^5$	1838
30	∞	∞	$1.35 \cdot 10^{16}$	$2.60 \cdot 10^{12}$	$3.22 \cdot 10^9$	$1.33 \cdot 10^5$
40	∞	∞	∞	$2.70 \cdot 10^{16}$	$3.29 \cdot 10^{12}$	$4.71 \cdot 10^6$

computed for $\eta = 0.95$

RANSAC Notes

Pros:

- extremely popular (>17000 citations in Google Scholar)
- used in many applications
- percentage of inliers not needed and not limited
- a probabilistic guarantee for the solution
- mild assumptions: σ known

Cons:

- slow if inlier ratio low
- It was observed experimentally that RANSAC takes several times longer than theoretically expected. This is due to noise – not every all-inlier sample generates a good hypothesis:

$$P(\text{inlier sample}) \neq P(\text{good model estimate})$$

RANSAC: case closed?

Extremely simple algorithm... should we stop the talk here?

RANSAC: case closed? NO

- **Cost function:** MSAC, MLESAC, Huber loss, ...
- **Outlier threshold** σ: Least median of Squares, MINPRAN, MAGSAC, ...

- **Correctness of the results. Degeneracy.**
  Solution: DegenSAC.

- **Accuracy** (parameters are estimated from minimal samples).
  Solution: Locally Optimized RANSAC, Graph-Cut RANSAC

- **Speed:** Running time grows with
  1. number of data points,
  2. number of iterations (polynomial in the inlier ratio)
     Addressing the problem:
     RANSAC with SPRT (WaldSAC), PROSAC

RANSAC variants (on 25.02.2019)

allintitle: "RANSAC"

Nagyjából 1 080 találat (0,03 másodperc)

[Locally optimized **RANSAC**](#) — [PDF] puc-rio.br
O Chum, J Matas, J Kittler - Joint Pattern Recognition Symposium, 2003 - Springer
A new enhancement of **ransac**, the locally optimized **ransac** (lo-**ransac**), is introduced. It has been observed that, to find an optimal solution (with a given probability), the number of samples drawn in **ransac** is significantly higher than predicted from the mathematical model ...
Idézetek száma: 511 Kapcsolódó cikkek Mind a(z) 14 változat Web of Science: 170

Efficient **RANSAC** for point-cloud shape detection — [PDF] psu.edu
R Schnabel, R Wahl, R Klein - Computer graphics forum, 2007 - Wiley Online Library
In this paper we present an automatic algorithm to detect basic shapes in unorganized point clouds. The algorithm decomposes the point cloud into a concise, hybrid structure of inherent shapes and a set of remaining points. Each detected shape serves as a proxy for a ...
Idézetek száma: 1129 Kapcsolódó cikkek Mind a(z) 9 változat Web of Science: 545

[PDF] Performance evaluation of **RANSAC** family — [PDF] bmva.org
S Choi, T Kim, W Yu - Journal of Computer Vision, 1997 - bmva.org
Random Sample Consensus (**RANSAC**)[3] has been popular in regression problem with samples contaminated with outliers. M-estimator, Hough transform, and others had been utilized before **RANSAC**. However, **RANSAC** does not use complex optimization as like M ...
Idézetek száma: 299 Kapcsolódó cikkek Mind a(z) 6 változat

Preemptive **RANSAC** for live structure and motion estimation — [PDF] springer.com Full View
D Nistér - Machine Vision and Applications, 2005 - Springer
A system capable of performing robust live ego-motion estimation for perspective cameras is presented. The system is powered by random sample consensus with preemptive scoring of the motion hypotheses. A general statement of the problem of efficient preemptive scoring is ...
Idézetek száma: 679 Kapcsolódó cikkek Mind a(z) 19 változat Web of Science: 144

Optimal randomized **RANSAC** — [PDF] cvut.cz
O Chum, J Matas - IEEE Transactions on Pattern Analysis and ..., 2008 - ieeexplore.ieee.org
A randomized model verification strategy for **RANSAC** is presented. The proposed method finds, like **RANSAC**, a solution that is optimal with user-specified probability. The solution is found in time that is close to the shortest possible and superior to any deterministic ...
Idézetek száma: 311 Kapcsolódó cikkek Mind a(z) 12 változat Web of Science: 132

Overview of the **RANSAC** Algorithm
KG Derpanis - Image Rochester NY, 2010 - rmozone.com
The RANdom SAmple Consensus (**RANSAC**) algorithm proposed by Fischler and Bolles [1] is a general parameter estimation approach designed to cope with a large proportion of outliers in the input data. Unlike many of the common robust estimation techniques such as ...
Idézetek száma: 143 Kapcsolódó cikkek Mind a(z) 6 változat

[PDF] Randomized **RANSAC** with Td, d test — [PDF] cvut.cz
O Chum, J Matas - Proc. British Machine Vision Conference, 2002 - cmp.felk.cvut.cz
Many computer vision algorithms include a robust estimation step where model parameters are computed from a data set containing a significant proportion of outliers. The **RANSAC** algorithm is possibly the most widely used robust estimator in the field of computer vision. In ...
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RANSAC variants (on 25.02.2019)

RANSAC

Nagyjából 53 400 találat (0,04 másodperc)

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Kapcsolódó keresések

ransac **algorithm**	ransac **matching**
point ransac	ransac **plane**
random sample consensus ransac	ransac **fitting**
sift ransac	ransac **point cloud registration**

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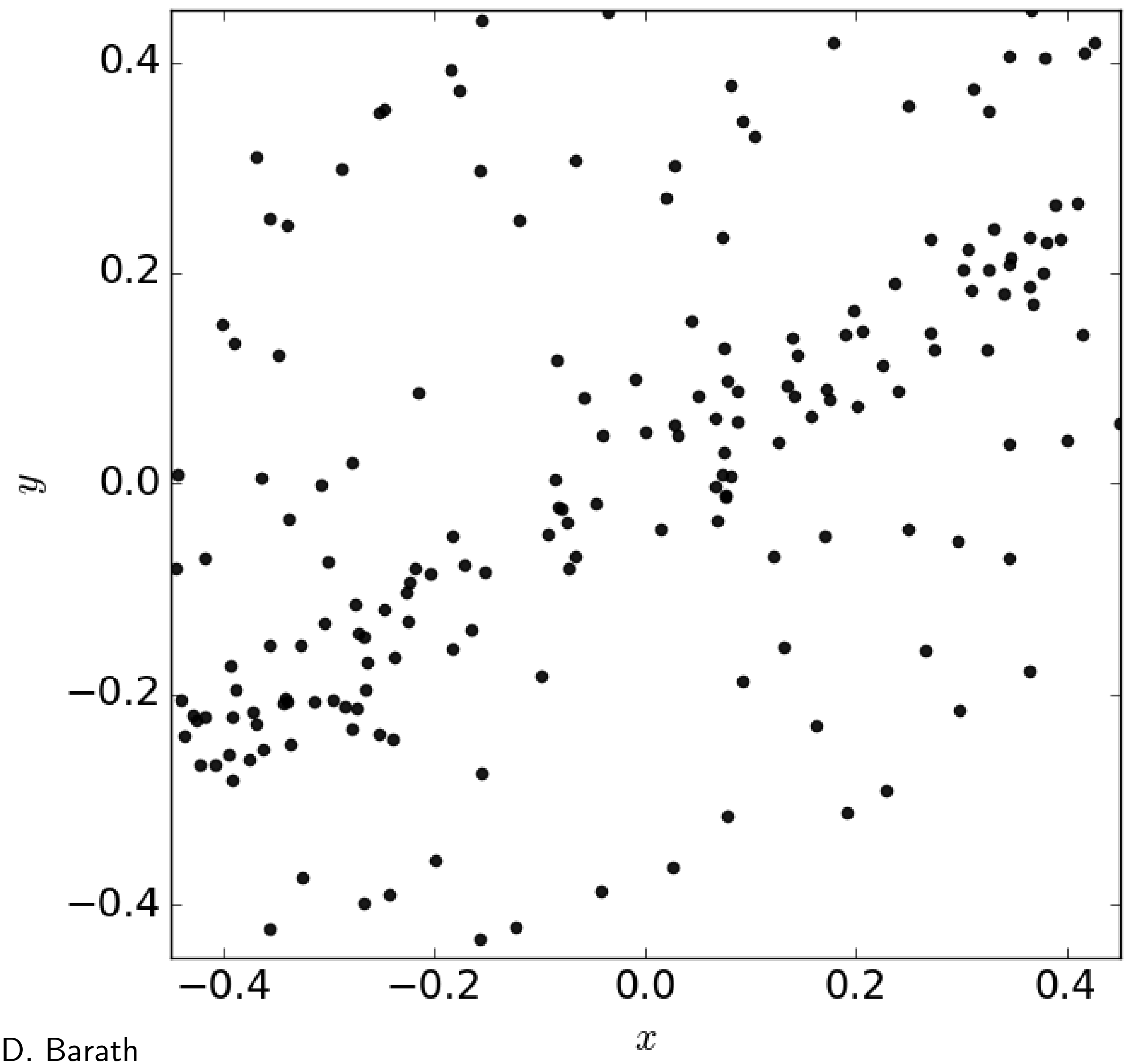
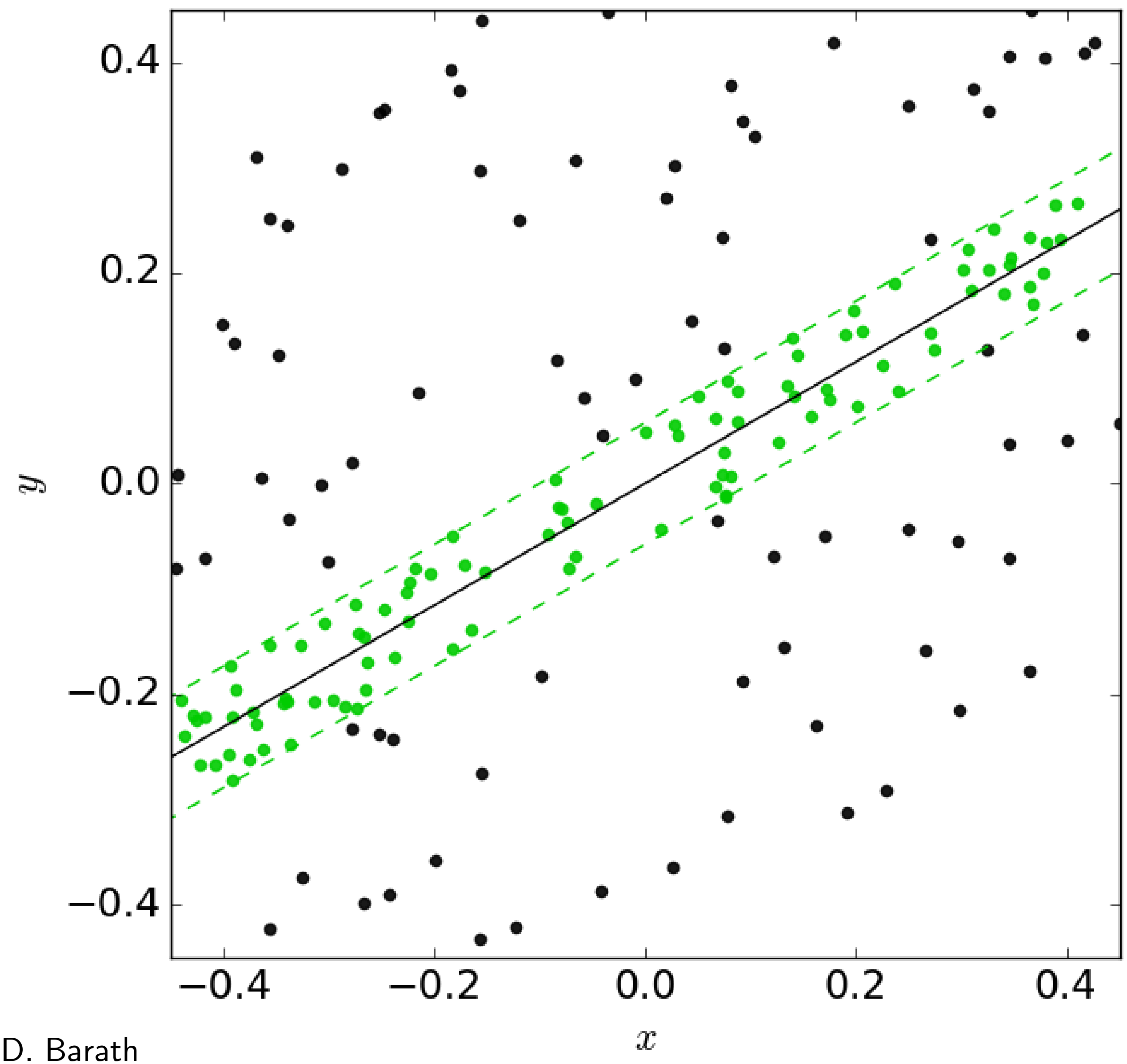
Locally Optimized RANSAC (LO-RANSAC): Problem Intro



