The Role of Learning in Occupational Decisions and Wage Dynamics

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Abstract

If workers are uncertain about their labor market productivity, this uncertainty could have crucial implications for occupational sorting and wage dynamics. This paper starts by documenting the key features in the US labor market that can be explained by the presence of uncertainty about workers' own skills and the learning process that unfolds depending on specific experiences. A structural model is built on these empirical patterns and characterizes that workers in their initial occupations would decide whether to stay or quit in response to the wage outcomes as they learn about their skills. The dynamic discrete choice model, extending conventional discrete-time duration analysis, is estimated using the Kalman filter and a conditional choice probability estimator. I find supporting evidence that workers experience significant uncertainty about their skills. The results depict that the difference in the dynamics of wage dispersion across occupations can be attributed to workers' sorting as well as human capital accumulation in different occupations. Counterfactual analysis shows that, in occupations with high probabilities of mismatch due to incorrect beliefs, an information provision policy would reduce the share of mismatched workers, resulting in a shorter employment duration in the initial occupations, higher wage growth, and lower wage dispersion.

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1 Introduction

The human capital framework over the life cycle (e.g., Ben-Porath, 1967; Mincer, 1974) has been the basis for modeling earnings and provided good explanations consistent with observed earnings patterns. The human capital investment decision problem is inherently characterized by uncertainty over future returns, similar in kind to any risky investment decisions. Such uncertainty, particularly individual-specific uncertainty about one's abilities, can lead to suboptimal investment decisions that may be viewed as mistakes. For instance, a worker might switch occupations upon discovering that their performance is rewarded differently than expected.[1] This raises several key questions: Do individuals learn about their abilities through experience and, in turn, achieve better economic outcomes? To what extent does imperfect information serve as a barrier to leveraging their comparative advantages? And what impact would providing information to individuals have on wage distribution and its dynamics?

This paper addresses these research questions while accounting for occupational mobility, which is frequent, especially among young workers, and has significant implications for wage levels, growth, and dispersion. Occupational decisions may not only allow individuals to seek better matches for their skills but also play a crucial role in skill development and, consequently, earning potential. To understand occupational decisions and mobility, recent research suggests that workers' learning is crucial (e.g., Papageorgiou, 2014; Groes et al., 2015). They highlight that the uncertainty about own abilities and learning process can explain workers' sorting patterns across occupations. This paper, focusing on male workers in the US in their first full-time occupation after completing formal education, shows a pattern that more experienced workers are less likely to quit their occupations, suggesting that learning about one's abilities could play a role in occupational decisions. I present an additional empirical pattern that the decision to stay in or quit their initial occupations is associated with new information obtained from wages earned in the previous period. These patterns suggest that learning about one's earning potential based on the observed wages could be a relevant mechanism of occupational decisions.

[1] Other examples relevant to uncertainty about one's attribute may include dropping out of high school track or college and working in occupations that typically do not require a college degree while having graduated from college.

Next, I consider a structural model that jointly treats the wages and duration of staying in an initial occupation in learning environments. The model accounts for selective attrition arising due to the Bayesian learning process, which generalizes conventional discrete-time duration models by incorporating richer time-series properties for unobservables. Specifically, this illustrates how workers learn about their abilities based on individual-specific signals and subsequently make occupational decisions—to either stay in or quit their current occupation. Workers are assumed to maximize their expected lifetime gains. Given the uncertain ability that determines the initial level of human capital stock (hereafter simply "skill"), workers compute the value of staying working in an occupation by forming expectations with respect to their current skill and its accumulation in the future in the occupation. I assume that the flow utility of working in the current period is the CRRA function of the wage with a relative risk aversion parameter set to one, and the realized wage is closely tied to output production. Then, the observed wage at the end of each period signals the worker's skill, and beliefs are updated in a Bayesian manner.[2] The skill accumulates stochastically through learning by doing. The econometric specification allows for explicitly testing the hypothesis of Bayesian learning against initially known ability.

Identification of the parametric model built in this paper is a special case of the non-parametric identification results in the literature. Bunting, Diegert, and Maurel (2024) provide identification results for potential outcome equations and ability distributions in a general class of learning models. Their identification results can be extended and applied in the presence of attrition. Then, under an identification assumption that workers do not possess private information conditional on what the econometricians can observe, standard identification results in the dynamic discrete choice literature hold in this learning model. The identification assumption relies on the main virtue of the NLSY79 which contains selection-free measurements that allow workers' skills before entering the labor market to be controlled for. Once identified, the model is estimated using the Kalman filter and conditional choice probability (CCP) estimation method. My approach enables the

[2] In each occupation, the more output depends on the unobserved ability and the less volatile productivity shock is, the more information about the abilities would be revealed on the job. This may generate experimentation incentives for the workers who value information because it increases the probability that they will sort themselves into the occupation at which they are the most productive. In this paper, while the productivity shock is of interest, the occupation-specific dependence between the latent ability and its effect on the output production is implicitly abstracted, not being separately identified from the dispersion of the ability.

unobserved skills to be addressed, while the burden of solving the dynamic programming problem is avoided by exploiting the terminal state nature inherent in this optimal stopping problem.

The estimates provide statistically significant results rejecting the null hypothesis that workers initially know about their own ability, so the uncertainty and learning affect occupational decisions in their initial occupations. The occupations show different features in wage and skill accumulation processes, providing different learning environments. The differences can explain the dynamics of the mean wage and within-occupation wage dispersion across occupations. To separate the effect of sorting attributed to learning, counterfactual simulations are carried out. A nontrivial share of workers is found to be mismatched by either staying in or quitting the initial occupations because the choices are made based not on their true skills but on their beliefs. I consider a policy, such as counseling, that can remove the uncertainty from the start of the workers' careers. Informing the workers perfectly mitigates workers' mismatch, reducing the duration of the first employment relationship while enhancing wage growth and reducing wage dispersion through sorting.

This paper contributes to two strands of literature that are central to understanding labor market outcomes: occupational decisions in learning environments and wage dynamics. The first strand of literature deals with uncertainty particularly about individuals' abilities and learning processes. Recent research using data that directly elicits individuals' beliefs has confirmed that they have uncertainty about their own personal traits and form biased beliefs (e.g., Stinebrickner and Stinebrickner, 2014; Hoffman and Burks, 2020; Bobba and Frisancho, 2022), which significantly influences decision-making. However, the idea of imperfect information and learning is not new to understanding occupational choice and career progression; Jovanovic (1979) pioneered the learning model that captures the job transition patterns, and Miller (1984) generalizes the model allowing different job types to provide different benefit of expected per-period rewards and information about future returns. Antonovics and Golan (2012) extend the model by incorporating correlations between occupation-specific productivity determined by the characteristics of the occupations.

There are several views to understanding the observed patterns of occupational mobility; skill accumulation (Jovanovic and Nyarko 1997; Sanders, 2014), a search process for a better match (Jovanovic, 1979; Neal 1999), and a process of learning about one's own ability (James, 2012; Gorry et al., 2019). This paper focuses on learning about own ability, keeping the other mechanisms implicit, and abstracts from general equilibrium effects considered in Papageorgiou (2014) and Groes et al. (2015). By building an easily comprehensible model that links learning to occupational choice in the early career, I provide direct evidence that supports the presence of uncertainty and learning about own ability. Two papers, James (2012) and Arcidiacono et al. (2024), are the closest to this work, but I carry out a more detailed investigation into earnings dynamics within and across different occupations through workers' sorting based on learning and human capital accumulation.

The second strand of literature is about the sources behind earnings distribution and its dynamics. Rubinstein and Weiss (2006) provide an extensive review of the studies about various economic explanations of earnings growth over the life cycle: human capital investment, search and learning. Whereas not everyone accepts the human capital framework as the basis for modeling earnings, the approach is surprisingly robust compared to other models in explaining earnings patterns (Polachek, Tirthatanmoy, and Thamma-Apiroam 2015). Building on Ben-Porath (1967) who established a framework that links wage growth to human capital accumulation, Magnac et al. (2018) further take into account individual heterogeneity influencing human capital investment decisions. This extension links individual heterogeneity in the human capital process to wage dispersion as well as wage growth, offering an explanation about the decrease in wage dispersion in workers' early careers followed by an increase. Taber and Vejlin (2020) quantify the wage variations attributed to several different sources incorporated in one model and find that heterogeneous premarket skills are the most important source of wage variation. This paper considers individual heterogeneity not only in premarket skills but also in its evolution through learning by doing along with uncertainty and learning about the component. I find that wage variation explained by ability initially unknown to the workers is comparable to the one attributed to idiosyncratic shocks. The decrease in earnings dispersion in some occupations in the early career could come from the sorting of workers whose skills distribution becomes less dispersed as they advance through their career choices to stay in or quit their initial occupation.

The remainder of the paper is organized as follows. Section 2 presents the data and provides descriptive evidence suggestive of learning. Section 3 describes a dynamic model of occupational decisions, where workers have imperfect information about their labor market productivity, and update their beliefs through the wage observations while accumulating skills through learning by doing. The model identification and relevant econometrics issues are also discussed. of the model, Section 4 details the estimation procedure and presents the estimation results. Section 5 studies the role of informational frictions on educational and labor market outcomes. Finally, Section 6 concludes.

2 Data and descriptive statistics

In this paper, I focus on male workers in the US labor market and document the patterns of earnings dynamics and occupational choices, specifically whether they stay in or quit their occupations. The primary data source for this analysis is the NLSY79, which tracks a nationally representative sample of individuals starting in 1979 and provides detailed employment information for each worker. I begin by briefly describing the data sources and presenting key sample statistics, with additional details regarding the sample selection criteria provided in the Appendix. In the following subsections, I present descriptive statistics that highlight the empirical features motivating and shaping the theoretical framework of this paper. These statistics illustrate the diverse patterns of earnings dynamics across occupations and suggest that learning about one's own skill could be crucial in understanding workers' occupational choices. Additionally, I emphasize that wages, based on workers' responses to realized wage shocks, may serve as a key source of information for this learning process.

2.1 Data: 1979 National Longitudinal Survey of Youth (NLSY79)

I utilize the Work History Data File of the NLSY79 to construct annual panels that track individuals aged 14 to 22 as of January 1, 1979. This single cohort with a large sample size allows researchers to study individuals who have faced relatively similar economic conditions over their life cycles.[3] The dataset provides information on the usual hours worked per week and the number of weeks worked per job within a year, enabling the calculation of total hours worked for each job annually. I define the primary job for each individual in a given year as the one with the highest number of hours worked. This primary job is supplemented with detailed employment information, including occupation title and earnings. To ensure consistency in occupational classifications across years, all occupational codes are converted into the Census 1990 Three-Digit Occupation Codes. Worker wages are measured by the rate of pay for the primary job, including tips, overtime, and bonuses before deductions, and are converted to a weekly rate. All wages are deflated by the Personal Consumption Expenditures Price Index and expressed in 2000 dollars.

Based on responses to employment questions, each individual is assigned to one of seven mutually exclusive activities each year: nonemployment or full-time employment in one of six occupational categories. These categories are defined by aggregating occupations that encompass the entirety of US employment. I follow Autor and Dorn (2013) and others to classify the occupational titles coded in three digits into broader groups.[4] Two groups, Executive/Managerial and Professional/Technical occupations, represent highly educated and well-paid jobs. In 1979, between one-quarter and two-thirds of workers in these occupations held at least a four-year college degree. Sales/Administrative Support represents a middle-skilled, white-collar occupational group predominantly held by women with a high school diploma or some college education. Additionally, there are groups of middle- and low-skilled blue-collar occupations, typically held by men with a high school degree or less education, including Production/Operators and Transportation/Construction. Lastly, the Low-skill Service category encompasses occupations in protective services, food preparation, cleaning, and personal care. Most workers in service occupations have no post-secondary education, although employment in this group has grown rapidly over the past several decades when post-secondary education has expanded fast.

[3] A common challenge in this context is to distinguish between cohort, time, and age effects. Because of the relatively narrow age range of participants initially surveyed by the NLSY79, it would be appropriate to view this data as capturing outcomes for one specific cohort of individuals over their lifecycle. Conditional on the single cohort, the model in the next section will incorporate time effect in the form of variation in the output price across calendar years. Then, the age effect only through occupational tenure can be identified by using the variation in different ages of entry into the labor market.

[4] For example, Autor and Dorn (2013) divide the occupations into the groups of Managers, professionals, technicians, finance, public safety; Production and craft; Transportation, construction, mechanics, mining, farm; Machine operators, assemblers; Clerical, retail sales; Service. The authors show that the distinct groups require different abstract routine, manual, task inputs.

