The Environmental Cost of Ivory Trade in Kenya

A model estimating the effects of elephant poaching on the Kenyan wildlife and economy

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Introduction and Methodology

The African elephant population has been decimated by poaching over the past ten years. Elephant tusks are a significant source of ivory, which is used in ornaments and jewelry worldwide. In 2016, the price of ivory was as high as 1,500 USD per pound due to increased demand coming from Asia (Fears 2016). Poaching has several negative effects, not only on the elephant species but also on the economic stability of many African nations that rely heavily on wildlife tourism (Kahumbu 2014). In particular, between 1970 and 1977, Kenya lost more than half of its elephants (Douglas-Hamilton 1979, page 19). Though banned in 1973 (Morell 2017), illegal elephant hunting continues and its effects are more and more detrimental for both the Kenyan wildlife and its internal economy.

The topic is of relevance and interest because of the current endangerment of the elephant population in Kenya. This project provides a framework to model ivory trade and policy analysis in Kenya. This is done by adapting environmental models and economic growth models available in the recent literature on this field. In particular, the models used are the logistic population growth model, the Gordon-Schaefer model, a modified Hotelling's model, and the Solow-Swan model.

In this report, we shall aim to provide three models for estimating the effects of poaching. The first shall focus on the Kenyan ecosystem. The model explores the predator-prey interaction and the effects that a reduced number of elephants has on wildlife equilibrium. The second provides a framework to model the behaviour of ivory producers. We present several scenarios: in particular, the case of sustainable harvesting, the case where firms maximise their profits regardless of their environmental impact, and the case where firms consider how elephants' natural growth is impacted by their choice. The third and final model provides a framework for the government or environmental protection agency to optimally determine spending in environmental conservation efforts. The models together will also demonstrate scope for policy-making.

1. The Ecosystem Model

Predator-Prey Interaction and Ecosystem Equilibrium

Elephant Population

Although elephants have no natural enemies in the wild, they can still become prey to large cats such as lions or hyenas. Specifically, weak, sick, young, or injured elephants may be targeted by such predators (Estes 1992, page 259). As the number of elephants changes, we would expect to see the populations of its predators following the same path due to the food chain.

We shall start our analysis by looking at the effects of decreased levels of elephants on the wildlife equilibrium. The model takes into account the predator-prey interaction in the ecosystem. Since predators depend on the number of elephants available, the growth of predators is affected by the number of elephants as well. We shall thus define two different growth functions.

The first growth model represents the growth of Kenyan elephants, which is affected by both poaching and predation:

$$\partial_t E[t] = g_E \left(1 - \frac{E[t]}{E_{\max}}\right) E[t] - \varepsilon_E E[t] P[t] - h[t] E[t] \tag{1.1}$$

where $\partial_t E[t]$represents the marginal change in the number of elephants at time t, $E[t]$is the stock of elephants in Kenya at time t. $g_E$represents the intrinsic growth rate of elephants, E_{max} is the carrying capacity of the Kenyan environment and $P[t]$ is the total stock of predators at time t. Lastly, $h[t]$ and $\varepsilon_E$respectively capture the degree of elephant poaching at time t and the effect that predators have on the population of elephants.

The growth model takes the form of a standard logistic growth model (Bjørndal, Munro 2012, page 16) with two additional linear terms to take into account the effects of hunting and predation. As such, the growth model demonstrates pure compensation, that is the proportional growth rate is a strictly decreasing function of stock.

We have assumed that ε_E is constant and exogenously determined. $h[t]$ is, instead, chosen by the ivory producers. For tractability, this study assumes that these two parameters feature in the equation as linearly dependent on the current stock of elephants. That is, $h[t]$ and $\varepsilon_E P[t]$ can be interpreted as the instantaneous proportion of the elephant stock that is hunted and eaten respectively in the period under analysis. This proportional hunting specification also assumes that the elephant population is uniformly mixing in the spatial area of interest.

The proportion of elephants subject to predation, $\varepsilon_E\, E[t]\, P[t]$, is also increasing with the stock of predators to model the idea that, with more predators, elephants are more often hunted. In reality, the predator-prey interaction would most likely be an unknown non-linear function. Specifying such a function would be beyond the scope of this study and the model would mostly result in an abstraction of such complication. For these reasons, this model assumes a linear predator-prey interaction.

Predator Population

The second growth model describes the growth of predators, which depends on the number of elephants available. A simplifying assumption that this study makes is that the two species that prey on Kenyan elephants, namely lions and hyenas, have similar growth models expressed as an aggregate of the two species's stock. In reality, the two species would be likely to have different growth rates and carrying capacities. Their ability to hunt elephants and rely on them for food may also differ.

The model takes a similar form to the elephant growth model, a standard logistic growth model (Bjørndal, Munro 2012, page 16) with an additional linear term:

$$\partial_t P[t] = g_P \left(1 - \frac{P[t]}{P_{\max}}\right) P[t] + \varepsilon_P E[t] P[t]$$

(1.2)

where $\partial_t P[t]$is the marginal change in the number of predators at time t, $P[t]$ is the stock of predators in Kenya at time t. g_P represents the intrinsic growth rate of predators and P_{max} is maximum carrying capacity of predators in the Kenyan environment if there are 0 elephants. Lastly, $\varepsilon_P$is a coefficient determining the effect of the elephant population on the growth of predators.

According to Howard (2017): "Due to their size, African elephants are not easy prey for many predators". Hence, the model is designed such that predators do not solely rely on elephants for food, having part of their growth model independent of the elephant stock. The predators' population is, nonetheless, supplemented by the available option of hunting elephants. This is captured by the coefficient ε_P. As per the elephant growth model, ε_P is similarly assumed to be constant and exogenously determined.

For tractability, the predation interaction term, $\varepsilon_P\ E[t]\ P[t]$, is once again unrealistically linear. However, unlike in the elephant growth model, the predation interaction term is positive because the study assumes that the more the food (in the form of elephants) that is available to the predators, the higher their growth. Similar to the elephant growth model, the predation effect is proportional to the stock of elephants.

In this study, the models are constructed as continuously differentiable functions: the smoothness makes the models more tractable than discrete-time models when solving for solutions. However, the limitation of such an approach is that data are often recorded in discrete periods and animals are measured in the space of natural numbers. Hence, an appropriate modification to these models would be required if extended to applied work. The smooth functions also imply that neither threshold effects nor irreversibilities can be modelled. For example, if elephant or predator populations fall below a certain level, the specification of the growth model may change in a non-continuous manner, and may not be able to return to previous levels ex ante.

Equilibrium Without Poaching

In order to determine the effect that poaching has on the growth of both elephants and predators, we simulated their dynamic path with and without poaching. The parameters chosen were as much as possible trying to match the current stock of each species. According to Nzwili (2013) there are between 30,000 and 38,000 elephants currently in Kenya. As such, we have approximated the number of elephants at 34,000. Scriber (2014) claims that elephant loss by natural predation is approximately 3% per year. As for the number of predators, following Planz (2009) we have approximated lions at 2,000 and following Mills (1998) we have approximated the number of hyenas at 1,500. Hence, aggregate number of predators is approximated to 3,500.

The model, at the without-poaching equilibrium, will try to simulate these figures:

(i) Elephant stock is 136,000
(ii) Predator stock is 3,500

The first assumption is based on Nzwili (2013) according to which, in 1979, approximately 1.2 million elephants roamed in Africa whilst today there remain 300,000. That is about 4 times less than in the past. We thus assume that Kenya's elephant population today saw a similar proportional decline to that faced by the whole continent. Hence, we assume it to be at 34,000, roughly 4 times less than it would at equilibrium, 136,000. The second assumption tells us that, at equilibrium, predators would be constant at 3,500 as they are not directly affected by humans.

For graphical modelling, the constant parameters are defined in such a way to emulate the information above. The intrinsic growth rate of elephants is set at 0.09, slightly lower than the intrinsic growth rate of predators, 0.1. The carrying capacity of elephants, set at 200,000, is much higher than that of predators, set at 3,000. Lastly, following the above-stated reasoning, the coefficients ε_E and ε_P, in the elephants' and predators' growth models, are small, set at 0.0086 and 0.000123 respectively, as elephants are difficult to hunt and they are not the only prey for lions and hyenas. For the rest of this study, the parameters will be assigned these values.

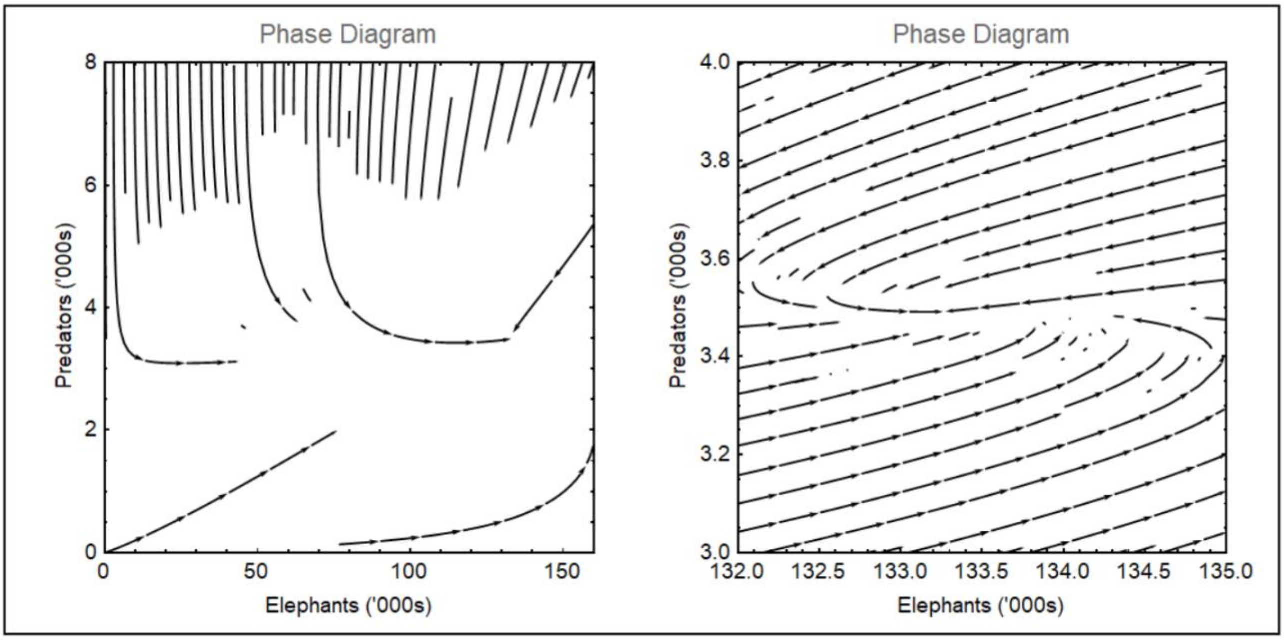

Figure 1.1: Ecosystem without poaching

Setting these parameters, the predator-prey dynamic equilibria may be represented in a phase space flow diagram as shown in Figure 1.1 (see Appendix for the *Mathematica* inputs used for all subsequent diagrams). The right panel is a closer view of the equilibrium in the left panel.