Data: 200 points

LO-RANSAC: Problem Introduction

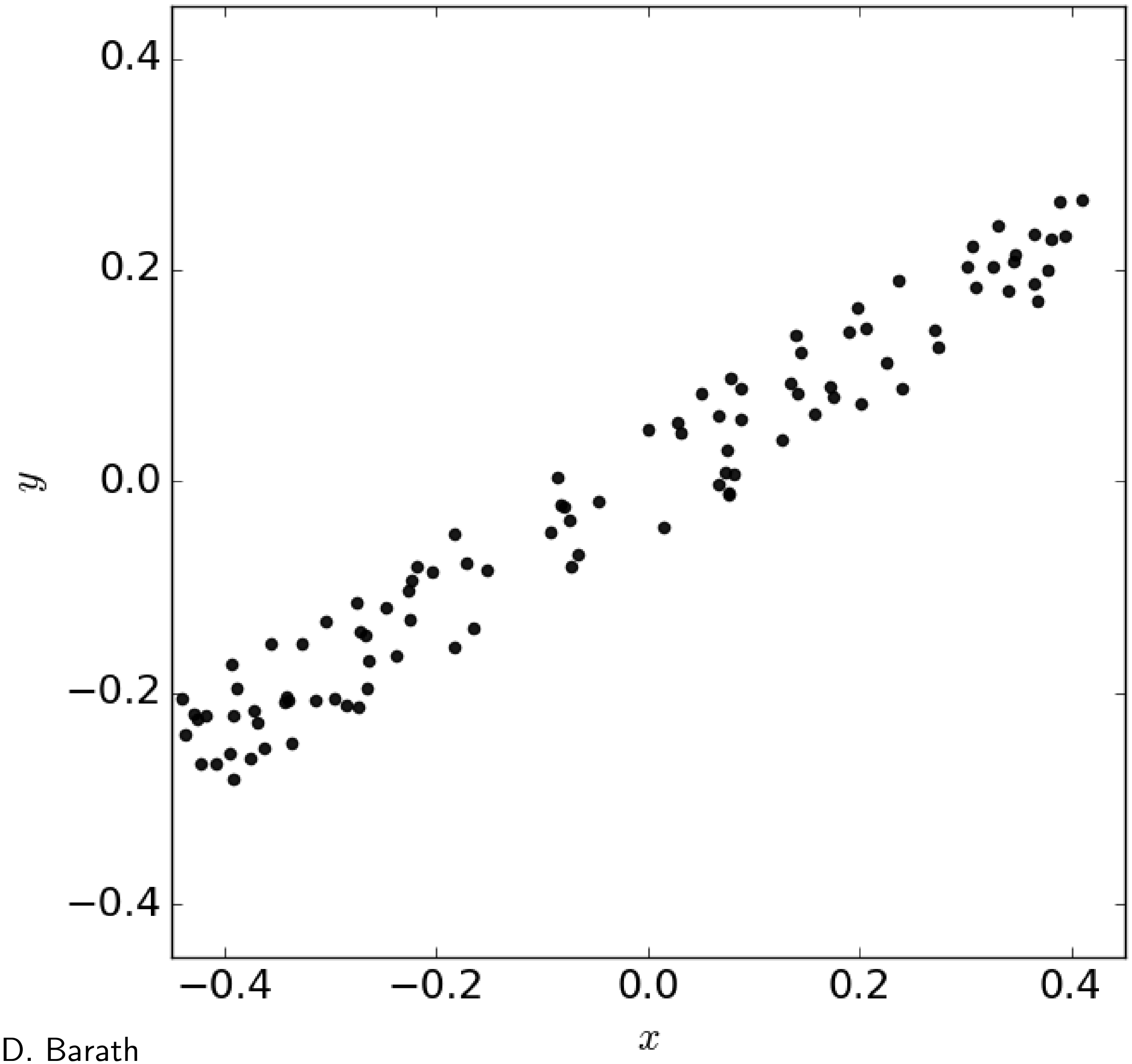


Data: 200 points

Model, 100 inliers

LO-RANSAC: Problem Introduction

For simplicity, consider only points belonging to the model (100 points)



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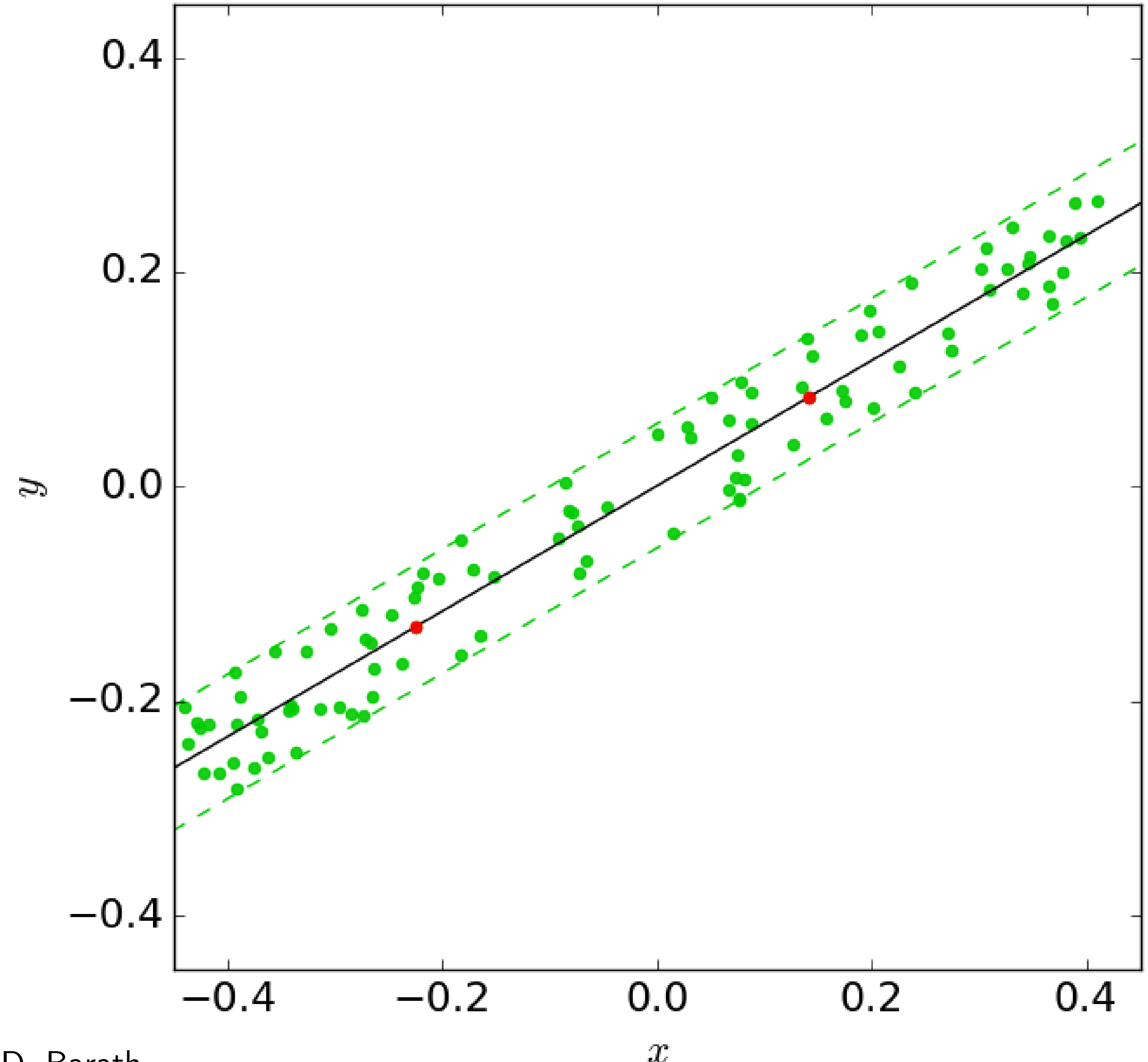


