선형대수 [02] - 과제 #02 소프트웨어학부 20213015 송규원

Exercise Set 1.3

In Exercises 1–2, suppose that A, B, C, D, and E are matrices with the following sizes:

A	B	C	D	E
(4×5)	(4×5)	(5×2)	(4×2)	(5×4)

In each part, determine whether the given matrix expression is defined. For those that are defined, give the size of the resulting matrix.

1. **a.** BA **b.** AB^T **c.** $AC + D$
d. $E(AC)$ **e.** $A - 3E^T$ **f.** $E(5B + A)$

$\underset{m\times r}{A}\ \underset{r\times n}{B} = \underset{m\times n}{AB}$ 를 이용

a. $(4\times5)(4\times5)$ 이므로 undefined

b. 4×4 matrix

c. 4×2 matrix

d. 5×2 matrix

e. 4×5 matrix

f. 5×5 matrix

In Exercises 7–8, use the following matrices and either the row method or the column method, as appropriate, to find the indicated row or column.

$$A = \begin{bmatrix} 3 & -2 & 7 \\ 6 & 5 & 4 \\ 0 & 4 & 9 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 6 & -2 & 4 \\ 0 & 1 & 3 \\ 7 & 7 & 5 \end{bmatrix}$$

$$AB = \begin{bmatrix} 3 & -2 & 7 \\ 6 & 5 & 4 \\ 0 & 4 & 9 \end{bmatrix}\begin{bmatrix} 6 & -2 & 4 \\ 0 & 1 & 3 \\ 7 & 7 & 5 \end{bmatrix} = \begin{bmatrix} 67 & 41 & 41 \\ 64 & 21 & 59 \\ 63 & 67 & 57 \end{bmatrix}$$

$$BA = \begin{bmatrix} 6 & -2 & 4 \\ 0 & 1 & 3 \\ 7 & 7 & 5 \end{bmatrix}\begin{bmatrix} 3 & -2 & 7 \\ 6 & 5 & 4 \\ 0 & 4 & 9 \end{bmatrix} = \begin{bmatrix} 6 & -6 & 70 \\ 6 & 17 & 31 \\ 63 & 41 & 122 \end{bmatrix}$$

$$AA = \begin{bmatrix} 3 & -2 & 7 \\ 6 & 5 & 4 \\ 0 & 4 & 9 \end{bmatrix}\begin{bmatrix} 3 & -2 & 7 \\ 6 & 5 & 4 \\ 0 & 4 & 9 \end{bmatrix} = \begin{bmatrix} -3 & 12 & 76 \\ 48 & 29 & 98 \\ 24 & 56 & 97 \end{bmatrix}$$

7. **a.** the first row of AB **b.** the third row of AB
c. the second column of AB **d.** the first column of BA
e. the third row of AA **f.** the third column of AA

a. $[67\ 41\ 41]$ b. $[63\ 67\ 57]$ c. $\begin{bmatrix} 41 \\ 21 \\ 67 \end{bmatrix}$

d. $\begin{bmatrix} 6 \\ 6 \\ 63 \end{bmatrix}$ e. $[24\ 56\ 97]$ f. $\begin{bmatrix} 76 \\ 98 \\ 97 \end{bmatrix}$

11. **a.**
$$\begin{aligned} 2x_1 - 3x_2 + 5x_3 &= 7 \\ 9x_1 - x_2 + x_3 &= -1 \\ x_1 + 5x_2 + 4x_3 &= 0 \end{aligned}$$

b.
$$\begin{aligned} 4x_1 \qquad - 3x_3 + x_4 &= 1 \\ 5x_1 + x_2 \qquad - 8x_4 &= 3 \\ 2x_1 - 5x_2 + 9x_3 - x_4 &= 0 \\ 3x_2 - x_3 + 7x_4 &= 2 \end{aligned}$$

