

실습문제 7-10

집합  $A=\{1,2,3,4,5\}$  에 대한 다음 관계들이 어떤 성질을 갖고 있는 관계인지 판별하라.

- (1)  $R_1=\{(1,2),(1,3),(1,4),(1,5),(2,3),(2,4),(2,5),(3,4),(3,5),(4,5)\}$   
(2)  $R_2=\{(1,1),(1,2),(2,1),(2,2),(3,1),(3,3),(3,5),(4,4),(4,5),(5,4),(5,5)\}$

(1)

$$\begin{matrix} & 1 & 2 & 3 & 4 & 5 \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

(1, 2)  
(2, 3)  $\rightarrow$  (1, 3)  $\in R_1$   
(2, 4)  $\rightarrow$  (1, 4)  $\in R_1$   
(2, 5)  $\rightarrow$  (1, 5)  $\in R_1$

(2, 3)  
(3, 4)  $\rightarrow$  (2, 4)  $\in R_1$   
(3, 5)  $\rightarrow$  (2, 5)  $\in R_1$

(3, 4)  
(4, 5)  $\rightarrow$  (3, 5)  $\in R_1$   
(4, 5)  $\rightarrow$  (4, 5)  $\in R_1$

(1, 3)  
(3, 4)  $\rightarrow$  (1, 4)  $\in R_1$   
(3, 5)  $\rightarrow$  (1, 5)  $\in R_1$

(1, 3)  
(2, 4)  $\rightarrow$  (1, 5)  $\in R_1$   
(3, 5)  $\rightarrow$  (1, 5)  $\in R_1$

(1, 4)  
(4, 5)  $\rightarrow$  (1, 5)  $\in R_1$   
(4, 5)  $\rightarrow$  (4, 5)  $\in R_1$

(1, 5)  
(5, 4)  $\rightarrow$  (1, 4)  $\in R_1$   
(5, 5)  $\rightarrow$  (1, 5)  $\in R_1$

(1, 5)  
(2, 5)  $\rightarrow$  (1, 5)  $\in R_1$   
(3, 5)  $\rightarrow$  (1, 5)  $\in R_1$

(1, 5)  
(4, 5)  $\rightarrow$  (1, 5)  $\in R_1$   
(5, 5)  $\rightarrow$  (1, 5)  $\in R_1$

$\therefore$  비반사 관계, 반대칭 관계, 추이관계

(2)

$$\begin{matrix} & 1 & 2 & 3 & 4 & 5 \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{matrix}$$

(1, 1)  
(1, 1)  $\rightarrow$  (1, 1)  $\in R_2$

(2, 2)  
(2, 2)  $\rightarrow$  (2, 2)  $\in R_2$

(3, 3)  
(3, 3)  $\rightarrow$  (3, 3)  $\in R_2$

(1, 2)  
(2, 1)  $\rightarrow$  (1, 1)  $\in R_2$   
(2, 2)  $\rightarrow$  (1, 2)  $\in R_2$

(1, 2)  
(2, 1)  $\rightarrow$  (3, 1)  $\in R_2$   
(1, 2)  $\rightarrow$  (3, 2)  $\notin R_2$

(1, 2)  
(2, 1)  $\rightarrow$  (4, 1)  $\in R_2$   
(1, 2)  $\rightarrow$  (4, 2)  $\notin R_2$

(1, 2)  
(2, 1)  $\rightarrow$  (5, 1)  $\in R_2$   
(1, 2)  $\rightarrow$  (5, 2)  $\in R_2$

(1, 2)  
(2, 1)  $\rightarrow$  (5, 1)  $\in R_2$   
(1, 2)  $\rightarrow$  (5, 2)  $\in R_2$

(1, 2)  
(2, 1)  $\rightarrow$  (5, 1)  $\in R_2$   
(1, 2)  $\rightarrow$  (5, 2)  $\in R_2$

$\therefore$  추이관계 x, 반사관계 o

실습문제 7-11

집합  $A = \{1, 2, 3\}$ 에 대한 관계  $R$ 이 다음과 같을 때,  $R^2, R^3, R^4$ 를 구하라.

$$M_R = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R^2 = M_{R^2} = M_R \odot M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R^3 = M_{R^3} = M_{R^2} \odot M_R = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R^4 = M_{R^4} = M_{R^3} \odot M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

## 실습문제 7-12

❖ 집합  $A=\{w,x,y,z\}$ 에 대한 관계가 다음과 같을 때 관계행렬의 거듭제곱을 이용해 추이관계인지 판별하라.

$$R = \{(w,z), (x,y), (z,w), (z,x)\}$$

$$n=4, \quad R^2, R^3, R^4 \leq R \quad \text{증명}$$

$$M_R = \begin{matrix} & \begin{matrix} w & x & y & z \end{matrix} \\ \begin{matrix} w \\ x \\ y \\ z \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \end{matrix} \quad R^2 = M_R \odot M_R = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$R_2 = \{(w,w), (w,x), (z,y), (z,z)\}$$

$\therefore R_2 \neq R, \quad R$ 은  $2$ 이관계가 아니다.

