Journal of King Saud University – Computer and Information Sciences 34 (2022) 7735–7750

Contents lists available at ScienceDirect

Journal of King Saud University – Computer and Information Sciences

journal homepage: www.sciencedirect.com

A hybrid two-stage robustness approach to portfolio construction under uncertainty

Ebenezer Fiifi Emire Atta Mills [a,*], Siegfried Kafui Anyomi [b]

[a] School of Mathematical Sciences, Wenzhou-Kean University, Wenzhou, China
[b] Tippie College, Department of Finance & the Vaughan Institute of Risk Management & Insurance, University of Iowa, USA

ARTICLE INFO

Article history:
Received 4 April 2022
Revised 3 June 2022
Accepted 23 June 2022
Available online 9 July 2022

Keywords:
Portfolio selection
Data envelopment analysis
Portfolio optimization
Entropic Value-at-Risk
Hybrid model

ABSTRACT

This paper proposes a hybrid two-stage robustness approach to portfolio construction under data uncertainty. In the first stage, a stock's efficiency performance from candidate stocks is assessed and selected using an integrated dynamic slack-based measure data envelopment analysis model. We discuss the stability of efficiency estimates using the leave-one-out method. In the second stage, a "robust" stable and scaled mean-variance-Entropic Value-at-Risk model is used to determine the optimal weights allocated to qualified stocks in the presence of proportional transaction costs. The proposed method reduces computational complexity, increases robustness, and provides a comprehensive evaluation of stocks under different financial decisions, thereby increasing conservatism in the investment process. We demonstrate the applicability of the proposed hybrid two-stage approach to stock data from the Shenzhen and Shanghai Stock Exchanges. Results show that with increasing required returns, the proposed method improves the capital amount for investment and lowers transaction costs at the expense of additional risk. The study concludes by comparing the computational performance of the proposed approach to that of existing methods.

1. Introduction

One of the most influential approaches to modern portfolio theory was proposed by Markowitz (1952). The model suggests that investors should use the related means and variances of the rate of return to determine potential portfolio allocations. Though inspiring many variants and extensions to come into fruition, Markowitz's modeling approach is often criticized due to the lack of consideration of transaction costs in the wealth allocation process. Borkovec et al.'s (2010) findings show that unexpected returns to transaction costs contribute to about 40% of losses in the financial market.

However, the challenge remains that selecting an investable portfolio of assets and allocating funds from the portfolios chosen by a rational investor is more complex than using the rate of return's mean and variance. Thakur et al. (2018), Atta Mills et al. (2020), and Peykani et al. (2020) argue for the applicability of a multi-criteria decision-making (MCDM) approach than just two-way criteria in selecting a portfolio of stocks. Key attributes in selecting a stock portfolio include but are not limited to the rate of return, company size, systematic risk, earnings per share growth rate, market trends, and quick ratio.

One of the most powerful MCDM methodologies for achieving this aim is data envelopment analysis (DEA). When various inputs and outputs are considered, DEA measures the relative efficiency of decision-making units (DMUs), i.e., listed companies. Thus, DEA can be used in portfolio construction to identify good stocks and filter out undesirable ones by measuring stock efficiency. After eliminating the undesirable stocks and identifying the most desired stocks, another stage is required to re-evaluate the eligible stocks for fund allocation.

Uncertainty in the data of the criteria parameters for portfolio construction should also be addressed (Bhattacharyya et al., 2014). Even with the use of just two parameters, mean and variance, there is still the problem of parameter uncertainty and estimation risk, as determined by Kao and Steuer (2016). Therefore, it might lead to significant structural changes in the presence of

* Corresponding author.
E-mail addresses: eattamil@kean.edu (E.F.E. Atta Mills), siegfried-anyomi@uiowa.edu (S.K. Anyomi).

Peer review under responsibility of King Saud University.

Production and hosting by Elsevier

https://doi.org/10.1016/j.jksuci.2022.06.016
1319-1578/

perturbed input parameters as shown in Filomena and Lejeune (2012) and Atta Mills et al. (2016). In the worst-case scenario, infeasible solutions will be presented. In using the DEA approach, the underlying assumption is that parameters are certain. It is essential to develop a technique and models for evaluating stocks and, as a result, decision-making about portfolio weights capable of being used under uncertainty, especially when efficiencies of units are close. Therefore, robust techniques should be considered to deal with data uncertainty.

Peykani et al. (2020) proposed a two-phase robust portfolio construction approach under uncertainty. The first stage entails an attempt to measure the efficiency of all candidate stocks using a so-called robust data envelopment analysis (RDEA) method. The second stage also attempts to allocate funds in each qualified stock using robust mean-semi variance-liquidity (RMSVL) and robust mean-absolute deviation-liquidity (RMADL) models. A robust optimization approach was used in all stages to reduce the unfavorable effect of estimation risk. However, the RDEA approach is static, and the RMSVL and RMADL models do not consider transaction costs.

This paper proposes a hybrid two-stage robustness approach to portfolio construction under uncertainty. The portfolio optimization approach, similar to DeMiguel et al. (2009), is studied to exploit the norm ball constraint model as a unifying framework to address problems associated with extreme portfolio weights and as under-diversification of a portfolio. Specifically, this paper studies the portfolio selection and fund allocation problems with proportional transaction costs by proposing a novel two-stage approach under uncertainty.

The first stage entails portfolio selection. An integrated dynamic slack-based measure data envelopment analysis (dynamic SBM-DEA) is used select qualified stocks based on stock efficiency performance from a menu of stocks. Under robustness testing, the dynamic SBM-DEA model employs a leave-one-out method to assess the stability of the efficiency estimates. In the second stage, entailing portfolio optimization, we mirror the stable and scaled mean–variance-Entropic Value-at-Risk model by Atta Mills et al. (2017) to define a modified scaled mean–variance-Entropic Value-at-Risk model for allocating funds in the presence of transaction costs. Using two risk measures reflects both symmetric and asymmetric risks, balancing the requirements of a fund manager and a regulator.

The practical points in this research revolve around two essential investment decisions an investor or a fund manager can pursue in the presence of proportional transaction costs. First, the significance of incorporating proportional transaction costs in constructing a new portfolio or revising an existing one is recognized. Proportional transaction costs, according to Muthuraman and Kumar (2006) and Zhu (2017), are induced by taxes on capital gains, liquidity costs, and brokerage fees (Wang et al., 2017). Thus, the proposed "robust" scaled mean–variance-Entropic Value-at-Risk model implicitly assumes that transaction costs will be paid at the start of the planning phase. Second, net capital gains or losses from the sale of stocks can be carried over from period to period and dynamically aid in the selecting stocks or the re-selection of stocks from candidate options.

The proposed two-stage hybrid approach for selecting and optimizing a portfolio is effectuated on the Shenzhen and Shanghai Stock Exchanges. The annualized performance of the proposed optimization strategy is compared to others in the literature using several performance metrics. These metrics include Sharpe and Sortino ratios. We apply the difference test of Ledoit and Wolf (2008) to ensure statistical stability and make inferences on whether the Sharpe and Sortino ratios of a portfolio are significantly different from that of a benchmark. This approach is a sophisticated and computationally intensive bootstrap test that does not necessitate strong distributional assumptions. It is a robust test that considers returns' non-normality and time-series characteristics.

This study provides at least three contributions. First, by establishing a framework for measuring dynamic stock efficiency, this research equips listed companies with a comprehensive outlook of their relative position on stock performance, leading to competitive management decisions. Second, the efficiency framework allows investors to assess companies before investing. Third, this study adds to the literature by presenting a stable and scaled portfolio under uncertain conditions with increased conservatism while meeting regulators' and fund managers' requirements.

There are several benefits to the proposed two-stage approach. To begin, the approach can be used in the face of data uncertainty. Second, the first stage reduces the computational complexity of portfolio optimization to satisfy the squared-Euclidean norm ball constraint. Third, by utilizing a two-stage approach, the level of conservatism in the investment process is increased. Lastly, in constructing a strategy appealing to non-conventional finance enthusiasts, all candidate investment choices are thoroughly evaluated under various financial criteria and aspects using multi-criteria decision-making approaches.

The rest of the research is presented in this manner. Section 2 presents the background and framework of portfolio optimization models and data envelopment analysis. Section 3 presents the methods used. Section 4 provides an illustrative application. Section 5 highlights the proposed approach's computational performance compared to existing portfolio optimization approaches. The last section provides the concluding remarks.

2. Background and Framework

2.1. Classic portfolio optimization models

Markowitz's (1952) mean-variance (MV) model has been discussed in diverse directions. The MV approach generates an optimization problem of the form:

$$\begin{array}{ll} \min & x^T Q x \\ \text{subject to} & \mu^T x \geqslant R, \\ & e^T x = 1, \end{array} \tag{1}$$

where x is the weight or allocation assigned to risky assets, μ is the expected return of assets, Q is the covariance matrix of portfolio return, R is a required return level, T is a transpose indicator, and e is a vector of ones. The second constraint is the budget constraint.

The presumption that the dynamics of financial securities can be correctly modeled with historical data underpins Markowitz's MV model. Some proponents of portfolio theory have shown the high sensitivity of MV estimates (Klein and Bawa, 1976). Jobson and Korkie (1980) discuss some fundamental sensitivity issues and propose using of shrinkage estimators. Additionally, random fuzzy models provide an alternative strategy for dealing with return uncertainty. Vercher et al. (2007) use fuzzy numbers to compensate for the unpredictable returns on financial assets. Filomena and Lejeune (2012) employed probabilistic distributions to address estimation risk. This study tackles estimation risk through the modification of portfolio weights by adding regularizers as a penalty term to the portfolio optimization problem (Atta Mills et al., 2016). This approach is prudent because, first, estimation risk is bounded by a function of the portfolio weights' norm, and hence constraining portfolio weights norms is identical to constraining estimation risks (Fan et al., 2012).

The MV model and its related risk measure have also been analyzed in a number of studies. The mean absolute deviation (MAD) was introduced by Konno et al. (1993) as an option for using squared deviations in the mean-variance model. Some extensions have sought to minimize the portfolio's semi-variance (Huang, 2008; Zhang et al., 2021). Other measures of risk are entropy

(Mercurio et al., 2020), lower partial moments (Nesaz et al., 2020), Gini mean difference (Sehgal and Mehra, 2021), Value-at-Risk (Mohammadi and Nazemi, 2020), and the Conditional Value-at-Risk (CVaR) (Khanjani Shiraz et al., 2020), just to mention a few. Some authors have proposed a combination of risk metrics to obtain better results (Roman et al., 2007; Atta Mills et al., 2016) which seems to provide answers to the discrepancies underlying the use of the MV and mean-CVaR models. The mean-variance-CVaR approach discussed by Roman et al. (2007) is presented as:

$$\begin{array}{ll} \min & x^T Q x \\ \text{subject to} & \mathrm{CVaR}_{(1-\epsilon)}(x) \leqslant b, \\ & \mu^T x \geqslant R, \\ & e^T x = 1, \end{array} \tag{2}$$

where ϵ is the confidence level, b and R are required risk and required returns, respectively. Although CVaR has been proven to be a coherent risk measure (Artzner et al., 1999), it tends to depend only on the tail of the distribution. It has also been shown not to be smooth (Cherny and Madan, 2006). In other interesting contribution, Ahmadi-Javid (2012) demonstrated that when Entropic-Value-at-Risk (EVaR) is considered, stochastic optimization problems that are computationally intractable with CVaR can be deduced. Akin to Ahmadi-Javid (2012), the portfolio optimization approach based on EVaR was analyzed by Zheng and Chen (2014), and the findings were compared to other downside risk measures such as VaR and CVaR. Their results indicate that EVaR has the highest risk resolution. They address the need to consider variance in addition to EVaR since both measures help manage symmetric and asymmetric risk on portfolio returns. Therefore, this paper investigates EVaR as part of a consolidated risk measure with variance in a portfolio optimization framework to better monitor the portfolio return's downside risk.

Practitioners in modern portfolio theory believe that using the mean-variance as a portfolio optimization technique is just a starting point. As a result, incorporating a portfolio structure with various constraints considering firm characteristics and specific investment regulations makes sense. A practical portfolio optimization strategy should consider real-world constraints like minimum transaction lots, transaction costs, cardinality constraints, and investment thresholds. Kellerer et al. (2000) explored the significance of incorporating certain realistic features that may exist, in fact, in a portfolio optimization model. It has been shown that using transaction costs reduces the number of assets chosen. Transaction lots significantly alter the composition of the resulting portfolio when selected assets and capital investments are taken into account. Pogue (1970) investigates the significance of including transaction costs in the MV context, in the spirit of how costs could affect portfolios. Other variants were examined in Lobo et al. (2007) and Mitchell and Braun (2013).

