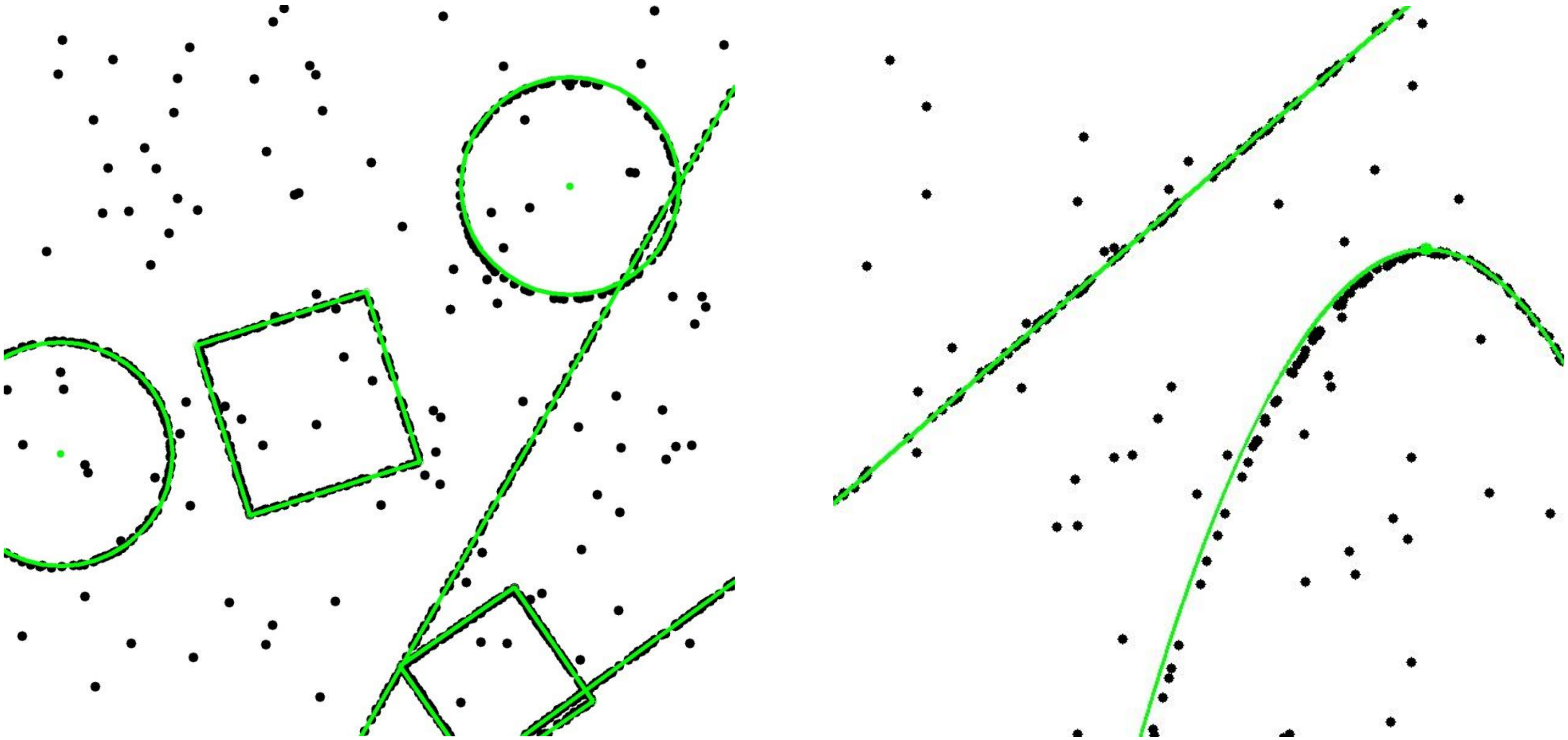
Robust multi-class multi-model fitting

Lecturer: Dániel Baráth

Multi-Class Multi-Instance Fitting – example 1

The problem of interpreting the input as mixture of noisy observations originating from **multiple** model **instances** of **different classes**.



Example problem: simultaneous fitting of circles, lines, squares, and parabolas.

Multi-Class Multi-Instance Fitting – example 2

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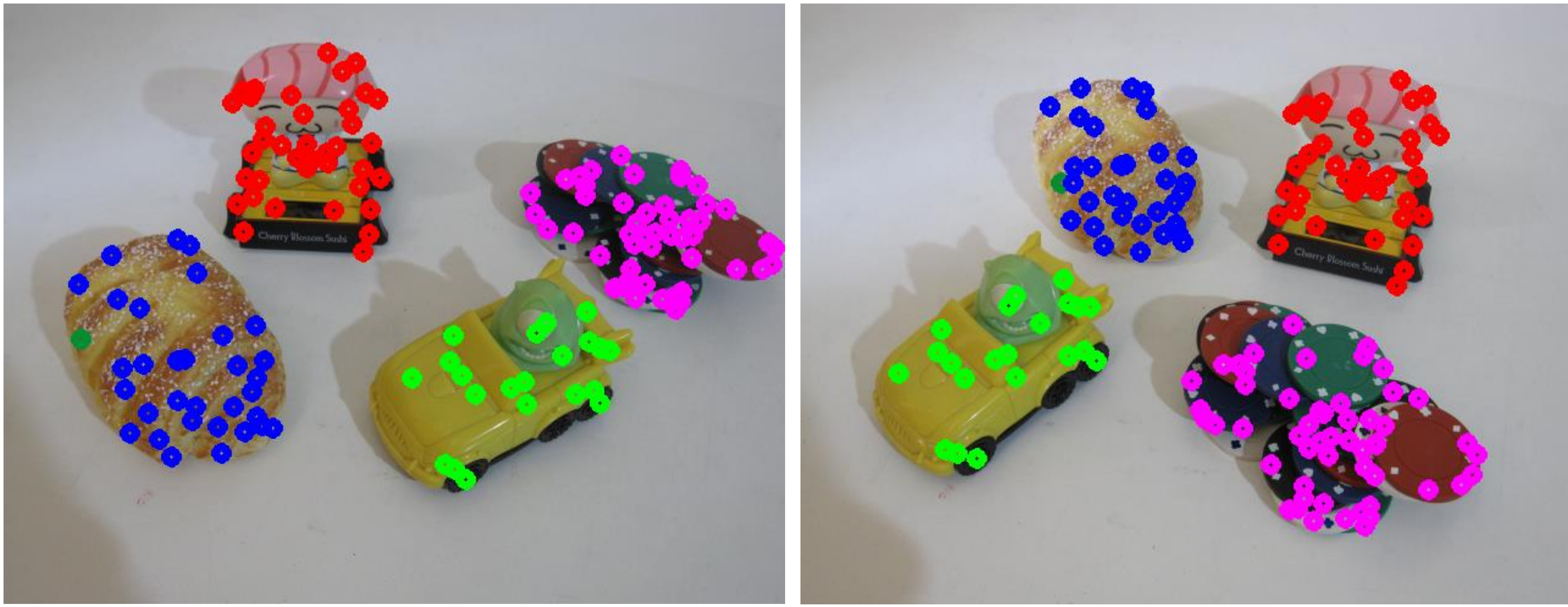
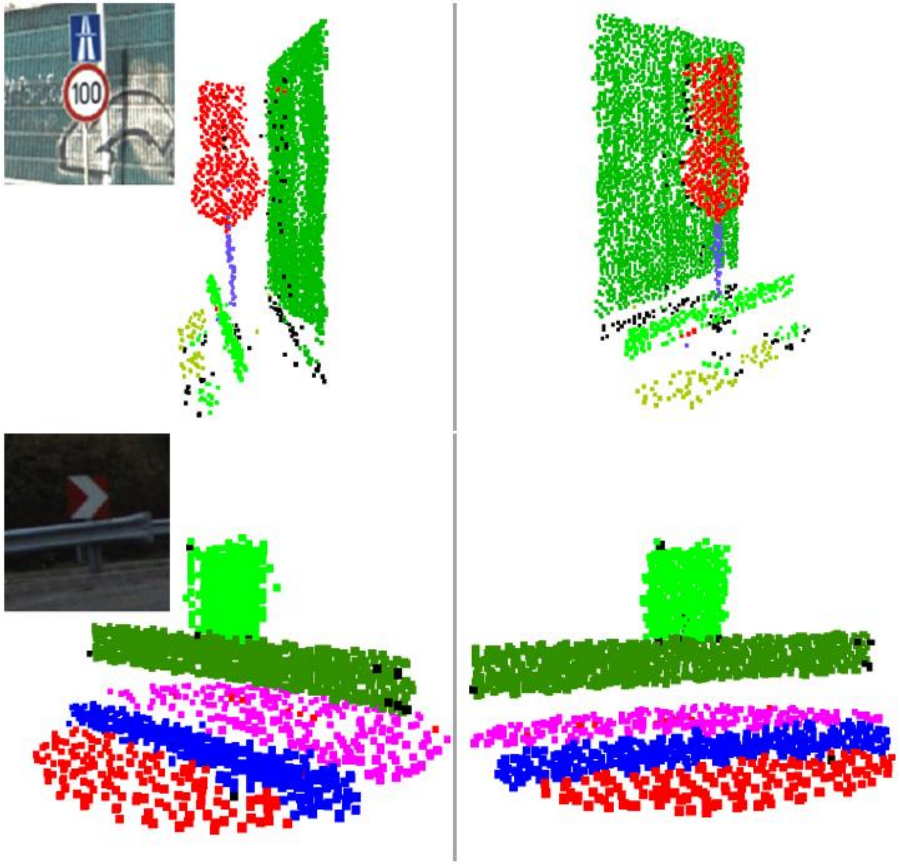


Example problem: simultaneous fitting of two-view rigid motions.

Multi-Class Multi-Instance Fitting – example 3

The problem of interpreting the input as mixture of noisy observations originating from **multiple** model **instances** of **different classes**.



Example problem: simultaneous fitting of planes and cylinders.

Multi-Class Multi-Instance Fitting – example 4

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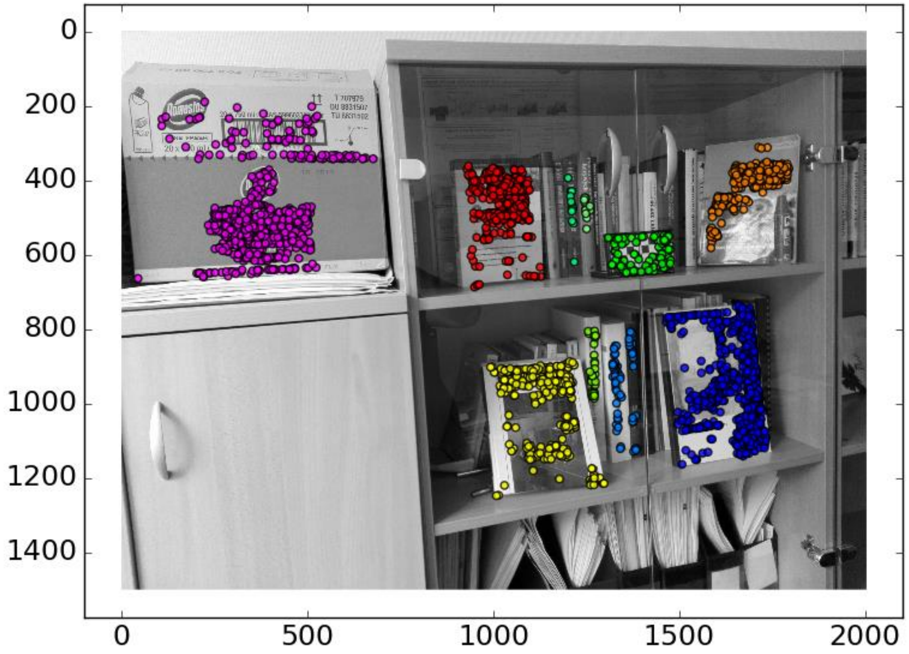
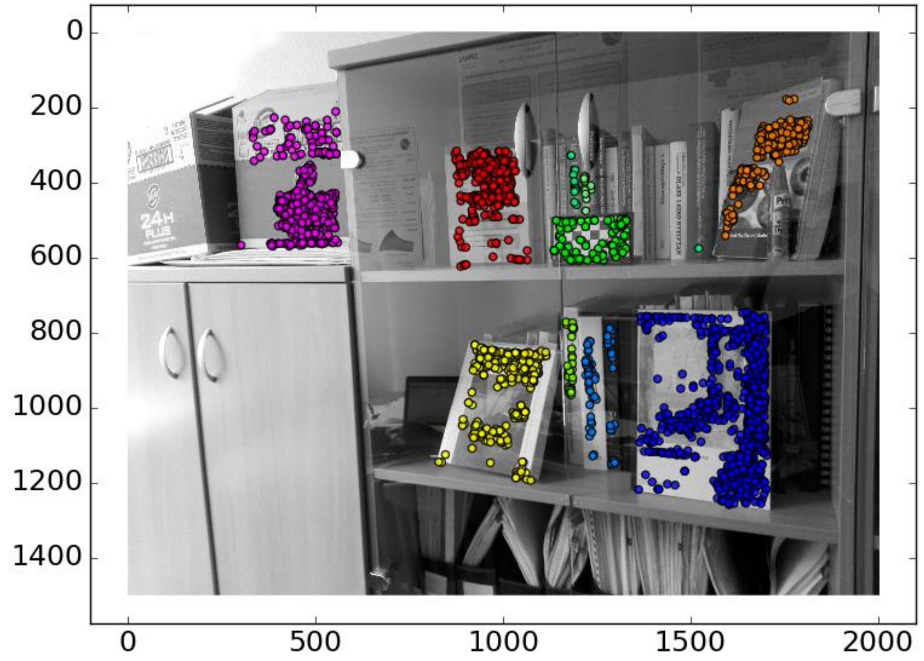




Example problem: simultaneous fitting of homographies.

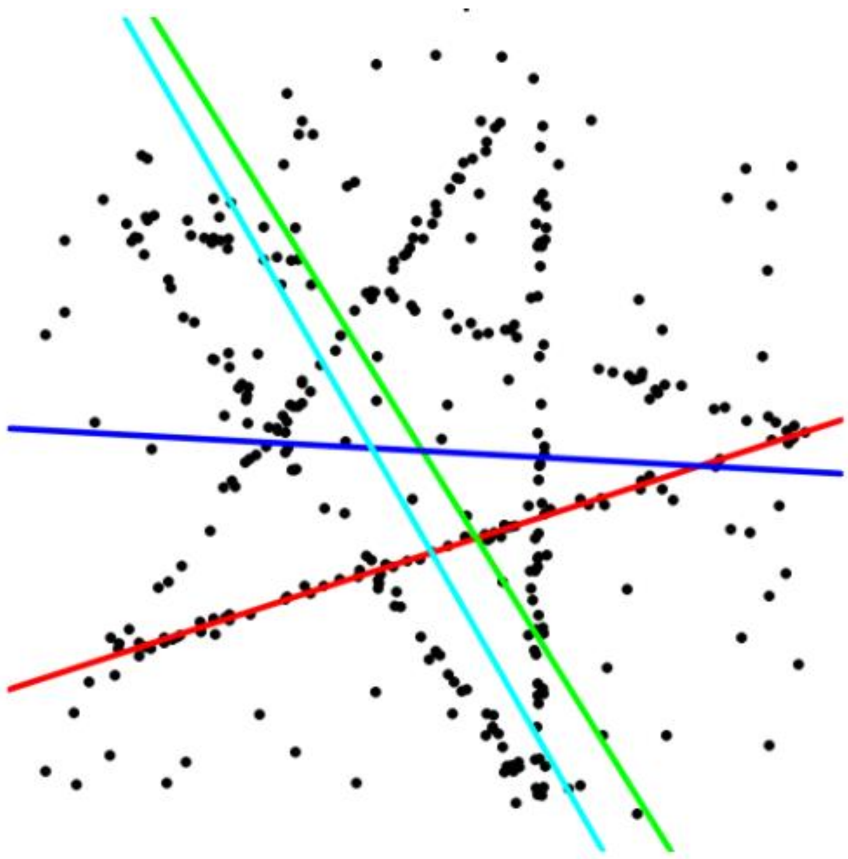
The problem of interpreting the input as mixture of noisy observations originating from **multiple** model **instances** of **different classes**.

Example problem: simultaneous fitting of rigid motions.

(Play the video!)

Traditional approaches

Traditional approaches fail in the presence of multiple models and outliers; objective functions fail to provide sensible goals.



Ordinary Least Squares (O.L.S.), Total Least Squares (T.L.S.) (viaPCA), Least Median of Squares (LMedS), Random Sample Consensus (RANSAC)

Source: https://web.eecs.umich.edu/~fouhey/earlier/thesis/dfouhey_thesisPresentation.pdf

Two definitions for model fitting

- **Minimal sample set** (MSS): a set of data points with the minimum cardinality to fit a model. For instance, $|\mathrm{MSS}| = 2$ for fitting a 2D line.
- **Consensus set** of model θ: set of data points that fit, i.e. their residuals are lower than a threshold, to model θ.

$$CS(\theta, P, \epsilon) = \{p \in P \mid \delta(\theta, p) < \epsilon\},$$

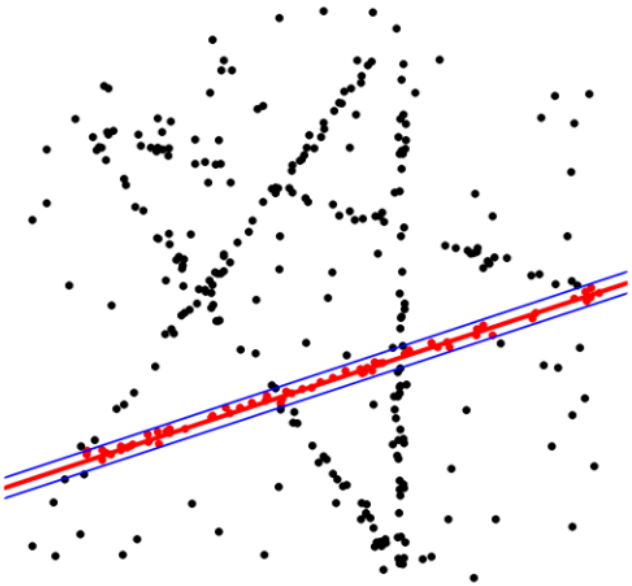
where ϵ is a threshold and δ is a function measuring the distance of model θ and point p.

Source: https://web.eecs.umich.edu/~fouhey/earlier/thesis/dfouhey_thesisPresentation.pdf

Intuitively: the problem

Intuitively:

- Optimize $|CS|$ by
- picking minimum sample to generate models.



Note: using minimal sample models is required due to the combinatorics of the problem. Think about RANSAC's iteration number.

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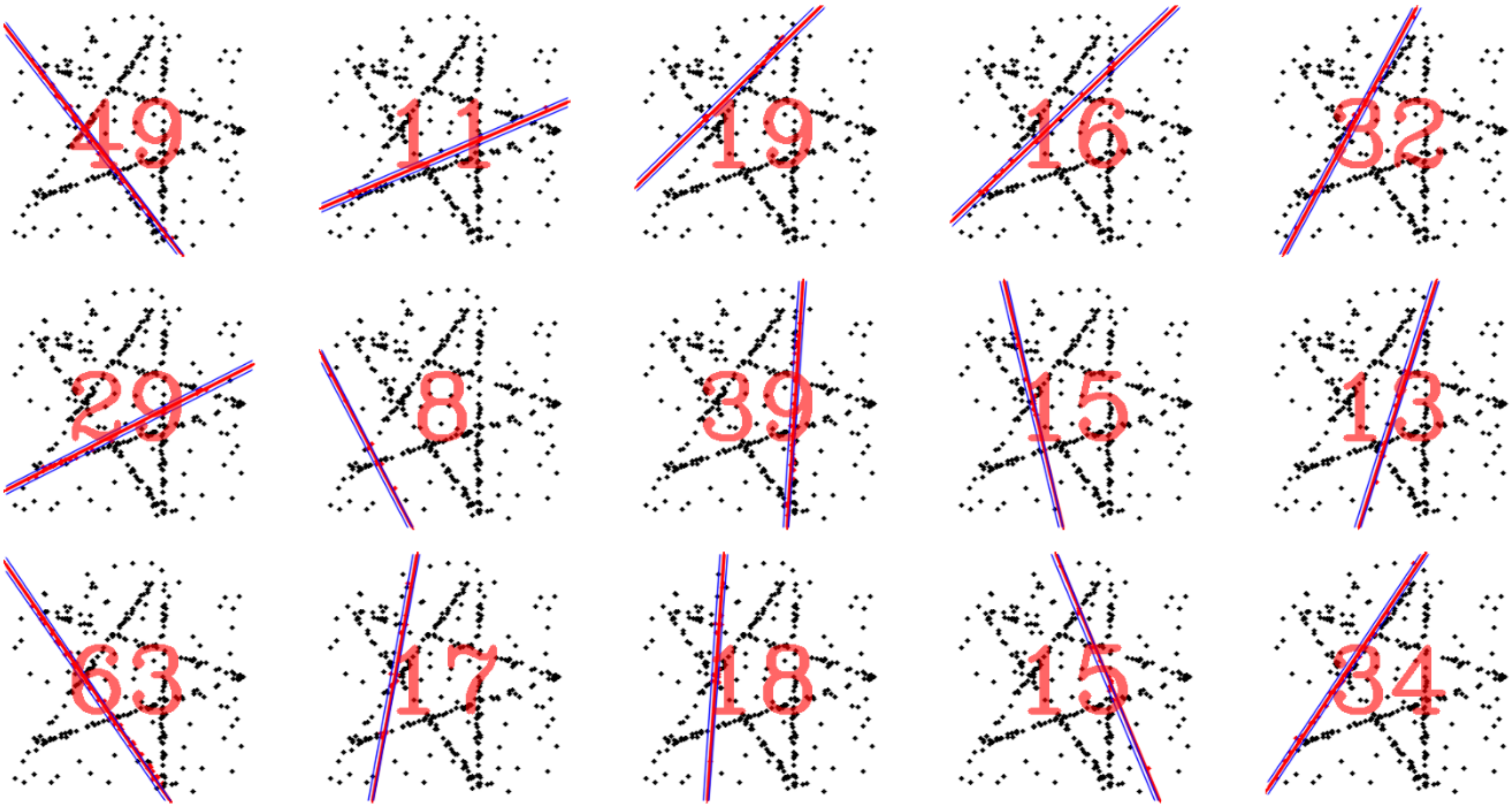
Some minimal sample models



