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실습과제 6-1

행렬들이 다음과 같을 때, 식을 연산하라. 연산할 수 없는 식은 연산할 수 없는 이유를 써라

$$A = \underset{3\times2}{\begin{bmatrix} 1 & 3 \\ 5 & 1 \\ 8 & 0 \end{bmatrix}} \quad B = \underset{2\times2}{\begin{bmatrix} 7 & 1 \\ 4 & 5 \end{bmatrix}} \quad C = \underset{3\times3}{\begin{bmatrix} 4 & 5 & 3 \\ 8 & 0 & 1 \\ 2 & 7 & 2 \end{bmatrix}} \quad D = \underset{2\times3}{\begin{bmatrix} 0 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}}$$

(1) A X B (2) (D x A) + B (3) C X B (4) C – (A X D) (5) A + D

(1)

$$= \begin{bmatrix} 7+12 & 1+15 \\ 35+4 & 5+5 \\ 56+0 & 8+0 \end{bmatrix}$$

$$= \begin{bmatrix} 19 & 16 \\ 39 & 10 \\ 56 & 8 \end{bmatrix}$$

(2)

D x A

$$= \begin{bmatrix} 0+5+16 & 0+1+0 \\ 0+15+32 & 0+3+0 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 1 \\ 47 & 3 \end{bmatrix}$$

(D x A) + B

$$= \begin{bmatrix} 28 & 2 \\ 51 & 8 \end{bmatrix}$$

(3)

C의 열 크기와 B의 행 크기가 달라서 연산 불가능함

(4)

A x D

$$= \begin{bmatrix} 0+0 & 1+9 & 2+12 \\ 0+0 & 5+3 & 10+4 \\ 0+0 & 8+0 & 16+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 10 & 14 \\ 0 & 8 & 14 \\ 0 & 8 & 16 \end{bmatrix}$$

C - (A x D)

$$= \begin{bmatrix} 4 & -5 & -11 \\ 8 & -8 & -13 \\ 2 & -1 & -14 \end{bmatrix}$$

(5)

두 행렬의 크기가 달라 연산 불가능함

실습과제 6-2

행렬들이 다음과 같을 때, 식을 연산하라. 연산할 수 없는 식은 연산할 수 없는 이유를 써라
(0 : 영행렬, I : 단위행렬)

$$A = \begin{bmatrix} 1 & 3 \\ 5 & 1 \\ 8 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 7 & 1 \\ 4 & 5 \end{bmatrix} \quad C = \begin{bmatrix} 4 & 5 & 3 \\ 8 & 0 & 1 \\ 2 & 7 & 2 \end{bmatrix} \quad D = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$

(1) B + 0 (2) C X I (3) 3A

(1) = B

(2) = C

(3)

$$= \begin{bmatrix} 3 & 9 \\ 15 & 3 \\ 24 & 0 \end{bmatrix}$$

실습과제 6-3

다음 행렬의 전치행렬을 구하고, 원래 대칭행렬인지 구별하라.

$$A = \begin{bmatrix} 1 & 5 & 4 & 8 \\ 5 & 3 & 2 & 0 \\ 4 & 2 & 5 & 3 \\ 8 & 0 & 3 & 7 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 3 & 1 & 8 \\ 8 & 2 & 3 & 7 \\ 5 & 1 & 1 & 5 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 5 & 4 & 8 \\ 5 & 3 & 2 & 0 \\ 4 & 2 & 5 & 3 \\ 8 & 0 & 3 & 7 \end{bmatrix} \qquad B^T = \begin{bmatrix} 0 & 0 & 8 & 5 \\ 1 & 3 & 2 & 1 \\ 1 & 1 & 3 & 1 \\ 0 & 8 & 7 & 5 \end{bmatrix}$$

∴ 대칭행렬임 ∴ 대칭행렬이 아님

실습과제 6-4

다음 부울행렬을 이용해 식을 연산하라.