When securities are transacted and executed, transaction fees such as bid-ask spreads and broker fees occur. The total transaction costs for each execution are calculated as follows:

$$c(x) = \sum_{k=1}^{N} c_k(x_k), \tag{3}$$

where c_k is the transaction cost function for asset k. Making the assumption that there exists a perfect market, that is, $c(x) = 0$, is a special case for a transaction cost model.

For a transaction cost model, Oksendal and Sulem (2002) considers a scenario where a fixed cost per transaction is imposed. Alternatively, transaction costs may be described as a proportional cost. We pay transaction costs $c(x - x^0)$ to obtain portfolio x from an initial portfolio x^0, where c is a convex non-negative function with $c(0) = 0$. A cost function can be used to describe this as:

$$c(x - x^0) = \sum_{k=1}^{N} c_k(x_k - x_k^0), \text{where } c_k(x_k - x_k^0) = \begin{cases} c_k^b(x_k - x_k^0), & \text{if } x_k > x_k^0 \\ c_k^s(x_k^0 - x_k), & \text{otherwise.} \end{cases} \tag{4}$$

As described earlier in the introduction, this paper assumes proportional transaction costs. To authors' understanding, transaction cost functions have no special significance over one another. The authors conclude that the choice of which transaction model to use is up to the researcher.

2.2. Coherent risk measure

CVaR has a drawback in that it only depends on the distribution's tail. That is, it is a 0–1 risk measure, as shown by Cherny and Madan (2006), and therefore not smooth. In terms of computational efficiency, Ahmadi-Javid (2012) also shows that CVaR is hard to control in stochastic optimization problems. EVaR, on the one hand, has been shown by Axelrod et al. (2016) to be computationally efficient and a coherent risk measure for quantifying risks. Please see Rockafellar and Uryasev (2000) and Ahmadi-Javid (2012) for a detailed and comprehensive discussion on CVaR and EVaR, respectively.

Because variance represents the difference from the expected return, incorporating EVaR into the mean-variance framework aids in controlling the downside risk of portfolio returns. This technique satisfies the regulator's criteria for small downside risk and the fund manager's need for a small variance.

Definition 1 Ahmadi-Javid, 2012. For a given level of confidence $\epsilon \in (0, 1]$, the EVaR of a random variable X whose distribution $\mathbb{P}$ belongs to set $\mathbb{Q}$ of distributional ambiguity is defined as:

$$\mathrm{EVaR}_{(1-\epsilon)}(X) := \inf_{z>0} \left\{ \frac{1}{z} \left[ln\mathbb{E}_{\mathbb{P}}[e^{(zX)}] - ln\epsilon \right] \right\}. \tag{5}$$

The EVaR approach to quantifying risks has been shown by Ahmadi-Javid (2012) to possess all the desirable properties of a coherent risk measure as enumerated above. In particular, Ahmadi-Javid (2012) proves that the entropic nature of EVaR under a known distribution corresponds to the tightest possible upper bound obtained from the Chernoff inequality for both VaR and CVaR.

This paper assumes that X follows a Gaussian distribution with mean μ and standard deviation σ. As a result, Eqn. (5) can be expressed as:

$$\begin{aligned} \mathrm{EVaR}_{1-\epsilon}(X) &= \inf_{z>0} \left\{ \tfrac{1}{z} \left[(ln\mathbb{E}_{X\sim\mathcal{N}(\mu,\sigma^2)}[e^{(zX)}] - ln\epsilon \right] \right\} \\ &= \inf_{z>0} \left\{ \mu + \tfrac{\sigma^2}{2z} - \tfrac{1}{z} ln\epsilon \right\} \\ &= \mu + \sqrt{2ln\tfrac{1}{\epsilon}}\sigma. \end{aligned} \tag{6}$$

2.3. Data Envelopment Analysis

Charnes et al. (1978) pioneered the use of data envelopment analysis (DEA) by building on Farrell (1957). DEA is a non-parametric instrument for evaluating and ranking the performance of homogeneous decision-making units (DMU). Cooper et al. (2004) asserted that a DMU is 100% efficient when the performances of other DMUs indicate that neither inputs nor outputs can be improved without sacrificing other inputs or outputs. Certain modifications are possible to the basic Charnes, Cooper, and Rhodes (CCR) (Charnes et al., 1978) and Banker, Charnes, and Cooper (BCC) models (Banker et al., 1984). One of them, dubbed the slack-based measure (SBM) model, was developed by Tone

(2001). Here, additional variables, s^{+} and s^{-}, known as slacks, are used to measure efficiency.

Consider n DMUs: listed companies ($j = 1, \ldots, n$). Each listed company has m inputs ($i = 1, \ldots, m$) and s outputs ($i = 1, \ldots, s$). Let $X = [x_{ij}, i = 1, \ldots, m, j = 1, \ldots, n]$ be an input matrix and $Y = [y_{ij}, i = 1, \ldots, s, j = 1, \ldots, n]$ be output matrix. The q-th line – i.e. X_q and Y_q of these matrices represent quantified inputs and outputs of unit DMU$_q$. The essence of DEA models in measuring the efficiency of unit DMU$_q$ lies in maximizing its efficiency rate through minimizing inputs (input-oriented) or/and maximizing outputs (output-oriented).

The optimization system provided constant returns to scale is:

$$
\begin{aligned}
\min \quad & \frac{1-\frac{1}{m}\sum_{i=1}^{m}\frac{s_i^-}{x_{iq}}}{1+\frac{1}{s}\sum_{i=1}^{s}\frac{s_i^+}{y_{iq}}} \\
\text{subject to} \quad & x_{iq} = \sum_{j=1}^{n} x_{ij}\lambda_j + s_i^- \qquad (1,\ldots,m) \\
& y_{iq} = \sum_{j=1}^{n} x_{ij}\lambda_j - s_i^+ \qquad (1,\ldots,s) \\
& \lambda_j, s_i^-, s_i^+ \geqslant 0.
\end{aligned}
\tag{7}
$$

In the model above, the objective function's numerator and denominator indicate the mean distance between the inputs and outputs from the efficiency threshold. For variable returns to scale (VRS) assumption, the constraint $\sum_{j=1}^{n}\lambda_j$ is added to the system. A DMU unit that is SBM efficient is also CCR efficient.

This SBM model serves as a basis for this study's proposed dynamic SBM-DEA model. Dynamic models' structures' primary distinction and uniqueness is the consideration of transition or carry-over indicators between consecutive periods, demonstrating the periods' interdependence. Failure to account for the dynamic nature of activities leads to an overestimation of efficiency (Tone and Tsutsui, 2010). Also, in the context of data availability, a dynamic model is preferable to a static one.

2.4. Robust optimization

In general, optimization system parameters are clouded by real-world uncertaimty (Peykani et al., 2019; Atta Mills et al., 2019). If accurate historical data are unavailable, there is an increasing imprecision in the input parameters. It is vital to maintain the robustness of the solution obtained from portfolio selection and optimization models in this condition; otherwise, the efficiency and ranking of the DMUs may be unreliable, portions allocated for investments may also become unreliable, imposing significant costs on various stakeholders. To avoid such an unfavorable outcome, stochastic, dynamic, and robust optimization techniques can be used (Atta Mills et al., 2019).

Another conventional technique is sensitivity analysis (Saltelli et al., 2019), which addresses uncertainty after finding an optimal solution. It is a technique incorporated into the DEA framework, as demonstrated by Avkiran (2015). Under robustness testing, this paper follows suit by discussing the stability of efficiency estimates through re-sampling (leave-one-out method). Therefore, the authors use the aforementioned approach as a robustness test to measure and rank efficient stocks under the portfolio selection stage.

Recently, the robust optimization method has become a widely accepted approach to dealing with uncertainty to reduce estimation risk. Another method that is used in this study that provides robustness is the modification of portfolio weights by adding regularizers as a penalty term to the portfolio optimization problem (Atta Mills et al., 2016). This approach to achieving robustness is prudent because, first, estimation risk is bounded by a function of the portfolio weights' norm. Hence, constraining portfolio weight norms is identical to constraining estimation risks (Fan et al., 2012). Estimation errors may also occur due to mathematical techniques used to solve for the portfolio weights. Thus, to achieve stability in the desired shapes and features of the portfolio, a direct modification of the portfolio weights may be necessary. Additionally, the magnitude of portfolio weights serves as an indicator of transaction costs. Atta Mills et al. (2016) find that the portfolios achieved via the norm ball constrained methodology tend to have better out of sample performance than the naive $1/N$ diversification and other portfolio optimization models in literature.

Robust portfolio optimization is achieved when the optimization model is feasible for virtually all possible values of uncertain parameters, and the resulting investment allocations exhibit minimum variation accordingly.

3. The two-stage technique: robustness approach to portfolio construction

This section discusses a robustness approach to the portfolio selection and optimization problem in financial markets. This approach is divided into two stages, each of which is explained thoroughly. The first stage measures the efficiency of all candidate stocks, whereas the second stage allocates the amount of funds in each qualified stock. Fig. 1 presents a summary of the proposed two-stage portfolio construction methodology used in this research.

3.1. Stage I: Portfolio selection

Over five stages, this stage evaluates and analyses the performance of all stocks available to investors. Only stocks that proceed through the investor's filter qualify as candidates for investment in the second stage.

Step 1a. Select a DEA model.
In the first step of this stage, the authors select a DEA model. For this study, an integrated dynamic SBM-DEA model is used to evaluate and measure the performance of stocks. This study pursues dynamic DEA because classical DEA methods neglect the dynamic effect of carry-over indicators between consecutive periods, but these indicators measure the efficiency of decision-making units in each period as well as over the entire period based on the long-term viewpoint. Failure to account for the dynamic nature of activities leads to overestimating efficiency estimates.
Step 1b. Determine a financial criteria for stock performance evaluation.
At this step, financial criteria for stock performance evaluation are chosen from the literature (Peykani et al., 2020; Atta Mills et al., 2020) and experts' opinions based on return, risk, valuation, liquidity, leverage, profitability, growth, and profit carried forward. The input, output, and carry-over indicators used are shown in Table 1.
Step 1c. Check for model validity and relevance of financial criteria.
To further substantiate the validity of the suggested model in Step 1a, this study examines for the minimal number of DMUs and the relevance of input, carry-over, and output indicators selected in Step 1b via robust regression analysis to additionally corroborate its validity.
Step 1d. Investigate a dynamic SBM-DEA model.
In the third step of stage I, the dynamic SBM-DEA model is used to derive stock performance estimates for ranking. This paper

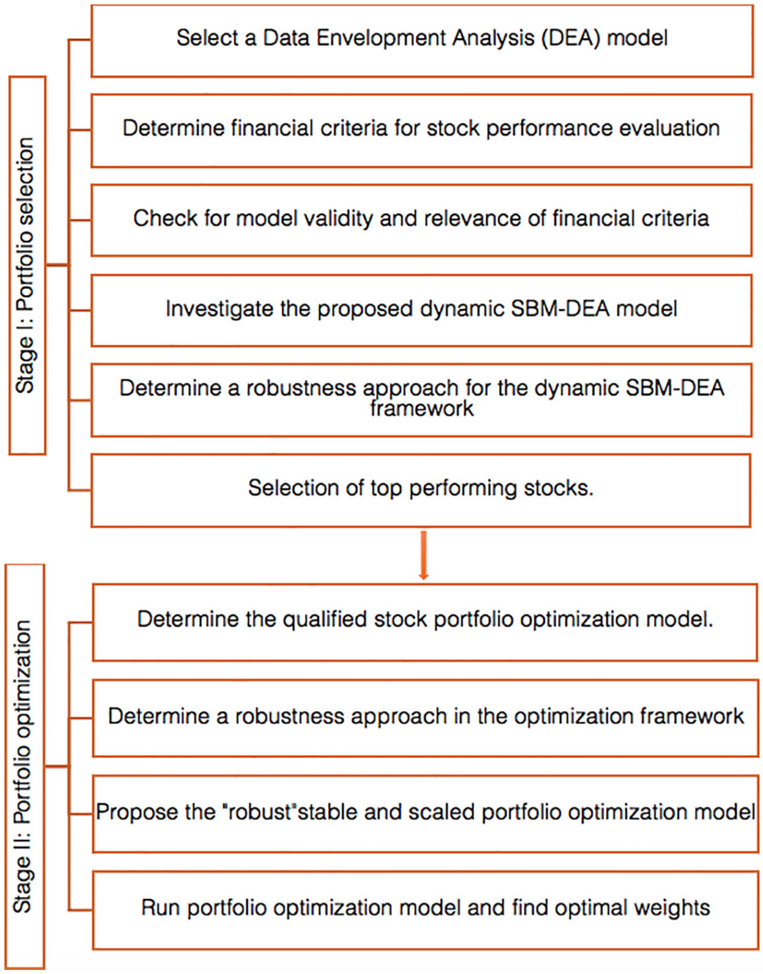

Fig. 1. Schematic summary of the proposed two-stage robustness approach.