RANSAC

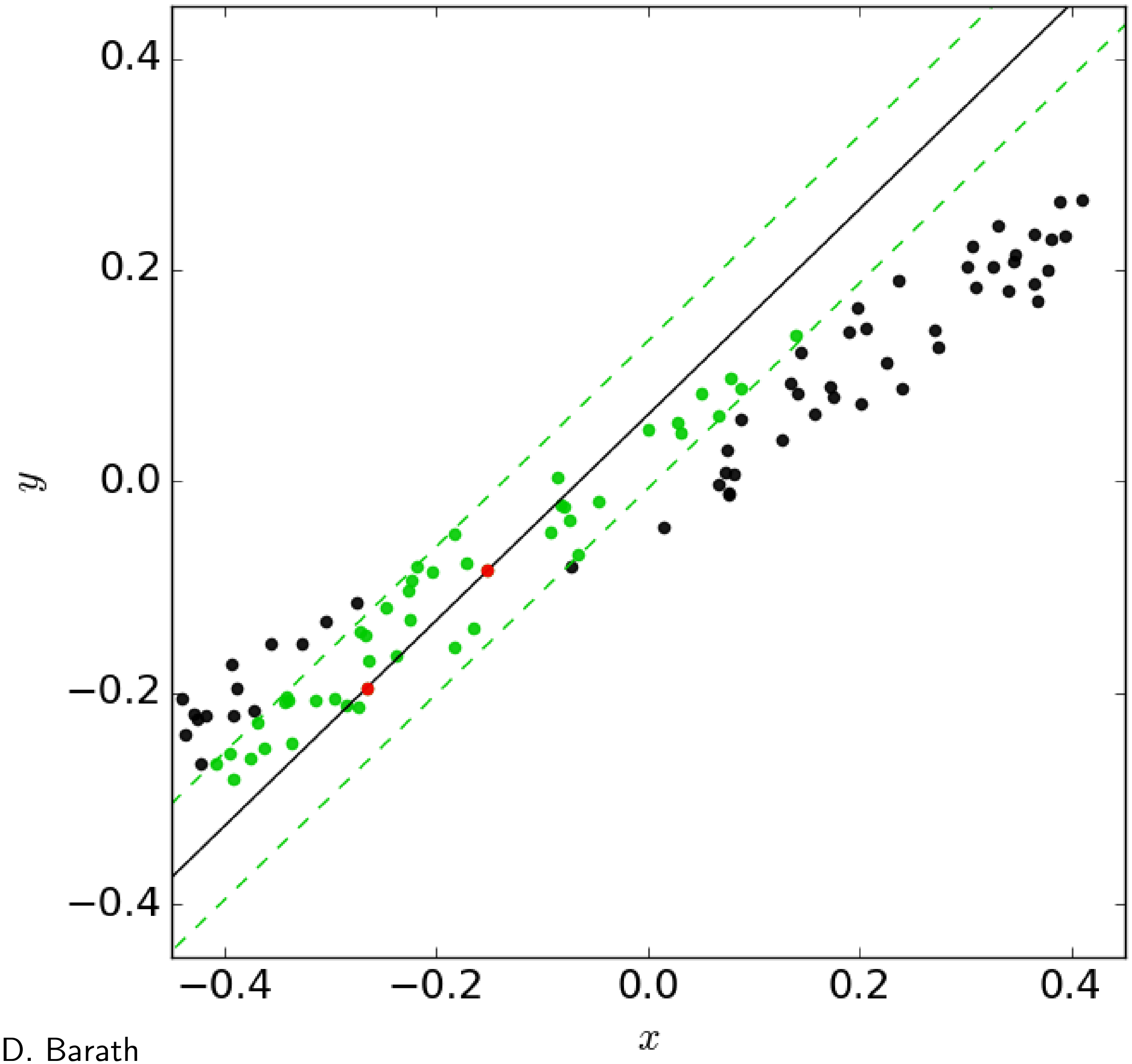
Hypothesis generation from 2 points

Will every two points generate the whole inlier set?

This sample:
YES. 100 inliers.

LO-RANSAC: Problem Introduction

For simplicity, consider only points belonging to the model (100 points)



RANSAC

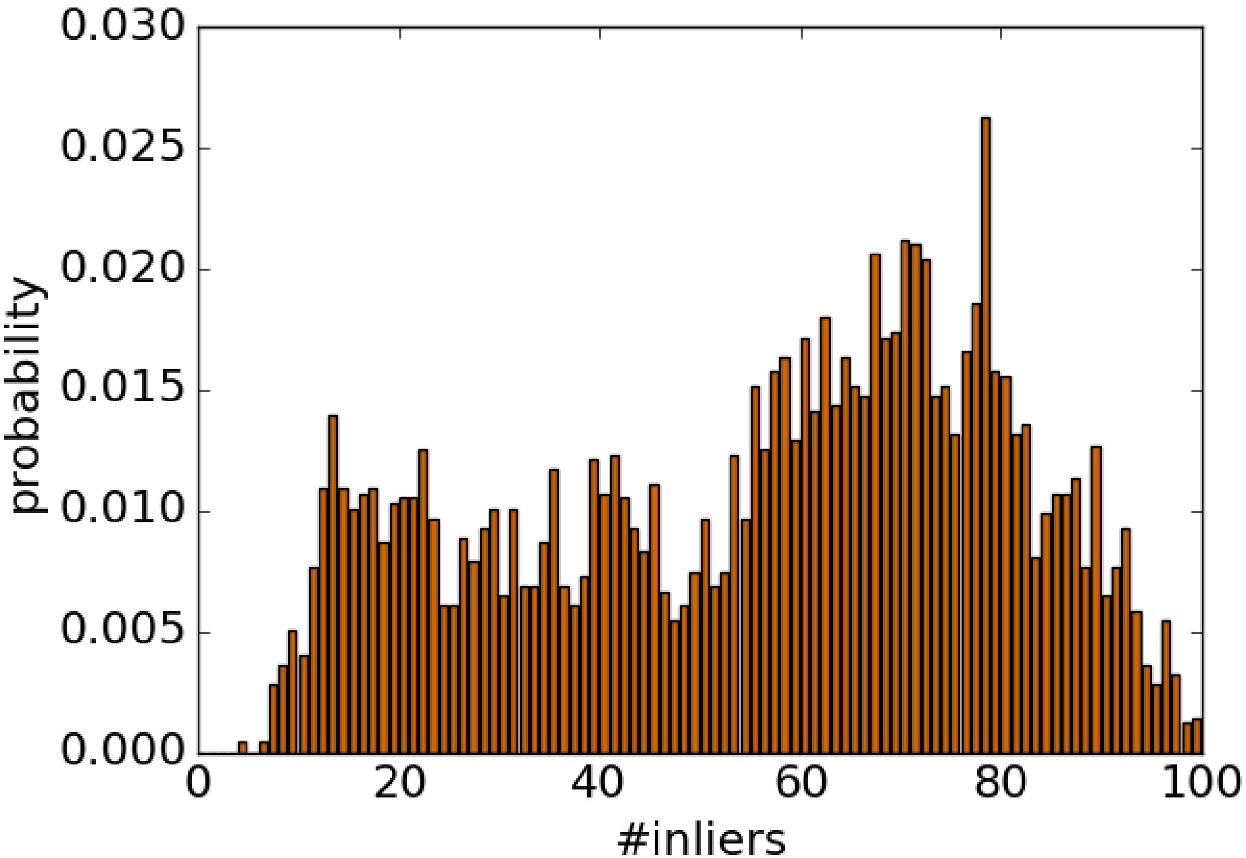
Hypothesis generation from 2 points

Will every two points generate the whole inlier set?

This sample:
NO. 45 inliers.

LO-RANSAC: Problem Introduction

For simplicity, consider only points belonging to model (100 points)



The distribution of the number of inliers obtained while randomly sampling points pairs

RANSAC

Hypothesis generation from 2 points

Will every two points generate the whole inlier set?

LO-RANSAC

Input: $\mathcal{X} = \{\mathbf{x}_j\}_{j=1}^{N}$ data points

$e(S) = \theta$ estimates *model parameters* θ given sample $S \subseteq \mathcal{X}$

$$f(\mathbf{x}, \theta) = \begin{cases} 0, & \text{if distance to model } \leq \text{ threshold } \sigma \\ 1, & \text{otherwise} \end{cases}$$

Cost function for single data point $\mathbf{x}$

$\Rightarrow J(\theta) = \sum_{\mathbf{x} \in \mathcal{X}} f(\mathbf{x}, \theta)$ is #outliers

η – required confidence in the solution, σ – outlier threshold

Output: θ^* parameter of the model minimizing the cost function

1: $iter \leftarrow 0,\ J^* \leftarrow \infty$
2: **repeat**
3: Select *random* $S \subseteq \mathcal{X}$ (sample size $m = |S|$) SAMPLING
4: Estimate parameters $\theta = e(S)$
5: Evaluate $J(\theta) = \sum_{\mathbf{x} \in \mathcal{X}} f(\mathbf{x}, \theta)$ VERIFICATION
6: If $J(\theta) < J^*$ then SO-FAR-THE-BEST
$\theta^* \leftarrow \theta,\ J^* \leftarrow J(\theta)$
7: $iter \leftarrow iter + 1$
8: **until** $P(\text{better solution exists}) = f(|\mathcal{X}|, J^*, iter) < \eta$
9: Compute θ^* from all inliers $\mathcal{X}_{in}$: $\theta^* \leftarrow$ LocalOptimization$(\mathcal{X}_{in}, \theta^*)$

LO-RANSAC

Input: $\mathcal{X} = \{\mathbf{x}_j\}_{j=1}^{N}$ data points

$e(S) = \theta$ estimates *model parameters* θ given sample $S \subseteq \mathcal{X}$