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$(C \odot B) \vee A$$

$(C \odot B)$

$$= \begin{bmatrix} \underset{0}{(1\wedge 0)} \vee \underset{1}{(1\wedge 1)} \vee \underset{0}{(0\wedge 1)} & \underset{0}{(1\wedge 0)} \vee \underset{1}{(1\wedge 1)} \vee \underset{0}{(0\wedge 1)} \\ \underset{0}{(0\wedge 0)} \vee \underset{0}{(0\wedge 1)} \vee \underset{1}{(1\wedge 1)} & \underset{0}{(0\wedge 0)} \vee \underset{0}{(0\wedge 1)} \vee \underset{1}{(1\wedge 1)} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$(C \odot B) \vee A$

$$= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

6-5

$$(C \odot B) \wedge A$$

$$= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \wedge \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

6-6

$$(B \odot C) \wedge D$$

$(B \odot C)$

$$= \begin{bmatrix} \underset{0}{(0\wedge 1)} \vee \underset{0}{(0\wedge 0)} & \underset{0}{(0\wedge 1)} \vee \underset{0}{(0\wedge 0)} & \underset{0}{(0\wedge 0)} \vee \underset{0}{(0\wedge 1)} \\ \underset{1}{(1\wedge 1)} \vee \underset{0}{(1\wedge 0)} & \underset{1}{(1\wedge 1)} \vee \underset{0}{(1\wedge 0)} & \underset{0}{(1\wedge 0)} \vee \underset{1}{(1\wedge 1)} \\ \underset{1}{(1\wedge 1)} \vee \underset{0}{(1\wedge 0)} & \underset{1}{(1\wedge 1)} \vee \underset{0}{(1\wedge 0)} & \underset{0}{(1\wedge 0)} \vee \underset{1}{(1\wedge 1)} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$(B \odot C) \wedge D$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \wedge \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

실습과제 6-7

다음 정사각행렬의 행렬식을 구하라.

$$A = \begin{bmatrix} 3 & -1 & -2 \\ -4 & 2 & 1 \\ 1 & 4 & -3 \end{bmatrix} \begin{matrix} 3 & -1 \\ -4 & 2 \\ 1 & 4 \end{matrix}$$

$$\det(A) = (-18 - 1 + 32) - (-12 + 12 - 4)$$
$$= 13 + 4 = 17$$

실습과제 6-8

다음 정사각행렬의 가능한 소행렬을 모두 구하고 각각의 행렬식을 구하라.

$$A = \begin{bmatrix} 3 & -1 & -2 \\ -4 & 2 & 1 \\ 1 & 4 & -3 \end{bmatrix}$$

$M_{11} = \begin{bmatrix} 2 & 1 \\ 4 & -3 \end{bmatrix}$ $\det(M_{11}) = -6 - 4 = -10$

$M_{12} = \begin{bmatrix} -4 & 1 \\ 1 & -3 \end{bmatrix}$ $\det(M_{12}) = 12 - 1 = 11$

$M_{13} = \begin{bmatrix} -4 & 2 \\ 1 & 4 \end{bmatrix}$ $\det(M_{13}) = -16 - 2 = -18$

$M_{21} = \begin{bmatrix} -1 & -2 \\ 4 & -3 \end{bmatrix}$ $\det(M_{21}) = 3 + 8 = 11$

$M_{22} = \begin{bmatrix} 3 & -2 \\ 1 & -3 \end{bmatrix}$ $\det(M_{22}) = -9 + 2 = -7$

$M_{23} = \begin{bmatrix} 3 & -1 \\ 1 & 4 \end{bmatrix}$ $\det(M_{23}) = 12 + 1 = 13$

$M_{31} = \begin{bmatrix} -1 & -2 \\ 2 & 1 \end{bmatrix}$ $\det(M_{31}) = -1 + 4 = 3$

$M_{32} = \begin{bmatrix} 3 & -2 \\ -4 & 1 \end{bmatrix}$ $\det(M_{32}) = 3 - 8 = -5$

$M_{33} = \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$ $\det(M_{33}) = 6 - 4 = 2$

실습과제 6-9

4차 정사각행렬 $C=\begin{bmatrix} 1 & -5 & 2 & -3 \\ 4 & 2 & -1 & 1 \\ -2 & -4 & 5 & 3 \\ 1 & 0 & -2 & -1 \end{bmatrix}$의 행렬식을 구하라.