A main virtue of the NLSY79 is that it includes measures that allow workers' skills before entering the labor market to be potentially controlled for. All respondents took the Armed Services Vocational Aptitude Battery (ASVAB) test at the start of the survey, which measures various cognitive skills. I construct each measure of verbal, math, and craftsmanship skills using the following two test components, respectively: Word Knowledge, Paragraph Comprehension, Arithmetic Reasoning, Mathematics Knowledge, Mechanical Comprehension, and Electronics Information. In addition, respondents were surveyed in 1979 and 1980 regarding their non-cognitive attitudes. Following literature emphasizing the importance of non-cognitive skills on earnings, I use the Rotter Locus of Control Scale and the Rosenberg Self-Esteem Scale to measure social skills. To construct scalar values for these different skill dimensions, I first adjust the mean and variance of each score across ages to account for systematic age effects, following the method of Altonji, Bharadwaj, and Lange (2012). I then apply Principal Component Analysis (PCA), normalizing the score associated with the first principal component to have a mean of 0 and a variance of 1.

I restrict the sample to males working in their first occupation after completing their education in order to examine occupational decision—stay in or quit— and wage patterns for a relatively homogeneous group over a period of time. This allows me to control for social differences (e.g., fertility) and not to delve into the relationship between wages and hours worked. Following Guvenen et al. (2020), I consider only workers "attached" to the labor force, restricting the sample to full-time workers with more than 1,200 imputed working hours in a given year. Additionally, to avoid issues with left truncation and to construct complete work histories, I exclude individuals who were already working at the start of the sample period. Further details on the sample selection process are provided in Appendix A. My final sample includes 2755 individuals and 17,036 individual-year observations.

2.2 Descriptive statistics

I present the key features of the data. Numerous researchers have documented the properties of earnings over the life cycle (see, for example, Huggett, Ventura, and Yaron (2006) for the U.S. or Magnac and Roux (2021) for France). However, the analysis in this paper

Table 1: Summary Statistics by Occupation

	Executive/ Managerial	Professional/ Technical	Sales/ Admin support	Production/ Operators	Transportation/ Construction	Low-skill Service
# Obs	2939	3372	1662	2081	5568	1414
# Individuals	313	358	340	378	991	375
Average age at entry	23.39	24.788	22.374	21.087	20.936	21.413
Education						
$\leq$ High School	.16	.084	.412	.786	.781	.667
Some college	.243	.17	.279	.138	.172	.24
$\geq$ 4-year college	.597	.746	.309	.077	.047	.093
Race						
Hispanic	.15	.151	.185	.18	.172	.136
Black	.153	.154	.238	.257	.233	.355
Non-Hispanic/Non-Black	.696	.696	.576	.563	.595	.509
Skill measures						
Average verbal skill	.647	.735	.283	-.322	-.261	-.276
Average math skill	.706	.905	.251	-.322	-.323	-.297
Average social skill	.331	.276	.117	-.172	-.108	-.14
Duration (year)						
Average duration	9.39	9.419	4.888	5.505	5.619	3.771
Median duration	6	6	3	3	3	2

Note: Annual records could stop being observed in the sample, which yields incomplete duration. They are treated as missing at random.

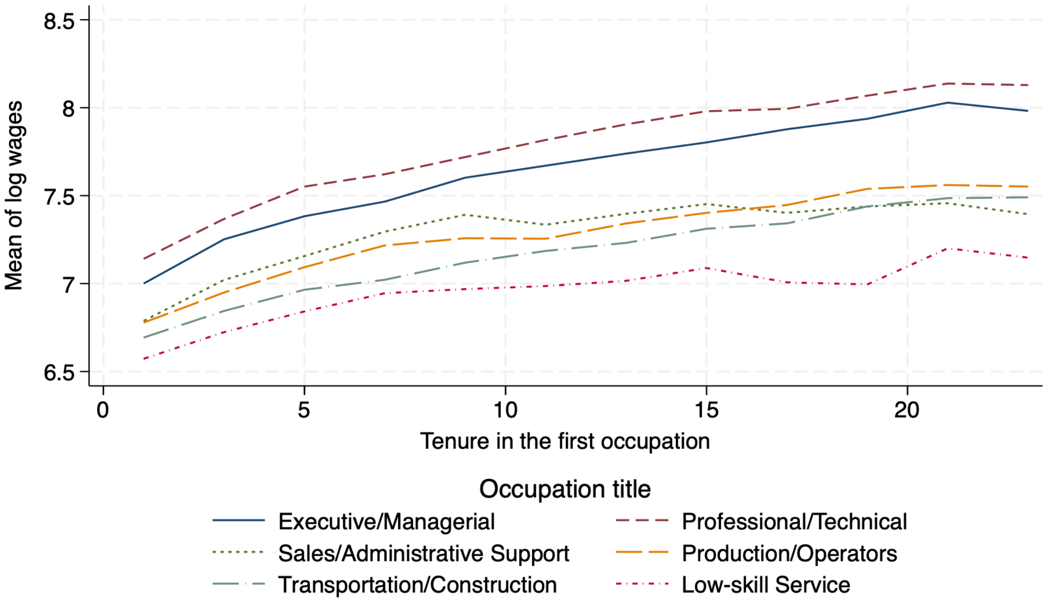
differs in that I focus specifically on the wage dynamics of workers in their initial full-time occupations and their decision to sort themselves out of the occupations. This focus naturally directs, rather than exploring labor market outcomes over the entire lifecycle, the analysis towards the wage dynamics during the initial employment relationship, the duration of its spell, and the reasons behind the decision to quit the initial occupations. In this subsection, I depict the wage dynamics that vary across occupations, which motivates the classification of the more detailed occupations than simply distinguishing white-collar and blue-collar occupations.

Wage dynamics across different occupations

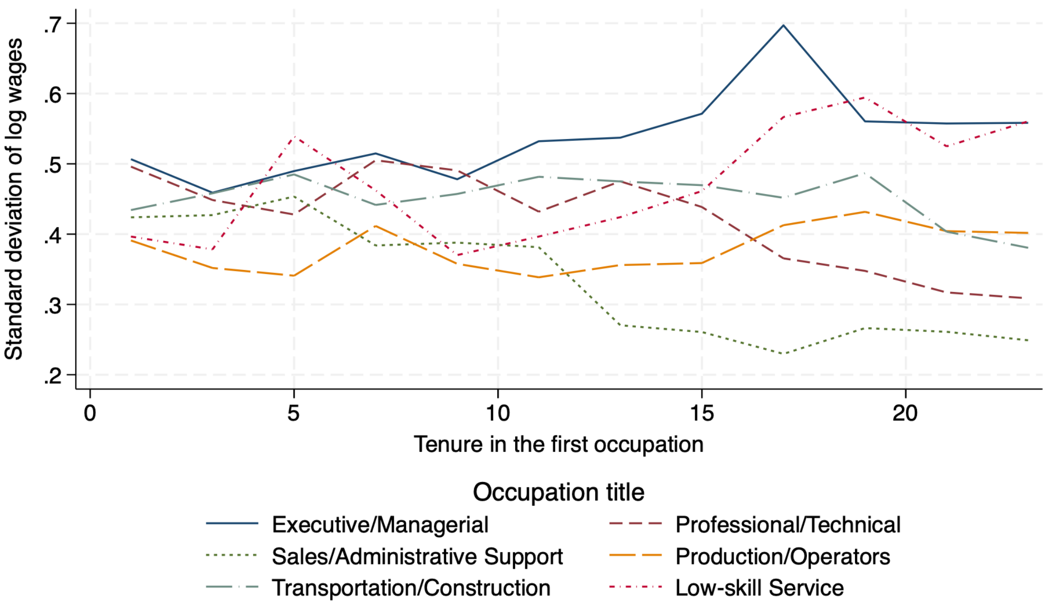
The upper panel in Figure 1 displays the mean log wages over tenure across different occupations. A common pattern observed is that wages increase rapidly during the first 10 years of tenure, reflecting the period of fast skill accumulation. This early-stage wage growth is consistent across all occupational categories. However, the log wage profiles tend to flatten after this rapid increase. Despite this general pattern, there are apparent differences in both the levels and growth rates of wages across occupations. Executive/Managerial and Professional/Technical occupations exhibit both the highest initial wage levels and the steepest wage growth over time, reflecting greater returns

Figure 1: Wage dynamics

(a) Mean of log wages across different occupations



(b) Standard deviation of log wages across different occupations



to experience in these high-skill occupations. In contrast, Production/Operators and Transportation/Construction occupations show lower starting wages and more modest wage growth over time, and workers in Low-skill Service occupations experience the lowest overall wages and the slowest wage growth, emphasizing the limited potential for human capital accumulation in this occupation.

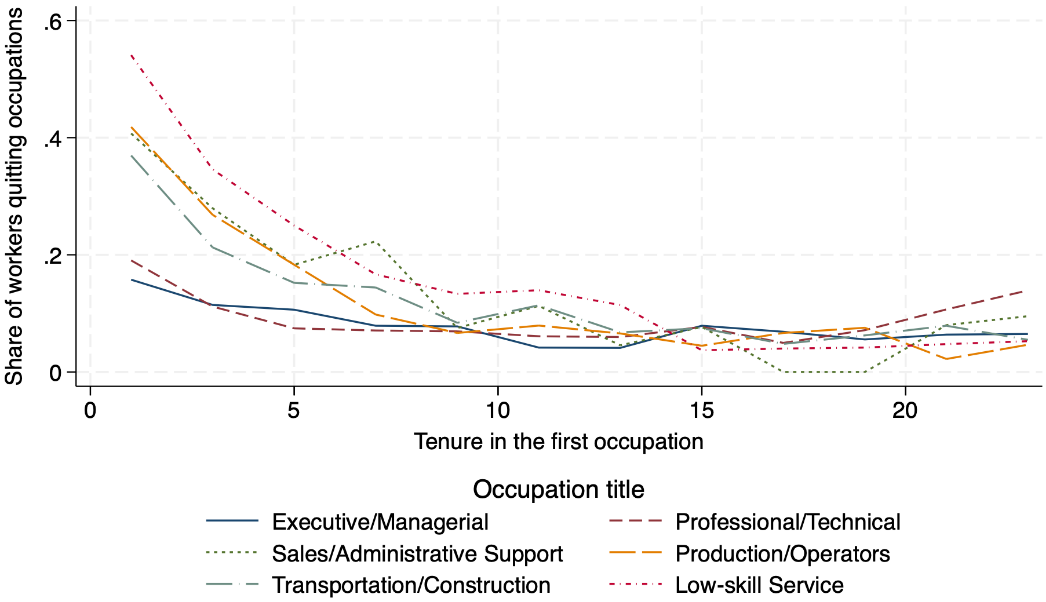
The lower panel of the figure illustrates the standard deviation of log wages over tenure, offering insights into wage dispersion across occupations. The patterns of wage dispersion differ notably between occupations with no distinctive common feature. Even across the occupations classified equally into white-collar occupations, the wage dispersion within Executive/Managerial occupation increases over tenure, while it decreases in Professional/Technical and Sales/Administrative Support occupations. Among the blue-collar occupations, the wage dispersion remains relatively stable over tenure in Production/Operators and Transportation/Construction, whereas Low-skill Service occupation display more significant fluctuations and increasing patterns. Those different patterns stress the need to define the refined occupational categories to better understand the wage dynamics.

Two main forces could generate the observed wage dynamics: human capital accumulation and workers' sorting. As human capital accumulates at different rates across occupations, the mean wage increases while yielding different trajectories. At the same time, workers are sorted, which may lower the wage dispersion if workers who decide to stay in their occupations get homogeneous. The increase in dispersion over tenure can still be understood within this framework through the stochastic components potentially embedded in human capital accumulation and choice processes, both of which add heterogeneity across the workers working in the initial occupations. In the next section, I build the structural model on the two forces that can explain the different patterns of wage dynamics.

Occupational decision: Stay/Quit

Figure 2 illustrates a decline, as tenure increases, in the share of workers who remain in their first occupations but decide to quit. This pattern supports the hypothesis that early in their careers, workers face uncertainty about their abilities, which they gradually resolve through experience in their occupations. Workers could experiment with different

Figure 2: Hazard rate:
Share of workers leaving the occupations conditional on working until the previous period



occupations in the initial stages of their careers to discover the one that best matches their skills and comparative advantages. As they accumulate experience and gain a better understanding of their strengths and preferences, they may tend to settle into occupations that better align with their abilities, resulting in decreased occupational mobility over time.[5]

Then, the higher rate of occupational switching early in careers could be explained by several factors: (a) workers initially face greater uncertainty about their own abilities and, consequently, may not be well-matched to their first occupation; (b) certain occupations may facilitate faster learning, enabling workers to find a better fit earlier in their careers; or (c) outside offer may be more attractive in some occupations, such as those where occupation-switching is easier or where occupations serve as stepping stones. For instance, occupations with lower wage growth, such as Low-skill Service and Production/Operators, may offer opportunities for workers to develop skills and enhance productivity, making them attractive stepping stones early in a career despite their lower initial wages to the worker who could had experienced more penalty in the other occupations.

