

# Household Debt Relief and the Debt Laffer Curve\*

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November 2023

## Abstract

Debt relief programs are often implemented in debt crises to reduce debt overhang and promote economic recovery. If debt overhang is severe, then debt relief can even benefit creditors by increasing repayment rates. This paper studies the impact of a large-scale household debt relief program in Hungary that reduced outstanding debt burdens by 20% for over 600,000 housing loans. Using regression discontinuity and difference-in-differences research designs, we find that debt relief persistently lowered default rates, especially for heavily indebted borrowers. We estimate the Debt Laffer Curve, which relates the net present value of debt to its face value, and find that it is hump-shaped, inverting for high levels of indebtedness. Debt relief generates an increase in labor income that accounts for part of the increase in repayment rates. A structural model of household debt and default can account for these patterns.

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\*We thank Christopher Palmer and Judit Rariga for helpful comments. The views expressed in this paper do not reflect the views of the Central Bank of Hungary.

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# 1 Introduction

Debt crises are episodes of high debt burdens and widespread defaults (Reinhart and Rogoff, 2009; Müller and Verner, 2023). A common, and often contentious, policy response in these crises is to implement debt relief to alleviate high debt burdens and promote economic recovery. Examples of debt relief programs abound in economic history. The Code of Hammurabi in Ancient Mesopotamia prescribed debt amnesty in the event of natural disasters. In the Great Depression, the Roosevelt administration abrogated gold clauses in debt contracts, and states enacted debt moratoria on mortgages (Alston, 1983, 1984; Kroszner, 1998; Edwards, 2018). The Less-Developed Country debt crisis of the 1980s resulted in debt restructuring through the “Brady Bonds.” The 2008 Financial Crisis led to several debt reduction programs for indebted mortgage borrowers in the U.S. (Ganong and Noel, 2020).<sup>1</sup> In 2022, the Biden administration cancelled \$200 billion dollars in student debt, citing “crushing” effects of debt on vulnerable borrowers.

Theory offers several reasons for why debt relief may be optimal in a debt crisis. One prominent argument is that debt relief can alleviate debt overhang (Myers, 1977).<sup>2</sup> High debt burdens distort the incentives to generate cash flows to service the debt, as these cash flows disproportionately benefit creditors. Debt relief can mitigate this distortion and thereby expand economic activity. While debt relief directly reduces the face value of debt, it can indirectly increase the net present value of debt by increasing repayment rates. As a result, the Debt Laffer Curve, which relates the net present value of repayment to the face value of debt, can be hump-shaped.

This paper examines the impact of a large-scale debt relief program for mortgage borrowers on borrower behavior. We pose the following questions: What is the impact of debt relief on repayment and borrower outcomes? How does this vary across the distribution of indebtedness? Are there circumstances under which debt relief increases the net present value of repayment, thereby benefitting both the borrower and the lender?

Our empirical setting is a large-scale debt relief program for foreign currency (FC)

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<sup>1</sup>Laryea and Laeven (2009) provides a brief description of some country episodes of large scale household debt restructuring programs.

<sup>2</sup>There are other important related arguments for debt relief. There can be positive general equilibrium effects of reducing debt burdens, as debt can boost aggregate demand and mitigate costly fire sales (Mian and Sufi, 2015; Korinek and Simsek, 2016). Moreover, debt relief can complete incomplete debt contracts and mitigate an inefficient fall in consumption and investment caused by liquidity constraints (Bolton and Rosenthal, 2002).

loans implemented in Hungary in 2015. There are several appealing features of this setting for studying the impact of debt relief. First, the program was large. It lowered debt burdens by 20% for about 600,000 FC housing loans. Second, the program contains a discrete eligibility threshold based on a loan's date of origination, which allows us to credibly estimate response to debt relief. Third, there is substantial *ex ante* variation in the degree of indebtedness. The policy occurred in the aftermath of a severe FC household debt crisis. The default rate on FC loans exceeded 20%, and more than half of borrowers had higher debt balances in local currency terms than they originally borrowed. This allows us to estimate the slope of the Debt Laffer Curve at for a wide range of indebtedness, including for heavily indebted borrowers.

We begin by presenting a conceptual framework to organize our empirical analysis. In a standard model of household debt overhang, we show that high debt burdens can reduce the incentives for households to generate income to service the debt. The cost of effort is born by the household, but the benefits of effort largely go to the lender when debt is high and the borrower is likely to default. The model thus generates a hump-shaped Debt Laffer Curve. The framework highlights that a key empirical objects to assess the impact of debt relief on the net present value of a loan is the effect of debt relief on the probability of default.

Motivated by the conceptual framework, we estimate the effect of debt relief on default. We use administrative credit registry data with detailed information on loan characteristics, debt positions, repayment, and default. Using regression discontinuity and difference-in-difference research designs, we find that the program reduced the probability of default by 6.5 percentage points. A one percent reduction in debt is associated with a 0.25–0.4 percentage point reduction in the probability of default. Moreover, the sensitivity of default with respect to debt relief is heterogeneous and largest for borrowers with the highest initial debt burdens.

Next, we estimate the household Debt Laffer Curve—the relation between the net present value of repayment and the face value of debt across the distribution of indebtedness. Our proxy for indebtedness is the outstanding debt balance before the policy is implemented relative to the originated amount. Our estimation uses as inputs the probability of default, the effect of debt on default, the loss-given-default (LGD), and the effect of debt on the LGD. For the first two inputs, we use our estimates from the data based on the quasi-experimental variation described above. For the LGD, we leverage loan-level estimates of LGD reported by the banks to the Central Bank of Hungary as inputs to bank capital requirements.

We find that the Debt Laffer Curve is hump-shaped in the degree of indebtedness. For low levels of indebtedness, the Debt Laffer Curve is upward sloping, and the slope is close to unity. However, once debt exceeds the originated amount it flattens. When debt exceeds originated amount by at least 20%, the Debt Laffer Curve inverts. Our estimates suggest that for borrowers who owe significantly more than they originally borrowed, debt relief improves the welfare of both the borrower and the lender.

What explains the large impact of debt relief on repayment, especially for heavily indebted borrowers? To answer this, we use individual level administrative income data that is matched to the household credit registry. We find that registered labor income increases by about 2% for borrowers eligible for debt relief. This can reflect both an increase actual labor income, as well as higher participation in the formal sector.

We also examine other borrower outcomes. We find that debt relief is not associated with significantly different borrowing behavior after the policy. There is also no differential take-up of COVID-era mortgage moratoria. We interpret this as evidence against the hypothesis that debt relief increases moral hazard.

In the final part of the paper, we build a quantitative life-cycle model of household debt and default. In the model, households have long-term debt. They can choose to default, subject to a utility cost and to wage garnishment by lenders, an assumption that is motivated by our empirical setting. The model predicts that default the probability of default falls following a debt relief program. Moreover, because households face a cost of default, the cash flows to the lender fall sharply at default. Therefore, the model generates a hump-shaped Debt Laffer Curve that matches the empirical evidence.

**Related Literature.** This paper is related to three strands of the literature.

First, this paper is most closely related to research examining the effect of household debt relief on default, consumption, and other borrower outcomes (Agarwal et al., 2017; Dobbie and Song, 2017; Aydin, 2021). This literature generally finds that debt relief improves repayment and boosts consumption. When borrowers are liquidity constrained, the effects are larger when relief is front-loaded (Ganong and Noel, 2020). A related strand of literature finds that debt relief has sizable effects on aggregate demand and output (Auclert et al., 2019; Auclert and Mitman, 2022). There is also evidence that debt relief for distressed borrowers boosts income, potentially by mitigating debt overhang (Dobbie and Song, 2015; Di Maggio et al., 2020).

Other studies, focusing on developing countries, find smaller benefits of debt relief. In contrast, Kanz (2016) and Giné and Kanz (2018) study the effect of a debt relief program targeting farmers in India. They find limited positive effects on productivity or income, as well as an increase in strategic default. Karlan et al. (2019) conduct experiments that paid off high-interest moneylender debt in India and the Philippines. They find that borrowers returned to the same debt levels as control households within a year. In contrast to these studies, we find that debt relief persistently reduced debt and default, and we find limited evidence of increased strategic default.

Second, our paper is also related to the literature on debt restructuring and the Debt Laffer Curve. This literature initially focused on debt overhang for sovereign borrowers (Krugman, 1988; Sachs, 1989), and previous empirical estimates of the Debt Laffer Curve used macro-level data (e.g., Claessens, 1990; Cohen, 1993). We do not know of studies estimating the Debt Laffer Curve using micro data and quasi-experimental variation. To our knowledge, we are the first to show empirically that Debt Laffer Curve can invert for highly indebted borrowers.

Third, this paper relates to research on the impact of debt and balance sheet shocks on households. A large literature finds that household net worth and debt shocks have substantial effects on household consumption and default (Mian et al., 2013; Fuster and Willen, 2017; Di Maggio et al., 2017). Moreover, these responses have real effects on employment and output through a demand channel (Mian and Sufi, 2014; Verner and Gyöngyösi, 2020). Household debt positions can also affect labor supply through wealth effects and debt overhang (Donaldson et al., 2019; Bernstein, 2021; Gyöngyösi et al., 2022).

**Roadmap.** The paper is organized as follows. We discuss the institutional context in Section 2. Section 3 describes the data. Section 4 presents a theoretical model of debt relief and the Debt Laffer Curve. Section 5 presents the empirical results for default. Section 6 presents our empirical estimate of the Debt Laffer Curve. ?? examines the mechanisms. Section 7 presents our quantitative model of the response to debt relief and the Debt Laffer Curve, and section 8 concludes.

## 2 Context for the Hungary’s 2015 Debt Relief Policy

In this section, we provide background on the Settlement and Conversion Program, which provided debt relief for FC debtor households. The program was implemented in the aftermath of a severe household foreign currency debt crisis. We first provide background context for the crisis and then describe the institutional details of the debt relief program.

### 2.1 Foreign Currency Credit Boom and Crisis

Hungary experienced a large credit boom prior to the 2008 Global Financial Crisis. A large fraction of household loans originated during the boom were denominated in foreign currency. Foreign currency lending was widespread for several reasons (Verner and Gyöngyösi, 2020; Gyöngyösi and Verner, 2022). First, there was a large spread between Hungarian forint and Swiss franc interest rates.<sup>3</sup> A stable exchange rate implied a persistent deviation from uncovered interest parity. Second, foreign banks expanded credit supply in order to gain market share in the newly contestable Central and Eastern European markets. Third, Hungary joined the EU in 2004, and many expected that Hungary would join the euro several years later (Fidrmuc et al., 2013). Before the outbreak of the Global Financial Crisis in September 2008, more than 60 percent of household debt was denominated in foreign currency, primarily Swiss franc.

The 2008 Global Financial Crisis led to a large and unexpected depreciation of the Hungarian forint. The crisis also resulted in a strong appreciation in the Swiss franc. By 2014, the forint depreciated by about 70% against the Swiss franc. This led to a substantial revaluation in outstanding FC debt burdens and monthly debt service payments. In aggregate, household debt-to-GDP ratio was revalued by 6-10 percentage points.

Figure 1 plots the distribution of the ratio of outstanding debt in 2014m12 to the originated amount. We refer to this ratio as  $\tilde{D}_{2014:12}$ . The sample is all FC housing loans originated between 2000 and 2008 in the Hungarian household credit registry. The figure assumes full repayment when calculating the outstanding amount to abstract from the mechanical increase in outstanding amount that arise from default.