The phase space flow diagram shows that, without poaching, equilibrium for elephants and predators dynamically converges to the assumed results (approximately 133,269 and 3,492). In order to remove the effect of poaching on the model, $h[t]$, which captures the degree of hunting, has been set at 0 thus showing what should happen naturally within the Kenyan ecosystem. Linearising the system, the eigenvalues of the corresponding Jacobian both have real negative values and, hence, stable (see Appendix).

Equilibrium With Constant Poaching

Let us now allow $h[t]$ to take a constant non-zero value in all periods, so as to estimate the damage on elephant and predator growth with poaching. This assumption will be loosened when ivory producers, who hunt elephants, are introduced in the model and $h[t]$will be variable based on the amount of effort supplied by producers. Setting $h = 0.0642$, the phase plot may be shown in Figure 1.2 (see Appendix). Again, the right panel is a closer view of the equilibrium.

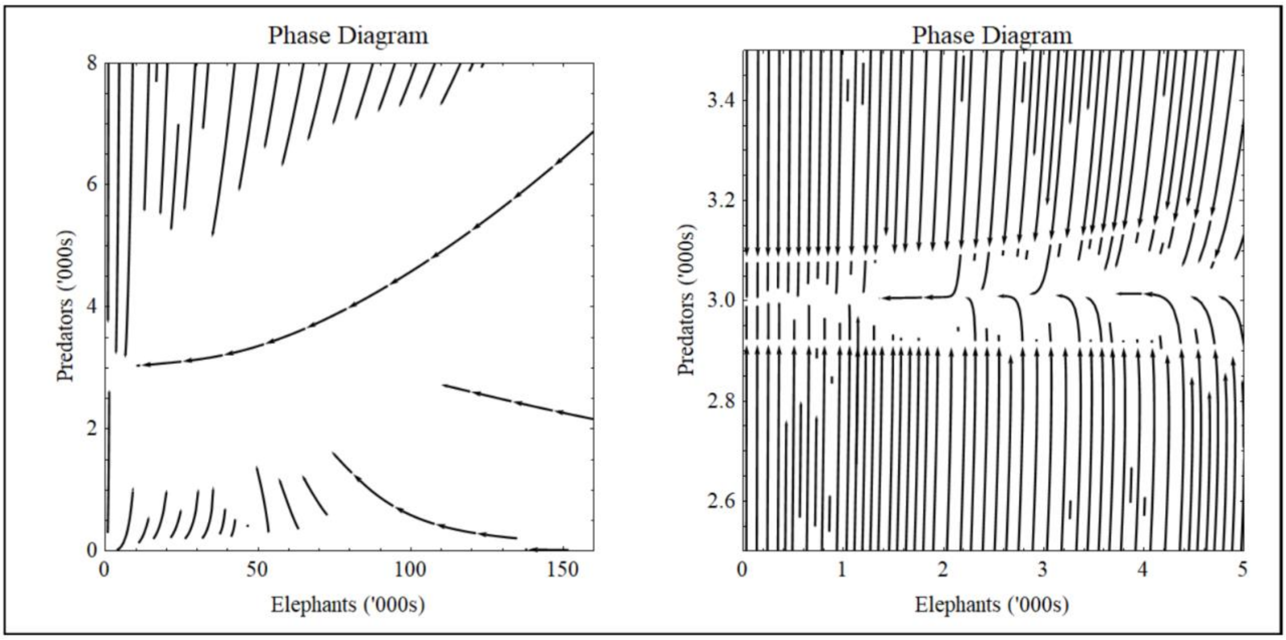

Figure 1.2: Equilibrium with unsustainable poaching

These phase space flow diagrams show that, with this level of $h[t]$, both the equilibrium number of elephants and predators have decreased (reaching approximately 0 and 3,000, respectively). Poaching has a dramatic effect on wildlife equilibrium and represents one of the biggest threats to Kenya's natural capital.

2. Modelling Firms

Profit Function

In this section, we shall look at what ivory traders do in order to maximise their profits. In the model, we will rely on the following three assumptions:

- Ivory producers are profit-maximising firms.
- Prices are exogenously determined and firms are price-takers. This is because illegal trade on the black market of endangered species is controlled by a large number of small criminal groups (Damania, Bulte 2007, page 464), and ivory is sold on an international market (Nelson 2017).
- Ivory producers do not have information about, or take into consideration, the predator-prey interaction in the ecosystem.

In the modelling, we will make the assumption that the industry behaves as a single firm, although it is a price taker, a local monopoly participating in an international market. First, we define the ivory producer's profit function:

$$\pi = pq[t] - c_1 F[t]^{\eta} \tag{2.1}$$

where p is the price of a unit of ivory, $q[t]$ is the quantity of ivory sold at time t, c_1 is the unit marginal cost of producing ivory, $F[t]$ is the amount of effort required to hunt and produce ivory as a function of time, and η is the output elasticity of effort.

We, then, define the quantity of ivory produced:

$$q[t] = h[t]E[t] = \varphi F[t]E[t] \tag{2.2}$$

where φ is a constant that captures the proportion of elephants hunted by $F[t]$ units of effort, thus representing hunting efficiency and corresponding to the catchability coefficient in the fishery management literature (Bjørndal, Munro 2012, page 17). Consequently, the amount of ivory extracted at time t depends on this efficiency coefficient, the units of effort employed, and the number of elephants. We assume that $q[t]$ is linear in effort. A more realistic model would use a production function exhibiting decreasing returns to effort.

Substituting (2.1) into (2.2), we then obtain:

$$\pi = p\varphi F[t]E[t] - c_1 F[t]^{\eta} \tag{2.3}$$

Imposing the restriction $\eta \geq 1$, the profit function is weakly concave. In the static model analysis, η will be set to 1. This simplification is such that the profit function is linearly increasing in effort. A more realistic model would include a specification that exhibits strict concavity following from increasing marginal cost to production.

2.1 Static Model

Sustainable Poaching

Following a modified Schaefer model that takes into account predator-prey interaction, we consider what happens if ivory production occurs on a sustainable yield basis. This entails that instantaneous extraction is equal to the instantaneous growth of the elephant stock. Mathematically, the stock of elephants is stable, $\partial_t E[t] = 0$:

$$h[t]E^* = g_E\left(1 - \frac{E^*}{E_{\max}}\right)E^* - \varepsilon_E E^* P[t] \tag{2.1.1}$$

where E^* denotes the steady-state stock of elephants. Substituting (2.2) into (2.1.1):

$$\varphi F[t]E^* = g_E\left(1 - \frac{E^*}{E_{\max}}\right)E^* - \varepsilon_E E^* P[t] \tag{2.1.2}$$

In the steady state, the stock of predators is also stationary, that is, $\partial_t P[t] = 0$. Let P^*denote the steady-state stock of predators. From (1.2):

$$0 = g_P\left(1 - \frac{P^*}{P_{\max}}\right)P^* - \varepsilon_E E[t]P^* \tag{2.1.3}$$

Using (2.1.2) and (2.1.3), solving for steady-state equilibrium level of elephants as a function of effort $F[t]$:

$$E^* = g_P E_{\max}\left(\frac{g_E - F[t]\varphi - \varepsilon_E P_{\max}}{g_E g_P + \varepsilon_E \varepsilon_E E_{\max} P_{\max}}\right) \tag{2.1.4}$$

Going back to our profit function, we now substitute the result obtained in (2.1.4) into (2.3) in order to derive an expression for sustainable profit. Profit is then maximised with respect to effort $F[t]$ and solved for F_{MEY}, the maximum economic yield (MEY):

$$\partial_{F[t]}\pi = 0 \tag{2.1.5}$$

MEY is the maximum resource rent that can be sustainably extracted from the elephant stock at equilibrium. The market structure needed to arrive at such a condition would be that the industry behaves like a single profit-maximising firm, that is, a monopoly.

If the industry is perfectly competitive and firms can freely enter or exit the market, industry profit will be driven to 0. The corresponding level of effort is F_{BE}, where there is no resource rents. This is known as the bionomic equilibrium (BE). Alternatively, the bionomic equilibrium is found where total revenue is equal to total costs. BE represents an over-allocation of effort as the marginal cost of effort exceeds the value of the marginal product of effort. As such, BE is an example of a market failure.

This level of effort occurs at F_{BE}, which, as stated above, represents the solution to the following condition:

$$\pi = 0 \tag{2.1.6}$$

We can also compare these results with the maximum sustainable yield (MSY), that is the maximum sustainable extraction of the resource. This is found by obtaining the maximum natural growth rate of the elephant stock. From (1.1), the first order condition becomes:

$$\partial_{E[t]}\left(g_E\left(1-\frac{E[t]}{E_{\max}}\right)E[t]-\varepsilon_E E[t]P^*\right)=0$$

(2.1.7)

Substituting (2.1.3) into (2.1.7) and solving for $E[t]$ yields the level of elephant stock, E^{**}, that maximises the elephant natural growth rate. This may be illustrated in Figure 2.1.1 (see Appendix for input).

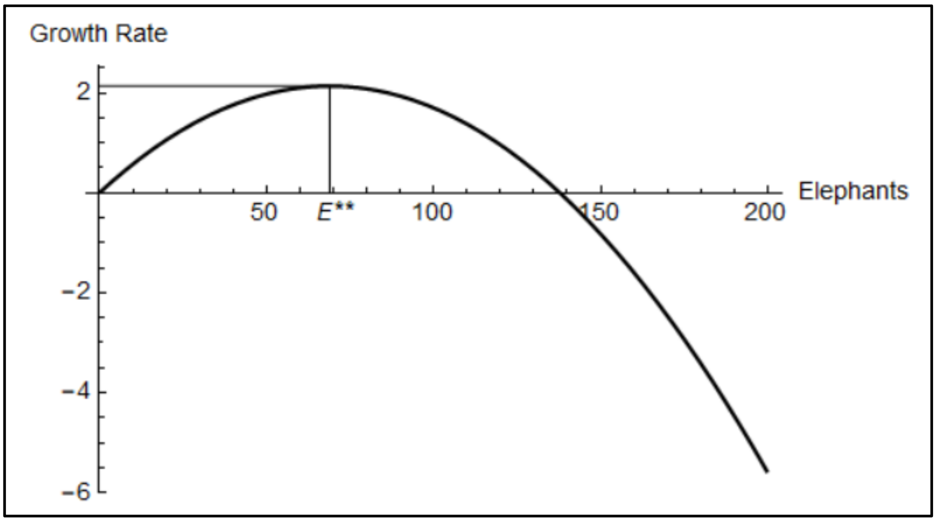

Figure 2.1.1: Elephant Growth Rate

Equating the maximum growth rate (at E^{**}) to the level of hunting and solving for effort $F[t]$, we obtain F_{MSY}, the level of effort corresponding to the maximum natural growth rate of elephants:

$$g_E\left(1-\frac{E^{**}}{E_{\max}}\right)E^{**}-\varepsilon_E E^{**}\,P^*=\varphi\,F[t]\,E^{**}$$

(2.1.8)

Specifying the ivory price, cost, and hunting efficiency parameters shall enable us to numerically find the maximum economic yield, maximum sustainable yield, and bionomic equilibrium levels of effort (see Appendix). Assigning ivory price as unity, cost parameter c_1 as 3, and hunting efficiency as 0.065, the corresponding levels of effort may be shown on revenue/cost space as per Figure 2.1.2.