[5]Although the decreasing mobility with age or tenure could be evidence of learning (Neal 1999), the feature can be also explained by a search model that does not even account for learning about match-specific productivity (called match quality).

2.3 Suggestive evidence of selection and learning

This paper focuses on the spell of the workers in their initial occupations, which would be the period when uncertainty about their abilities is likely to be most prevalent, so the learning process could play a significant role. Given that the sample consists of an unbalanced panel by construction, I first examine whether the duration of being in the sample (i.e., working in the initially assigned occupations) is endogenously determined. After exploring the selective attrition from the sample, I provide suggestive evidence that learning, particularly through wage observations, could serve as a relevant mechanism driving the non-random sample.

Presence of selective attrition

To investigate the presence of selective attrition, I conduct a series of simple tests applying the variable addition test proposed by Nijman and Verbeek (1992). Specifically, I regress log wages on proxies for human capital (e.g., skill measures, occupational tenure, education) and, importantly, include a dummy variable indicating whether the worker leaves his initial occupation in the next period. A significant coefficient on this attrition dummy implies the presence of selectivity bias, suggesting that the sample could be non-randomly selected through attrition. Table 2 below presents the estimation results and supports the presence of selective attrition underlying the wage process through the workers' decisions to stay in their initial occupations. More details regarding the practice are described in Appendix B.

Not just rejecting the hypothesis of no selective attrition in the sample, the negative estimates of the attrition dummy indicate that workers who leave the sample tend to have lower wages compared to those who remain. This alludes to the potential mechanism of selective attrition, meaning that workers having lower wages than average or expected are more likely to exit the sample. The estimates controlling for occupation-specific effects through the mean, tenure, and other individual characteristics in different specifications further confirm that this interpretation holds even after accounting for these factors. In the next section, I build a structural model that accounts for the selective attrition mechanism explicitly in conjunction with the wage process.

Table 2: Test for Selective Attrition

	(1)	(2)	(3)	(4)
Attrition next period	-0.131***	-0.111***	-0.119***	-0.116***
	(0.0122)	(0.0122)	(0.0122)	(0.0121)
Controls				
Individual characteristics	o	o	o	o
Occupation-specific mean	x	o	o	o
Occupation-specific tenure effect	x	x	o	o
Occupation-specific effects of individual characteristics	x	x	x	o
# of obs	14100	14100	14100	14100
Adjusted R-squared	0.420	0.430	0.434	0.447

Note: Standard errors are in parentheses. *** denotes significance at the 1% level.

Possible selection mechanism: Learning from wage observations

With the pieces of evidence in the previous figures and table where it is found that the wage process could be subject to selective attrition and the attrition can be viewed as an outcome of learning, the following question arises: What is the source of the information that workers use in learning and subsequent decision whether to quit their initial occupations? A likely candidate is the wage, which would be closely tied to a worker's productivity on the job. If workers gain new information from their own realized wages, their occupational decisions may respond to it, which is the hypothesis studied in the following.

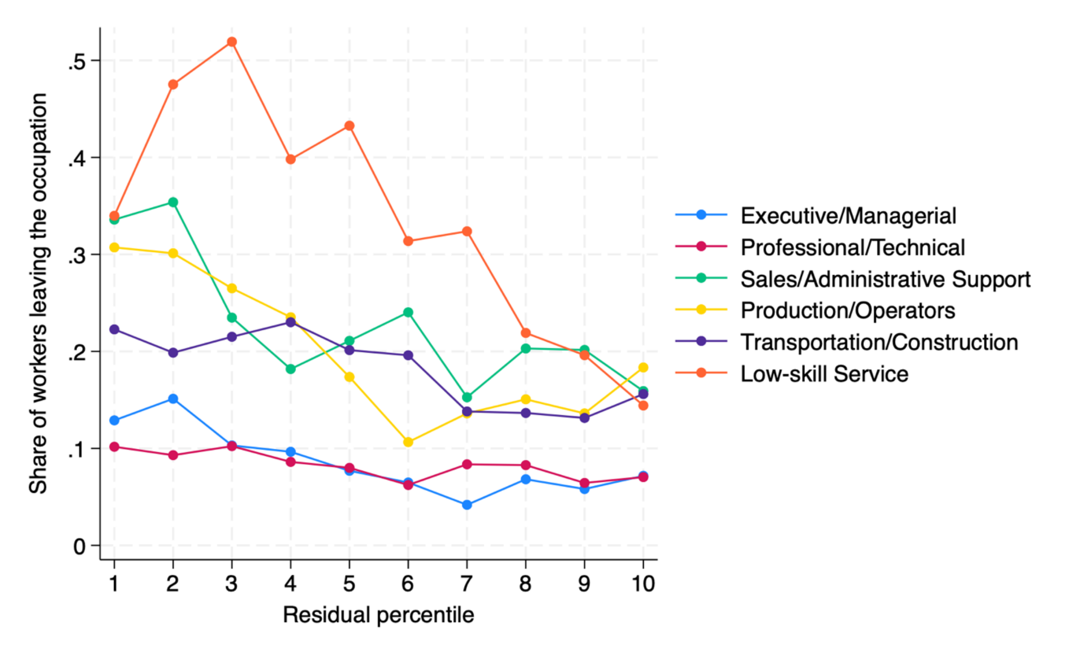
To investigate if wages provide information that influences the workers' decision to quit their occupations, the first task would be to capture the "new information" the workers obtain from the realized wages. Given that the analysis focuses on workers in their initial occupation after the transition from school to the labor market, it could be expected that the private information that workers hold is relatively homogeneous at the outset. I regress log wages on individual characteristics, as done in the specification (4) in Table 2, but this time without the dummy variable indicating the attrition next period.[6] Then, the new information would be contained in the wage residuals computed by comparing the actual wage and the predicted wage based on observable characteristics.[7]

[6] Note that individual fixed effects are not included, which is to get the residuals reflecting individual deviations from the general pattern rather than the deviations around the individual mean. individual fixed effects impose that all residual profiles sum to zero for each person, which is an undesirable feature in this practice.

[7] Considering heterogeneous effects over tenure, additional features found in the analysis are, as having higher tenure, (i) the effects of observed initial individual characteristics except for the ASVAB test scores on the log wage fall, and (ii) the occupational decision gets less responsive to the wage residual.

Figure 3 displays the relationship between the wage residuals and the probability of quitting the occupations in the next period. The figure shows that the fraction of workers quitting their initial occupations is higher for those experiencing lower residuals in percentile which are negative values indeed, and it decreases as they face higher residuals. This trend indicates that workers whose productivity, assumed to be reflected in their wages, is below the expected one based on their observable characteristics are more likely to quit. Conversely, workers with positive residuals—those producing above expectations—are less likely to leave their occupations. The pattern suggests that if wages serve as an accurate reflection of productivity, workers whose performance falls short of expectations are more inclined to leave the occupations for better matches.

Figure 3: Share of workers leaving the occupations in response to the wage shock



3 Structural model

Given the descriptive statistics, I build a structural model that enables separation of the potential channels bringing about the observed patterns of occupational choice and wage dynamics. Disentangling the mechanisms is essential to understanding if uncertainty about one's own ability plays a role in the labor market and quantifying its effects in different occupations. I consider a model of employment dynamics that features uncertainty in a learning environment. The model provides intuition for how information about ability is produced and how the learning process works, which offers a perspective on the interaction between beliefs, wages, and occupational sorting. Focusing on the worker's first occupation spell, in which uncertainty could prevail the most, the model aims to investigate whether learning about one's own ability in the occupation actually plays a role in workers' decisions to stay working or quit. The worker's optimization problem is viewed as an optimal stopping decision formulated in a familiar dynamic programming problem where a finite dependence property (Hotz and Miller 1993; Arcidiacono and Miller 2011) holds. After the model is introduced, I discuss the identification and econometric considerations of the model under the assumptions imposed in the structural model.

3.1 Discrete time duration model

A basic building block for the analysis is a dynamic discrete choice model that extends conventional discrete-time duration analysis. Individual workers are assumed to start spells in an exogenously given occupation, which abstracts from the initial conditions problem conditional on the observables. With t denoting the number of years since worker i started working in occupation j he is assigned, let $d_{ijt} = 1$ if the worker stays at time t and $d_{ijt} = 0$ if he quits. The exogenously specified initial condition $d_{ij1} = 1$ means that every worker starts with zero experience in the occupations they are assigned. At the end of each period, he receives a wage based on which he updates his belief about his own skill (or called productivity, human capital stock) which is initially determined by one's ability. The skill accumulates based on his occupation through learning by doing. Initially unknown ability, in conjunction with a permanent shock in the skill accumulation, makes the worker not exactly know his skill level, so he forms and revises the belief about his own skill and gains associated with staying in or quitting the occupation. The discrete choice, modeled starting from period t=2, has a terminal period $T_i + 1$ (≥ 2) at the beginning of which the worker decides to quit. Then, the choice $d_{iT_i+1} = 0$ signifies that work i has remained working up to period T_i.

Human capital and Output production

Following the human capital models, worker i's potential productivity in occupation j in period t is directly related to skill level H_ijt. The worker initially assigned to the occupation is uncertain about his skill which he learns through a sequence of noisy signals from the output production.[8] I assume that potential output production may be represented by

$$Y_{ijt} = H_{ijt}\text{exp}\{\sigma_p(j)\varepsilon^p_{it}\} \tag{1}$$

where $\varepsilon^p_{it} \sim N(0,1)$ represents transitory variation in the productivity at work. Along with its variability, $\sigma_p(j)$, varying across the occupations, the realized idiosyncratic productivity shock is denoted by ε^p_{ijt}.

The initial skill and its accumulation via the experience in the occupation is specified as follows:

$$logH_{ij1} = X_i\gamma_j + \kappa_{ij} + A_{ij} \tag{2}$$

$$logH_{ijt+1} = (1-\delta_j)logH_{ijt} + \alpha_j + \sigma_h(j)\varepsilon^h_{it+1} \tag{3}$$

where X_i depicts one's observed characteristics before entering the labor market with their contributions to the initial skill varying across occupations by γ_j.[9] There is an individual heterogeneity component, κ_{ij}, known to the workers. Given the quality measures regarding the worker's observed skills, I assume $\kappa_{ij} = \eta_i\rho_j$ where η_i is the observed skill endowments vector. A_{ij} reflects, using Heckman et al. (1998) terminology, the ability to "earn" in occupation j and embodies the contribution of the initially unknown endowment to subsequent earnings. It is unknown to the worker and learned throughout the tenure in the corresponding occupation. Log of the skill accumulates by $\alpha_j > 0$ and depreciates

[8] Uncertainty in human capital investments could arise from various sources. For example, the future price of different skills in the production function would be another source because the market demand and the production technology progress stochastically over time. This paper focuses only on the uncertainty rooted in one's attribute, specifically ability.

[9] To be clear, X_i includes an intercept, which captures the initial human capital common to the workers.

by its rate $\delta_j \in (0, 1)$, both of which are specific to current occupations but common across the workers within the same occupation. Skill accumulation shock, $\varepsilon_{it}^h \sim N(0, 1)$, has individual-specific and persistent effects on workers' productivity through the newly produced skill.

Learning-by-doing type of model in this paper shares similar features with the seminar Ben-Porath model (1967) and its extensions (e.g., Heckman et al. 1998; Magnac et al. 2018), even though in their framework, differently from this paper, workers choose the amount of investment in skill accumulation; worker's productivity is directly proportional to skill level, and each period worker's skill is augmented by the amount of new human capital created along with its depreciation by the amount proportional to the current human capital. The key feature of the Ben-Porath model that skill rises over the life cycle at a diminishing rate can be captured in this learning-by-doing model, on top of the constant depreciation rate, either by human capital depreciation rates varying over age or/and more complicated functional forms of skill accumulation. However, in this paper, which focuses on the early career when wage growth mainly happens, I restrain the model from those specifications that are aimed to replicate the concave earnings profile in the latter lifecycle.

