ELEC 390	Final Exam Review 5/9/2012	Yifan Ge

Effective area:

$$A_e = \frac{P_{int}}{S_i} \Rightarrow A_e = \frac{\lambda^2 D}{4\pi} \quad \text{(matched impedance condition)}$$

short dipole: $A_e = \frac{3\lambda^2}{8\pi}$ halfwave. $D = 1.64$.

Friis Transmission Formula

$$\frac{P_{rec}}{P_t} = G_t G_r \left(\frac{\lambda}{4\pi R}\right)^2 \quad \boxed{\text{power transfer ratio}}$$

Signal-to-noise ratio:

$P_n = K T_{sys} B$ where $K = 1.38 \times 10^{-23}$ J/k B is the receiver bandwidth in Hz.

$$SNR = \frac{P_{rec}}{P_n} \quad \text{(dimensionless)}.$$

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electric and magnetic field vectors.

$$\tilde{E} = \hat{x}\,\tilde{E}_{ox}\,e^{-jkz} + \hat{y}\,\tilde{E}_{oy}\,e^{-jkz}$$ wave propagate in $+z$ direction

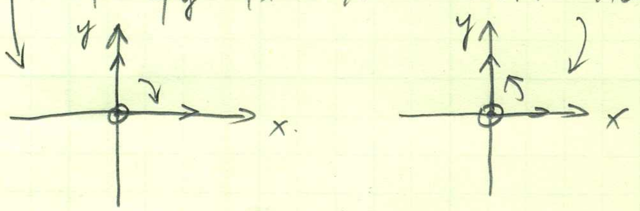
$$\tilde{H} = \frac{1}{\eta}\hat{k} \times \tilde{E}$$ ($\hat{k}$ is the wave propagation direction)

$$k = \omega\sqrt{\mu\epsilon} = \frac{\omega}{\mu_p}$$ wavenumber

$$\Rightarrow \tilde{E} = (\hat{x}\,a_x e^{j\varphi_x} + \hat{y}\,a_y e^{j\varphi_y})\,e^{-jkz}$$ $+z$ direction

If $\varphi_y - \varphi_x = 90^\circ$. Right Hand. y leads x.

If $\varphi_y - \varphi_x = -90^\circ$ Left Hand. x leads y



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Maxwell's Equations:

	time-domain	phasor
Gauss's Law	$\nabla \cdot \vec{D} = \rho_v$	$\nabla \cdot \hat{D} = \hat{\rho}_v$
Faraday's Law	$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\nabla \times \hat{E} = -j\omega \hat{B}$
"Magnetic Gauss's" Law	$\nabla \cdot \vec{B} = 0$	$\nabla \cdot \hat{B} = 0$
Ampere's Law:	$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$	$\nabla \times \hat{H} = \hat{J} + j\omega \hat{D}$

Constituitive relations: $\vec{D} = \epsilon \vec{E}$, $\vec{B} = \mu \vec{H}$, $J = \sigma E$

Antenna:

far fields $d\hat{E} = \hat{\theta} \frac{jk\eta \hat{I}_0 dz}{4\pi R} e^{-jkR} \sin\theta$, $d\hat{H} = \hat{\varphi} \frac{jk\hat{I}_0 dz}{4\pi R} e^{-jkR} \sin\theta$

$F(\theta) = \sin^2\theta$

$D = 1.5$ (1.76 dBi)

Hertzian Dipole:

Exact: $\hat{E} = \hat{R} \frac{\hat{I}_0 k^2 l}{4\pi} \eta e^{-jkR} \left[\frac{2}{(kR)^2} - \frac{j2}{(kR)^3}\right] \cos\theta + \hat{\theta} \frac{\hat{I}_0 k^2 l}{4\pi} \eta e^{-jkR} \left[\frac{j}{kR} + \frac{1}{(kR)^2} - \frac{j}{(kR)^3}\right] \sin\theta$

$\hat{H} = \hat{\varphi} \frac{\hat{I}_0 k^2 l}{4\pi} e^{-jkR} \left[\frac{j}{kR} + \frac{1}{(kR)^2}\right] \sin\theta$