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repeat
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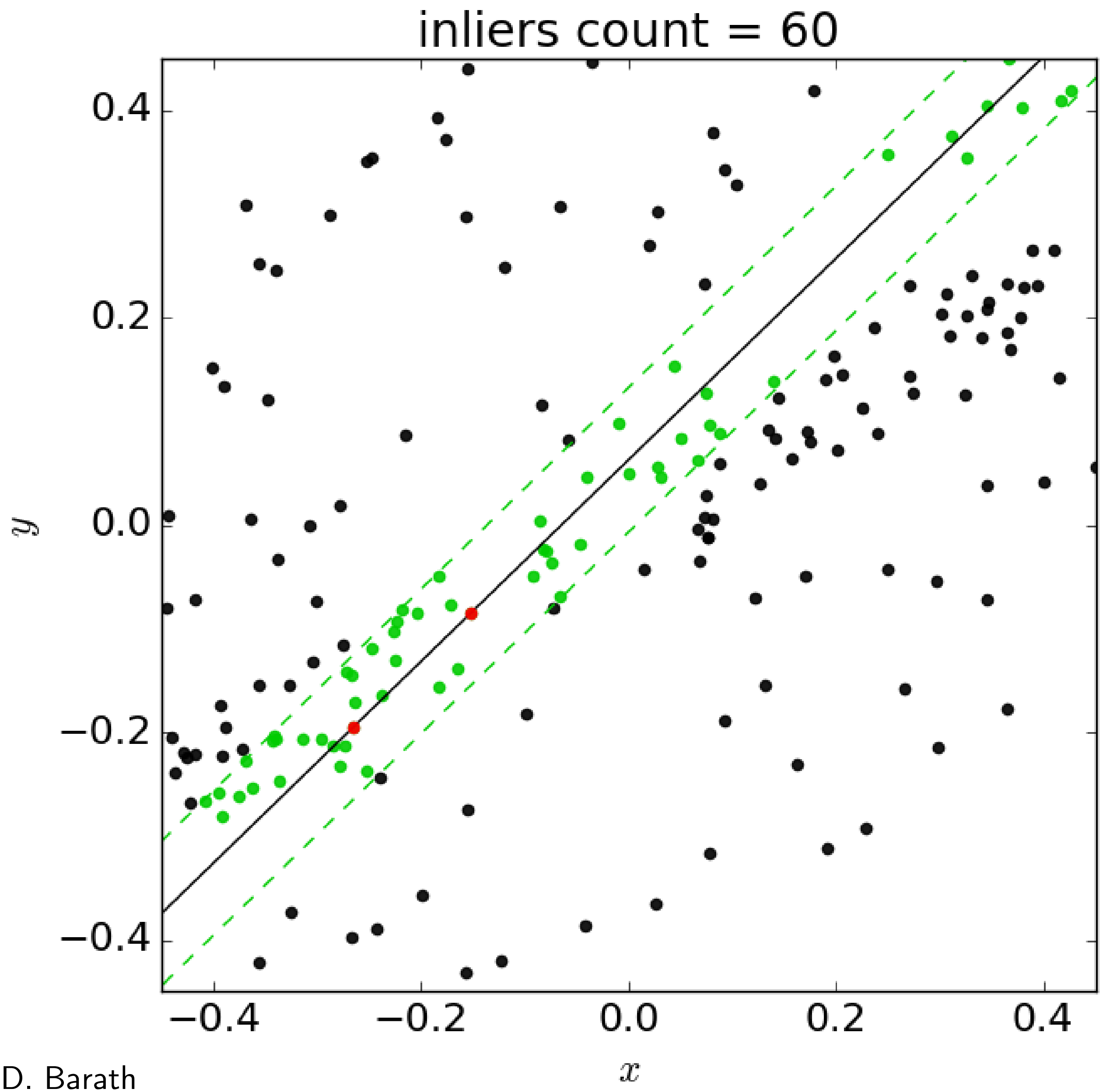
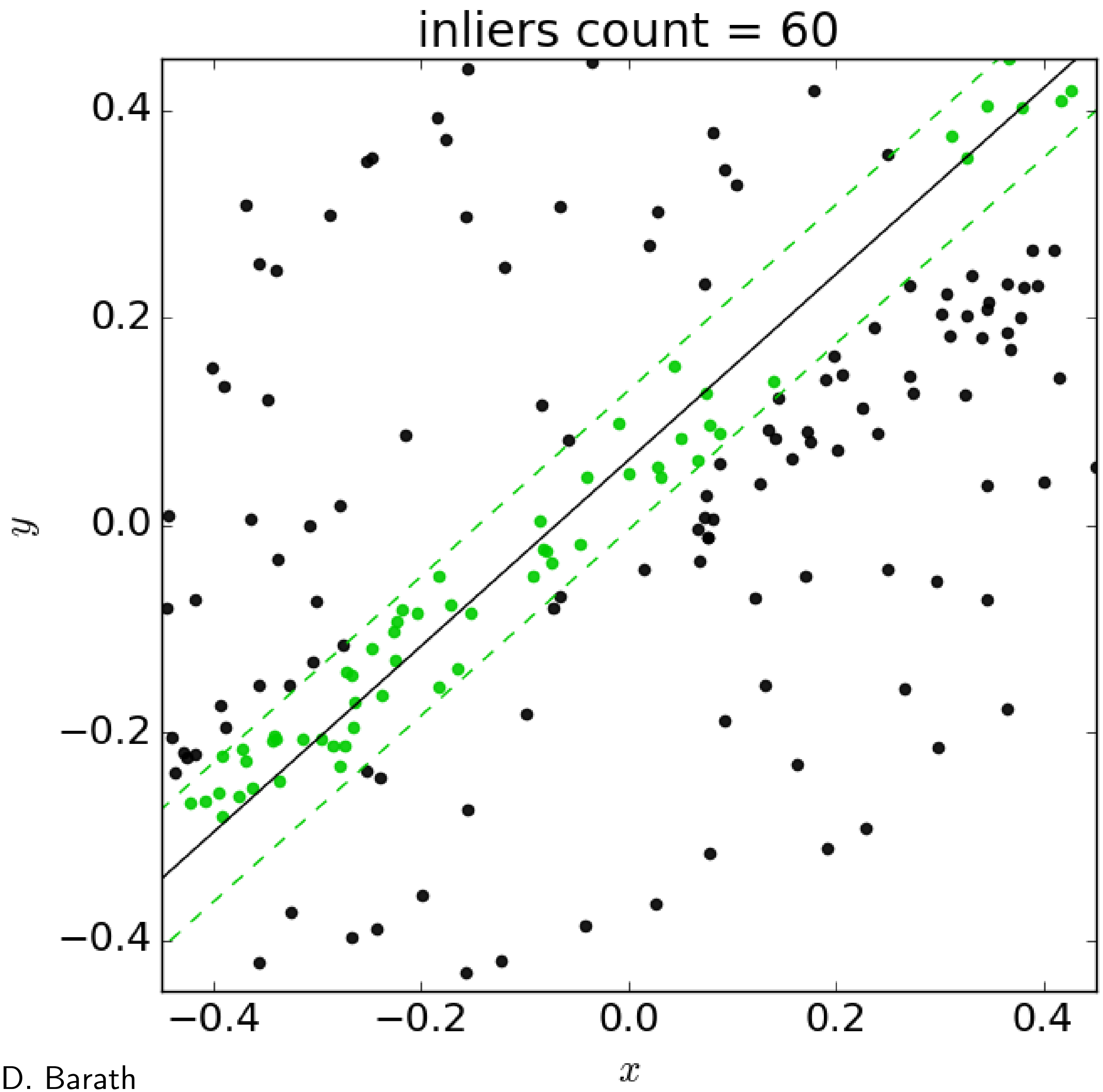
    Estimate parameters $\theta = e(S)$
    Evaluate $J(\theta) = \sum_{\mathbf{x} \in \mathcal{X}} f(\mathbf{x}, \theta)$ VERIFICATION
    If $J(\theta) < J^*$ then SO-FAR-THE-BEST
        $\theta^* \leftarrow \text{LocalOptimization}(\mathcal{X}_{in}, \theta),\ J^* \leftarrow J(\theta)$
    $iter \leftarrow iter + 1$
until $P(\text{better solution exists}) = f(|\mathcal{X}|, J^*, iter) < \eta$
gone

