Machine Learning for Mechanical Engineering

Homework 3

Due Wednesday, 04/04/2018, 9:30 AM

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POSTECH

- For your code, email your .ipynb file to (`iai.postech@gmail.com`)
- When you send an e-mail, write down [Machine Learning HW3] on the title
- And please write your NAME on your .ipynb files. ex) 김지원_20182315_HW2.ipynb
- Do not submit a printed version of your code. It will not be graded.

In this homework, we want to demonstrate an image panorama as an example of linear regression. A panorama is any wide-angle view or representation of a physical space.

Problem 1 (Load images)

You need to install opencv packages https://pypi.python.org/pypi/opencv-python and download images from this [link](https://www.dropbox.com/sh/xpgcmxvvzplxqzq/AAAcjqJP3ekpjSn0isdFVoQxa?dl=0). If you are able to plot three pictures like below, you will get full scores.

In [1]:

```
import numpy as np
import matplotlib.pyplot as plt
import cv2
```

In [2]:

```
imag1 = cv2.imread('./data2/1.jpg')
imag1 = cv2.cvtColor(imag1, cv2.COLOR_BGR2RGB)
imag2 = cv2.imread('./data2/2.jpg')
imag2 = cv2.cvtColor(imag2, cv2.COLOR_BGR2RGB)
imag3 = cv2.imread('./data2/3.jpg')
imag3 = cv2.cvtColor(imag3, cv2.COLOR_BGR2RGB)
```

In [3]:

```
plt.figure(figsize=(10, 6))
plt.imshow(imag1)
plt.axis('off')
plt.show()
```

In [4]:

```
plt.figure(figsize=(10, 6))
plt.imshow(imag2)
plt.axis('off')
plt.show()
```

In [5]:

```
plt.figure(figsize=(10, 6))
plt.imshow(imag3)
plt.axis('off')
plt.show()
```