Information structure and earnings determination

I assume that workers and employers have symmetric information about workers' ability to earn, A_{ij}, which is not directly observed. At the beginning of the employment relationship, the agents have a prior about A_{ij}, which is consistent with its population distribution: $N(\mu_A(j), \sigma_A(j)^2)$.[10] They know with certainty the parameters of output production and human capital production functions, and workers' wages are determined by a one-period contract that firms offer by specifying it as a function of the output realized in the period. As in Gibbons and Murphy (1992) and more recently in Camargo, Lange, and Pastorino (2024), a linear contract along with competition among firms (e.g., $W_{ijt} = (1 - b_{ijt})E[Y_{ijt}|I_{it}] + b_{jt}Y_{ijt}$) is a popular specification that allows complex issues (e.g., productivity-enhancing contract, risk sharing) to be studied by yielding systematic differences between wages and

[10] Ability is occupation-specific, leaving the covariance between the components unrestricted. Assuming one-dimensional ability implies that the best workers are better at all occupations. Even though many empirical articles in the literature (Gibbons et al. 2005; Antonovics and Golan 2012; Groes et al. 2015) pay attention to a one-dimensional model of ability, the empirical results in the literature favor the model of comparative advantage (see, for example, Papageorgiou 2014; Arcidiacono et al. 2024). This paper does not add complexity by focusing on the ability associated with the initial occupation.

output production.[11] However, in the absence of workers' amount of output production or compensation policy in data, it is usually challenging to identify the piece rate. This paper focuses on the specific case of $b_{jt} = 1$, which links observed compensation and unobserved workers' output production.[12]

Workers are assumed to be rewarded completely on the basis of output production. Namely, the piece rate in the linear contract set to one:

$$W_{ijt} = P_{jm(t)} Y_{ijt} \tag{4}$$

where $P_{jm(t)}$ is the occupation-specific output price in calendar year m at time t which denotes duration in the first occupation. Its variation over time by aggregate shock is assumed to be perfectly foreseen by the workers for simplicity.[13] This yields, by replacing the skill accumulation process recursively, the following equation for log wages:

$$\begin{aligned} w_{ijt} &= p_{jm(t)} + h_{ijt} + \varepsilon^{p}_{ijt} \\ &= p_{jm(t)} + (1-\delta_j)^{t-1}(X_i\gamma_j + \eta_i\rho_j) + \frac{1-(1-\delta_j)^{t-1}}{1-(1-\delta_j)}\alpha_j + \{(1-\delta_j)^{t-1}A_{ij} + \xi_{ijt}\} \end{aligned} \tag{5}$$

where $w_{ijt} \equiv logW_{ijt}$, $p_{jm} \equiv logP_{jm}$, and $h_{ijt} \equiv logH_{ijt}$. The last bracket stands for the ex-ante unpredicted part of the log wage. Ability A_ij underlies the uncertainty in the initial skill that lasts to the subsequent periods while its effect decreases over time. On the other hand, the variation attributed to the skill production shocks $\xi_{ijt} \equiv \sum_{\ell=1}^{t}(1-\delta_j)^{\ell}\varepsilon^{h}_{ij\ell} + \varepsilon^{p}_{ij\ell}$ accumulates over time.

Bayesian learning and belief update

Working in a certain occupation provides the accumulation of occupation-specific skill and information to learn how good one is in the occupation. Workers form beliefs about their skill levels and update them based on the wage observed at the end of the period. That is, at the start of period t, given one's prior belief about h_{ijt} characterized $N(\mu^{*}_{ijt}, \sigma^{2*}_{ijt})$, the

[11]The specification nests particular specifications found in empirical studies. Gibbons et al. (2005) assume that wages are determined by the expected output (i.e., $b_{jt} = 0$), and in Arcidiacono et al. (2024), wages depend only on the realized output (i.e., $b_{jt} = 1$).

[12]This is in line with the suggestive evidence that wages contain information about workers' abilities.

[13]The process of the output prices could be modeled using an AR(1) process, which would be aligned with the literature that assumes aggregate shocks in the labor market follow an AR(1) process. For simplicity, I adopt the perfect foresight assumption.

worker observes a signal $s_{ijt} = w_{ijt} - p_{jm} (= h_{ijt} + \varepsilon^p_{ijt})$.[14] Since the human capital stock cannot be separated from the idiosyncratic productivity shock in the signal, the worker forms posterior in the Bayesian fashion based on the signal. Considering the skill that is supposed to be accumulated through learning by doing, the belief is updated. Given the normality assumptions, the posterior (or the prior in the next period) is characterized by its mean and variance:

$$\mu^*_{ijt+1} = (1-\delta_j)\Big[\frac{\sigma^2_p(j)}{\sigma^{2*}_{ijt} + \sigma^2_p(j)}\mu^*_{ijt} + \big\{1 - \frac{\sigma^2_p(j)}{\sigma^{2*}_{ijt} + \sigma^2_p(j)}\big\} s_{ijt}\Big] + \alpha_j \tag{6}$$

$$\sigma^{2*}_{ijt+1} = (1-\delta_j)^2 \frac{\sigma^2_p(j)}{\sigma^{2*}_{ijt} + \sigma^2_p(j)}\sigma^{2*}_{ijt} + \sigma^2_h(j) \tag{7}$$

with their initial values: $\mu^*_{ij1} = X_i\gamma_j + \rho_j\eta_i + \mu_A(j)$ and $\sigma^{2*}_{ij1} = \sigma^2_A(j)$.

Note that the magnitude of the noise in the signal, $\sigma^2_p(j)$, is constant over time whilst it varies across the occupations. Intuitively, workers in different occupations learn about their skill levels at different speeds of learning. Namely, new information about own ability in the signal is more reliable when the noise is small (i.e., the signal-to-noise ratio is high), so less weight on the prior and more weight on the informative signal are placed. Therefore, learning is not independent of occupation j to which the worker is initially assigned. Note that the precision of the posterior beliefs does not necessarily monotonically decrease over tenure due to the shock to skill production. Instead, the variance converges to a value determined by the variability of the idiosyncratic shocks: $\sigma^2_p(j)$ and $\sigma^2_h(j)$.

Preference and dynamic programming

Let the vector of state variables be denoted by Ω_{it}, where $\Omega_{it} \equiv (\mu^*_{ijt}, \sigma^*_{ijt}, m(t))$ contains the variables characterizing the belief about h_{it} and the calendar year. At the beginning of each period, the worker faces an outside option whose mean value is $v_0(\Omega_{it})$ but with an idiosyncratic component $\varepsilon^u_{0,ijt}$. He decides, maximizing lifetime expected utility, to stay working if the value of continued employment is greater than the outside option and quits otherwise. The worker's per-period utility is separable in the expected log wage $\mathbb{E}_{h,\varepsilon^p}[w_{ijt}(h_{ijt}, \varepsilon^p_{ijt})]$ with the productivity shock ε^p_{ijt} being realized at the end of the period while the idiosyncratic utility shock $\varepsilon^u_{1,ijt}$ is observed before making the choice. I further

[14] Keeping track on workers' beliefs about skill level rather than about ability itself makes the notation more simple, still yielding the same learning process; for example, the signal about "ability" at time t is $A_{ij} + \frac{\varepsilon_{ijt}}{(1-\delta_j)^t} = w_{ijt} - \{p_{jm(t)} + (1-\delta_j)^{t-1}(X_i\gamma_j + \eta_i\rho_j) + \frac{1-(1-\delta_j)^{t-1}}{1-(1-\delta_j)}\alpha_j\}/(1-\delta_j)^{t-1}$.

assume that $\varepsilon^u_{ijt} = (\varepsilon^u_{0,ijt}, \varepsilon^u_{1,ijt})$ is distributed i.i.d. Type 1 Extreme Value with mean zero, and variance equal to $(\pi/\sqrt{6})\sigma_u(j)$[15] This shock, interpreted as a shock at the worker level or idiosyncratic worker characteristics, captures search frictions as they are treated in the form of exogenous shocks in this model.

While the choice to stay working yields the current wage gain and the continuation value associated with the optimal decision next period based on the posterior, the choice process ends when the worker chooses to quit. This allows the worker's problem to be framed by an optimal stopping problem, which can be written in the form of a Bellman equation with the value function:

$$\begin{aligned} V_t(\Omega_{it}, \varepsilon^u_{ijt}) = \max\{ & v_{0t}(\Omega_{it}) + \varepsilon^u_{0,ijt}, \mathbb{E}_{h,\varepsilon^p}[w_{ijt}(h_{ijt}, m(t), \varepsilon^p_{ijt})] + \varepsilon^u_{1,ijt} \\ & + \beta \int_{\varepsilon^u} \int_{\varepsilon^h} \int_{\varepsilon^p} V_{t+1}(\Omega_{it+1}, \varepsilon^u_{ijt+1}) dF(\Omega_{it+1}|\Omega_{it}, \varepsilon^p_{ijt}, \varepsilon^h_{ijt+1}) dF(\varepsilon^p_{ijt}) dF(\varepsilon^h_{ijt+1}) dF(\varepsilon^u_{ijt+1})\} \end{aligned} \tag{8}$$

Define $v_{1t}(\Omega_{it})$ as the value function summarizing the lifetime expected utility the worker would expect to receive from choosing a stay, net of the idiosyncratic component of the current period. Then, his optimization problem is viewed to choose an alternative by comparing $v_{0t}(\Omega_{it}) + \varepsilon^u_{0,ijt}$ and $v_{1t}(\Omega_{it}) + \varepsilon^u_{1,ijt}$

3.2 Identification

The parameters associated with each occupation are identified separately within the workers having different series of choices and wage outcomes in the same occupation. Therefore, for ease of notation, I abstract from the j subscript denoting the occupation from here on. I state the following standard normalization:

(a) Population distribution of A_i follows $N(\mu_A, \sigma^2_A)$ where μ_A is normalized to 0

(b) A_i is independent of X_i and η_i

(c) Idiosyncratic shocks are independent of each other and over time

(d) Discount factor $\beta = 0.95$

Additionally, since skill level is an unobserved object that does not have natural measures, observed high wages can be rationalized by either a large human capital stock or high

[15]This is equivalent to normalizing the location parameter to $-\sigma_u(j)$ multiplied by the Euler's constant, where $\sigma_u(j)$ is the scale parameter.

returns to human capital. I normalize the wage return to human capital stock to one, which is already imposed in the model. Due to scale indeterminacy inherent in the utility specification, the scale is normalized by setting the coefficient of the log wage to one. Given the normalization and the parametric assumptions in the structural model, this subsection is devoted to explaining the crucial identification assumption and the source of variations to discuss the identification of the distributions of ability, skill accumulation shock, and productivity shock.

The identification problem is, from the selected population distribution of choices and wages ($f_{(w_1,...,w_T),(d_2,...,d_{T+1})}$, where $T = \max_i\{T_1, ..., T_i\}$)) to recover the conditional distributions of the choice probabilities ($f_{d_t|A}$) and potential wages ($f_{w_t|A}$). Identification of the model in this paper is based on Bunting, Diegert, and Maurel (2024), who provide identification results for the distribution of potential outcomes in a general class of learning models that accounts for the presence of selection issues through learning. The key idea is to characterize the relationship using Bayes' rule:

$$f_{w_1,...,w_T} = f_{w_1,...,w_T|d_2,...,d_{T+1}} \frac{f_{d_2,...,d_{T+1}}}{f_{d_2,...,d_{T+1}|w_1,...,w_T}} \tag{9}$$

where the conditional density $f_{w_1,...,w_T|d_2,...,d_{T+1}}$ is directly identified from the data and weighted by a selection adjustment term to recover $f_{w_1,...,w_T}$. Once the selection weights are identified relying on the learning framework, the parameters determining the potential wage distribution, including the distributions of ability and the other unobservables, can be identified.[16]

The identification strategy in Bunting, Diegert, and Maurel (2024) can be extended to the case of attrition, where no outcome is observed in the sample once an individual chooses to quit and fall into the absorbing state.[17] Additionally, I assumed that each worker with initially unknown ability does not possess persistent private information that affects their decision, which implies that the choice depends on beliefs about ability updated only through previously observed wages and covariates. Namely, the learning

[16]The residual denoted by ξ_{it} in equation (5) is a sum of the skill accumulation shocks and idiosyncratic productivity shock, of which variance varies over tenure. Under the assumptions imposing normality, independence between them, and identical distribution over time, the distributions of the shocks can be separately identified once the variance of the residual distributions are identified in different tenures.