a. $A = \begin{bmatrix} 2 & -3 & 5 \\ 9 & -1 & 1 \\ 1 & 5 & 4 \end{bmatrix}$, $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $b = \begin{bmatrix} 7 \\ -1 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} 2 & -3 & 5 \\ 9 & -1 & 1 \\ 1 & 5 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ -1 \\ 0 \end{bmatrix} = \left[\begin{array}{ccc|c} 2 & -3 & 5 & 7 \\ 9 & -1 & 1 & -1 \\ 1 & 5 & 4 & 0 \end{array}\right]$$

b. $A = \begin{bmatrix} 4 & 0 & -3 & 1 \\ 5 & 1 & 0 & -8 \\ 2 & -5 & 9 & -1 \\ 0 & 3 & -1 & 7 \end{bmatrix}$, $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ 3 \\ 0 \\ 2 \end{bmatrix}$

$$\begin{bmatrix} 4 & 0 & -3 & 1 \\ 5 & 1 & 0 & -8 \\ 2 & -5 & 9 & -1 \\ 0 & 3 & -1 & 7 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 0 \\ 2 \end{bmatrix} = \left[\begin{array}{cccc|c} 4 & 0 & -3 & 1 & 1 \\ 5 & 1 & 0 & -8 & 3 \\ 2 & -5 & 9 & -1 & 0 \\ 0 & 3 & -1 & 7 & 2 \end{array}\right]$$

33. Suppose that type I items cost \$1 each, type II items cost \$2 each, and type III items cost \$3 each. Also, suppose that the accompanying table describes the number of items of each type purchased during the first four months of the year.

TABLE Ex-33

	Type I	Type II	Type III
Jan.	3	4	3
Feb.	5	6	0
Mar.	2	9	4
Apr.	1	1	7

What information is represented by the following product?

$$\begin{bmatrix} 3 & 4 & 3 \\ 5 & 6 & 0 \\ 2 & 9 & 4 \\ 1 & 1 & 7 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} \text{1월에 구입한 총 금액} \\ \text{2월에 구입한 총 금액} \\ \text{3월에 구입한 총 금액} \\ \text{4월에 구입한 총 금액} \end{bmatrix}$$

Exercise Set 1.4

25. $3x_1 - 2x_2 = -1$

$4x_1 + 5x_2 = 3$

$$\begin{bmatrix} 3 & -2 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -2 \\ 4 & 5 \end{bmatrix}^{-1}\begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\frac{1}{15+8}\begin{bmatrix} 5 & 2 \\ -4 & 3 \end{bmatrix}\begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\therefore\ x_1 = \frac{-5+6}{23} = \frac{1}{23},\quad x_2 = \frac{9+4}{23} = \frac{13}{23}$$

27. $6x_1 + x_2 = 0$

$4x_1 - 3x_2 = -2$

$$\begin{bmatrix} 6 & 1 \\ 4 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 1 \\ 4 & -3 \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ -2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\frac{1}{-18-4}\begin{bmatrix} -3 & -1 \\ -4 & 6 \end{bmatrix}\begin{bmatrix} 0 \\ -2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\therefore\ x_1 = -\frac{2}{22} = -\frac{1}{11},\quad x_2 = -\frac{-12}{22} = \frac{6}{11}$$

37. $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ $\quad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$Ax = I$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$r_2 \leftarrow r_2 - r_1$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$r_3 \leftarrow r_3 - r_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & \cancel{2}\,1 & 1/2 & -1/2 & 1/2 \end{array}\right] \Rightarrow r_3 \leftarrow \tfrac{1}{2} r_3$$

$r_2 \leftarrow r_2 + r_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1/2 & 1/2 & 1/2 \\ 0 & 0 & 1 & 1/2 & -1/2 & 1/2 \end{array}\right]$$