Table 1
Indicators used in stock performance measurement.

Indicators	Type	Definition
Price to Earnings (P/E) ratio	Input	Indicates the Renminbi (RMB) amount an investor can expect to invest in a company in order to receive one RMB of that company's earnings.
Quick ratio	Input	Indicator of a company's short-term liquidity position and measures a company's ability to meet its short-term obligations with its most liquid assets.
Solvency ratio	Input	Indicator used to measure a company's ability to meet its long-term debt obligations.
Standard deviation	Input	Statistical measure of market volatility, measuring how widely stock prices are dispersed from the average stock price.
Rate of return	Output	Proportion of gain or loss on an investment over a specified period (continuously compounded).
Earnings per share	Output	Company's profit divided by the outstanding shares of its common stock. Indicator of a company's profitability.
Earnings per share growth rate	Output	It gives a good picture of the rate at which a company has grown its profitability per unit of equity.
Accumulated dividend	Carry-over	Dividend on a share of cumulative preferred stock that has not yet been paid to the shareholder. Accumulated dividends are the result of dividends that are carried forward from previous periods.

considers discretionary inputs, and a carry-over indicator in measuring stock performance. The authors conceptualize the framework of the integrated dynamic SBM-DEA model from a set of stock performance indicators illustrated in Table 1.

Taking into consideration the dynamic structure presented in Figs. 2 and 3, this paper assumes n DMUs: listed companies $(j = 1, \ldots, n)$ over T terms $(t = 1, \ldots, T)$. At period t, each listed company has m inputs $(i = 1, \ldots, m)$ and s outputs $(i = 1, \ldots, s)$ with a carry-over to the following term $t + 1$. Let $x_{ijt}(i = 1, \ldots, m)$ and $y_{ijt}(i = 1, \ldots, s)$ be the input and output indicators of listed company j at period t, respectively. To specify the carry-over by period t, listed company j and item i, the symbolization $z_{ijt}^{free}(i = 1, \ldots, nfree; j = 1, \ldots, n; t = 1, \ldots, T)$ for depicting carry-over values where $nfree$ is the number of free carry-overs.

Each listed company uses m input indicators with inter-period $nfree$ free carry-over to produce s number of outputs for which can be defined as $X = [x_1, \ldots, x_n] \in \mathbb{R}^{m\times n}, Y = [y_1, \ldots, y_n] \in \mathbb{R}^{s\times n}$, and $Z^{free} = [z_1, \ldots, z_n] \in \mathbb{R}^{nfree\times n}$. The production possibility set can be expressed as:

$$G(x) = \{(x, y, z^{free}) | x \geqslant X\lambda, y \geqslant Y\lambda, z^{free} : free, \lambda \geqslant 0\}, \quad (8)$$

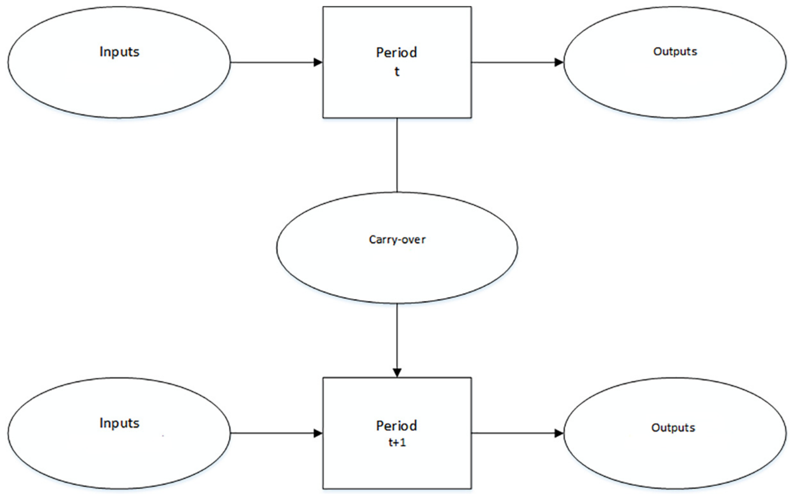

Fig. 2. Dynamic framework of stock performance indicators.

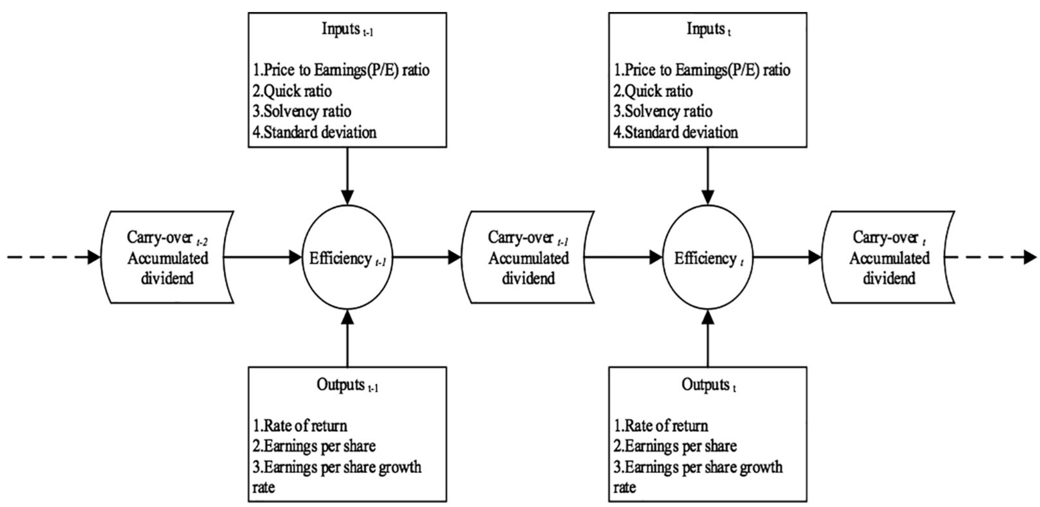

Fig. 3. Dynamic process for stock efficiency measurement.

where λ is the intensity vector of weights for the period t. The continuity of carry-over activities between period t and $t+1$ can be met with the condition:

$$\sum_{j=1}^{n} z_{ijt}^{free} \lambda_j^t = \sum_{j=1}^{n} z_{ijt}^{free} \lambda_i^{t+1} \quad (\forall i; t = 1, \ldots, T-1), \tag{9}$$

The authors can express DMU_q in terms of production possibility as:

$$\begin{aligned}
x_{iqt} &= \sum_{j=1}^{n} x_{ij} \lambda_j^t + s_{it}^- \quad (i = 1, \ldots, m; t = 1, \ldots, T),\\
y_{iqt} &= \sum_{j=1}^{n} y_{ijt} \lambda_j^t - s_{it}^+ \quad (i = 1, \ldots, s; t = 1, \ldots, T),\\
z_{iqt}^{free} &= \sum_{j=1}^{n} z_{ijt}^{free} \lambda_j^t + s_{it}^{free} \quad (i = 1, \ldots, nfree; t = 1, \ldots, T),\\
&\sum_{j=1}^{n} \lambda_j^t = 1 \quad (t = 1, \ldots, T),\\
&s_{it}^- \geqslant 0,\\
&s_{it}^+ \geqslant 0,\\
&s_{it}^{free} : free \quad \forall (i, t),
\end{aligned} \tag{10}$$

where s_{it}^-, s_{it}^+, and s_{it}^{free} are slacks representing input excess, output shortfall, and carry-over deviation respectively.

The dynamic SBM-DEA model is assumed to be non-oriented as employees of listed companies have considerable discretion in handling the levels of inputs and final outputs. As a result, various weights can be assigned to inputs and outputs based on their relative positions (Avkiran, 2015). Likewise, variable returns-to-scale accede to the variety of listed companies' sizes. The non-oriented overall efficiency, according to production theory, is:

$$\theta_o^* = \min \frac{\frac{1}{T}\sum_{t=1}^{T} w^t \left[1 - \frac{1}{m}\left(\sum_{i=1}^{m} \frac{w_i^- s_{it}^-}{x_{iqt}}\right)\right]}{\frac{1}{T}\sum_{t=1}^{T} w^t \left[1 + \frac{1}{s}\left(\sum_{i=1}^{s} \frac{w_i^+ s_{it}^+}{y_{iqt}^+}\right)\right]} \tag{11}$$

$$\text{subject to} \quad \text{Eqn.(9)}, \quad \text{Eqn.(10)},$$

where w^t, w_i^-, and w_i^+ are weights to term t, input i and output i respectively and satisfies the conditions:

$$\sum_{t=1}^{T} w^t = T, \quad \sum_{i=1}^{m} w_i^- = m, \quad \text{and} \quad \sum_{i=1}^{s} w_i^+ = s. \tag{12}$$

The non-oriented period efficiency is calculated using an optimal solution, $\left(\{\lambda_q^{t*}\}, \{s_{qt}^{-*}\}, \{s_{qt}^{+*}\}, \text{ and } \{s_{qt}^{free*}\}\right)$, as follows:

$$\theta_{qt} = \frac{1 - \frac{1}{m}\left(\sum_{i=1}^{m} \frac{w_i^- s_{iqt}^{-*}}{x_{iqt}}\right)}{1 + \frac{1}{s}\left(\sum_{i=1}^{s} \frac{w_i^+ s_{iqt}^{+*}}{y_{iqt}^+}\right)} \qquad (t = 1, \ldots, T). \tag{13}$$

DMU_q is period efficient for non-oriented case if $\theta_{qt} = 1$, i.e. $s_{iqt}^{-*} = 0$, and $s_{iqt}^{+*} = 0(\forall i)$. The overall efficiency of DMU_q holds if $\theta_q^* = 1$, i.e. $s_{iqt}^{-*} = 0$, and $s_{iqt}^{+*} = 0$. DMU_q is overall efficient, if and only if it is period efficient for all periods.

One can solve Eqn. (11) by using the efficient algorithms for solving non-linear fractional programming problems discussed by Dolatnezhadsomarin et al. (2019). However, the authors of this study preferably used MaxDEA software to tackle this problem.

Step 1e. Determine a robustness approach for the dynamic SBM-DEA framework.

In the fourth step of stage I, robustness testing is pursued to deal with uncertainty in the integrated dynamic SBM-DEA model. The illustrative application discusses the stability of estimates through re-sampling (leave-one-out method). The fourth constraint of Eqn. (10) corresponds to the variable returns-to-scale assumption. If this constraint is deleted, then the constant returns-to-scale model is assumed.

Step 1f. Selection of top performing stocks.

In the final step of stage I, top-performing stocks, called herein as qualified stocks, are selected from candidate stocks based on a ranking metric of the stocks deduced from the overall efficiency scores of the dynamic SBM-DEA framework. These top-performing stocks will be subjected to a portfolio optimization model for asset allocation in stage II.

3.2. Stage II: Portfolio Optimization

The amount to invest in each qualified stock is determined in this four-step process, which culminates in the creation of the portfolio. The portfolio optimization model determines the weights of qualified stocks of listed companies (DMUs) from the first stage.

Step 2a. Determine the qualified stock portfolio optimization model.

In the first step of stage II, a portfolio optimization model inspired by Atta Mills et al. (2017) with the consideration of return, risk associated with the deviation around the expected return, downside risk, capital for investment, and transaction costs will be discussed.