[17]For example, Assumption L4 in their paper needs to be relaxed when the selection issues are posed by attrition.

model can be seen as a model of selection on observables, where the conditional choice probability function does not depend on any latent variable and is thus identified directly from the data. Having identified the CCPs along with the wage parameters, I can apply standard identification arguments from the dynamic discrete choice literature (see, e.g., Hotz and Miller, 1993; Magnac and Thesmar 2002) to identify conditional value functions and recover the primitives of the choice model under the standard assumptions imposed above.[18]

Lastly, I add the intuition that the panel dimension of the choice process provides additional information to disentangle the variances of the two unobservable in the wage process: the skill accumulation shock and idiosyncratic productivity shock. Without the skill accumulation shock, the variance of workers' beliefs about their own skills converges to zero. This means that any occupational choice after some periods is barely attributed to learning and responses to the realized wages. Hence, if the skill accumulation shock is significant, the variations of occupational choices in the latter periods in the panel can help the variance of the skill accumulation shock to be identified. The choices in response to wage observations enable the variance of the productivity shock to be identified. For example, consider worker A and worker B, whose skills and beliefs are the same, and there are two signals, say, a good signal and a bad signal, for simplicity. Worker A receive receives a good signal first, and a bad signal next, and worker B receives it in the other order around. Abstracting from the duration dependence through the different outside option values in the two periods, the probability of quitting after having the information through the signals would not be equal in the learning environment. Hence, when they learn about their ability, one would be more likely to quit than the other, which provides information about the weight placed on the prior which is a function of the variance of the idiosyncratic productivity shock.

3.3 Econometric Issues

Selective attrition and potential bias

[18] Note that, in conjunction with the distributional assumptions and conventional Bayesian learning, the tractable sequence of beliefs characterized by only two parameters makes the model possess a first-order Markov structure.

Central to the structural model in the paper is that, in the presence of learning about ability on the job, luck (i.e., realizations of ε^P and ε^h) leads to different career trajectories. That is, the duration of working in the occupation provides information about the realized luck in the preceding periods. Consequently, the decision to stay up to a certain period is not random and is dependent on past skill accumulation and productivity shocks through the learning process, even under independence between the unobservables in the wage and choice equations.[19] Estimating wage parameters raises the econometric issue of correcting the selection bias resulting from selective attrition. The approach in this paper is to estimate the wage equation jointly with the explicit selection process specified. The usual panel data approaches, both fixed effects and random effects models, applied to the data generated by such a model yield biased estimates of the wage parameters. To make the specification comparable to the usual Mincer wage equation, let's first assume no skill depreciation $\delta = 0$ and no skill accumulation shock $\varepsilon_{it}^h = 0$ (i.e., deterministic human capital accumulation). Once I define the deviation of the observed vector of time-varying variable x_{it} from its individual mean by $\tilde{x}_{it} \equiv x_{it} - \sum_{\ell=1}^{T_i} x_{i\ell}/T_i$, and analogously for the other variables, it is straightforward to show that the condition for consistency of the fixed effects estimator (i.e., within estimator), $\hat{\theta}_{FE} = \sum_i \sum_{\ell}^{T_i} [\tilde{x}'_{i\ell}\tilde{x}_{i\ell}d_{i\ell}]^{-1}[\tilde{x}'_{i\ell}\tilde{w}_{i\ell}d_{i\ell}]$,is

$$\mathbb{E}[\tilde{\varepsilon}_{it}^p | d_{i1} = \cdots = d_{iT_i} = 1, d_{iT_i+1} = 0] = 0. \tag{10}$$

The sufficient condition would not generally hold because d_{it} is a function of belief which is determined by $\{\varepsilon_{il}^p\}_{l=1}^{t-1}$ and $\{\varepsilon_{il}^h\}_{l=2}^{t-1}$ through the dynamics of learning.[20] Intuitively, the condition for staying for T_i periods is not independent of the signals so the realized shocks; For example, suppose that a worker who believes himself to have high productivity as a result of positive signals he has received is more likely to stay in his initial occupation (i.e., $v_{1t}(\Omega_{it}) - v_{0t}(\Omega_{it})$ increases in μ_{ijt} for all t). Then, the worker staying for T_i period may not have experienced too negative wage signal in the periods before T_i. Namely, the condition for staying in up until T_i would imply a truncation of the unconditional distribution of $\{\varepsilon_{il}^p\}_{l=1}^{T_i-1}$. Therefore, selective attrition due to the learning process would

[19] Nonrandom selection and attrition problems coming from a nonzero correlation between the unobservable in the wage and the one in utility is well understood in the literature. In this paper, I focus on the bias generated by learning about ability while maintaining the assumption that the idiosyncratic productivity shocks, skill accumulation shocks, and utility shocks are independent.

[20] A case in which the consistency condition holds with the sufficient condition being not necessarily met is the situation where observations are missing deterministically given the explanatory variables.

yield a bias in the within estimator.[21] More discussion and a simulation result is found in Appendix D. Since the condition for consistency of the random effects estimator is stronger than the one for the within estimator, I do not discuss the bias in the random effects estimator here.

Selection at entry and unobserved heterogeneity

Even though econometricians can control for some individual characteristics (e.g., years of education before entering the labor market) usually observed in data, there could be initial unobserved heterogeneity that is known to the economic agents (i.e., workers and potential employers) but unknown to the econometricians. Namely, it could be unlikely that initial human capital stock is exogenous conditional on the observed variables at the time of labor force participation. The examples include that initially observed schooling is an outcome of a stochastic process containing measurement error or that observed proxies for the academic/cognitive abilities may omit to capture the productivity in the workplace, in which cases the exogeneity conditioning only on observable would be problematic.

This problem with selection at entry can be addressed by allowing for unobserved heterogeneity types in the spirit of Heckman and Singer (1984) and Keane and Wolpin (1997). Specifically, workers are defined as one of K types with type-specific components that capture permanent characteristics. The finite mixture approach assumes that econometricians know that there are K types. The type-specific unobserved heterogeneity known to the agents and the unobserved abilities initially unknown to the agents are independent. Then, the fact that discrete choice depends on one's known type as well as beliefs about unknown skill levels prevents the distribution of the factors from being easily identified. However, exploiting a main virtue of the NLSY79 that contains auxiliary selection-free measurements of various skills, along with the panel dimension of the data, will allow the mixture model to be identified.

[21] The instrument proposed in Gibbons et al. (2005) exploits the property of a learning model that Bayesian beliefs are a martingale. However, the IV estimator could still fail to correct the attrition bias. To see this, rewrite the current wage using the previous wage linked by the human capital production. It leads unobservables in the equation to be a composite of skill accumulation and idiosyncratic productivity shocks:

$$w_{it} = p_{m(t)} - (1-\delta)p_{m(t-1)} + (1-\delta)w_{it-1} + \alpha + [\varepsilon^h_{it} + \{\varepsilon^p_{it} + (1-\delta)\varepsilon^p_{it-1}\}]. \quad (11)$$

The suggested instruments of the lagged wages, $\{w_{i\ell}\}_{\ell=1,2,\cdots,t-1}$, are correlated with w_{it-1} but uncorrelated with the unobservable component in the equation. However, the IVs are still susceptible to selective attrition; the condition to have valid IVs is to stay longer than three periods, which yields the restricted estimation sample conditioning on the observability. This implies that the moment conditions for the IVs could still result in biased estimates.

Specificity of skills and value of quitting

Skill, determined by initially unknown ability and stochastic skill accumulation process, could be general in the sense that productive workers in one occupation could be more likely to be productive in some other occupations. This can potentially have the value of an outside option dependent on either one's skill or ability in the current occupation.[22] In this case, beliefs about one's own skill in the initial occupation affect not only the value of staying in but also the value of quitting. Exclusion restrictions may need to be imposed to identify, without relying on or further invoking parametric assumptions, the value of the outside option separately from the effect associated with the variables in the expected wage.

One of the possible restrictions would be occupation-specific regressors that generate variations only in the utility of outside options; for example, occupation-specific demand for labor. This is a common strategy found, for example, in Heckman and Sedlacek (1985) and D'Haultfœuille and Maurel (2013). More specifically in the present setting, occupation-specific vacancies in occupation j' ($\neq j$) are excluded from the wage equation for occupation j, and the vacancies in j' are uncorrelated with unobserved components of wages in j conditional on vacancies in j.[23] For simplicity, throughout the paper, I simplify the model by assuming that skills are occupation-specific and uncorrelated to each other, which rationalizes the specification that the value of quit is independent of one's own skill in the current occupation.

4 Estimation

In this section, I detail the maximum likelihood estimation procedure for the dynamic discrete choice model, which incorporates learning about one's own skill based on observed wages along with its stochastic evolution. Given unobserved state variables of skills, the likelihood is computed using the Kalman filter that recursively estimates the unobserved skill from observed signals of wages. It is worth noting that, given a set of parameter values, the estimate of the skill each period indeed corresponds to the belief formed by

[22]For simplicity, I assume that the value of quitting is deterministic and independent of one's belief about own skill, which plays the role of an exclusion restriction.

[23]Once general equilibrium effects through labor supply have an effect within a period, the condition could be violated, but the feedback effect may have a time lag.

the workers. Then, this algorithm allows me to calculate the conditional distribution of skills, wages, and occupational choices sequentially from the first period to the last period. When the likelihood contribution of the choices is computed, I use the CCP estimation exploiting the terminal state nature. Parameter estimates and the model fit are discussed in the following subsections.

The idea of using the Kalman filter is that the model can be framed within the context of a state-space model with hidden state variables and observable measurements. Additionally, it is a dynamic linear model in which the system evolves over time according to linear stochastic equations, with all latent variables having normal distributions. Optimal estimates of the hidden states $\{h_{it}\}$ based on noisy observations $\{w_{it}\}$ are provided by the recursive algorithm that consists of two main phases: Prediction step and Update phases. Prediction phase is to predict the state at the start of each period before making a decision and observing a new measurement:

$$\begin{aligned}
\text{State prediction}: \hat{h}_{it} &= E[h_{it}|I_{it-1}] \\
&= (1-\delta)\hat{h}_{it|t-1} + \alpha \\
\text{Prediction uncertainty}: s_{it} &= Var[h_{it}|I_{it-1}] \\
&= (1-\delta)^2 s_{it|t-1} + \sigma_h^2
\end{aligned}$$

where $I_{it-1} = \{X_i, d_{i1} = \cdots = d_{it-1} = 1, w_{i1}, \ldots, w_{it-1}\}$ summarizes all the information up to period $t-1$. Update phase incorporates the information in the measurement based on the current priori prediction and the wage observation, which refines the estimate of the skill level:

$$\begin{aligned}
\text{State update}: \hat{h}_{it|t-1} &= \hat{h}_{it-1} + K_{it}\nu_{it} \\
\text{Update uncertainty}: s_{it|t-1} &= (1-K_{it})s_{it-1}
\end{aligned}$$

where $\nu_{it} = w_{it} - \{p_{m(t)} + \hat{h}_{it-1}\}$ is innovation (or prediction error) coming from the realization of measurement equation. The Kalman gain, $K_{it} = s_{it-1}/(s_{it-1}+\sigma_p^2)$, determines the weight given to the new measurement relative to the predicted state.

4.1 Maximum Likelihood Estimation

The likelihood function for the model involves the joint probability of the observed wages and occupational decisions, considering the unobserved skills. The Kalman filter facilitates the computation of the likelihood by providing the necessary components to evaluate the probability of the observed wage and the probability of the observed choice made on the belief about own skill. Note that the choice is assumed to depend on the workers' beliefs about their skills updated through the observed sequence of wages, implying that, conditional on the model parameters, each worker's belief matches the estimate of the skill updated through the Kalman filter. Suppose the likelihood contribution of individual i who makes the choice to stay working until period T_i and quit in period $T_i + 1$. Given the sequence of T_i number of binary decisions and T_i number of wage outcomes, his contributions to the likelihood can be written by:[24]

$$\begin{aligned} L_i(d_{i2} = \cdots = d_{it-1} = 1, w_{i1}, \ldots, w_{it-1}|X_i, d_{i1} = 1) = L_i(w_{i1}|X_i)L_i(d_{i2} = 1|I_{i1})L_i(w_{i2}|I_{i1}, d_{i2} = 1) \\ \cdots L_i(d_{iT_i} = 1|I_{iT_i-1})L_i(w_{iT_i}|I_{iT_i-1}, d_{iT_i} = 1)L_i(d_{iT_i+1} = 0|I_{iT_i}) \end{aligned} \tag{12}$$

Likelihood of the wages

Kalman filter facilitates the construction of the likelihood function of worker i by calculating the conditional distribution of human capital and wages sequentially from the first period $t = 1$ to the last period $t = T_i$. I perform the prediction and update steps recursively. where the log wage at t is a linear function of normal random variables given information up to $t - 1$. Thus, the likelihood of observing log wage w_{it} is given by the normal density:

$$L_i(w_{it}|I_{it-1}, d_{it} = 1) = (2\pi s_{it-1})^{-1/2} exp\Big[- \frac{\{w_{it} - (p_{m(t)} + \hat{h}_{it-1})\}^2}{2s^2_{it-1}}\Big] \tag{13}$$

Likelihood of the choices

The worker's expected values associated with staying in or quitting the current occupation are constructed by exploiting the insights from Hotz and Miller (1993) and, more generally, Arcidiacono and Miller (2011). Taking advantage of the terminal state nature of the

