```
import torch
from torch.distributions.normal import Normal
import math
import matplotlib.pyplot as plt
### (a) log-derivative trick
def df_dmu(x, mu, sigma):
  return x*torch.sin(x)*(x-mu)/sigma**2 + mu-1
def df_dt(x, mu, sigma):
  return x*torch.sin(x)*(-1+(x-mu)**2/sigma**2) + sigma - 1
epochs = 200
Ir = 1e-2
batch = 16
x_batch = torch.zeros(256,1)
# initialize
mu = torch.tensor([0.0])
t = torch.tensor([0.0])
mu_lst_a =[]
t_lst_a = []
for epoch in range(epochs+1):
  x_batch = Normal(loc=mu, scale=torch.exp(t)).sample(x_batch.size())
  mu -= df_dmu(x=x_batch, mu=mu, sigma=torch.exp(t)).mean() * Ir
  t -= df_dt(x=x_batch, mu=mu, sigma=torch.exp(t)).mean() * Ir
  if epoch%2==0:
     mu_lst_a.append(mu.item())
     t_lst_a.append(t.item())
  if epoch%100==0:
     print('epoch:', epoch, 'mu:',mu.item(), 'sigma:',math.exp(t.item()), 'tau:', t.item())
     X = Normal(loc=mu.item(), scale=math.exp(t.item())).sample((1,5000))
                     print('f:',((X*torch.sin(X)).mean()+ 0.5*(mu.item()-1)**2 +
                                                                                     math.exp(t.item())
math.log(math.exp(t.item()))).item())
```

(b) reparameterization trick

```
def dphi_dx(x):
  return torch.sin(x)+x*torch.cos(x)
def df_dmu(y, mu, sigma):
  return dphi_dx(mu+sigma*y) + mu - 1
def df_dt(y, mu, sigma):
  return dphi_dx(mu+sigma*y)*sigma*y + sigma - 1
y_batch = torch.zeros(256,1)
# initialize
mu = torch.tensor([0.0])
t = torch.tensor([0.0])
mu_lst_b = []
t_lst_b = []
for epoch in range(epochs+1):
  y_batch = Normal(loc=0, scale=1).sample(x_batch.size())
  mu -= df dmu(y=y batch, mu=mu, sigma=torch.exp(t)).mean() * lr
  t -= df dt(y=y batch, mu=mu, sigma=torch.exp(t)).mean() * lr
  if epoch%2==0:
     mu_lst_b.append(mu.item())
     t_lst_b.append(t.item())
  if epoch%100==0:
     print('epoch:', epoch, 'mu:',mu.item(), 'sigma:',math.exp(t.item()), 'tau:', t.item())
     X = Normal(loc=mu.item(), scale=math.exp(t.item())).sample((1,5000))
                      print('f:',((X*torch.sin(X)).mean()+ 0.5*(mu.item()-1)**2 +
                                                                                      math.exp(t.item())
math.log(math.exp(t.item()))).item())
plt.subplot(1, 2, 1)
plt.scatter(0,0,s=20)
plt.plot(mu_lst_a, t_lst_a, color='k')
plt.title('log derivative trick')
plt.subplot(1, 2, 2)
plt.scatter(0,0,s=20)
plt.plot(mu_lst_b, t_lst_b, color='k')
plt.title('reparametrization trick')
plt.savefig('log derivative vs reparametrization.png')
plt.show()
```