4행 선택 : $\det(C) = c_{41}C_{41} + c_{42}C_{42} + c_{43}C_{43} + c_{44}C_{44}$

$= 1\times(-1)\times\begin{vmatrix} -5 & 2 & -3 \\ 2 & -1 & 1 \\ -4 & 5 & 3 \end{vmatrix} + (-2)\times(-1)\times\begin{vmatrix} 1 & -5 & -3 \\ 4 & 2 & 1 \\ -2 & -4 & 3 \end{vmatrix} + (-1)\times1\times\begin{vmatrix} 1 & -5 & 2 \\ 4 & 2 & -1 \\ -2 & -4 & 5 \end{vmatrix}$

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$= -1\times2 + 2\times116 + (-1)\times72 = 158$

① 의 행렬식 - 2행 이용

$\det(A) = a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23}$

$= 2\times(-1)\times\begin{vmatrix} 2 & -3 \\ 5 & 3 \end{vmatrix} + (-1)\times1\times\begin{vmatrix} -5 & -3 \\ -4 & 3 \end{vmatrix} + 1\times(-1)\times\begin{vmatrix} -5 & 2 \\ -4 & 5 \end{vmatrix}$

$= -2(6+15) + (-1)\times(-15-12) + (-1)\times(-25+8)$

$= -42 + 27 + 17 = 2$

② 의 행렬식 - 2행 이용

$\det(A) = a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23}$

$= 4\times(-1)\times\begin{vmatrix} -5 & -3 \\ -4 & 3 \end{vmatrix} + 2\times1\times\begin{vmatrix} 1 & -3 \\ -2 & 3 \end{vmatrix} + 1\times(-1)\times\begin{vmatrix} 1 & -5 \\ -2 & -4 \end{vmatrix}$

$= -4\times(-15-12) + 2(3-6) + (-1)\times(-4-10)$

$= 108 - 6 + 14 = 116$

③ 의 행렬식 - 1행 이용

$\det(A) = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$

$= 1\times1\times\begin{vmatrix} 2 & -1 \\ -4 & 5 \end{vmatrix} + (-5)\times(-1)\times\begin{vmatrix} 4 & -1 \\ -2 & 5 \end{vmatrix} + 2\times1\times\begin{vmatrix} 4 & 2 \\ -2 & -4 \end{vmatrix}$

$= (10-4) + 5(20-2) + 2(-16+4)$

$= 6 + 90 - 24 = 72$

실습과제 6-10

다음 행렬의 행렬식을 구하시오.

$\begin{bmatrix} -1 & 3 & 0 \\ -2 & 1 & 1 \\ 5 & 4 & -3 \end{bmatrix}$

1행 선택 : $\det(A) = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$

$= -1\times1\times\begin{vmatrix} 1 & 1 \\ 4 & -3 \end{vmatrix} + 3\times(-1)\times\begin{vmatrix} -2 & 1 \\ 5 & -3 \end{vmatrix}$

$= -(-3-4) + (-3)\times(6-5)$

$= 7 + (-3) = 4$

실습과제 6-11

det(A)가 ϕ이면 특이, ϕ이 아니면 가역

다음 행렬이 가역행렬인지 특이행렬인지 구분하고, 가역행렬이라면 역행렬을 구하라.

(1) $A=\begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & -3 \\ 4 & 1 & 1 \end{bmatrix}$ (2) $B=\begin{bmatrix} 1 & 2 & 0 \\ 3 & 4 & 0 \\ 5 & 6 & 0 \end{bmatrix}$

$\det(A) = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$

$= 1\times1\times\begin{vmatrix} 1 & -3 \\ 1 & 1 \end{vmatrix} + (-1)\times(-1)\times\begin{vmatrix} 2 & -3 \\ 4 & 1 \end{vmatrix} + 1\times2\times\begin{vmatrix} 2 & 1 \\ 4 & 1 \end{vmatrix}$

