

공학기초수학 6주차 온라인 과제

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7.1절

$\int \cos^{-1} x \, dx$ 을 구하여라.

$$\cos^{-1} x = t$$

$$\cos t = x$$

$$-\sin t = dx$$

$$\begin{aligned}\int \cos^{-1} x \, dx &= \int -t \cdot \sin t \, dt \\ &= -t \cdot (-\cos t) - \int (-\cos t)(-1) \, dt \\ &= t \cdot \cos t - \sin t + C\end{aligned}$$

$$t = \cos^{-1} x, \cos t = x, \sin t = \sqrt{1-x^2} \text{ 이므로,}$$

$$\begin{aligned}\int \cos^{-1} x \, dx &= t \cdot \cos t - \sin t + C \\ &= \cos^{-1} x \cdot x - \sqrt{1-x^2} + C\end{aligned}$$

7.2절

#20. $\int_{\pi/4}^{\pi/2} \cot^5 \varphi \csc^3 \varphi \, d\varphi$ 을 구하여라.

$$\begin{aligned}\int_{\pi/4}^{\pi/2} \cot^5 \varphi \csc^3 \varphi \, d\varphi \\ = \int_{\pi/4}^{\pi/2} \cot^4 \varphi \cdot \csc \varphi \cdot (\csc^2 \varphi - 1)^2 \cdot \csc^2 \varphi \, d\varphi\end{aligned}$$

$$\csc \varphi = t$$

$$-\cot \varphi \cdot \csc \varphi = dt$$

$$\varphi \rightarrow \frac{\pi}{4} \text{ 일 때, } t \rightarrow \sqrt{2}$$

$$\varphi \rightarrow \frac{\pi}{2} \text{ 일 때, } t \rightarrow 1 \text{ 이므로}$$

$$\begin{aligned}\int_{\pi/4}^{\pi/2} \cot^4 \varphi \cdot \csc \varphi \cdot (\csc^2 \varphi - 1)^2 \cdot \csc^2 \varphi \, d\varphi \\ = - \int_{\sqrt{2}}^1 (t^2 - 1)^2 \cdot t^2 \, dt \\ = \int_1^{\sqrt{2}} (t^6 - 2t^4 + t^2) \, dt \\ = \left[\frac{t^7}{7} - \frac{2}{5} t^5 + \frac{t^3}{3} \right]_1^{\sqrt{2}} \\ = \frac{22\sqrt{2}}{105} - \frac{8}{105}\end{aligned}$$