$r_1 \leftarrow r_1 - r_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & 1/2 & -1/2 \\ 0 & 1 & 0 & -1/2 & 1/2 & 1/2 \\ 0 & 0 & 1 & 1/2 & -1/2 & 1/2 \end{array}\right]$$

$$\therefore\ x = \begin{bmatrix} 1/2 & 1/2 & -1/2 \\ -1/2 & 1/2 & 1/2 \\ 1/2 & -1/2 & 1/2 \end{bmatrix} = A^{-1}$$

39. $(AB)^{-1}(AC^{-1})(D^{-1}C^{-1})^{-1}D^{-1}$

$$(AB)^{-1} = A^{-1}B^{-1}$$

$$(D^{-1}C^{-1})^{-1} = DC$$

x=1

$$A^{-1}B^{-1} \cdot AC^{-1} \cdot DC \cdot D^{-1} \qquad \therefore\ B^{-1}$$

x=1 x=1

Exercise Set 1.5

In Exercises 7–8, use the following matrices and find an elementary matrix E that satisfies the stated equation.

$$A = \begin{bmatrix} 3 & 4 & 1 \\ 2 & -7 & -1 \\ 8 & 1 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 8 & 1 & 5 \\ 2 & -7 & -1 \\ 3 & 4 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 3 & 4 & 1 \\ 2 & -7 & -1 \\ 2 & -7 & 3 \end{bmatrix}, \quad D = \begin{bmatrix} 8 & 1 & 5 \\ -6 & 21 & 3 \\ 3 & 4 & 1 \end{bmatrix}$$

$$F = \begin{bmatrix} 8 & 1 & 5 \\ 8 & 1 & 1 \\ 3 & 4 & 1 \end{bmatrix}$$

7. **a.** $EA = B$ **b.** $EB = A$

c. $EA = C$ **d.** $EC = A$

a.
$$E\begin{bmatrix} 3 & 4 & 1 \\ 2 & -7 & -1 \\ 8 & 1 & 5 \end{bmatrix} = \begin{bmatrix} 8 & 1 & 5 \\ 2 & -7 & -1 \\ 3 & 4 & 1 \end{bmatrix}$$

$r_1 \leftarrow r_3$, $r_2 \leftarrow r_2$, $r_3 \leftarrow r_1$

$$\therefore \quad E = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

b.
$$E\begin{bmatrix} 8 & 1 & 5 \\ 2 & -7 & -1 \\ 3 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 4 & 1 \\ 2 & -7 & -1 \\ 8 & 1 & 5 \end{bmatrix}$$

$r_1 \leftarrow r_3$, $r_2 \leftarrow r_2$, $r_3 \leftarrow r_1$

$$E = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

c.
$$E\begin{bmatrix} 3 & 4 & 1 \\ 2 & -7 & -1 \\ 8 & 1 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 4 & 1 \\ 2 & -7 & -1 \\ 2 & -7 & 3 \end{bmatrix}$$

$r_1 \leftarrow r_1$, $r_2 \leftarrow r_2$, $r_3 \leftarrow r_3 - 2r_1$

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

d.
$$E\begin{bmatrix} 3 & 4 & 1 \\ 2 & -7 & -1 \\ 2 & -7 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 4 & 1 \\ 2 & -7 & -1 \\ 8 & 1 & 5 \end{bmatrix}$$

$r_1 \leftarrow r_1$, $r_2 \leftarrow r_2$, $r_3 \leftarrow r_3 + 2r_1$

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

25.
$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 4 & 3 \\ 0 & 0 & 1 \end{bmatrix} = A$$

$r_1 \leftarrow r_1 + 2r_3$

$$E_1 = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$r_2 \leftarrow \frac{1}{4} r_2$

$$E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$r_2 \leftarrow r_2 - \frac{3}{4} r_3$

$$E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -3/4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/4 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -3/4 \\ 0 & 0 & 1 \end{bmatrix} = E_1E_2E_3$$

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3/4 \\ 0 & 0 & 1 \end{bmatrix} = E_1^{-1}E_2^{-1}E_3^{-1}$$