LO-RANSAC: Example



Init

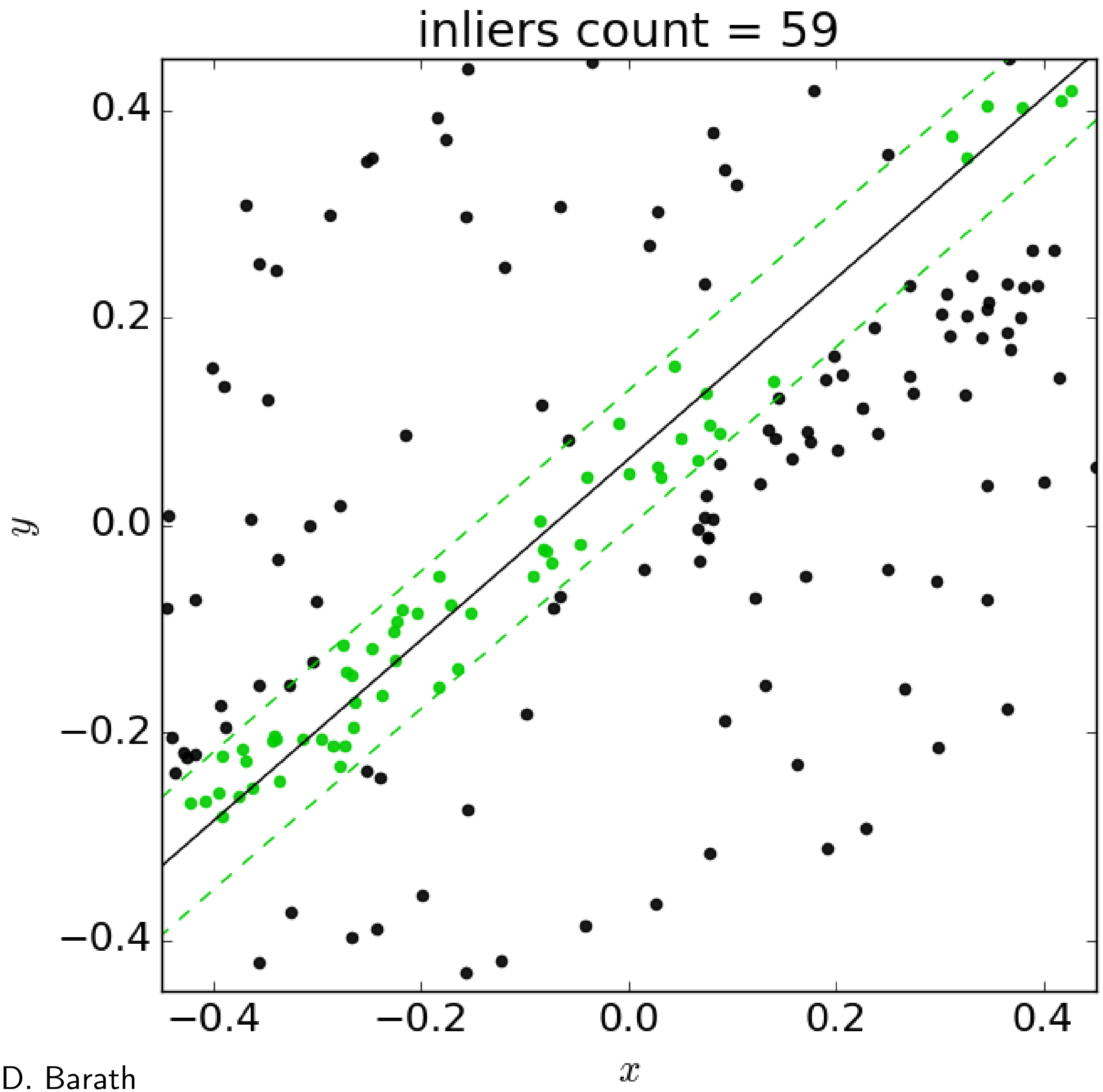
LO-RANSAC: Example



Init

Iteration 1

LO-RANSAC: Example

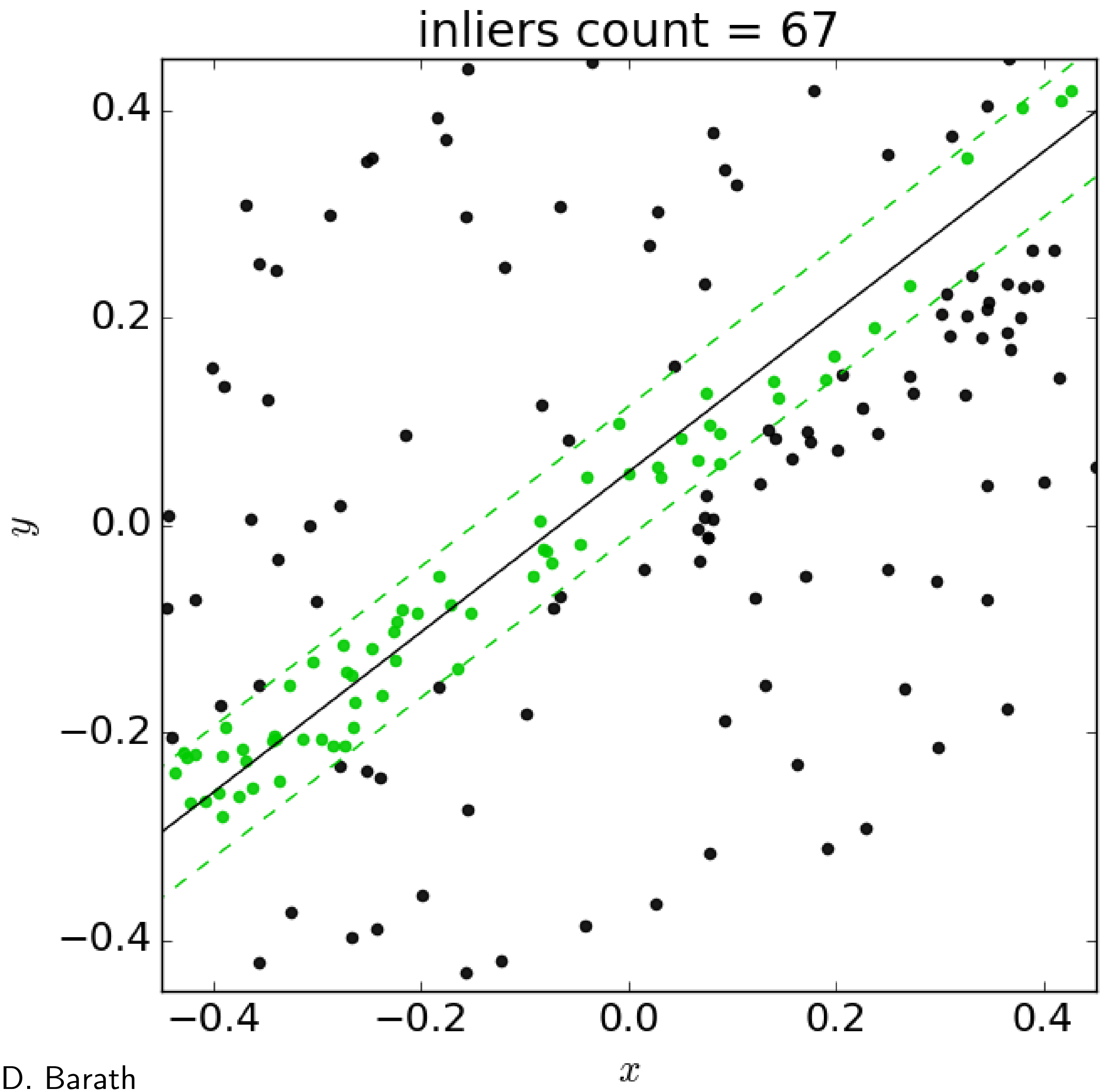


Init

Iteration 1

Iteration 2

LO-RANSAC: Example



Init

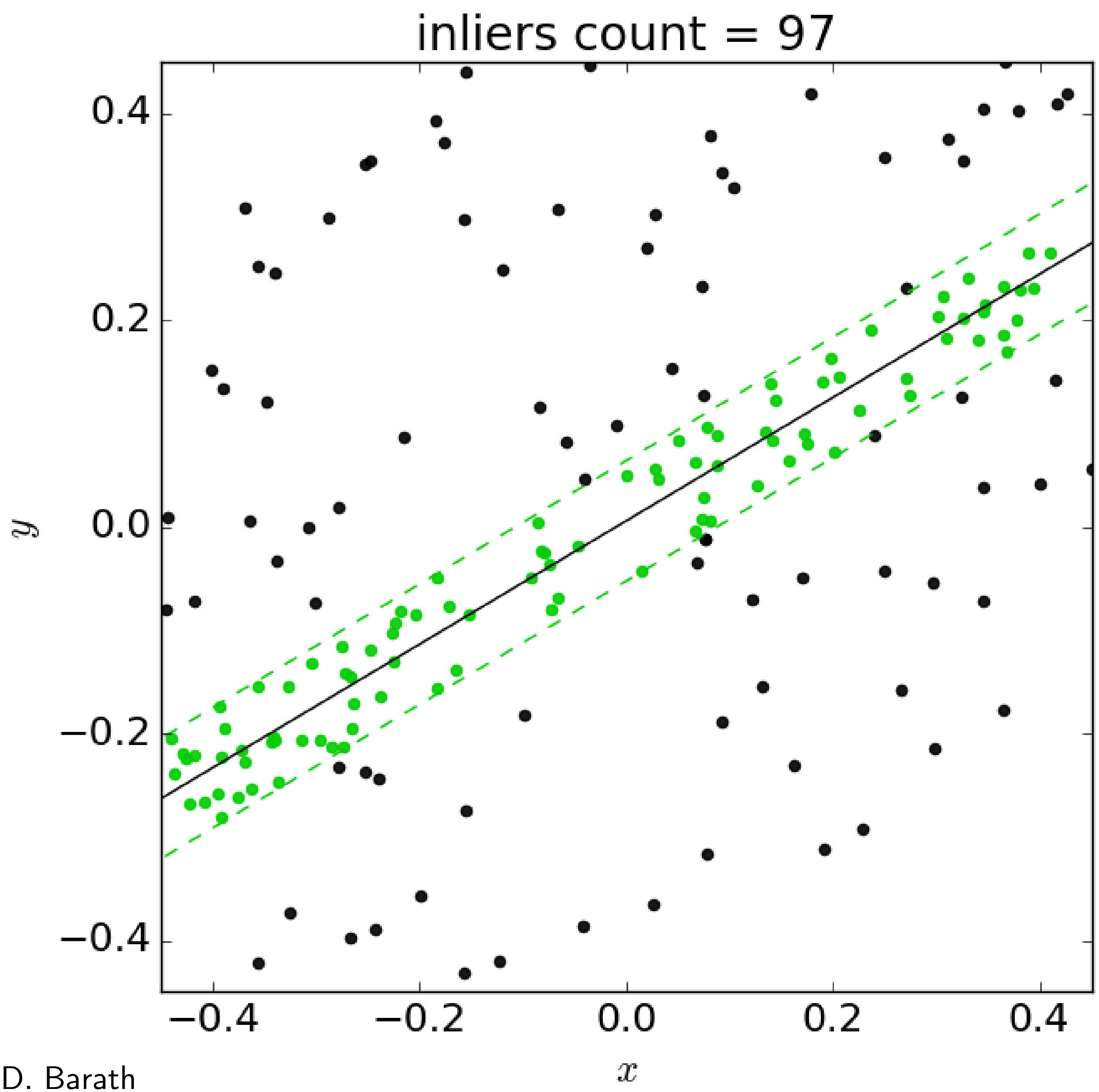
Iteration 1

Iteration 2

...

Iteration 7

LO-RANSAC: Example



Init

Iteration 1

Iteration 2

...

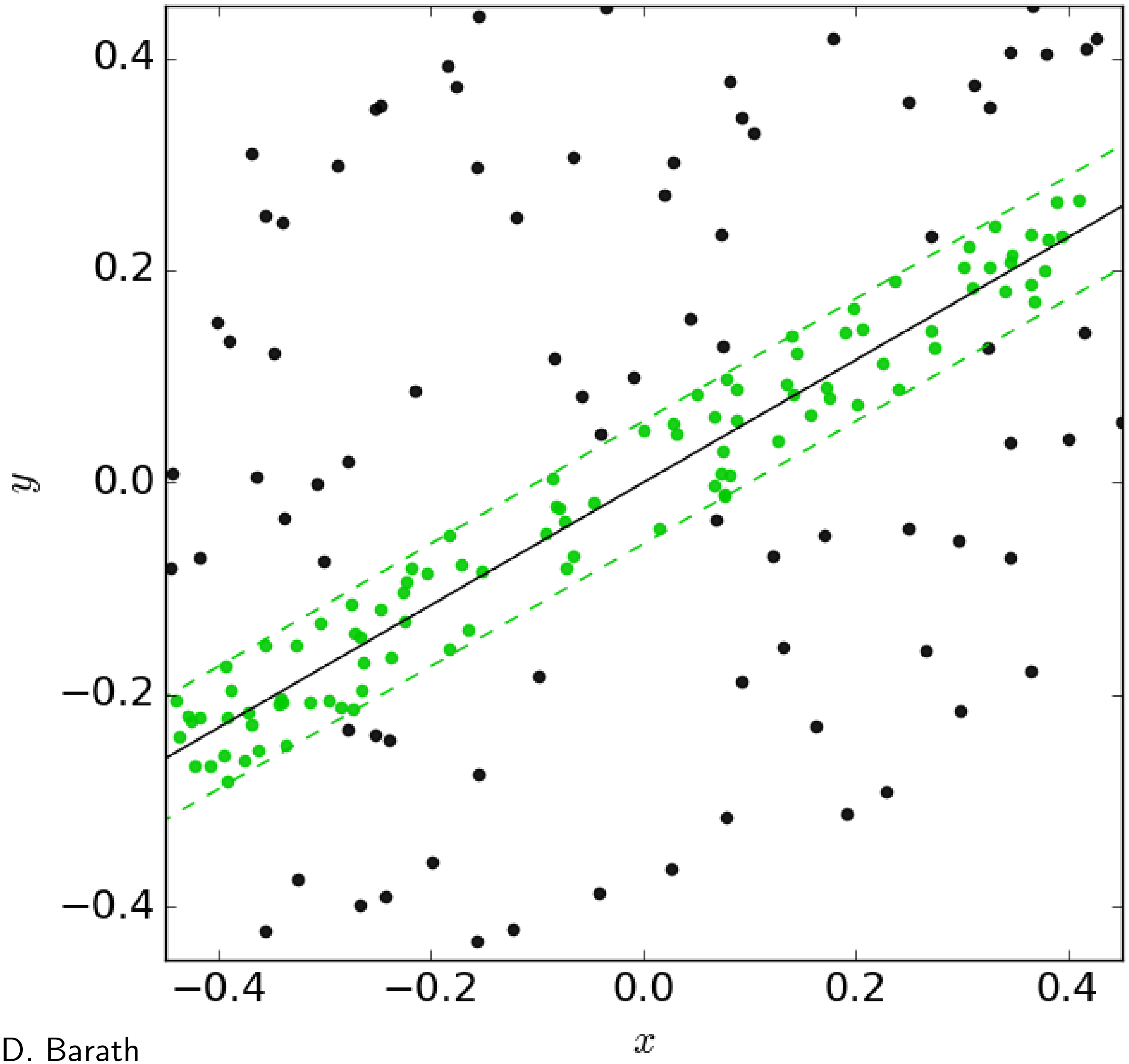
Iteration 7

...

Itration 15

LO-RANSAC: Example

Comparison with model (100 inliers):



LO-RANSAC: Problem Summary

It was observed experimentally that RANSAC takes several times longer than theoretically expected. This is due to the noise – not every all-inlier sample generates a good hypothesis.

By applying local optimization (LO) to the-best-so-far hypotheses:

(i) a near perfect agreement with theoretical performance

(ii) lower sensitivity to noise and poor conditioning.

The LO is shown to be executed so rarely, log(*iter*) times, that it has minimal impact on the execution time.