Suppose R is the required return level, N the number of stocks and x^0 as the initial stocks before revision: x_k^0 is the proportion of capital initially allocated to asset $k, k = 1, 2, 3, \ldots, N$. Let x, x^b and x^s be N dimensional vectors of controllable variables: x_k is the portfolio invested in stock k after revision, x_k^b are purchases of stock k and x_k^s are sales of stock k. The transaction costs incurred when selling stocks is c^s and that of buying stocks is c^b. The financial portfolio is characterized by a N-dimensional vector of random returns ζ. Let μ_k be the expected value of RMB 1 invested in stock k at the end of the period. The variance is set to be $\sigma^2(\zeta_x) = E[(\zeta - E(\zeta))^2]$. Also, set $(1-\epsilon)$ to be the probability risk level. Historical data of returns for each stock k at T successive time frames is used to estimate the mean return μ and the covariance matrix, $Q : Q = E[(\zeta - \mu)(\zeta - \mu)]^T]$. Thus, the mean–variance-EVaR portfolio optimization model with proportional transaction costs is given as:

$$\begin{array}{ll} \min & x^T Q x + \mathrm{EVaR}_{(1-\epsilon)}(x) \\ \text{subject to} & \mu^T x \geqslant R, \\ & \sum_{k=1}^{N} x_k + c^b \sum_{k=1}^{N} x_k^b + c^s \sum_{k=1}^{N} x_k^s \leqslant 1, \\ & x_k = x_k^0 + x_k^b - x_k^s, \quad \forall k \\ & x_k^b \cdot x_k^s = 0, \\ & x, x^b, x^s \in \mathbb{R}_+^N. \end{array} \tag{14}$$

For non-zero transaction costs, the capital (RMB 1) is not available for investment in its entirety. Costs would be incurred to rebalance the portfolio to the point that the entire capital pool is no longer available for investment. The objective function can therefore be expressed as:

$$\frac{x^T Q x}{(e^T x)^2} + \frac{\mathrm{EVaR}_{(1-\epsilon)}(x)}{e^T x}.$$

The transaction costs incurred is $1 - (e^T x)$, so $e^T x$ is the capital available for investment. The authors opt to scale the standard risk measurement - variance by the square of the actual capital invested and the downside risk - entropic value-at-risk by the actual capital invested. The risk per RMB invested is the scaled objective function. The complementarity constraint, $x_k^b \cdot x_k^s = 0$, and non-negative constraint impede the likelihood of simultaneous sales and purchases. However, this leads to a combinatorial form on the problem making it difficult to solve. Mitchell and Braun (2013) show that it is not necessary to impose complementarity constraints if the portfolio has a risk-less asset. This will result in a tractable convex problem. Therefore, this study considers a risk-less asset as part of the portfolio. Let y be the fraction invested in a risk-less asset after revision, and μ_f be the value of the risk-less asset at the end of the period. The authors constrained the risk-less asset by this constraint $A \leqslant y \leqslant B$, where A and B are the minimum and maximum weight allocated to the risk-less asset respectively. This constraint is imposed due to the tendency of the portfolio system to favor a risk-less investment.

To this end, the scaled mean–variance-EVaR model is:

$$\begin{array}{ll} \min & \frac{x^T Q x}{(e^T x + y)^2} + \frac{\mathrm{EVaR}_{(1-\epsilon)}(x)}{e^T x + y} \\ \text{subject to} & \mu^T x + \mu_f y \geqslant R, \\ & \sum_{k=1}^{N} x_k + c^b \sum_{k=1}^{N} x_k^b + c^s \sum_{k=1}^{N} x_k^s + y \leqslant 1, \\ & x_k = x_k^0 + x_k^b - x_k^s, \quad \forall k \\ & A \leqslant y \leqslant B, \\ & x, y, x^b, x^s \in \mathbb{R}_+^N. \end{array} \tag{15}$$

Analytically, in case there is no transaction costs then $e^T x + y = 1$ and the two risk measurements are regained. Therefore, the choice passes the test necessitated of any theoretical augmentation; regain the erstwhile result.

Step 2b. Determine a robustness approach in the optimization framework.

In the second step of stage II, the modification of portfolio weights by adding regularizers as a penalty term to the portfolio optimization problem (Atta Mills et al., 2016) is pursued to deal with uncertain data and parameters in the scaled mean–variance-EVaR model. The weight constrained portfolio optimization is discussed by defining the general norm as squared l_2-norm ball.

Step 2c. Propose the "robust" stable and scaled portfolio optimization model.

In the third step of stage II, "robust" stable and scaled portfolio optimization model will be proposed. This step is vital. The scaled mean variance-EVaR portfolios are made robust by adding the constraint that the norm of the portfolio weight vector is less than a predetermined threshold, κ. These portfolios can be viewed as the result of shrinking portfolio weights in the scaled mean–variance- EVaR portfolio. Shrinking portfolio weights is prudent, as indicated in the previous section and supported by DeMiguel et al. (2009). We tackle estimation risks using portfolio norms. Atta Mills et al. (2016) used the squared-Euclidean norm ball constraint to alleviate estimation risk and obtained better computational results than other strategies. The general p-norm, $p > 1$, is defined as $\|x\|_p = \sqrt[p]{\left(\sum_{k=1}^{N}|x_k|^p\right)}$.

By specifying the portfolio norm as squared-l_2-norm ball, we investigate weight constrained portfolios.

According to the squared- Euclidean norm-constrained approach, the "robust" stable and scaled mean–variance-EVaR is proposed as:

$$\begin{aligned} \min \quad & \frac{x^TQx}{(e^Tx+y)^2} + \frac{\text{EVaR}_{(1-\epsilon)}(x)}{e^Tx+y} \\ \text{subject to} \quad & \mu^T x + \mu_f y \geqslant R, \\ & \sum_{k=1}^{N} x_k + c^b \sum_{k=1}^{N} x_k^b + c^s \sum_{k=1}^{N} x_k^s + y \leqslant 1, \\ & x_k = x_k^0 + x_k^b - x_k^s, \quad \forall k \\ & \|x - x^0\|_2^2 \leqslant \kappa^2, \\ & A \leqslant y \leqslant B, \\ & x, y, x^b, x^s \in \mathbb{R}_+^N, \end{aligned} \tag{16}$$

where κ is a given threshold called a tuning parameter, with $\|x - x^0\|_2^2 = (x - x^0)^T (x - x^0)$ as the squared l_2-norm of a vector. The tuning parameter is estimated by a method of cross-validation. The authors perform cross-validation for feasible values and chose the value that generated the least cross-validation mean error. This enables the aforementioned model to incorporate robustness in the presence of continuously evolving or misspecified data.

The technique introduced by Charnes and Cooper (1962) in solving fractional optimization problems is used. Denote

$$w := \frac{1}{e^Tx + y}, \quad w \geqslant 1 \tag{17}$$

Therefore,

$$\begin{aligned} & e^T xw + yw = 1, \\ & \hat{y} = yw, \quad \hat{x} = xw, \\ & \hat{x}^b = x^b w, \quad \hat{x}^s = x^s w. \end{aligned} \tag{18}$$

Constraints of Eqn. (16) are multiplied by w and also add the constraint $\sum_{k=1}^{N}\hat{x}_k + \hat{y} = 1$. Then, the stable and scaled optimization problem can be reformulated in the form:

$$\begin{aligned} \min \quad & \hat{x}^T Q \hat{x} + \text{EVaR}^T_{(1-\epsilon)}(\hat{x}) \\ \text{subjectto} \quad & \mu^T \hat{x} + \mu_f \hat{y} \geqslant Rw, \\ & \sum_{k=1}^{N} \hat{x}_k + c^b \sum_{k=1}^{N} \hat{x}_k^b + c^s \sum_{k=1}^{N} \hat{x}_k^s + \hat{y} \leqslant w, \\ & \hat{x}_k = x_k^0 w + \hat{x}_k^b - \hat{x}_k^s, \quad \forall k \\ & \sum_{k=1}^{N} \hat{x}_k + \hat{y} = 1, \\ & \|\hat{x} - x^0 w\|_2^2 \leqslant \kappa^2 w, \\ & Aw \leqslant \hat{y} \leqslant Bw, \\ & \hat{x}, \hat{y}, \hat{x}^b, \hat{x}^s \in \mathbb{R}_+^N. \end{aligned} \tag{19}$$

The solution $\hat{y}^*, \hat{x}^*, \hat{x}^{*s}, \hat{x}^{*b}$, and w^* is deduced and get an optimal solution to Eqn. (19) by rescaling $\hat{y}^*, \hat{x}^*, \hat{x}^{s*}$, and $\hat{x}^{b*}$. Thus, $y^* = \frac{\hat{y}^*}{w^*}, x^* = \frac{\hat{x}^*}{w^*}, x^{s*} = \frac{\hat{x}^{s*}}{w^*}$, and $x^{b*} = \frac{\hat{x}^{b*}}{w^*}$.

Step 2d. Run portfolio optimization model and construct optimal weights.

In the final step of stage II, the investor's desired portfolio will be constructed with weights of top performing stocks and a risk-less asset obtained from the optimal solution provided by the stable "robust" and scaled portfolio optimization model. With changing required return level, an efficient frontier is constructed.

4. Illustrative application

This section describes the applicability of the paper's proposed hybrid robustness approach to the portfolio construction problem using a real-world case study from the Shenzhen and Shanghai Stock Exchanges. Based on the total market value of all outstanding shares and data availability, the first 100 stocks are investigated. Financial data from January 31, 2013, to 2019 are extracted via the Wind Financial Terminal. The comprehensive list of the selected stocks is available in Appendix A.1.

The Shanghai Stock Exchange began operations on December 19, 1990, and as of July 2021, it was the world's third-largest stock market by market size, with a market value of US$7.62 trillion. It is also the largest stock exchange in Asia. The exchange currently has about 2037 companies listed. On the Shanghai Stock Exchange, there are two categories of stocks: "A" shares and "B" shares. The local Renminbi (RMB) currency is used to price "A" shares, whereas "B" shares are offered in US dollars. Initially, "A" shares were solely available to domestic investors, whilst "B" shares were available to domestic and international investors.

The Shenzhen Stock Exchange started business on December 1, 1990. According to the World Federation of Stock Exchanges (WFE) statistics, on December 31, 2021, Shenzhen ranked third, third, and fourth, respectively, regarding the annual transaction amount, financing amount, and the number of IPO companies in the world. As of October 11, 2020, the liquidity level of the Shenzhen "A" share market was higher than that of the New York Stock Exchange (NYSE) market and more stable.

4.1. Portfolio selection: Dynamic stock efficiency analysis

Table 1 presents definitions of the indicators used in this study for stock performance measurement. The input, output, and carry-over indicators are key financial indicators that examine the stock performance of companies listed in the Shenzhen and Shanghai Stock Exchanges. As indicated in Step 1c, the authors check the model validity via a minimal number of DMUs and the relevance of stock efficiency indicators used in the proposed dynamic SBM-DEA model.

First, the 100 companies used are twelve and a half times that of the total input, carry-over, and output indicators of eight in this study. According to Golany and Roll (1989), the minimum needed ratio is two. Hence the authors underline that the construct validity of the dynamic DEA model used in this investigation is reliable and stable. Secondly, the authors utilized robust regression to validate the model by explaining the contributions of each input indicator in generating outputs via a carry-over for performance evaluation. Initial tests on the indicators of stock efficiency using Grubbs' test or the extreme studentized deviate technique reveal that they may contain some outliers. Hence, outlier-resistant robust regression is suitable. The method of moment (MM) estimator of the robust regression technique, which also deals with possible heteroscedasticity, was incorporated.

Table 2 presents the regression results on the relevance of stock indicators. The results show that the input indicators and inputted carry-over indicator of accumulated dividend explain 63.8%, 71.3%, and 70.8% of the change in the rate of return, earnings per share, and earnings per share growth rate, respectively. The p-values indicate that the indicators are statistically significant. An increase in the P/E ratio and solvency ratio would increase the rate of return. Meanwhile, the input indicators for the outputs indicate the same. The inputs significantly and positively affect earnings per share and earnings per share growth rate, which have a collective explanatory power of approximately 71.3% and 70.3%, respectively. The P/E ratio, solvency ratio, standard deviation, and the log of accumulated dividend carry-over inputs positively correlate with the mean return rate, earnings per share, and earnings per share growth rate. The findings validate our model by explaining the significance and contributions of each input and the carry-over in generating outputs to measure and evaluate the efficiency of the listed companies.

Table 3 presents summary statistics of all the indicators. The dataset should ideally have positive values. The authors cope with negative values by adopting Cheng et al.'s (2013) variant radial measure approach and replacing original values with absolute values to quantify the proportion of improvements required to achieve the frontier. The average P/E ratio from 2013 to 2019 for the listed companies is RMB 36.163. The highest P/E ratio is associated with company 600536.SH in 2019. This maximum value of RMB 573.387 could mean that company 600536.SH stock is overvalued, or investors expect high growth rates in the future. On average, a company has RMB 1.395 of liquid assets available to cover each RMB 1 of its current liabilities. The lowest quick ratio of 0.082 is associated with company 600663.SH in 2015. This minimum measures the inability of company 600663.SH to pay all of its outstanding liabilities with only assets convertible to cash quickly. The average solvency ratio of 1.395 means that the financial capacity of the listed companies can meet long-term debt obligations. On average, the proportion of gain in stock investments is 0.044, and its accompanying risk is 0.424. Company 000661.SZ made RMB 10.270 for each share of its stock in 2019, making it the highest in the period. On average, companies make RMB 0.901 for each share of stock. If investors believe a company's profits are higher than its share price, they will pay more for its shares. Stocks with at least a 25% increase in earnings per share over the prior year indicate that a company's goods or services are in high demand. The average value of 22.24% is decent. Company 002493.SZ in 2014 had the worst EPS growth rate throughout the sample period.