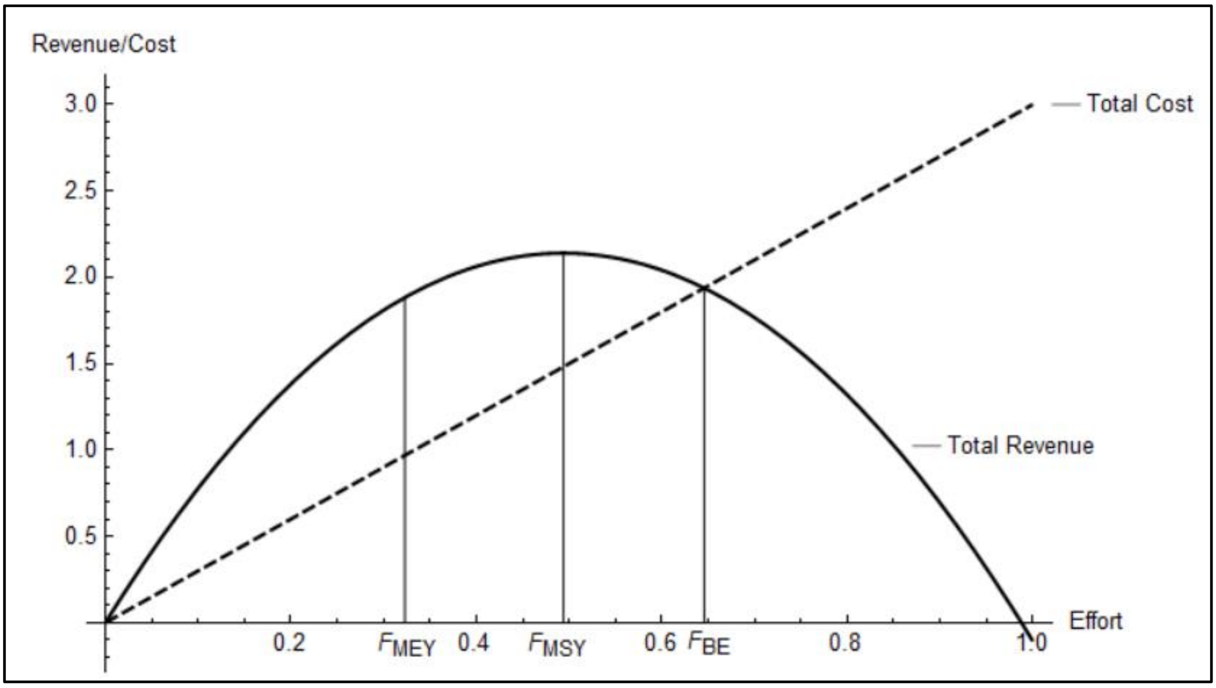

Figure 2.1.2: MEY, MSY, and BE

Figure 2.1.2 is the familiar diagram of the Gordon-Schaefer model. For the parameters assumed, it is the case that $F_{MEY} < F_{MSY} < F_{BE}$. Increasing the cost (parameter c_1) raises the steepness of the total cost line, while increasing ivory price (parameter p) raises the total revenue curve. As such it could be the case where $F_{MEY} < F_{BE} < F_{MSY}$ if cost were high enough. F_{MEY} and F_{BE} represent two extreme market structures. From the literature (Damania and Bulte 2007), the actual ivory market would tend to the competitive extreme, so even if successful policy ensures sustainable poaching ceteris paribus, effort level would still be close to F_{BE}. At this bionomic equilibrium level of effort, resource rents are driven to 0, hence there is an over-allocation of effort leading to overexploitation of the resource. On the other hand in the monopoly case at F_{MEY} , resource rents are maximised. It is also worth noting that $F_{MEY} < F_{MSY}$ the maximum sustainable yield level of effort.

In order to observe the effect of these effort levels on the equilibrium number of elephants and predators, we substitute the found levels of effort into the ecosystem model of Figure 1.2 and plot phase space flow diagrams. The assumption is that firms fix their level of effort in all time periods. This is a limitation of the static model that will be later loosened.

At the bionomic equilibrium level of effort, F_{BE} = 0.646, approximately corresponding to $h = 0.042$, the equilibrium number of elephants and predators is stable at (46.1538, 3.17031). This is shown in Figure 2.1.3.

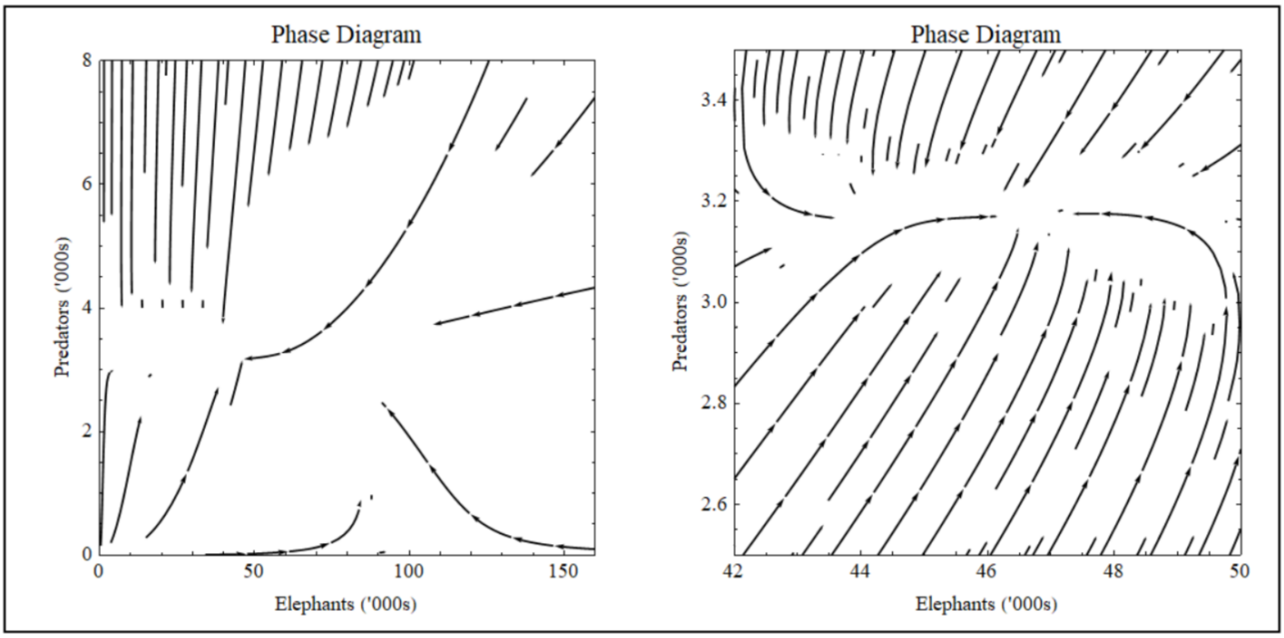

Figure 2.1.3: Equilibrium at BE

At $F_{MSY} = 0.494$, approximately corresponding to $h = 0.031$, the equilibrium number of elephants and predators is stable at (66.6345, 3.24588). This is shown in Figure 2.1.4.

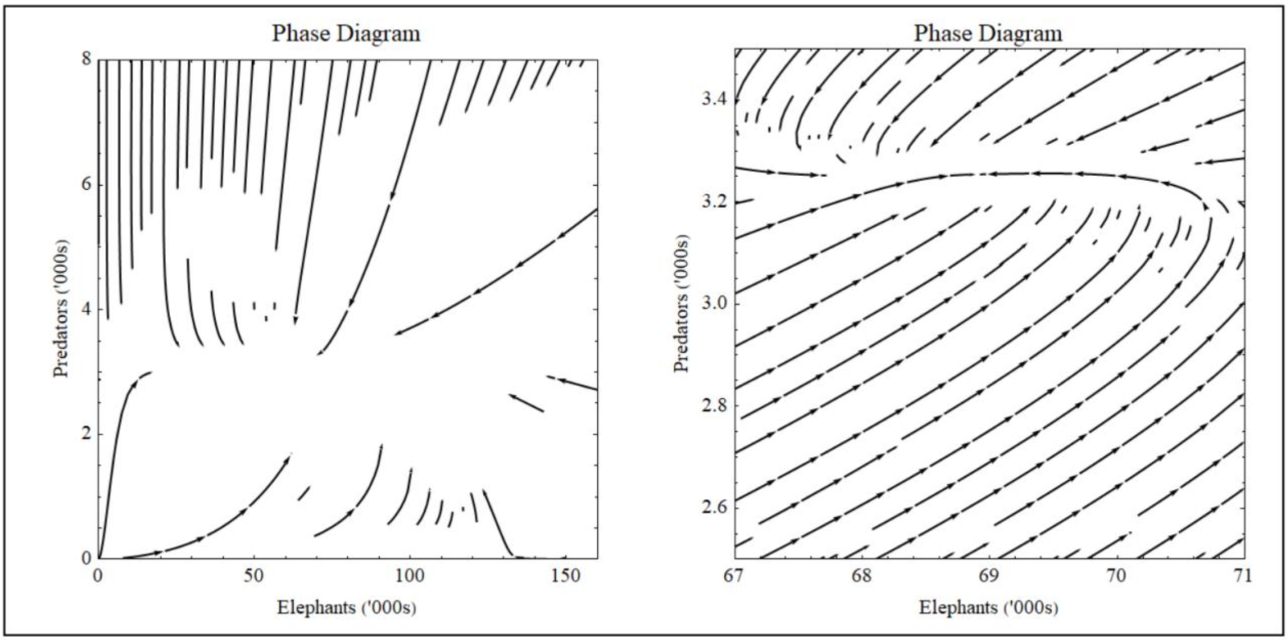

Figure 2.1.4: Equilibrium at MSY

At $F_{MEY} = 0.323$, approximately corresponding to $h = 0.021$, the equilibrium number of elephants and predators is stable at (89.7112, 3.33103). This is shown in Figure 2.1.5.