## 실습문제 7-13

❖ 집합  $A=\{w,x,y,z\}$ 에 대한 관계가 다음과 같을 때 관계행렬의 거듭제곱을 이용해 추이관계인지 판별하라.

$$R = \{(w,w), (y,y), (x,z)\}$$

$$n=4, \quad R^2, R^3, R^4 \leq R \quad \text{증명}$$

$$M_R = \begin{matrix} & \begin{matrix} w & x & y & z \end{matrix} \\ \begin{matrix} w \\ x \\ y \\ z \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix} \quad R^2 = M_R \odot M_R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\rightarrow R^3 = \{(w,w), (y,y)\}, \quad R^3 \leq R$$

$$R^2 = M_R \odot M_R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R^4 = M_{R^2} \odot M_R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\rightarrow R^2 = \{(w,w), (y,y)\}, \quad R^2 \leq R \quad \rightarrow R^4 = \{(w,w), (y,y)\}, \quad R^4 \leq R$$

$\therefore$  관계  $R$ 은  $2$ 이관계

## 실습문제 7-14

집합  $A = \{a, b, c, d\}$ 에 대한 관계  $R = \{(a,b), (b,b), (b,c), (d,a), (d,c)\}$ 의 반사폐포  $S$ 를 구하라.

$$(a, a), \quad \cancel{(b, b)}, \quad (c, c), \quad (d, d)$$

$$S = R \cup \{(a, a), (c, c), (d, d)\}$$

$$S = \{(a, a), (a, b), (b, b), (b, c), (c, c), (d, a), (d, c), (d, d)\}$$

$$M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{matrix}$$

## 실습문제 7-15

집합  $A = \{a, b, c, d\}$ 에 대한 관계  $R = \{(a, b), (b, b), (b, c), (c, a)\}$ 에 대한 추이폐포인 관계  $S$ 를 구하라.

$$S = R \cup \{(a, a), (a, c), (c, b), (c, c), (c, a)\}$$

$$S = \{(a, a), (a, b), (a, c), (b, a), (b, b), (b, c), (c, a), (c, b), (c, c)\}$$

$$M_R = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad M_{R^2} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} = M_R \quad \therefore S^2 \subseteq S$$

$$M_{R^3} = M_R \circ M_{R^2} = M_R \quad \therefore R^3 \subseteq R$$

$\therefore S$ 는 축이관계

$$M_{R^4} = M_R \circ M_{R^3} = M_R \quad \therefore R^4 \subseteq R$$

## 실습문제 7-16

❖ 집합  $A = \{a, b, c, d\}$ 에 대한 관계가 다음과 같을 때 관계  $R$ 에 대해 추이폐포인 관계  $S$ 를 연결관계를 이용해 구하라.

$$R = \{(a, b), (b, c), (c, d)\}$$

$$n = 4 \quad M_R = \begin{matrix} a & b & c & d \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

$$M_{R^2} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad M_{R^3} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$M_{R^4} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R^* = \begin{matrix} a & b & c & d \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

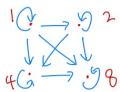
$$\therefore S = \{(a, b), (a, c), (a, d), (b, c), (b, d), (c, d)\}$$

## 실습문제 7-17

다음의 집합  $A$ 에 대한 관계  $R$ 이 부분순서관계이다 하세도표를 그리시오

$$A = \{1, 2, 4, 8\}$$

$$R = \{(1, 1), (1, 2), (1, 4), (1, 8), (2, 2), (2, 4), (2, 8), (4, 4), (4, 8), (8, 8)\}$$



해설 ①



해설 ②

$$\begin{aligned} 1 &\leq 2, 1 \leq 4, 1 \leq 8 \\ 2 &\leq 4, 2 \leq 8 \\ 4 &\leq 8 \end{aligned}$$

해설 ③