We evaluate the overall performance of 100 stocks and assess each stock's period performance. The period stock efficiency scores using Step 1d were obtained using MaxDEA software. Table 4 displays the overall and period stock efficiency scores of the 100 listed companies, complemented by Fig. 4. The average overall efficiency score is 45.3% for the entire period, indicating a 54.7% loss in productivity due to inefficiencies. The period efficiencies outperformed the average overall efficiency. The highest average period efficiency was in 2015. Twenty-six (26) listed companies experienced sustained and stable stock performance and were DEA efficient during the entire period. Two companies, 002311.SZ and 600536.SH were inefficient initially but recovered in 2014 and maintained efficiency afterward. Eight (8) companies experienced inefficiency in only one period throughout the time horizon. Companies 000021.SZ and 601766.SH were efficient at the start of the period but became DEA inefficient afterward and never recovered. The five listed companies with the lowest overall stock efficiency scores were 600118.SH, 000967.SZ, 600895.SH, 601800.SH, and 002353.SZ. Most listed companies experienced sporadic variations in stock efficiency over the entire period.

According to Statman (1987), a well-diversified portfolio of stocks should have at least 30 stocks for borrowing investors and 40 for lending investors. Most financial experts claim that 20 to 30 stocks are an ideal bound for most portfolios and that investors who go beyond 30 usually don't see too much of an incremental reward for increasing amounts of diversification (How many stocks should you own? why there's no single 'right' answer, 2021). Informed by the aforementioned studies, the qualified set of stocks that are selected from the rankings of the stock efficiency scores for the next stage, i.e., portfolio optimization is the DEA efficient ones.

4.1.1. Robustness of stock efficiency results

In general, optimization system parameters are generally clouded by uncertainty in real-world situations. Therefore, it is key to maintain the robustness of the solution obtained from portfolio selection in this condition. By doing this, the efficiency and ranking of the candidate stocks become a reliable and solid foundation for portfolio optimization. This leads to an empirical evaluation of efficiency score stability by re-sampling as part of the robustness investigation.

Table 2
Regression results on the relevance of indicators.

Inputs/Outputs	Rate of return	Earnings per share	Earnings per share growth rate
Constant	0.076***	−0.088***	0.061***
	(0.265)	(-0.167)	(0.134)
P/E ratio	0.001**	0.002 ***	0.002***
	(0.001)	(0.000)	(0.000)
Quick ratio	0.000	0.059 ***	0.013***
	(0.020)	(0.012)	(0.010)
Solvency ratio	0.004*	0.044**	0.025**
	(0.021)	(0.013)	(0.011)
Standard deviation	0.016***	0.130***	0.017***
	(0.131)	(0.083)	(0.067)
Log(Accumulated dividend)	0.005*	0.029**	0.000**
	(0.012)	(0.007)	(0.006)
Adjusted R-squared	0.638***	0.713**	0.708***

Note: Superscripts *, **, and *** denote the statistical significance at the 10%, 5% and 1% level respectively.

Table 3
Summary statistics of indicators for stock performance measurement.

Indicators	Mean	Median	Maximum	Minimum	Standard Deviation
Price to Earnings (P/E) ratio	36.163	24.202	573.387	−38.858	47.773
Quick ratio	1.395	0.937	15.198	0.082	1.534
Solvency ratio	1.395	0.959	11.029	0.035	1.408
Standard deviation	0.424	0.377	1.625	0	0.208
Rate of return	0.044	0.018	1.468	−1.841	0.435
Earnings per share	0.901	0.641	10.270	−0.613	0.922
Earnings per share growth rate	0.224	0.070	18.667	−2.476	1.276
Accumulated dividend	1.877×10^9	4.332×10^8	6.054×10^{10}	0	5.380×10^9

Table 4
Dynamic stock efficiency scores of listed companies.

Listed Companies	Overall Efficiency	Rank	Period Efficiency Score						
			2013	2014	2015	2016	2017	2018	2019
000002.SZ	1	1	1	1	1	1	1	1	1
000021.SZ	0.028	93	1	0.163	0.097	0.362	0.360	0.005	0.098
000050.SZ	0.224	58	1	1	0.312	0.079	0.162	0.371	0.132
000100.SZ	0.125	74	1	0.287	1	0.082	1	0.033	0.086
000301.SZ	0.164	64	1	0.026	1	1	1	1	1
000538.SZ	0.199	62	1	1	1	1	1	0.038	0.172
000661.SZ	1	1	1	1	1	1	1	1	1
000671.SZ	0.223	60	1	1	0.243	0.174	0.226	0.087	0.203
000739.SZ	1	1	1	1	1	1	1	1	1
000786.SZ	1	1	1	1	1	1	1	1	1
000858.SZ	0.700	34	1	1	1	1	1	0.234	0.870
000860.SZ	0.140	69	1	1	0.136	0.061	0.068	0.301	0.170
000895.SZ	0.621	36	1	1	1	1	1	1	0.180
000938.SZ	0.426	40	1	1	1	1	1	0.123	0.230
000961.SZ	1	1	1	1	1	1	1	1	1
000967.SZ	0.426×10^{-3}	99	0.237	0.000	0.471	0.094	0.120	0.434	0.297
000977.SZ	0.102	80	0.150	0.052	0.161	0.079	0.096	0.172	0.310
001872.SZ	1	1	1	1	1	1	1	1	1
002001.SZ	0.338	46	1	0.137	1	1	0.291	0.300	0.218
002007.SZ	1	1	1	1	1	1	1	1	1
002032.SZ	1	1	1	1	1	1	1	1	1
002050.SZ	0.060	87	0.168	0.175	0.444	0.218	0.021	0.031	0.087
002120.SZ	0.375	44	1	1	1	1	1	0.106	0.205
002146.SZ	1	1	1	1	1	1	1	1	1
002153.SZ	1	1	1	1	1	1	1	1	1
002180.SZ	0.281	51	1	1	1	1	1	0.056	0.213
002185.SZ	0.697	35	0.386	0.374	1	1	1	1	1
002202.SZ	0.063	86	1	1	0.900	0.018	0.093	0.020	1
002223.SZ	1	1	1	1	1	1	1	1	1
002241.SZ	0.125	75	0.201	0.401	0.277	0.104	0.163	0.138	0.046
002294.SZ	0.895	28	1	1	1	1	1	1	0.549
002304.SZ	1	1	1	1	1	1	1	1	1
002311.SZ	0.840	29	0.426	1	1	1	1	1	1
002352.SZ	0.136	71	0.993	0.297	1	1	0.044	0.048	0.584
002353.SZ	0.008	96	0.773	1	0.001	0.925	0.241	1	1
002405.SZ	1	1	1	1	1	1	1	1	1
002415.SZ	0.121	77	0.317	0.088	0.922	0.071	0.085	0.124	0.123
002422.SZ	0.223	59	1	1	0.054	1	1	0.218	0.169
002460.SZ	0.087	83	1	1	1	1	1	0.572	0.012
002475.SZ	0.135	72	0.177	0.237	0.282	0.141	0.045	0.271	0.295
002493.SZ	0.722	32	1	1	1	1	1	0.242	1
300012.SZ	1	1	1	1	1	1	1	1	1
300144.SZ	1	1	1	1	1	1	1	1	1
300251.SZ	0.325	49	0.345	0.338	1	0.265	0.108	1	1
600004.SH	1	1	1	1	1	1	1	1	1
600018.SH	0.278	52	1	0.231	1	1	0.773	0.084	0.185
600019.SH	0.709	33	0.253	1	1	1	1	1	1
600028.SH	1	1	1	1	1	1	1	1	1
600031.SH	0.103	79	0.128	0.187	0.023	0.078	1	0.571	0.575
600039.SH	0.029	92	0.286	0.360	0.276	0.012	0.024	0.012	0.477
600048.SH	0.385	43	1	0.478	0.134	1	0.174	1	1
600066.SH	0.155	66	0.311	0.289	1	0.089	0.116	0.088	0.128
600068.SH	0.147	68	0.051	0.147	0.087	0.098	1	1	1
600089.SH	0.076	84	0.167	0.037	0.519	0.081	0.045	0.118	0.122
600118.SH	0.253×10^{-3}	100	0.192	0.070	1	1	0.043	0.000	0.303
600143.SH	0.040	90	0.075	0.062	0.346	0.008	0.110	0.071	0.394
600160.SH	1	1	1	1	1	1	1	1	1
600177.SH	0.436	39	0.346	1	1	1	0.103	1	1
600188.SH	0.222	61	0.106	0.109	0.099	0.266	0.568	1	1
600196.SH	0.098	82	0.333	0.038	0.392	0.123	0.042	0.464	0.298
600219.SH	0.328	47	0.194	0.271	0.253	0.249	0.319	1	1
600236.SH	0.162	65	1	1	1	0.285	0.028	1	1
600276.SH	1	1	1	1	1	1	1	1	1
600295.SH	0.111	78	0.274	0.225	0.025	0.237	0.094	0.920	0.462
600298.SH	0.402	42	1	1	1	1	1	1	0.076
600340.SH	1	1	1	1	1	1	1	1	1
600350.SH	0.425	41	1	0.343	0.920	0.108	1	1	1
600372.SH	0.026	94	0.006	0.108	0.048	0.019	0.701	1	0.433
600383.SH	0.498	37	1	1	1	1	0.965	0.111	1
600415.SH	0.323	50	1	1	1	1	1	0.106	0.118
600436.SH	0.450	38	1	1	0.485	0.200	0.622	0.294	0.489
600438.SH	0.174	63	1	0.119	0.040	1	1	1	0.311

Table 4 (*continued*)

Listed Companies	Overall Efficiency	Rank	Period Efficiency Score						
			2013	2014	2015	2016	2017	2018	2019
600487.SH	0.147	67	0.177	0.293	0.305	0.373	0.262	0.044	0.261
600536.SH	0.827	30	0.371	1	1	1	1	1	1
600549.SH	0.274	53	0.187	1	1	0.161	0.946	0.157	0.150
600585.SH	1	1	1	1	1	1	1	1	1
600600.SH	0.058	88	0.482	0.015	0.711	0.134	0.148	0.057	0.176
600612.SH	0.927	27	1	0.631	1	1	1	1	1
600663.SH	0.273	54	1	1	1	0.114	1	0.072	0.971
600690.SH	0.123	76	1	0.113	0.548	0.047	0.432	1	0.041
600795.SH	1	1	1	1	1	1	1	1	1
600801.SH	1	1	1	1	1	1	1	1	1
600867.SH	0.770	31	1	0.298	1	1	1	1	1
600886.SH	0.245	55	1	1	1	1	1	0.187	0.045
600887.SH	0.130	73	0.110	0.192	1	1	0.036	0.149	0.441
600893.SH	0.326	48	0.144	0.374	0.083	1	1	1	1
600895.SH	0.001	98	0.000	0.230	0.359	0.518	0.189	0.263	0.490
600900.SH	1	1	1	1	1	1	1	1	1
601018.SH	0.242	57	1	1	0.136	0.188	0.249	0.094	1
601238.SH	0.358	45	1	1	0.223	0.221	1	0.597	0.151
601607.SH	0.051	89	0.077	0.322	1	0.041	0.050	0.015	0.065
601668.SH	0.243	56	1	0.252	0.602	0.060	1	0.170	1
601766.SH	0.064	85	1	0.205	0.250	0.022	0.055	0.026	0.257
601800.SH	0.008	97	0.001	0.694	0.990	0.798	1	1	1
601857.SH	1	1	1	1	1	1	1	1	1
601877.SH	0.031	91	0.738	0.429	0.841	0.006	0.018	1	0.052
601888.SH	0.009	95	0.923	1	1	1	0.001	1	1
601899.SH	0.139	70	0.080	0.110	0.119	1	1	0.048	1
601933.SH	0.101	81	0.549	0.125	0.214	0.067	0.127	0.048	0.077
601992.SH	1	1	1	1	1	1	1	1	1
Mean	0.453		0.727	0.659	0.733	0.653	0.664	0.607	0.638

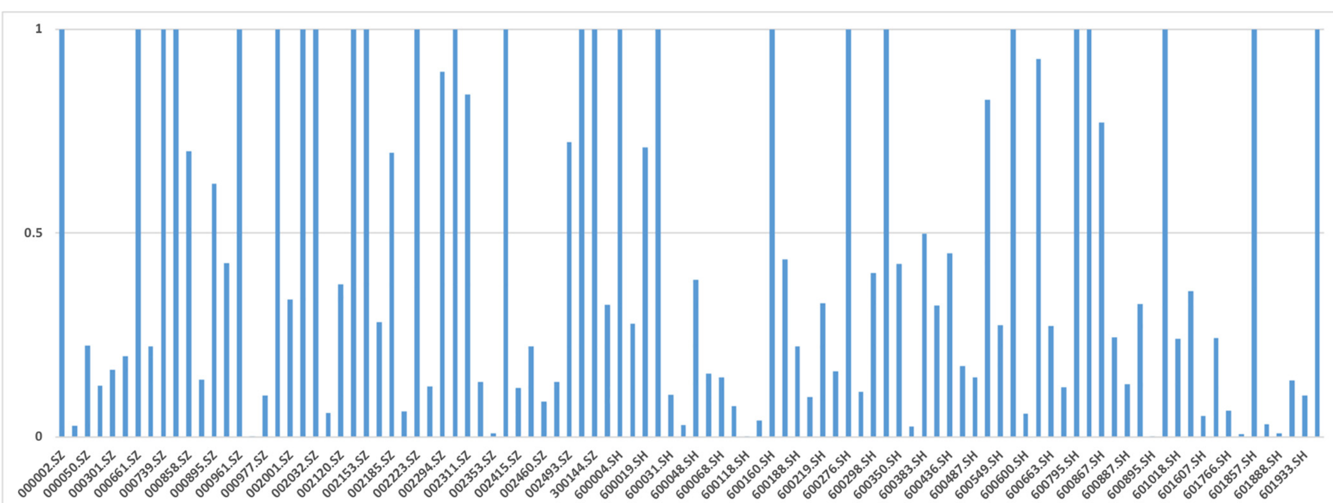

Fig. 4. Overall dynamic efficiency scores of listed companies.

