

# A General Framework for Action-Oriented Process Mining

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## 1 Event data

**Definition 1 (Basic Notations).** Let  $X$  denote an arbitrary set.  $\mathcal{P}(X)$  denote the power set of  $X$ , i.e.,  $\mathcal{P}(X) = \{X' \mid X' \subseteq X\}$ . Let  $\perp$  be the null element. We denote  $\hat{X} = X \cup \{\perp\}$  *to additionally include the null element*. Let  $X_a, \dots, X_z$  be arbitrary sets and  $\bar{x}_1 = (x_{a_1}, \dots, x_{z_1}) \in X_a \times \dots \times X_z$  a tuple of these sets. Given a partial function  $f \in X \rightarrow Y$ ,  $\text{dom}(f) = X'$  is the domain of  $f$  such that  $X' \subseteq X$ .

**Definition 2 (Universes).** We define the following universes ~~to be used~~ in the remaining of this paper:

- $\mathcal{U}_{ei}$  is the universe of event identifiers
- $\mathcal{U}_{proc}$  is the universe of process identifiers,
- $\mathcal{U}_{act}$  is the universe of activities,
- $\mathcal{U}_{res}$  is the universe of resources,
- $\mathcal{U}_{time}$  is the universe of timestamps,
- $\mathcal{U}_{oc}$  is the universe of object classes,
- $\mathcal{U}_{oi}$  is the universe of object identifiers,
- $\mathcal{U}_{omap} = \mathcal{U}_{oc} \rightarrow \mathcal{P}(\mathcal{U}_{oi})$  is the universe of object mappings where, for  $omap \in \mathcal{U}_{omap}$ , we define  $omap(oc) = \emptyset$  if  $oc \notin \text{dom}(omap)$ ,
- $\mathcal{U}_{attr}$  be the universe of attribute names,
- $\mathcal{U}_{val}$  the universe of attribute values,
- $\mathcal{U}_{vmap} = \mathcal{U}_{attr} \rightarrow \mathcal{U}_{val}$  is the universe of value mappings where, for  $vmap \in \mathcal{U}_{vmap}$ , we define  $vmap(attr) = \perp$  if  $attr \notin \text{dom}(vmap)$ .
- $\mathcal{U}_{event} = \mathcal{U}_{ei} \times \mathcal{U}_{proc} \times \mathcal{U}_{act} \times \mathcal{U}_{res} \times \mathcal{U}_{time} \times \mathcal{U}_{omap} \times \mathcal{U}_{vmap}$  is the universe of events.

We assume these universes are pairwise disjoint, e.g.,  $\mathcal{U}_{ei} \cap \mathcal{U}_{proc} = \emptyset$ .

**Definition 3 (Event Projection).** Given an event  $e = (ei, proc, act, res, time, omap, vmap) \in \mathcal{U}_{event}$ ,  $\pi_{ei}(e) = ei$ ,  $\pi_{proc}(e) = proc$ ,  $\pi_{act}(e) = act$ ,  $\pi_{res}(e) = res$ ,  $\pi_{time}(e) = time$ ,  $\pi_{omap}(e) = omap$ , and  $\pi_{vmap}(e) = vmap$ .

**Definition 4 (Event Stream).** An event stream  $S$  is a (possibly infinite) set of events, i.e.,  $S \subseteq \mathcal{U}_{event}$  such that  $\forall e_1, e_2 \in S \pi_{ei}(e_1) = \pi_{ei}(e_2) \implies e_1 = e_2$ . We let  $\mathcal{U}_{stream}$  denote the universe of all possible event streams.

not consider  
but this is correct

close  
is it for 2  
could universe be wrong?

first defn of the universe?

**Definition 5 (Time Window).** A time window  $tw = (t_s, t_e) \in \mathcal{U}_{time} \times \mathcal{U}_{time}$  is a pair of timestamps such that  $t_s \leq t_e$ . Given a time window  $tw = (t_s, t_e)$ ,  $\pi_{start}(tw) = t_s$  and  $\pi_{end}(tw) = t_e$ .  $\mathcal{U}_{tw}$  is the set of all possible time windows.

**Definition 6 (Time Moment).** A time moment  $tm = (t, tw) \in \mathcal{U}_{time} \times \mathcal{U}_{tw}$  is a pair of a timestamp  $t$  and a time window  $tw$  such that  $t \geq \pi_{end}(tw)$ . Given  $tm = (t, tw)$ , we indicate  $\pi_t(tm) = t$  and  $\pi_{tw}(tm) = tw$ .  $\mathcal{U}_{tm}$  is the set of all possible time moments.