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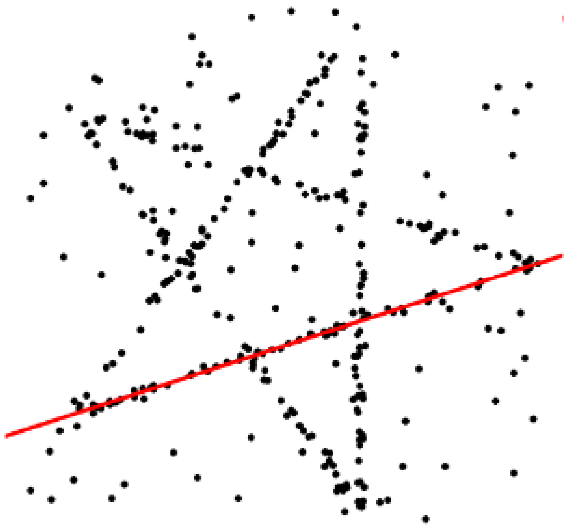
Multi-model case: sequential RANSAC



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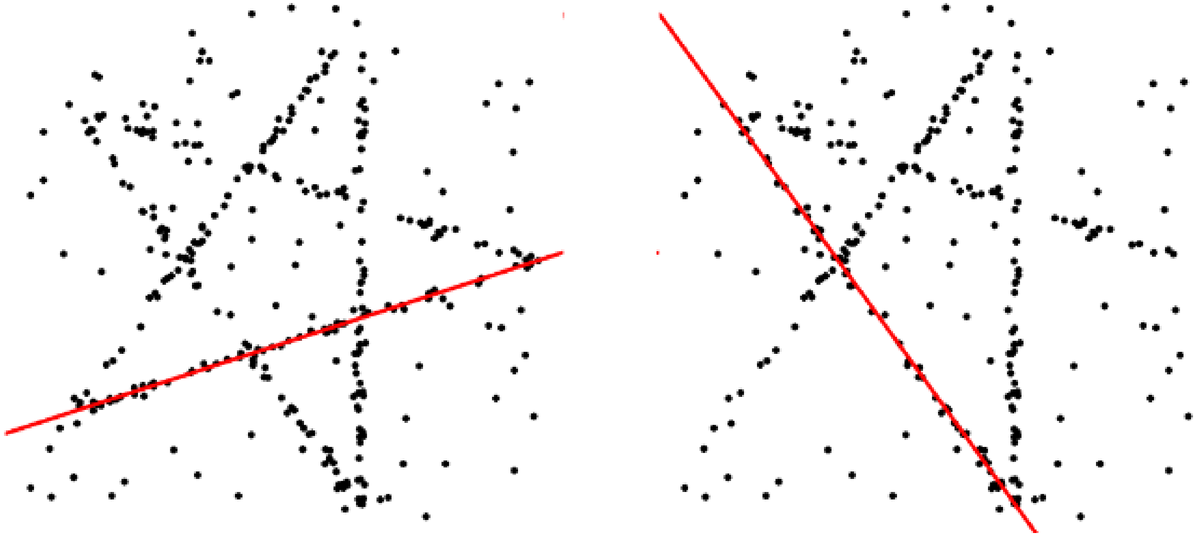

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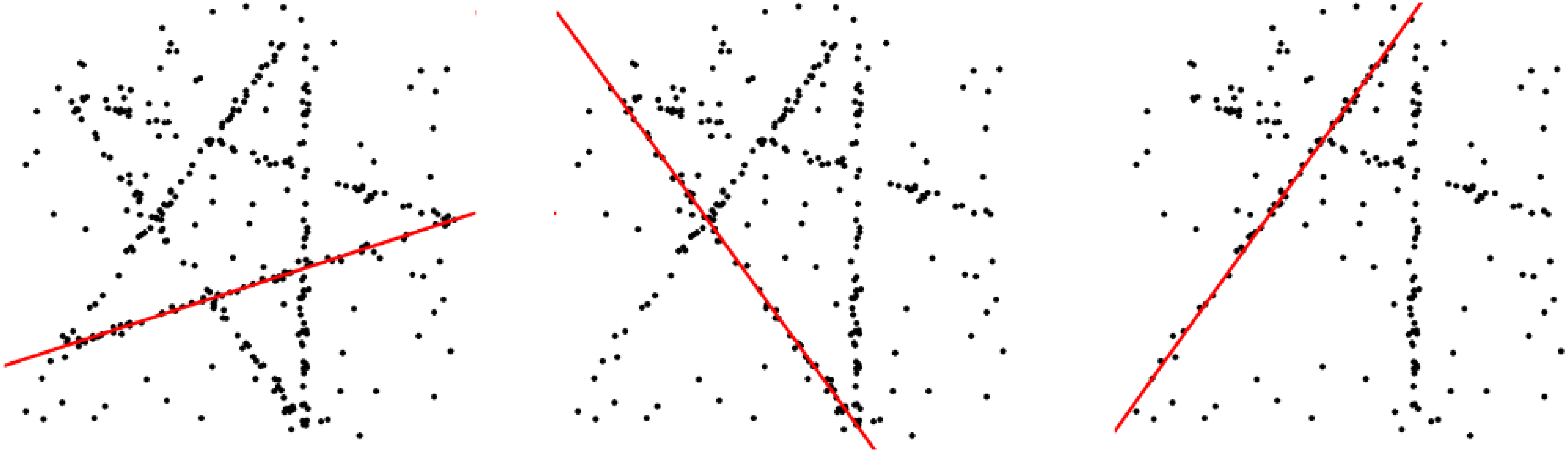

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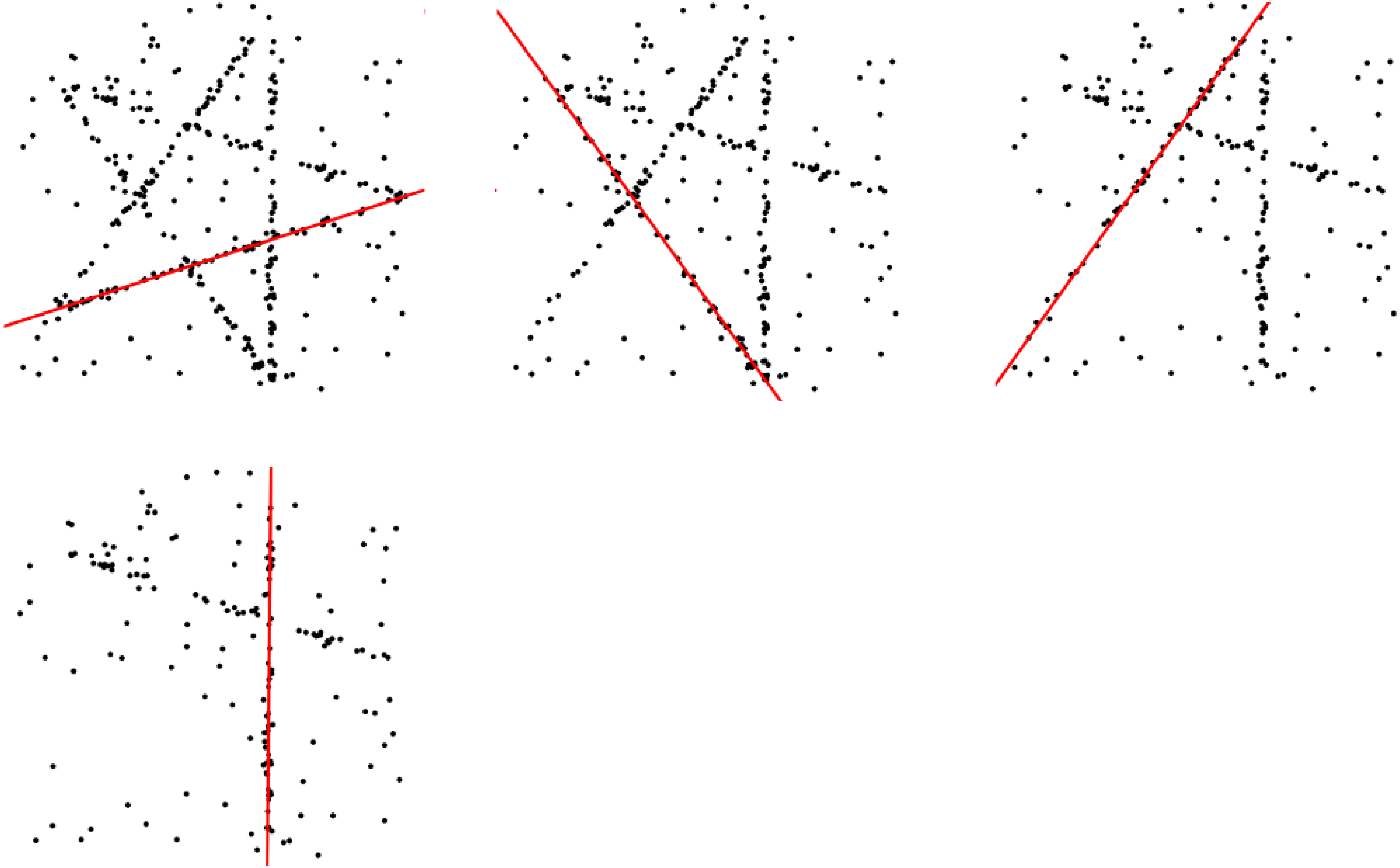

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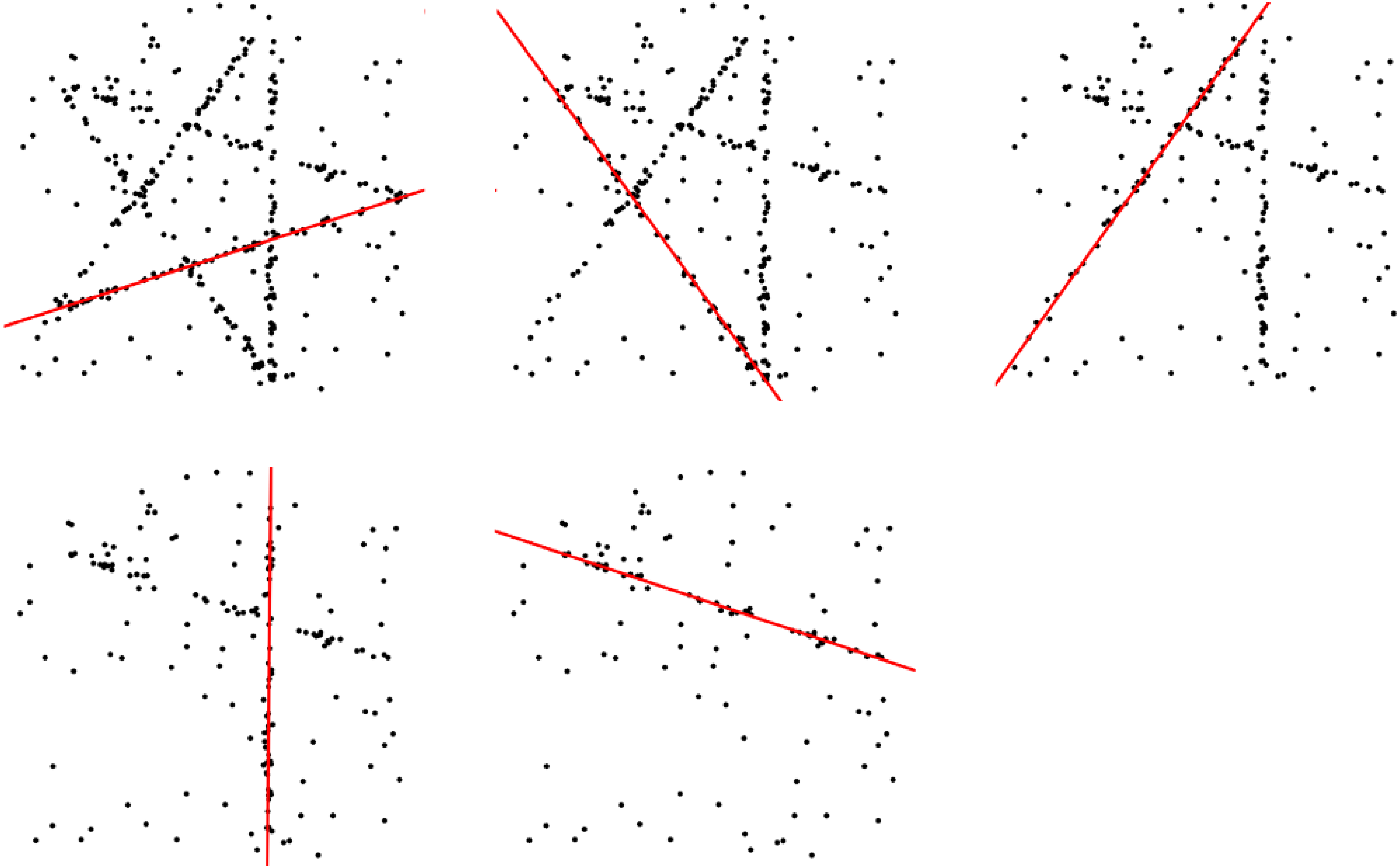

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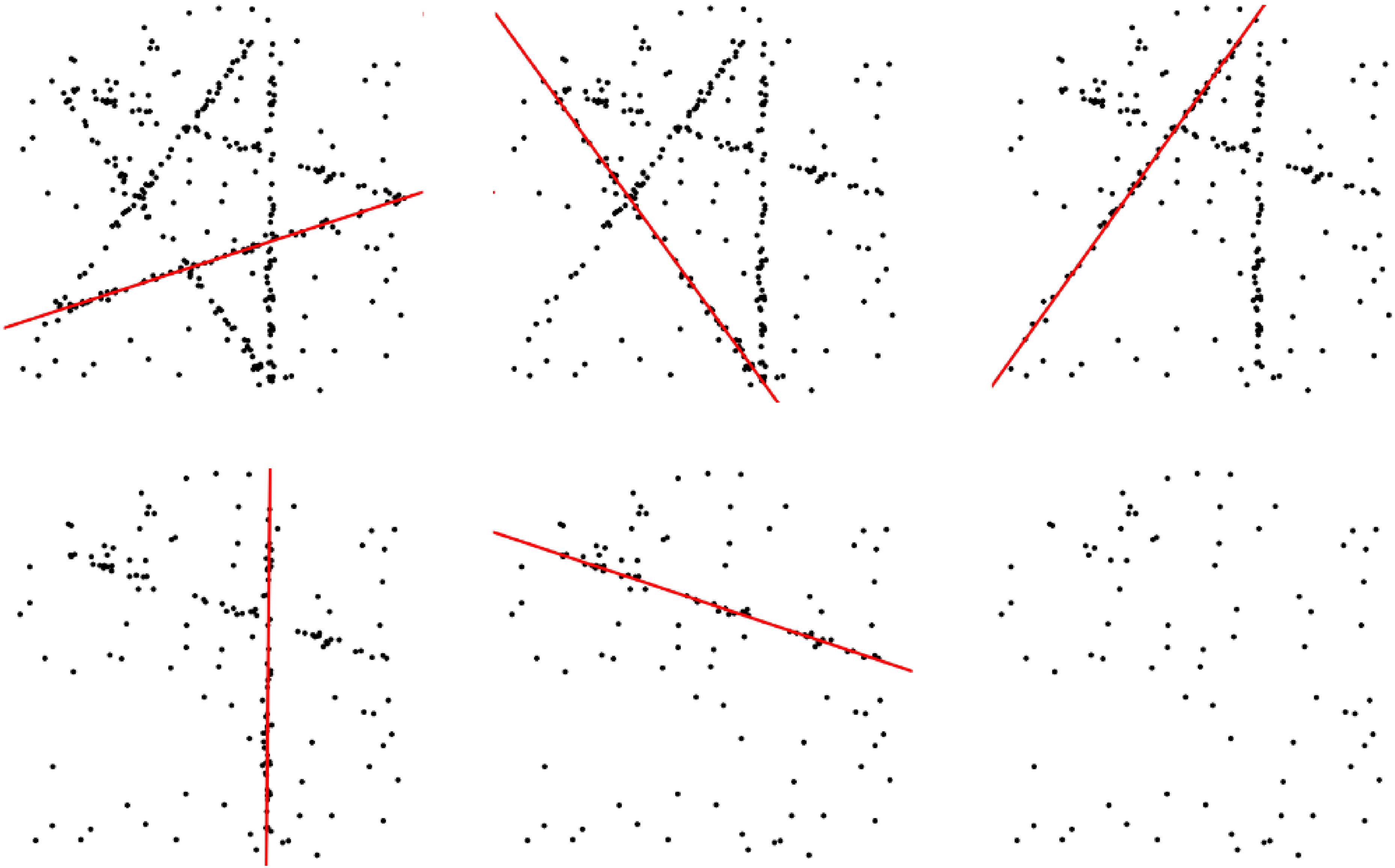

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Sequential RANSAC: pros and cons

Pros:

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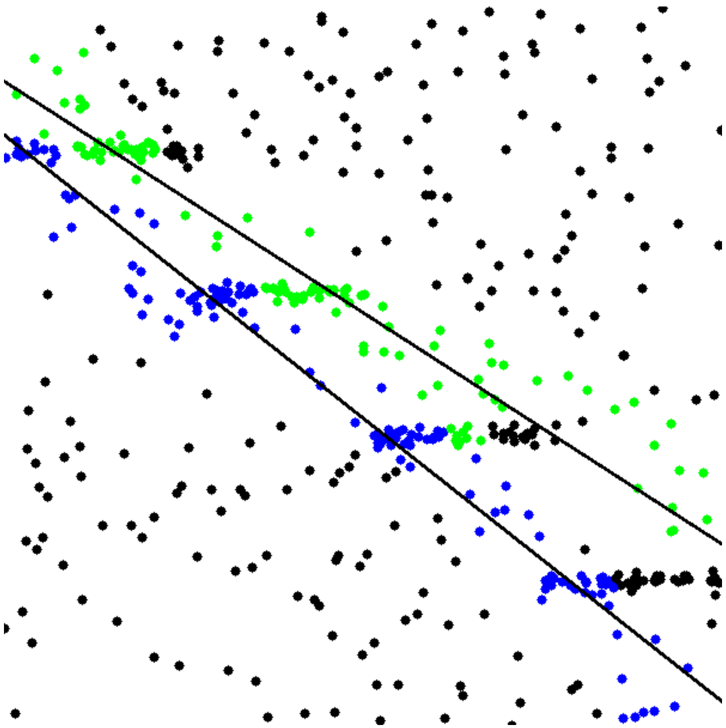


Multi-RANSAC

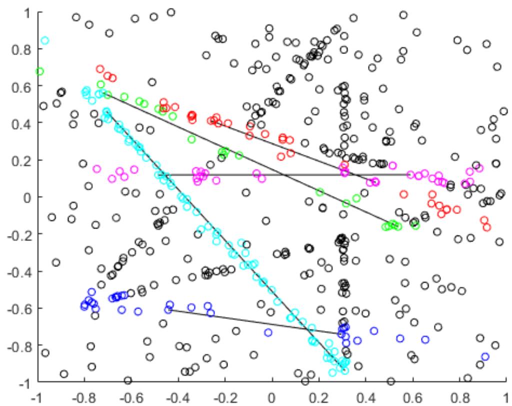
Input:

- P - Set of data points
- ϵ - Threshold
- K - **Number of desired models**

Algorithm:

- Select $m * K$ points, where m is the size of a minimal sample.
- Fit m models and count their inliers.
- Update the so-far-the-best models and, then, restart.