The leave-one-out process, demonstrated in the context of dynamic SBM-DEA for portfolio selection, is used for re-sampling. The fact that overall and period efficiency scores fall within the confidence intervals produced (at 95%) suggests that dynamic SBM-DEA stock efficiency scores can be generalized to a larger population, i.e., efficiency scores are stable when subjected to re-sampling. Re-sampling results are available upon reasonable request.

4.2. Portfolio optimization: allocation weights for qualified stocks

In this section, the authors present an illustrative example of the proposed "robust" stable and scaled portfolio optimization model. The monthly stock price from January 31, 2013, to December 31, 2019, of the 26 qualified listed companies were extracted from Wind Financial Terminal. The log-returns of stocks are deduced by the natural logarithm of the price ratio for two consecutive months. For the risk-less asset, we consider China's 1-Year Bond yield on a monthly basis for the same horizon. The log-returns for the bond are deduced by the natural logarithm of the sum of 1 and the stated rate of interest.

Following Chen et al.'s (2012) approach, the initial portfolio is normalized i.e. equally-weighted portfolio, hence $x_k^0 = 0.038$ for $k = 1, \ldots, 26$. Considering different required returns based on average log stock returns, $R = (0.007, 0.009, 0.012, 0.014, 0.016, 0.018)$, for $\epsilon = 0.05$, and transaction costs, $c^b = c^s = 0.01$, the optimal

Table 5
Optimal weights of stocks and bond with related results under different required returns.

Required Return: R		0.005	0.007	0.009	0.012	0.014	0.016	0.018
000002.SZ	x_1	0.030	0.047	0.048	0.001	0.001	0.001	0.001
000661.SZ	x_2	0.001	0.001	0.001	0.055	0.184	0.280	0.664
000739.SZ	x_3	0.001	0.001	0.001	0.001	0.056	0.001	0.001
000786.SZ	x_4	0.002	0.001	0.001	0.019	0.040	0.001	0.001
000961.SZ	x_5	0.001	0.001	0.001	0.001	0.001	0.001	0.001
001872.SZ	x_6	0.031	0.004	0.052	0.122	0.184	0.326	0.256
002007.SZ	x_7	0.001	0.001	0.001	0.001	0.001	0.001	0.001
002032.SZ	x_8	0.019	0.002	0.020	0.012	0.001	0.001	0.001
002146.SZ	x_9	0.045	0.065	0.088	0.090	0.047	0.001	0.001
002153.SZ	x_{10}	0.013	0.001	0.002	0.003	0.001	0.001	0.001
002223.SZ	x_{11}	0.001	0.001	0.001	0.001	0.001	0.001	0.001
002304.SZ	x_{12}	0.059	0.097	0.126	0.140	0.115	0.011	0.001
002405.SZ	x_{13}	0.003	0.001	0.001	0.007	0.007	0.001	0.001
300012.SZ	x_{14}	0.001	0.001	0.001	0.001	0.001	0.001	0.001
300144.SZ	x_{15}	0.001	0.001	0.001	0.001	0.001	0.001	0.001
600004.SH	x_{16}	0.001	0.001	0.001	0.007	0.018	0.001	0.001
600028.SH	x_{17}	0.018	0.009	0.014	0.001	0.001	0.001	0.001
600160.SH	x_{18}	0.001	0.001	0.001	0.001	0.001	0.001	0.001
600276.SH	x_{19}	0.066	0.118	0.147	0.160	0.135	0.131	0.001
600340.SH	x_{20}	0.014	0.001	0.007	0.028	0.014	0.001	0.001
600585.SH	x_{21}	0.001	0.001	0.001	0.001	0.001	0.001	0.001
600795.SH	x_{22}	0.019	0.004	0.018	0.001	0.001	0.001	0.001
600801.SH	x_{23}	0.001	0.001	0.001	0.001	0.001	0.001	0.001
600900.SH	x_{24}	0.100	0.221	0.238	0.213	0.134	0.178	0.001
601857.SH	x_{25}	0.036	0.067	0.063	0.001	0.001	0.001	0.001
601992.SH	x_{26}	0.054	0.103	0.113	0.080	0.001	0.001	0.001
China 1-Year Bond	y	0.050	0.050	0.050	0.050	0.050	0.050	0.050
Risk (variance): x^TQx		0.001	0.002	0.003	0.004	0.007	0.010	0.024
Downside risk: $\mathrm{EVaR}_{(0.95)}$		0.115	0.147	0.199	0.241	0.310	0.327	0.435
Transaction cost: $1-(y+e^Tx)$		0.430	0.198	0.002	0.002	0.002	0.003	0.006
Available capital: $y+e^Tx$		0.570	0.802	0.998	0.998	0.999	0.997	0.994

weights of the 26 qualified stocks are obtained by solving Eqn. (19). The transaction cost assumption of 1% was based on a study by Zhang (2018) that the implicit trading cost of the Shanghai and Shenzhen markets are generally within the range of 0.5% to 1%. Given the characteristics of the risk-less asset, China's 1-Year Bond is restricted through this constraint: $0 \leqslant y \leqslant 0.050$. To promote diversification in the presence of no short selling, the authors introduce this constraint, $0.001 \leqslant x \leqslant 1$, on the stock portfolio. The results of the second stage are displayed in Table 5.

The results presented in Table 5 show that the efficient frontier of the "robust" stable and scaled mean–variance-EVaR portfolio is graphically displayed in Fig. 5. Different optimal weights are allocated to stocks for a set of required returns. An increase in required returns increases downside risk and the risk associated with the deviation around the expected return; thus, a higher risk leads to higher returns, which agrees with extant literature and practice. If investors want to have a large portion of their capital invested and pay lower transaction costs, they will have to take on additional risk to control downside risk and the deviation around the expected return.

However, when compared to the unscaled portfolio optimization model in Eqn. (14) but with the norm constraint, investors will incur additional risk in controlling for downside risk but will be compensated with lower transaction costs, larger capital available for investment, and lower symmetric risk. This is in line with results obtained by Atta Mills et al. (2017).

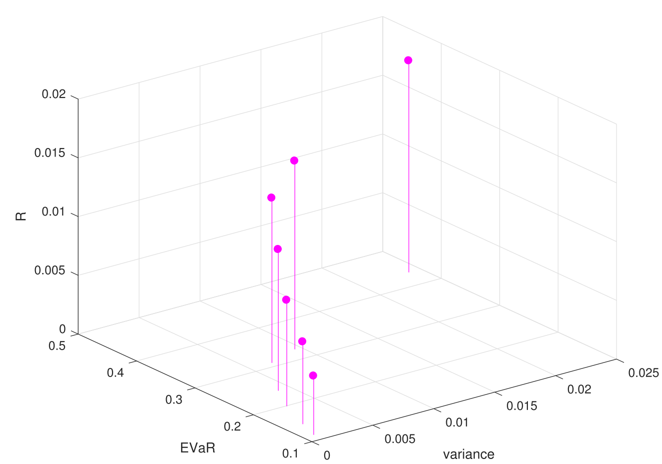

Fig. 5. Efficient frontier of "robust"stable and scaled mean–variance-EVaR model.

5. Performance Measures

This section presents the annualized computational performance of the proposed "robust" stable and scaled mean–variance-EVaR (RSSMVE) portfolio (see Eqn. 19) in comparison to the baseline method, mean–variance-EVaR (MVE) portfolio (see Eqn. 14), mean–variance-CVaR (MVC) portfolio, mean–variance (MV) portfolio, global minimum-variance (MinV) portfolio, naive equally-weighted $(1/N)$ portfolio, l_1-penalized mean–variance (l_1-MV) portfolio Brodie et al. (2009), and minimum variance portfolio resulting from using a diagonal covariance matrix (DCM) portfolio (Kirby and Ostdiek, 2012). The understudy period is from January 31, 2013, to December 31, 2019.

Each problem instance is modelled with Matlab package CVX on a 64-bit Macbook Pro with 2.3 GHz Intel Core i7 and 16 GB of RAM. CVX supports covex optimization usung a particular approach called disciplined convex programming (see Grant et al., 2006). CVX contains solvers SeDuMi, SDPT3, Gurobi, MOSEK, and GLPK, and one can change to any preferred solver. For improved efficiency and time, the solvers pursue the dual problem. This study

Table 6
The results of performance metrics for the proposed approach and others in literature.

Models	Sharpe ratio	Sortino ratio	Allocation for Transaction costs	Available Capital
RSSMVE	0.286**	0.513**	0.007	0.993
MVE	0.201*	0.395*	0.362	0.638
MVC	0.198*	0.388**	0.275	0.725
MV	0.215*	-	0.512	0.488
MinV	0.234	-	0.307	0.653
$1/N$	0.113	-	0.557	0.443
l_1-MV	0.231*	-	0.402	0.598
DCM	0.121	-	0.413	0.587

Note: Superscripts *, **, and *** denote the statistical significance at the 10%, 5% and 1% level respectively.

used the standard CVX distribution and reported the average CVX CPU times of SeDuMi and SDPT3 for 100 instances.

Two portfolios from literature, MVE and MVC, seek optimal compromise between short tails and the risk associated with the deviation around expected return. So comparing them with the proposed model in this study is prudent. It should be noted that the minimum required return and transaction costs when selling or buying stocks are set to 0.13 and 0.01 respectively. China's deposit interest rate of 0.015 is used as a riskless return rate.

To evaluate the computational performance of the eight portfolios, we consider the portfolio allocation problem for transaction costs and available capital and analyze the Sharpe and Sortino ratios. The Sortino ratio (SoR) is the excess risk-adjusted returns of a portfolio above the risk-free rate relative to its downside risk. The Sharpe ratio (ShR) uses the risk-adjusted returns of a portfolio above the risk-free but relative to its standard deviation. When comparing two portfolios, the one with a higher Sharpe or Sortino ratio provides a better return for the same risk. The higher the Sharpe (Sortino) ratio of an investment, the higher an investor is compensated per risk unit. Sharpe ratio is deduced by

$$ShR = \frac{\mathbb{E}(R_p) - R_f}{\sigma_p}, \tag{20}$$

where $\mathbb{E}(R_p)$ is the expected portfolio return, R_f is the riskless asset rate, and $\sigma_p = \sqrt{x^T Q x}$ is the standard deviation of the portfolio return. Given that the models under study have a measure for downside risk in EVaR and CVaR, the authors identify Sortino ratio to reflect that. Thus,

$$SoR = \frac{\frac{1}{T}\Sigma_{t=1}^{T}(R_p(t) - R_I(t))}{V}, \tag{21}$$

where R_p is portfolio return and R_I is the Shanghai Composite Index return. V is either EVaR or CVaR of the portfolio. The Sortino ratio is a metric that compares a portfolio's risk-adjusted returns to an investment target while taking into account downside risk. This is similar to the Sharpe ratio, which uses standard deviation to measure risk-adjusted returns relative to the risk-free rate. When comparing two portfolios, the one with higher Sharpe and Sortino ratios provide better return for the same risk, which is attractive to investors.