Far field: $kR \gg 1$

$\hat{E} = \hat{\theta} \frac{jk\eta \hat{I}_0 l}{4\pi R} e^{-jkR} \sin\theta \qquad \hat{H} = \hat{\varphi} \frac{jk\hat{I}_0 l}{4\pi R} e^{-jkR} \sin\theta$

$F(\theta) = \sin^2\theta$

Short Dipole:

far field: $\hat{E} = \hat{\theta} \frac{jk\eta \hat{I}_0 l}{8\pi R} e^{-jkR} \sin\theta \qquad \hat{H} = \hat{\varphi} \frac{jk\hat{I}_0 l}{4\pi R} e^{-jkR} \sin\theta$

Triangular current distribution

→ Poynting vector: $\underline{\vec{S}_{av}} = \frac{1}{2} \mathrm{Re}\{\hat{E} \times \hat{H}^*\} = \hat{R}\, S_{max}\, F(\theta, \varphi)$

max magnitude (S_{max}); normalized Radiation Pattern (F), Max = 1

→ Directivity and Gain:

Radiated Power: $P_{rad} = S_{max} R^2 \int_0^{2\pi}\int_0^{\pi} F(\theta,\varphi) \sin\theta \, d\theta \, d\varphi$.

$D = 4\pi R^2 \frac{S_{max}}{P_{rad}} = \frac{S_{max}}{|S_{iso}|} = \frac{4\pi}{\iint F(\theta,\varphi)\sin\theta \, d\theta\, d\varphi}$ $\left(S_{iso} = \hat{R} \frac{P_{in}}{4\pi R^2} \right)$

$\xi = \frac{R_{rad}}{R_{rad} + R_{loss}} \Rightarrow G = \xi D$, $P_{rad} = \xi P_{in}$ $\boxed{S_{max} = \frac{P_{in} G}{4\pi R^2}}$ ☆

R_{rad}: Hertzian: $80\pi^2 \left(\frac{l}{\lambda}\right)^2$; short: $20\pi^2 \left(\frac{l}{\lambda}\right)^2$

→ Radiation resistance: $R_{rad} = 2P_{rad}/|I_0|^2$

→ Loss resistance:

$R_{loss} = \frac{2P_{loss}}{|I_0|^2}$, Hertzian: $R_{loss} = \frac{1}{2\pi a}\sqrt{\frac{\pi f \mu_c}{\sigma_c}}$; short: $R_{loss} = \frac{1}{6\pi a}\sqrt{\frac{\pi f \mu_c}{\sigma_c}}$

$\lambda/2$ dipole: $R_{loss} = \frac{\lambda}{8\pi a}\sqrt{\frac{\pi f \mu_c}{\sigma_c}}$; arbitrary Length: $R_{loss} = \frac{1}{2\pi a |I_0|^2}\sqrt{\frac{\pi f \mu_c}{\sigma_c}} \int_{-l/2}^{l/2} |I(z)|^2 dz$.

Center-fed Arbitrary length:

$I(z) = \begin{cases} I_0 \sin\left[k\left(\frac{l}{2} - |z|\right)\right] & -\frac{l}{2} < z < \frac{l}{2} \\ 0 & \text{else where} \end{cases}$

$\hat{E} = \hat{\theta}\, j \frac{\eta}{2\pi} \hat{I}_0 \frac{e^{-jkR}}{R} \left[\frac{\cos(0.5kl\cos\theta) - \cos(0.5kl)}{\sin\theta}\right]$ $|\hat{H}| = \frac{|\hat{E}|}{\eta}$

$F(\theta) = \frac{1}{[1-\cos(0.5kl)]^2} \left| \frac{\cos[0.5kl\cos\theta] - \cos(0.5kl)}{\sin\theta} \right|^2$

Half-wave dipole:

$D = 1.64$ (2.15 dBi)