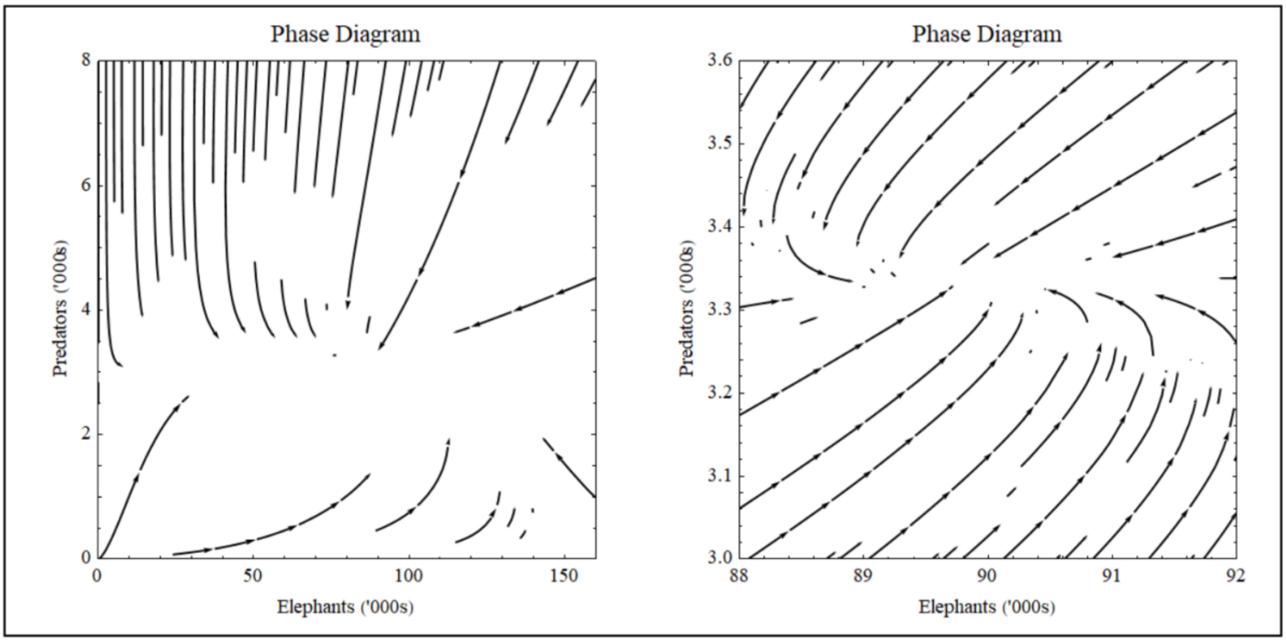

Figure 2.1.5: Equilibrium at MEY

The three phase space flow diagrams show the effect that the ivory market structure has on the ecosystem. A lower amount of effort that arises from the case of a single firm increases both the number of Kenyan elephants and predators, compared to the level of effort arising from a competitive market.

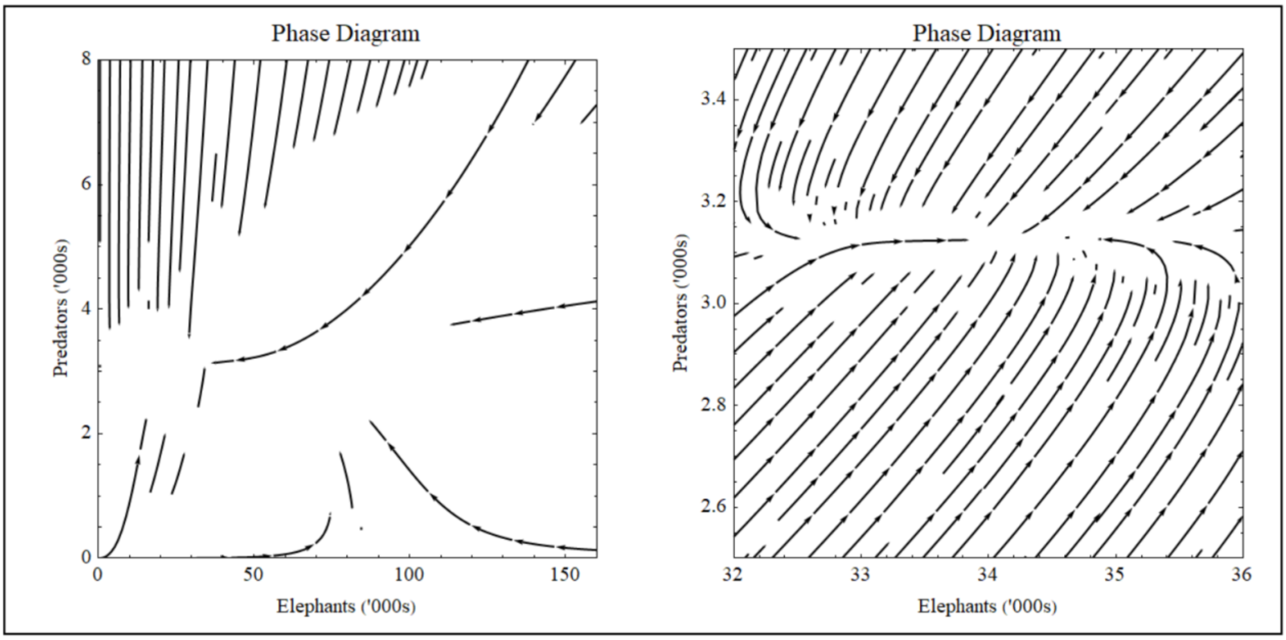

Figure 2.1.6: Equilibrium at BE with lower cost

The parameters chosen in the Gordon-Schaefer model led to the BE number of elephants (approx. 46,138), significantly larger than the true number of elephants in Kenya (34,000). Therefore, the true level of effort is predicted to be above F_{BE} (0.644533). A reason behind this discrepancy between the estimated and true values may be an overspecification of the cost of hunting elephants. A lower cost for producers (c_1) in the Gordon-Schaefer model would lead to a higher F_{BE}, and thus to a lower equilibrium number of elephants (see Appendix for adjusted Gordon-Schaefer model). With a reduced c_1(from 3 to

2.2), the new F_{BE} is larger (0.73684). This level of effort leads to the equilibrium number of elephants approximately approaching the current number of elephants and predators within Kenya, at (33.8562, 3.12489). This is shown above in Figure 2.1.6.

Lastly, the Gordon-Schaefer model assumes sustainable poaching. Alternatively, the current situation of the ecosystem could be the case of unsustainable poaching as per Figure 1.2 and the elephant stock is on a path toward zero.

Unsustainable Extraction

The Gordon-Schaefer analysis shows the impact given a sustainable exploitation of the resource. Unsustainable poaching may be also modelled using the static model. Firms maximise profit with respect to effort level $F[t]$according to condition (2.1.5). However, unlike in the Gordon-Schaefer model, sustainable harvest does not occur. Given that the profit function (2.3) is linear with respect to effort (having set $\eta = 1$), firms will supply optimal level of effort $F^* = +\infty$, or realistically speaking, the supremum of the set of all possible effort levels. As such, the elephant population will be driven to zero as previously shown in the Figure 1.2. For a more rigorous illustration of this mismanagement, a dynamic model may be used.

2.2 Dynamic Model

While the static analysis is able to provide insight on the equilibrium, it does not provide information about the path needed to reach that equilibrium. In order to assess more rigorously whether over-allocation of effort would also imply the overexploitation of Kenyan elephants, the dynamic path of elephants and predators must be considered. Our analysis will consist of two parts. The first shall look at ivory producers who do not take into account their influence on the elephants stock. The second shall look at ivory producers who do take into account the elephant stock growth.

No Consideration of Elephant Growth

Let us first set up the ivory producers' dynamic problem. We assume that the ivory production industry behaves as a single firm. This producer maximises the present value of the stream of future profits with respect to the control variable, effort $F[t]$:

$$\max_{E[t],F[t]} \left\{ \int_{t=0}^{+\infty} e^{-\rho t}\, \pi\big[E[t], F[t]\big]\, dt \right\} \tag{2.2.1}$$

Profit $\pi[E[t], F[t]]$ is defined as per (2.3), and ρ is the continuous discount rate or social interest rate. We also assume that the firm always observes the elephant stock, $E[t]$, but perceives it as exogenously determined. An explanation for such a treatment would be that poachers are ignorant of, or do not consider in their decision making, the effect of poaching on the elephant stock.

With these assumptions, the firm maximises instantaneous profit with respect to effort $F[t]$ and the first order condition may be written as per (2.1.5). This may be interpreted as the firm's best response to the current stock of elephants. Unlike in the static model analysis, the restriction $\eta = 1$is relaxed to $\eta \geq 1$. For numerical modelling, it is set to $\eta > 1$ to avoid the case that optimal level of effort $F^* = +\infty$ as discussed above. The first order condition is evaluated as:

$$F^* = \left(\frac{\varphi p E[t]}{\eta c_1}\right)^{\frac{1}{\eta-1}}$$

(2.2.2)

An intuitive depiction of the model is that the firm observes, at every point of time, the instantaneous elephant stock level, and chooses the effort level which maximises its profit according to (2.2.2). The result of its choice of effort then feeds back into the ecosystem, affecting the elephant stock growth rate through the elephants poached employing that effort level. Defining parameters with fixed constants, the ecological impact of such a industry can be modelled by numerically solving (1.1), (1.2), and (2.2.2). The initial stock of elephants and predators have been approximately set to their current estimated levels, 34,000 and 3,000 respectively. For this and subsequent dynamic plots, baseline parameters for the ecosystem are set as per Figure 1.1, while setting ivory price and cost as unity, hunting efficiency to 0.065, and output elasticity to 1.5. With these parameters, the dynamic path of variables are plotted in Figure 2.2.1 (see Appendix for input).