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<sup>3</sup>The spread on rates faced by borrowers opened after the unexpected removal of government subsidies for LC loans in late 2003. The removal of interest subsidies for domestic currency loans quickly resulted in a switch to foreign currency lending. From 2004 to 2008, the majority (79.3%) of new housing loans were denominated in foreign currency.

Variation in  $\tilde{D}_{2014:14}$  is driven by the time of origination, loan maturity, and loan currency. For example, loans originated closer to the crisis, with longer maturities, at a stronger forint exchange rate, and in Swiss franc see have the highest  $\tilde{D}_{2014:12}$ . Figure 1 shows that, because of the strong depreciation and limited amortization before the depreciation, most FC debtors owed significantly more in 2014 in domestic currency terms than they had initially borrowed nearly a decade earlier.

The FC debt revaluation led to a large increase in household default rates. The vast majority of households were not hedged against the depreciation, so the depreciation represented a substantial negative shock to FC borrowers' liquidity and net worth.<sup>4</sup> Figure 2a plots the evolution of default on housing loans, where default is defined as 90 days in arrears. Following the exchange rate depreciation that started in late 2008, the default rate on housing loans gradually increased, reaching almost 20 percent for FC mortgage loans and 30 percent for FC home equity loans by 2014. In contrast, the default rate for LC loans rose more gradually, peaking at 8%.<sup>5</sup>

Figure 2b relates default in December 2014 against outstanding debt relative to the originated amount,  $\tilde{D}_{2014:12}$ . Borrowers whose outstanding balance was a larger fraction of the originated amount were much more likely to be in financial distress and to default. Figure 2b suggests that this is a setting where the Debt Laffer curve may be concave and even inverted, as many households were in severe financial distress.

## 2.2 Settlement and Conversion Program

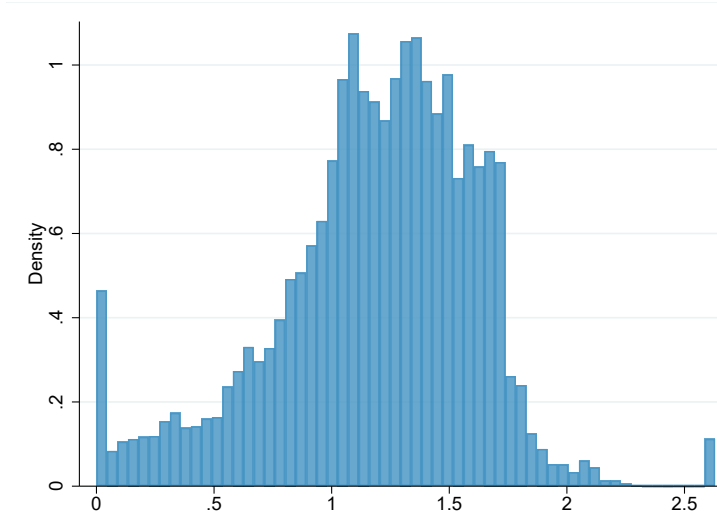
The large increase in household financial distress and the ensuing economic cost of the FC debt crisis led to a series of policies to address the high debt burdens of FC debtors. The Settlement and Conversion program, the focus of this paper, was the largest policy in terms of scope and scale.<sup>6</sup>

<sup>4</sup>Gyöngyösi, Rariga, and Verner (2022) provide survey evidence showing that most households had negligible income and assets in foreign currency. As a result, consumption of foreign currency debtors was highly sensitive to exchange rate shocks.

<sup>5</sup>In addition to the large increase in non-performing loans, the revaluation of household debt depressed household consumption and significantly contributed to the severity of the recession. Verner and Gyöngyösi (2020) and Gyöngyösi, Rariga, and Verner (2022) provide a detailed overview of the credit boom and explore the consequences of the debt revaluation shock on the real economy and household consumption at the household and regional level.

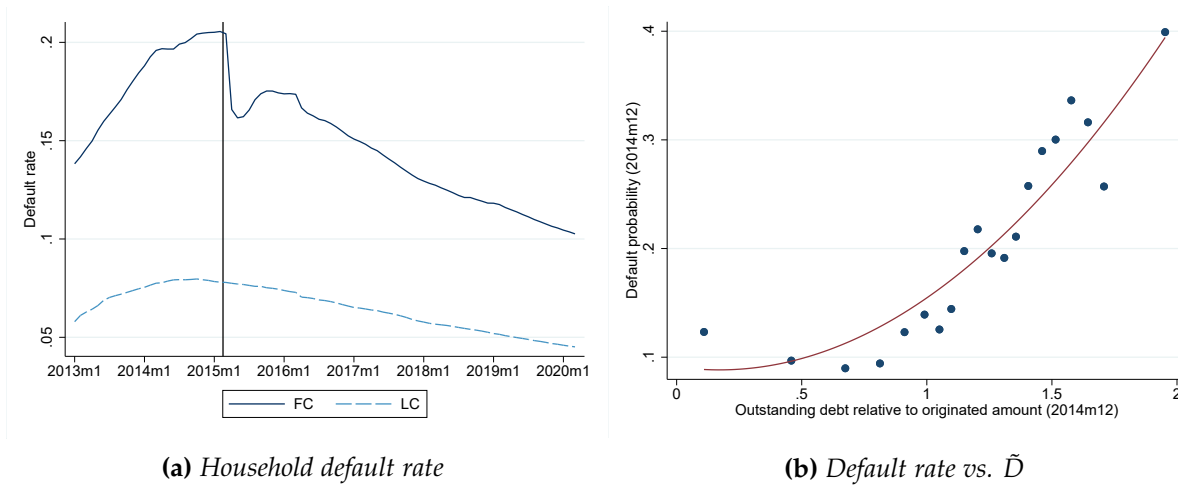
<sup>6</sup>Beyond the Settlement and Conversion, there were several other policies targeting foreign currency debtors. The Exchange Rate Cap, introduced in 2012, allowed debtors to pay their debt at a preferential exchange rate for a grace period of 5 years. The difference between the fixed, preferential rate and the actual market rate was collected in a special account as an HUF loan. The Early Repayment Program in 2011 also provided debt relief to FC debtor households in the form of a prepayment option at a

**Figure 1:** *Distribution of outstanding debt as a fraction of amount borrowed in December 2014*



Notes: This figure plots the distribution of outstanding debt in 2014m12 relative to the originated amount ( $\tilde{D}$ ) for foreign currency mortgage and home equity loans.

**Figure 2:** *Household Default Rate*



Notes: Panel (a) plots the time series of the default rate on housing loans around the Settlement and Conversion Program. The figure is based on a balance sample of housing (mortgage and home equity) loans that were outstanding at the end of 2014. Panel (b) presents a binned scatterplot of the default rate in 2014 against  $\tilde{D}$ , the outstanding balance in 2014m12 relative to the originated amount. The sample is all housing loans originated between 2000m1 and 2008m9. The figure shows that borrowers with higher outstanding amount relative to originated amount have higher default rates. Note that we calculate the outstanding balance so as to exclude the mechanical effect of default on outstanding amount.

preferential exchange rate. However, because the policy required full prepayment, it mostly benefited wealthier households who could afford to participate. See Gyöngyösi and Verner (2022) for a brief review these other policies.



**Timeline of the policy.** In June 2014, following inquiries by the government on the constitutionality of features of FC loans, the Supreme Court (Curia) ruled that specific contract features of FC loans were unfair.<sup>7</sup> First, a common practice was for banks to charge an exchange rate spread when converting loan installments on FC loans into domestic currency. Second, banks unilaterally changed loan contract terms by increasing the interest rate spread and introducing other fees. Housing loans in Hungary were variable rate loan, but did not have a pre-specified basis rate. Instead, banks could adjust rates unilaterally based on changes in their cost of capital.<sup>8</sup> These unilateral contract term changes were deemed unlawful if the loan contract did not specify the reason for passing through various bank costs to loan terms.

The Supreme Court decision opened the legal basis for FC borrowers to file lawsuits against banks for these unfair practices. However, given that there were about 600,000 FC loans outstanding in 2014, the large volume of litigation that would have ensued would have paralyzed the Hungarian courts and led to significant delays for debtors to receive compensation. Therefore, the government passed the Settlement and Conversion Program (SCP) in November 2014 (Act LXXVII/2014) to regulate the compensation of foreign currency debtors.

**Design and eligibility.** The SCP required banks to compensate households for excess payments from the application of the exchange rate spread and unilateral loan term modifications. These excess payments were treated as pre-payments, so the compensation would take the form of debt relief following a specific formula. The debt relief was first used to repay penalties and other fees, the remaining amount was used for principal reduction. Households who had already repaid their loans received cash compensation. The average compensation for FC borrowers was about 20% of outstanding debt balances (Dancsik et al., 2015). Overall, the program led to a reduction in household debt of 3% of GDP. The cost of the SCP was borne by the banking sector.<sup>9</sup>

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<sup>7</sup>This followed a separate decision by the Constitutional Court in March 2014, which declared that the Basic Law imposes an obligation on the state to create and maintain the institutional system that protects the interests of consumers and acts against the abuse of dominant power, as well as to create legislation that ensures the rights of consumers.

<sup>8</sup>For example Szigel (2012) notes that foreign currency debtors interest payments significantly increased despite the monetary easing of the Swiss National Bank and estimates that 25 percent of the increasing household debt burden can be attributed to higher interest rates charged by banks in the wake of the crisis.

<sup>9</sup>Dancsik et al. (2015) provide a comprehensive overview of the impact of the debt relief program on outstanding debt using survey and administrative data.

Although the the policy was designed to help foreign currency debtors, not all of them were eligible to the program. The program targeted mortgage and home equity loans.<sup>10</sup> Importantly, the eligibility was restricted to loans originated after May 1, 2004, the date when Hungary joined the European Union.

The second part of the SCP was the Conversion Program. It converted eligible loans to local currency at an exchange rate fixed in November 2014.<sup>11</sup> This policy effectively ended the exchange rate exposure of the household sector as almost the entire stock of foreign currency household debt was converted to domestic currency.

**Macroeconomic context.** The SCP was implemented during a relatively stable macroeconomic environment. The economy was recovering from the severe recession that started in 2008. Real GDP surpassed its pre-crisis peak in 2014. Unemployment declined to 7.7% in 2014, from a peak of 11.2% in 2010.

### 3 Data

We rely on several administrative data sources. Below, we introduce them in detail.

#### 3.1 Household Credit Registry

Our main data source is the household credit registry (KHR-L10) at the Central Bank of Hungary. The household credit registry provides loan level information on all household loans outstanding as of May 2012 or later. The data contains granular information on loan characteristics, including the date of origination, amount borrowed, currency denomination, maturity, and identity of the creditor institution. Moreover, it provides monthly updates on the current outstanding debt and debt service cost. The credit registry also contains default status for all loans starting in 2008. Default is defined as unpaid installments exceed the minimum wage for at least 90 days. There is also borrower-level information on borrower age and zip code.

A supplement to the household credit registry (KHR-L11) contains information on banks' estimates of the loss given default (LGD) for each loan starting in January

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<sup>10</sup>Auto loans would later also be included in the program.

<sup>11</sup>The Swiss Central Bank lifted the euro-Swiss franc peg in January 2015. While this affected Swiss franc borrowers in other countries like Poland and Austria, the Conversion Program prevented the further rise of debt burdens for FC borrowers in Hungary, as the exchange rate for the conversion was fixed in November 2014. The Central Bank of Hungary used its reserves to provide the necessary funding to banks for the conversion.