Multi-RANSAC: an example



iter #1

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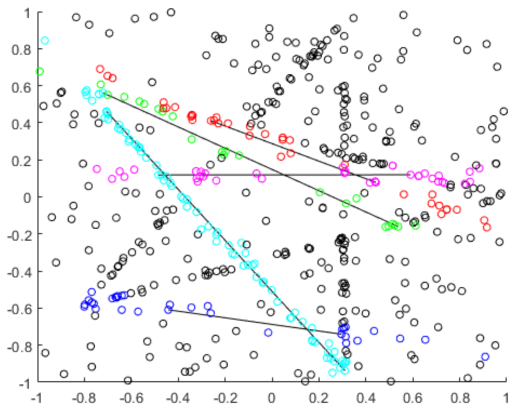

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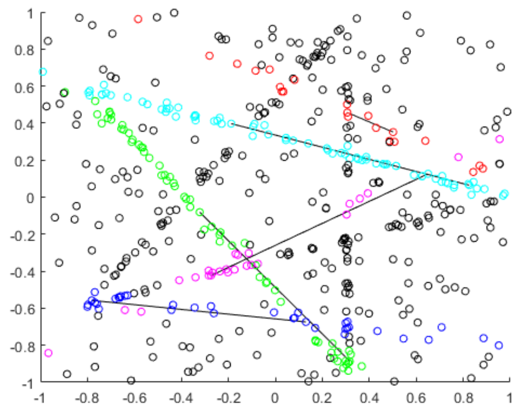

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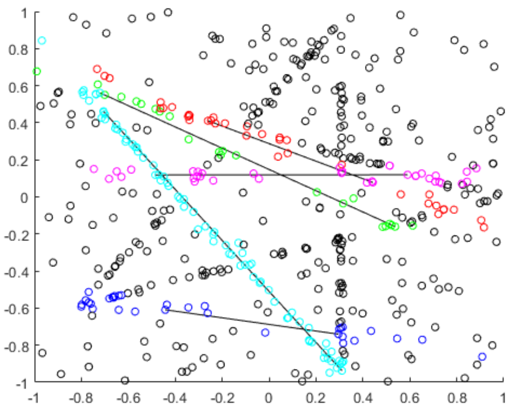

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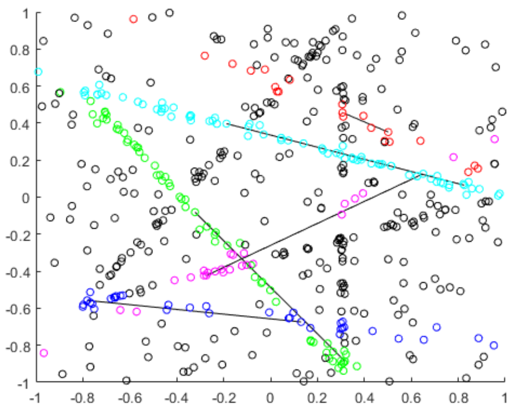

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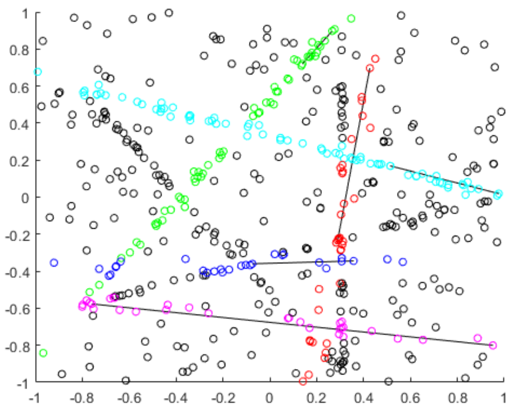

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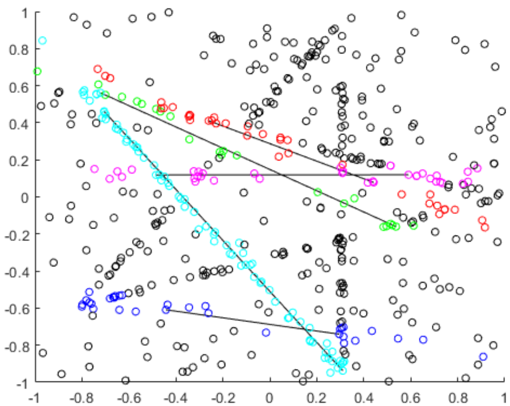

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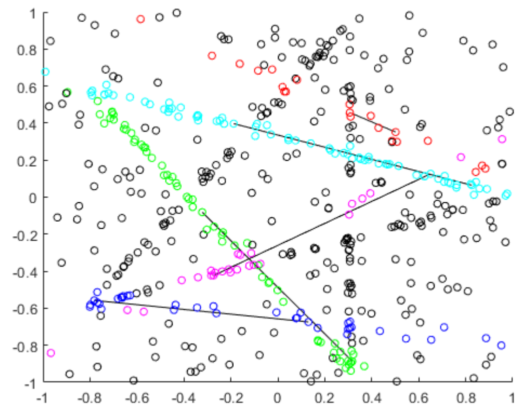

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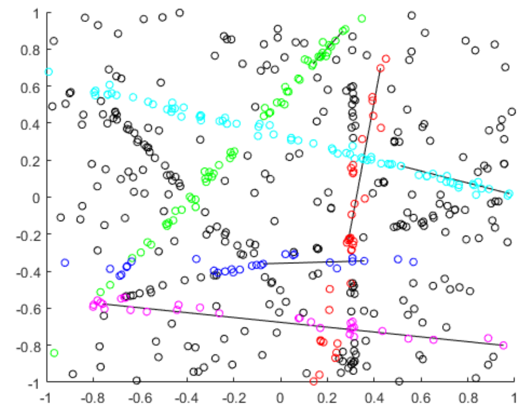

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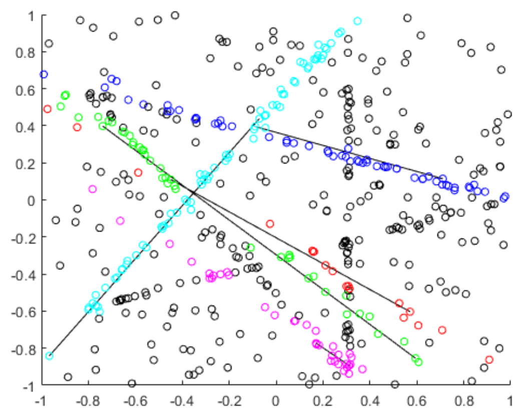

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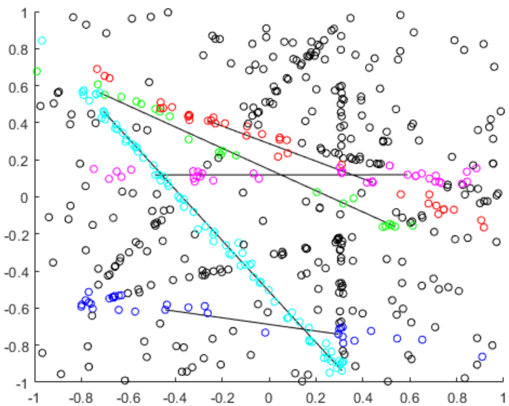

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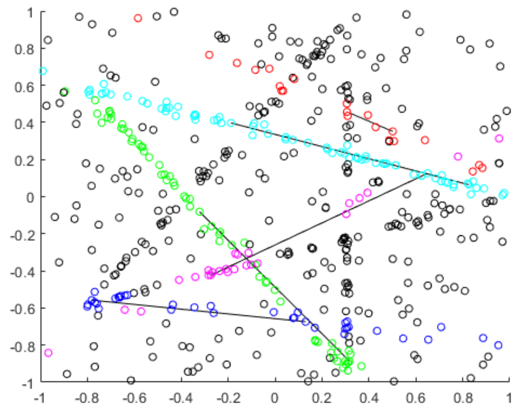

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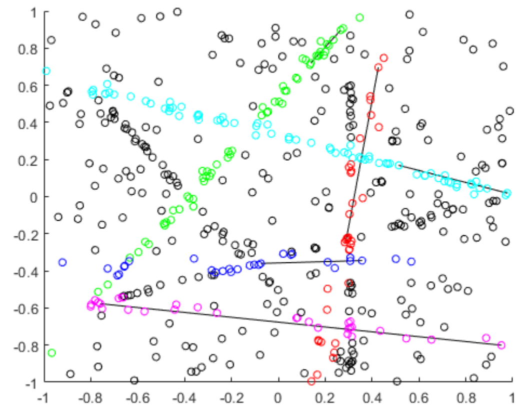

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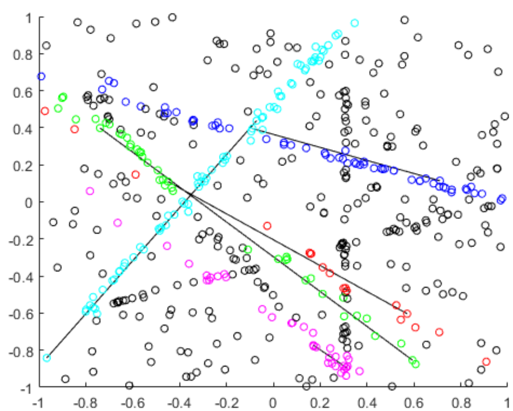

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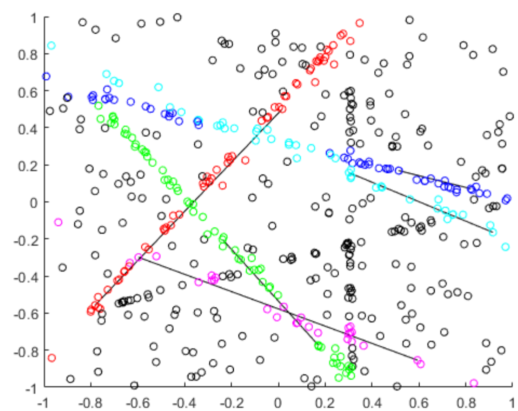

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Inlier ratio $\epsilon = Q/N$ [%]

Size of the sample m

	15%	20%	30%	40%	50%	70%
2	132	73	32	17	10	4
4	5916	1871	368	116	46	11
7	$1.75 \cdot 10^6$	$2.34 \cdot 10^5$	$1.37 \cdot 10^4$	1827	382	35
8	$1.17 \cdot 10^7$	$1.17 \cdot 10^6$	$4.57 \cdot 10^4$	4570	765	50
12	$2.31 \cdot 10^{10}$	$7.31 \cdot 10^8$	$5.64 \cdot 10^6$	$1.79 \cdot 10^5$	$1.23 \cdot 10^4$	215
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computed for $\eta = 0.95$

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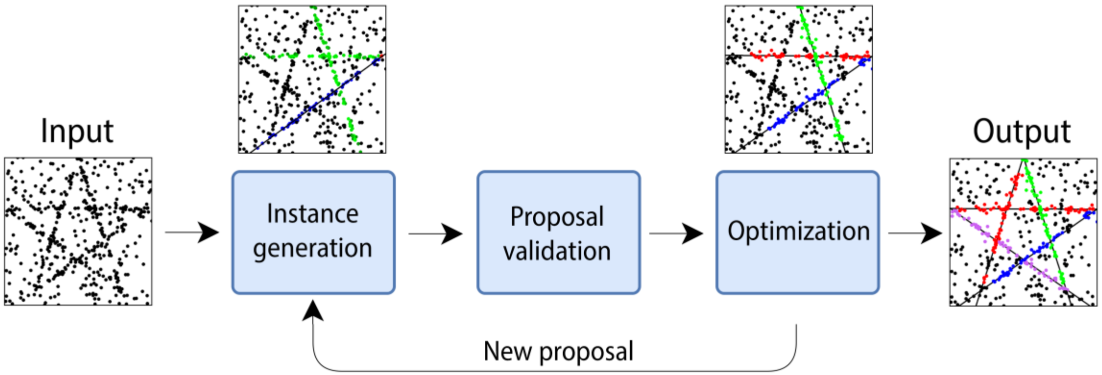
State-of-the-art pipeline



- Generate thousands of initial models.

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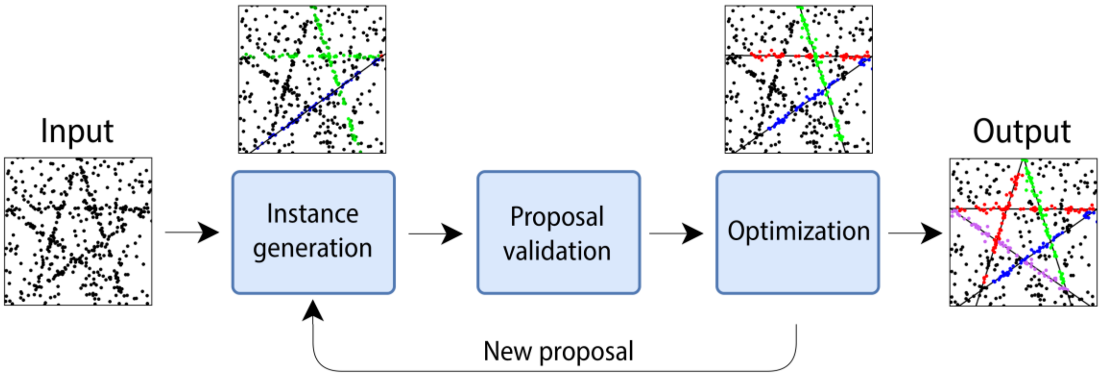

- Generate thousands of initial models.
- Apply an (iterated) optimization to
  – select a subset of models interpreting the scene the most,
  – refine the parameters of selected models.

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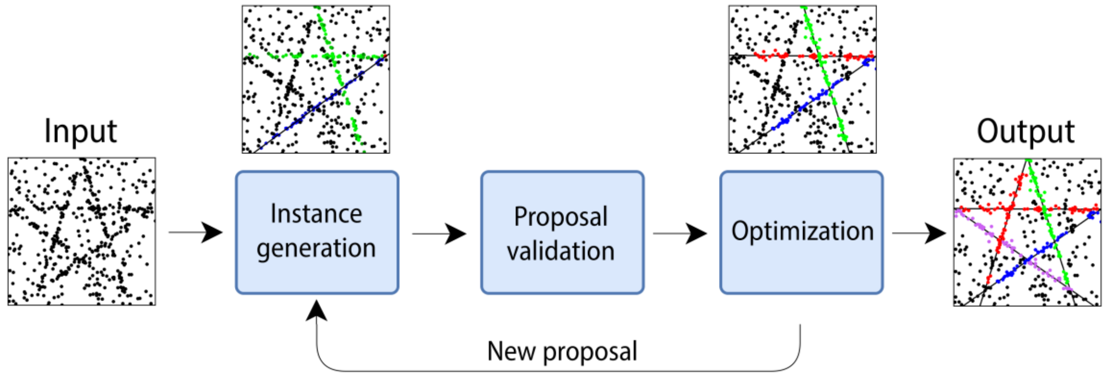

- Generate thousands of initial models.
- Apply an (iterated) optimization to
  - select a subset of models interpreting the scene the most,
  - refine the parameters of selected models.
- Post-processing of the obtained model, e.g. to remove statistically insignificant models.

State-of-the-art techniques

Three major categories:

- Energy-minimization-based techniques
- Preference analysis-based techniques
- Hypergraph partitioning-based techniques

  (we will not talk about them)

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Multi-model fitting as a labeling problem

We formulate the task as follows:

Find

- Set of N_i instances $\{\mathcal{I}_j\}_{j=1}^{N_i}$, and
- $\mathcal{L}^*$ labeling ($\mathcal{L} \colon \mathcal{X} \to \{\mathcal{I}_j\}_{j=0}^{N_i}$) such that:

$$\mathcal{L}^*, \theta^* = \arg \min_{\mathcal{L}, \{\theta_j\}_{j=1}^{N_i}} \sum_{\mathbf{x} \in \mathcal{X}} f_{\mathcal{L}(\mathbf{x})}(\mathbf{x}, \theta_{\mathcal{L}(\mathbf{x})})$$

Notes:

- This is a labeling problem with $N_i + 1$ labels:
  - N_i labels are instance indices, instances are from class $\mathcal{C} = (\Theta, f)$ (e.g. lines)
  - Label 0 is assigned to outliers. Outlier class is $\mathcal{C}_0 = (\varnothing, f_0)$, with:
    - Single instance $\mathcal{I}_0 = (\mathcal{C}_0, \varnothing)$, and
    - Cost function $f_0(\mathbf{x}, \varnothing) = \mathrm{const}$

Find

- Instance $\mathcal{I}$
- $\mathcal{L}^*$ labeling (($\mathcal{L} : \mathcal{X} \rightarrow \{\mathcal{I}, \mathcal{I}_0\}$) such that:

$$\mathcal{L}^*, \theta^* = \arg\min_{\mathcal{L},\theta} \sum_{\mathbf{x} \in \mathcal{X}} f_{\mathcal{L}(\mathbf{x})}(\mathbf{x}, \theta_{\mathcal{L}(\mathbf{x})})$$

$\mathrm{E}(\mathbf{x}) = f(\mathbf{x})$

- $\mathbf{x}$ is an inlier
- $f(\mathbf{x}) = 0$ (RANSAC)
- $f(\mathbf{x}) = [\rho(\mathbf{x})]^2$

$\mathrm{E}(\mathbf{x}) = f_0(\mathbf{x})$

- $\mathbf{x}$ is an outlier
- $f_0(\mathbf{x}) = \text{const}$

Note that this formulation is consistent with the previously discussed algorithms.

The question is how to **solve** the labeling problem.

1. **Data (unary) term:**

   penalize point-to-model residual (distance, f(x,θ))

2. **Spatial Regularization, pair-wise term:**

   close points are more likely belong to the same model instance.

3. **Complexity term:**

   penalize the introduction of new labels.

Energy – Spatial Coherence Term

Term penalizing neighbors with different labels:

labels

weights

$$E_s(\mathcal{L}) = \sum_{(p,q)\in\mathcal{A}} w_{pq} [\![\mathcal{L}(p) \neq \mathcal{L}(q)]\!]$$

$$[\![\text{true}]\!] \stackrel{\text{def}}{=} 1, \quad [\![\text{false}]\!] \stackrel{\text{def}}{=} 0$$

Edges in the
neighborhood graph

Energy – Complexity term

The term to suppress weak model instances by *penalizing* the introduction of *new labels*.

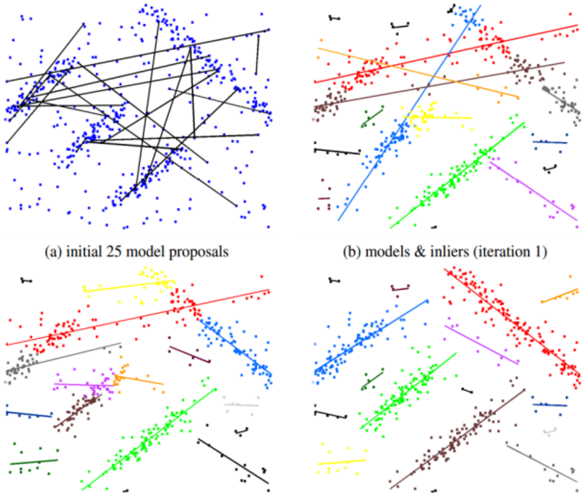
$$E_c(\mathcal{L}) = \sum_{l \in \mathcal{L}_L} \psi_l$$

Penalty of introducing a new model

Set of distinct labels

The PEARL Algorithm

1. Propose:
   - $i \leftarrow 0$, randomly sample data to get labeling $\mathcal{L}^0$
   - $*$ (optional for $i > 0$) sample more or merge/split current models in $\mathcal{L}^i$
2. Expand:
   - run α-expansion for $\alpha \in \mathcal{L}^i$
   - if the energy does not decrease, stop
3. Re-estimate labels:
   - obtain new set of labels $\mathcal{L}^{i+1}$
   - $i \leftarrow i + 1$, goto 2 (optionally to $*$)



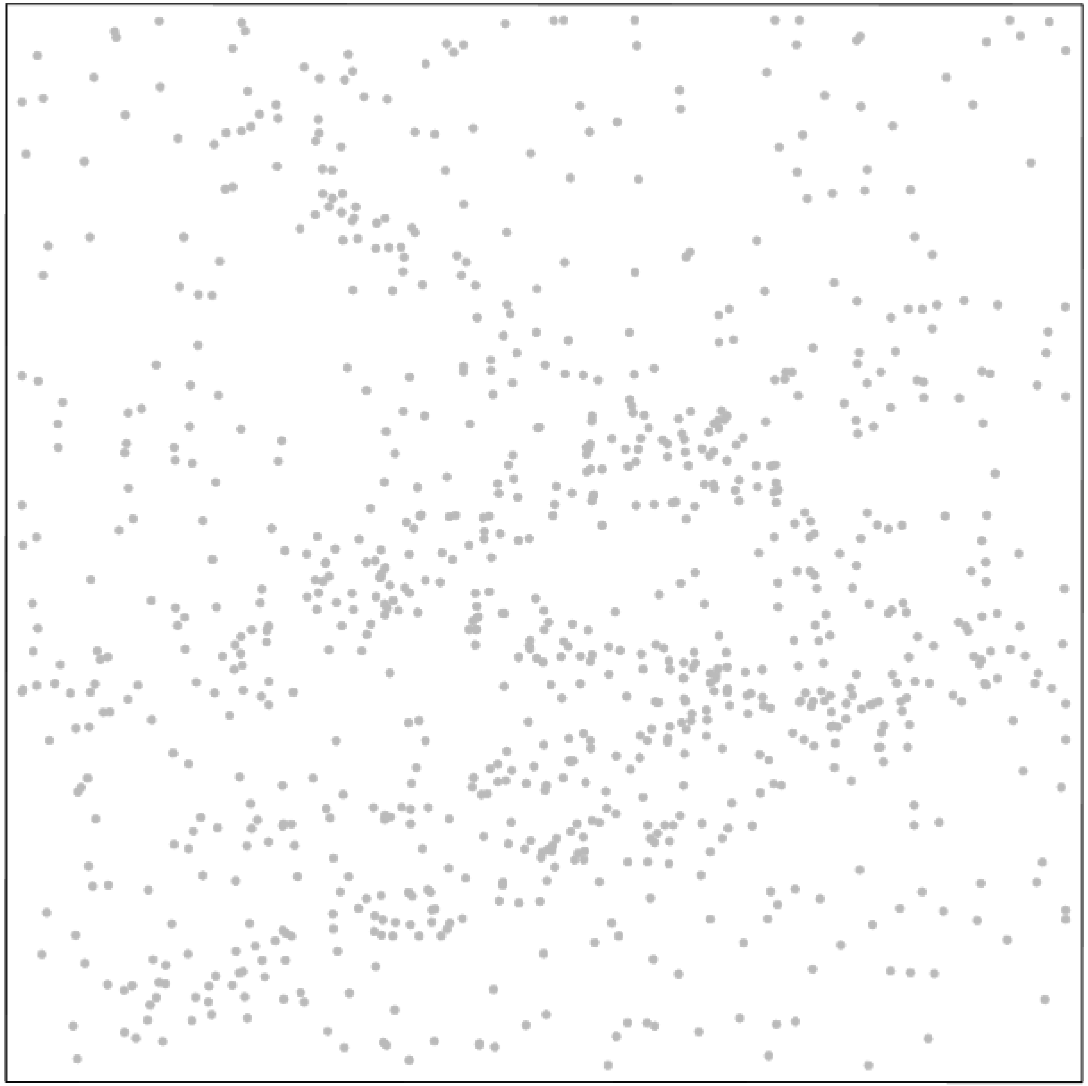
(a) initial 25 model proposals (b) models & inliers (iteration 1) (c) models & inliers (iteration 2) (d) at convergence (iteration 5)

Notes:

- Initialization similar to RANSAC, instances processed in parallel.
- Starts with number of instance much bigger than in the solution.
- Introduces pairwise terms:
  - promotes spatial coherence
  - elimination of needless instances.