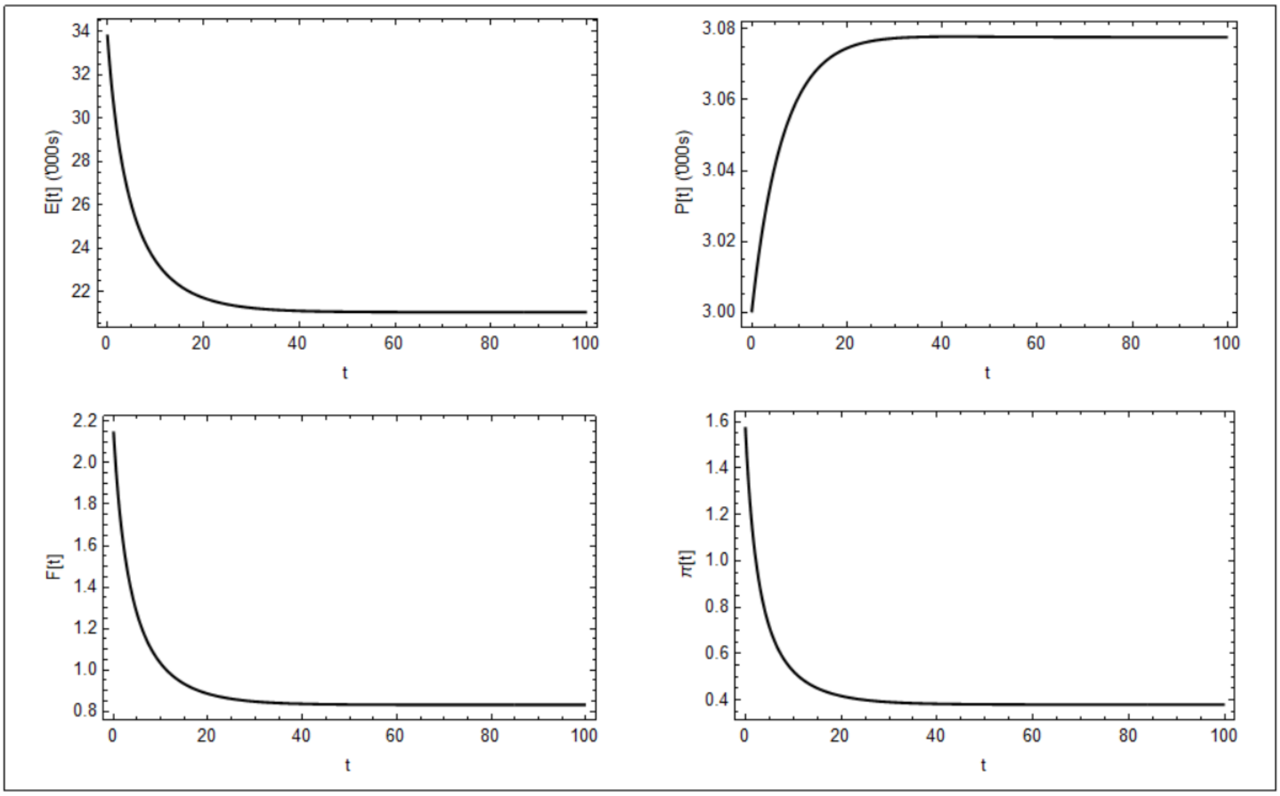

Figure 2.2.1: Dynamic path without consideration of elephant growth

It can be seen from the graphs that effort $F[t]$ experiences a similar evolution as the elephant stock$E[t]$. This can be deduced from the first order condition given by (2.2.2). Profit $\pi[t]$, thus, sees a falling evolution as hunting becomes less efficient with the shrinking elephant stock (resulting from the proportional extraction assumption). Because of increasing marginal cost in the profit function specification, $\eta > 1$, economic incentives are such that the elephant stock is not driven to zero, albeit reduced by about a third of its initial level. The equilibrium is hence sustainable, although at a lower elephant stock level than the initial one.

Several observations can also be made by perturbing the parameters (see Appendix for plots of these cases):

1. Increasing cost (parameter c_1 from 1 to 1.5) raises the equilibrium elephant stock (from about 21 to 30 thousand). Equilibrium predator stock increases slightly, while effort and profit both fall for all t
2. Increasing the output elasticity (parameter η from 1.5 to 2) greatly increases the time period to obtain equilibrium, and equilibrium elephant stock is slightly higher (from about 21 to 25 thousand).
3. Increasing the price of ivory (parameter$p$ from 1 to 10) drives the equilibrium elephant stock much lower (from about 21 to 2 thousand), in a shorter time period. Effort and profit are much higher in earlier periods, but are left only slightly higher at equilibrium (equilibrium effort rises from about 0.83 to 0.97).

The economic intuitions behind these observations are straightforward. Increasing the cost parameter increases the marginal cost of effort, hence the firm would cut back on production and elephant stock would be higher. The higher cost would necessarily mean a lower profit. Increasing the price of ivory increases the marginal revenue of the firm, hence incentivises higher production of ivory. The corresponding higher level of effort drives the elephant stock down. However, equilibrium profit with a higher price is only slightly higher because the lower equilibrium elephant stock means that poaching is much less efficient (having assumed proportionate extraction).

Firm Considers Elephant Growth

We now consider a model where firms take into account the growth rate of elephants. With regard to the model, the predator stock is always observed, but is perceived by the firm to be exogenously determined. An explanation for such a treatment would be that poachers do know how predation and their activity, explicitly affect the elephant stock. However, they do not have any information on predator growth.

This study again makes the assumption that the industry behaves as 1 firm. While still maximising the present value of the stream of future profits with respect to the control variable, effort $F[t]$, the firm faces the constraint, the growth of elephants, describing the evolution of the state variable, elephant stock E[t]:

$$\max_{E[t],F[t]} \left\{ \int_{t=0}^{+\infty} e^{-\rho t}\, \pi\big[E[t], F[t]\big]\, dt \right\} \; s.t.\; \partial_t E[t] = G\big[E[t], P[t]\big] - H\big[E[t], F[t]\big]$$

(2.2.3)

$\partial_t E[t]$ and $\pi[E[t], F[t]]$ are specified as jointly concave in $E[t]$and $F[t]$in order to satisfy Mangsarian sufficient conditions. The natural growth rate of elephants and the effect of poaching on the elephant growth rate are defined as per (1.1), $G[E,P] = g_E\,(1 - \frac{E[t]}{E_{max}})\,E[t] - \varepsilon_E\, E[t]\, P[t]$ and $H[E,F] = p\,\varphi\, F[t]\, E[t]$ respectively. Profit $\pi[E[t], F[t]]$ is defined as per (2.3). Again, the restriction in the static model analysis $\eta = 1$is relaxed to $\eta \geq 1$. The producer's problem (2.2.3) may be written with these specified functions:

$$\max_{E[t],F[t]} \left\{ \int_{t=0}^{+\infty} e^{-\rho t}\, (p\varphi\, F[t]\, E[t] - c_1 F[t]^{\eta})\, dt \right\}$$
$$s.t.\, \partial_t E[t] = g_E \left(1 - \frac{E[t]}{E_{\max}}\right) E[t] - \varepsilon_E E[t] P[t] - p\varphi F[t] E[t]$$

(2.2.4)

For notational brevity, the function names will be used instead. Defining the current-value Hamiltonian:

$$\mathcal{H} = \pi\big[E[t], F[t]\big] + \lambda[t]\big(G\big[E[t], P[t]\big] - H\big[E[t], F[t]\big]\big)$$

(2.2.5)

$\lambda[t]$ is a costate function. By Pontryagin's maximum principle, first order conditions yield:

$$\rho\lambda[t] - \partial_t\lambda[t] = \partial_{E[t]}\mathcal{H} = \partial_{E[t]}\pi + \lambda[t]\left(\partial_{E[t]}G - \partial_{E[t]}H\right)$$

(2.2.6)

$$0 = \partial_{F[t]}\mathcal{H} = \partial_{F[t]}\pi - \lambda[t]\partial_{F[t]}H$$

(2.2.7)

At equilibrium, the costate function is stable, $\partial_t\,\lambda[t]$ = 0. Substituting (2.2.6) into (2.2.7) yields the the fundamental equation or "Golden Rule" for renewable resource management:

$$\rho = \partial_{E[t]}(G - H) - \frac{\partial_{E[t]}\pi \times \partial_{F[t]}(G - H)}{\partial_{F[t]}\pi}$$

(2.2.8)

The left-hand side is the interest rate and the right-hand side represents the rate of return on marginal investment in the resource, the elephant stock. In particular, $\partial_E(G - H)$represents the marginal product of natural capital, the impact of resource investment on sustainable extraction. The second term $\frac{\partial_E\pi\partial_F(G-H)}{\partial_F\pi}$is the marginal stock effect, that is, the impact of resource investment on extraction rent. The second term may be further analysed. The denominator $\partial_{F[t]}\pi$ is the marginal resource investment cost, namely the opportunity cost of marginal profit forgone, and the numerator is the marginal effect of resource investment on stock growth $\partial_{F[t]}(G - H)$ multiplied by the marginal effect of stock on extraction rent $\partial_{E[t]}\pi$.

An intuitive depiction of the model is that the firm observes, at every point of time, the instantaneous predator stock level. With this information, the firm maximises the present value of profit in the form of a dynamic path, taking into consideration the evolution of the elephant stock. Although the producer does not take the predator growth model into consideration, the producer's choice of effort level affects the elephant stock which influences and is influenced by the predator stock. Hence it is not a "complete" dynamic optimisation.

We thus proceed to plot the dynamic solution. Parameters are defined with the same baseline fixed constants as that of the previous dynamic model of the firm not taking the elephant growth into consideration. The continuous discount rate is also set at 0.035 (or 3.5%). The first order conditions (2.2.6) and (2.2.7) along with the constraint in (2.2.4) are numerically solved for the optimal path. Figure 2.2.2 shows the optimal path of the control variable, effort, along with the corresponding profit, stock of elephants and predators, and value of the costate function (see Appendix).