2015. These estimates are provided by banks to the Central Bank of Hungary as part of capital regulation. The LGD is defined based on a threshold of default of 90 days, consistent with the definition of default in the data.

### 3.2 Income Data from Pension Contributions

To study labor supply decisions, we use income recorded in individual-level pension contributions data (ONYF). These data cover the period 2000 to 2017 and are provided by the Hungarian State Treasury. The data contain information on all income sources that require a pension contribution payment. This includes all types of employment contracts and various types of social transfers ranging from unemployment benefits to child related transfers.<sup>12</sup> There is no cap on pension contribution payments, so the data measure the total amount of income without any top-coding. These data also allow us to observe the start and end date of the employment or social transfer spell, as well as the amount received. For the employment contracts, we observe the tax identifier of the employer, the occupation code, and the reason for spell breaks in employment, ranging from workplace accidents to sick leave.

We rely on a matching of the credit registry and pension contribution data performed by the Central Bank of Hungary in 2019. The matching was based on personal information that are only available in the original data, including name, date of birth, and mother's maiden name. Section B.1 in the data appendix provides further details on the matching.

## 4 Theory and Empirical Framework

### 4.1 Motivating Theory

To motivate the empirical analysis, we write down a simple static model of the household debt default that generate a Debt Laffer Curve. In section 7, we expand this framework into a richer quantitative life-cycle model of default.

Consider a borrower with linear utility over consumption and outstanding debt  $D$ .<sup>13</sup> The borrower chooses effort  $n$  with convex utility cost  $\phi(n)$  in order to obtain a labor income  $wn$ . The wage  $w$  is stochastic, with support  $[\underline{w}, \bar{w}]$  and probability

<sup>12</sup>The main drawback in terms of coverage is that income from self-employment is not included, as this does not require a pension contribution.

<sup>13</sup>The exposition here builds on Obstfeld and Rogoff (1996).

density function  $\pi(w)$  and corresponding cumulative distribution function  $\Pi(w)$ . We also assume that the expected value of  $w$  is one,  $\int w\pi(w)dw = 1$ . Effort  $n$  is chosen before the realization of the wage shock.

After the realization of the wage shock, the borrower can choose to repay or default. If the borrower defaults, the lender can garnish  $\xi$  fraction of labor income  $wn$ . Therefore, the payment to the creditor is a debt contract with a put option,  $\min\{D, \xi wn\}$ , and the borrower will choose to default when  $w < \frac{D}{\xi n}$ .

The borrower's problem is

$$\max_n -\phi(n) + \mathbb{E} [wn - \min\{D, \xi wn\}],$$

which can be rewritten as

$$\max_n -\phi(n) + n - \int_{\frac{D}{\xi n}}^{\frac{D}{\xi n}} \xi n w \pi(w) dw - D \int_{\frac{D}{\xi n}}^{\bar{w}} \pi(w) dw.$$

The first order condition for  $n$  is

$$\phi'(n) = 1 - \xi \int_{\frac{D}{\xi n}}^{\frac{D}{\xi n}} w \pi(w) dw, \quad (1)$$

which implicitly defines effort as a function of debt,  $n(D)$ .<sup>14</sup> The efficient level of labor is  $\phi'(n^*) = 1$ . However, since  $\phi''(n) > 0$ , effort is decreasing in debt,  $n'(D) < 0$ . Therefore, with debt the actual level of effort is below the first best,  $n < n^*$ .

Given the policy function  $n(D)$ , the value of debt is  $V(D, n(D))$ . This depends on both the face value of debt  $D$  and the effort decision  $n$ :

$$V(D, n(D)) = D \int_{\frac{D}{\xi n(D)}}^{\bar{w}} \pi(w) dw + \xi n(D) \int_{\frac{D}{\xi n(D)}}^{\frac{D}{\xi n(D)}} w \pi(w) dw \quad (2)$$

We refer to  $V$  as a function of  $D$  as the Debt Laffer Curve.

The derivative of  $V$  with respect to  $D$  is

$$\begin{aligned} \frac{dV(D, n(D))}{dD} &= \int_{\frac{D}{\xi n(D)}}^{\bar{w}} \pi(w) dw + \xi n'(D) \int_{\frac{D}{\xi n(D)}}^{\frac{D}{\xi n(D)}} w \pi(w) dw \\ &= 1 - \Pi\left(\frac{D}{\xi n(D)}\right) + \xi n'(D) \int_{\frac{D}{\xi n(D)}}^{\frac{D}{\xi n(D)}} w \pi(w) dw \end{aligned}$$

<sup>14</sup>See Appendix C.1 for detailed derivations of all equations and results in this section.

The first term is one minus the probability of default. This term is positive, as a higher face value of debt implies higher payment to creditors, conditional on full repayment. In contrast, the second term is negative, since  $n'(D) < 0$ . Increasing debt burdens lowers the incentive to exert effort, which increases the probability of default and lowers the repayment given default. For sufficiently high levels of debt, the Debt Laffer Curve inverts, and reducing debt can benefit both the borrower and the lender.

## 4.2 Empirical Framework

The simple model suggests a reduced form framework that relates the net present value of debt  $V$  to the face value of debt  $D$ . The net present value of debt can be represented as the face value of debt in repayment, times the probability of repayment, plus the value of debt in default, times the probability of default:

$$V(D) = D(1 - p(D)) + p(D)D(1 - L(D)), \quad (3)$$

where  $p(D)$  is the probability of default and  $L(D)$  is the loss given default, both of which can be functions of the face value of debt  $D$ .

Taking the total derivative of (3) yields the slope of the Debt Laffer Curve as a function of  $D$ :<sup>15</sup>

$$\frac{dV(D)}{dD} = 1 - (p(D) + Dp'(D))L(D) - L'(D)Dp(D). \quad (4)$$

To estimate the slope of the Debt Laffer Curve at a given  $D$ , one thus needs to estimate  $p(D)$ ,  $p'(D)$ ,  $L(D)$ , and  $L'(D)$ . Note that  $Dp'(D) = \frac{dp(D)}{dD/D}$  is the semi-elasticity of the probability of default with respect to debt. It measures the percentage point change in the probability of default from a 1% change in outstanding debt.

Given estimates of  $\widehat{\frac{dV(D)}{dD}}$  at a series of points  $\{D_1, \dots, D_K\}$ , with  $D_0 = 0$ , we can

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<sup>15</sup>See the appendix C.2 for a simple framework that provides the assumptions under which this equation holds for a long-term loan. In particular, (4) is the derivation of the value of debt up to a discount rate adjustment. If debt  $D$  is priced at fair value, then the discount rate adjustment is larger than one, as the required flows to service the debt  $D$  should be discounted at a higher rate than the expected cash flows. The former accounts for default in the discount rate, while the latter accounts for default in the expected cash flows.

then trace out the entire Debt Laffer Curve by integrating over these points

$$\widehat{V(D)} = \sum_{j=1}^K \frac{d\widehat{V(D)}}{dD} \Big|_{D=D_j} (D_j - D_{j-1}), \quad D_0 = 0 \quad (5)$$

## 5 The Effect of Debt Relief on Default

A key input to estimating the Debt Laffer Curve is an estimate of the marginal effect of debt on default,  $p'(D)$ , across the distribution of indebtedness,  $D$ . As else equal, a higher value of  $p'(D)$  implies a lower slope of the Debt Laffer Curve. If the default is sufficiently sensitive to outstanding debt, then the Debt Laffer Curve may even invert.

Estimating the sensitivity of default with respect to debt requires exogenous variation in outstanding debt,  $D$ . Our source of exogenous variation in  $D$  comes from an eligibility threshold for the 2015 Settlement and Conversion Program (SCP). Loans originated before May 1, 2004, were not eligible for the program and therefore did not receive debt relief. Since foreign currency household lending started in early 2004, this provides a control group of FC borrowers who did not benefit from debt relief. In this section, we utilize both regression discontinuity and difference-in-difference designs to estimate the impact of debt relief on default.

### 5.1 Regression Discontinuity Design Evidence

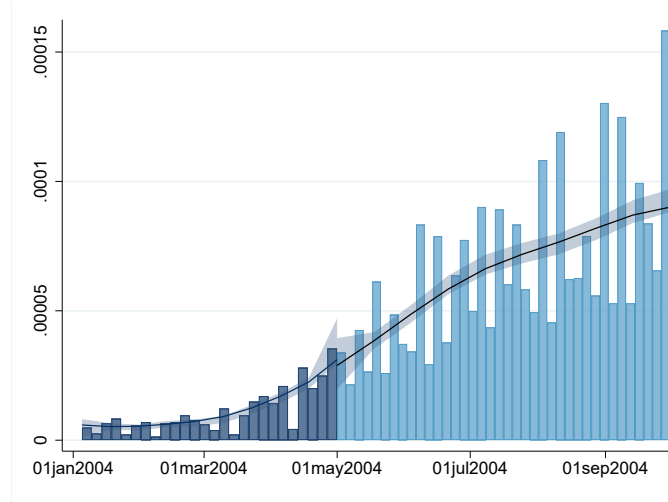
**Manipulation and permutation tests.** We start by presenting evidence from a regression discontinuity design (RDD). The eligibility threshold allows us to credibly identify the effect of the debt relief policy using a RDD for two reasons.

First, at the time of borrowing, households could not plausibly have foreseen the currency crisis in 2008, let alone the Settlement in 2015. As a result, households borrowing in early 2004 were unlikely to postpone their borrowing to be eligible for the debt relief program. Moreover, when the program was announced, households could not manipulate the date of origination retroactively to become eligible for the program.

Consistent with this, Figure 3 shows that the distribution of loan origination dates is smooth around the May 1, 2004 cutoff for program eligibility. There is no evidence of bunching in the running variable (date of origination) around the cutoff. More formally, we cannot reject the null of no systematic manipulation in the running variable applying the test for density discontinuity proposed by Cattaneo et al. (2018),

which builds on the idea of testing for manipulation proposed by McCrary (2008). Furthermore, we also examine whether the date of the origination of individual loans reported by banks to the credit registry changed from before the announcement of the policy to after it was implemented. All the dates of origination were unchanged.