PEARL

Propose
Expand
Re-estimate labels



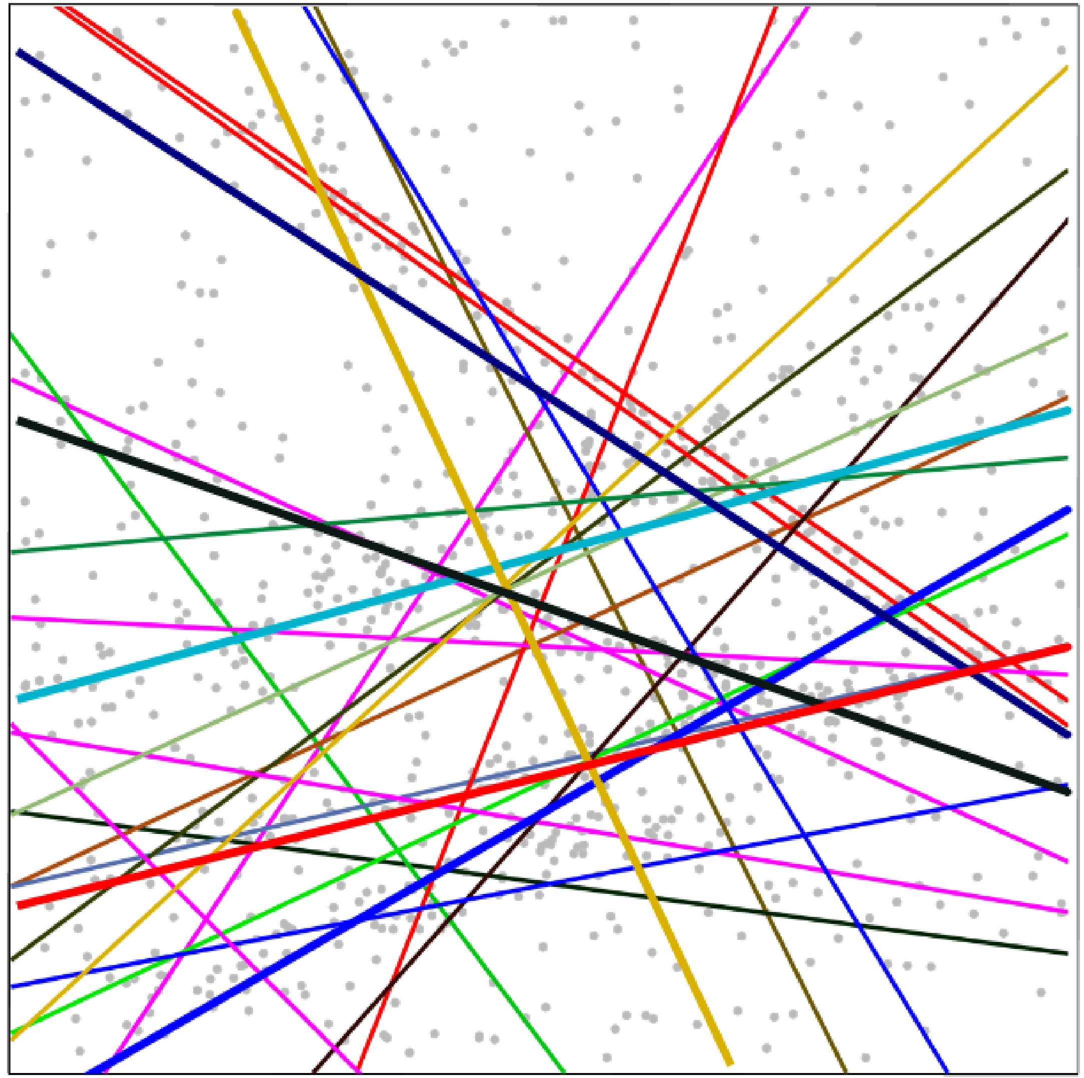
data points

Source: https://www.csd.uwo.ca/Courses/CS9629a/Lectures/lec09_geometricModels.pdf

Propose

Expand

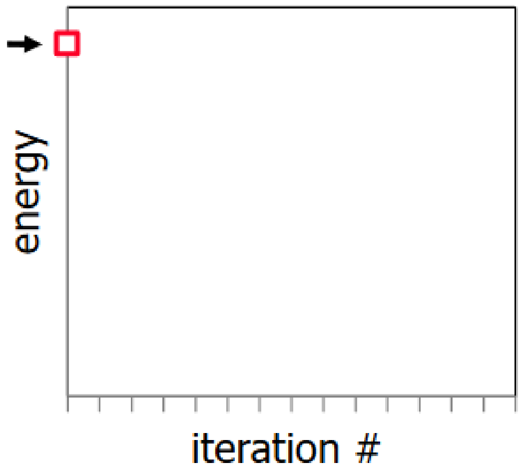
Re-estimate labels

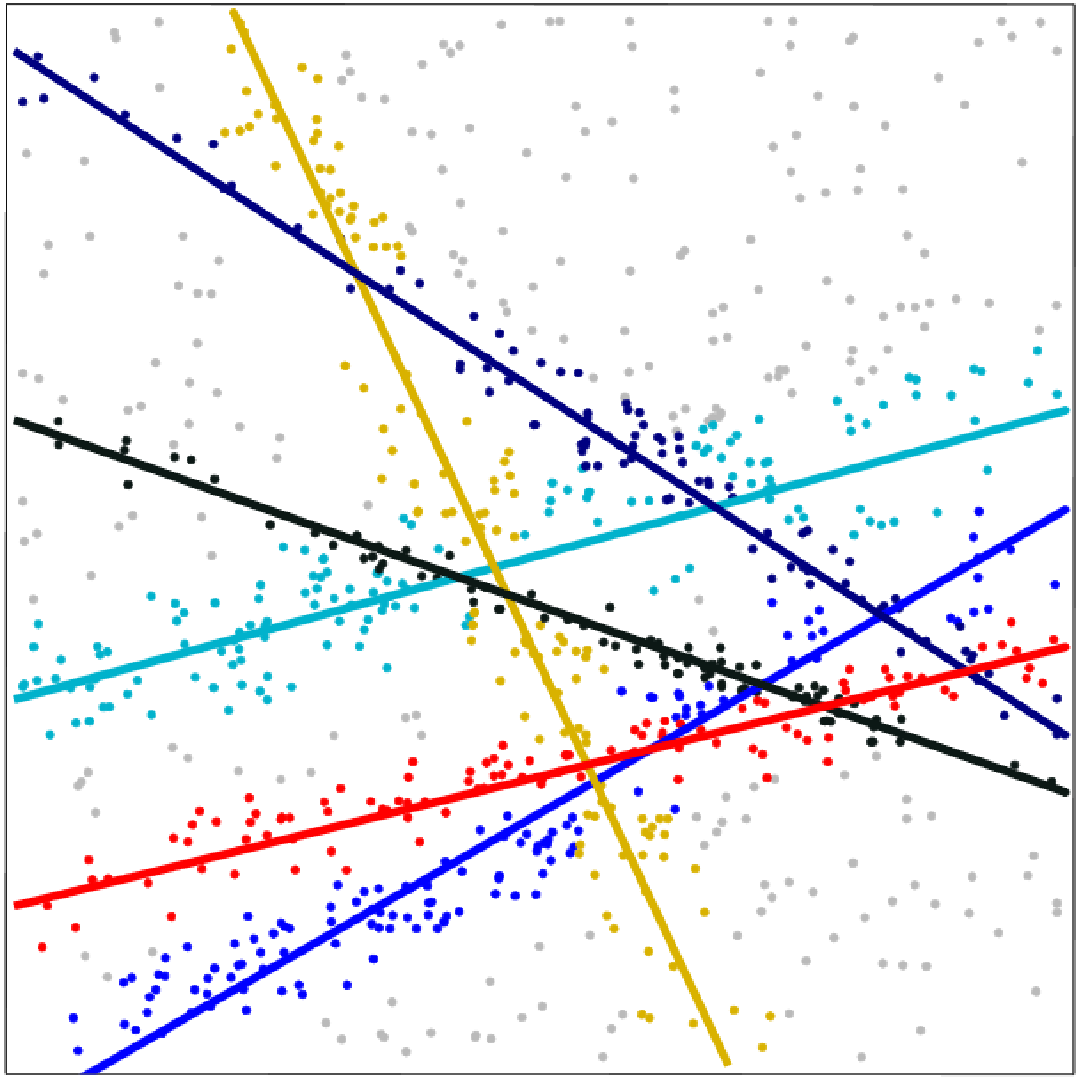


data points + randomly sampled models

Source: https://www.csd.uwo.ca/Courses/CS9629a/Lectures/lec09_geometricModels.pdf

Propose
Expand
Re-estimate labels





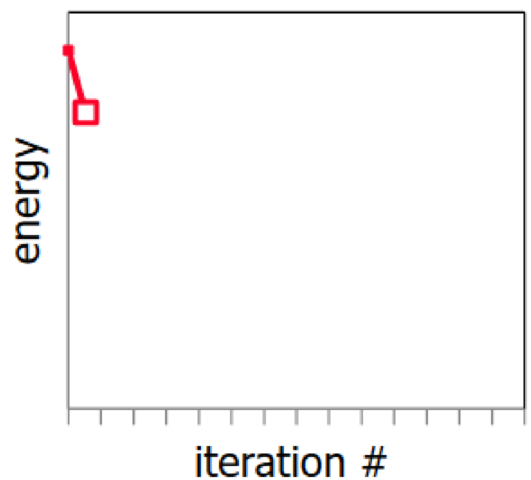
iteration 1: optimize labeling L

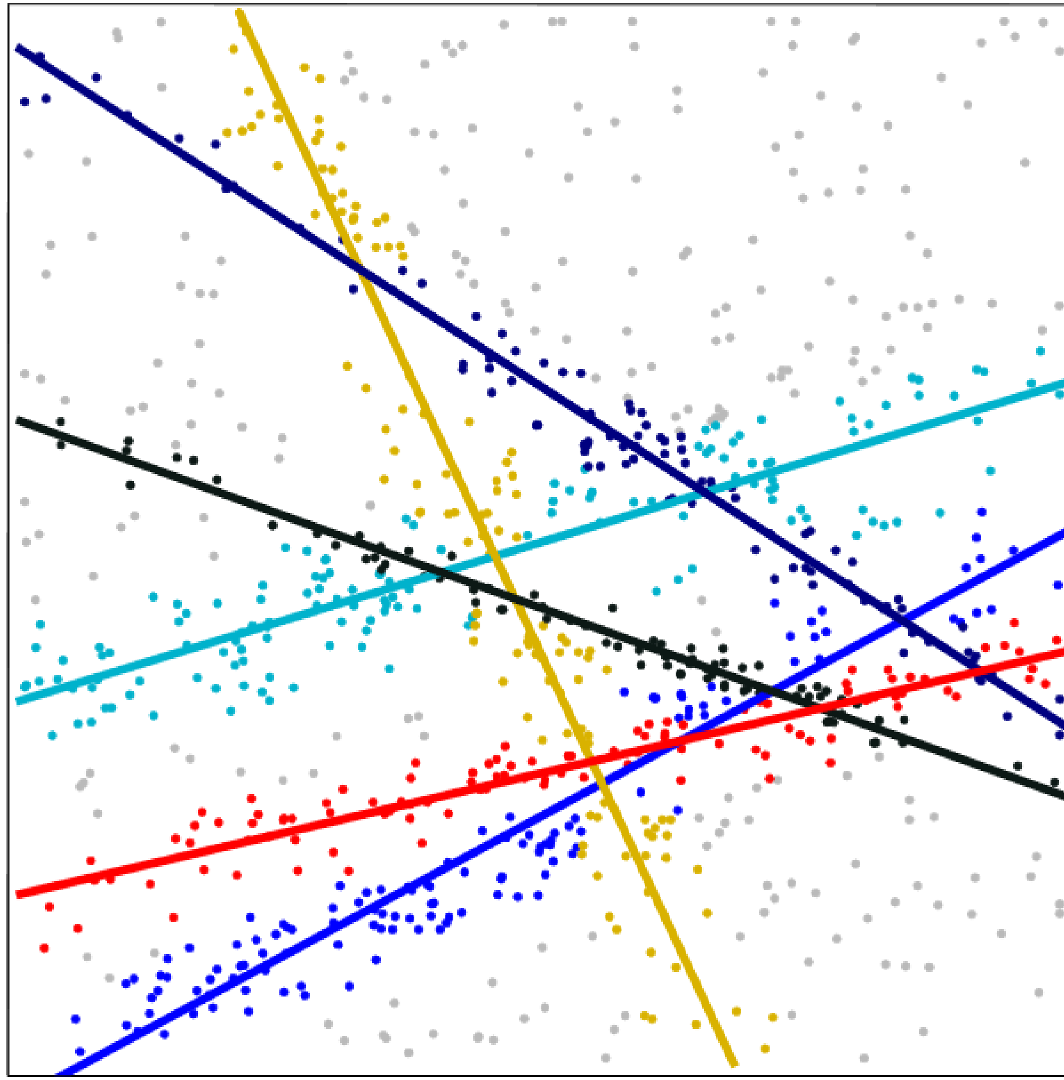
Source: https://www.csd.uwo.ca/Courses/CS9629a/Lectures/lec09_geometricModels.pdf

Propose

Expand

Re-estimate labels



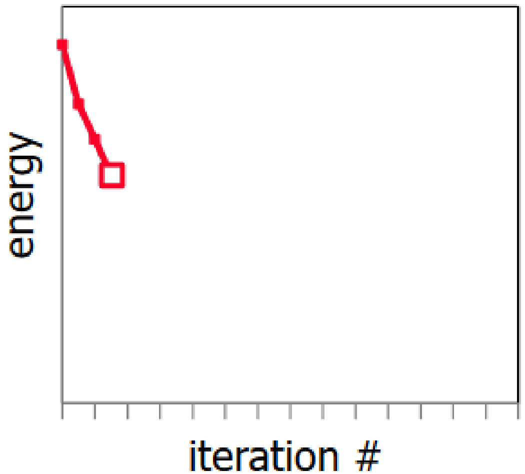


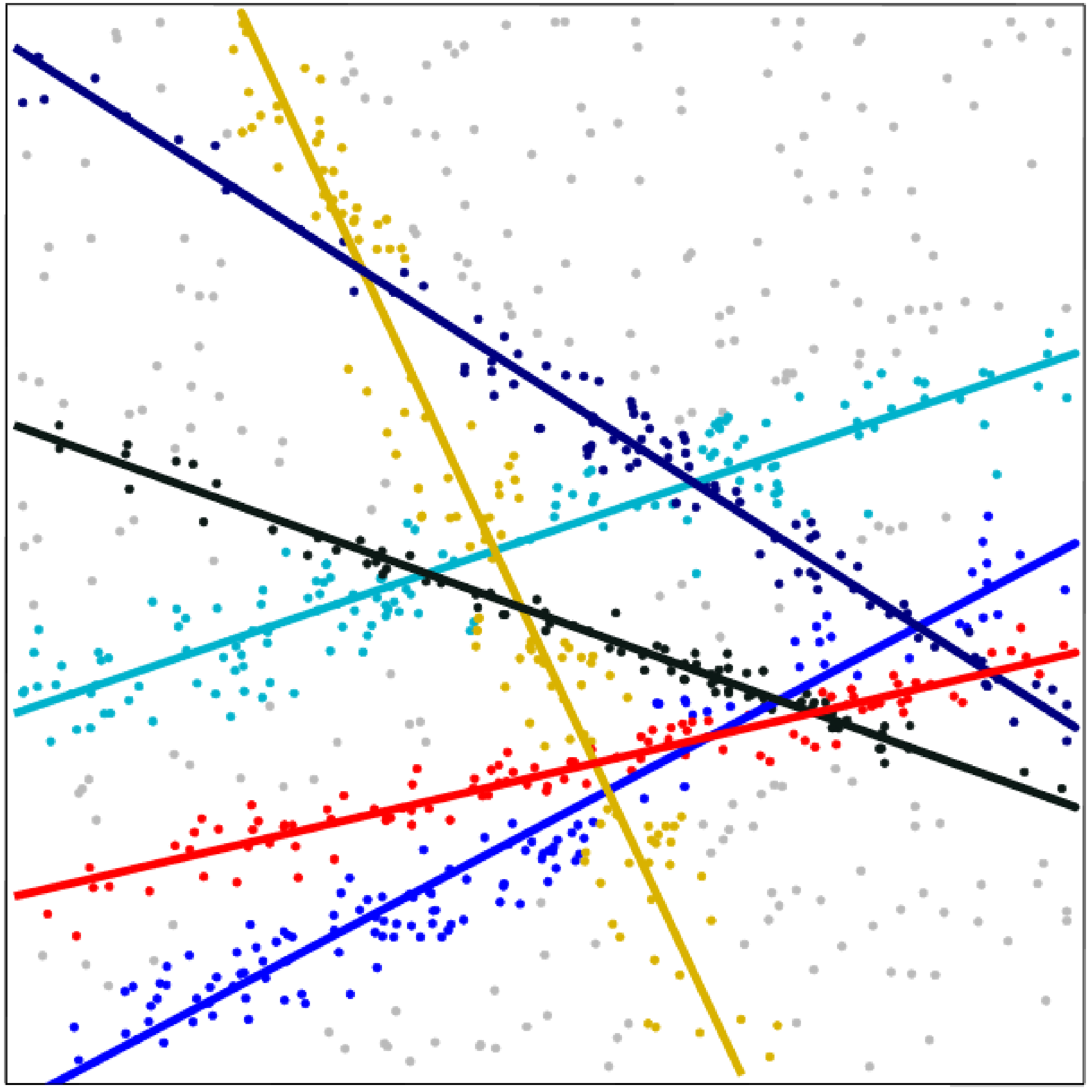
iteration 1: reestimate models

Source: https://www.csd.uwo.ca/Courses/CS9629a/Lectures/lec09_geometricModels.pdf

PEARL

Propose
Expand
Re-estimate labels





iteration 2: reestimate models

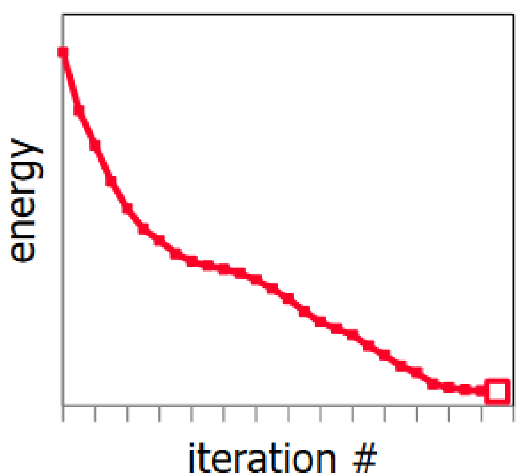
D. Barath

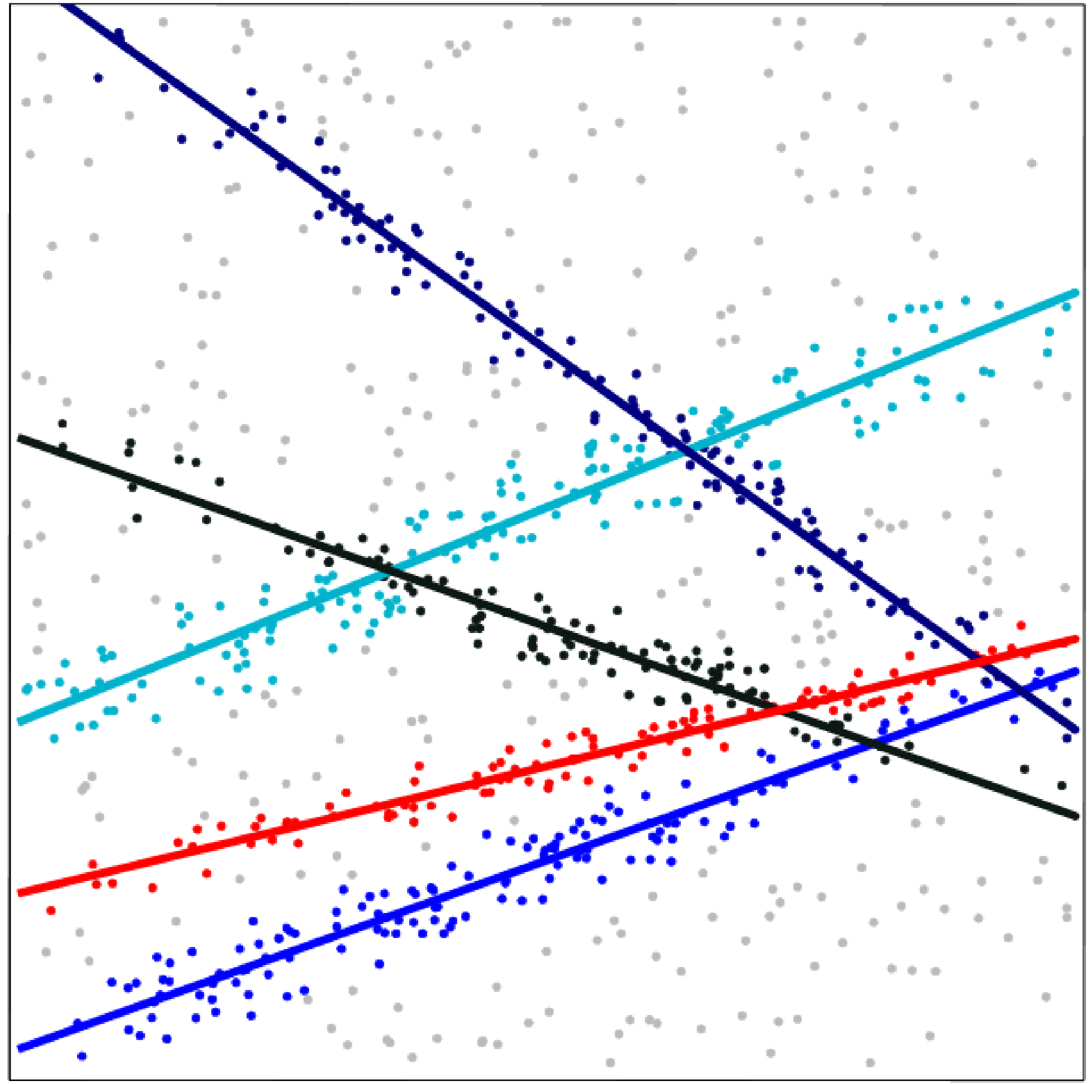
Source: https://www.csd.uwo.ca/Courses/CS9629a/Lectures/lec09_geometricModels.pdf