$\hat{E} = \hat{\theta}\, j \frac{\eta}{2\pi} \hat{I}_0 \frac{e^{-jkR}}{R} \left[\frac{\cos(0.5\pi\cos\theta)}{\sin\theta}\right]$ $|\hat{H}| = \frac{|\hat{E}|}{\eta}$

Phase approximation:

$e^{-jk|R - R'|} \approx e^{-jkR} e^{jkz'\cos\theta}$

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Input Impedance

$$Z_{in} = Z_0 \left(\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} \right) \qquad Y_{in} = Y_0 \left(\frac{Y_L + jY_0 \tan\beta l}{Y_0 + jY_L \tan\beta l} \right)$$ Loss less

Lossy line:

$$Z_{in} = Z_0 \left(\frac{Z_L + Z_0 \tanh\gamma l}{Z_0 + Z_L \tanh\gamma l} \right) \qquad \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

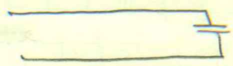
Special Cases:

1) $Z_L = Z_0 \Rightarrow \Gamma = 0 \Rightarrow \boxed{Z_{in} = Z_0}$
2) $l \ll \lambda \to \beta l \ll 1$
   $\Rightarrow Z_0 \tan\beta l \ll Z_L \Rightarrow Z_{in} = Z_L$
3) $Z_L = 0 \Rightarrow \boxed{Z_{in} = jZ_0 \tan\beta l}$ = closed end.
4) $Z_L \to \infty \Rightarrow \boxed{Z_{in} = -jZ_0 \cot\beta l}$ = open end
5) $l = \lambda/2 \Rightarrow \boxed{Z_{in} = Z_L}$
6) $l = \lambda/4 \Rightarrow \beta l = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2} \Rightarrow \boxed{\frac{Z_0^2}{Z_L}}$ ← important

When load is a reactance $Z_L = jX$

$$Z_{in} = jZ_0 \frac{X + Z_0 \tan\beta l}{Z_0 - X\tan\beta l} \qquad |\Gamma| = 1.$$

Wave Trap



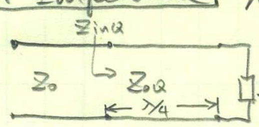
$Z_{in} = 0$ @ Designed frequency.

$\Rightarrow X + Z_0 \tan\beta l = 0 \Rightarrow X = -Z_0 \tan\left(\frac{2\pi l}{\lambda}\right)$

Match Impedance

① Maximum power transfer ② prevent reflection ③ specified Z_L

①



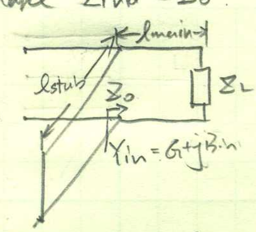
If $Z_L \Rightarrow$ Pure Real $\quad Z_{0Q} = \sqrt{Z_{in} Z_L}$.

Otherwise, add at V_{max} or V_{min}. $\quad 2\beta z + \theta_r = 2\pi n$ or $(2n+1)\pi$

$$Z(-d_{max}) = Z_0 \frac{1+|\Gamma|}{1-|\Gamma|} \qquad Z(-d_{min}) = Z_0 \frac{1-|\Gamma|}{1+|\Gamma|}$$

make $Z_{in} = Z_0$

②



Let $Y_{in} = Y_0 = 1/Z_0$.

$$l^{shunt}_{main} = \frac{\lambda}{4\pi}\left[\theta_r \pm \cos^{-1}(-|\Gamma|)\right]$$

$B_{stub} = -B_{in}$

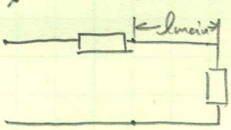
$X_{stub} = -X_{in}$

$$B^{shunt}_{in} = \pm \frac{2|\Gamma|}{\sqrt{1-|\Gamma|^2}} Y_0$$

$$l_{stub,sc} = \frac{\lambda}{2\pi} \tan^{-1}\left(\frac{-Y_0}{B_{stub,sc}}\right)$$

$$l_{stub,oc} = \frac{\lambda}{2\pi} \tan^{-1}\left(\frac{-Z_0}{X_{stub,OC}}\right)$$