Graph-Cut RANSAC: Problem Introduction

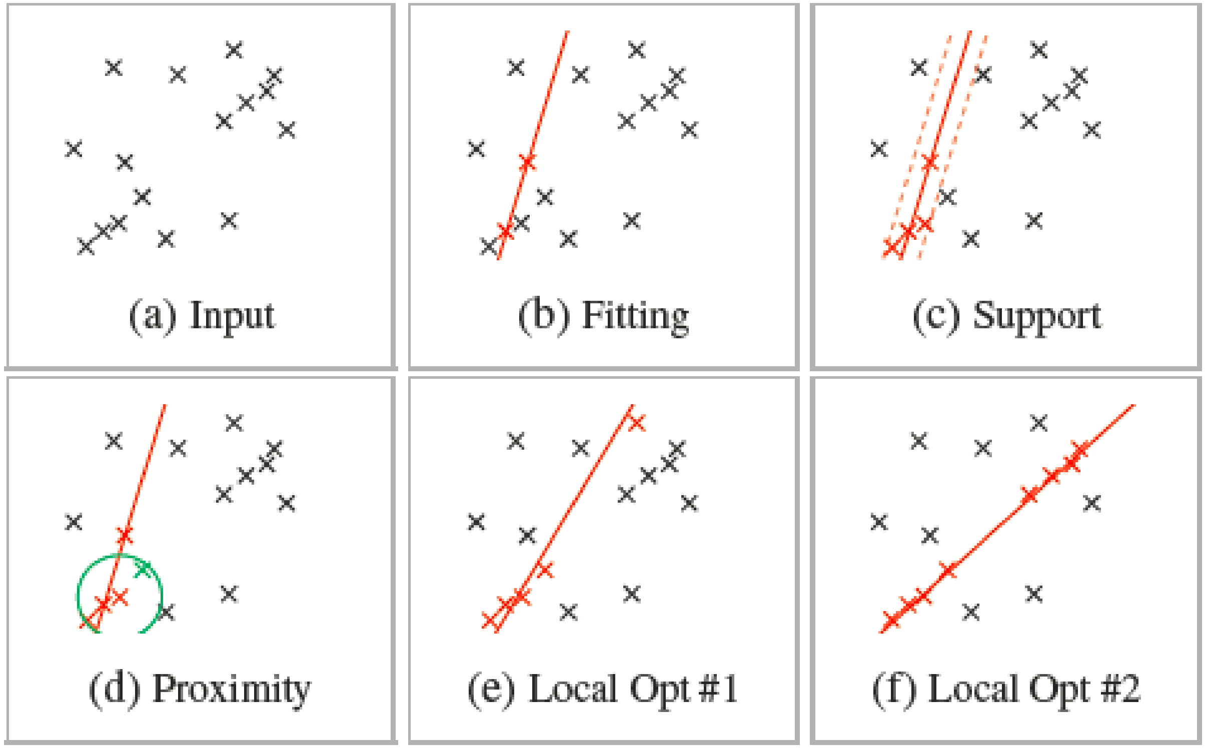


Figure 1: The proposed graph-cut based local optimization converging from a "not-all-inlier" sample, i.e. it is contaminated by an outlier, to the desired model. (a) The input data points, (b) RANSAC-like sampling and model fitting, (c) computation of model support, e.g. counting the inliers, (d) considering spatial proximity by graph-cut, (e-f) iterated local optimization using least-squares fitting and graph-cut.

Graph-Cut RANSAC

Input: $\mathcal{X} = \{\mathbf{x}_j\}_{j=1}^{N}$

$e(S) = \theta$ estimates model parameters θ, given sample $S \subseteq \mathcal{X}$

$$f(\mathbf{x}, \theta) = \begin{cases} 0, \text{if distance to model } \leq \text{threshold} \\ 1, \text{otherwise} \end{cases}$$

Output: θ^* parameter of the model minimizing the cost function

1. $iter = 0,\ J^* = \infty$
2. **repeat**
3. Select random $S \subseteq \mathcal{X}$ (sample size $m = |S|$)
4. Estimate parameter $\theta = e(S)$
5. Evaluate $J(\theta) = \sum_{x \in \mathcal{X}} f(\mathbf{x}, \theta)$
6. If $J(\theta) < J^*$ then
7. $\theta^*, \mathcal{L}^* \leftarrow \arg\min_{\theta, \mathcal{L}} \sum_{\mathbf{x} \in \mathcal{X}} f(\mathbf{x}, \theta) + \lambda \sum_{(\mathbf{x},\mathbf{y}) \in \mathcal{A}} [\![\mathcal{L}(\mathbf{x}) \neq \mathcal{L}(\mathbf{y})]\!]$
8. $J^* \leftarrow J(\theta^*)$
9. $iter \leftarrow iter + 1$
10. **until** P(better solution exists) $< \mu(|\mathcal{X}|, J^*, iter)$

Getting a θ^* model and an $\mathcal{L}^*$ labeling, i.e. assigns each point to the inlier or outleir classes, such that
(i) the assignment cost is minimal and
(ii) the spatial coherence holds.

D. Barath

GC-RANSAC: Terminating from a „bad” sample

Table 2: Percentage of "not-all-inlier" minimal samples leading to the correct solution during line (**L**) and fundamental matrix (**F**) fitting. For lines, the average over 1000 runs on three different outlier percentage (100%, 500%, 1000%) and noise levels 0.0 – 9.0 px, thus 15000 runs were performed. For fundamental matrices, the mean of 1000 runs on the `AdelaideRMF` dataset is shown.

	LO	LO$^+$	LO'	GC
L	6%	5%	4%	**15%**
F	29%	30%	24%	**32%**

GC-RANSAC: Problem Summary

It was observed that real world data in computer vision often is spatially coherent. Therefore, close points are more likely to belong to the same model.

By applying graph-cut local optimization (GC) to the-best-so-far hypotheses:

(i) the algorithm often terminates before the theoretically required iteration number due to finding the model from a non-all-inlier sample.

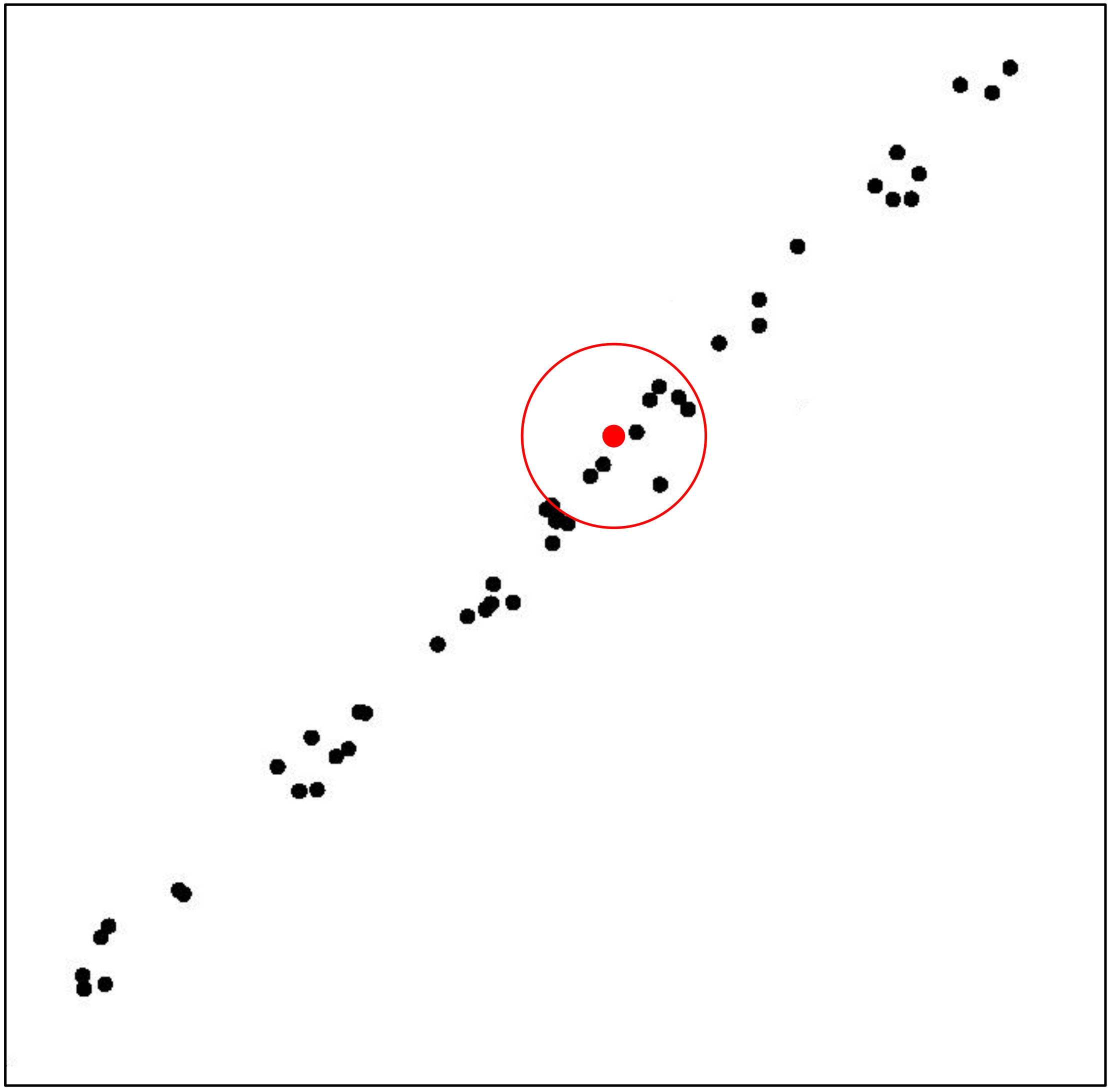
(ii) lower sensitivity to noise and poor conditioning.

The GC is shown to be executed so rarely, log(*iter*) times, that it has minimal impact on the execution time.