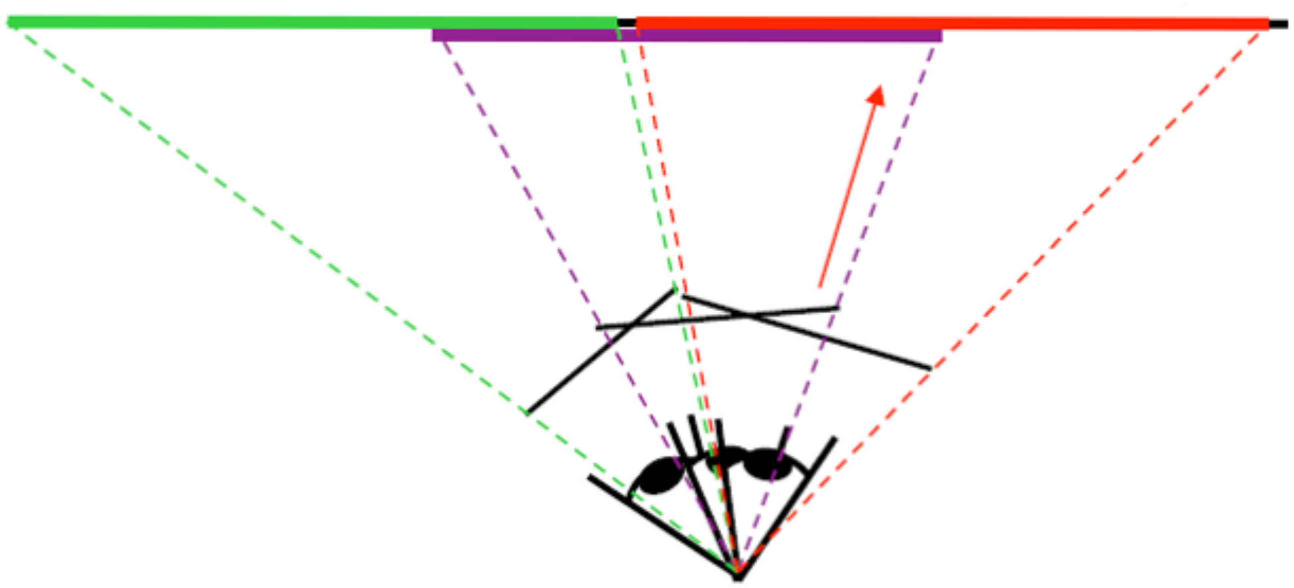
Problem 2

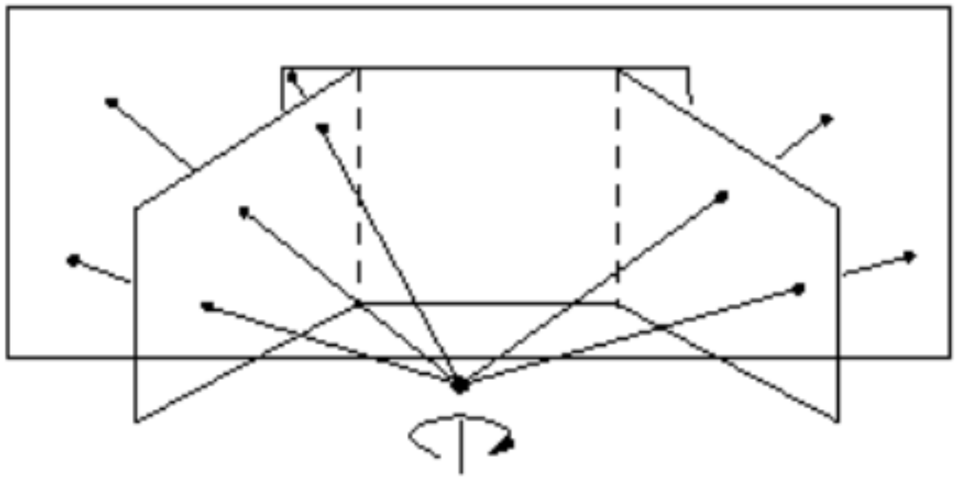
Panorama

Here, we are explaining the basic concept of homography (i.e., perspective transformation).

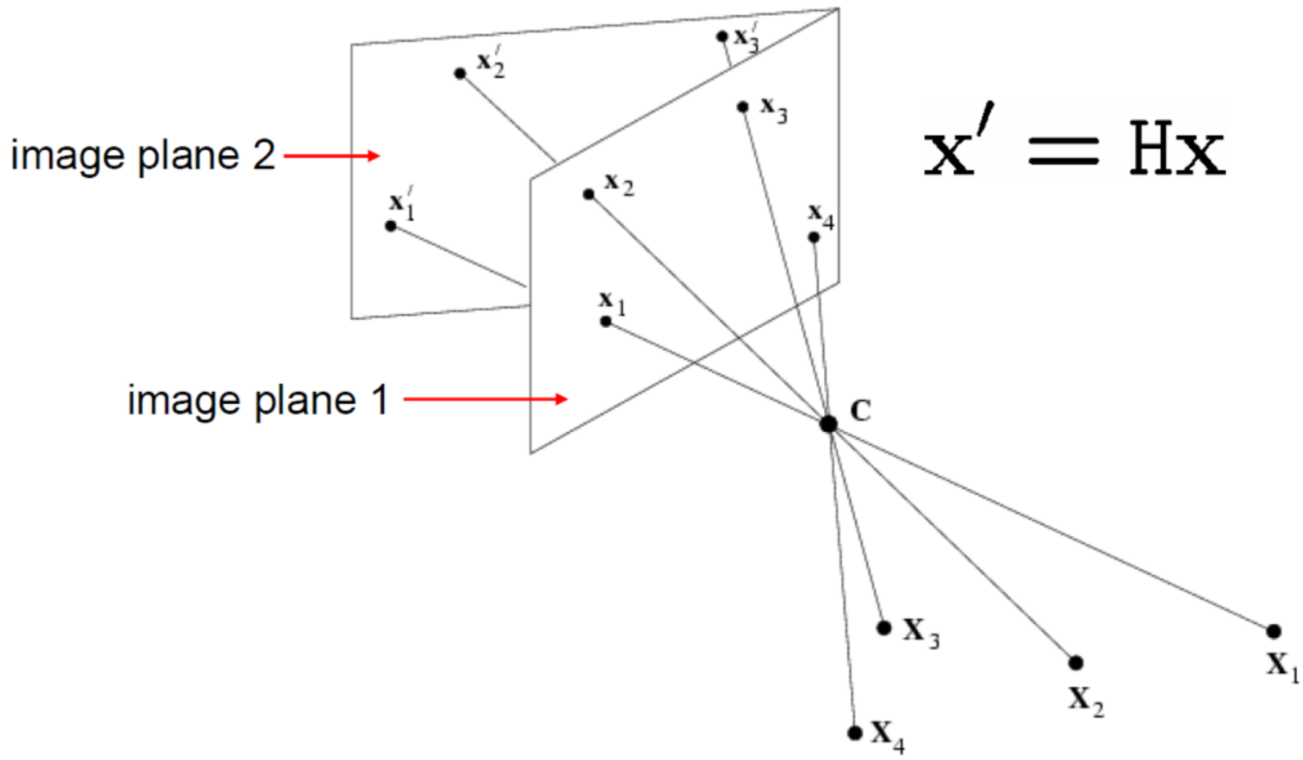
- Any wide-angle view or representation of a physical space
- images with horizontally elongated fields of view