**Figure 3:** *Distribution of Loan Origination Dates around Debt Relief Program Eligibility*

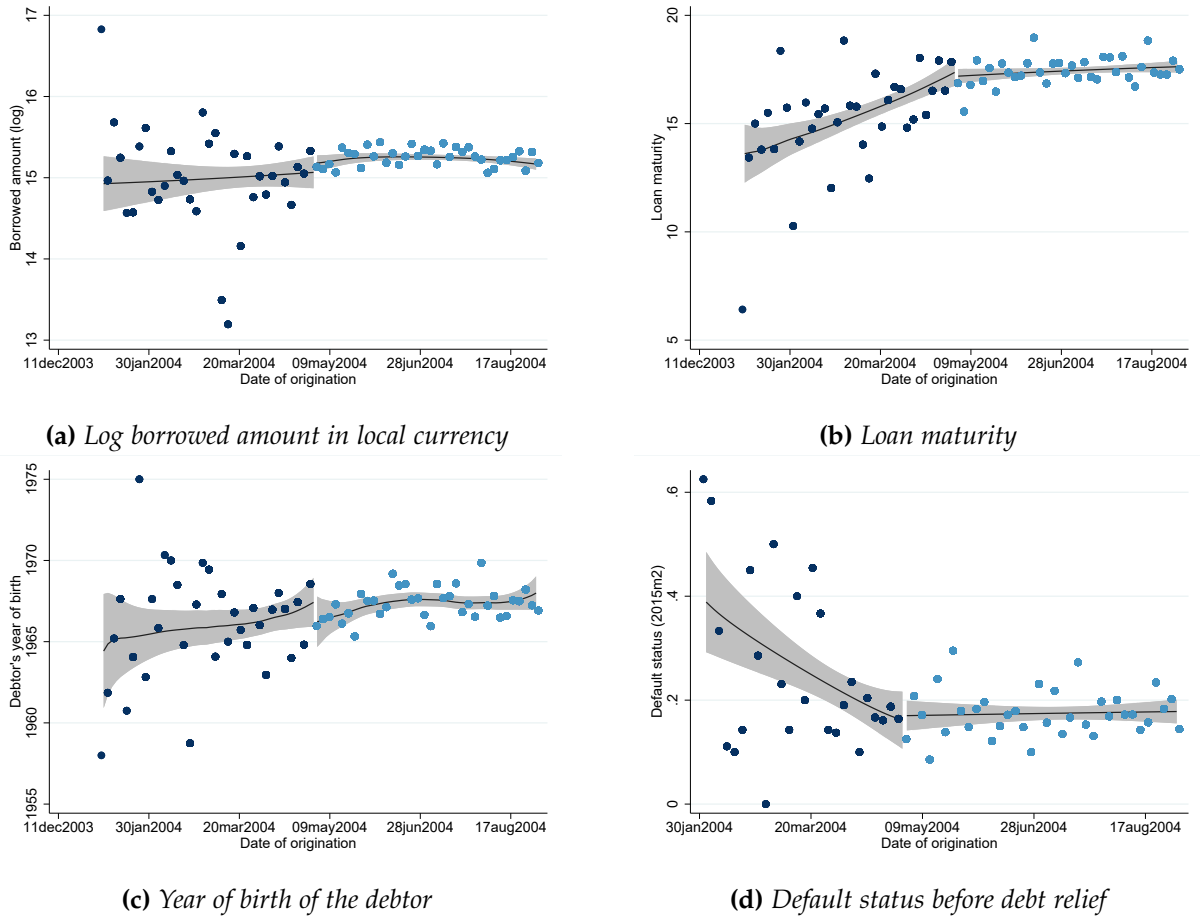


*Notes:* This figure plots a histogram of the number of foreign currency housing loan originations by calendar time in 2004. Loans originated before May 1, 2004 were not eligible for debt relief program, while loans originated after May 1, 2004 were eligible. The figure shows that the distribution of mortgage originations is smooth through this cutoff.

Second, loans and their borrowers have similar observable characteristics around the eligibility cutoff. Figure 4 plots several loan and borrower characteristics as a function of the origination date. Characteristics are measured as of December 2014. Borrowers to the left and right of the cutoff have similar principal, loan maturity, age, and default status before the program was implemented. These results confirm that there are no discrete jumps in loan and borrower characteristics around the May 1, 2004 cutoff. Hence, borrowers that do not receive debt relief are plausibly comparable to borrowers receiving debt relief around the eligibility cutoff.

**Effect of debt relief on default.** Figure 5 presents a graphic visualization of the RDD estimates of the effect of debt relief on debt, debt service, and default. Table 1 presents the corresponding RD estimates and standard errors. In Table 1, we report specifications both with and without control variables. We select MSE-optimal common bandwidth and report bias-corrected RD estimates with robust variance estimator.

**Figure 4: RDD Balance Tests: Loan and Borrower Characteristics by Date of Origination**



Notes: These figures plot loan and borrower characteristics against the date of origination around the Settlement eligibility threshold. Loans originated before May 1, 2004 were not eligible for the Settlement.

Figure 5a plots the change in debt from February 2015 to April 2015 as a function of the date of origination. The change in debt for loan  $i$  is calculated as

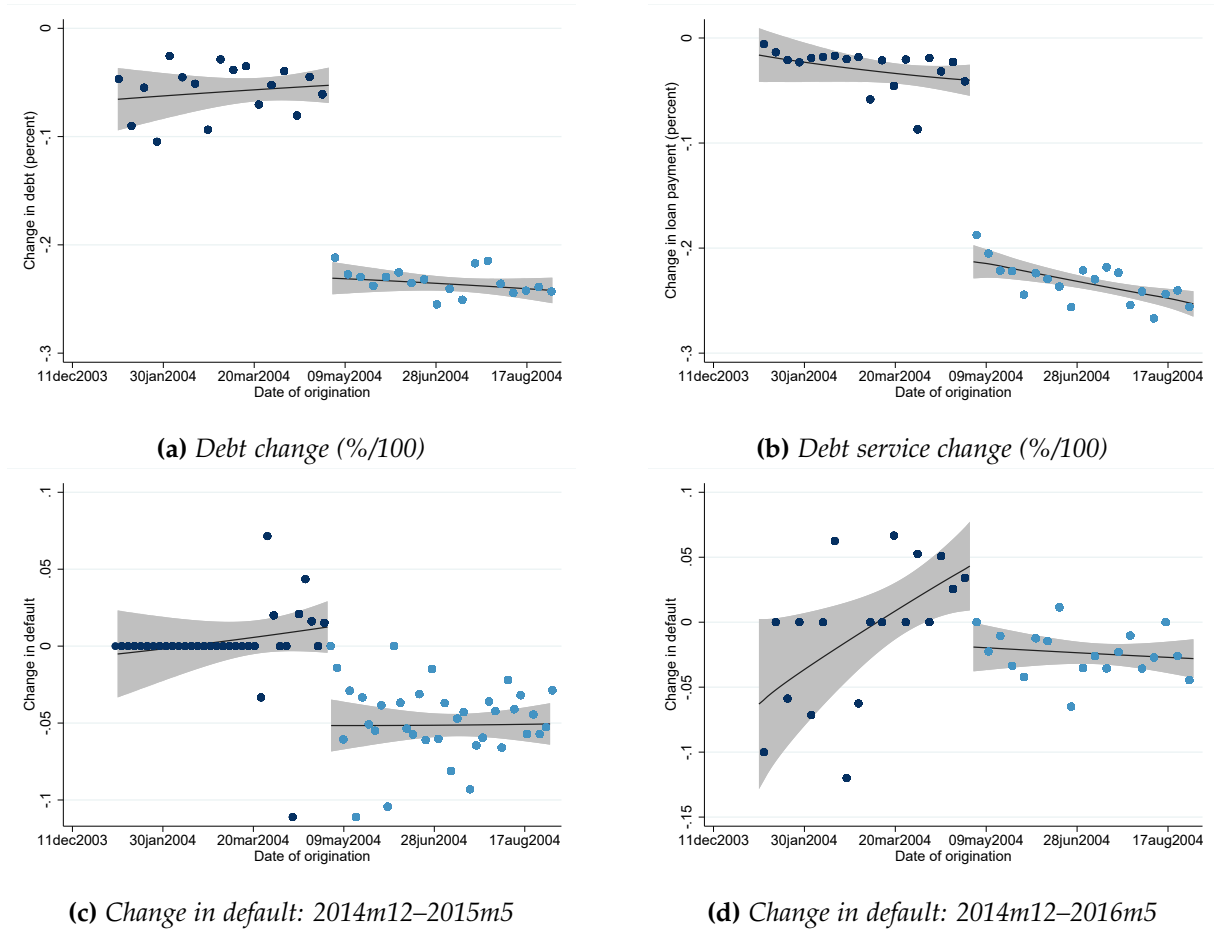
$$\Delta D_i = \frac{D_{i,c,2015m2} \times e_c - D_{i,HUF,2015m4}}{D_{i,c,2015m2} \times e_c},$$

where  $D_{i,c,t}$  is the outstanding debt in currency  $c$  at time  $t$  and  $e_c$  is the conversion exchange rate set by the Settlement and Conversion Program. We calculate debt relief using the change in debt from February to April, as banks were allowed to implement the policy either in March or April. Figure 5a shows that eligible borrowers receive a 19% reduction in debt, and there is a clear jump around the eligibility threshold.

Figure 5b shows the program as led to a similar decline in monthly debt service,



**Figure 5: Effect of Debt Relief on Debt, Debt Service, and Default: Regression Discontinuity Evidence**



*Notes:* These figures present estimates of the the impact of the debt relief on outstanding debt, debt service, and default using a regression discontinuity design around the Settlement Program eligibility cutoff. The horizontal axis shows the date of origination and the vertical axis show various outcomes. Loans originated before May 1, 2004 were not eligible for debt relief through the Settlement.

defined as monthly interest and amortization, for eligible borrowers. The estimates in columns (3) and (4) in Table 1 reveal that eligible borrowers receive a 17.5% reduction in monthly debt service. The reduction in debt service is slightly smaller than in debt, as the converted loans carried slightly higher local currency interest rates.

Panel (c) and (d) in Figure 5 show the results for default rates. Figure 5c shows that default rates fell by about 6.5 percentage points from before the policy in 2014m12 to shortly after the implementation of the policy in 2015m5. Part of this effect comes from a mechanical curing of loans in default with arrears smaller than the debt relief. Figure 5d therefore considers the change in default from 2014m12 to 2016m5, over on

year after the policy. The effect on default is similar, with an estimate of 5.8%.<sup>16</sup>

**Table 1:** *Effect of Debt Relief on Debt, Debt Service, and Default: Regression Discontinuity Evidence*

	$\Delta \ln(D)$		$\Delta$ debt service		$\Delta$ default 2015m2-2015m5		$\Delta$ default 2015m2-2016m5	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Eligible	-0.190** (0.0169)	-0.191** (0.0169)	-0.175** (0.0137)	-0.175** (0.0139)	-0.0649** (0.0153)	-0.0646** (0.0153)	-0.0582* (0.0285)	-0.0578* (0.0282)
Bandwidth	103	103	103	103	103	103	103	103
Controls		Yes		Yes		Yes		Yes
Observations	3535	3535	3438	3438	3510	3510	3170	3170

*Notes:* This table shows the effect of being eligible to the Settlement Program by exploiting the May 1, 2004 eligibility cutoff. MSE-optimal common bandwidth, bias-corrected regression discontinuity estimates with robust variance estimator. Controls are year of birth, log originated amount, loan currency (Swiss franc or euro), and loan type (mortgage or home equity). +, \*, and \*\* indicate significance at the 10%, 5%, and 1% level, respectively.

## 5.2 Dynamic difference-in-differences estimates

The RDD evidence provides compelling evidence that debt relief lowered default rates. This section presents estimates from a difference-in-differences research design. The DID design allows for more precise estimates of the effect of default relief on default. The increased precision is valuable for estimating the effect of debt relief on default over time and for estimating heterogeneous treatment effects. The benefit of increased precision from the DID comes at the expense of making a stronger functional form assumption about the counterfactual evolution of outcomes in the absence of treatment based on the control group of ineligible borrowers, as is standard in the DID framework.

We analyze the dynamic impact of debt relief eligibility on debt and default by estimating the following specification:

$$y_{it} = \alpha_i + \delta_t + \sum_{k \neq 2015m2} (\beta_k \text{Eligible}_i \times \mathbf{I}(t = k) + \Gamma_k X_i \times \mathbf{I}(t = k)) + \varepsilon_{it}, \quad (6)$$

where  $y_{it}$  an outcome variable, such as the default status of loan  $i$  at time  $t$ .  $\text{Eligible}_i$  is the eligibility status of the loan based on the time of origination; it is equal to 1 for

<sup>16</sup>In the next subsection, we present additional evidence from a difference-in-differences research design that the decline in default is not purely mechanical. We also present the full dynamic effect of the policy on default.

loans originated after May 1, 2004 and zero otherwise.  $I(t = k)$  is an indicator variable that equals one when  $t = k$  and zero otherwise, and  $X_i$  is a set of loan and borrower characteristics, which we interact with time fixed effects.