Propose

Expand

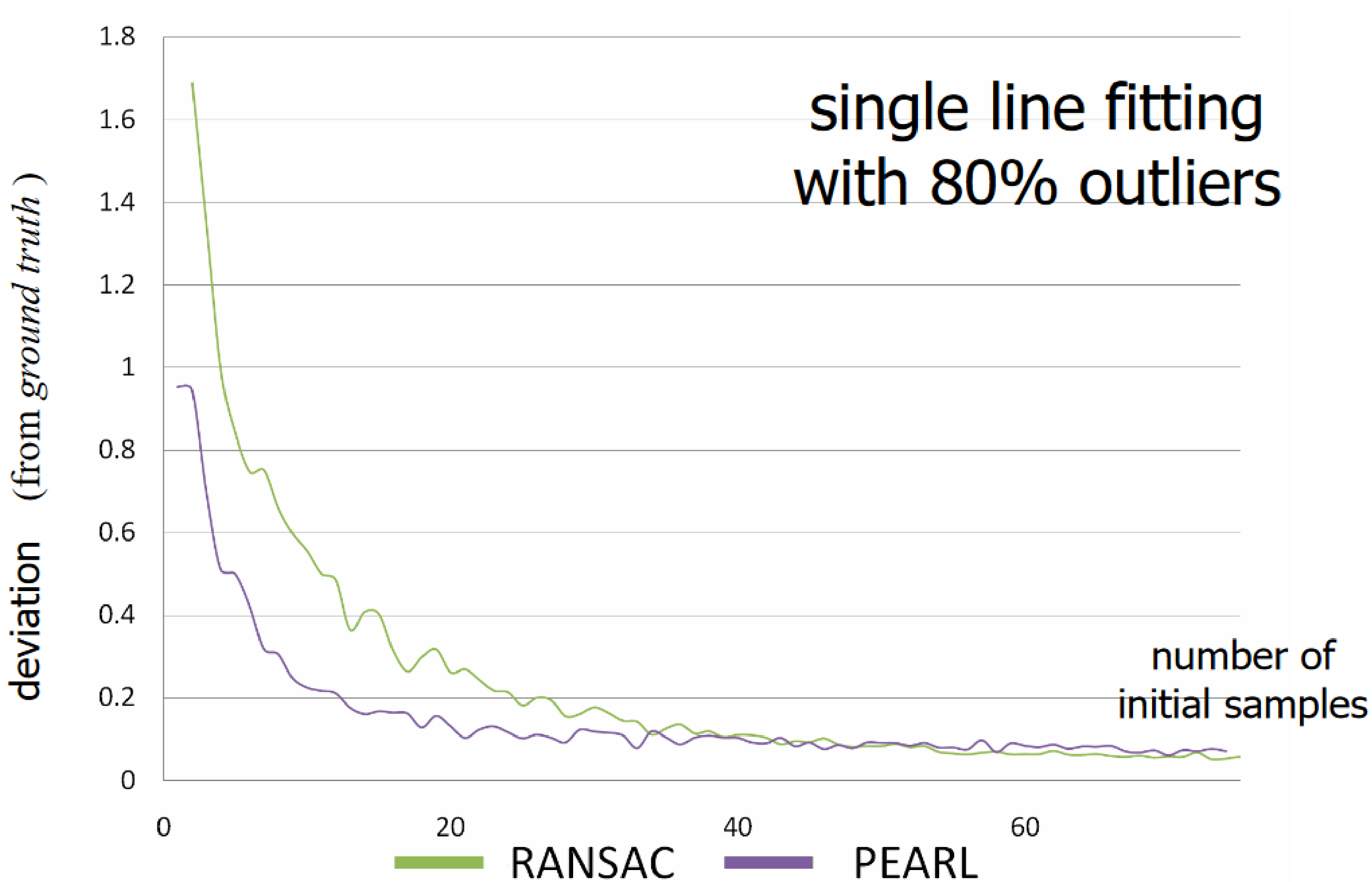
Re-estimate labels





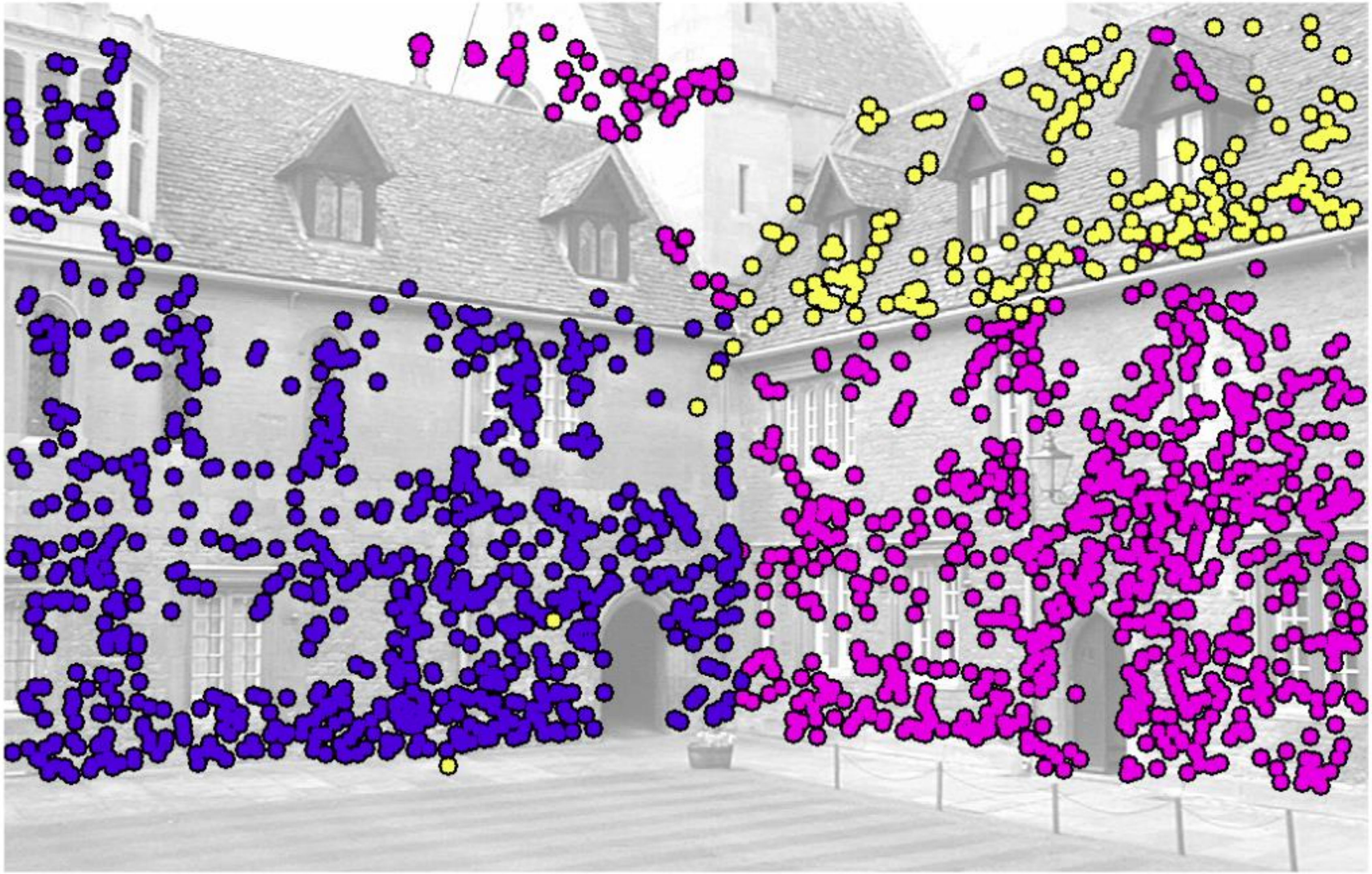
iteration 15... converged.

Number of initial models



Source: https://www.csd.uwo.ca/Courses/CS9629a/Lectures/lec09_geometricModels.pdf

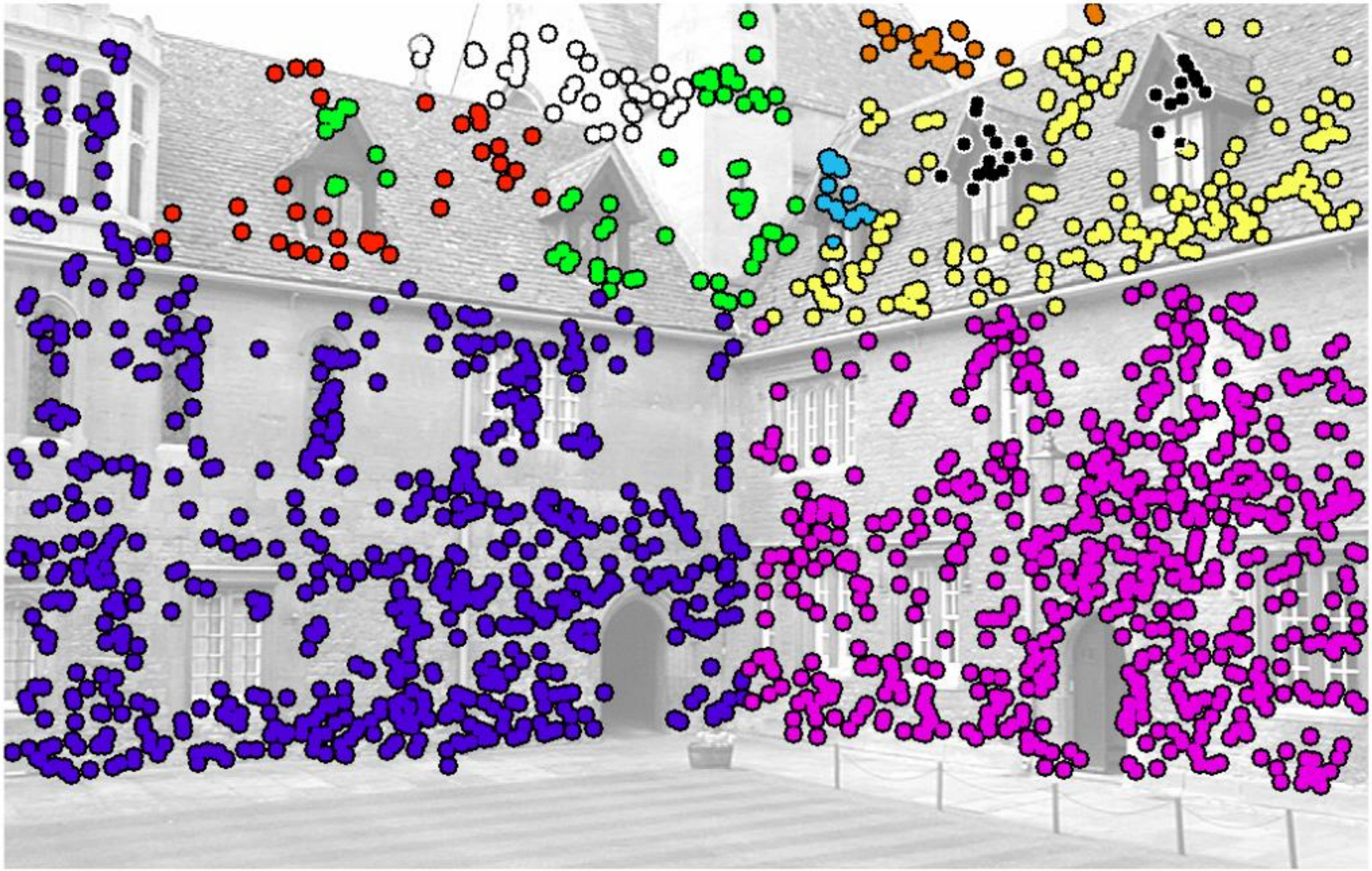
Effect of energy terms: label cost



(a) Label costs only

Source: https://www.csd.uwo.ca/Courses/CS9629a/Lectures/lec09_geometricModels.pdf

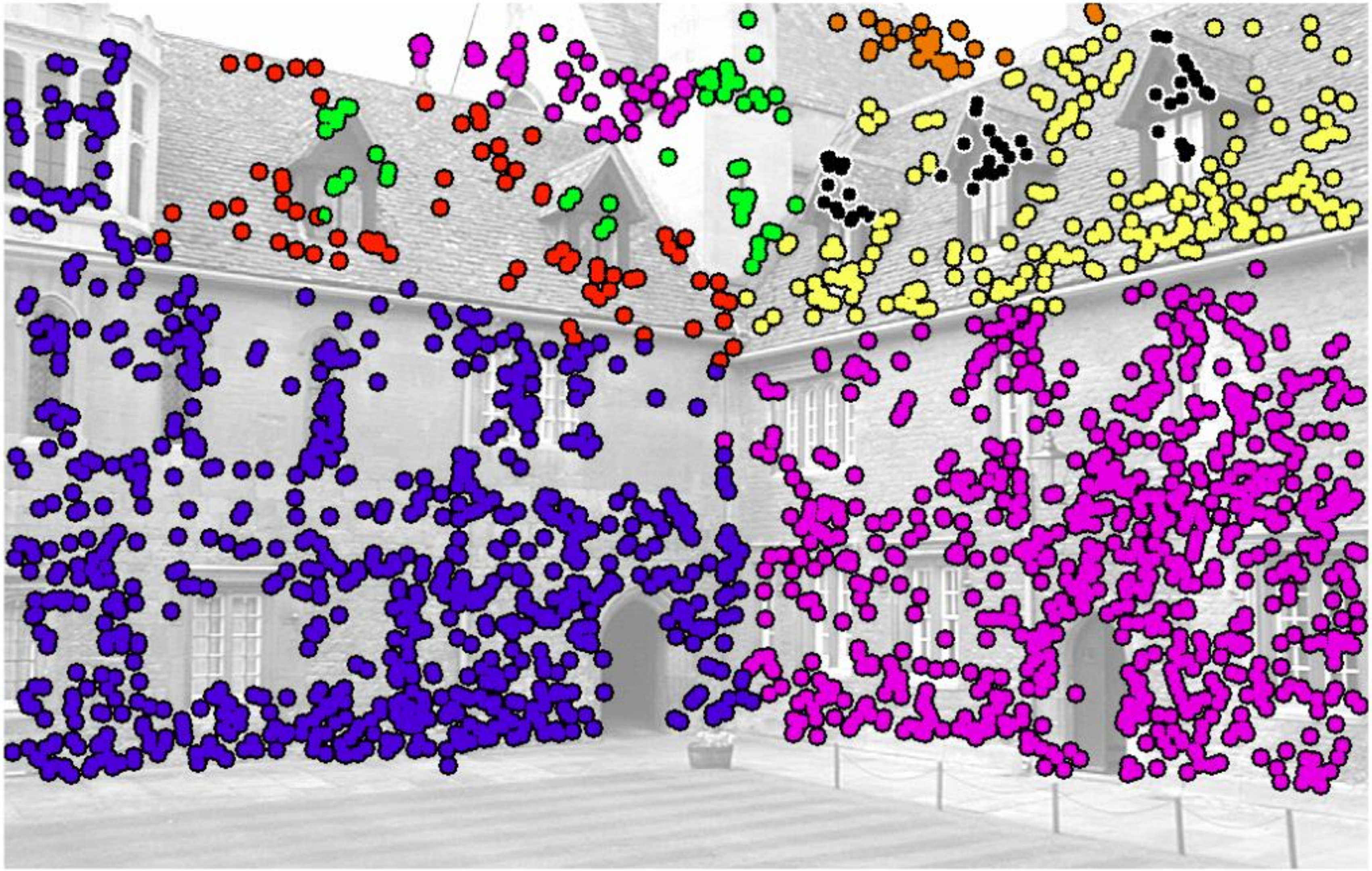
Effect of energy terms: spatial coherence



(b) Spatial regularity only

Source: https://www.csd.uwo.ca/Courses/CS9629a/Lectures/lec09_geometricModels.pdf

Spatial coherence and label cost



(c) Spatial regularity + label costs

D. Barath

Source: https://www.csd.uwo.ca/Courses/CS9629a/Lectures/lec09_geometricModels.pdf

Multi-X

Input: $\mathcal{X} = \{\mathbf{x}_j\}_{j=1}^{N}$ data points

$\mathcal{C}_1, \mathcal{C}_2, ..., \mathcal{C}_M$ classes

$e_{\mathcal{C}}(S) = \theta$ estimates *model parameters* θ for class $\mathcal{C}$, given $S \subseteq \mathcal{X}$

$f_{\mathcal{C}}(\mathbf{x}, \theta)$ cost function for class $\mathcal{C}$

$\mathcal{C}_0,\ \mathcal{I}_0,\ f_0(\mathbf{x}, \varnothing) = \text{const}$ outlier class, instance and cost function

Generate instances $\{\mathcal{I}\}^0$ (by PROSAC sampling). $i \leftarrow 1$.

repeat

$\quad \{\mathcal{I}\}^i \leftarrow \text{ModeSeeking}(\{\mathcal{I}\}^{i-1})$

$$\mathcal{L}^i \leftarrow \arg\min_{\mathcal{L}} \sum_{\mathbf{x} \in \mathcal{X}} f_{\mathcal{L}(\mathbf{x})}(\mathbf{x}, \theta_{\mathcal{L}(\mathbf{x})}) + \lambda_1 \sum_{(\mathbf{x},\mathbf{y}) \in \mathcal{A}} [\![\mathcal{L}(\mathbf{x}) \neq \mathcal{L}(\mathbf{y})]\!] + \lambda_2 |\mathcal{L}|$$

$\quad \{\mathcal{I}\}^{i+1} \leftarrow \text{ReestimateInstances}(\{\mathcal{I}\}^i, \mathcal{L}^i, \mathcal{X})$

$\quad i \leftarrow i + 1$

until !Convergence$(\{\mathcal{I}\}^i, \mathcal{L}^i)$

$\{\mathcal{I}\}^* \leftarrow \{\mathcal{I}\}^{i-1}, \mathcal{L}^* \leftarrow \mathcal{L}^{i-1}$

$\{\mathcal{I}\}^*, \mathcal{L}^* \leftarrow \text{RemoveStatisticallyInsignificantModels}(\{\mathcal{I}\}^*, \mathcal{L}^*)$

Multi-X

Input: $\mathcal{X} = \{\mathbf{x}_j\}_{j=1}^{N}$ data points

$\mathcal{C}_1, \mathcal{C}_2, ..., \mathcal{C}_M$ classes

$e_{\mathcal{C}}(S) = \theta$ estimates *model parameters* θ for class $\mathcal{C}$, given $S \subseteq \mathcal{X}$

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Generate instances $\{\mathcal{I}\}^0$ (by PROSAC sampling). $i \leftarrow 1$.

repeat

$\{\mathcal{I}\}^i \leftarrow \text{ModeSeeking}(\{\mathcal{I}\}^{i-1})$

New label move

$$\mathcal{L}^i \leftarrow \arg\min_{\mathcal{L}} \sum_{\mathbf{x} \in \mathcal{X}} f_{\mathcal{L}(\mathbf{x})}(\mathbf{x}, \theta_{\mathcal{L}(\mathbf{x})}) + \lambda_1 \sum_{(\mathbf{x},\mathbf{y}) \in \mathcal{A}} [\![\mathcal{L}(\mathbf{x}) \neq \mathcal{L}(\mathbf{y})]\!] + \lambda_2 |\mathcal{L}|$$

$\{\mathcal{I}\}^{i+1} \leftarrow \text{ReestimateInstances}(\{\mathcal{I}\}^i, \mathcal{L}^i, \mathcal{X})$

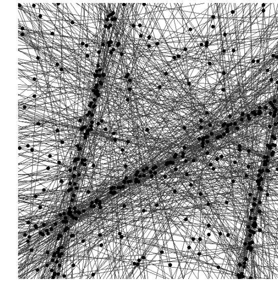
$i \leftarrow i + 1$

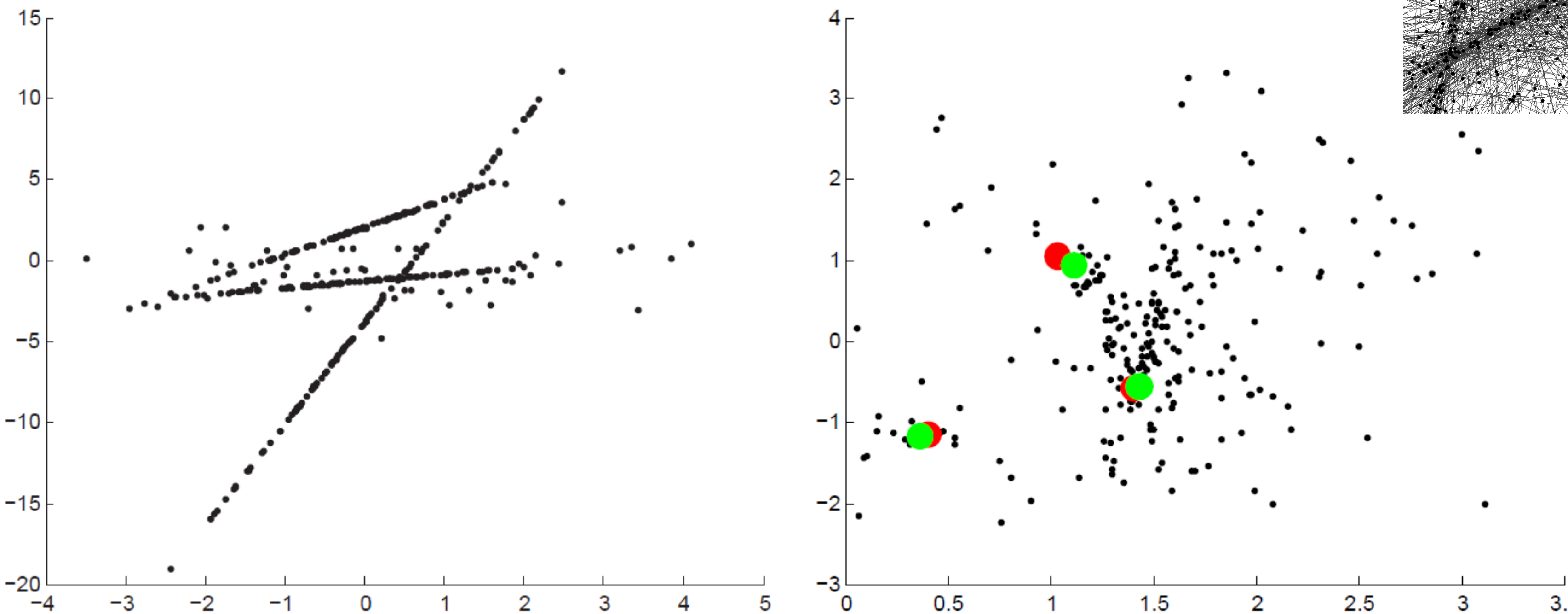
until !Convergence$(\{\mathcal{I}\}^i, \mathcal{L}^i)$

$\{\mathcal{I}\}^* \leftarrow \{\mathcal{I}\}^{i-1}, \mathcal{L}^* \leftarrow \mathcal{L}^{i-1}$

$\{\mathcal{I}\}^*, \mathcal{L}^* \leftarrow \text{RemoveStatisticallyInsignificantModels}(\{\mathcal{I}\}^*, \mathcal{L}^*)$

Replacing label sets, motivation





(Left) Three lines each generating 100 points with zero-mean Gaussian noise added + 50 outliers. **(Right)** 1000 line instances generated from random point pairs, the GT instance parameters (red) and the modes (green) provided by Mean-Shift in the model parameter domain: angle – vertical, offset – horizontal.