- idea: projecting images onto a common plane





- Camera rotating about its center



- Two image planes are related by a homography $H H$

  Do not worry about a homography transformation. (out of this course's scope)

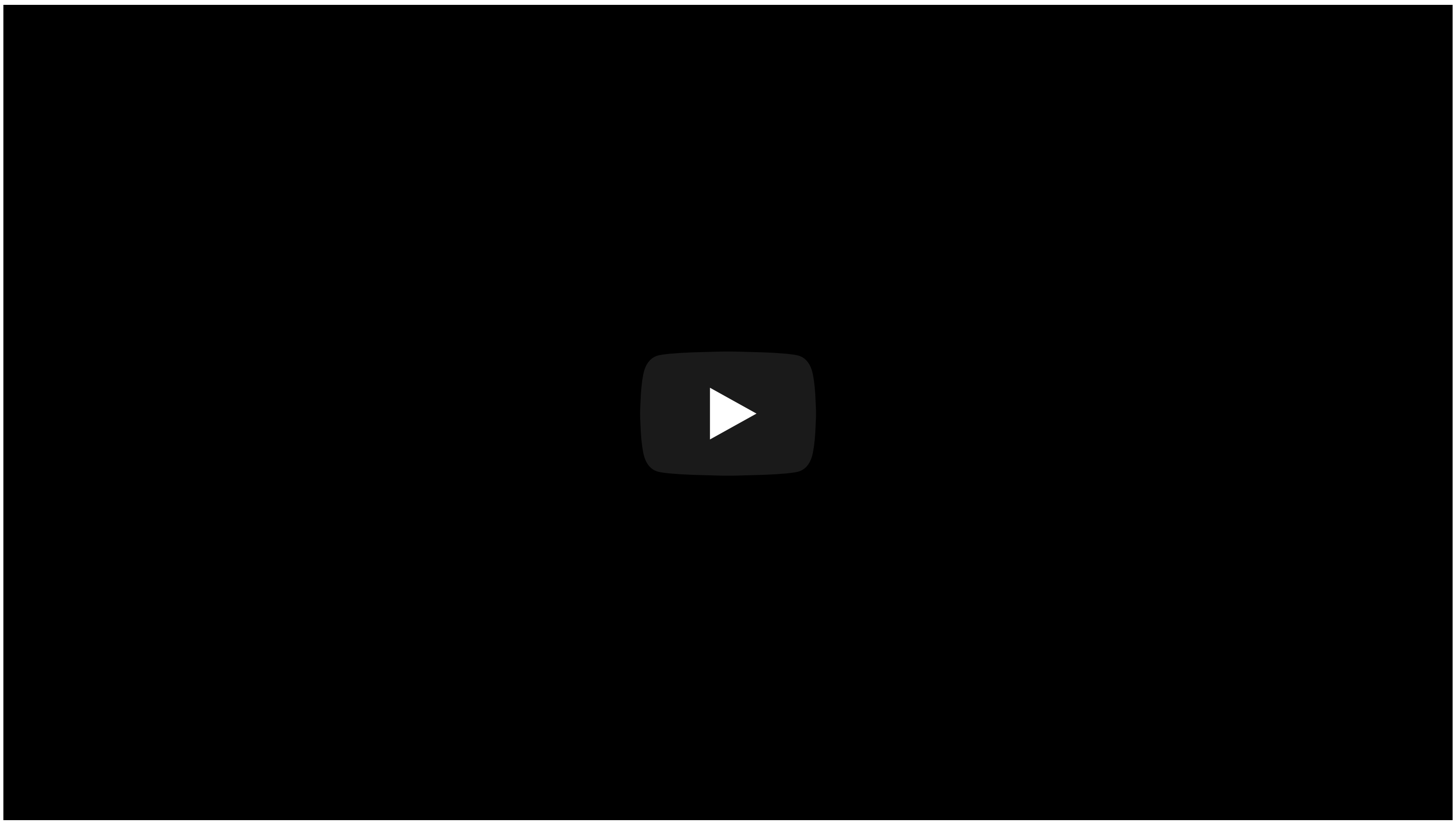
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} \sim \begin{bmatrix} \omega x' \\ \omega y' \\ \omega \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} \sim \begin{bmatrix} \omega x' \\ \omega y' \\ \omega \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- For the advanced learner, watch the following online lecture by Prof. Aaron Bobick