To compare similar borrowers, we restrict our sample to a relatively tight window around the May 1, 2004 eligibility threshold. In particular, we restrict the sample to loans originated between March 1 and June 30, 2004. Non-eligible loans were originated in March and April, while eligible loans originated in May and June.<sup>17</sup> We refer to this as the *RDD-DID Sample*. We estimate (6) by OLS and cluster standard errors at the loan level. The identifying assumption for consistent estimation of  $\{\beta_k\}$  is that, in the absence of treatment, treated borrowers' outcomes would have evolved in parallel to control borrowers' outcomes.

Figure 6 plots the estimates of  $\{\beta_k\}$  from equation (6) for debt and default. The estimation period is 2013m1 to 2018m12, and the omitted based period is 2015m2. Panel (a) presents the estimates for outstanding debt. There is no differential pre-trend in the evolution of debt for eligible and ineligible borrowers in the two years leading up to the policy. Eligible borrowers then see an approximately 20% decline in outstanding debt after the policy, consistent with the RDD estimate.

Panel (b) Figure 6 presents the estimates of (6) with default as the outcome variable. There is no difference in the evolution of default status between eligible and non-eligible loans before the policy. This supports the parallel trends assumption. Following the implementation of the debt reduction, there is a significant decline in the default rate of about 6 percentage points. The effect is persistent, lasting for at least four years until the end of our sample.

Columns 1-3 of Table 2 confirms these estimates based on a difference-in-differences model where we estimate the average effect over the entire post-reform period:

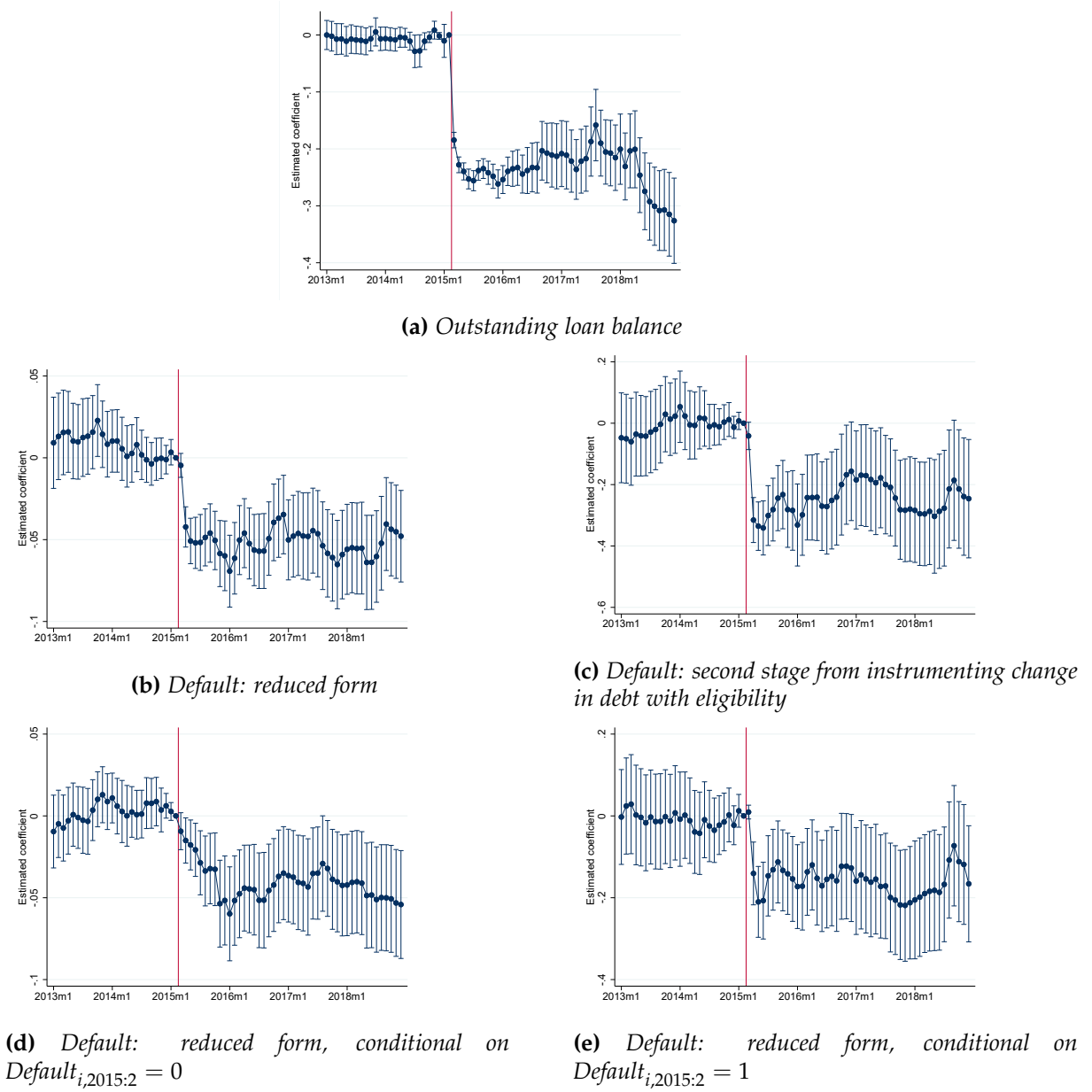
$$\text{Default}_{it} = \alpha_i + \delta_t + \beta \text{Eligible}_i \times \text{Post}_t + \Gamma X_i \times \text{Post}_t + \varepsilon_{it}, \quad (7)$$

In this specification,  $\text{Post}_t$  is an indicator variable that equals one after 2015m2 and zero otherwise. Being eligible for debt relief reduces default rates by 6.5 percentage points in our preferred specification that includes loan controls and subregion fixed effects, both interacted with  $\text{Post}_t$ .

Is the decline in the default rate large in magnitude? The pre-policy average of 11

<sup>17</sup>Figure A.1 shows that the results are very similar when allowing for a wider bandwidth and including all loans originated in 2004.

**Figure 6:** *Effect of Debt Relief on Outstanding Debt, Debt Service, and Default: Difference-in-Differences Evidence Based on Program Eligibility*



*Notes:* This figure plots the dynamic impact of eligibility in the Settlement Program on debt and default from estimating equation (6) (all panels except (c)) and equation (9) (panel (c)). Error bars represent 95% confidence intervals based on standard errors clustered at the loan level.

percent default rate for this sample loans.<sup>18</sup> In terms of magnitudes, the 6.5 percentage

<sup>18</sup>This sample has a lower default rate than the full portfolio of FC loans in Figure 2a for two reasons. First, loans originated in 2004 had a longer time to amortize before being hit by the exchange rate shock. Second, lending standards may have deteriorated during the lending boom.

**Table 2:** *Effect of Debt Relief on Default: Difference-in-Differences Evidence Based on Program Eligibility*

	Reduced form			IV		
	(1)	(2)	(3)	(4)	(5)	(6)
Eligible $\times$ Post	-0.0391** (0.00970)	-0.0693** (0.0115)	-0.0658** (0.0113)			
Change in debt $\times$ Post				0.240** (0.0583)	0.417** (0.0703)	0.408** (0.0707)
Individual & month FE	Yes	Yes	Yes	Yes	Yes	Yes
Loan controls $\times$ Post		Yes	Yes		Yes	Yes
Subregion $\times$ Post FE			Yes			Yes
F-statistic				2099.4	915.1	829.1
$R^2$	0.780	0.782	0.786			
Observations	225962	225962	225962	221674	221674	221674

*Notes:* Columns 1-3 in this table presents estimates of (10). Columns 4-6 instrument the percentage change in debt with program eligibility. Loan controls are loan type, borrower year of birth, log originated amount, loan maturity, and indicators for loan currency, all interacted with  $Post_t$ . Subregion fixed effects are fixed effects at the NUTS-1 level, interacted with  $Post_t$ . Standard errors are clustered at the loan level. +, \*, and \*\* indicate significance at the 10%, 5%, and 1% level, respectively.

point reduction represents a 59% reduction in the default rate.

Is the decline in the default rate large relative to the treatment? As we saw from equation (4), a key input to the slope of the Debt Laffer Curve is  $D \cdot p'(D) = \frac{dp(D)}{dD/D}$ , the semi-elasticity of default with respect to debt. We estimate this parameter by two-stage least squares, instrumenting the percentage change in debt from debt relief with program eligibility:

$$\Delta D_i \times Post_t = \alpha_i^{FS} + \delta_t^{FS} + \pi^{FS} Eligible_i \times Post_t + \Gamma^{FS} X_i \times Post_t + \varepsilon_{it}^{FS} \quad (8)$$

$$Default_{it} = \alpha_i^{SS} + \delta_t^{SS} + \beta_k^{SS} \widehat{\Delta D_i \times Post_t} + \Gamma^{SS} X_i \times Post_t + \varepsilon_{it}^{SS} \quad (9)$$

Columns (4)-(6) of Table 2 present the estimates of  $\beta^{SS}$  from this two-stage least-squares specification. The estimates imply that a one-percentage point reduction in debt reduces default rates by about 0.4 percentage points. Figure 6c presents the evolution of two-stage least-squares estimates over time, showing that the treatment effect is stable over time.

Finally, we note that the estimated effect of debt relief on default based on program

eligibility is broadly similar to a simpler analysis based on the evolution of default rates for FC and LC loans, as reported in Figure 2a. In particular, Figure 2a shows the debt relief program led to an immediate fall in FC default rates in 2015m3, as loans in default were mechanically cured. FC default rates then rise slightly at the end of 2015, as some loans enter default again. However, from 2016 onward FC default rates decline rapidly. Relative to the peak default rate in 2015m1, the average decline in the default rate over the next four years, relative to the change in the LC default rate, is about 6 percentage points, similar to our DID estimate of 6.5 percentage points.

**Effect by initial default status.** Given the high initial default rates, a natural question is whether the reduction in default in Figure 6b is driven by loans in default curing or reduced entry into default by loans that were not in default at the time of the program. Panels (d) and (e) in Figure 6 present estimates of (6) separately by default status in 2015m2. For loans that were not initially in default, debt relief leads to a gradual decline in the likelihood of default to about 4 percentage points. This is driven by lower entry into default relative to the control group.

Panel (e) Figure 6 shows that, for loans that were initially in default, there is a large initial decline in default of nearly 20 percentage points. Importantly, this effect is persistent. This suggests that loans are cured by debt relief are able to continue making payments to remain current.

Given the initial default rate of 11% for this sample, this implies that about 33% of the overall decline in default in Figure 6b is explained by loans that were initially in default, while 66% is explained by reduced entry into default from loans that were current before the program.

**Heterogeneity based on indebtedness.** The basic theoretical framework outlined in section 4 suggests that highly indebted borrowers are more likely to respond to debt relief by improving repayment performance. These borrowers face the most severe debt overhang problem, so reducing debt burdens can increase repayment performance. If the sensitivity of default with respect to debt relief is sufficiently large, then the Debt Laffer Curve can even invert.