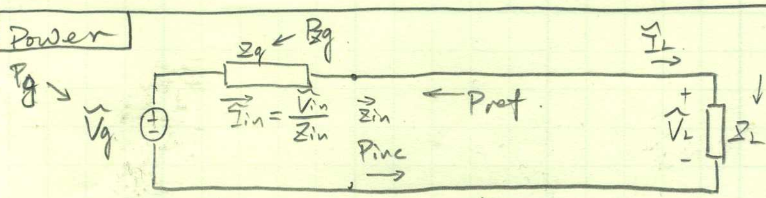
③



$$l^{series}_{main} = \frac{\lambda}{4\pi}\left[\theta_r \pm \cos^{-1}(|\Gamma|)\right]$$

$$X^{series}_{in} = \mathrm{Im}\{Z_{in}(-l^{series}_{main})\} = \mp \frac{2|\Gamma|}{\sqrt{1-|\Gamma|^2}} Z_0$$

Power



$P_L = P_{inc} - P_{ref}$

$$V^+ = \hat{V}_{in} \frac{1}{e^{j\beta l} + \Gamma e^{-j\beta l}} \qquad \hat{V}_{in} = \hat{V}_g \left(\frac{Z_{in}}{Z_g + Z_{in}}\right)$$

$$\hat{V}_L = V_0^+(1+\Gamma) \qquad \hat{I}_L = \frac{V_0^+}{Z_0}(1-\Gamma). \quad \Rightarrow P_L = \frac{1}{2}\mathrm{Re}\{\hat{V}_L \hat{I}_L^*\} = \frac{|V_0^+|^2}{2Z_0}(1-|\Gamma|^2)$$

$$P_{inc} = \frac{|V_0^+|^2}{2Z_0} \qquad P_{ref} = |\Gamma|^2 \frac{|V_0^+|^2}{2Z_0} \qquad P_g = -\frac{1}{2}\mathrm{Re}\{\hat{V}_g \hat{I}_{in}^*\}$$

$$P_{Z_g} = \frac{1}{2}\mathrm{Re}\{(\hat{I}_{in} Z_g)\hat{I}_{in}^*\} = \frac{1}{2}|\hat{I}_{in}|^2 Z_g$$

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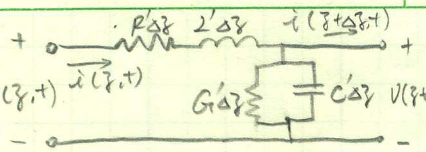

|Xmn line equations|

① $-\frac{\partial v(z,t)}{\partial z} = R' i(z,t) + L' \frac{\partial i(z,t)}{\partial t} \Rightarrow -\frac{d\hat{V}(z)}{dz} = (R'+j\omega L')\hat{I}(z)$

② $-\frac{\partial i(z,t)}{\partial z} = G' v(z,t) + C' \frac{\partial v(z,t)}{\partial t} \Rightarrow -\frac{d\hat{I}(z)}{dz} = (G'+j\omega C')\hat{V}(z)$

Wave Equations:

$d①/dz$

$$\Rightarrow \frac{d^2 \hat{V}(z)}{dz^2} - \gamma^2 \hat{V}(z) = 0$$

$d②/dz$

$$\Rightarrow \frac{d^2 \hat{I}(z)}{dz^2} - \gamma^2 \hat{I}(z) = 0$$

complex propagation constant → $\gamma^2 = (R'+j\omega L')(G'+j\omega C') = \alpha + j\beta$ ← phase constant; α ← attenuation constant

$$\hat{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \qquad \hat{I}(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

$$\frac{V_0^+}{I_0^+} = Z_0 = \frac{-V_0^-}{I_0^-} = \frac{R'+j\omega L'}{\gamma} = \sqrt{\frac{R'+j\omega L'}{G'+j\omega C'}}$$ $\boxed{\text{Characteristic impedance}}$

$$v(z,t) = Re(\hat{V}(z)e^{j\omega t}) = |V_0^+|e^{-\alpha z}\cos(\omega t - \beta z + \varphi^+) + |V_0^-|e^{\alpha z}\cos(\omega t + \beta z + \varphi^-)$$