In [6]:

```
%%html
<center><iframe src="https://www.youtube.com/embed/pU4NorC7lb0?rel=0"
width="560" height="315" frameborder="0" allowfullscreen></iframe></center>
```



Find key points between two images

- Suppose these matching points are given.
  - We have manually found the matching points for you, although there is a technique to do this automatically.

- pos1 and pos2 are matching points between img01 and img02
- pos3 and pos4 are matching points between img02 and img03

In [7]:

```
pos1 = np.array([[2121, 2117, 2749, 3095, 3032, 3375, 3677, 3876],
                 [1431, 2034, 2033, 1885, 2017, 2037, 1885, 2279]], dtype=np.i
nt64)
pos2 = np.array([[188, 58, 828, 1203, 1121, 1437, 1717, 1817],
                 [1217, 1909, 1952, 1827, 1952, 1991, 1870, 2226]], dtype=np.i
nt64)
pos3 = np.array([[2338, 2379, 2658, 2899, 2977, 3272, 2716, 2786],
                 [1948, 1874, 2000, 1837, 1964, 1966, 2143, 2317]], dtype=np.i
nt64)
pos4 = np.array([[109, 178, 497, 795, 851, 1144, 534, 580],
                 [1907, 1828, 1988, 1834, 1971, 1993, 2145, 2333]], dtype=np.i
nt64)
```

1) Visualization of key points

In [8]:

```
## your code here
## Write down your own code to mark the key points (red dots) on the locations of the given images
```

Estimation of homography H

$$X' = HX$$

$$X' = HX$$

where X and X' are position vectors of key points, and H is a Perspective Transformation

Goal: we need to estimate homography H via matching points between two images

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} \sim \begin{bmatrix} \omega x' \\ \omega y' \\ \omega \end{bmatrix} = \begin{bmatrix} \theta_1 & \theta_2 & \theta_3 \\ \theta_4 & \theta_5 & \theta_6 \\ \theta_7 & \theta_8 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} \sim \begin{bmatrix} \omega x' \\ \omega y' \\ \omega \end{bmatrix} = \begin{bmatrix} \theta_1 & \theta_2 & \theta_3 \\ \theta_4 & \theta_5 & \theta_6 \\ \theta_7 & \theta_8 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Show the following equations from the above homography H

$$\begin{aligned} x' &= \frac{\theta_1 x + \theta_2 y + \theta_3}{\theta_7 x + \theta_8 y + 1} \\ y' &= \frac{\theta_4 x + \theta_5 y + \theta_6}{\theta_7 x + \theta_8 y + 1} \end{aligned}$$