NAPSAC Sampling

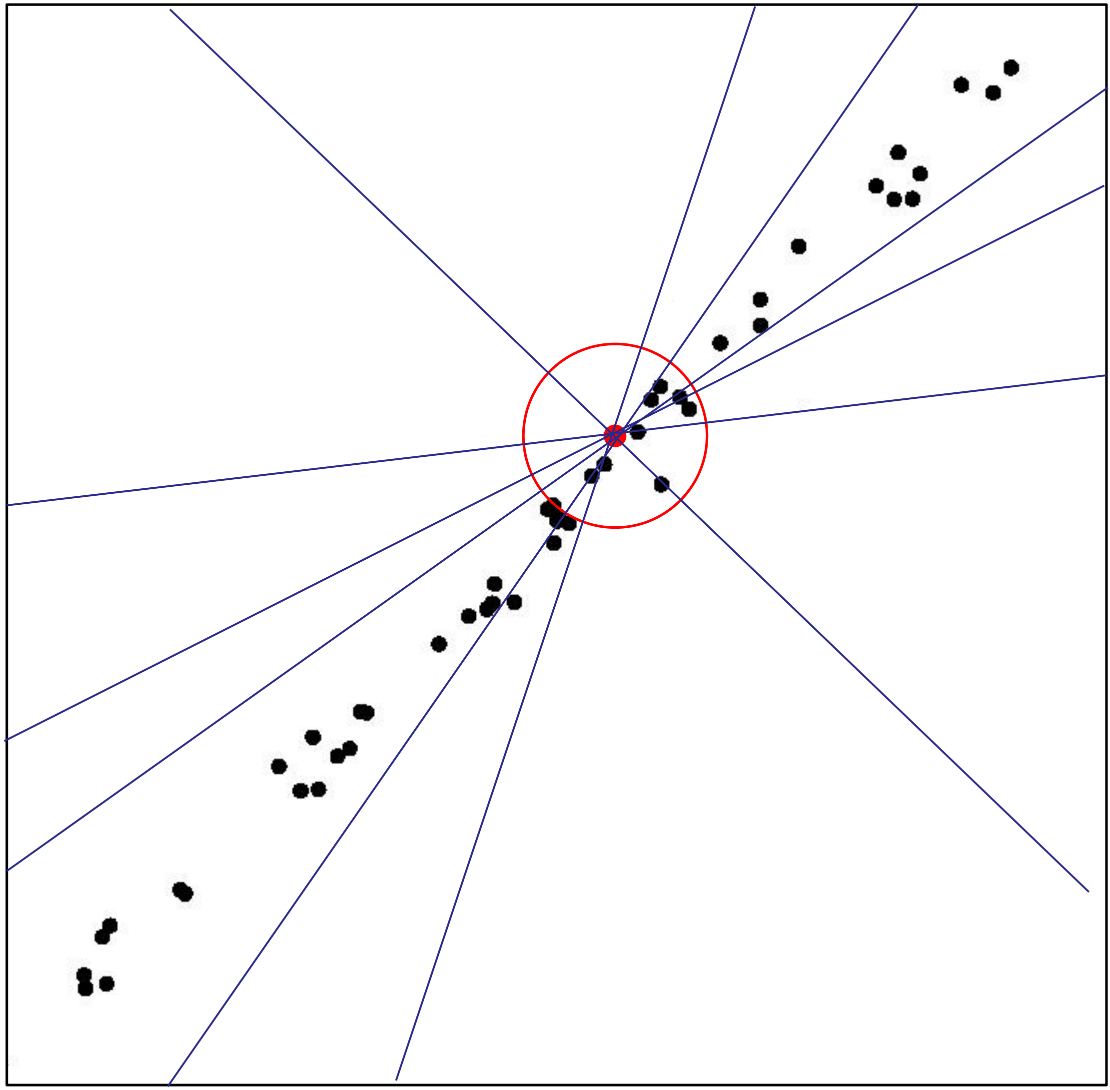
Considering spatial coherence was also introduced to the sampling part.

NAPSAC draws samples from the n-neighborhood of a point.



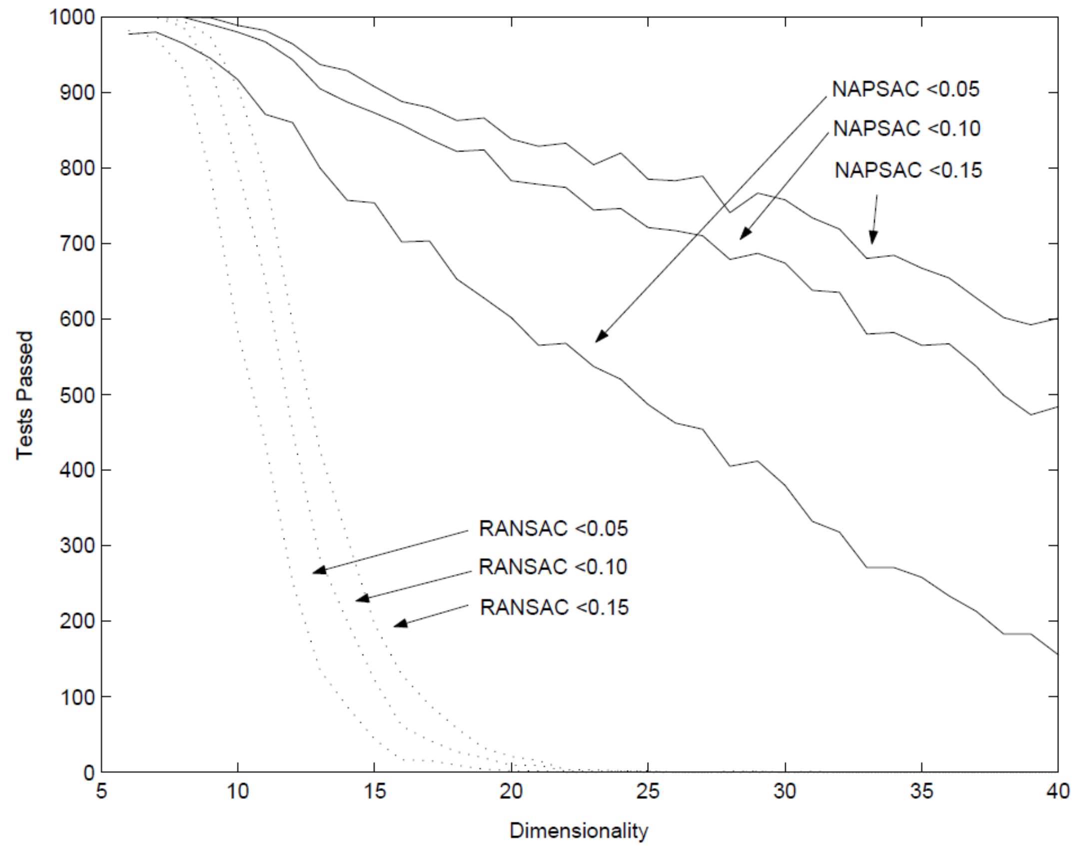
NAPSAC Issue

Even though this assumptions often holds, NAPSAC samples are „bad" due to the poor conditioning of the points.



NAPSAC Sampling in High-dimensions

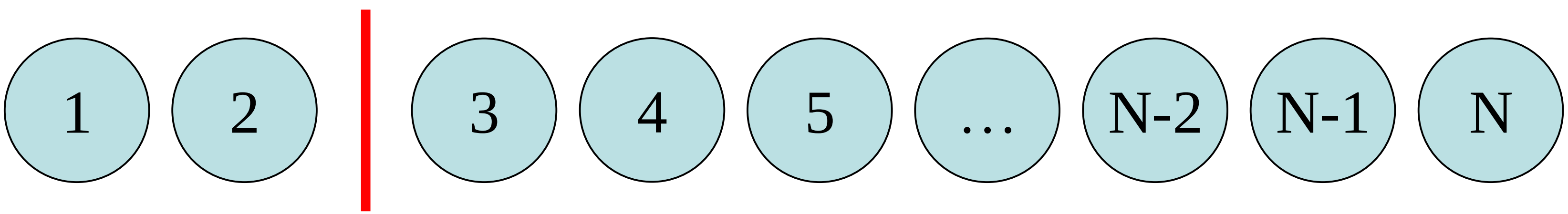
However, in case of model fitting in high-dimensional spaces, it appears to be good.



PROSAC – Progressive Sample Consensus

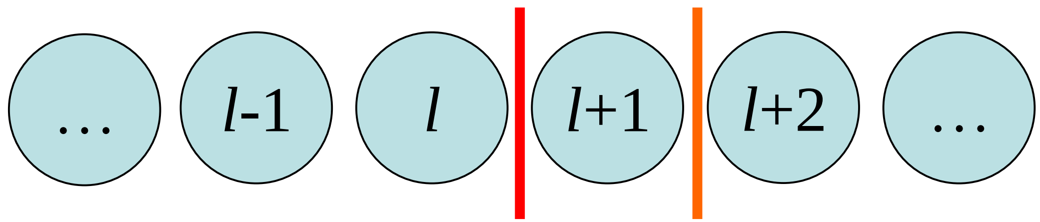
- Not all correspondences are created equally
- Some are better than others
- Sample from the best candidates first

Data points ordered by their „probabilities of being inlier”.



Sample from here

PROSAC – Progressive Sample Consensus



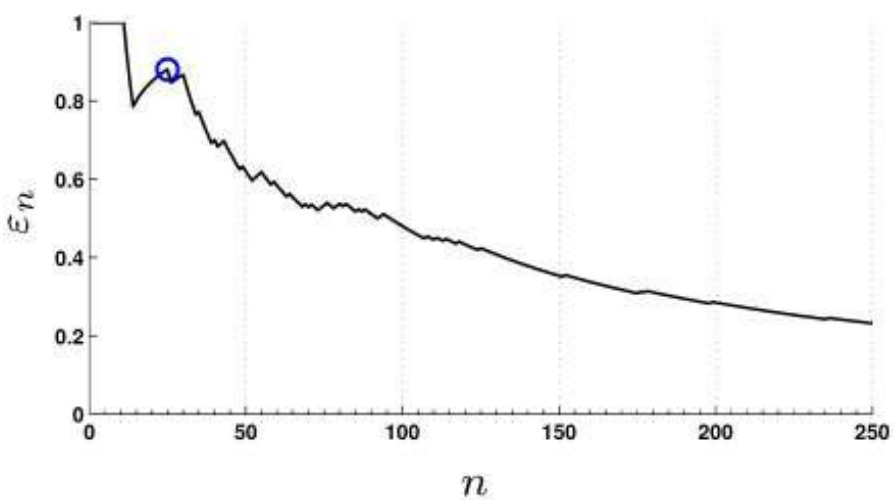
Draw T_l samples from (1 … l)
Draw T_{l+1} samples from (1 … l+1)

Samples from (1 … l+1) that are not from (1 … l) contain l+1

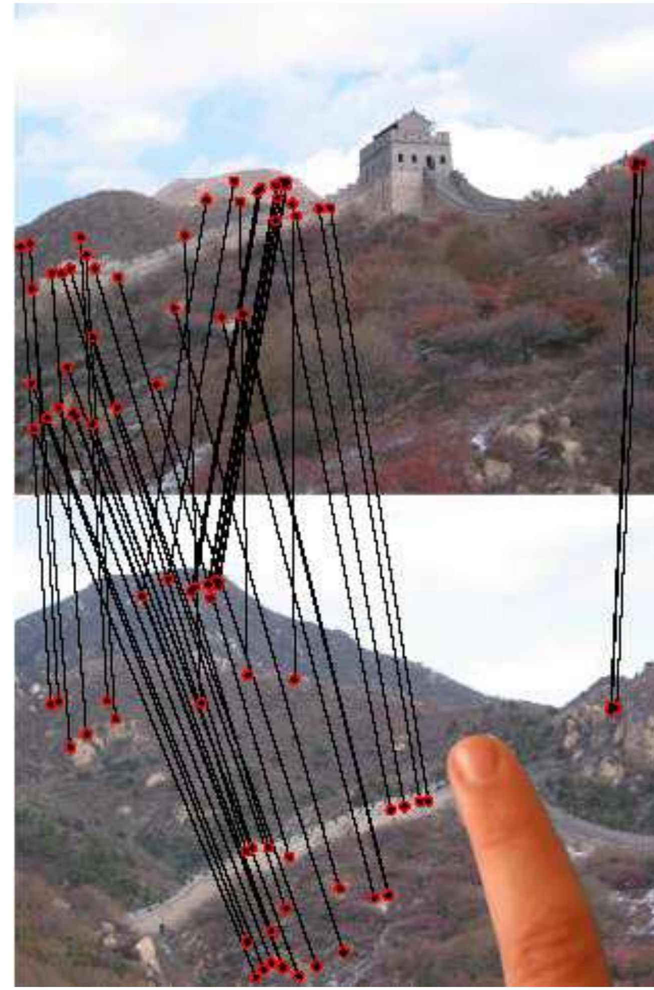
Draw T_{l+1} - T_l samples of size m-1 and add l+1

PROSAC – Problem Summary

Instead of drawing samples randomly, samples containing tentative correspondences are drawn preferably. The sampling procedure gradually progresses towards uniform sampling of standard RANSAC.