$= 1\times(1+3) + 1\times(2+12) + 2\times(2-4) = 4+14-4 = 14 \rightarrow$ 가역행렬

$A_{11} = 1\times\begin{vmatrix} 1 & -3 \\ 1 & 1 \end{vmatrix} = 1+3 = 4$

$A_{12} = -1\times\begin{vmatrix} 2 & -3 \\ 4 & 1 \end{vmatrix} = -(2+12) = -14$

$A_{13} = 1\times\begin{vmatrix} 2 & 1 \\ 4 & 1 \end{vmatrix} = 2-4 = -2$

$A_{21} = -1\times\begin{vmatrix} -1 & 2 \\ 1 & 1 \end{vmatrix} = -(-1-2) = 3$

$A_{22} = 1\times\begin{vmatrix} 1 & 2 \\ 4 & 1 \end{vmatrix} = 1-8 = -7$

$A_{23} = -1\times\begin{vmatrix} 1 & -1 \\ 4 & 1 \end{vmatrix} = -(1+4) = -5$

$A_{31} = 1\times\begin{vmatrix} -1 & 2 \\ 1 & -3 \end{vmatrix} = 3-2 = 1$

$A_{32} = -1\times\begin{vmatrix} 1 & 2 \\ 2 & -3 \end{vmatrix} = -(-3-4) = 7$

$A_{33} = 1\times\begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = 1+2 = 3$

$[A_{ij}] = \begin{bmatrix} 4 & -14 & -2 \\ 3 & -7 & -5 \\ 1 & 7 & 3 \end{bmatrix}$

$\therefore [A_{ij}]^T = \begin{bmatrix} 4 & 3 & 1 \\ -14 & -7 & 7 \\ -2 & -5 & 3 \end{bmatrix}$

$\det(B) = b_{11}B_{11} + b_{12}B_{12}$

$= 1\times1\times\begin{vmatrix} 4 & 0 \\ 6 & 0 \end{vmatrix} + (-1)\times2\times\begin{vmatrix} 3 & 0 \\ 5 & 0 \end{vmatrix}$

$= 1\times(0-0) + (-1)\times2\times(0-0) = 0$ ∴ 특이행렬

실습과제 6-12

다음 연립1차방정식을 가우스 소거법으로 해를 구하라

$$\left[\begin{array}{cccc|c} 4 & 4 & 5 & 3 & 7 \\ 3 & 3 & 3 & 3 & 9 \\ 2 & 1 & 6 & 3 & 10 \\ 2 & 0 & 4 & 2 & 4 \end{array}\right] \qquad \begin{cases} 4w+4x+5y+3z = 7 \\ 3w+3x+3y+3z = 9 \\ 2w+x+6y+3z = 10 \\ 2w+4y+2z = 4 \end{cases}$$

① 2행 $\times \frac{1}{3}$

$$\left[\begin{array}{cccc|c} 4 & 4 & 5 & 3 & 7 \\ 1 & 1 & 1 & 1 & 3 \\ 2 & 1 & 6 & 3 & 10 \\ 2 & 0 & 4 & 2 & 4 \end{array}\right]$$

② 1행 2행 교환

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 3 \\ 4 & 4 & 5 & 3 & 7 \\ 2 & 1 & 6 & 3 & 10 \\ 2 & 0 & 4 & 2 & 4 \end{array}\right]$$

③ $-4\times$1행 + 2행

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 3 \\ 0 & 0 & 1 & -1 & -5 \\ 2 & 1 & 6 & 3 & 10 \\ 2 & 0 & 4 & 2 & 4 \end{array}\right]$$

④ $-2\times$1행 + 3행

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 3 \\ 0 & 0 & 1 & -1 & -5 \\ 0 & -1 & 4 & 1 & 4 \\ 2 & 0 & 4 & 2 & 4 \end{array}\right]$$

⑤ $-2\times$1행 + 4행

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 3 \\ 0 & 0 & 1 & -1 & -5 \\ 0 & -1 & 4 & 1 & 4 \\ 0 & -2 & 2 & 0 & -2 \end{array}\right]$$

⑥ 2행 3행 교환

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 3 \\ 0 & -1 & 4 & 1 & 4 \\ 0 & 0 & 1 & -1 & -5 \end{array}\right]$$

⑦ -2×2행 + 4행

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 3 \\ 0 & -1 & 4 & 1 & 4 \\ 0 & 0 & 1 & -1 & -5 \\ 0 & 0 & -6 & -2 & -10 \end{array}\right]$$

⑧ 6×3행 + 4행

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 3 \\ 0 & -1 & 4 & 1 & 4 \\ 0 & 0 & 1 & -1 & -5 \\ 0 & 0 & 0 & -8 & -40 \end{array}\right]$$

$\therefore\ -8z = -40,\ z = 5$

$y - 5 = -5,\ y = 0$

$-x + 5 = 4,\ x = 1$

$w + 1 + 5 = 3,\ w = -3$

실습문제 6-12-1

가우스 소거법을 이용해 다음 연립 1차 방정식의 해를 구하라.