„Replacing label sets" move

Input: $\{\mathcal{I}\}^i$ instance set in the ith iteration

$\mathcal{C}_1, \mathcal{C}_2, ..., \mathcal{C}_M$ classes

$\{\mathcal{I}\}^i_{\mathcal{C}_j} \subseteq \{\mathcal{I}\}^i$ instances of class $\mathcal{C}_j$, $j \in [1, M]$

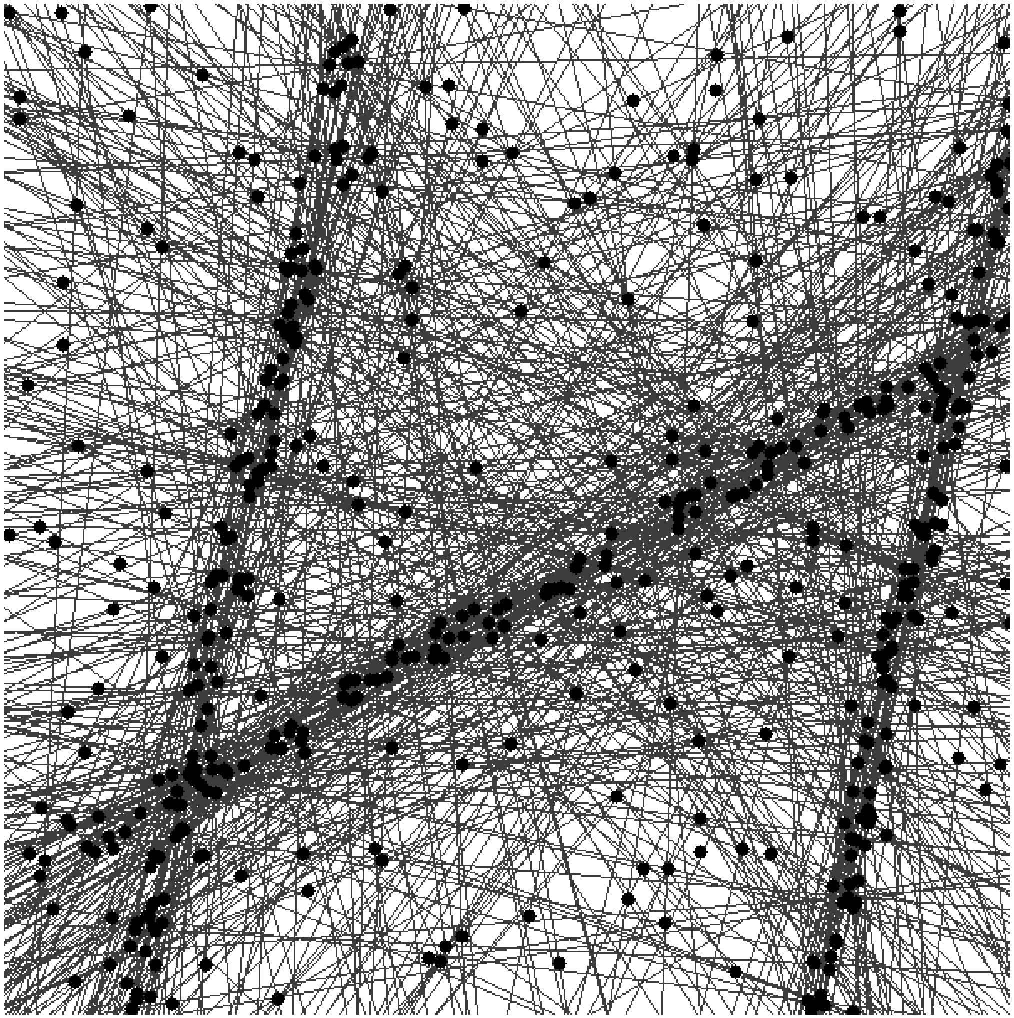
$\Theta : \mathcal{I}^* \to \mathcal{I}^*$ mode-seeking function

$E(L) = e$ energy function

1: $\{\mathcal{I}\}^\Theta = \Theta(\{\mathcal{I}\}^i_{\mathcal{C}_1}) \cup \Theta(\{\mathcal{I}\}^i_{\mathcal{C}_2}) \cup ... \cup \Theta(\{\mathcal{I}\}^i_{\mathcal{C}_M})$,

2: $\{\mathcal{I}\}^{i+1} = \begin{cases} \{\mathcal{I}\}^\Theta & \text{if } E(L_{\{\mathcal{I}\}^\Theta}) \leq E(L_i), \\ \{\mathcal{I}\}^i & \text{otherwise} \end{cases}$,

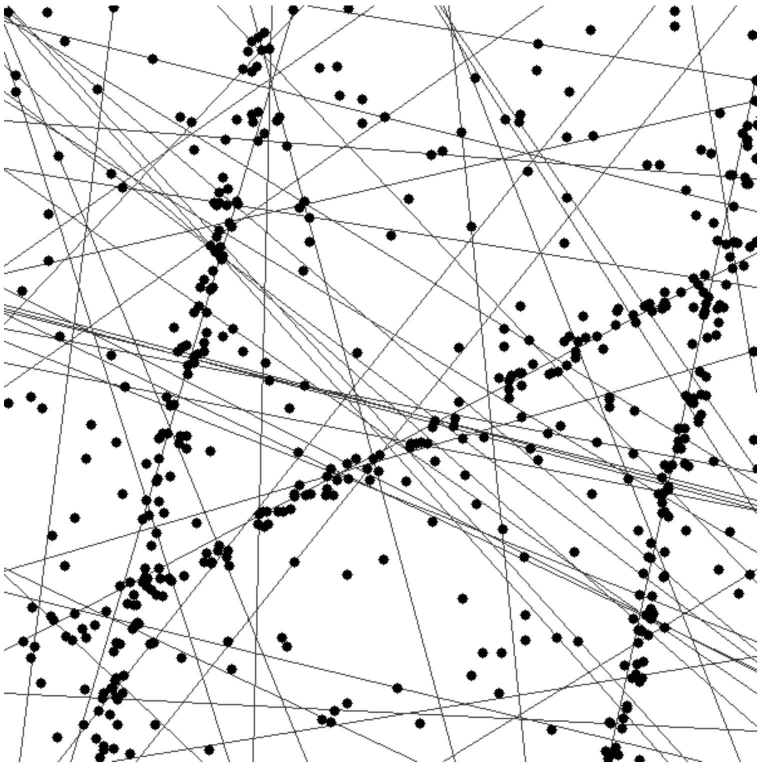
Example experiment: PEARL and Multi-X



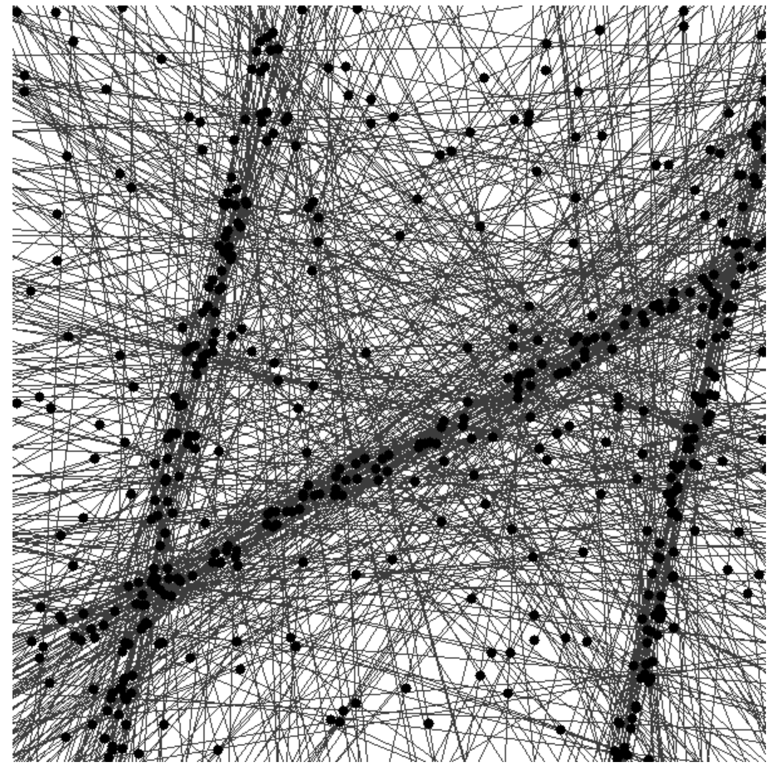
Experiment: 1st iteration, mode-seeking

Mode-seeking significantly reduces the instance number for Multi-X.

Multi-X



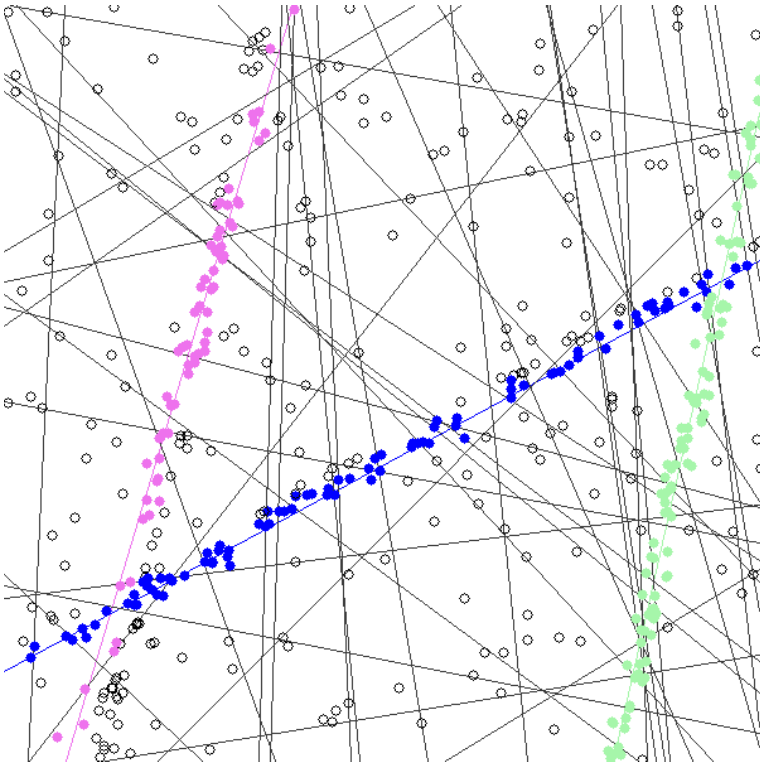
PEARL



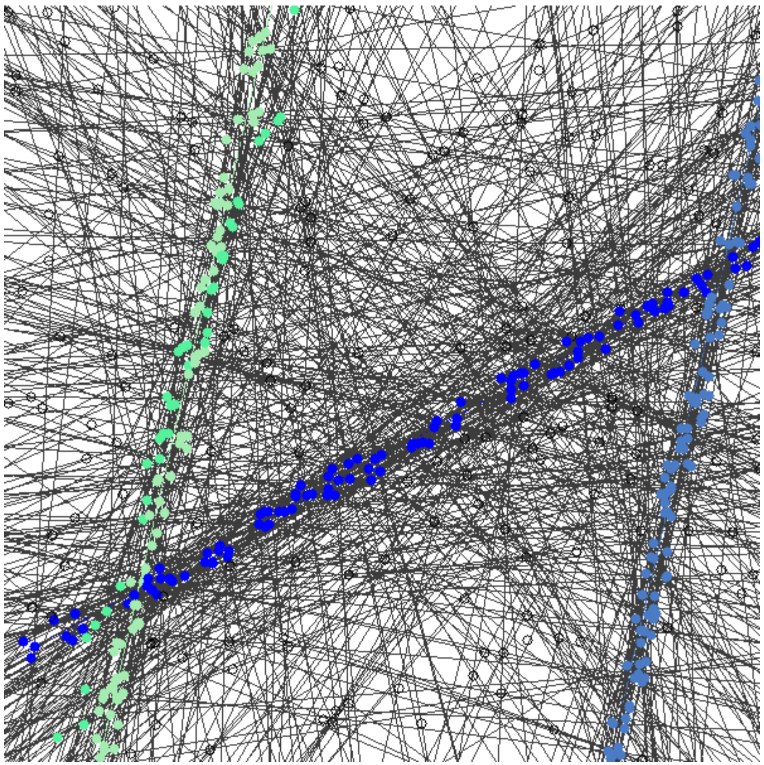
Experiments: 1st iteration, labeling

Colors denote instances. Non-filled circles are labelled outliers.

Multi-X



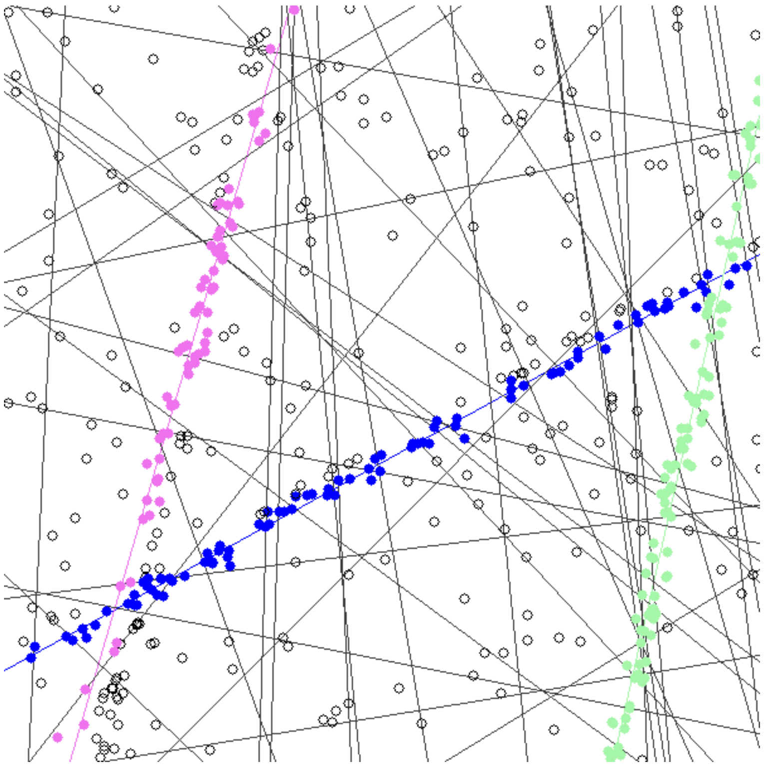
PEARL



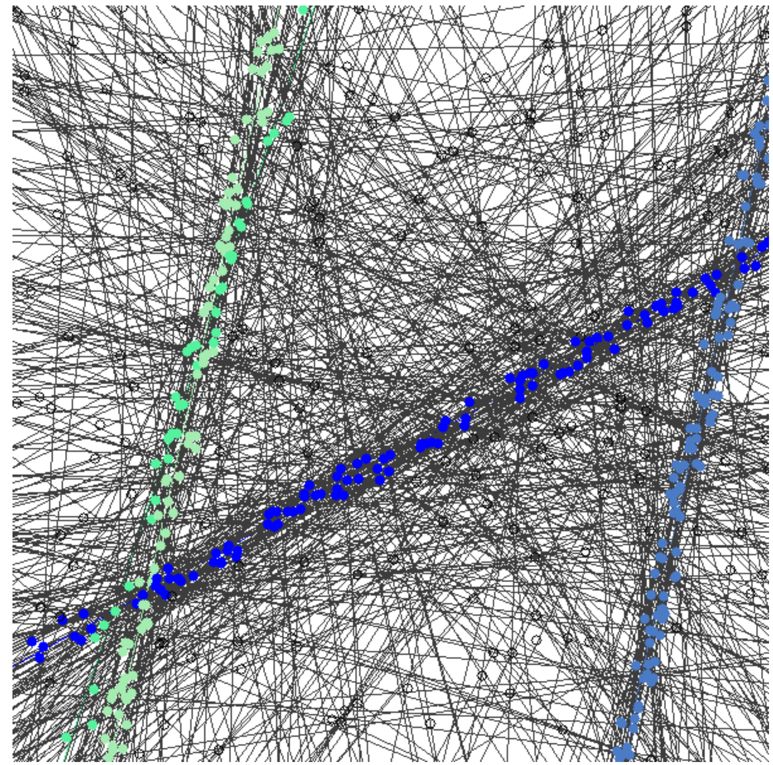
Experiment: 1^{st} iteration, re-estimation

Re-estimation of instance parameters based on the inliers.

Multi-X



PEARL



Experiment: 1st iteration, instance validation

All instances are removed for which no points are assigned.

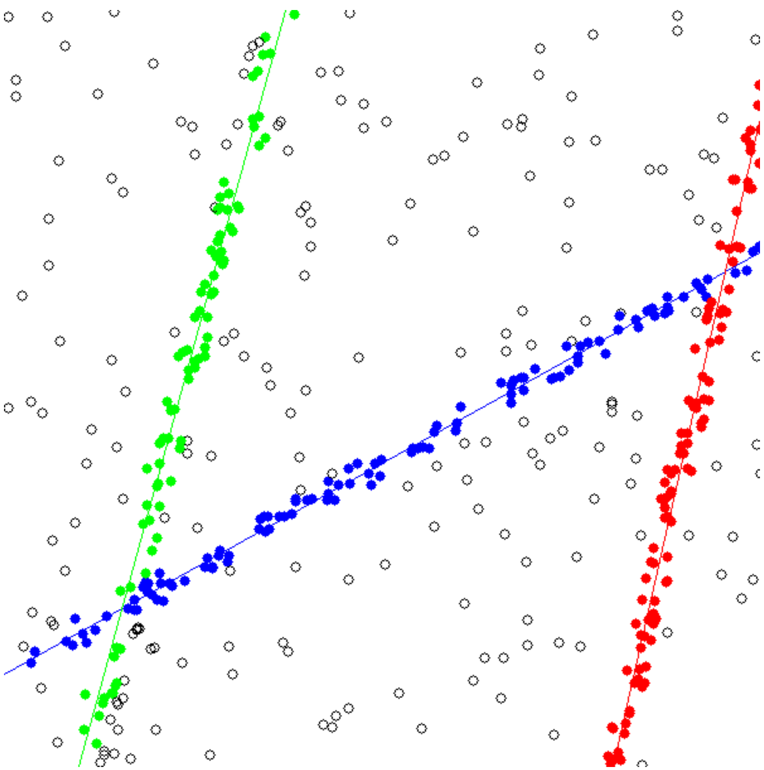
Multi-X

PEARL

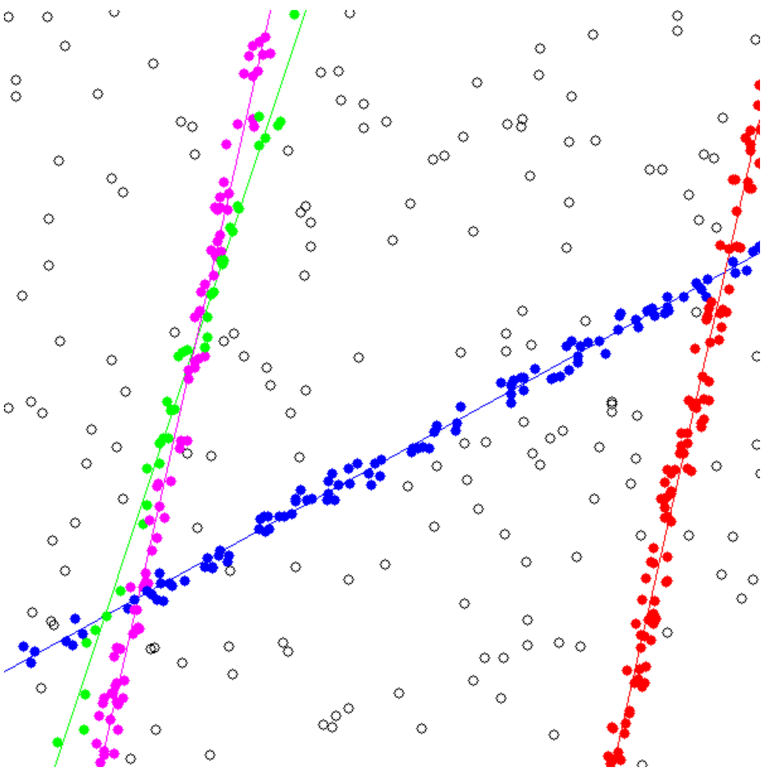
Experiment: 2nd iteration, labeling

Colors denote instances. Non-filled circles are labelled outliers.