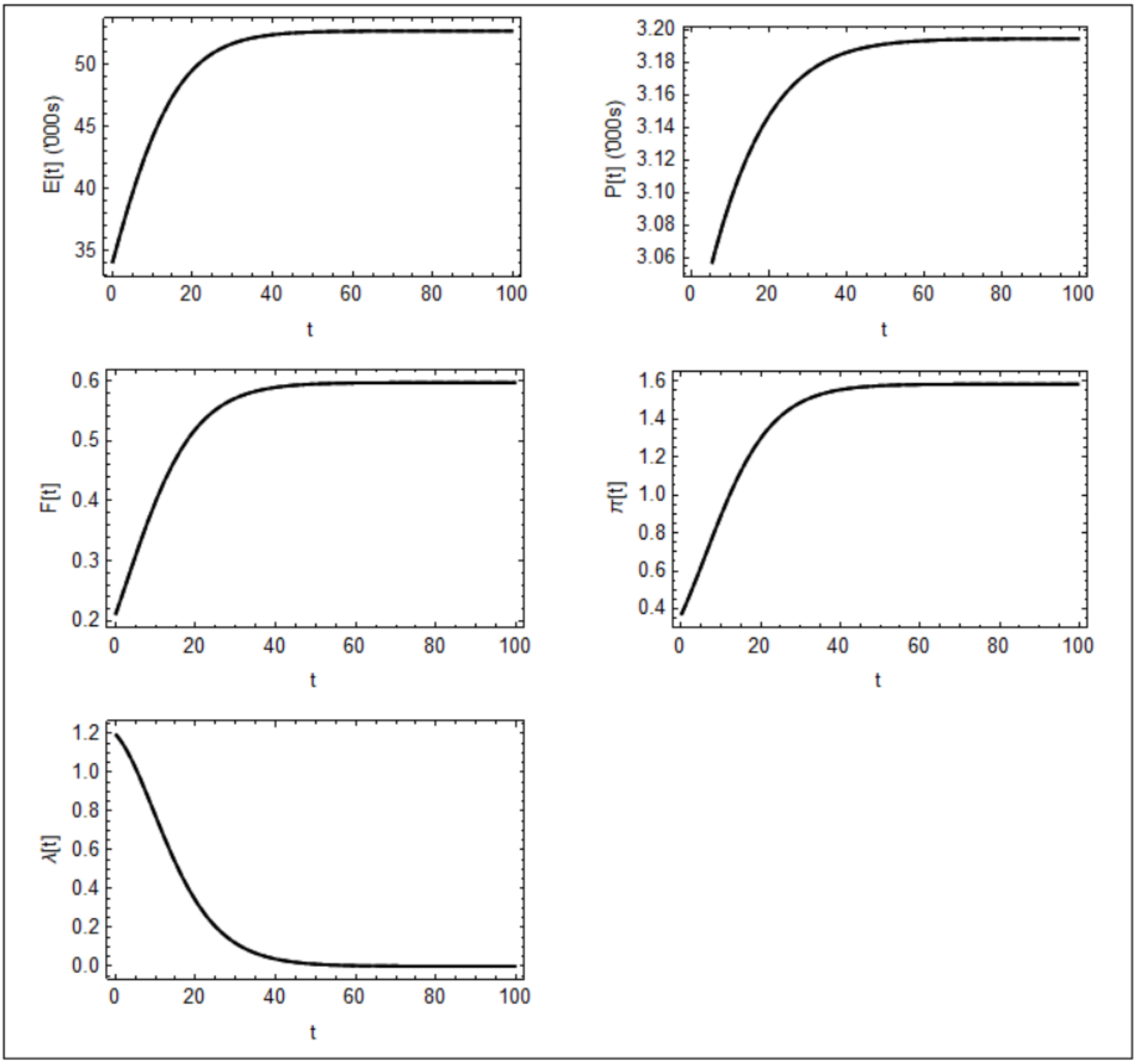

Figure 2.2.2: Dynamic path with consideration of elephant growth

Despite being defined with the same constant parameters, the dynamic path of the variables considered in this model is very different from the previous model, where the firm did not take elephant stock growth into consideration. In early periods, effort level is low, which can be interpreted as the firm investing in the elephant stock resource by withholding extraction. Doing so allows the stock to gradually recover, and also increases its natural growth rate. As the natural growth rate increases, the firm gradually increases effort until extraction is equal to the natural growth rate of elephants.

In this model, the firm is also noticeably better off than in the previous model. Instantaneous profits are sustainable at a much higher level (about 1.6 as compared to 0.37 previously). A smaller equilibrium effort is also employed (at 0.6 as compared to 0.83 previously). Economically, this means that larger rents are being obtained from the elephant resource, with a smaller amount of factor inputs. Such firm behaviour would hence be an aggregate improvement in efficiency, as factor inputs may be employed elsewhere in other industries. These results also show that the previous model is a case of resource overexploitation and mismanagement. However, the loser in this model may be the labour that could have found employment in the ivory industry as per the previous model, but because of factor immobility, would be unemployed instead.

Perturbing parameters such as cost and ivory price yields similar results as per the previous model, and will not be repeatedly discussed (see Appendix for plots of these cases). In the previous model, because the firm was maximising instantaneous profit, the discount rate was insignificant. However, in this model the parameter would be of interest. The dynamic path with an increased discount rate (parameter ρ from 0.035 to 0.1), which is higher than the intrinsic growth rate of elephants g_E (set at 0.09), is plotted in Figure 2.2.3.

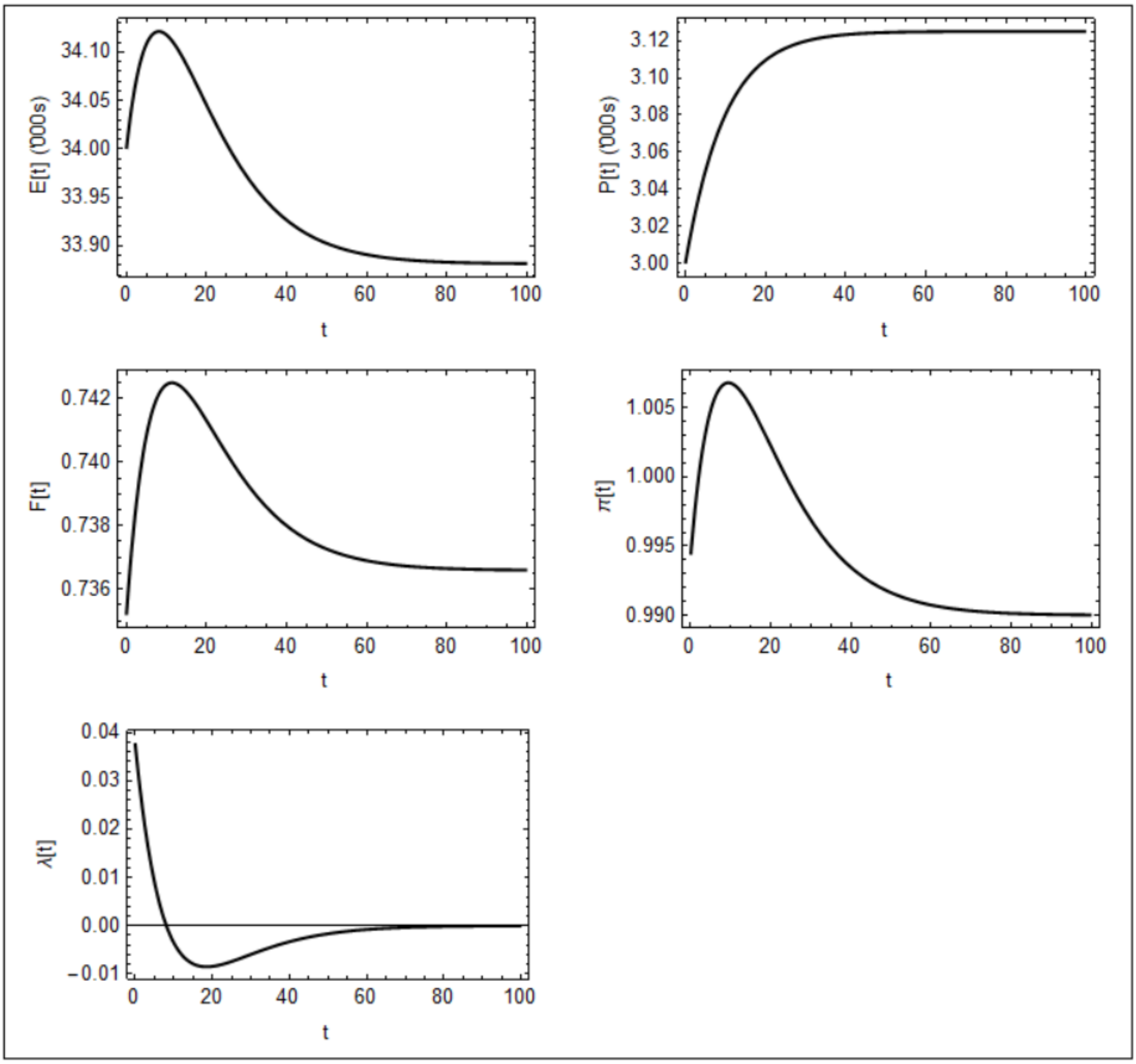

Figure 2.2.3 Increased discount rate

Unlike in the case with a lower discount rate, equilibrium elephant stock is lower than the initial level. This is consistent with findings in fishery management, where "with [intrinsic growth rate < discount rate], the economic theory tells us that the rational de facto private resource owners should drive the resource to extinction" (Bjørndal, Munro 2012, page 162). Although it is not the case in this model because the industry behaves as a single firm, a discount rate that is higher than the intrinsic growth rate of the elephant stock does place higher risk on the elephant population.

The economic intuition behind the higher discount rate result is that firms discount future gains by a greater degree, and hence would prefer to extract more from the resource in earlier periods than invest in it. This can be seen from the higher effort level in earlier periods compared to in the previous case of a lower discount rate. Another perspective is that from capital theory using the fundamental equation of renewable resource management (2.2.8): a higher discount rate would require a higher rate of return on the marginal investment in the resource. But even with the higher discount rate, the elephant stock is still higher than in the case of Figure 2.2.1. The case of Figure 2.2.1 may be alternatively interpreted as the firm having a positively infinite discount rate, hence it maximises instantaneous profit without regard to future profit flows.

Dynamic Model Limitations

A limitation of this model is that the dynamic path is (locally) saddle-point stable. The optimal paths above were plotted using uniquely chosen initial values of effort. With these initial values, the system evolves toward a stable equilibrium, as seen from the stability attained by the costate function $\lambda[t]$. This study will not attempt to rigorously prove the existence of the saddle-point stability, but will show graphically that perturbing the uniquely chosen initial value of effort (solid line path), keeping all other specifications the same, would destabilise the system (dotted line paths). This is done for the case of Figure 2.2.2, illustrated in Figure 2.2.4.

Given saddle-point stability, this model is assuming that the firm chooses the single path toward the stable equilibrium. This is because any slight deviation from this path would lead to a divergent solution. This assumption is hardly realistic. However, one may argue that such a outcome is a result of mathematical construction. The firm, in reality, once it realises that it is diverging from this optimal path, would correct itself to be on the planned optimal path again.

In this study's treatment of the dynamic model, the case where the firm fully optimises with respect to both the elephant and predator growth specifications is not in scope. However, this study predicts that such behaviour will bring a small increase in profits of firms. The increase in profits would be due to the firm's better predictive ability of the elephant stock growth. The reason why this effect would be small is because the constant parameters were chosen, particularly in the predator growth model, such that predator stock is little affected by the predator-prey interaction, given that elephants are difficult prey as previously discussed. It can be seen from the plots of the two models in this study that the predator population sees only slight changes (less than 0.3 thousand) as a result of a larger change in the elephant stock. Given that the predator population is roughly constant, the second model could be a good approximation for any results from behaviour that fully optimises with respect to the full ecosystem interaction.