Table 3 presents estimates of the effect of debt relief on default across the distribution of indebtedness. Our measure of the degree of borrower indebtedness before the debt relief program is the ratio of outstanding debt to the originated amount,  $\tilde{D}$ . As we saw in Figure 1, most FC borrowers owed significantly more than they originally

**Table 3:** *Effect of Debt Relief on Default: Heterogeneity by Indebtedness*

Decile of $\tilde{D}$	$\beta^{SS}$	S.E.	F-statistic	N
(1)	0.128	(0.108)	73.64	16246
(2)	0.077	(0.120)	105.8	19797
(3)	0.115	(0.093)	129.8	19864
(4)	0.146	(0.232)	50.02	19818
(5)	0.151	(0.159)	66.13	19881
(6)	0.308	(0.274)	97.85	19837
(7)	0.540*	(0.240)	92.95	19777
(8)	0.965*	(0.384)	94.84	19879
(9)	0.540	(0.404)	57.55	19770
(10)	2.076**	(0.667)	91.24	19787

*Notes:* This table presents estimates of the semi-elasticity of default with respect to debt from estimation of (9) separately by decile of  $\tilde{D}$ .  $\tilde{D}$  is defined as the outstanding loan amount in 2014m12 relative to the originated amount. See Figure 1 for the distribution of  $\tilde{D}$ . Standard errors are clustered at the loan level. +, \*, and \*\* indicate significance at the 10%, 5%, and 1% level, respectively.

borrowed due to exchange rate movements and the speed with which they amortized before the series of exchange rate depreciations (loan maturity).

Table 3 shows that, for borrowers with relatively low indebtedness, the effect of debt relief on default is modest. For example, for borrowers in the first decile of  $\tilde{D}$ , a one percent reduction in outstanding debt lowers default rates by 0.13 percentage points. However, the estimated effect rises across higher deciles of  $\tilde{D}$ . For example, the estimate is 0.54 for borrowers in the seventh decile, and 2.1 for borrowers in the tenth decile. These estimates indicate the highly indebted borrowers are much more sensitive to debt relief.

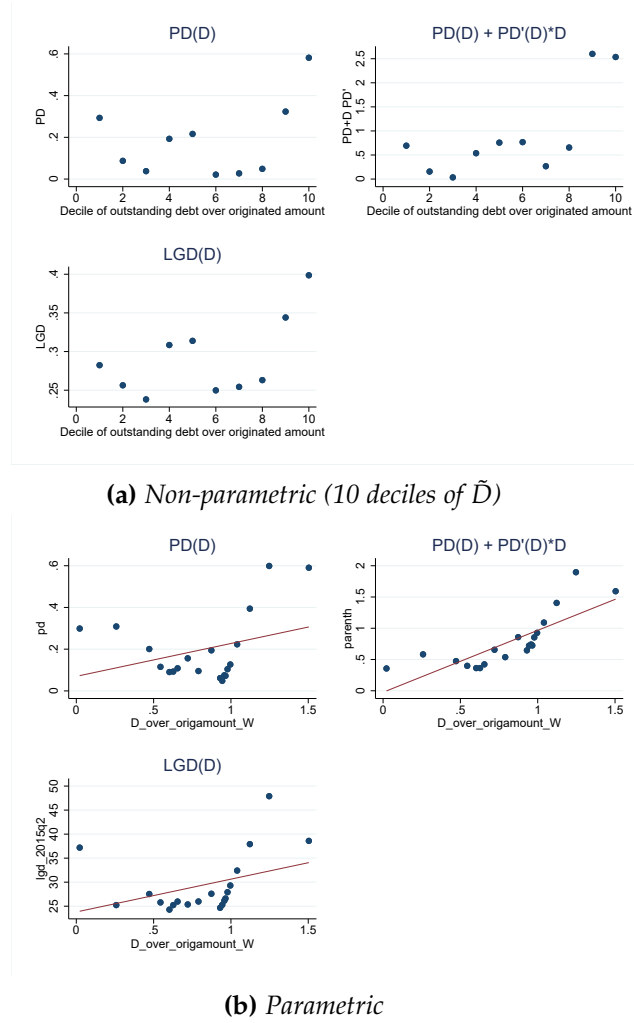
## 6 The Empirical Debt Laffer Curve

In this section, we construct an estimate of the Debt Laffer Curve using the framework outlined in section 4. We present results from two approaches: a non-parametric and a parametric approach.

## 6.1 Debt Laffer Curve: Non-parametric Estimation

We partition borrowers in  $K = 10$  deciles based on our proxy of household indebtedness, outstanding debt relative to the originated amount,  $\tilde{D}$ . For each decile, we estimate the pre-policy default probability,  $p(\tilde{D})$ , as the average default rate in that sample. The top-left panel in Figure 7a shows that the probability of default in the *RDD-DID Sample* is highest for borrowers with the highest values of  $\tilde{D}$ , consistent with the full sample evidence in Figure 2b.

**Figure 7: Inputs to the Estimating the Debt Laffer Curve**



Notes: This figure plots the inputs into constructing the Debt Laffer Curve.

Figure 7a also plots estimates of the term  $\widehat{p(\tilde{D}_k)} + \hat{\tilde{D}}_k \widehat{p'(\tilde{D}_k)}$  from (4). This term is also increasing across deciles of  $\tilde{D}$ . This reflects that more indebted borrowers have



both a higher probability of default and a higher sensitivity of default to debt, as we saw in the previous section.

In Figure 7a we plot estimates of the loss given default,  $\widehat{L(\tilde{D}_k)}$  across deciles of  $\tilde{D}$ . We use the loan-level loss-given-default (LGD) reported by banks to the Central Bank of Hungary in 2015Q2 as part of capital requirements regulation. The figure shows that the the LGD ranges from 25% to 40%. It is highest for loans with the highest indebtedness. Intuitively, these borrowers would be the least likely to cure after entering default. Moreover, these loans have the smallest equity cushion and would therefore entail the largest losses for banks in the event of foreclosure.

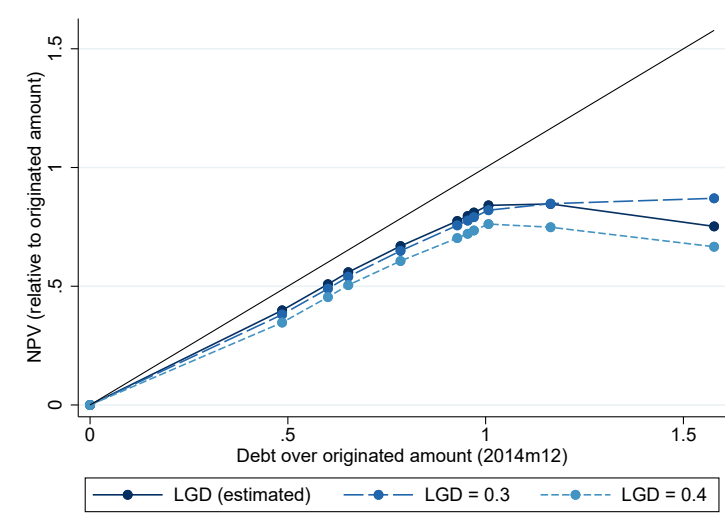
The final input to the slope of the Debt Laffer Curve in equation (4) is the marginal effect of debt on the LGD,  $L'(\tilde{D})$ . From both a theoretical perspective and based on Figure 7a, we would expected  $L'(\tilde{D}) > 0$ . However, because the LGD data is only available starting in 2015Q2, we cannot estimate  $L'(\tilde{D})$  using our difference-in-differences design, as the debt relief program was implemented in 2015Q1.<sup>19</sup> For our calculation, we make the simplifying assumption that  $L'(\tilde{D}) = 0$ . This is conservative, in the sense that it makes it more difficult to find an inverted Debt Laffer Curve.

Figure 8a plots the estimated Debt Laffer Curve using the non-parametric approach. In particular, we integrate the estimates of the slope of the Debt Laffer Curve,  $\widehat{\frac{dV(\tilde{D})}{d\tilde{D}}}$ , for each decile of  $\tilde{D}$  using (5) to obtain the Debt Laffer Curve estimate,  $\widehat{V(\tilde{D})}$ . The solid line presents Debt Laffer Curve estimates using the banks' estimated LGD, while the dashed lines assume a fixed LGD.

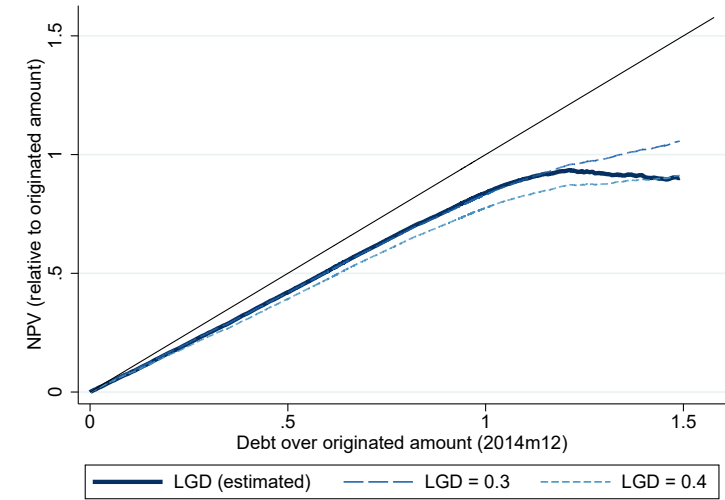
There are two takeaways from Figure 8a. First, the Debt Laffer Curve is concave in indebtedness  $\tilde{D}$ . This implies that, on the margin, increase outstanding debt balances has a diminishing effect on the net present value of expected repayment. Second, the Debt Laffer Curve flattens sharply and even inverts for highly indebted borrowers. This inversion happens roughly once  $\tilde{D}$  exceeds one, i.e., when borrowers owe more than they initially borrowed a decade earlier. At this high level of indebtedness, reducing debt burdens does not reduce the expected net present value of the loan to the lender; it slightly increases it.

<sup>19</sup>Figure A.2 presents evidence from the RD design debt relief lowered the LGD. In this figure, we analyze the loss-given-default in levels, rather than the change around the program. The RD design, however, does not give us the power to estimate heterogeneity in  $L'(\tilde{D})$  across the distribution of  $\tilde{D}$ .

**Figure 8: Debt Laffer Curve Estimates**



**(a) Non-parametric estimate**



**(b) Parametric estimate**

*Notes:* This figure presents the estimate Debt Laffer Curve. The x-axis is outstanding debt relative to the originated amount  $\tilde{D}$ . The y-axis is the expected net present value of repayment. See text for details on the construction of the DLC.

## 6.2 Debt Laffer Curve: Parametric Estimation

For the parametric approach, we first estimate the impact of debt relief on default  $p'(\tilde{D})$  across the distribution of  $\tilde{D}$  by interacting debt relief,  $\Delta D_i$ , with a polynomial

in  $\tilde{D}_i$ :

$$\text{Default}_{it} = \alpha_i + \delta_t + \sum_{j=0}^J \beta_j \Delta D_i \times \tilde{D}_i^j \times \text{Post}_t + \Gamma X_i \times \text{Post}_t + \varepsilon_{it}. \quad (10)$$

We set  $J = 5$  to flexibly capture heterogeneity in  $p'(\tilde{D})$ . Consistent with the difference-in-differences approach in section 5, we instrument  $\{\Delta D_i \times \tilde{D}_i^j \times \text{Post}_t\}_{j=0}^J$  with  $\{\text{Eligible}_i \times \tilde{D}_i^j \times \text{Post}_t\}_{j=0}^J$ . This yields our estimate of  $p'(\tilde{D})_i$  for each loan  $i$ .