Multi-X



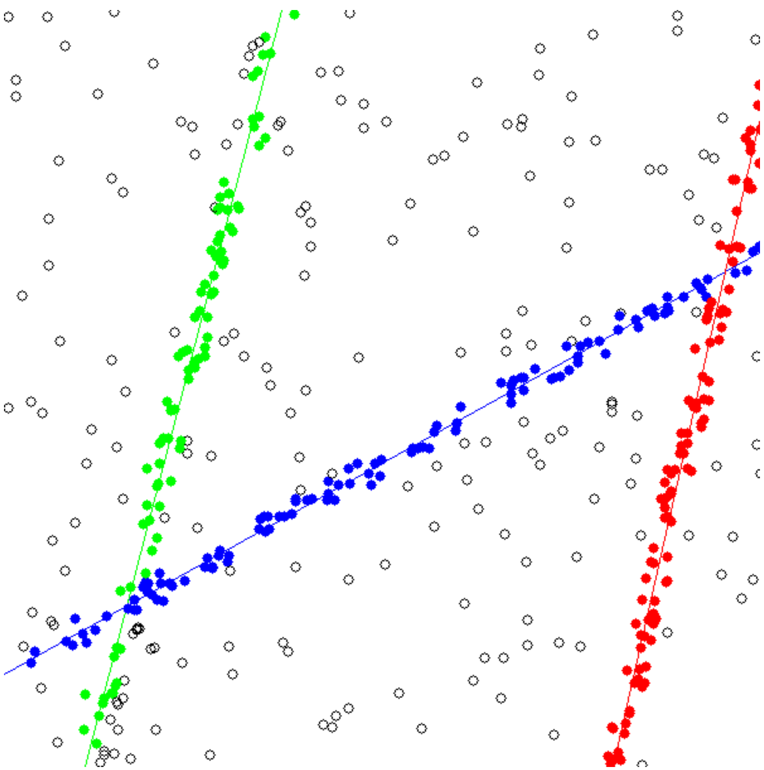
PEARL



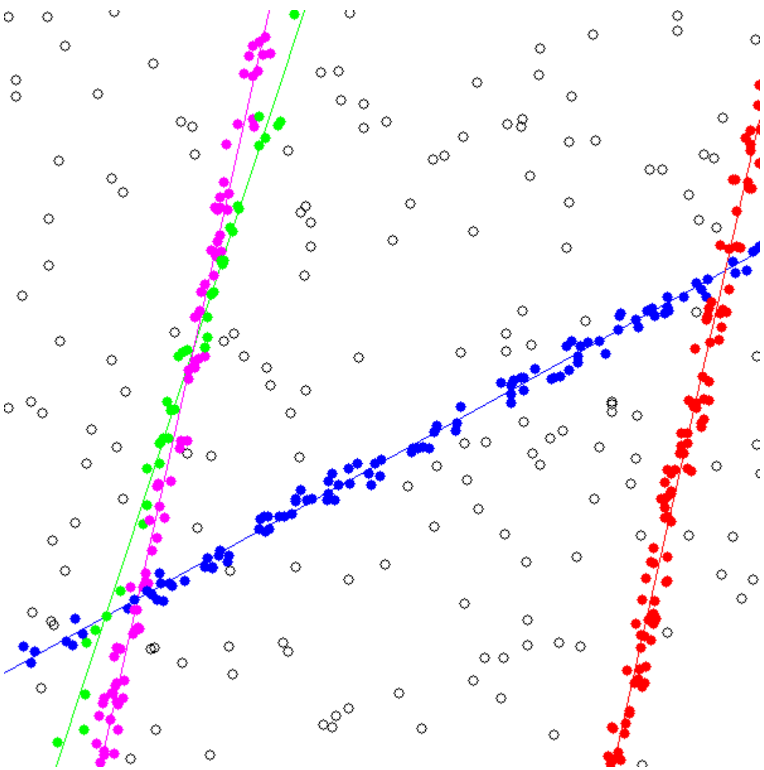
Experiment: 2nd iteration, re-estimation

Re-estimation of instance parameters based on the inliers.

Multi-X



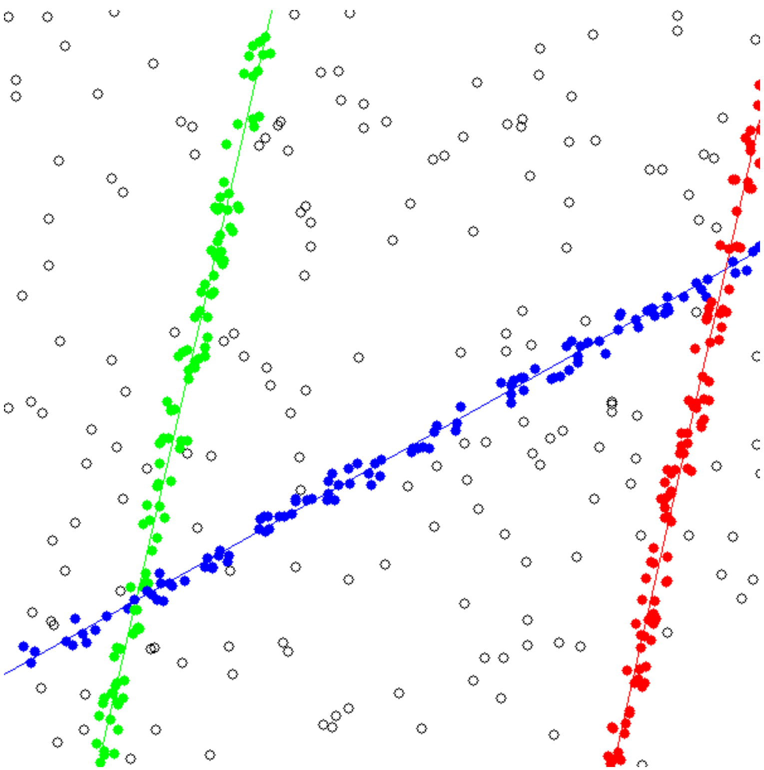
PEARL



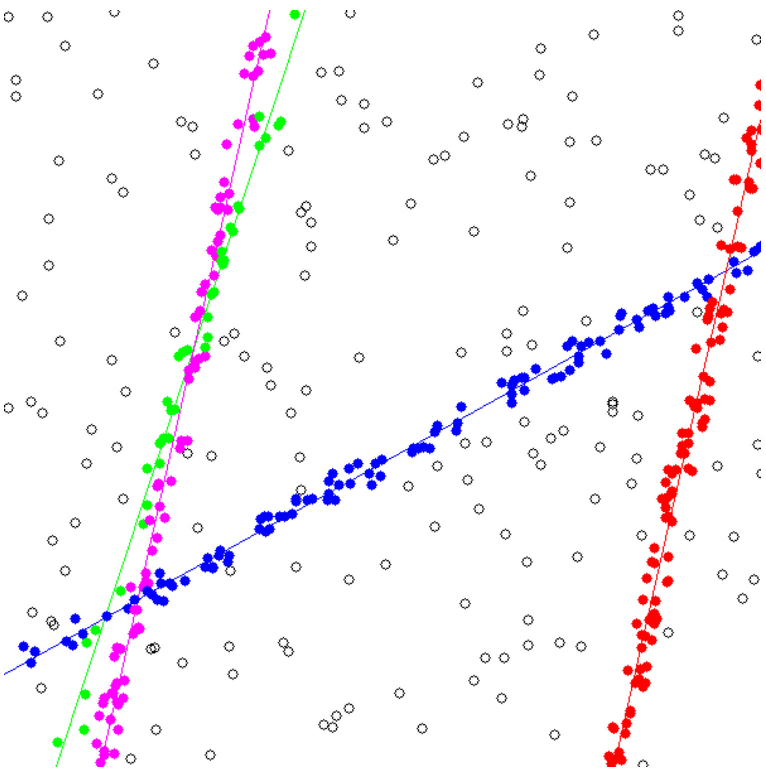
Experiment: Final iteration, labeling

Colors denote instances. Non-filled circles are labelled outliers.

Multi-X



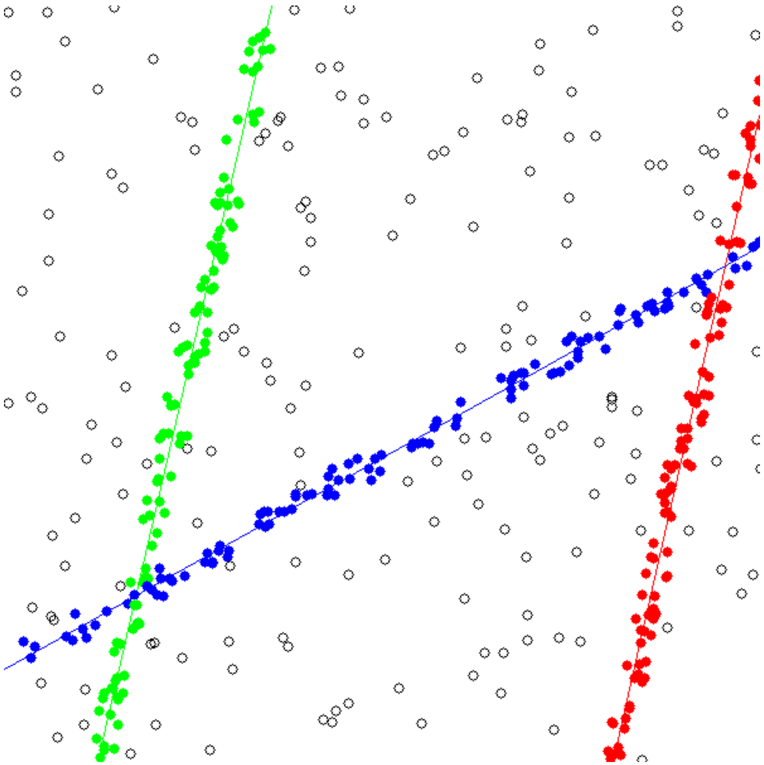
PEARL



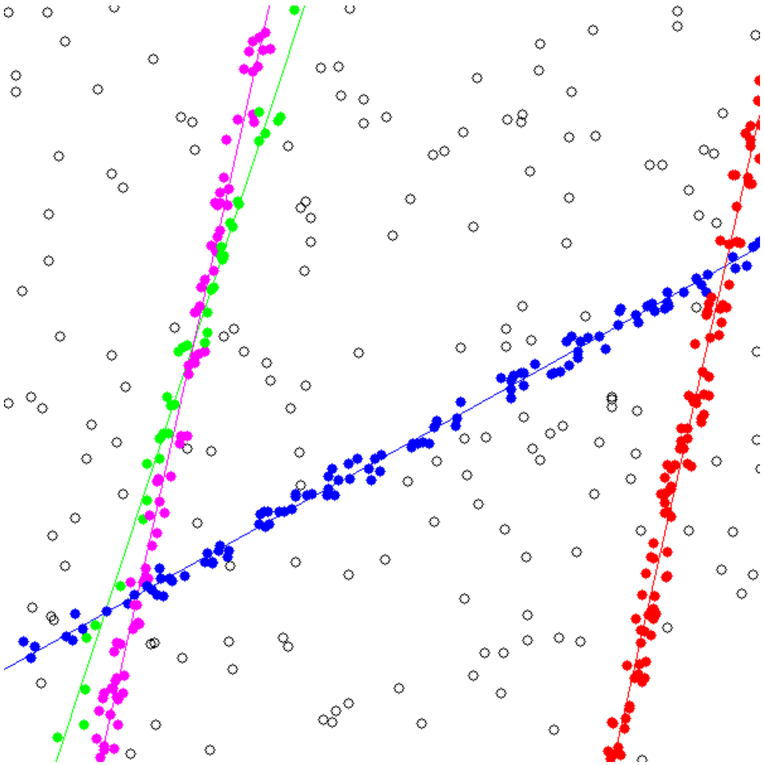
Experiment: Final iteration, re-estimation

Re-estimation of instance parameters based on the inliers.

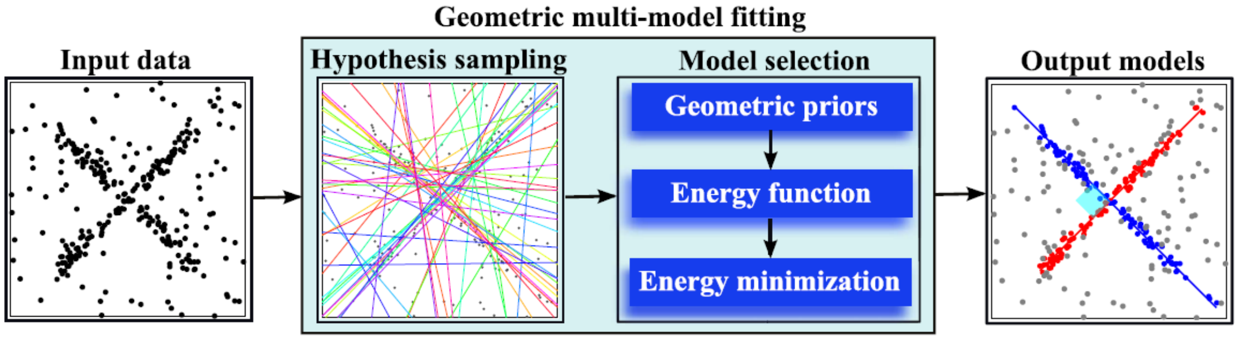
Multi-X



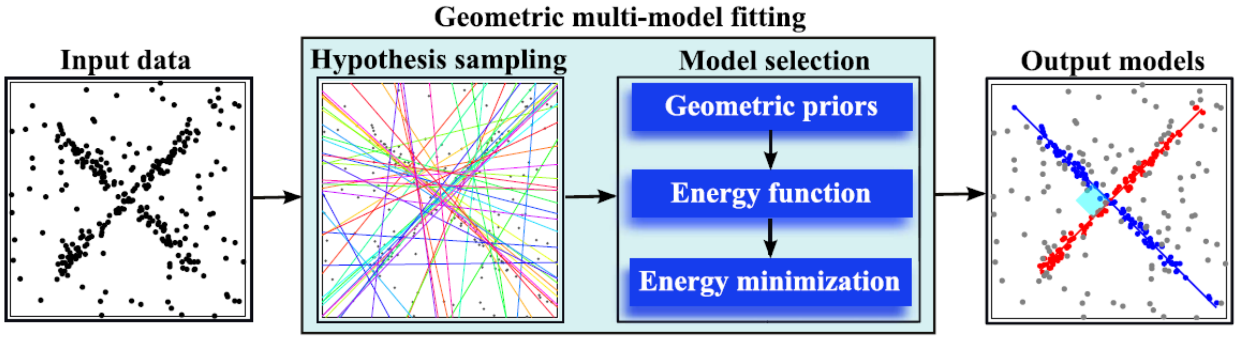
PEARL



Model fitting with geometric priors (MFIGP)



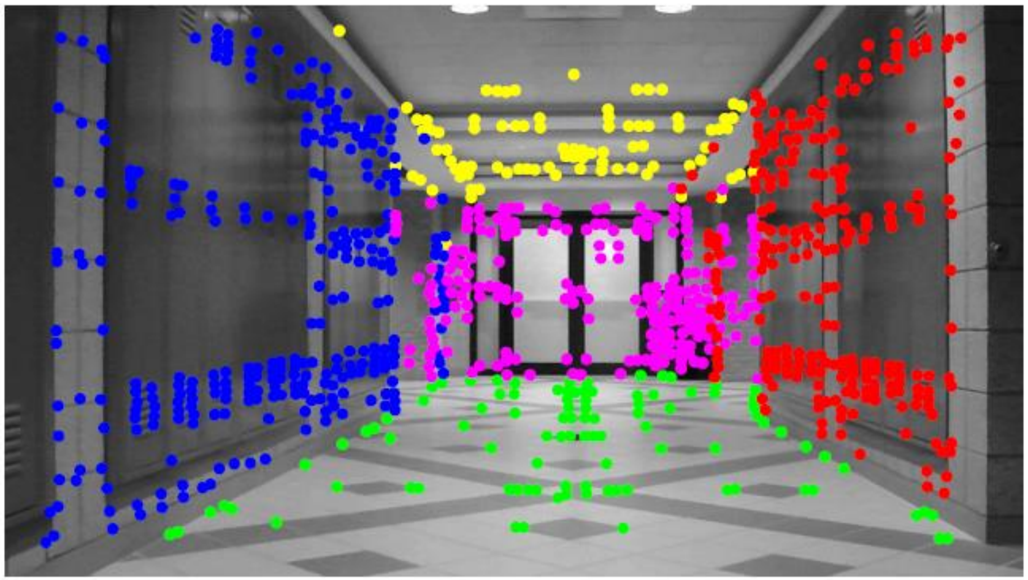
Model fitting with geometric priors (MFIGP)



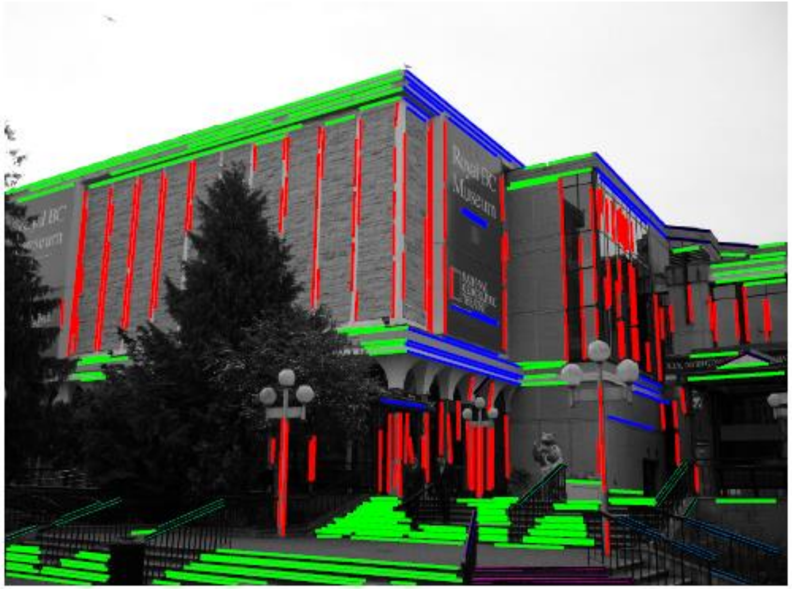
$$E(f) = \sum_{n=1}^{N} D_{\mathbf{x}_n}(f_n) + w_1 \sum_{\{p,q\} \in \mathcal{N}} V(f_p, f_q) + w_2 G(\Theta(f))$$

New energy term: geometric prior,
e.g. orthogonality of planes

Examples for geometric priors



(a) Plannar surface detection.



(b) Vanishing point detection.

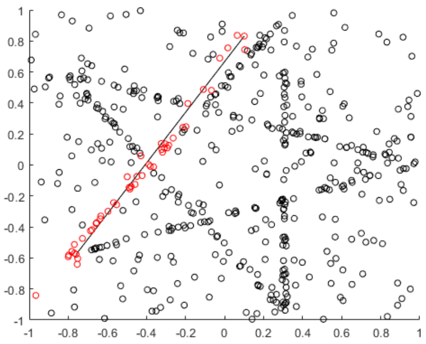
State-of-the-art techniques

Three major categories:

- Energy-minimization-based techniques
- **Preference analysis-based techniques**
- Hypergraph partitioning-based techniques

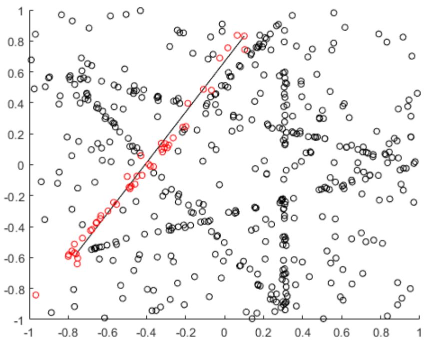
  (we will not talk about them)

J-Linkage (example)



- Generate a random model

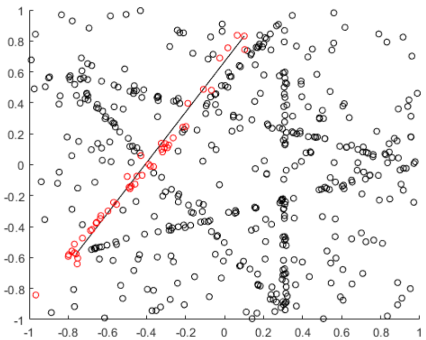
J-Linkage (example)



- Generate a random model
- Estimate its preference vector.
  A row for each point (0 – outlier, 1 – inlier)

	L1
P1	0
P2	0
P3	1
P4	1
P5	1
P6	1
P7	0
P8	0
P9	0
P10	0
P11	1
P12	1
P13	1
P14	0
P15	0
P16	1
P17	1
P18	1
P19	1
P20	1
P21	0
P22	0

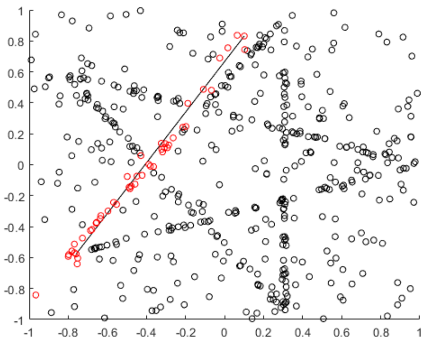
J-Linkage (example)

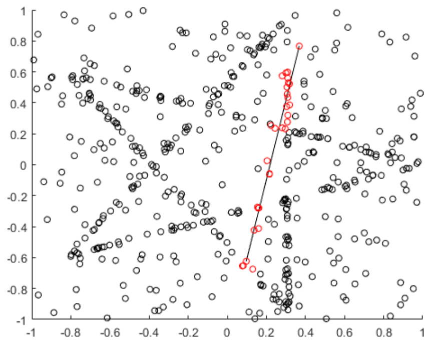


- Generate a random model
- Estimate its preference vector.
  A row for each point (0 – outlier, 1 – inlier)
- Repeat