## 2 A General Framework for Action-oriented Process Mining

### 2.1 Overview

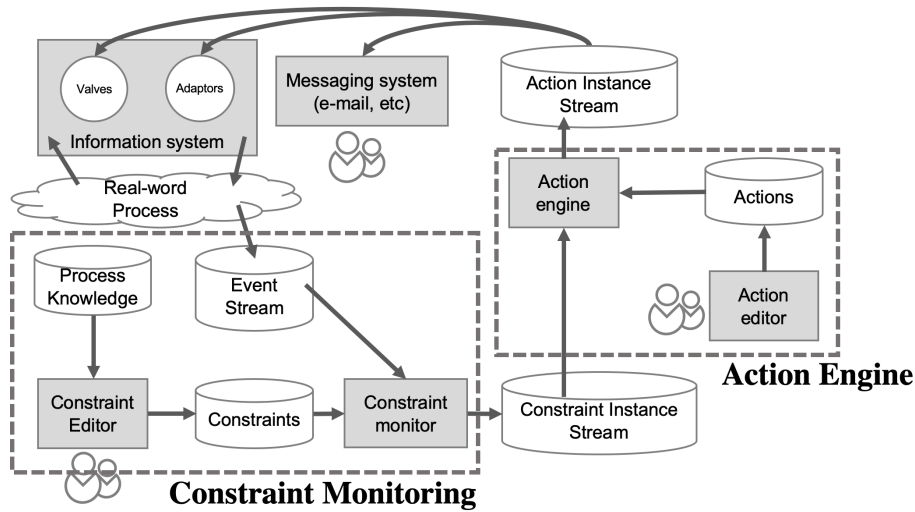


Fig. 1. Overview of a general framework for action-oriented process mining (AOPM)

### 2.2 Constraint Monitoring

**Definition 7 (Constraint Formula).** We define  $\mathcal{U}_{outc} = \{OK, NOK\}$  to be the universe of outcomes.  $cf \in \mathcal{U}_{stream} \times \mathcal{U}_{tw} \rightarrow \hat{\mathcal{U}}_{proc} \times \hat{\mathcal{U}}_{act} \times \hat{\mathcal{U}}_{res} \times \mathcal{U}_{omap} \times \mathcal{U}_{vmap} \times \mathcal{U}_{outc}$  is a constraint formula such that, for any  $S \in \mathcal{U}_{stream}$  and  $tw \in \mathcal{U}_{tw}$ ,

$$\neg \exists_{(proc, res, act, omap, vmap, outc) \in cf(S, tw)} (proc = \perp \wedge res = \perp \wedge act = \perp \wedge dom(omap) = \emptyset \wedge dom(vmap) = \emptyset), \text{ and}$$

space space space

—  $\forall (proc, res, act, omap, vmap, outc), (proc', res', act', omap', vmap', outc') \in cf(S, tw) : proc = proc' \wedge act = act' \wedge res = res' \wedge omap = omap' \wedge vmap = vmap' \implies outc = outc'.$

universe?

$\mathcal{U}_{cf}$  is the set of all possible constraint formulas.

**Definition 8 (Constraint).** A constraint  $c = (cf, TM) \in \mathcal{U}_{cf} \times \mathcal{P}(\mathcal{U}_{tm})$  is a pair of a constraint formula  $cf$  and a set of time moments  $TM$ .  $\mathcal{U}_c$  is the set of all possible constraints.

universe?

**Definition 9 (Constraint Instance).** A constraint instance  $ci \in \mathcal{U}_{cf} \times \hat{\mathcal{U}}_{proc} \times \hat{\mathcal{U}}_{act} \times \hat{\mathcal{U}}_{res} \times \mathcal{U}_{omap} \times \mathcal{U}_{vmap} \times \mathcal{U}_{time} \times \mathcal{U}_{outc}$  is a tuple of a constraint formula  $cf$ , a process identifier  $proc$ , an activity  $act$ , a resource  $res$ , a object mapping  $omap$ , a value mapping  $vmap$ , a timestamp  $time$ , and an outcome  $outc$ .  $\mathcal{U}_{ci}$  is the set of all possible constraint instances.

$G = (cf, act, res, omap, vmap, time, outc)$   
↑  
write it out if you have the variable parts like

**Definition 10 (Constraint Instance Projection).** Given a constraint instance  $ci = (cf, proc, act, res, omap, vmap, time, outc) \in \mathcal{U}_{ci}$ ,  $\pi_{cf}(ci) = cf$ ,  $\pi_{proc}(ci) = proc$ ,  $\pi_{act}(ci) = act$ ,  $\pi_{res}(ci) = res$ ,  $\pi_{omap}(ci) = omap$ ,  $\pi_{vmap}(ci) = vmap$ ,  $\pi_{time}(ci) = time$ , and  $\pi_{outc}(ci) = outc$ .

again where is  $\pi$  coming from?