[24] In period 1, all workers are assigned to their initial occupations, which is not modeled in this paper, so the discrete choices of either staying or quitting from the second period on contribute to the likelihood.

dynamic discrete choice problem allows the future value conditional on staying in the current period to be expressed as follows:

$$
\begin{aligned}
v_{1t}(\hat{h}_{it}, s_{it}) &= \mathbb{E}_{h,\varepsilon^p}[w_{it}(\hat{h}_{it}, \varepsilon^p_{it})] + \beta \int_{\varepsilon^h}\int_{\varepsilon^p} \sigma_u \log\Big[\frac{exp\{v_{0t+1}\}}{\sigma_u} + \frac{exp\{v_{1t+1}(\hat{h}_{it+1}, s_{it+1})\}}{\sigma_u}\Big] \\
&\qquad\qquad dF(\hat{h}_{it+1}, s_{it+1}|\hat{h}_{it}, s_{it}, \varepsilon^p_{it}, \varepsilon^h_{it+1}) dF(\varepsilon^p_{it}) dF(\varepsilon^h_{it+1}) \\
&= p_{m(t)} + \hat{h}_{it} + \beta\Big[v_{ot+1} + \int_{\varepsilon^h}\int_{\varepsilon^p} \big\{ -\sigma_u \log P_{0t+1}(\hat{h}_{it+1}, s_{it+1})\big\} \\
&\qquad\qquad dF(\hat{h}_{it+1}, s_{it+1}|\hat{h}_{it}, s_{it}, \varepsilon^p_{it}, \varepsilon^h_{it+1}) dF(\varepsilon^p_{it}) dF(\varepsilon^h_{it+1})\Big].
\end{aligned}
$$

where the state variables other than those characterizing the beliefs are abstracted away in the expression. In the first equality, the ex-ante value of making the optimal decision next period is expressed as the sum of the conditional values associated with each choice using the property of the i.i.d. type 1 extreme value utility shocks. $P_{0,t+1}(\hat{h}_{it+1}, s_{it+1}) \equiv Prob(d_{it+1} = 0|\hat{h}_{it+1}, s_{it+1})$ in the second equality denotes the probability to make the quitting choice next period, conditional on the next period's belief that is determined by the realization of ε^p_{it} and ε^h_{it}. The ex-ante value is expressed by the conditional value from quitting and a function of conditional choice probabilities (CCPs). The latter can be interpreted as a non-negative adjustment term that adjusts for the possibility that the quitting decision could not be the optimal choice. This interpretation can be illustrated by the relation that as the probability of quitting goes to zero, the adjustment term grows infinitely.

The difference in value functions, $v_{1t} - v_{0t}$, matters for the observed choice probability: $P_{1t} = \exp(v_{1t} - v_{0t})/\{1 + \exp(v_{1t} - v_{0t})\}$. By substituting v_{1t} with the expression above, the probability can be expressed only by the utility parameters and the one-period ahead choice probability. This CCP estimation method provides a way to estimate the utility parameters not solving the dynamic programming problem, which consists of the following two steps. The first step involves estimating and predicting the CCPs. The second step then takes the first-step CCPs as data and estimates the remaining structural parameters in the multinomial logit with an offset term. The CCP estimation that estimates a model in stages does not affect the consistency of the estimates, but does reduce efficiency.[25].

[25]To calculate the valid standard errors that account for the multiple stage estimation procedure, a bootstrap procedure can be used. CCPs in the first stage can be estimated in any flexible way (e.g., neural network) as long as it provides consistent estimates.

Then, the likelihood of observing choice d_{it} is given by

$$L_i(d_{it}|I_{it-1}) = \Big[\frac{\exp\{(v_{1t}-v_{0t})/\sigma_u\}}{1+\exp\{(v_{1t}-v_{0t})/\sigma_u\}}\Big]^{d_{it}}\Big[1-\frac{\exp\{(v_{1t}-v_{0t})/\sigma_u\}}{1+\exp\{(v_{1t}-v_{0t})/\sigma_u\}}\Big]^{1-d_{it}} \tag{14}$$

where

$$v_{1t}-v_{0t} = p_{m(t)} + \hat{h}_{it} + \beta v_{ot+1} - v_{ot} + \beta \int_{\varepsilon^h}\int_{\varepsilon^p} \big\{ -\sigma_u \log P_{0t+1}(\hat{h}_{it+1}, s_{it+1})\big\}$$
$$dF(\hat{h}_{it+1}, s_{it+1}|\hat{h}_{it}, s_{it}, \varepsilon^p_{it}, \varepsilon^h_{it+1})dF(\varepsilon^p_{it})dF(\varepsilon^h_{it+1}).$$

4.2 Estimation Results

In this subsection, I examine the estimation results in Table 3. The distribution of abilities across occupations reflects the initial beliefs of workers as they enter the labor market. The findings indicate that the standard deviations of ability distributions (σ_A) are statistically significant across all occupations, confirming that workers face uncertainty regarding their abilities at the onset of their careers. An increase in ability by one standard deviation leads to a wage increase of approximately 38-40%, except in Professional/Technical and Low-skilled Service occupations. In the Professional/Technical occupation, abilities are initially more dispersed, resulting in an approximate 55% wage increase with each standard deviation rise in ability. It is noteworthy that the standard deviation in the ability component is higher than that in the productivity shock (σ_p), suggesting that unexplained wage variation is initially more explained by ability than by idiosyncratic productivity shocks. However, comparing the relative importance gets complicated, as the effect of ability on wages is compounded by skill shocks impacting newly accumulated human capital and workers' sorting based on beliefs.[26]

The growth of log wages is determined by the rate of skill accumulation (α) and its depreciation (δ). In other words, wage growth can be substantial due either to significant gains from experience and/or minimal losses over tenure. Two high-skill occupations, Executive/Managerial and Professional/Technical, exhibit distinct mechanisms in this

[26]The magnitude of the idiosyncratic productivity shock also reflects the degree of noise, and its relative magnitude with respect to the uncertainty in skill determines the speed of learning. However, in the presence of skill accumulation shocks that add uncertainty and wage dispersion., the small variability of productivity shock does not necessarily guarantee a faster learning environment.

respect. In Executive/Managerial occupation, skill accumulation occurs at a relatively low rate but experiences less depreciation, making initial human capital more critical in determining future wage potential. In Professional/Technical occupation where the variance of skill accumulation shock (σ_h^2) is large, on top of the high wage growth, the risk-neutral workers are induced to stay because of the expectation that a large skill accumulation shock could be realized in the future. This incentive is larger for younger workers since skill accumulation shock permanently affects their skill levels.

The last row indicates the scale of the utility shock, modeled as a type I extreme value distribution. A high utility shock value would suggest that worker sorting could occur significantly through factors outside the scope of this model. Specifically, sorting through learning is less prevalent in blue-collar occupations, such as Production/Operators and Transportation/Construction. This implies that other labor theories, such as job search and the stepping-stone model, may be relevant to these occupations. The estimates for other wage parameters align with prior literature. For instance, even where some estimates are not statistically significant, math skills tend to be more highly rewarded in high-skill occupations like Executive/Managerial and Professional/Technical, whereas verbal skills are more advantageous in low-skill occupations.

4.3 Model Fit

The point estimates show the existence and relevance of the learning process with occupational decision. Before delving into more quantitative practice using the structural estimates, I check for the model fit. Using the estimated parameters of the model in the previous subsection, I simulate a data set from the explanatory variables in the original data and repeat the simulation ten times. In each simulated data set, individuals enter at the calendar time of their actual entry in the occupation, and their true ability and idiosyncratic shocks are drawn from the corresponding distributions characterized by the estimated model parameters. Then, the decision to stay in or quit the first occupation is endogenously determined along with the simulated wages.

Figure 4 presents the standard deviation of the log wages in each occupation. Note that the moments associated with the choice probability and the mean of log wages are matched in the maximum likelihood procedure. I simulate and draw the figure to see whether the

Table 3: Parameter Estimates by Occupation

	Executive/ Managerial	Professional/ Technical	Sales/ Admin Support	Production/ Operators	Transportation/ Construction	Low-skill Service
Ability distribution						
σ_A, SD of ability	0.3984	0.5480	0.3923	0.3868	0.3806	0.3366
	(0.0357)	(0.0474)	(0.0280)	(0.0234)	(0.0146)	(0.0198)
Human capital production and productivity shock						
α, New human capital production	0.4207	1.5266	0.4584	0.4083	0.5091	0.0231
	(0.0474)	(0.0730)	(0.0465)	(0.0331)	(0.0213)	(0.0121)
δ, Depreciation of log human capital	0.0486	0.1945	0.0571	0.0525	0.0672	0.0210
	(0.0373)	(0.0514)	(0.0435)	(0.0385)	(0.0241)	(0.0000)
σ_h, SD of Skill shock	0.1800	0.2997	0.1532	0.0939	0.1833	0.1032
	(0.0000)	(0.0391)	(0.0000)	(0.0432)	(0.0400)	(0.0395)
σ_p, SD of Productivity shock	0.2680	0.1802	0.3145	0.1986	0.2174	0.2661
	(0.0000)	(0.0593)	(0.2558)	(0.0983)	(0.0588)	(0.0493)
Initial human capital						
Constant	6.7944	6.8899	6.7127	6.7794	6.6619	6.5443
	(0.0000)	(0.0001)	(0.0000)	(0.0041)	(0.0000)	(0.0017)
Observed Verbal Skill	-0.0333	0.0398	0.0207	0.0491	0.0613	0.0471
	(0.0262)	(0.0516)	(0.0310)	(0.0766)	(0.0419)	(0.0577)
Observed Math Skill	0.2694	0.1355	0.1137	0.0983	-0.0005	0.0321
	(0.0644)	(0.0728)	(0.0893)	(0.0253)	(0.1475)	(0.0338)
Value of quitting (outside option)						
Constant	29.9826	29.9722	29.9556	29.9688	29.9638	29.8170
	(0.1351)	(0.1674)	(0.1018)	(0.5746)	(0.2520)	(0.1709)
Observed Verbal Skill	1.7521	-1.3253	-0.3935	1.9502	-0.0418	-2.0419
	(0.0000)	(0.3969)	(0.4368)	(0.4303)	(0.2963)	(Inf)
Observed Math Skill	-0.2384	0.3129	-0.0305	-0.7557	-0.4844	0.9639
	(0.0255)	(0.1422)	(0.0389)	(0.0299)	(0.0284)	(Inf)
Tenure (duration dependence)	0.6533	-2.2945	0.8804	1.7653	2.0439	2.5653
	(0.0534)	(0.0424)	(0.1721)	(0.1019)	(0.0488)	(0.0905)
σ_u, Scale of utility shock	0.2905	0.1523	0.2469	0.3372	0.3428	0.3011
	(0.0413)	(0.0460)	(0.0603)	(0.0468)	(0.0302)	(0.0281)

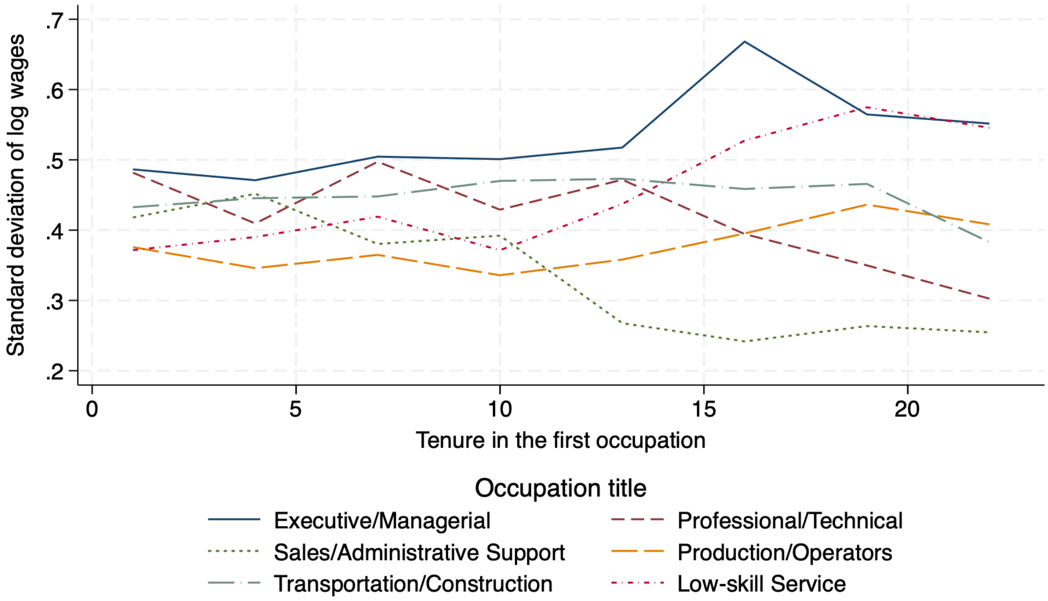
* Standard errors in parentheses.

estimated structural model can generate the observed pattern that is not directly taken into account in the estimation. Figure 4 illustrates that, in terms of wage dispersion, the estimated model can replicate the patterns of trajectories over tenure in different occupations presented in Figure 1 (b). With a ten times larger number of workers in the simulated data, the trajectories are smoother than the observed one in each occupation, while the general trend fits well.