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & -2 \\ -2 & -1 & -2 & 3 & 29 \\ 1 & 3 & -1 & -1 & -16 \\ 1 & -2 & 3 & 2 & 11 \end{array}\right] \qquad \begin{cases} w + x + y + z = -2 \\ -2w - x - 2y + 3z = 29 \\ w + 3x - y - z = -16 \\ w - 2x + 3y + 2z = 11 \end{cases}$$

① $2\times$1행 + 2행

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & -2 \\ 0 & 1 & 0 & 5 & 25 \\ 1 & 3 & -1 & -1 & -16 \\ 1 & -2 & 3 & 2 & 11 \end{array}\right]$$

② $(-1)\times$1행 + 3행

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & -2 \\ 0 & 1 & 0 & 5 & 25 \\ 0 & 2 & -2 & -2 & -14 \\ 1 & -2 & 3 & 2 & 11 \end{array}\right]$$

③ $(-1)\times$1행 + 4행

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & -2 \\ 0 & 1 & 0 & 5 & 25 \\ 0 & 2 & -2 & -2 & -14 \\ 0 & -3 & 2 & 1 & 13 \end{array}\right]$$

④ -2×2행 + 3행

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & -2 \\ 0 & 1 & 0 & 5 & 25 \\ 0 & 0 & -2 & -12 & -64 \\ 0 & -3 & 2 & 1 & 13 \end{array}\right]$$

⑤ 3×2행 + 4행

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & -2 \\ 0 & 1 & 0 & 5 & 25 \\ 0 & 0 & -2 & -12 & -64 \\ 0 & 0 & 2 & 16 & 88 \end{array}\right]$$

⑥ 3행 + 4행, $-\frac{1}{2}\times3$행

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & -2 \\ 0 & 1 & 0 & 5 & 25 \\ 0 & 0 & 1 & 6 & 32 \\ 0 & 0 & 0 & 4 & 24 \end{array}\right]$$

⑦ $\frac{1}{4}\times4$행

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & -2 \\ 0 & 1 & 0 & 5 & 25 \\ 0 & 0 & 1 & 6 & 32 \\ 0 & 0 & 0 & 1 & 6 \end{array}\right]$$

$\therefore\ z = 6$

$y + 36 = 32,\ y = -4$

$x + 30 = 25,\ x = -5$

$w - 5 - 4 + 6 = -2,\ w = 1$

실습과제 6-13

가우스 조르단 소거법을 이용해 다음 연립1차 방정식의 해를 구하시오

$$\begin{cases} w + x + y + z = -2 \\ -2w - x - 2y + 3z = 29 \\ w + 3x - y - z = -16 \\ w - 2x + 3y + 2z = 11 \end{cases}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & -2 \\ 0 & 1 & 0 & 5 & 25 \\ 0 & 0 & 1 & 6 & 32 \\ 0 & 0 & 0 & 1 & 6 \end{array}\right]$$

① 3행 = 3행 − 4행 , 2행 = 2행 − 5×4행 , 1행 = 1행 − 4행

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & -8 \\ 0 & 1 & 0 & 0 & -5 \\ 0 & 0 & 1 & 0 & -4 \\ 0 & 0 & 0 & 1 & 6 \end{array}\right]$$

② 1행 = 1행 − 3행

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 0 & -5 \\ 0 & 0 & 1 & 0 & -4 \\ 0 & 0 & 0 & 1 & 6 \end{array}\right]$$

③ 1행 = 1행 − 2행

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -5 \\ 0 & 0 & 1 & 0 & -4 \\ 0 & 0 & 0 & 1 & 6 \end{array}\right]$$