**Definition 11 (Constraint Instance Stream).** A constraint instance stream CIS is a (possibly infinite) set of constraint instances, i.e.,  $CIS \subseteq \mathcal{U}_{ci}$ .  $\mathcal{U}_{CIS}$  is the set of all possible constraint instance streams.

**Definition 12 (Constraint Monitoring).** Let  $C \subseteq \mathcal{U}_c$  be a set of constraints to be used for monitoring.  $cm_C \in \mathcal{U}_{stream} \rightarrow \mathcal{U}_{CIS}$  is a constraint monitoring such that, for any  $S \in \mathcal{U}_{stream}$ ,  $cm_C(S) = \{(cf, proc, act, res, omap, vmap, time, outc) \in \mathcal{U}_{ci} \mid \exists TM, tm (cf, TM) \in C \wedge tm \in TM \wedge time = \pi_t(tm) \wedge (proc, act, res, omap, vmap, outc) \in cf(S, \pi_{tw}(tm))\}$ .

### 2.3 Action Engine

**Definition 13 (Transaction).** Let  $\mathcal{U}_{op}$  be the universe of operations that are executed by information systems (e.g., send emails). A transaction  $tr = (op, vmap) \in \mathcal{U}_{op} \times \mathcal{U}_{vmap}$  is a pair of an operation  $op$  and a parameter mapping  $vmap$ .  $\mathcal{U}_{tr} \subseteq \mathcal{U}_{op} \times \mathcal{U}_{vmap}$  denotes the set of all possible transactions.

universe  $\mathcal{U}_{op}$  missing

**Definition 14 (Action Formula).** An action formula  $af \in (\mathcal{U}_{CIS} \times \mathcal{U}_{tw}) \rightarrow \mathcal{P}(\mathcal{U}_{tr})$  is a function that generates a set of transactions by analyzing the constraint instances in a constraint instance stream that belong to a time window.

? not mathematically clear to me

**Definition 15 (Action).** An action  $\alpha = (af, TM) \in \mathcal{U}_{af} \times \mathcal{P}(\mathcal{U}_{tm})$  is a pair of an action formula  $af$  and a set of time moments  $TM$ .  $\mathcal{U}_\alpha$  denotes the set of all possible actions.

should be closer together if possible

**Definition 16 (Action Instance).** An action instance  $ai = (af, tr, time) \in \mathcal{U}_{af} \times \mathcal{U}_{tr} \times \mathcal{U}_{time}$  is a tuple of an action formula  $af$ , a transaction  $tr$ , and a timestamp  $time$ .  $\mathcal{U}_{ai}$  is the set of all possible action instances. Given an action instance  $ai = (af, tr, time) \in \mathcal{U}_{ai}$ ,  $\pi_{af}(ai) = af$ ,  $\pi_{tr}(ai) = tr$ , and  $\pi_{time}(ai) = time$ .

$\pi_{time}(ai) = \pi_{tw}(time)$

You use  $i$  by universes above for identifier universes. It is not so nice to have it for instance, too.

*to define this already above unco-plel.*

**Definition 17 (Action Instance Projection).** Given an action instance  $ai = (af, tr, time) \in \mathcal{U}_{ai}$ ,  $\pi_{af}(ai) = af$ ,  $\pi_{tr}(ai) = tr$ , and  $\pi_{time}(ai) = time$ .

**Definition 18 (Action Instance Stream).** An action instance stream  $AIS$  is a (possibly infinite) set of action instances, i.e.,  $AIS \subseteq \mathcal{U}_{ai}$ .  $\mathcal{U}_{AIS}$  is the set of all possible action instance streams.

**Definition 19 (Action Engine).** Let  $A \subseteq \mathcal{U}_\alpha$  be a set of actions to be used by an action engine.  $ae_A \in \mathcal{U}_{CIS} \rightarrow \mathcal{U}_{AIS}$  is an action engine such that, for any  $CIS \in \mathcal{U}_{CIS}$ ,  $ae_A(CIS) = \{(af, tr, time) \in \mathcal{U}_{ai} \mid \exists TM, tm (af, TM) \in A \wedge tm \in TM \wedge time = \pi_t(tm) \wedge tr \in af(CIS, \pi_{tw}(tm))\}$ .

## References

*General note:*

*It would be a little bit easier if you would have an example showing what as a constraint is etc. Your concept helps to define the definitions, but still it is not completely clear from a real-life point of view.*

*For the rest I think the definitions are good (will already accept) so key should be :) There are some slight inconsistencies, but nothing really wrong.*

*;) :*