수치해석 기말고사 2차 - 소프트웨어학부 2021741016 송규원

1. 라그랑주 보간법을 사용하여 다음의 노드를 갖는 최소 차수의 다항식을 구하시오.

x	0	2	3	4
y	7	11	28	63

$$l_0(x) = \frac{(x-2)(x-3)(x-4)}{(0-2)(0-3)(0-4)} = -\frac{1}{24}(x-2)(x-3)(x-4)$$

$$l_1(x) = \frac{(x-0)(x-3)(x-4)}{(2-0)(2-3)(2-4)} = \frac{1}{4}x(x-3)(x-4)$$

$$l_2(x) = \frac{(x-0)(x-2)(x-4)}{(3-0)(3-2)(3-4)} = -\frac{1}{3}x(x-2)(x-4)$$

$$l_3(x) = \frac{(x-0)(x-2)(x-3)}{(4-0)(4-2)(4-3)} = \frac{1}{8}x(x-2)(x-3)$$

$$\therefore P_3(x) = 7 - 2x + x^3$$

2. 1번 문제를 뉴턴 보간법을 사용하여 보간 다항식을 구하시오.

$P_0(x) = 1$

$P_1(x) = P_0(x) + c(x - x_0) = 1 + cx$

$P_1(2) = 1 + 2c = 11 \quad \therefore c = 5$

$P_2(x) = P_1(x) + c(x - x_0)(x - x_1)$

$= 1 + 5x + c(x - 0)(x - 2)$

$P_2(3) = 1 + 15 + 3c = 28 \quad \therefore c = 4$

$P_3(x) = P_2(x) + c(x - x_0)(x - x_1)(x - x_2)$

$= 1 + 5x + 4x(x - 2) + c(x - 0)(x - 2)(x - 3)$

$P_3(4) = 1 + 20 + 40 + 8c = 61 \quad \therefore c = 1$

$\therefore P_3(x) = 4x^2 - 8x + 1 + x^3 - 4x^2 + 5x$

$= x^3 - 2x + 1$

3. 주어진 데이터의 뉴턴 보간 다항식을 구하고, 두 가지 축소형태로 변환하여 f(4.2)의 근사값을 계산하시오.(소수점 셋째 자리까지 계산)

x	0	1	2	4	6
y	1	9	23	93	259

$P_0(x_1) = 1$

$P_1(x_1) = P_0(x_1) + C_1(x - x_{0_1}) = 1 + C_1(x - 0_1) = 1 + C_1 x$

$P_1(1_1) = 1 + C_1 = 9 \quad \therefore C_1 = 8$

$P_2(x_1) = P_1(x_1) + C_2(x - x_{0_1})(x - x_{1_1})$

$= 1 + 8x + C_2 x(x - 1_1)$

$P_2(2_1) = 1_1 + 2C_2 = 23 \quad \therefore C_2 = 11$

$P_2(x_1) = 1 + 8x + 11x(x - 1_1)$

$P_3(x_1) = P_2(x_1) + C_3(x - x_{0_1})(x - x_{1_1})(x - x_{2_1})$

$= 1 + 8x + 11x^2 - 11x + C_3 x(x - 1_1)(x - 2_1)$

$P_3(4_1) = 1 + 11_2 + 48 - 12 + 24C_3 = 93 \quad \therefore C_3 = 1$

$P_3(x_1) = x^3 + 0x + 1$

마찬가지로 하면 $P_4(x_1)$ 의 C_4 값은 0 이므로 써줄 필요가 없다.

$P(4.2_1) = (4.2_1)^3 + 0(4.2_1) + 1 = 104.488$

다항 풀이 축소 ①	축소 ②
$P_4(x_1) = P_3(x_1) + 0 \cdot (x - x_{0_1})(x - x_{1_1})(x - x_{2_1})(x - x_{3_1})$	$P_4(x_1) = 1 + 0x + x^3$
$P_3(x_1) = P_2(x_1) + (x - x_{0_1})(x - x_{1_1})(x - x_{2_1})$	$= 1 + x(0 + x^2)$
$\therefore 1 + 8x + 11x(x - 1_1) + x(x - 1_1)(x - 2_1)$	$P_4(4.2_1) = 1 + 103.488 = 104.488$
$= 1 + x(8 + 11(x - 1_1) + (x - 1_1)(x - 2_1))$	
$= 1 + x(8 + (x - 1_1)(11 + (x - 2_1)))$	
$P_3(4.2_1) = 1 + 4.2(8 + 3.2(11 + 2.2_1)) = 104.488$	