	L1
P1	0
P2	0
P3	1
P4	1
P5	1
P6	1
P7	0
P8	0
P9	0
P10	0
P11	1
P12	1
P13	1
P14	0
P15	0
P16	1
P17	1
P18	1
P19	1
P20	1
P21	0
P22	0

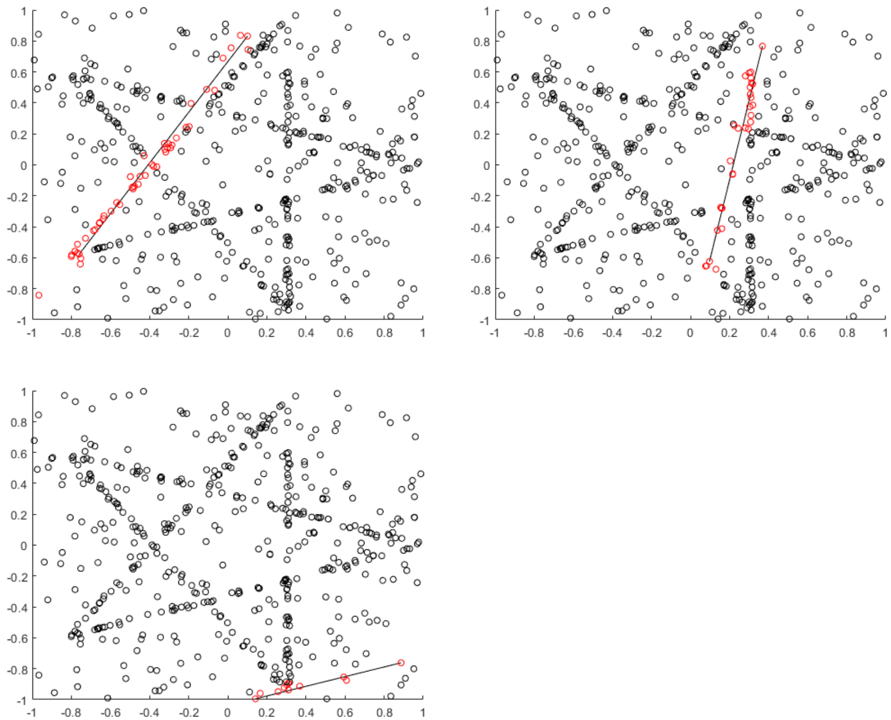
J-Linkage (example)





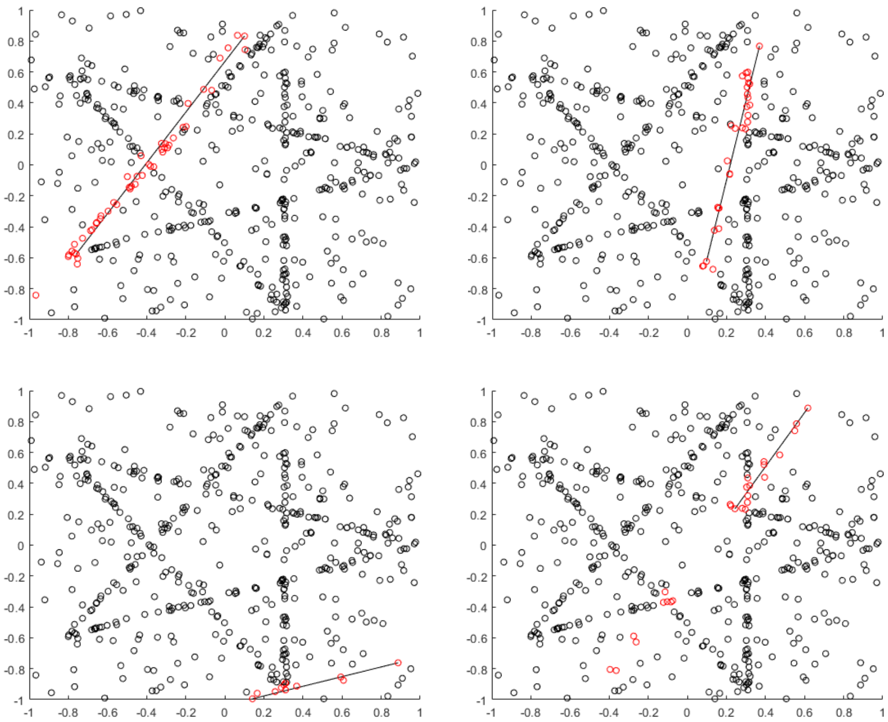
	L1	L2
P1	0	0
P2	0	0
P3	1	0
P4	1	0
P5	1	0
P6	1	0
P7	0	0
P8	0	0
P9	0	0
P10	0	0
P11	1	0
P12	1	0
P13	1	0
P14	0	0
P15	0	0
P16	1	0
P17	1	0
P18	1	0
P19	1	0
P20	1	0
P21	0	0
P22	0	0

J-Linkage (example)



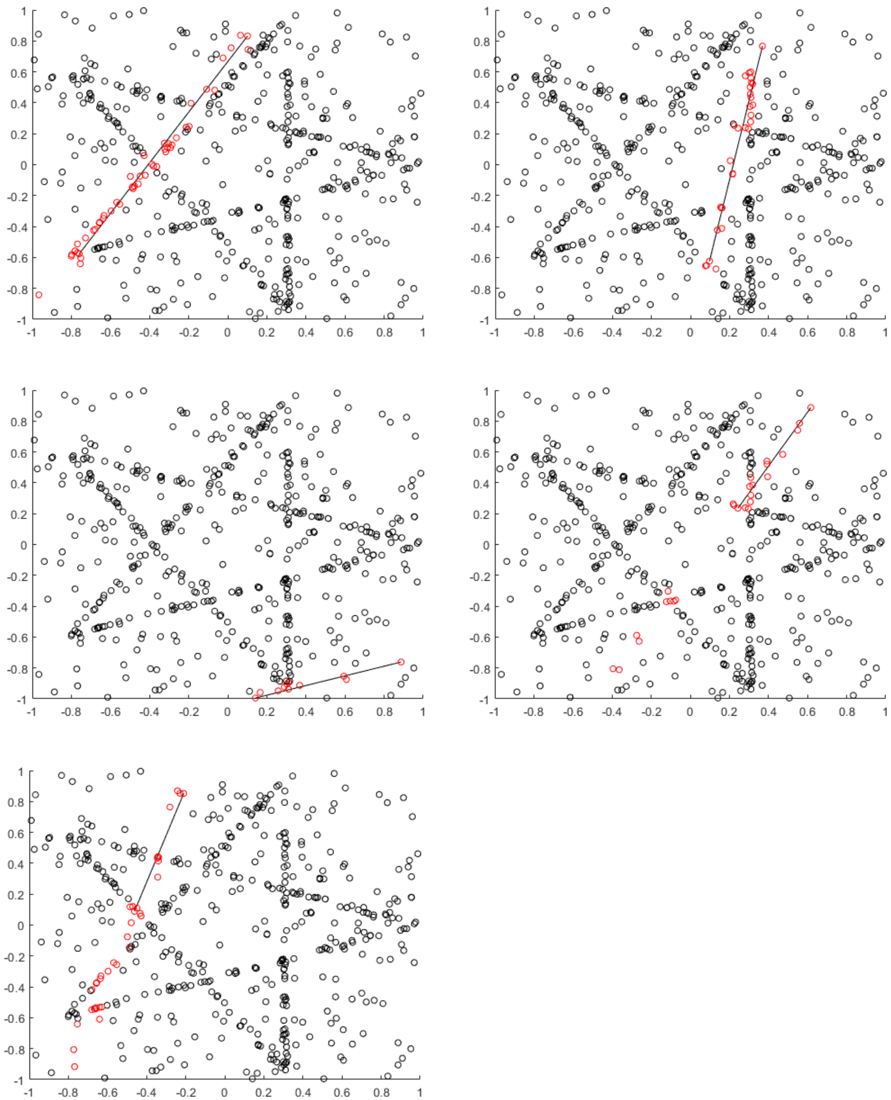
	L1	L2	L3
P1	0	0	0
P2	0	0	0
P3	1	0	0
P4	1	0	0
P5	1	0	0
P6	1	0	0
P7	0	0	0
P8	0	0	1
P9	0	0	1
P10	0	0	0
P11	1	0	1
P12	1	0	0
P13	1	0	0
P14	0	0	0
P15	0	0	0
P16	1	0	0
P17	1	0	0
P18	1	0	0
P19	1	0	0
P20	1	0	0
P21	0	0	0
P22	0	0	1

J-Linkage (example)



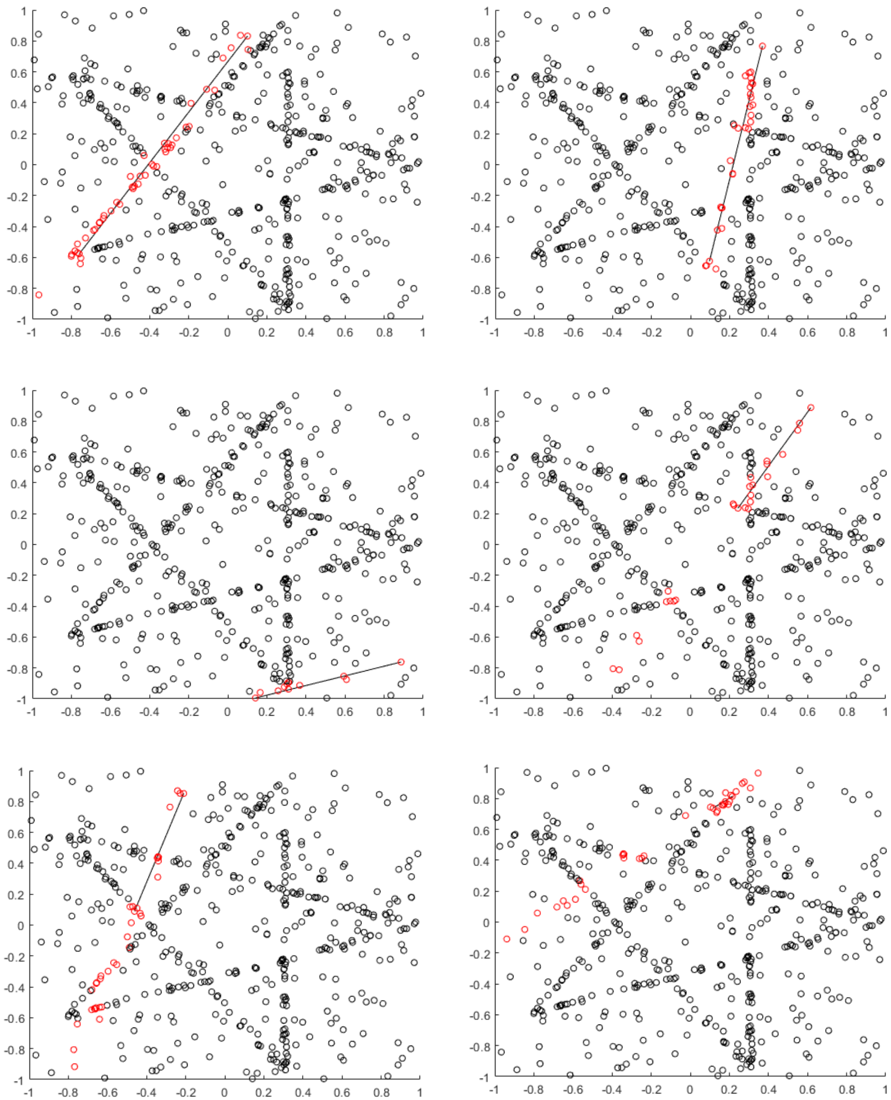
	L1	L2	L3	L4
P1	0	0	0	0
P2	0	0	0	1
P3	1	0	0	0
P4	1	0	0	1
P5	1	0	0	0
P6	1	0	0	0
P7	0	0	0	0
P8	0	0	1	1
P9	0	0	1	1
P10	0	0	0	0
P11	1	0	1	1
P12	1	0	0	0
P13	1	0	0	0
P14	0	0	0	0
P15	0	0	0	0
P16	1	0	0	0
P17	1	0	0	1
P18	1	0	0	0
P19	1	0	0	0
P20	1	0	0	0
P21	0	0	0	0
P22	0	0	1	1

J-Linkage (example)



	L1	L2	L3	L4	L5
P1	0	0	0	0	0
P2	0	0	0	1	0
P3	1	0	0	0	0
P4	1	0	0	1	0
P5	1	0	0	0	0
P6	1	0	0	0	1
P7	0	0	0	0	0
P8	0	0	1	1	0
P9	0	0	1	1	0
P10	0	0	0	0	0
P11	1	0	1	1	0
P12	1	0	0	0	0
P13	1	0	0	0	1
P14	0	0	0	0	1
P15	0	0	0	0	0
P16	1	0	0	0	0
P17	1	0	0	1	0
P18	1	0	0	0	0
P19	1	0	0	0	0
P20	1	0	0	0	0
P21	0	0	0	0	0
P22	0	0	1	1	0

J-Linkage (example)



	L1	L2	L3	L4	L5	L6
P1	0	0	0	0	0	0
P2	0	0	0	1	0	0
P3	1	0	0	0	0	0
P4	1	0	0	1	0	0
P5	1	0	0	0	0	0
P6	1	0	0	0	1	0
P7	0	0	0	0	0	0
P8	0	0	1	1	0	0
P9	0	0	1	1	0	0
P10	0	0	0	0	0	0
P11	1	0	1	1	0	0
P12	1	0	0	0	0	0
P13	1	0	0	0	1	1
P14	0	0	0	0	1	0
P15	0	0	0	0	0	0
P16	1	0	0	0	0	0
P17	1	0	0	1	0	0
P18	1	0	0	0	0	0
P19	1	0	0	0	0	0
P20	1	0	0	0	0	0
P21	0	0	0	0	0	0
P22	0	0	1	1	0	0

J-Linkage

- Merge the two most similar models

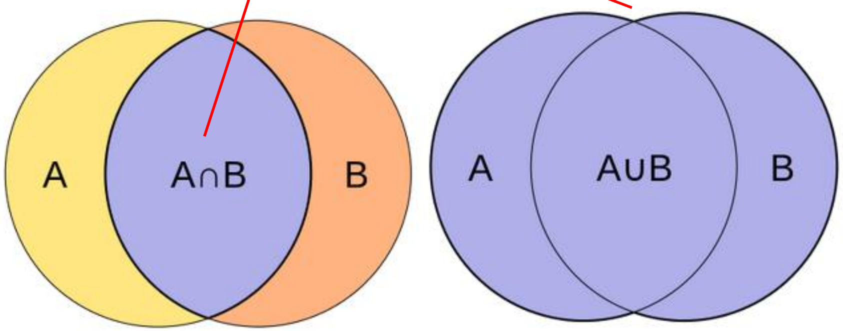
	L1	L2	L3	L4	L5	L6
P1	0	0	0	0	0	0
P2	0	0	0	1	0	0
P3	1	0	0	0	0	0
P4	1	0	0	1	0	0
P5	1	0	0	0	0	0
P6	1	0	0	0	1	0
P7	0	0	0	0	0	0
P8	0	0	1	1	0	0
P9	0	0	1	1	0	0
P10	0	0	0	0	0	0
P11	1	0	1	1	0	0
P12	1	0	0	0	0	0
P13	1	0	0	0	1	1
P14	0	0	0	0	1	0
P15	0	0	0	0	0	0
P16	1	0	0	0	0	0
P17	1	0	0	1	0	0
P18	1	0	0	0	0	0
P19	1	0	0	0	0	0
P20	1	0	0	0	0	0
P21	0	0	0	0	0	0
P22	0	0	1	1	0	0

J-Linkage

- Merge the two most similar models.

Jaccard-similarity:

$$J(A,B) = \frac{|A \cap B|}{|A \cup B|} = \frac{|A \cap B|}{|A| + |B| - |A \cap B|}.$$



	L1	L2	L3	L4	L5	L6
P1	0	0	0	0	0	0
P2	0	0	0	1	0	0
P3	1	0	0	0	0	0
P4	1	0	0	1	0	0
P5	1	0	0	0	0	0
P6	1	0	0	0	1	0
P7	0	0	0	0	0	0
P8	0	0	1	1	0	0
P9	0	0	1	1	0	0
P10	0	0	0	0	0	0
P11	1	0	1	1	0	0
P12	1	0	0	0	0	0
P13	1	0	0	0	1	1
P14	0	0	0	0	1	0
P15	0	0	0	0	0	0
P16	1	0	0	0	0	0
P17	1	0	0	1	0	0
P18	1	0	0	0	0	0
P19	1	0	0	0	0	0
P20	1	0	0	0	0	0
P21	0	0	0	0	0	0
P22	0	0	1	1	0	0

J-Linkage

- Merge the two most similar models.

Jaccard-similarity:

$$J(A,B) = \frac{|A \cap B|}{|A \cup B|} = \frac{|A \cap B|}{|A| + |B| - |A \cap B|}.$$

Example:
$|L_3 \cup L_4| = |\{P_1, P_4, P_8, P_9, P_{11}, P_{17}, P_{22}\}| = 7$

	L1	L2	L3	L4	L5	L6
P1	0	0	0	0	0	0
P2	0	0	0	1	0	0
P3	1	0	0	0	0	0
P4	1	0	0	1	0	0
P5	1	0	0	0	0	0
P6	1	0	0	0	1	0
P7	0	0	0	0	0	0
P8	0	0	1	1	0	0
P9	0	0	1	1	0	0
P10	0	0	0	0	0	0
P11	1	0	1	1	0	0
P12	1	0	0	0	0	0
P13	1	0	0	0	1	1
P14	0	0	0	0	1	0
P15	0	0	0	0	0	0
P16	1	0	0	0	0	0
P17	1	0	0	1	0	0
P18	1	0	0	0	0	0
P19	1	0	0	0	0	0
P20	1	0	0	0	0	0
P21	0	0	0	0	0	0
P22	0	0	1	1	0	0

J-Linkage

- Merge the two most similar models.

Jaccard-similarity:

$$J(A,B) = \frac{|A \cap B|}{|A \cup B|} = \frac{|A \cap B|}{|A| + |B| - |A \cap B|}.$$

Example:

$|L_3 \cup L_4| = |\{P_1, P_4, P_8, P_9, P_{11}, P_{17}, P_{22}\}| = 7$

$|L_3 \cap L_4| = |\{P_8, P_9, P_{11}, P_{22}\}| = 4$

	L1	L2	L3	L4	L5	L6
P1	0	0	0	0	0	0
P2	0	0	0	1	0	0
P3	1	0	0	0	0	0
P4	1	0	0	1	0	0
P5	1	0	0	0	0	0
P6	1	0	0	0	1	0
P7	0	0	0	0	0	0
P8	0	0	1	1	0	0
P9	0	0	1	1	0	0
P10	0	0	0	0	0	0
P11	1	0	1	1	0	0
P12	1	0	0	0	0	0
P13	1	0	0	0	1	1
P14	0	0	0	0	1	0
P15	0	0	0	0	0	0
P16	1	0	0	0	0	0
P17	1	0	0	1	0	0
P18	1	0	0	0	0	0
P19	1	0	0	0	0	0
P20	1	0	0	0	0	0
P21	0	0	0	0	0	0
P22	0	0	1	1	0	0

J-Linkage

- Merge the two most similar models.

Jaccard-similarity:

$$J(A,B) = \frac{|A \cap B|}{|A \cup B|} = \frac{|A \cap B|}{|A| + |B| - |A \cap B|}.$$

Example:

$|L_3 \cup L_4| = |\{P_1, P_4, P_8, P_9, P_{11}, P_{17}, P_{22}\}| = 7$

$|L_3 \cap L_4| = |\{P_8, P_9, P_{11}, P_{22}\}| = 4$

$J(L_3, L_4) = 4/7$

	L1	L2	L3	L4	L5	L6
P1	0	0	0	0	0	0
P2	0	0	0	1	0	0
P3	1	0	0	0	0	0
P4	1	0	0	1	0	0
P5	1	0	0	0	0	0
P6	1	0	0	0	1	0
P7	0	0	0	0	0	0
P8	0	0	1	1	0	0
P9	0	0	1	1	0	0
P10	0	0	0	0	0	0
P11	1	0	1	1	0	0
P12	1	0	0	0	0	0
P13	1	0	0	0	1	1
P14	0	0	0	0	1	0
P15	0	0	0	0	0	0
P16	1	0	0	0	0	0
P17	1	0	0	1	0	0
P18	1	0	0	0	0	0
P19	1	0	0	0	0	0
P20	1	0	0	0	0	0
P21	0	0	0	0	0	0
P22	0	0	1	1	0	0

J-Linkage

- Merge the two most similar models.

Jaccard-similarity:

$$J(A,B) = \frac{|A \cap B|}{|A \cup B|} = \frac{|A \cap B|}{|A| + |B| - |A \cap B|}.$$

Example:

$|L_3 \cup L_4| = |\{P_1, P_4, P_8, P_9, P_{11}, P_{17}, P_{22}\}| = 7$

$|L_3 \cap L_4| = |\{P_8, P_9, P_{11}, P_{22}\}| = 4$

$J(L_3, L_4) = 4/7$

	L1	L2	L3	L4	L5
P1	0	0	0	0	0
P2	0	0	0	0	0
P3	1	0	0	0	0
P4	1	0	0	0	0
P5	1	0	0	0	0
P6	1	0	0	1	0
P7	0	0	0	0	0
P8	0	0	1	0	0
P9	0	0	1	0	0
P10	0	0	0	0	0
P11	1	0	1	0	0
P12	1	0	0	0	0
P13	1	0	0	1	1
P14	0	0	0	1	0
P15	0	0	0	0	0
P16	1	0	0	0	0
P17	1	0	0	0	0
P18	1	0	0	0	0
P19	1	0	0	0	0
P20	1	0	0	0	0
P21	0	0	0	0	0
P22	0	0	1	0	0

- Repeat until there is something to merge.

T-Linkage

Similar to J-Linkage, but

- the preference vectors is not binary.
- the Tanimoto-distance is used as the similarity function
- different way to merge the preference vectors

Thank You!

Do not remove (macro defs for TexPoint)

common.tex