$u_p = f\lambda = \frac{\omega}{\beta}$ $\qquad \lambda_g$ = guide wavelength

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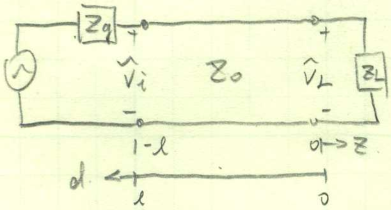
Lossless XMN LINE $\quad R' \ll \omega L' \quad G' \ll \omega C' \Rightarrow \gamma = j\omega\sqrt{L'C'} = j\beta \qquad Z_0 = \sqrt{\frac{L'}{C'}}$

$\lambda f = u_p = \frac{1}{\sqrt{\mu_0 \varepsilon_r \varepsilon_0}} = \frac{c}{\sqrt{\varepsilon_r}} \qquad \mu_0 = 4\pi \times 10^{-7}$ H/m $\qquad \varepsilon_0 = 8.854 \times 10^{-12}$ F/m $\qquad u_p = \frac{1}{\sqrt{L'C'}}$

$$\hat{V} = \underbrace{V_0^+ e^{-j\beta z}}_{\text{incident wave}} + \underbrace{V_0^- e^{j\beta z}}_{\text{reflected wave}} \qquad \hat{I} = I_0^+ e^{-j\beta z} + I_0^- e^{j\beta z}$$



$\hat{V}_L = \hat{V}(z=0) = V_0^+ + V_0^- \qquad \hat{I}_L = \hat{I}(z=0) = \frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0}$

$Z_L = \left(\frac{V_0^+ + V_0^-}{V_0^+ - V_0^-}\right) Z_0 \Rightarrow V_0^- = \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right) V_0^+$

$-\frac{I_0^-}{I_0^+} = \Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{z_L - 1}{z_L + 1}$ $\boxed{\text{Voltage reflection coefficient}}$

$\Gamma = |\Gamma| e^{j\theta_r} \quad (|\Gamma| \le 0) \qquad$ ① $\Gamma = 0 \ (Z_L = Z_0) \qquad$ ② $\Gamma = 1 \ (Z_L = \infty) \qquad$ ③ $\Gamma = -1 \ (Z_L = 0)$

$$|\hat{V}(z)| = [\hat{V}(z)\hat{V}^*(z)]^{1/2} = \{[V_0^+(e^{-j\beta z} + |\Gamma|e^{j\theta_r}e^{j\beta z})]\cdot[V_0^{+*}(e^{j\beta z} + |\Gamma|e^{-j\theta_r}e^{-j\beta z})]\}^{1/2}$$

$$= |V_0^+|\left[1 + |\Gamma|^2 + 2|\Gamma|\cos(2\beta z + \theta_r)\right]^{1/2}$$

$|\hat{V}(z)|_{max}$ @ $(2\beta z + \theta_r) = 2n\pi \Rightarrow z = \frac{2n\pi - \theta_r}{2\beta} \qquad |\hat{V}|_{max} = |V_0^+|[1 + |\Gamma|]$

$|\hat{V}(z)|_{min}$ @ $(2\beta z + \theta_r) = (2n+1)\pi \Rightarrow z = \frac{(2n+1)\pi - \theta_r}{2\beta} \qquad |\hat{V}|_{min} = |V_0^+|[1 - |\Gamma|]$

Voltage Standing Wave Ratio $\quad S = \frac{|\hat{V}|_{max}}{|\hat{V}|_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|} \quad$ (SWR)

$$|\Gamma| = \frac{S-1}{S+1}$$

$R_s = \sqrt{\frac{\pi f \mu_c}{\sigma_c}} \qquad L'C' = \mu\varepsilon \qquad \frac{G'}{C'} = \frac{\sigma}{\varepsilon}$