$$\begin{aligned} x' &= \frac{\theta_1 x + \theta_2 y + \theta_3}{\theta_7 x + \theta_8 y + 1} \\ y' &= \frac{\theta_4 x + \theta_5 y + \theta_6}{\theta_7 x + \theta_8 y + 1} \end{aligned}$$

$$\begin{aligned} \theta_1 x + \theta_2 y + \theta_3 - \theta_7 x' x - \theta_8 x' y - x' &= 0 \\ \theta_4 x + \theta_5 y + \theta_6 - \theta_7 y' x - \theta_8 y' y - y' &= 0 \end{aligned}$$

$$\begin{aligned} \theta_1 x + \theta_2 y + \theta_3 - \theta_7 x' x - \theta_8 x' y - x' &= 0 \\ \theta_4 x + \theta_5 y + \theta_6 - \theta_7 y' x - \theta_8 y' y - y' &= 0 \end{aligned}$$

For m pairs of matching potins, show that a feature matrix Φ can be expressed as follows:

- Φ is a feature matrix

$$\Phi = \begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1' x_1 & -x_1' y_1 \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -y_1' x_1 & -y_1' y_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_m & y_m & 1 & 0 & 0 & 0 & -x_m' x_m & -x_m' y_m \\ 0 & 0 & 0 & x_m & y_m & 1 & -y_m' x_m & -y_m' y_m \end{bmatrix}$$

$$\Phi = \begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1' x_1 & -x_1' y_1 \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -y_1' x_1 & -y_1' y_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_m & y_m & 1 & 0 & 0 & 0 & -x_m' x_m & -x_m' y_m \\ 0 & 0 & 0 & x_m & y_m & 1 & -y_m' x_m & -y_m' y_m \end{bmatrix}$$

- θ is a column vector for unknown parameters in a perspective transformation H

$$\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \\ \theta_6 \\ \theta_7 \\ \theta_8 \end{bmatrix}$$

$$\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \\ \theta_6 \\ \theta_7 \\ \theta_8 \end{bmatrix}$$

- b is a column vector for corresponding positions in the base image

$$b = \begin{bmatrix} x'_1 \\ y'_1 \\ x'_2 \\ y'_2 \\ \vdots \\ x'_m \\ y'_m \end{bmatrix}$$

$$b = \begin{bmatrix} x'_1 \\ y'_1 \\ x'_2 \\ y'_2 \\ \vdots \\ x'_m \\ y'_m \end{bmatrix}$$

- It ends up becoming a linear regression problem

$$\min_{\theta} \|\Phi\theta - b\|_2^2$$

$$\min_{\theta} \|\Phi\theta - b\|_2^2$$

$$\theta^* = (\Phi^T\Phi)^{-1}\Phi^T b$$

$$\theta^* = (\Phi^T\Phi)^{-1}\Phi^T b$$

2) Perspective homography for image 1 and image 2

In [9]:

```
## your code here
## Construct feature matrix using homography H, and a vector having entries of matching points in image 2




## your code here
## Define perspective_theta using linear regression

perspective_theta =
```

3) Perspective homography for image 2 and image 3

In [10]:

```
## your code here
## Construct feature matrix using homography H, and a vector having entries of matching points in image 2



## your code here
## Define perspective_theta2 using linear regression

perspective_theta3 =
```

Image warping

- Again, do not worry about the image warping (outside lecture's scope)

In [11]:

```
cv2.warpPerspective?
```

In [12]:

```
## Apply image warping on image 1 & image 3 using cv2.warpPerspective function

## do translation to fit the warping image into a size of (18000, 6500) screen.
translation = np.matrix([[1, 0, 6000],
                         [0, 1, 2500],
                         [0, 0, 1]])

warpedImage = cv2.warpPerspective(imag1, translation*perspective_theta, (18000, 6500))
warpedImage3 = cv2.warpPerspective(imag3, translation*perspective_theta3, (18000, 6500))
```

In [13]:

```
screen = warpedImage.copy()
screen[screen==0] = warpedImage3[screen==0]
screen[2500:3024+2500,6000:4032+6000] = imag2

## Visualize panorama image
plt.figure(figsize=(20, 12))
plt.imshow(screen)
plt.show()
```