∴ $w = 1$, $x = -5$, $y = -4$, $z = 6$

실습과제 6-14

다음 정사각행렬에 대해 가우스 조르단 소거법을 이용해 역행렬을 구하라.

$$B = \begin{bmatrix} 2 & 1 & 1 \\ 4 & 3 & 5 \\ 2 & 1 & 2 \end{bmatrix} \qquad C = \begin{bmatrix} 3 & 3 & 0 & 5 \\ 4 & 1 & -3 & 0 \\ 2 & 10 & 3 & 0 \\ 2 & 2 & 0 & -2 \end{bmatrix} \qquad D = \begin{bmatrix} 3 & -8 & 3 \\ -1 & 7 & -3 \\ 0 & -2 & 1 \end{bmatrix}$$

1행 = 1행 × 1/2

$$\left[\begin{array}{ccc|ccc} 1 & 1/2 & 1/2 & 1/2 & 0 & 0 \\ 4 & 3 & 5 & 0 & 1 & 0 \\ 2 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$$

2행 = 2행 − 4×1행 , 3행 = 3행 − 2×1행

$$\left[\begin{array}{ccc|ccc} 1 & 1/2 & 1/2 & 1/2 & 0 & 0 \\ 0 & 1 & 3 & -2 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

2행 = 2행 − 3×3행 , 1행 = 1행 − 1/2×3행

$$\left[\begin{array}{ccc|ccc} 1 & 1/2 & 0 & -1/2 & 0 & -1/2 \\ 0 & 1 & 0 & 1 & 1 & -3 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

1행 = 1행 − 1/2×2행

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & -1/2 & 1 \\ 0 & 1 & 0 & 1 & 1 & -3 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

∴ $B^{-1} = \begin{bmatrix} 1/2 & -1/2 & 1 \\ 1 & 1 & -3 \\ -1 & 0 & 1 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} 3 & -8 & 3 & 1 & 0 & 0 \\ -1 & 7 & -3 & 0 & 1 & 0 \\ 0 & -2 & 1 & 0 & 0 & 1 \end{array}\right]$$

−1×2행 후 , 1행 ↔ 2행 , 2행 ↔ 3행

$$\left[\begin{array}{ccc|ccc} 1 & -7 & 3 & 0 & -1 & 0 \\ 0 & -2 & 1 & 0 & 0 & 1 \\ 3 & -8 & 3 & 1 & 0 & 0 \end{array}\right]$$

3행 = 3행 − 3×1행 , 2행 = $-\frac{1}{2}$×2행

$$\left[\begin{array}{ccc|ccc} 1 & -7 & 3 & 0 & -1 & 0 \\ 0 & 1 & -1/2 & 0 & 0 & -1/2 \\ 0 & 13 & -6 & 1 & 3 & 0 \end{array}\right]$$

3행 = 3행 − 13×2행

$$\left[\begin{array}{ccc|ccc} 1 & -7 & 3 & 0 & -1 & 0 \\ 0 & 1 & -1/2 & 0 & 0 & -1/2 \\ 0 & 0 & 1/2 & 1 & 3 & 13/2 \end{array}\right]$$

3행 = 2×3행

$$\left[\begin{array}{ccc|ccc} 1 & -7 & 3 & 0 & -1 & 0 \\ 0 & 1 & -1/2 & 0 & 0 & -1/2 \\ 0 & 0 & 1 & 2 & 6 & 13 \end{array}\right]$$

2행 = 2행 + 1/2×3행 , 1행 = 1행 − 3×3행

$$\left[\begin{array}{ccc|ccc} 1 & -7 & 0 & -6 & -19 & -39 \\ 0 & 1 & 0 & 1 & 3 & 6 \\ 0 & 0 & 1 & 2 & 6 & 13 \end{array}\right]$$

1행 = 1행 + 7×2행

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 2 & 3 \\ 0 & 1 & 0 & 1 & 3 & 6 \\ 0 & 0 & 1 & 2 & 6 & 13 \end{array}\right]$$

∴ $D^{-1} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 6 \\ 2 & 6 & 13 \end{bmatrix}$