P2. $\displaystyle \operatorname*{minimize}_{\theta \in \mathbb{R}^P} \operatorname*{maximize}_{\phi \in \mathbb{R}^P} \; E_{X \sim P_{true}} \left[\log D_\phi (X) \right] + \lambda E_{\tilde{X} \sim P_\theta} \left[\log (1 - D_\phi(\tilde{X})) \right]$

$\Leftrightarrow \quad // \quad // \quad \int P_{true}(x) \log D_\phi(x) + \lambda P_\theta(x) \log (1 - D_\phi(\tilde{x})) \, dx$

The integral is maximized by

$$D_\phi^*(x) = \frac{P_{true}(x)}{P_{true}(x) + \lambda P_\theta(x)}$$

$\left(\begin{array}{l} \because g(y) = a \log y + b \log (1-y), \; 0 < y < 1 \text{ 에서} \\[2mm] g''(y) = -\dfrac{a}{y^2} - \dfrac{b}{(1-y)^2} < 0 \quad \text{where} \quad 0 < y < 1 \\[2mm] g'(y) = 0 \quad \Rightarrow \quad y^* = \dfrac{a}{a+b} \\[2mm] y^* = \dfrac{a}{a+b} \text{ 일때 } g(y) \text{ 최대} \end{array} \right)$

$\Leftrightarrow \displaystyle \operatorname*{minimize}_{\theta \in \mathbb{R}^P} E_{X \sim P_{true}} \left[\log \frac{P_{true}(x)}{P_{true}(x) + \lambda P_\theta(x)} \right] + \lambda E_{X \sim P_\theta} \left[\log \frac{\lambda P_\theta(x)}{P_{true}(x) + \lambda P_\theta(x)} \right]$

$\Leftrightarrow \displaystyle \operatorname*{minimize}_{\theta \in \mathbb{R}^P} \int P_{true}(x) \log \frac{P_{true}(x)}{P_{true}(x) + \lambda P_\theta(x)} + \lambda P_\theta(x) \log \frac{\lambda P_\theta(x)}{P_{true}(x) + \lambda P_\theta(x)} \, dx$

$\Leftrightarrow \displaystyle \operatorname*{minimize}_{\theta \in \mathbb{R}^P} \int \left(\frac{P_{true}(x)}{P_\theta(x)} \log \frac{\frac{P_{true}(x)}{P_\theta(x)}}{\frac{P_{true}(x)}{P_\theta(x)} + \lambda} + \lambda \log \left(\frac{\lambda}{\lambda + \frac{P_{true}(x)}{P_\theta(x)}} \right) + \underbrace{(1+\lambda) \log(1+\lambda) - \lambda \log \lambda}_{\text{Constant}} \right) P_\theta(x) \, dx$

$\Leftrightarrow \displaystyle \operatorname*{minimize}_{\theta \in \mathbb{R}^P} \int f\left(\frac{P_{true}(x)}{P_\theta(x)} \right) \cdot P_\theta(x) \, dx$

$\Leftrightarrow \displaystyle \operatorname*{minimize}_{\theta \in \mathbb{R}^P} D_f (P_{true} \| P_\theta)$