Next, we estimate the pre-policy probability of default  $p(\tilde{D})$  using a probit model with pre-policy default rate in 2014m12 as the outcome variable. As the predictors, we again use a 5<sup>th</sup> order polynomial in  $\tilde{D}$  and pre-determined loan and borrower characteristics. The probability-of-default model is:

$$p(\tilde{D})_i = \Phi \left( \sum_{j=0}^J \delta_j \tilde{D}_i^j + \phi X_i \right).$$

From this model, we obtain an estimate of the probability of default for each loan  $\widehat{p(\tilde{D})}_i$ .

Figure 7b plots a binned scatterplot of the predicted probability of default,  $\widehat{p(\tilde{D})}_i$ , against indebtedness,  $\tilde{D}$ . The probability of default is rising in indebtedness, especially for high levels of indebtedness. We also report the binned scatterplot of  $\left( \widehat{p(\tilde{D})}_i + \widehat{p'(\tilde{D})}_i \tilde{D}_i \right)$  against  $\tilde{D}_i$ . The relation is upward-sloping and strong, as both the probability of default and the sensitivity of default to debt is increasing in indebtedness. Finally, we use the banks' estimates of LGD for each loan as  $\widehat{L(\tilde{D})}_i$ . We again make the simplifying and conservative assumption that  $L'(\tilde{D}) = 0$ .

With these estimates of  $p(\tilde{D})$ ,  $p'(\tilde{D})$ , and  $L(\tilde{D})$  for each loan, we sort loans by  $\tilde{D}_i$  and integrate using (5) to obtain a parametric estimate of the Debt Laffer Curve  $\widehat{V(\tilde{D})}$ . Figure 8b plots the estimate of the Debt Laffer Curve using this parametric approach. The shape is similar to the non-parametric estimate in panel (a). The DLC is concave and inverts when outstanding debt exceeds the originated amount by 20%. Highly indebted borrowers with  $\tilde{D} > 1.2$  are on the wrong side of the DLC. Based on this estimate, providing debt relief for these borrowers would increase the value of these loans to the lender.

## 7 Quantitative Model

### 7.1 Model Set-up

We consider an economy with a continuum of *ex ante* identical homeowners with mortgage debt. Homeowners begin life with initial mortgage debt  $d_0 = (1 - LTV)H_i$ . For simplicity, we set  $H_i = 1$ , and we do not model housing or mortgage choice.<sup>20</sup> Time runs from  $t = 1, \dots, T$ .

**Preferences.** Households have preferences over consumption,  $c_t$  and labor,  $n_t$ , and and default,  $\varphi_t \in \{0, 1\}$ . We suppress the subscript for a household to lighten the notation. If the household chooses to default on the mortgage ( $\varphi_t = 1$ ), it incurs a utility cost  $\kappa$ . Preferences are given by

$$\mathbb{E}_1 \sum_{t=1}^T \beta^t u(c_t, n_t, \varphi_t),$$

where we will assume separable preferences between consumption, labor, and default

$$u(c_t, n_t, \varphi_t) = \left( \frac{c_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \phi \frac{n_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} - \varphi_t \kappa \right).$$

**Mortgage contract.** The mortgage maturity is assumed to be  $m = T$  periods. Repayment begins in period  $t = 1$  and continues until period  $T$ . Mortgages are denominated in foreign currency. Following our empirical setting, we assume the mortgage payment in foreign currency follows an annuity:

$$pmt_t^0 = d_t \frac{r_d}{1 - (1 + r_d)^{-T+t-1}}.$$

The evolution of a household's debt without default follows the standard annuity formula

$$d_{t+1} = d_t \left( (1 + r) - \frac{r}{1 - (1 + r)^{-T+t-1}} \right). \quad (11)$$

<sup>20</sup>Here, we follow Campbell and Cocco (2015), who also assume do not model housing choice. They set housing consumption for each household  $i$ ,  $H_i$ , to a fixed number. Moreover, they assume preferences are separable in housing and consumption, so they also don't explicitly model housing utility.

**Flow of funds constraints and default.** The household's flow of funds constraint without default ( $\varphi_t = 0$ ) is given by

$$c_t + a_{t+1} + \mathcal{E}_t pmt_t^0 = w_t n_t + (1 + r)a_t, \quad (12)$$

where  $a_t$  is a liquid asset that earns return  $r$ ,  $pmt_t^0$  is the mortgage payment out of default (in FC),  $\mathcal{E}_t$  is the exchange rate, and  $w_t$  is the wage. The wage follows an exogenous process

$$w_t = \rho w_{t-1} + u_t$$

where  $\alpha_t$  an exogenous age component and  $y_t$  is a persistent shock. Moreover, we assume that there is a borrowing constraint on the liquid asset:<sup>21</sup>

$$a_{t+1} \geq \underline{a}.$$

Default occurs when the household does not make the full payment  $\mathcal{E}_t pmt_t^0$ . In addition to the utility cost  $\kappa$ , we assume that if the borrower defaults, the lender can garnish  $\xi$  fraction of the borrowers' labor income and impose a penalty for late payment.<sup>22</sup> The payment in default is

$$pmt_t^1 = \xi w_t n_t$$

The flow of funds constraint for a household that is in default ( $\varphi_t = 1$ ) is given by

$$c_t + a_{t+1} + pmt_t^1 = w_t n_t + (1 + r)a_t. \quad (13)$$

When the household defaults, the outstanding debt balance (in FC) next period follows

$$d_{t+1} = d_t - \underbrace{(\mathcal{E}^{-1} \xi w_t n_t - r d_t)}_{\text{Partial amortization}} + \underbrace{pen[pmt_t^0 - \xi \mathcal{E}^{-1} w_t n_t]}_{\text{Penalty for partial payment}}, \quad (14)$$

where  $pen$  is a parameter that governs the penalty for partial repayment.

<sup>21</sup>We follow Campbell and Cocco (2015) set  $\underline{a} = 0$ .

<sup>22</sup>Berlinger et al. (2021) provides survey evidence that wage garnishment was common by banks on FC debtors in default.

**Bellman equations.** The exogenous state variables of the household are  $(t, \mathcal{E}_t, w_t)$ , and the endogenous state variables are  $a_t$  and  $d_t$ . We summarize the state as:  $\mathbf{s}_t = (\mathcal{E}_t, w_t, a_t, d_t)$ . The Bellman equation conditional on not defaulting ( $\varphi_t = 0$ ) is

$$V_{0,t}(\mathbf{s}_t) = \max_{c_t, n_t, a_{t+1}} u(c_t, n_t, 0) + \beta \mathbb{E}_t [V_{t+1}(\mathbf{s}_{t+1})]$$

s.t. (11) and (12).

The Bellman equation conditional on defaulting ( $\varphi_t = 1$ ) is

$$V_{1,t}(\mathbf{s}_t) = \max_{c_t, n_t, a_{t+1}} u(c_t, n_t, 1) + \beta \mathbb{E}_t V_{t+1}(\mathbf{s}_{t+1})$$

s.t. (13) and (14).

The period  $t$  value function is the maximum of the value functions without and with default

$$V_t(\mathbf{s}_t) = \max_{\varphi_t \in \{0,1\}} \{V_{0,t}(\mathbf{s}_t); V_{1,t}(\mathbf{s}_t)\}.$$

Given the household's optimal policy functions, the value of the loan for the lender (in LC) can be written recursively as

$$V_t^L(\mathbf{s}_t) = (1 - \varphi_t(\mathbf{s}_t)) \mathcal{E}_t p m t_t^0 + \varphi_t(\mathbf{s}_t) \zeta w_t n_t(\mathbf{s}_t) + \frac{1}{1 + r^L} \mathbb{E}_t V_{t+1}^L(\mathbf{s}_{t+1})$$

where  $r^L$  is the lender's discount rate.

**Model set-up comparison to the literature.** Our model most closely resembles the model of Campbell and Cocco (2015) (CC2015). However, we make several alternative assumptions to match our empirical setting. In contrast to CC2015, we endogenize labor supply and allow for wage garnishment in default.<sup>23</sup> This allows for the evolution of debt to be endogenous, adding an endogenous state variable to the household's problem. CC2015 instead assume that default makes the household exit homeownership and enter the rental market. We believe our assumptions are better suited to the empirical setting, as foreclosures were limited following a foreclosure moratorium imposed in 2010. Instead, banks pursued repayment by garnishing wages, but

<sup>23</sup>The appendix in CC2015 has a variant where lender can seize financial asset upon default, but they don't allow for lender to garnish labor income.

allowed households to stay in their homes. We also assume different mortgage contracts and exogenous shocks. For example, we allow for FC mortgages and exchange rate shocks.<sup>24</sup> Unlike CC2015, we also don't allow for pre-payment, which is rare in our empirical setting.

**Calibration.** We use standard parameters from the literature for our calibration. For the utility cost of default  $\kappa$ , we calibrate it to match the pre-policy default rate on FC loans of 20%. We assume the exchange rate  $\mathcal{E}_t$  follows a Markov process specified to match the exchange rate process expected before the 2008 crisis. In particular, we assume a positive but low probability of a large devaluation, based on Consensus Forecasts which the exchange rate was expected to remain stable.<sup>25</sup> We approximate the wage process using the Rouwenhorst (1995) method, which works well for discretizing persistent processes.

**Policy functions.**

## 7.2 Debt Relief and the Debt Laffer Curve

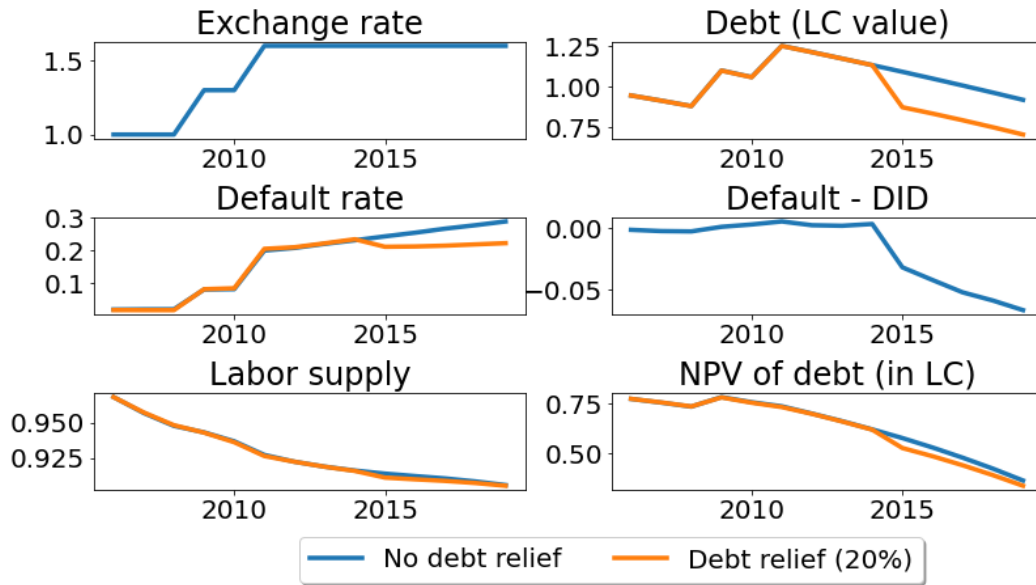
We simulate an exchange rate path to match the forint depreciation from 2008 to 2014 of 60%.

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<sup>24</sup>CC2015 allow for variable rate mortgages, and they model inflation shocks.

<sup>25</sup>Appendix ?? provides additional details on the calibration.