To ensure statistical stability throughout the period of study, we examine the statistical significance of economic gains. We use Ledoit and Wolf's (2008) bootstrapping approach based on 1000 simulations to test whether the differences in the Sharpe and Sortino ratios of a portfolio optimization strategy are different from that of a benchmark, S&P China A50 index. Table 6 summarizes the results of the metrics used for the benchmark comparison.

Specifically, we emphasize that proportional transaction cost is a significant factor in investment because costs impact the net aggregate returns on investment, given the risk involved. Low transaction costs can ensure that investors maximize returns and ultimately achieve wealth targets. Given the results in Table 6, the Sharpe ratio and the Sortino ratio are the highest under the proposed RSSMVE model, among all other models. Our proposed RSSMVE model outperforms the second-best model, MinV, by 22%, comparing the Sharpe ratios. RSSMVE performance could be attributed to the scaled and stable (regularized) approach

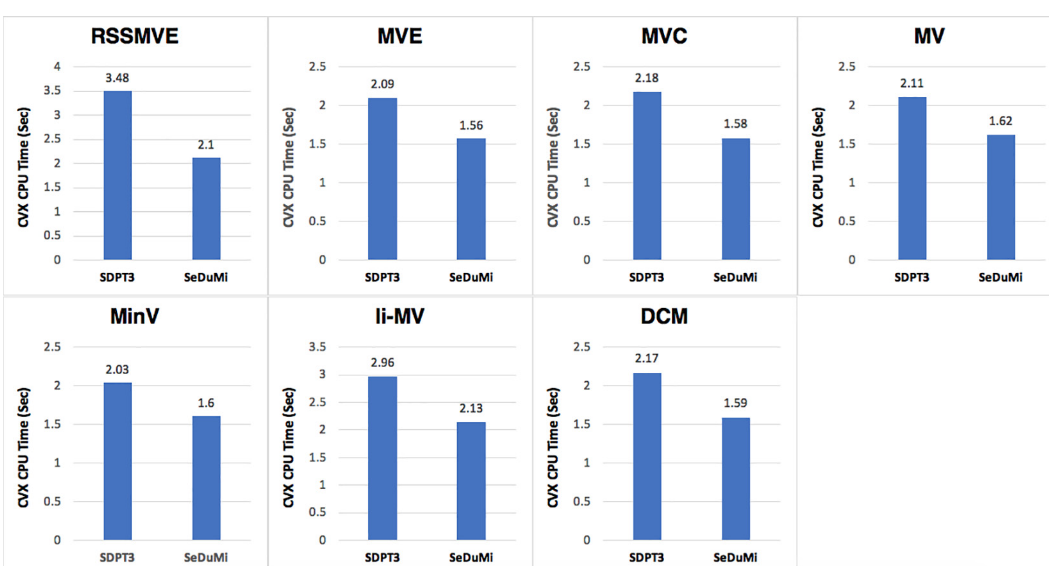

Fig. 6. Computational efficiency of SDPT3 and SeDuMi solvers for 100 instances.

incorporated in the model. The cost allocation to transactions is the lowest, and the available capital is the highest under the proposed RSSMVE model. Transaction cost decreases by 97% by comparing our proposed model to the MVC model, the second-best model for costs. The amount of accumulated capital increases by 37% compared to the MVC model, the second-best for capital availability.

The RSSMVE portfolio has a lower allocation for transaction costs and have a large portion of capital invested. The $1/N$ benchmark has the lowest turnover by construction and therefore the least Sharpe ratio. However, lower allocation for transaction costs also means lower turnover. With large turnovers, reasonable transaction costs wipe out the economic gains of many traditional portfolios as evidenced by MV portfolio. The proposed approach in this study has a better sample performance as evidenced by high Sharpe, high Sortino ratio, investors having a large portion of capital invested, smaller allocation to transaction costs, and lower turnover rates. Additionally, the proposed approach used in this research for optimal asset allocation is favorable as it overcomes unsteady and extreme portfolio weights induced by estimation error due to parameter uncertainty.

Fig. 6 sheds light on the computational tractability of all seven models under consideration using two solvers in Matlab. We solve the portfolio optimization problem in Eqn. (19) using semidefinite optimization solvers, SeDuMi, and SDPT3. We compare the computational times needed by each solver for the portfolio optimization problem. SDPT3 performs worse for all seven models compared to Solver SeDuMi. The CVX CPU computational time under Solver SDPT3 for the proposed model is about 3.48 seconds, the worst among all seven models. The computational CVX CPU time under SeDuMi is about 2.1 seconds for the proposed RSSMVE model, the sixth best performing model. Under SeDuMi, the most time-consuming model is l_1-MV. Since the proposed RSSMVE and the l_1-MV models are norm-constrained, we infer that the norm-constrained portfolio requires more execution times. The differences in computational times lie in the number of constraints and the dimensions involved for different portfolio strategies. We note that the execution times of the portfolio strategy should be in tandem with the financial performance.

6. Conclusion

This study proposes a hybrid two-stage portfolio construction approach to deal with data uncertainty, reduce computational complexity, and provide a comprehensive evaluation of stocks from various financial criteria that appeal to non-conventional finance enthusiasts. Using a two-stage approach, conservatism in the investment process increases. The first stage (portfolio selection) involves using an integrated dynamic data envelopment analysis to select qualified stocks based on stock efficiency performance from a menu of stocks. The second stage (portfolio optimization) relies on a proposed "robust" stable and scaled mean-variance-Entropic Value-at-Risk model for allocating funds in the presence of transaction costs. We demonstrate the applicability of our proposed model to stock data from the Shenzhen and Shanghai Stock Exchanges. This study uses Ledoit and Wolf's (2008) difference test to compare the performance of the different portfolios with that of a benchmark, the S&P China A50 index.

The analysis was conducted using data from 2013 to 2019, and 100 stocks based on market capitalization and data availability were selected. Under robustness testing, the first stage of the illustrative application discusses the stability of efficiency estimates through re-sampling (leave-one-out method). The average overall stock efficiency score is 0.453, indicating a 54.7% loss in productivity. Twenty-six (26) companies exhibited excellent financial-based activities that sustained stable stock performance. Most listed companies experienced sporadic variations in stock efficiency over the entire period. These 26 stocks are qualified stocks for the portfolio optimization stage. We exploit the squared norm ball constraint model to address problems associated with extreme portfolio weights and under-diversification of a portfolio. Optimal weights were allocated to these stocks under varying required returns.

Using our proposed approach compared to seven other portfolio choice models, we find that the Sharpe and Sortino ratios are the highest among all other models. Our proposed model also outperforms the second-best model, MinV, by 22%, comparing the Sharpe ratios and by 30% by using the Sortino ratio. Under our proposed model, the cost allocation to transactions is the lowest, and the available capital is the highest. The amount of accumulated capital increases by 37% using our proposed model compared to the MVC model, the second-best for capital availability. Lastly, we show the CVX CPU computational tractability of all eight models under consideration using solvers SeDuMi, and SDPT3 in the MATLAB programming language. Here, we document that more execution time is required since our proposed model is norm-constrained. The differences in computational time lie in the number of constraints and the dimensions involved for different portfolio strategies.

Our findings also have implications for fund managers and regulators. With increasing required returns, the proposed portfolio optimization approach increases the amount of capital for investment and lowers transaction costs at the expense of additional risk. Compared to the existing models, the proposed approach in this study has better returns relative to investment risk than the existing ones. The method described in this paper can be applied to a portfolio composed of other securities and subjected to an opposing financial market environment. Other forms of uncertainty programming approaches can be applied to tackle data uncertainty. Additionally, combining findings from various methods like fuzzy or grey multi-criteria decision-making approaches makes it possible to rank the stocks before portfolio optimization and compare the results.

Funding

The authors acknowledge support from Wenzhou-Kean University Internal Research Support Program. Grant/Award Number: IRSPG202204.

Availability of data and materials

Data is available upon reasonable request.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

The authors thank the reviewers for taking time off their busy schedules to add value to the manuscript. The authors also thank Li Honglin for her support.

Appendix A

Table A.1
Names of 100 listed companies used for this study.

Securities code	Company Name	Securities Code	Company Name
000002.SZ	China Vanke Co.,Ltd.	600048.SH	Poly Developments and Holdings Group Co., Ltd.
000021.SZ	Shenzhen Kaifa Technology Co., Ltd.	600066.SH	Zhengzhou Yutong Bus Co.,Ltd.
000050.SZ	Tianma Microelectronics Co.,Ltd.	600068.SH	China Gezhouba Group Company Limited
000100.SZ	TCL Technology Group Corporation	600089.SH	TBEA Co.,Ltd.
000301.SZ	Jiangsu Eastern Shenghong Co.,Ltd.	600118.SH	China Spacesat Co.,Ltd.
000538.SZ	Yunnan Baiyao Group Co., Ltd.	600143.SH	Kingfa Sci.&Tech. Co.,Ltd.
000661.SZ	Changchun High-tech Industry (Group) Co., Ltd.	600160.SH	Zhejiang Juhua Co.,Ltd
000671.SZ	Yango Group Co.,Ltd	600177.SH	Youngor Group Co.,Ltd.
000739.SZ	Apeloa Pharmaceutical Co.,Ltd	600188.SH	Yanzhou Coal Mining Company Limited
000786.SZ	Beijing New Building Materials Public Limited Company	600196.SH	Shanghai Fosun Pharmaceutical (Group) Co., Ltd.
000858.SZ	Wuliangye Yibin Co.,Ltd	600219.SH	Shandong Nanshan Aluminium Co., Ltd.
000860.SZ	Beijing Shunxin Agriculture Co.,Ltd	600236.SH	Guangxi Guiguan Electric Power Co.,Ltd.
000895.SZ	Henan Shuanghui Investment & Development Co.,Ltd.	600276.SH	Jiangsu Hengrui Pharmaceuticals Co.,Ltd
000938.SZ	Unisplendour Corporation Limited	600295.SH	Inner Mongolia Eerduosi Resources Co.,Ltd.
000961.SZ	Jiangsu Zhongnan Construction Group Co.,Ltd	600298.SH	Angel Yeast Co.,Ltd
000967.SZ	Infore Environment Technology Group Co. Ltd.	600340.SH	China Fortune Land Development Co.,Ltd.
000977.SZ	Inspur Electronic Information Industry Co.,Ltd	600350.SH	Shandong Hi-Speed Company Limited
001872.SZ	China Merchants Port Group Co., Ltd.	600372.SH	China Avionics Systems Co.,Ltd.
002001.SZ	Zhejiang NHU Company Ltd.	600383.SH	Gemdale Corporation
002007.SZ	Hualan Biological Engineering,Inc.	600415.SH	Zhejiang China Commodities City Group Co.,Ltd
002032.SZ	Zhejiang Supor Co., Ltd.	600436.SH	Zhangzhou Pientzehuang Pharmaceutical Co., Ltd.
002050.SZ	Zhejiang Sanhua Intelligent Controls Co.,Ltd.	600438.SH	Tongwei Co., Ltd.
002120.SZ	Yunda Holding Co., Ltd.	600487.SH	Hengtong Optic-Electric Co.,Ltd
002146.SZ	Risesun Real Estate Development Co.,Ltd	600536.SH	China National Software & Service Company Limited
002153.SZ	Beijing Shiji Information Technology Co.,Ltd.	600549.SH	Xiamen Tungsten Co.,Ltd.
002180.SZ	Ninestar Corporation	600585.SH	Anhui Conch Cement Company Limited
002185.SZ	Tianshui Huatian Technology Co.,Ltd.	600600.SH	Tsingtao Brewery Company Limited
002202.SZ	Xinjiang Goldwind Sci.&Tech. Co.,Ltd	600612.SH	Lao Feng Xiang Co.,Ltd.
002223.SZ	Jiangsu Yuyue Medical Equipment & Supply Co.,Ltd	600663.SH	Shanghai Lujiazui Finance & Trade Zone Development Co.,Ltd.
002241.SZ	Goertek Inc.	600690.SH	Haier Smart Home Co., Ltd.
002294.SZ	Shenzhen Salubris Pharmaceuticals Co., Ltd.	600795.SH	GD Power Development Co.,Ltd.
002304.SZ	Jiangsu Yanghe Brewery Joint-Stock Co., Ltd.	600801.SH	Huaxin Cement Co.,Ltd.
002311.SZ	Guangdong Haid Group Co.,Limited	600867.SH	Tonghua Dongbao Pharmaceutical Co.,Ltd.
002352.SZ	S.F. Holding Co., Ltd.	600886.SH	SDIC Power Holdings Co.,Ltd.
002353.SZ	Yantai Jereh Oilfield Services Group Co., Ltd.	600887.SH	Inner Mongolia Yili Industrial Group Co.,Ltd.
002405.SZ	Navinfo Co., Ltd.	600893.SH	Aecc Aviation Power Co,Ltd
002415.SZ	Hangzhou Hikvision Digital Technology Co.,Ltd.	600895.SH	Shanghai Zhangjiang Hi-Tech Park Development Co.,Ltd
002422.SZ	Sichuan Kelun Pharmaceutical Co.,Ltd.	600900.SH	China Yangtze Power Co., Ltd.
002460.SZ	Ganfeng Lithium Co., Ltd.	601018.SH	Ningbo Zhoushan Port Company Limited
002475.SZ	Luxshare Precision Industry Co., Ltd.	601238.SH	Guangzhou Automobile Group Co., Ltd
002493.SZ	Rongsheng Petrochemical Co., Ltd.	601607.SH	Shanghai Pharmaceuticals Holding Co.,Ltd
300012.SZ	Centre Testing International Group Co., Ltd.	601668.SH	China State Construction Engineering Corporation Limited
300144.SZ	Songcheng Performance Development Co.,Ltd	601766.SH	CRRC Corporation Limited
300251.SZ	Beijing Enlight Media Co., Ltd.	601800.SH	China Communications Construction Company Limited
600004.SH	Guangzhou Baiyun International Airport Company Limited	601857.SH	Petrochina Company Limited
600018.SH	Shanghai International Port (Group) Co., Ltd.	601877.SH	Zhejiang Chint Electrics Co.,Ltd.
600019.SH	Baoshan Iron & Steel Co.,Ltd.	601888.SH	China Tourism Group Duty Free Corporation Limited
600028.SH	China Petroleum & Chemical Corporation	601899.SH	Zijin Mining Group Company Limited
600031.SH	Sany Heavy Industry Co.,Ltd.	601933.SH	Yonghui Superstores Co., Ltd.
600039.SH	Sichuan Road&Bridge Co.,Ltd	601992.SH	BBMG Corporation