5 Counterfactual Analysis

In this section, I take advantage of the structural estimates to simulate workers' true ability and the progressions of their skills and beliefs while they are not observed in the data. Then, it is available to define and measure mismatch to study the implication of learning on workers' sorting. Next, I consider a counterfactual policy that removes uncertainty after the first period of the employment relationship. The comparison between data and counterfactuals enables the effect of learning to be studied, accounting for its dynamic effect

Figure 4: Model fit: Hazard rates



on the duration of spell working in the initial occupations and wage dynamics through sorting as well as human capital accumulation. In order to do the practice, additional assumptions are needed to fully solve the model and simulate data. To set the terminal period, I assume that all workers quit their initial occupation one year after the maximum age observed in the data, and it is known with certainty. The corresponding terminal values associated with staying and quitting are parametrically extrapolated. Then, I solve the model backward from the terminal age and get the solution (i.e., policy function). Given the initial characteristics and the calendar year of workers' entry into the occupations in the original data, I simulate data sets by drawing workers' true ability and idiosyncratic shocks from the corresponding distributions characterized by the parameter estimates. Then, the duration of the spell in the initial occupations through the choice of either staying or quitting, along with wage outcomes, is endogenously determined.

5.1 Types of Mismatch

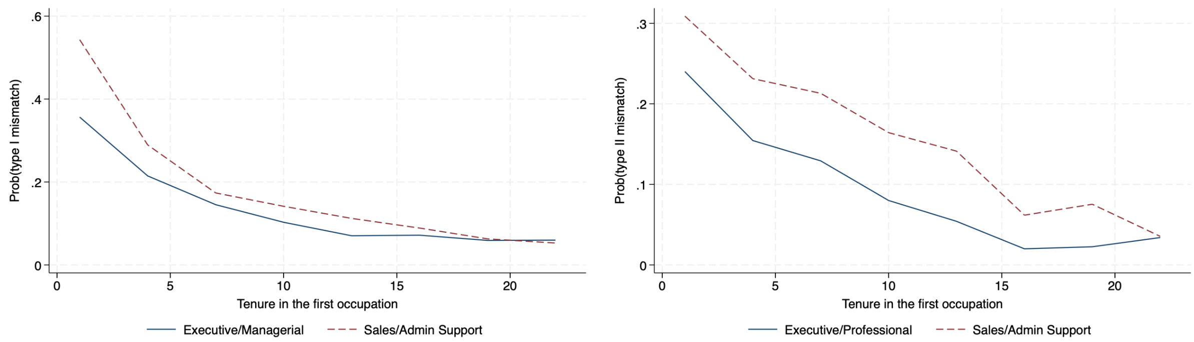
The decision to quit is based not on the true skill but on the belief that has been updated through the series of wage observations, so inevitably pure luck (i.e., realization of idiosyncratic shocks) plays a role. In this learning environment, there are two types of mistakes that the workers could make, which are defined here as mismatch. The first kind of mismatch is to quit because of the belief, even though the actual skill justifies a decision to stay. Namely, this type I mismatch occurs when the current occupation is mistakenly discarded: a false positive. The second kind, type II mismatch, is to decide to stay due to beliefs when the actual skill favors separation: a false negative. Within the structural model defined in Section 3, the probabilities of the mismatch can be defined as follows:[27]

$$\text{Type I mismatch} : P(v_{0t} + \varepsilon^u_{0,it} > v_{1t}(\mu^*_{it}, \sigma^{2*}_{it}) + \varepsilon^u_{1,it} \mid v_{0t} + \varepsilon^u_{0,it} < v_{1t}(h_{it}) + \varepsilon^u_{1,it})$$
$$\text{Type II mismatch} : P(v_{0t} + \varepsilon^u_{0,it} < v_{1t}(\mu^*_{it}, \sigma^{2*}_{it}) + \varepsilon^u_{1,it} \mid v_{0t} + \varepsilon^u_{0,it} > v_{1t}(h_{it}) + \varepsilon^u_{1,it})$$

Figure 5 shows the share of mismatched workers in the two occupations in which initially unknown abilities are distributed with very similar dispersion: Executive/Managerial and Sales/Admin Support. I find that, in the early tenure years, both types of mismatch are more frequent in Sales/Admin Support, which is attributable to its features in the learning process and outside options. In this occupation, the wage signal is relatively noisier, implying that workers have higher chances of experiencing extreme luck and the learning process is slow. In conjunction with this learning environment, the value of quitting increases faster. These features lead the Sales/Admin Support workers with initially high ability to wrongly quit the occupation and workers with initially low ability to stay driven by optimistic beliefs. The mismatch may limit the workers' chance to accumulate the skill in the occupation that could be potentially aligned with their true ability.[28] Once the beliefs are refined as the workers stay longer, the mismatch displays convergence, but the probabilities are not zero. This is because the underlying uncertainty never disappears when the skill accumulation is stochastic.

Figure 5: Dynamics of mismatch

(a) Type I mismatch (b) Type II mismatch



5.2 Counterfactual Policy: Information provision

I consider a counterfactual policy of getting the workers better informed about their ability during the first year of the employment relationship. A special case would be that workers have perfect information from the second period on, which can help them make better choices without the information friction. In practice, this could be an intervention to introduce a period and a chance for workers to inspect their skill level more seriously on the job.[29]

I simulate the occupational decisions and wage dynamics of the workers assigned to Sales/Admin Support whose probabilities of two types of mismatch are higher. In the counterfactual scenario, both types of mismatch do not happen. Hence, it can be expected that, with workers being better sorted in the early tenure years, wage dynamics would be different. In particular, in Figure 6, which illustrates the counterfactual labor outcomes, it is found that better sorting yields a higher mean and lower dispersion of log wages.

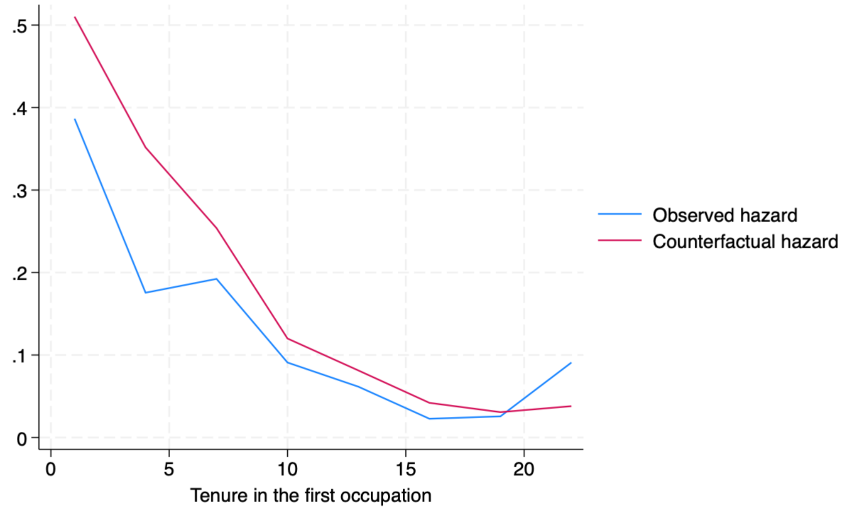
[27]The probabilities consider the fact that the current decision could be driven by the realization of the utility shocks, so the mistakes are ex-ante not deterministic. Also, note that the incentive to stay working while anticipating a potential large skill gain through the skill accumulation shock in the future is incorporated into the continuation value of staying.

[28]Both types of mismatch also have implications for the firm-side which is not accounted for in the structural model. For example, a high type I mismatch results in a higher probability of losing highly productive workers. while high type II mismatch leads to longer duration to stay working for low productivity workers, which can potentially incur (opportunity) costs to the firms.

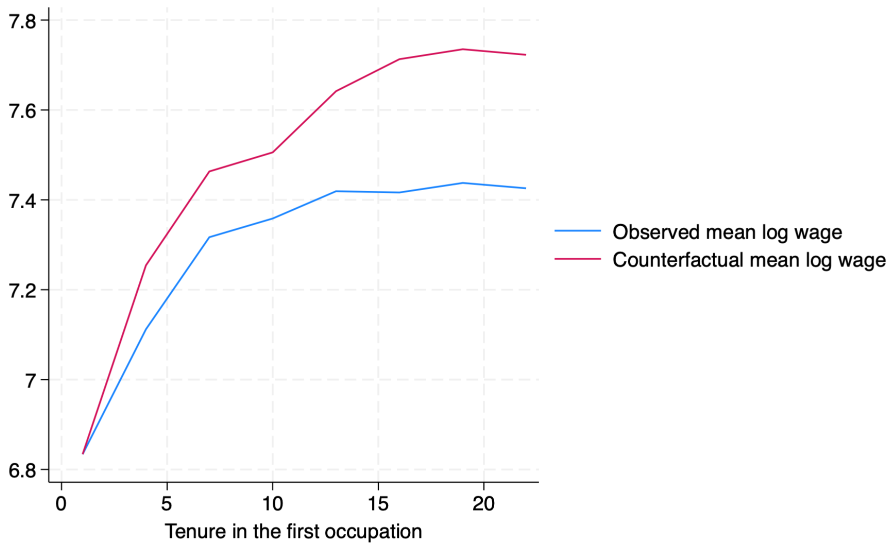
[29]This policy can cost additional effort to the workers and/or resources to the firms. For simplicity, I abstract from the additional changes, potentially induced by the policy, that alter the economic agents' behavior outside of the structural model.

Figure 6: Counterfactual labor market outcomes: Sales/Admin Support occupation

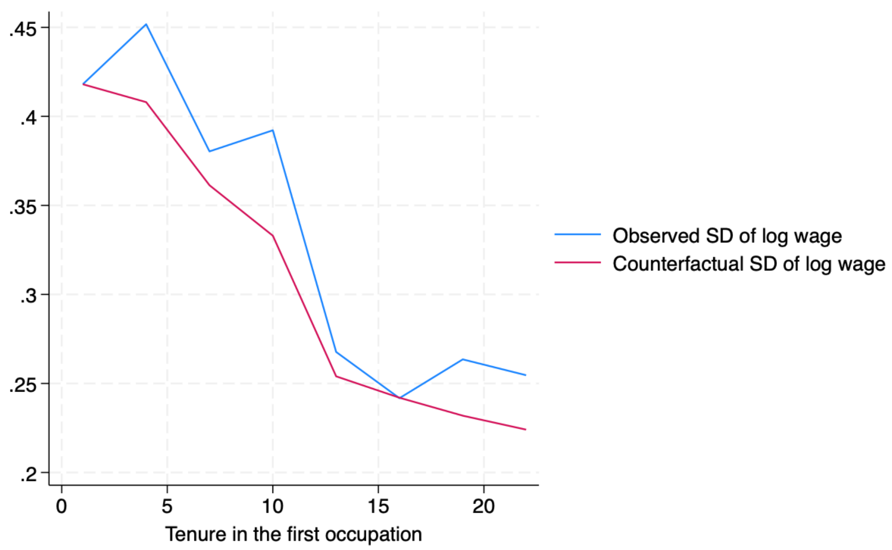
(a) Counterfactual hazard rate



(b) Counterfactual mean of log wage



(c) Counterfactual standard deviation of log wages



6 Conclusion

In this paper, I study the occupational decision and wage dynamics based on the human capital framework. I first present empirical motivation that different occupations show different patterns of wage dynamics. Along with the differences across the occupations, it is shown that the decision to stay in or quit their first occupations is associated with new information obtained from wages realized in the previous period. Given the hypothesis that workers' learning about their own skill could explain their response to the new information while influencing the wage dynamics, I built a structural model. The model incorporating the workers' learning in a Bayesian fashion in the labor market is estimated using the Kalman filter and the CCP estimation method. The estimates can mimic the data patterns fairly well. The estimation results are in line with the recent research suggesting the importance of workers' learning about their own abilities. First of all, I provide supporting evidence that the workers in their first occupation actually experience uncertainty about their own abilities. It is found to have significant implications on occupational decisions and workers' sorting, which, in turn, has an impact on wage dynamics. More specifically, I show that a non-trivial share of workers can make choices that could lead to a type of mismatch because of their beliefs, which are not made if the actual skill level is informed to the workers. The counterfactual policy of interest, information provision about workers' skills from the start, is found to have impacts on the workers by yielding their shorter duration in the initial occupation while increasing the mean and lowering the dispersion of the log wage distribution among the workers staying in the occupation. of the workers in the first occupations.

