Research Statement

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My most recent work has been in studying measurable-preserving dynamical systems. A continuous dynamical system is a map $\pi : \mathbb{R} \times X \to X$, where X is some topological space. If a system is measure-preserving, I can take any subset $A \subseteq X$, find it's measure, and apply $\pi(t, A)$ to that subset, for any value of t, and it's measure does not change. I am currently focusing on the conjecture that there exists a (C^r, C^∞, C^ω, piecewise-linear) measure-preserving dynamical system on $\mathbb{R}^3$, with all trajectories uniformly bounded.

The question originates with a problem of Ulam's in the Scottish Book [11]. A construction of Jones and Yorke [6] showed a technique for bounding all of the trajectories, but without uniform bound, and without measure preservation. Kuperberg and Reed [9] solved Ulam's original problem, by constructing a dynamical system on $\mathbb{R}^3$ with all trajectories uniformly bounded. This was eventually modified to avoid compact trajectories [10], but the measure preserving element was still missing. In fact, the construction of uniformly bounded trajectories specifically required that the system not preserve measure. I was able to construct on $\mathbb{R}^3$, C^0 and C^∞ dynamical systems with bounded trajectories [3] both of which are volume-preserving. These were achieved by modifying flows using plugs, as in [17] and [8]. I am currently supervising undergraduate research on a C^∞ volume-preserving dynamical system, with uniformly bounded trajectories, on $\mathbb{R}^5$. Extensions of this problem to either $\mathbb{R}^4$ or $\mathbb{R}^3$ would be appropriate for an undergraduate student who had previously studied multivariable calculus or differential equations.

In this same vein, I investigate piecewise-linear dynamical systems, using foliations. Since piecewise-linear dynamical systems do not have smooth vector fields, we have to study them as measured foliations, as described in [13] and [7]. To this end I have constructed a PL foliation of $\mathbb{R}^3$, which is measured, and where all leaves are contained in bounded sets [4]. I am working towards making this bound uniform. The entry point is a foliation of $\mathbb{R}^3$ by Vogt [16], where each leaf is a circle. I am investigating if there is a way to make this foliation measured, possible by inserting volume-preserving piecewise-linear plugs to change the foliation.

My second area of interest in in tiling spaces. A tiling is a subdivision of $\mathbb{R}^n$ into a possibly infinite collection of pairwise disjoint subsets, each of which is homeomorphic to an open ball. Depending on the placement of the origin in a particular tile, we can generate an infinite number of tilings, using the same set of tiles. It is possible to define a metric on two of these tilings, and by taking the completing of the set of all tilings under this metric, we get a tiling space [15]. The topology of this tiling space can give us information about the underlying tiling that generated the space. We have cohomology groups that can be calculated for the space as in [1], and a theory for homology computations [12]. There is a theory that a duality exists [5], but this is currently unproven. The homology theory extends to a larger class of spaces, called Smale Spaces [14], but there are not many computational techniques established for this theory. I am interested in how to develop a more robust theory of computation for these invariants. While the algebraic topology may be too advanced to involve undergraduate students, there are questions about how tiling spaces are constructed, and how symbolic dynamics can be used in studying tiling spaces, which would be excellent undergraduate research projects.

I am also interested in research in mathematics education. I study how active learning techniques can affect mathematical self-efficacy, and how that in turn can moderate anxiety's effect on performance. It is demonstrated in the literature that increasing a student's perceived mathematical self-efficacy can moderate the effects of high anxiety on test performance, for example in [2]. I am interested in measuring how active learning techniques, such as team-based or inquiry-based learning, and an emphasis on productive classroom struggle, such as standards-based grading, can increase self-efficacy. I have presented on the techniques used in these active learning methods, and am currently developing a study to gather data on self-efficacy and mathematics anxiety. It is my intention to continue this study for the foreseeable future, and undergraduate participation would be very welcome. Particularly students who are interested in becoming educators would benefit from joining in this study.

References

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