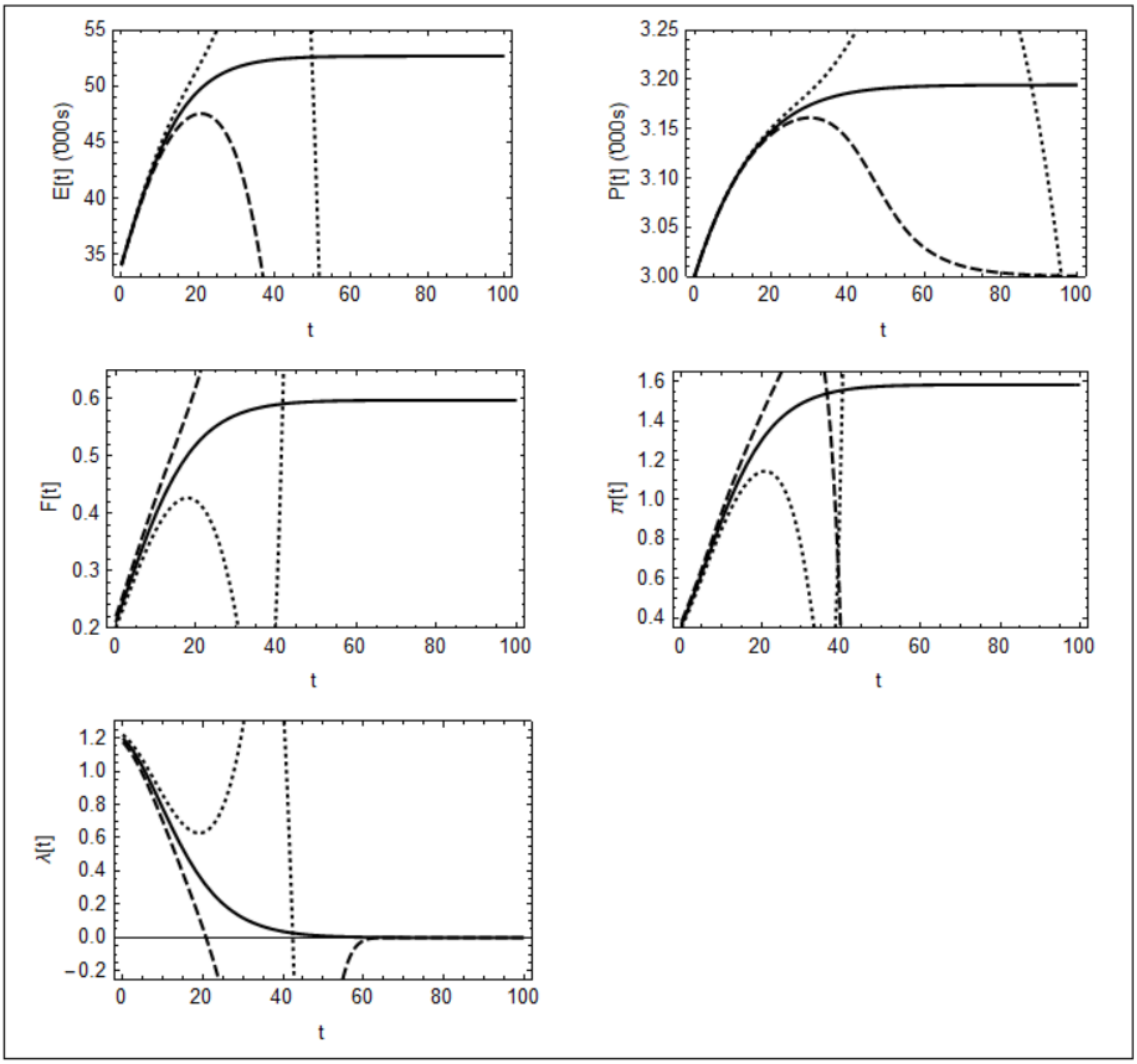

Figure 2.2.4: Saddle-point stability

3. The Economy Growth Model

We now turn to considering the choice of a social planner, who maximises the utility gained from non-natural capital, k_1, and natural (environmental) capital, k_2. We shall assume that there is a positive correlation between the stock of natural capital and the utility that the economy gains from wildlife tourism. As noted by Paula (2014), the Kenyan economy relies heavily on wildlife tourism, so we assume that the social planner gains substantial utility from natural capital preservation. Thus, we can proceed in specifying the utility function that the social planner maximises utility over time:

$$\int_{t=0}^{\infty} e^{-\rho t} u\left[k_1[t], k_2[t]\right] = \int_{t=0}^{\infty} e^{-\rho t}\left(\log[b] + \alpha_1 \log\left[k_1[t]\right] + \alpha_2 \log\left[k_2[t]\right]\right) \tag{3.1}$$

For simplicity reasons, we assume that b = 1, α_1=1 and α_2=1. For tractability reasons, we have designed the social planner's utility function in log form.

The social planner shall maximise his utility function subject to two constraints. The first is represented by the net growth of non-natural capital. The stock of non-natural capital evolves over time according to a modified Solow-Swan Growth Model:

$$\partial_t k_1[t] = s(1 - \Theta[t]) k_1[t]^a (AL)^{1-a} - \gamma k_1[t] \tag{3.2}$$

where $k_1[t]$ represents non-natural capital at time t, L represents labour, s is the saving rate in the economy, $\Theta[t]$ is the proportion of GDP spent on environmental conservation, A refers to labour-augmenting technology or "knowledge", and γ captures the depreciation of non-natural capital over time.

This model follows similar assumptions to the standard Solow Model. It is in continuous time, and the aggregate production function $Y = k_1[t]^a (A\,L)^{1-a}$, which may be interpreted as GDP, satisfies the Inada conditions. It is also assumed that saving rate and depreciation rate are exogenously determined and remain constant in all periods.

The second constraint corresponds to the net growth of natural capital. The natural stock evolves over time according to the following growth function:

$$\partial_t k_2[t] = \Phi\left(\Theta[t]^\delta - \Theta_1\right) k_2[t] \tag{3.3}$$

where $k_2[t]$ represents non-natural capital at time t, φ is a constant that measures how conservation policy translates into an effect on natural capital growth, $\Theta[t]$ is the proportion of GDP spent on environmental conservation, and δ captures the proportion of the environmental conservation expenditure that translates into the growth of natural capita. It is assumed that $\delta > 1$ as expenditure on environmental conservation may not translate into actual conservation effects. By this assumption, a higher δ is associated with a lower actual effect from abatement expenditure since $\Theta[t] < 1$. In the modelling, we set the parameter δ equal to 2.

The natural stock growth model for k_2 shows that if an economy's effective spending on environmental conservation as a proportion of their GDP, $\Theta[t]^\delta$, is higher than some threshold, Θ_1, the growth of natural stock is positive. However, if an economy's effective spending on environmental conservation as a proportion of their GDP is lower than the threshold, the growth of natural capital is negative.

Hence, the social planner's utility maximisation problem may be written as:

$$max\left\{\int_{t=0}^{\infty} e^{-\rho t}(\log[b]+\alpha_1\log[k_1[t]]+\alpha_2\log[k_2[t]])\right\}$$
$$s.t.\ \partial_t k_1[t]=s(1-\Theta[t])k_1[t]^a(AL)^{1-a}-\gamma k_1[t]\quad and\quad \partial_t k_2[t]=\Phi(\Theta[t]^\delta-\Theta_1)k_2[t]$$

(3.4)

where $\Theta[t]$ is the control variable that the social planner chooses, while facing state variables $k_1[t]$ and $k_2[t]$.

Solving the maximisation problem allows us to graph the optimal path for the conservation. We define the current value Hamiltonian:

$$\mathcal{H}=\log[b]+\alpha_1\log[k_1[t]]+\alpha_2\log[k_2[t]]+\lambda_1[t](s(1-\Theta[t])k_1[t]^a(AL)^{1-a})$$
$$+\lambda_2[t](\Phi(\Theta[t]^\delta-\Theta_1)k_2[t])$$

(3.5)

By Pontryagin's maximum principle, first order conditions yield (refer to Appendix for full expressions of derivatives):

$$\rho\lambda_1[t]-\partial_t\lambda_1[t]=\partial_{k_1[t]}\mathcal{H}$$

(3.6)

$$\rho\lambda_2[t]-\partial_t\lambda_2[t]=\partial_{k_2[t]}\mathcal{H}$$

(3.7)

$$0=\partial_{\Theta[t]}\mathcal{H}$$

(3.8)

For specific parameters (which we listed in the Appendix to this report) and low level of necessary conservation to sustain natural capital, equations (3.6), (3.7), and (3.8) are solved for the optimal path. The optimal paths of $\Theta[t]$, $k_1[t]$, and $k_2[t]$are plotted in Figure 3.1.

The optimal path may be useful for the social planner, i.e. the Kenyan government or an environmental protection agency, to establish its optimal environmental conservation expenditure. As we can see, the more the social planner spends on environmental conservation, the more his economy's natural capital shall grow. However, in doing that, the social planner must also take into account the effect that such increased environmental conservation expenditure has on non-natural capital. Indeed, expenditure on environmental conservation can be sustained until a certain threshold after which non-natural capital starts to decrease, negatively affecting the economy's proportion of GDP relying on $k_1[t]$.

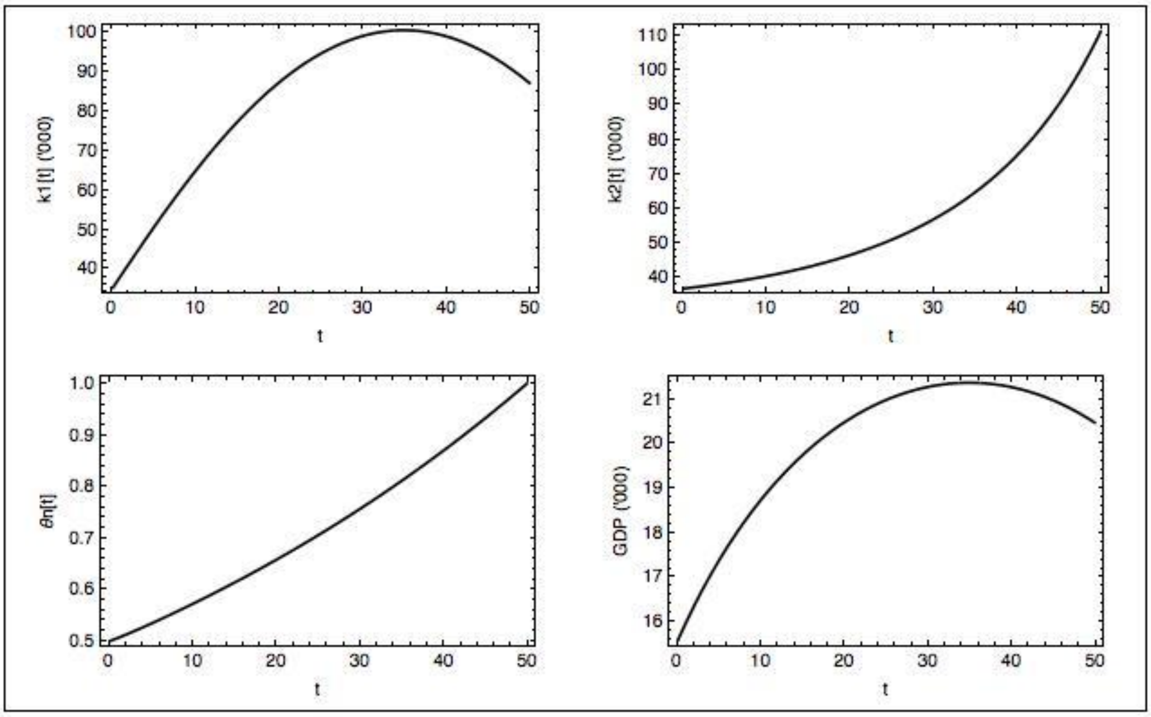

Figure 3.1: Low level of optimal conservation

When there is a higher level of spending necessary to keep the natural capital at its constant level, Θ_1, the optimal path evolves as shown in Figure 3.2. We can see from the graphs that, when the stabilising conservation spending is higher, the government would choose similar path in therm of the proportion of GDP spent on conservation. Nonetheless, this would result in an initial decrease of the natural capital, given the low level of conservation spending, which turns into a sharp increase after a considerable amount of time. This scenario perhaps captures a case where transaction costs of implementing policy are too high, which outweighs the benefit of implementing environmental conservation instruments.