References

Ahmadi-Javid, A., 2012. Entropic value-at-risk: A new coherent risk measure. J. Optim. Theory Appl. 155 (3), 1105–1123.
Artzner, P., Delbaen, F., Eber, J.M., Heath, D., 1999. Coherent measures of risk. Math. Finance 9 (3), 203–228.
Atta Mills, E.F.E., Yan, D., Yu, B., Wei, X., 2016. Research on regularized mean–variance portfolio selection strategy with modified roy safety-first principle. SpringerPlus 5 (1), 1–18.
Atta Mills, E.F.E., Yu, B., Yu, J., 2017. Scaled and stable mean-variance-evar portfolio selection strategy with proportional transaction costs. J. Business Econ. Manage. 18 (4), 561–584.
Atta Mills, E.F.E., Yu, B., Zeng, K., 2019. Satisfying bank capital requirements: A robustness approach in a modified roy safety-first framework. Mathematics 7 (7), 593.
Atta Mills, E.F.E., Baafi, M.A., Amowine, N., Zeng, K., 2020. A hybrid grey mcdm approach for asset allocation: evidence from china's shanghai stock exchange. J. Business Econ. Manage. 21 (2), 446–472.
Avkiran, N.K., 2015. An illustration of dynamic network dea in commercial banking including robustness tests. Omega 55, 141–150.
Axelrod, A., Carlone, L., Chowdhary, G., Karaman, S., 2016. Data-driven prediction of evar with confidence in time-varying datasets. In: 2016 IEEE 55th Conference on Decision and Control (CDC). IEEE, pp. 5833–5838.
Banker, R.D., Charnes, A., Cooper, W.W., 1984. Some models for estimating technical and scale inefficiencies in data envelopment analysis. Manage. Sci. 30 (9), 1078–1092.
Bhattacharyya, R., Hossain, S.A., Kar, S., 2014. Fuzzy cross-entropy, mean, variance, skewness models for portfolio selection. J. King Saud University-Computer Inform. Sci. 26 (1), 79–87.
Borkovec, M., Domowitz, I., Kiernan, B., Serbin, V., 2010. Portfolio optimization and the cost of trading. J. Investing 19 (2), 63–76.
Brodie, J., Daubechies, I., De Mol, C., Giannone, D., Loris, I., 2009. Sparse and stable markowitz portfolios. Proc. Nat. Acad. Sci. 106 (30), 12267–12272.
Charnes, A., Cooper, W.W., 1962. Programming with linear fractional functionals. Naval Res. Logistics Quarterly 9 (3–4), 181–186.
Charnes, A., Cooper, W.W., Rhodes, E., 1978. Measuring the efficiency of decision making units. Eur. J. Operational Res. 2 (6), 429–444.

Chen, A.H., Fabozzi, F.J., Huang, D., 2012. Portfolio revision under mean-variance and mean-cvar with transaction costs. Rev. Quant. Financ. Acc. 39 (4), 509–526.
Cheng, G., Zervopoulos, P., Qian, Z., 2013. A variant of radial measure capable of dealing with negative inputs and outputs in data envelopment analysis. Eur. J. Oper. Res. 225 (1), 100–105.
Cherny, A.S., Madan, D.B., 2006. Coherent measurement of factor risks. arXiv preprint math/0605062.
Cooper, W.W., Seiford, L., Thanassoulis, E., Zanakis, S.H., 2004. Dea and its uses in different countries. Eur. J. Oper. Res. 154 (2), 337–344.
DeMiguel, V., Garlappi, L., Nogales, F.J., Uppal, R., 2009. A generalized approach to portfolio optimization: Improving performance by constraining portfolio norms. Manage. Sci. 55 (5), 798–812.
Dolatnezhadsomarin, A., Khorram, E., Pourkarimi, L., 2019. Efficient algorithms for solving nonlinear fractional programming problems. Filomat 33 (7), 2149–2179.
Fan, J., Zhang, J., Yu, K., 2012. Vast portfolio selection with gross-exposure constraints. J. Am. Stati. Assoc. 107 (498), 592–606.
Farrell, M.J., 1957. The measurement of productive efficiency. J. R. Stat. Society: Series A (General) 120 (3), 253–281.
Filomena, T.P., Lejeune, M.A., 2012. Stochastic portfolio optimization with proportional transaction costs: Convex reformulations and computational experiments. Oper. Res. Letters 40 (3), 212–217.
Golany, B., Roll, Y., 1989. An application procedure for dea. Omega 17 (3), 237–250.
Grant, M., Boyd, S., Ye, Y., 2006. Disciplined convex programming. In: Global Optimization. Springer, pp. 155–210.
How many stocks should you own? why there's no single 'right' answer. URL: https://www.businessinsider.com/how-many-stocks-should-i-own. (Accessed on 04/20/2021).
Huang, X., 2008. Mean-semivariance models for fuzzy portfolio selection. J. Comput. Appl. Math. 217 (1), 1–8.
Jobson, J.D., Korkie, B., 1980. Estimation for markowitz efficient portfolios. J. Am. Stati. Assoc. 75 (371), 544–554.
Kao, C., Steuer, R.E., 2016. Value of information in portfolio selection, with a taiwan stock market application illustration. Eur. J. Oper. Res. 253 (2), 418–427.
Kellerer, H., Mansini, R., Speranza, M.G., 2000. Selecting portfolios with fixed costs and minimum transaction lots. Ann. Oper. Res. 99 (1), 287–304.
Khanjani Shiraz, R., Tavana, M., Fukuyama, H., 2020. A random-fuzzy portfolio selection dea model using value-at-risk and conditional value-at-risk. Soft. Comput. 24, 17167–17186.
Kirby, C., Ostdiek, B., 2012. It's all in the timing: simple active portfolio strategies that outperform naive diversification. J. Financial Quantitative Anal. 47 (2), 437–467.
Klein, R.W., Bawa, V.S., 1976. The effect of estimation risk on optimal portfolio choice. J. Financial Econ. 3 (3), 215–231.
Konno, H., Shirakawa, H., Yamazaki, H., 1993. A mean-absolute deviation-skewness portfolio optimization model. Ann. Oper. Res. 45 (1), 205–220.
Ledoit, O., Wolf, M., 2008. Robust performance hypothesis testing with the sharpe ratio. J. Empirical Finance 15 (5), 850–859.
Lobo, M.S., Fazel, M., Boyd, S., 2007. Portfolio optimization with linear and fixed transaction costs. Ann. Oper. Res. 152 (1), 341–365.
Markowitz, H., 1952. Portfolio selection. The. J. Finance 7 (1), 77–91.
Mercurio, P.J., Wu, Y., Xie, H., 2020. An entropy-based approach to portfolio optimization. Entropy 22 (3), 332.
Mitchell, J.E., Braun, S., 2013. Rebalancing an investment portfolio in the presence of convex transaction costs, including market impact costs. Optim. Methods Software 28 (3), 523–542.
Mohammadi, S., Nazemi, A., 2020. On portfolio management with value at risk and uncertain returns via an artificial neural network scheme. Cognitive Syst. Res. 59, 247–263.
Muthuraman, K., Kumar, S., 2006. Multidimensional portfolio optimization with proportional transaction costs. Math. Finance: Int. J. Math., Stat. Financial Econ. 16 (2), 301–335.
Nesaz, H.H., Jasemi, M., Monplaisir, L., 2020. A new methodology for multi-period portfolio selection based on the risk measure of lower partial moments. Expert Syst. Appl. 144, 113,032.
Oksendal, B., Sulem, A., 2002. Optimal consumption and portfolio with both fixed and proportional transaction costs. SIAM J. Control Optim. 40 (6), 1765–1790.
Peykani, P., Mohammadi, E., Rostamy-Malkhalifeh, M., Hosseinzadeh Lotfi, F., 2019. Fuzzy data envelopment analysis approach for ranking of stocks with an application to tehran stock exchange. Adv. Math. Finance Appl. 4 (1), 31–43.
Peykani, P., Mohammadi, E., Jabbarzadeh, A., Rostamy-Malkhalifeh, M., Pishvaee, M. S., 2020. A novel two-phase robust portfolio selection and optimization approach under uncertainty: A case study of tehran stock exchange. Plos one 15 (10), e0239,810.
Pogue, G.A., 1970. An extension of the markowitz portfolio selection model to include variable transactions' costs, short sales, leverage policies and taxes. J. Finance 25 (5), 1005–1027.
Rockafellar, R.T., Uryasev, S., et al., 2000. Optimization of conditional value-at-risk. J. Risk 2, 21–42.
Roman, D., Darby-Dowman, K., Mitra, G., 2007. Mean-risk models using two risk measures: a multi-objective approach. Quantitative Finance 7 (4), 443–458.
Saltelli, A., Aleksankina, K., Becker, W., Fennell, P., Ferretti, F., Holst, N., Li, S., Wu, Q., 2019. Why so many published sensitivity analyses are false: A systematic review of sensitivity analysis practices. Environ. Modelling Softw. 114, 29–39.
Sehgal, R., Mehra, A., 2021. Worst-case analysis of gini mean difference safety measure. J. Ind. Manage. Optim. 17 (4), 1613.
Statman, M., 1987. How many stocks make a diversified portfolio? J. Financial Quantitative Anal. 22 (3), 353–363.
Thakur, G.S.M., Bhattacharyya, R., Sarkar, S., 2018. Stock portfolio selection using dempster–shafer evidence theory. J. King Saud University-Computer Inform. Sci. 30 (2), 223–235.
Tone, K., 2001. A slacks-based measure of efficiency in data envelopment analysis. Eur. J. Oper. Res. 130 (3), 498–509.
Tone, K., Tsutsui, M., 2010. Dynamic dea: A slacks-based measure approach. Omega 38 (3–4), 145–156.
Vercher, E., Bermudez, J.D., Segura, J.V., 2007. Fuzzy portfolio optimization under downside risk measures. Fuzzy Sets Syst. 158 (7), 769–782.
Wang, X.T., Li, Z., Zhuang, L., 2017. Risk preference, option pricing and portfolio hedging with proportional transaction costs. Chaos, Solitons & Fractals 95, 111–130.
Zhang, Q. et al., 2018. The performance of china stock market, based on the analysis of implicit transaction cost. Am. J. Ind. Business Manage. 8 (04), 1050.
Zhang, P., Zeng, Y., Chi, G., 2021. Time-consistent multiperiod mean semivariance portfolio selection with the real constraints. J. Ind. Manage. Optim. 17 (4), 1663.
Zheng, C., Chen, Y., 2014. Portfolio selection based on relative entropy coherent risk measure. Systems Eng. -Theory Practice 34 (3), 648–655.
Zhu, J., 2017. Optimal financing and dividend distribution with transaction costs in the case of restricted dividend rates. ASTIN Bull.: J. IAA 47 (1), 239–268.