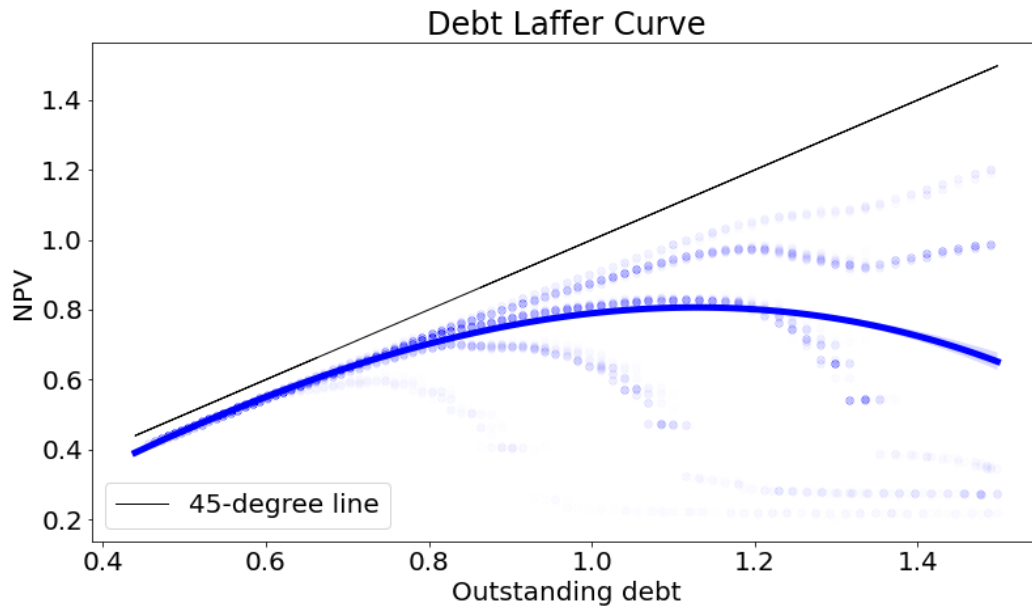
**Figure 9: Debt Relief Experiment in the Model**



*Notes:* Both the NPV of debt service and the outstanding debt are scaled by the originated amount, as in our empirical setting. The dots are the individual loans in the simulation. The solid line is the fitted polynomial based on the outstanding amount and NPV of the individual loans.



**Figure 10:** *Debt Laffer Curve in the Model*



*Notes:* Both the NPV of debt service and the outstanding debt are scaled by the originated amount, as in our empirical setting. The dots are the individual loans in the simulation. The solid line is the fitted polynomial based on the outstanding amount and NPV of the individual loans.

## 8 Conclusion

Why don't banks unilaterally provide debt relief to households on the wrong side of the Debt Laffer Curve? Banks may be concerned that raising the possibility of debt forgiveness will reduce repayment rates (e.g., Mayer et al., 2014). Banks may not be able to target borrowers on the wrong side of the Debt Laffer Curve. Even if banks could target those borrowers for whom it would be a Pareto improvement to reduce debt, such relief might be perceived to be unfair by other customers. Finally, the Debt Laffer Curve is relatively flat, where it is inverted, so the gains to the bank may be small relative to these costs. Therefore, it may require policy intervention that internalizes the benefits to borrowers and to the broader economy.

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# The Household Debt Laffer Curve

## Online Appendix

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Emil Verner<sup>‡</sup>

- Appendix A presents additional figures and tables.
- Appendix B provides additional data description
- Appendix C presents derivations

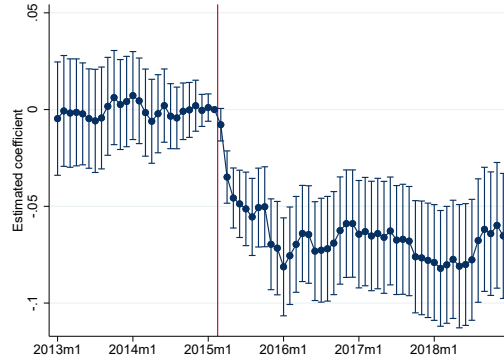
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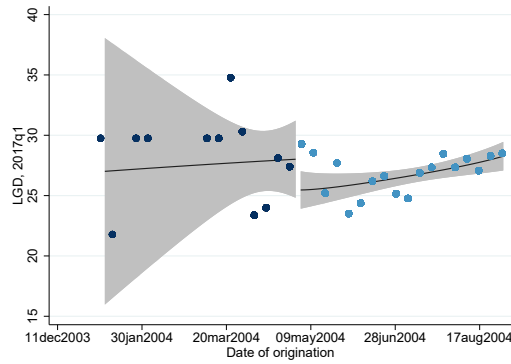
## A Appendix Figures and Tables

**Figure A.1:** *Effect of Debt Relief on Default: Robustness using a Wider Bandwidth around Program Eligibility*



*Notes:* This figure plots the effect of debt relief program eligibility on default from estimation of (6), as in Figure 6b. However, instead of restricting to loans originated between March 1, 2004 and June 30, 2004, we include all loans originated in 2004. This provides a larger treatment and control group.

**Figure A.2:** *RDD Estimates of the Effect of Debt Relief on Banks' Estimated Loss-Given-Default*



*Notes:*



## **B Data Appendix**

### **B.1 Matching Credit Registry with Pension Contribution Data**

The Pension Contribution Data and Credit Registry are were matched by the Central Bank of Hungary in 2019 using several pieces of personal information, including name, date of birth, and mother’s maiden name. BISZ, a subsidiary of the Central Bank of Hungary that maintains the household credit registry, has the legal obligation to delete all information on loans one year after the termination of loan contracts for non-default loans and five years after the termination for loans in default. This implies that matching of the two data sources can potentially incomplete as information on terminated loans is not available at BISZ.

We improve the matching quality by exploiting that if an individual has other existing loan products in 2019 observable in the credit registry, then we can link other loans, including loans affected by the SCP, to individuals using the anonymized identifier of the Central Bank of Hungary. Since we observe all loan contracts provided by all credit institutions, even those that were not matched, we document the matching process and its quality in detail in Appendix B.1.

## C Additional Derivations

### C.1 Derivation of Key Results in Section 4

The first-order condition in (1) is derived as follows:

$$\begin{aligned} -\phi'(n) + 1 - \xi \int_{\underline{w}}^{\frac{D}{\xi n}} w \pi(w) dw + \xi n \frac{D}{\xi n} \pi\left(\frac{D}{\xi n}\right) \frac{D}{\xi n^2} - D \pi\left(\frac{D}{\xi n}\right) \frac{D}{\xi n^2} &= 0 \\ -\phi'(n) + 1 - \xi \int_{\underline{w}}^{\frac{D}{\xi n}} w \pi(w) dw &= 0 \end{aligned}$$

The second-order condition is

$$\begin{aligned} -\phi''(n) + \xi \frac{D}{\xi n} \pi\left(\frac{D}{\xi n}\right) \frac{D}{\xi n^2} &< 0 \\ -\phi''(n) + \pi\left(\frac{D}{\xi n}\right) \frac{D^2}{\xi n^3} &< 0 \end{aligned}$$

By the implicit function theorem, the derivative of the policy function  $n(D)$  is:

$$\begin{aligned} n'(D) &= -\frac{f_n}{f_D} = -\frac{\left(-\phi''(n) + \pi\left(\frac{D}{\xi n}\right) \frac{D^2}{\xi n^3}\right)}{-\xi \frac{D}{\xi n} \pi\left(\frac{D}{\xi n}\right) \frac{1}{\xi n}} \\ &= \frac{\left(-\phi''(n) + \pi\left(\frac{D}{\xi n}\right) \frac{D^2}{\xi n^3}\right)}{\frac{D}{\xi n^2} \pi\left(\frac{D}{\xi n}\right)} < 0 \end{aligned}$$

where  $f$  is defined by the FOC (1). The term  $\left(-\phi''(n) + \pi\left(\frac{D}{\xi n}\right) \frac{D^2}{\xi n^3}\right)$  is negative by the second-order condition. The denominator is positive. Therefore, the derivative of  $n$  with respect to  $D$  is negative.

The derivative of  $V(D, n(D))$  in (2) with respect to  $D$  is

$$\begin{aligned} \frac{dV(D, n(D))}{dD} &= \int_{\frac{D}{\xi n(D)}}^{\bar{w}} \pi(w) dw - D \pi\left(\frac{D}{\xi n(D)}\right) \left(\frac{1}{\xi n(D)} - \frac{D}{\xi n(D)^2} n'(D)\right) \\ &\quad + \xi n'(D) \int_{\underline{w}}^{\frac{D}{\xi n(D)}} w \pi(w) dw + \xi n(D) \frac{D}{\xi n(D)} \pi\left(\frac{D}{\xi n(D)}\right) \left(\frac{1}{\xi n(D)} - \frac{D}{\xi n(D)^2} n'(D)\right) \\ &= \int_{\frac{D}{\xi n(D)}}^{\bar{w}} \pi(w) dw + \xi n'(D) \int_{\underline{w}}^{\frac{D}{\xi n(D)}} w \pi(w) dw. \end{aligned}$$

## C.2 Heuristic Derivation of Equation (4) for Long-Term Debt

Consider an annuity loan with outstanding balance  $D$  and installments  $I = F \cdot D$ , where  $F = \frac{r_B}{1-(1+r_B)^{-M}}$ . Assume that the value of this loan can be written as the discounted value of expected payments, where payments are discounted at the discount rate  $r$ . In particular, we assume that each period the expected payment is the probability of repayment  $(1 - p_j)$  times the installment, plus the probability of default  $(p_j)$  times the product installment and one minus the expected loss on the installment,  $L_j$ . This implies the following valuation equation:

$$V(D) = \sum_{j=1}^M \frac{(1 - p_j)I + p_j I(1 - L_j)}{(1 + r)^j}$$

Taking the total derivative with respect to  $D$ , we have

$$\frac{dV(D)}{dD} = \sum_{j=1}^M \frac{(-1)p'_j(D)I + (1 - p_j)I'(D) + (p'_j(D)I + p_j I'(D))(1 - L_j) - p_j I L'_j(D)}{(1 + r)^j}$$

Next, we assume that the impact of debt on the probability of default and loss given default is independent of remaining maturity:  $p'_j(D) = p'(D)$  and  $L'_j(D) = L'(D)$ . We think of this as debt changes shifting the probability of default and loss-given-default in parallel over the time dimension. This assumption holds approximately for  $p'(D)$  in our empirical setting, given the persistent effect on the probability of default. Further, assume that the probability of default and loss-given-default are independent of time  $p_j = p(D)$  and  $L_j = L(D)$ . With these assumptions, and using that  $I = FD$ , we have:

$$\begin{aligned} \frac{dV(D)}{dD} &= \sum_{j=1}^M \frac{(1 - p)F - p'(D)FD + (p'(D)FD + pF)(1 - L) - pFDL'(D)}{(1 + r)^j} \\ &= \sum_{j=1}^M F \frac{1 - (p(D) + Dp'(D))L - pFDL'(D)}{(1 + r)^j} \\ &= \frac{r_B}{1 - (1 + r_B)^{-M}} \cdot \frac{1 - (1 + r)^{-m}}{r} [1 - (p(D) + Dp'(D))L - pDL'(D)] \end{aligned}$$

Note that if  $r = r_B$ , this simplifies to equation (4). Otherwise, we would need to apply a discount rate adjustment. If the loan is originated at fair value, then  $r < r_B$ .