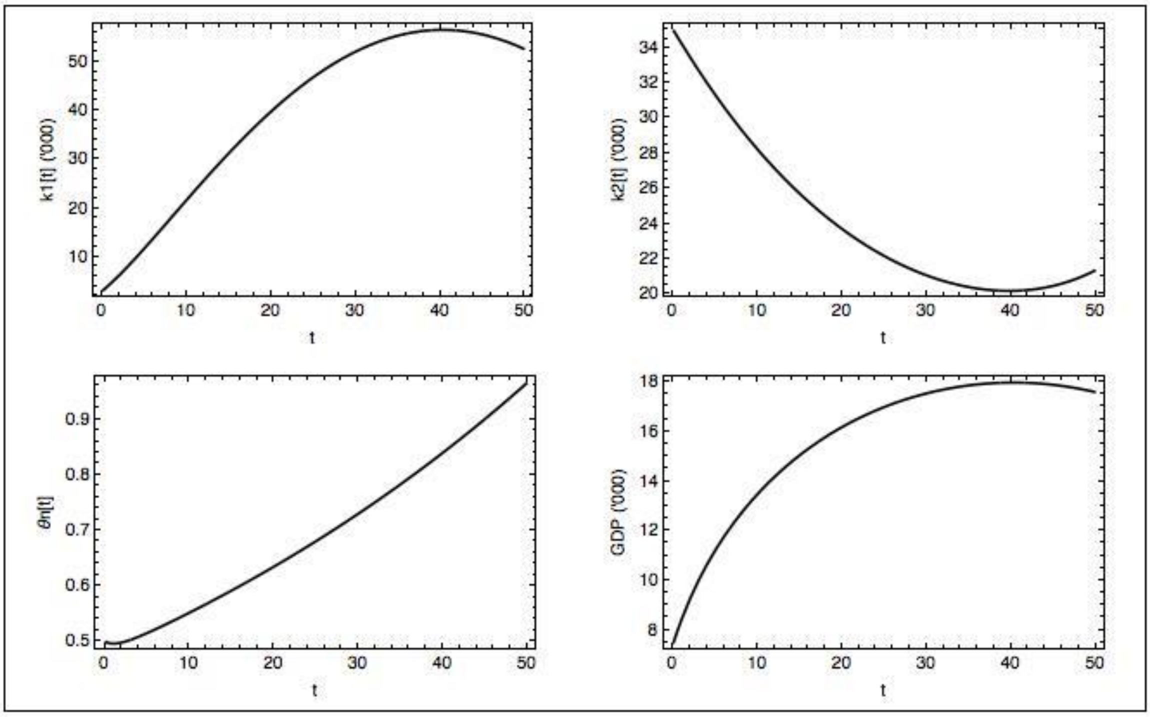

Figure 3.2: High level of optimal conservation

Lastly, an increase in elephant poaching, that is a decrease in $k_2[t]$, also leads to a decrease in $k_1[t]$ both endangering the Kenyan economy as a whole and calling for a strong increase in environmental conservation expenditure or environmental policy. The modelling of this interaction is not in the scope of this study, but will provide a framework for such analysis. The natural capital can be modelled as a increasing function of the stock of elephants and predators:

$$k_2[E[t], P[t], t]$$

As modelled above, the presence of poaching decreases the stock of elephants and predators through the ecosystem interaction and hence decreases natural capital. The environmental conservation expenditure may also be written as an increasing function of the cost parameters of producing ivory :

$$\Theta[c_1, \eta, t]$$

The reasoning behind such a function is that implementing environmental conservation policy in the form of incentive adjusting methods like regulation and taxation will have associated administrative and enforcement costs. Hence increasing the cost of ivory production would demand larger government expenditure.

A more robust model will explore the interaction between environmental conservation expenditure Θ more precisely affecting natural k_2 by increasing the cost of ivory production c_1 and thereby disincentivizing heavy elephant and predator poaching given a certain market structure. However, cost may not be the only avenue which affects poaching levels. As seen from the models in this study, competition policy that alters the market structure would have a significant effect. With regards to price, ivory is sold on an international market (Nelson 2017), which makes it difficult for the government alone to influence such demand.

4. Discussion and Conclusions

This study has provided a framework to model the impact that elephant poaching has on both the Kenyan wildlife equilibrium and its economy.

The Ecosystem Model

Through modelling the growth of elephant and predator populations, we have been able to show the dynamic interaction between elephants and predators. Given that elephants are difficult prey, we assumed a weak predation relationship between elephants and predators. This is reflected in the model by a small drop in the predator population even if the elephant population is driven to zero through unsustainable hunting. By making further assumptions, particularly about the carrying capacity of elephants, and the absence of threshold effects and irreversibilities, equilibrium elephant and predator population would converge approximately to the 1979 levels in the long run, if poaching were to completely cease.

A relevant limitation of this model is that elephants likely play a larger role in the ecosystem (Save the Elephants n.d.). Elephants provide water for other animals during dry seasons by "using their tusks to dig for water" themselves (ibid). They also "create gaps in the vegetation… [that] allow new plants to grow and create pathways for smaller animals to use" (ibid). In addition, "some species rely entirely upon elephants for seed dispersal" (ibid). Our model is an abstraction of all these interactions. Such omitted interactions would represent a source of error if they significantly influenced either the elephant population directly or the predator population and indirectly have a feedback effect on the elephant population.

Climate change is also another large confounding factor in the analysis, as "one of the main threats to wildlife in Africa" (Maoga 2017). In 2009, "by the end of the dry spell, conservationists estimated that 40 percent of Kenya's wildlife population was lost" (ibid). The effect of climate change would likely affect the growth model parameters over time, such as intrinsic growth rates and carrying capacities. In the model, these parameters are specified as constants over time. This would likely lead to an overestimation of the elephant and predator populations, assuming they are negatively affected.

Modelling Firms

From the static model analysis that assumes sustainable poaching, the market structure of the ivory industry plays a large role in determining the effect poaching has on the ecosystem. Under a monopoly, maximum economic yield is achieved, which results in maximum resource rent and higher equilibrium elephant and predator populations. It is also of note that this level of poaching effort is lower than that to attain maximum sustainable yield. Under a perfect competition structure, resource rents are driven to zero, and the equilibrium elephant and predator populations are lower. The analysis shows that local stakeholders are better off in the case of the monopoly. Firms obtain maximum resource rents, while the ecosystem is larger.

These results would be useful to a social planner that is considering implementing competition policy to improve social welfare. Although the assumption that sustainable poaching occurs, the stark contrast between the efficiency attained by a monopoly compared to a competitive market is seen.

The dynamic model analysis shows how perverse incentives lead to over-allocation of effort and overexploitation of the elephant resource. The case where the industry maximises instantaneous profit can be interpreted as an instance of information asymmetry, where the industry either does not take into consideration its effect on the ecosystem or it is extremely nearsighted with an infinite discount rate. This is compared to the case where the industry maximises its profits constrained by the elephant growth model. The former case results in a less efficient equilibrium than the latter, suggesting an overexploitation of the resource.

Although the dynamic model makes the limiting assumption that the industry behaves as a single monopolistic firm, the results show the degree to which incentive adjusting policy, such as increasing cost, price, and social discount rate, affects the optimising behaviour of the industry. If able to accurately estimate certain parameters with low uncertainty, the policymaker may make use of the dynamic analysis to simulate the effectiveness of policies such as Pigouvian taxes (that raise marginal cost) . Alternatively, the optimal dynamic path may be used to guide specification of incentive blocking policies such as quotas, to raise the elephant population to a more efficient level.

The policymaker may use both the static and dynamic analyses, taking their limitations into account, to design a policy that alters market structure and firm incentives. However, the current ban on elephant hunting must to be lifted in order to implement such market regulating policy. Given that the elephant population has been in decline despite the long-standing ban, it could be the case that the ban has been ineffective. This may be because the resource rents that could be reaped, even while the resource is inefficiently exploited, are much more attractive than the risk of being penalised for illegal activity. Hence, legalisation and regulation of the market may prove to be a more effective conservation effort compared to this pure incentive blocking policy.

A limitation of this analysis of firms is that only monopolistic and competitive markets are considered. Other market structures may arise due to differences in ivory product. In this study, it has been assumed that the industry produces a homogenous ivory product. In cases of oligopolistic structure, strategic interactions would require game theoretical modelling. Another limitation is that because the elephant hunting in Kenya is currently illegal (Morell 2017), greater information asymmetries may exist in the market. Hence even in a competitive market structure, conventional general equilibrium analysis may not be fully appropriate.

The Economy Growth Model

Lastly, through the 'Economy Growth Model' we have provided a framework for optimal environmental conservation expenditure that should be chosen by the Kenyan government. For the policy maker, the growth model answers the question, "how much and when should I spend on environmental policy?" for a given policy effectiveness, and not "in which policies should I spend?". The growth model would be of use to the policy maker, if able to estimate accurately the required parameters. The latter question is, in turn, answered by our model of the ivory firms and the ecosystem.

In the model, we analysed the chosen path for environmental conservation and GDP of social planner that maximises his utility both with respect to natural capital and to non-natural capital. We have seen how expenditure on environmental conservation can be sustained until a certain threshold after which non-natural capital starts to decrease. On contrary, the evolution of natural capital shows a different pattern. When the stabilising conservation spending is low, the government would choose a path in which natural capital constantly increases. When the spending required for stock maintenance is high, the government would choose to increase conservation spending at a lower level which first results in a decrease then an exponential increase of natural capital.

There are several limitations that prevents us to model the optimal path to be applied in real world setting. Firstly, the assumption about the social planner's preferences is an oversimplified reflection of reality. Social planners would also take into account the effects that their spending may have on society and economic indicators of the country. In particular, the open economy aspects of the problem are ignored in our study. Secondly, parameters may change over time, relating to the resilience of the ecosystem. We have assumed that there is a unique optimal portion of GDP that should be spent on conservation. This is highly superficial because this proportion would presumably change by the alteration of the stock of elephants, predators and the gross amount of GDP itself. In addition, technological changes can have a direct effect conservation effectiveness which in turn determines the amount required for the sustainability of natural capital. Furthermore, our model disregards the double dividend hypothesis which states that environmental policy can simultaneously improve the environment and increase economic efficiency.

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