The fraction of inliers in top *n* correspondences.



	k	min k	max k	time [sec]
PROSAC	**9**	5	29	0.06
RANSAC	**106,534**	97,702	126,069	10.76

Optimal Randomized Strategy

- Model Verification is Sequential Decision Making

$$H_g\colon\ P(x_i = 1 | H_g) \geq \varepsilon$$
$$H_b\colon\ P(x_i = 1 | H_b) = \delta$$
$x_i = 1 \quad x_i$ is consistent with the model

Where:

- H_g - hypothesis of a 'good' model ($\approx$ from an uncontaminated sample)
- H_b - hypothesis of a 'bad' model, ($\approx$ from a contaminated sample)
- δ - probability of a data point being consistent with an arbitrary model

- Optimal (the fastest) test that ensures with probability α that H_g is not incorrectly rejected is the *Sequential probability ratio test* (SPRT) [Wald47]

SPRT (Simplified from Wald)

- Compute likelihood ratio $$\lambda_i = \prod_{j=1}^{i} \frac{P(x_j|H_b)}{P(x_j|H_g)}$$
- If $\lambda_i > A$ reject the model
- If $i = N$ accept the model as 'good'

Two important properties of SPRT:

1. Probability of rejecting a good model $\alpha < \frac{1}{A}$
2. Average number of verifications $V = C \log A$

$$C \approx \left(P(0|H_b) \log \frac{P(0|H_b)}{P(0|H_g)} + P(1|H_b) \log \frac{P(1|H_b)}{P(1|H_g)} \right)^{-1}$$

WaldSAC

Repeat $\frac{k}{1-\frac{1}{A}}$ times:

1. Hypothesis generation
2. Model Verification

   use SPRT

Time:

t_M

$\overline{m}_s \cdot C \log A$

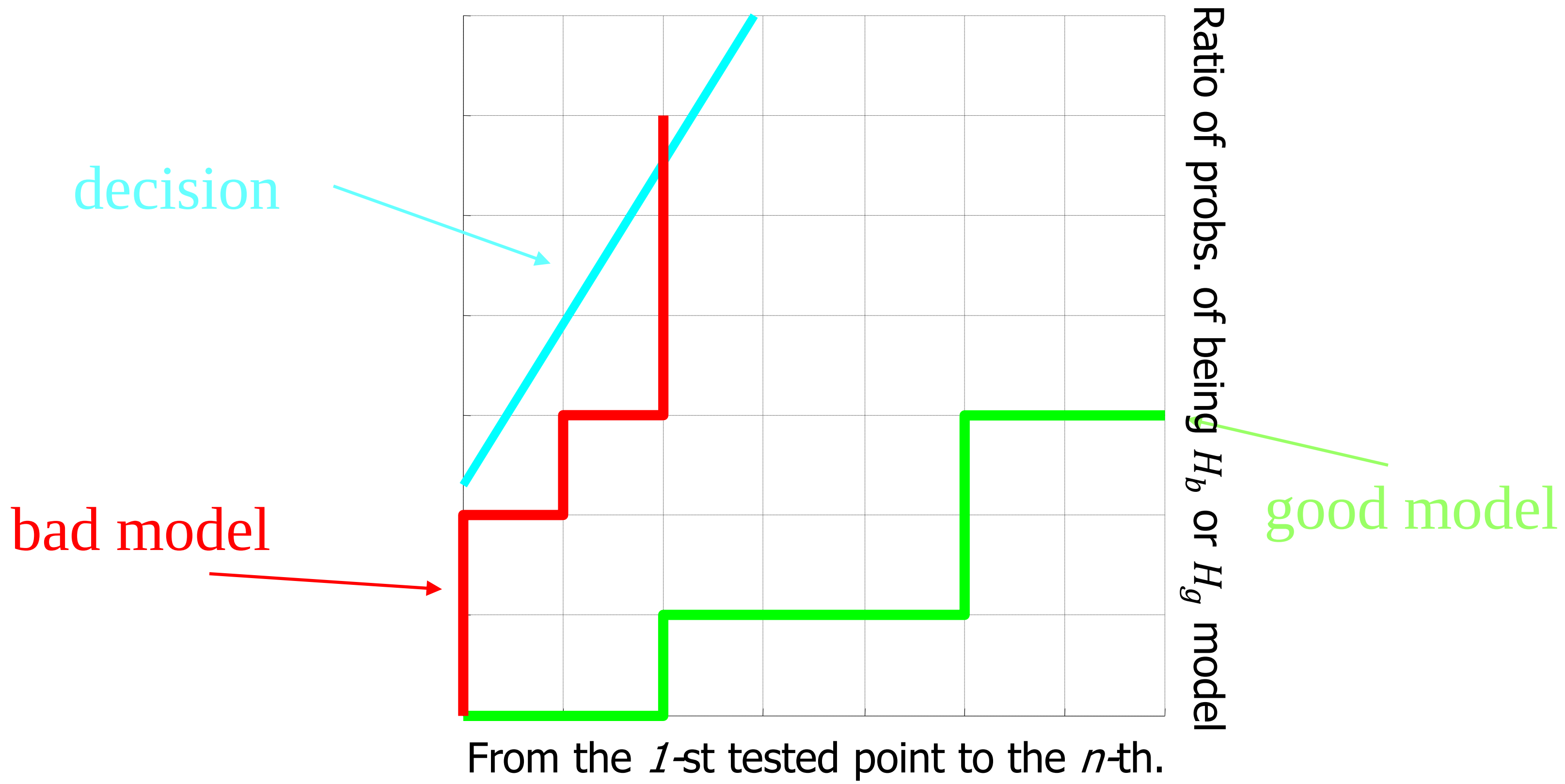
$C \approx ((1-\delta)\log\frac{1-\delta}{1-\varepsilon} + \delta \log\frac{\delta}{\varepsilon})^{-1}$

$$t(A) = \frac{k}{(1-1/A)}(t_M + \overline{m}_S C \log A)$$

In sequential statistical decision problem decision errors are traded off for time. These are two incomparable quantities, hence the constrained optimization.

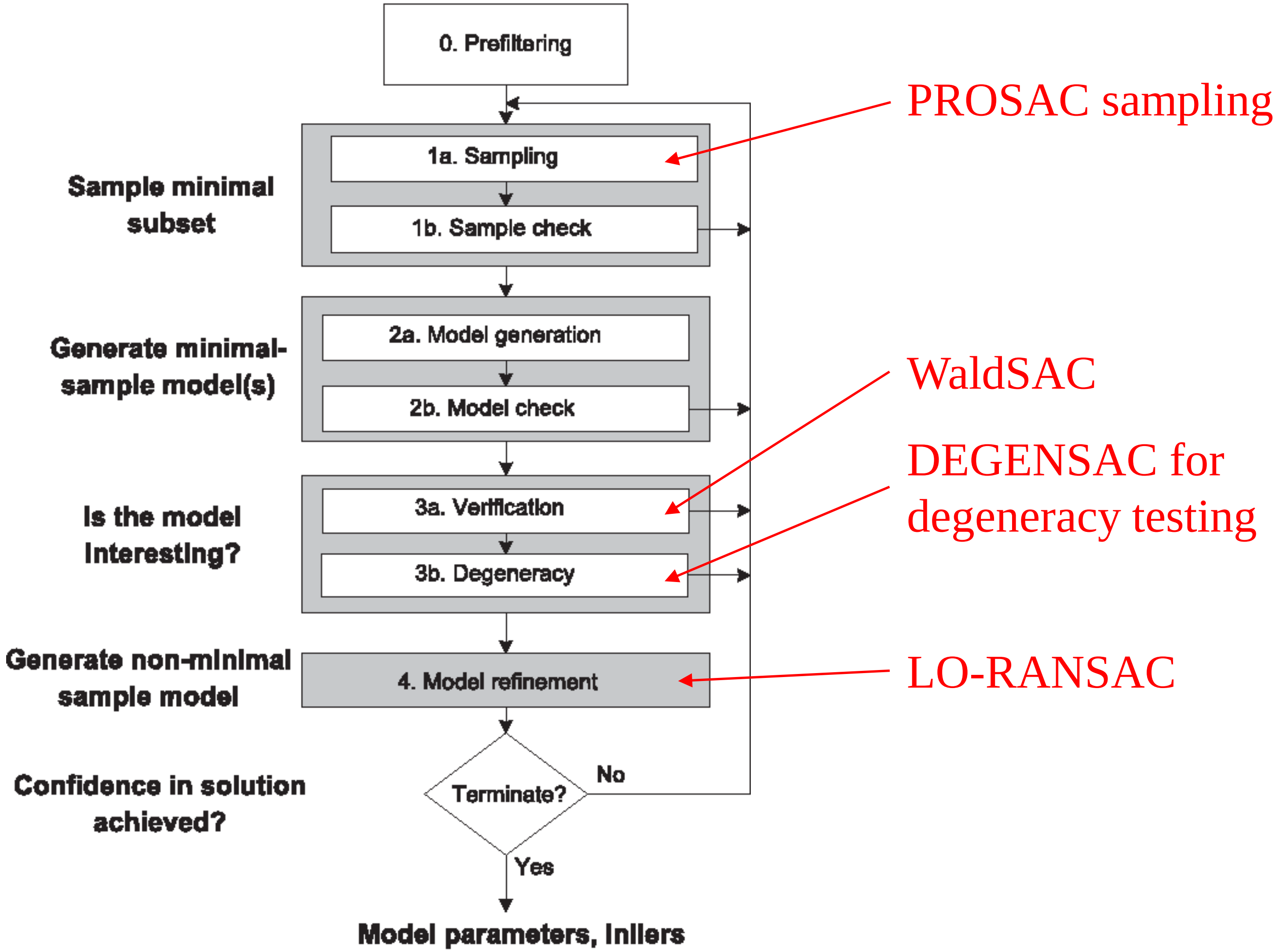
In WaldSAC, decision errors cost time (more samples) and there is a single minimised quantity, time $t(A)$, a function of a single parameter A.

SPRT



decision
bad model
good model
Ratio of probs. of being H_b or H_g model
From the *1*-st tested point to the *n*-th.

USAC: Universal framework for sample consensus



Conclusions

- RANSAC is in principle simple, but (as is often the case), a state-of-the-art implementation is not.
- Even though working accurately, RANSAC still has a number of issues to be solved.

Thank You!

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