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Appendix

A Data Construction

To construct annual panel data for our main analysis, we use NLSY79's Work History Data File, which records individuals' employment histories up to five jobs on a weekly basis from 1978 to 2010. Following Pavan (2011), total hours worked for each job within a year is calculated, based on which I define primary jobs for each year as the one for which an individual spent the most hours worked within the year. We construct panel data with annual frequency (from 1978 to 2010) from a series of observations of primary jobs.

Measurement error in occupational switching has received particular attention (e.g., Visschers and Carrillo-Tudela 2023). To avoid miscoded occupational switches, I address it by dropping transitions that immediately revert (which often indicates incorrect coding in the middle year). Arguably, conditioning occupation switches on simultaneous employer switches would reduce the measurement error, but I keep the possibility to change one's occupation under the same employer open while categorize the occupations broadly so that a minor measurement error could be address in this way.

Worker's wages are measured by the usual rate of pay for the primary job at the time of interview. These wages include tips, overtime and bonuses before deductions and are converted to an hourly rate using usual hours worked when not reported as such. All the wages are deflated by the price index for personal consumption expenditures into real term in the 2000 dollars. We drop the observations if their wages are missing.

I limit my sample to the individuals who make their initial long-term transition from school to labor markets during the survey period. In practice, those who work more than 1,200 hours in the initial year of the survey is dropped, and the year when they work more than 1,200 hours least for two following years during the survey period is set as his initial period considered in the sample. Those who are in the military service more than two years during the period are also eliminated from our sample. For the individuals who go back to school from the labor force during the survey period, I assume they start their career from the point they reenter labor markets, and drop the observations before that time. Being different from the wage growth literature excluding workers who are weakly attached to the labor force, I include them in the analysis. Finally, those with a valid occupation code and wage, demographics and ASVAB scores information above age 16 are kept.

B Test for Selection: Variable Addition

I discuss a simple way to test the presence of sample selection, attrition in particular, which motivates a model to control for and delve into the selection mechanism. Consider regressing the log wage received by worker i assigned to his initial occupation j having worked t years on observed characteristics. For simplicity, following the Mincer wage equation, consider the usual case where years of schooling, experience, and other observed ability measurements are in X_{it} as proxies for human capital, but time-invariant unobserved heterogeneity not captured by the observables is in the error term:

$$log(W_{ijt}) = X_{it}\gamma_j + \{A_{ij} + \varepsilon^p_{ijt}\} \tag{15}$$

Selection or attrition bias could occur because, with $d_i = (d_1, d_2, \ldots, d_{T_i}, d_{T_i+1})$ denoting the observability of the wages, the conditional expectation of the error term could be not equal zero: $E[A_{ij} + \varepsilon^p_{ijt}|d_i]$. If this conditional expectation is known, one could add it as an extra regressor so that the parameters in the extended model can be estimated consistently. This is the essence of the well-known two-step estimation procedure in the cross-sectional sample selection model proposed by Heckman (1976, 1979). However, the conditional expectation is not known before the selection process is known or jointly modeled. Building on Nijman and Verbeek (1992) who propose a simple test for selective nonresponse in panel data, the conditional expectation is approximated in a simple way: $E[A_{ij} + \varepsilon^p_{ijt}|d_i] = \kappa_{jt} d_{it+1}$.

The statistical result that says d_{it+1} significantly affects the log wage (i.e., κ_{jt} is significantly different from zero) rejects the hypothesis of no selection of sample. To make the test simple, I further assume $\kappa_{jt} = \kappa$. I start with regressing the log wage on observed individual characteristics such as race, education, the Armed Services Vocational Aptitude Battery (ASVAB) scores before labor market entry, and years of experience. The estimation results based on different specifications in this paper say d_{it+1} has an effect, which leads to the conclusion that the issues with selective attrition and the resulting non-random sample could underlie the wage process through the workers' decisions to stay in or quit their initial occupations.[30]

C Linear Wage Contract and Identification Issues

Suppose a linear wage contract $W_{it} = a_{it} + b_t Y_{it}$, where Y_{it} is the produced output. Related identification issues are discussed in the following. Equipped with competitive market assumption ensuring zero ex-ante expected profit, fixed pay a_{it} can be expressed by a function of the expected output: $a_{it} = (1 - b_t)E[Y_{it}|\Omega_{it}]$, where $\Omega_{it} = \{\mu^*_{it}, \sigma^{2*}_{it}, X_{it}, \eta_i\}$ is information set at the start of period t. This yields the following wage equation:

$$\begin{aligned} W_{it} &= (1 - b_t)E[Y_{it}|\Omega_{it}] + b_t Y_{it} \\ &= (1 - b_t)\{\mu^*_{it} + \frac{\sigma^{2*}_{it}}{2} + \frac{\sigma^2_p}{2}\} + b_t \exp(h_{it} + \varepsilon^p_{it}) \end{aligned} \tag{16}$$

Piece rate Note that the (raw) wages based on the linear contract are composed of two components: fixed pay $(1 - b_t)E[Y_{it}|\Omega_{it}]$ and variable pay $b_t Y_{it}$. When one of the two components is observed along with wages W_{it}, the piece rate can be directly identified. To see this, I use the following relationship $E[W_{it}(\Omega_{it}, \varepsilon^p_{it})] = (1 - b_t)E[E[Y_{it}|\Omega_{it}]] + bE[Y_{it}]$, which yields $E[W_{it}] = E[Y_{it}]$. Hence, by denoting the observed fixed pay and variable pay by W^F and W^V, respectively, the following ratios identify the piece rate:

$$\frac{E[W^V]}{E[W_{it}]} = 1 - b_t, \quad \frac{E[W^F]}{E[W_{it}]} = b_t \tag{17}$$

Given the absence of information about the share of fixed and variable pays, without an exogenous variation that enables the piece rate to be identified, a parametric specification needs to be imposed not to invoke an identification problem; piece rates b_t are fixed to a certain value.

[30]The result is robust with respect to the specification including either fixed or random effects with the same set of explanatory variables.

D Simulation Practice: Attrition Bias

Individual heterogeneity and endogenous quitting decisions can posit a serious econometric challenge. In the absence of learning and stochastic skill accumulation, a popular approach to address the endogeneity through the time-invariant unobserved heterogeneity would be to introduce individual fixed effects and estimate the observational equation on the subsample of workers whose wage outcomes are observed repeatedly.

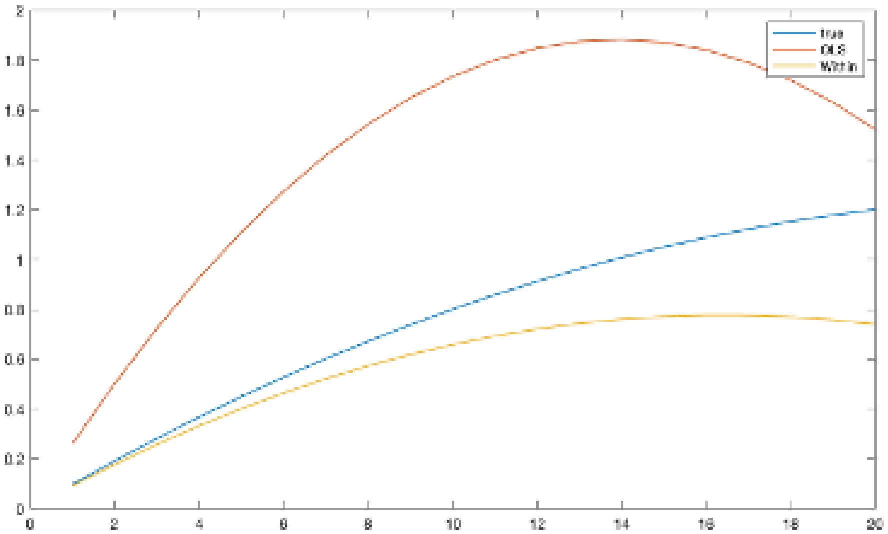
However, an issue of sample selection can lead the estimated coefficients of the time-varying variables using the fixed effects approach (i.e., within or first-difference estimators) to be biased. An example would be the presence of learning as in the model in this paper. In a learning environment, Gibbons et al. (2005) proposed instruments, in conjunction with the fixed effect approach, that exploit the key property of Bayesian learning models in which beliefs are a martingale.[31] The instruments, wages realized two or more periods ago, are based on the idea that new information obtained in the current period is orthogonal to the prior belief.

In learning environments where the choice in the current period (e.g., stay/quit, different occupations) contains information about the previously realized idiosyncratic components in the outcome equation, the instrumental variable (IV) estimator in Gibbons et al. (2005) would fail to correct the selection bias. This is regardless of the specification determining the wages; in two extreme cases of the linear contract, wages fully dependent either on the realized or expected output production, the instruments can not control for the sample selection appropriately.

A simulation practice below supposes a spell on a job and illustrates the biased estimates of the returns to tenure/seniority in the fixed effects approach in the uncertainty and learning environment. The direction of the sample selection bias is hard to predict in advance, though; in line with job search and matching theories of job mobility (e.g., Burdett 1978; Jovanovic 1979), it is plausible that workers perceiving themselves to be productive in the current job are likely to have longer tenure, predicting a positive bias. However, conversely, workers perceiving productivity may quit their current jobs at low tenures, which contributes to a negative bias in the tenure effect. It depends on the set of

[31]The wages in their paper are assumed to be dependent only on the expected productivity (i.e., a linear contract with the piece rate being equal to zero).

parameter values, but in a reasonable scenario yielding relatively short spells in the initial occupations, a comparison of wages for workers with different job tenures in a learning environment will understate the returns to tenure.



E CCP Estimator Employing Type I Extreme Value Shocks

Assuming the i.i.d. Type I Extreme Value distribution (TIEV) of the utility shocks in the structural model provides the following convenient closed-form. To see this, let's assume that $Y_1, \cdots, Y_J$ are independent, non-identically distributed extreme value random variables with their location parameters $\mu_1, \cdots, \mu_J$ and common scale parameter σ. The distribution of Y_j is given by $P(Y_j \leq y_j) = \exp[-\exp\{-(y_j - \mu_j)/\sigma\}]$. Then, its mean and variance are $E[Y_j] = \mu_j + \sigma\gamma$ and $(\pi\sigma)/\sqrt{6}$, respectively, where γ is the Euler's constant. This family is max-stable:

$$\begin{aligned} P(\max\{Y_1, \cdots, Y_J\} \leq c) &= \prod_j \exp\{-\exp(-\frac{c-\mu_j}{\sigma})\} \\ &= \exp\Big[-\exp\Big\{-\frac{c - \sigma \log \sum_j \exp(\mu_j/\sigma)}{\sigma}\Big\}\Big] \end{aligned} \tag{18}$$

From the expression, it can be found that $\max\{Y_1, \cdots, Y_J\}$ follows the extreme value distribution with location parameter $\sigma \log \sum_j \exp(\mu_j/\sigma)$ and scale parameter σ.

The structural model in Section 3 assumes that utility shocks are distributed i.i.d. Type 1 Extreme Value with scale parameter σ_u and location parameter $-\sigma_u\gamma$. Then, $\{v_j + \varepsilon_j^u)\}_{j=1,2,...,J}$ are extreme value random variables, with which the max-stable property yields that $\max_j\{v_j + \varepsilon_j^u\}$ also follows a TIEV distribution with the location parameter $\sigma_u \log \sum_j \exp\{(v_j - \sigma_u\gamma)/\sigma_u\}$ and scale parameter σ_u. It provides that, for an arbitrary alternative $q \in \{1, 2, ..., J\}$,

$$
\begin{aligned}
E_{\varepsilon^u}[\max_j\{v_j + \varepsilon_j^u\}] &= \sigma_u \log \Big[\sum_j \exp(\frac{v_j - \sigma_u\gamma}{\sigma_u})\Big] + \sigma_u\gamma \\
&= \sigma_u \log \Big[\frac{\sum_j \exp(v_j/\sigma_u)}{\exp(v_q/\sigma_u)} \exp(v_q/\sigma_u)\Big] \\
&= -\sigma_u \log P_q + v_q \qquad (19)
\end{aligned}
$$

Hence, with this particular distribution, the ex-ante value function can be rewritten with respect to the conditional value function associated with an arbitrarily selected choice q and the corresponding adjustment term which is a function of the CCP.